Politically Robust Financial Regulation: Online Appendix*

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Abstract

This is an online appendix to the paper "Politically Robust Financial Regulation". This appendix considers an alternative functional form, and re-derives the paper's main results using this alternative functional form. An additional exercise on bank value transfers is also included.

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1 Introduction

This is an online appendix to the paper "Politically Robust Financial Regulation". This appendix considers an alternative functional form, which is fully described below. That is, this appendix covers all the same steps as the original paper. It then derives the main paper's Propositions, including those for the extensions conducted in the main paper, using this alternative functional form. All proofs are contained at the end of this appendix.[1] At the end, after the proofs, an additional exercise is included on bank value transfers to the public, which could represent, for instance, an interest payment or an investment in the provision of banking services.

2 Model setup

There are three agents in the model: the bank, the government and the public, all of whom are risk neutral. We first describe their respective objectives and constraints in Sections 2.1-2.3, after which Section 2.4 lays out the timing of the game, which formalizes the interaction among the agents.

2.1 The bank

The financial sector is represented by a single bank that is managed by its owners. The bank's decisions center on its asset side, while its funding side is fixed and consists entirely of costless government-insured deposits. Bank owners are thus residual claimants that obtain the full upside of positive bank returns, and none of the downside in the event of bank insolvency. Furthermore, bank owners are considered a separate agent class that is not part of the public.[2]

[1] The algebraic derivations in the proofs are conducted in a *Mathematica* file that is available on request.

[2] Bank profits do not form a part of public welfare (Section 2.2). Implicitly, this can be seen as a Rawlsian representation of social welfare, centering on the well-being of the agents that are least well-off, which here are the members of the public that are not bank owners.

On its asset side, the bank takes two decisions. First, the bank chooses its initial asset risk profile. Second, in the event that the bank becomes insolvent, it can choose to engage in a gamble for resurrection. We describe these choices, in turn.

2.1.1 The bank's risk profile

A standard modeling approach is to let a bank choose between a safe and a risky project. However, for this paper's purposes we require a richer setting, in which a single parameter represents how much riskier the risky project is than the safe project, which this section describes in several steps.

The bank starts off with a retail deposit base of size 1. These deposits pay no interest and are never called (i.e., there are no bank runs). The bank chooses how to invest them. The net return on bank assets, r, is drawn from a uniform distribution with mean μ and width w, where w is defined such that the distribution ranges from $\mu - w$ to $\mu + w$.[3] Here, both μ and w depend on the risk profile, ρ, that the bank chooses. That is, the bank chooses from a continuum of possible distributions when determining ρ. In particular,

$$\mu(\rho) = 1 - \rho \tag{1}$$

$$w(\mu(\rho)) = \frac{1}{\mu(\rho)} = \frac{1}{1-\rho} \tag{2}$$

$$\rho \in [0, b] \text{ with } 0 < b < 1 \tag{3}$$

What we are modeling here is essentially the tradeoff faced by a call option investor, who likes a higher variance of returns. This is so, because the investor accrues the upside when the option is in-the-money, but has a bounded downside. A larger variance of returns is of most value to a call option owner when the underlying asset is near or below the call price: if the option is deep in-the-money then owning it becomes almost equivalent to owning the underlying asset, and the investor cares about the downside too.

[3] Thus, the gross return on bank assets is $(1 + r)$, and the expected gross return on the bank's investment of size 1 is $(1 + \mu)$.

Due to its limited liability, the return structure of a risk-neutral bank bears similarity to that of a call option owner. Instead of the usual mean-variance tradeoff associated with risk aversion, the risk-neutral bank trades off a higher variance against higher expected returns. This squarely centers the model on a choice between socially beneficial safer profiles and socially detrimental riskier profiles that destroy true project value but gain option value for the bank.

The crucial parameter in (1)-(3) is b, the upper bound of the possible values that ρ can take. This parameter indicates how socially adverse the tradeoff is that banks face, with a higher b implying a more perverse choice set that includes projects whose expected values to the bank are more reliant on the bank's limited liability protection.

The modeling in (1)-(3) provides us with a foundation for a binary choice, which parameterizes how much riskier the risky project is than the safe project. The binary nature of this choice stems from the bank's optimization, which can only have two possible optima: $\rho = 0$ and $\rho = b$. Either the bank chooses the safest project, $\rho = 0$, or it opts for the riskiest project, $\rho = b$. This is shown in the proof of Lemma 1, and derives from the fact that the bank's expected profit in (4) is convex in ρ. Intuitively, when ρ is close to zero, the bank gains little from a higher variance of project returns, because its limited-liability call option is deep in-the-money. Therefore, at $\rho = 0$ a marginal increase in ρ lowers the bank's expected profit, by reducing μ. But as ρ rises, the option value starts to play a greater role, and the marginal value of a higher variance of returns increases, until $E[\Pi]$ begins to rise in ρ.[4]

$$E[\Pi] = E[\max\{r, 0\}] \tag{4}$$

Lemma 1 *When considering (4) as the objective function of a one-shot optimization prob-*

[4]In (4) the bank's profit on its investment of size 1 is $(1 + r)1 - 1 = r$, conditional on the lower bound on returns that is implied by the bank's limited liability, namely 0.

lem, the bank's optimal risk choice is given by

$$\rho = \begin{cases} b \textit{ if } b > \bar{b} \\ 0 \textit{ otherwise} \end{cases} \tag{5}$$

where $\bar{b}$ is a threshold value, which satisfies the $b \in (0,1)$ constraint from (3). When b is larger than $\bar{b}$ the bank chooses $\rho = b$, and when b is smaller than $\bar{b}$, the bank chooses $\rho = 0$.[5]

Proof. On page 22. ■

Given the binary choice implied by Lemma 1 as well as the fact that $\rho = b$ is detrimental from a social perspective, we now refer to the bank's optimization as a choice between the good project, $\rho = 0$, and the bad project, $\rho = b$. The bank chooses the bad project when it is bad enough, namely when $b > \bar{b}$. When this is the case, the bad project offers the bank enough compensation in the form of higher option value to overcome the decline of μ.

Note that $\bar{b}$ represents the bank's decision threshold when its only objective is to maximize (4): other considerations, such as the possibility of gambling for resurrection and the constraints imposed by financial regulation, will shift this threshold.

2.1.2 Gambling for resurrection

Once the bank has chosen its project, and the return on that project has materialized, the bank may face an additional choice. In particular, if the bank has become insolvent, it could choose to engage in a gamble for resurrection. We model the bank's gamble for resurrection as a negative expected return activity.[6] Gambling for resurrection is therefore socially harmful, and would always be prevented by a government with the policy tools to do so, provided that government is solely concerned with social welfare. This provides a simple,

[5] In this Lemma, and consistently throughout the paper, we ignore knife-edge cases where an agent is indifferent between outcomes, because these make no qualitative difference to our results. That is, (5) could equivalently be written with if $\rho = b$ if $b \geq \bar{b}$ without affecting the rest of the paper's analysis.

[6] Note that the bank's initial risk choice in Section 2.1.1 centers on projects that have positive expected value. Given $b < 1$, even the bad project is certain to have a positive expected return.

stark setting in which to investigate the impact of political incentives.

We envisage the bank's gamble as zombie lending: a reinvestment in its failed project. We model this as a simple coin toss. With a probability of 50% the gamble succeeds and boosts the net return on the bank's project by $\frac{1}{1-b}$. Per (1)-(3), this always suffices to make the bank solvent again.[7] But with 50% probability the gamble fails, and the net return on the bank's project declines by $-\frac{\gamma}{1-b}$. Here, $\gamma > 1$, which means that the expected return on the gamble is negative. Taking account of the possibility to gamble when insolvent (but not yet of possible government regulation) the bank's optimization problem becomes to maximize to ρ the expected profit given by

$$E[\Pi] = E[r|\, r > 0] + E[\min\{r + g, 0\}|\, r < 0] \tag{6}$$

where

$$g = \begin{cases} \frac{1}{1-b} \text{ with probability } \frac{1}{2} \\ -\frac{\gamma}{1-b} \text{ with probability } \frac{1}{2} \end{cases} \tag{7}$$

Since an insolvent bank only sees an upside to the gamble for resurrection, it will always choose to engage in the gamble. Moreover, the option to gamble when the bad project fails, makes the bank more inclined to choose the bad project in the first place. The threshold value of b above which the bank chooses the bad project becomes lower due to the option to gamble, as recorded in Lemma 2.

Lemma 2 *The bank's solution to (6) in isolation (i.e., absent regulation) is*

$$\rho = \begin{cases} b \text{ if } b > \underline{b} \\ 0 \text{ otherwise} \end{cases} \tag{8}$$

where $\underline{b} \in \left(0, \overline{b}\right)$.

[7] From (1)-(3), the bank's lowest possible initial net return on the bad project is $r = 1 - b - \frac{1}{1-b}$, and therefore a successful gamble turns this net return into $1 - b > 0$ and this implies the gross return is $(1 + (1 - b)) > 1$, which is enough for the bank to repay depositors 1 and retain a profit.

Proof. On page 23. ■

2.2 The public

The public stands at the receiving end of the bank's risk taking. It bears the cost of bank insolvency, because the government's payout of deposit insurance is funded with (lump-sum) taxes levied on the public. From (4) the expected cost to the public from bank insolvency is $E[\min\{r, 0\}]$ when the bank does not have the option to gamble for resurrection. If the bank does have that option, then from (6) the expected loss to the public becomes $E[\min\{r+g, 0\}| \, r < 0]$.[8] We do not model here why the bank should exist, from a social perspective. The implicit assumption is that the bank performs a critical function, which could be in the payment system or credit provision.[9]

Although the public is aware that bank insolvency is costly to it, it has limited insight into the way that the financial sector works. The public faces two types of informational frictions in our model:

1. **Imperfect information**: the public is imperfectly informed about the health of the financial sector. As long as the bank remains active, the public receives an imperfect signal about its health. Instead, if the bank is closed and deposit insurance is paid out, its true solvency position is revealed to the public.

2. **Incomplete information**: the public does not have insight into the process (i.e., bank optimization subject to regulation) that determines bank risk, neither the initial risk profile nor gambles for resurrection. The only bank variable that is visible to the public, but imperfectly so, is realized bank solvency, r.

The public's imperfect information about the bank's solvency is represented by a signal,

[8] This abstracts from negative externalities on the economy associated with bank insolvency.

[9] An extension available on request makes this more explicit, by considering bank value transfers to the public, which could represent interest payments or an investment in the provision of banking services, which is costly to the bank but creates value for the public. The extension investigates how such value tranfsers interact wih the government's incentives to regulate the bank.

$\theta \in (0, 1)$. This signal is a revelation probability: the public has a θ chance of knowing the bank's true solvency, r. This is so, unless the bank is closed, in which case the public always knows the bank's solvency. Therefore, the limit cases of $\theta \to 1$ and $\theta \to 0$, respectively, represent the cases of a perfectly informed public and a public that is completely in the dark as long as the bank is open for business. We assume that the public's prior is that the bank is solvent ($r \geq 0$), unless the signal or bank closure reveal otherwise.

If and when the public learns that the bank is insolvent it imposes a punitive cost of x on the government, which could be an electoral loss or a mass demonstration, for instance. We do not explicitly model why the public imposes this action on the government. One interpretation is that when the public feels the financial pain associated with the bank's insolvency, it obtains an intrinsic value from punishing the government, to which it had delegated the task of financial stability. Overall, the public is a passive player here, with a known reaction function rather than an explicit optimization problem. The bank and the government both solve optimization problems, where they consider the public's reaction.

2.3 The government

We consider a single policy maker, the government, which represents the public's interests. That is, the government does not suffer from an agency bias that could, for instance, make it susceptible to lobbying by the bank. The fact that the government can end up playing along with the bank even though it has no preference to do so, is an important outcome. The public's blindness about the financial sector drives the distortions to the government's decision making, which ultimately harm the public itself.

Cannot the government share its knowledge of the financial sector with the public, and thereby overcome these distorted incentives? The public's incomplete information means that it does not understand the bank's optimization problem and risk profile. Instead, it thinks only in terms of outcomes - how much money the bank has lost - rather than exposures and probability distributions. That is, r has meaning to the public, while ρ

does not. However, by the time that outcomes are realized, the government already has an incentive to prevent their revelation, if negative. The government may wish to join the bank's gamble for resurrection, which, if successful, ensures that the public never learns about the bank's insolvency, and therefore does not take punitive action against the government.

The government's loss function is of the form $E\left[\min\left\{\text{Bank returns}, 0\right\} - x\right]$, where the size of x determines how sensitive the government is to the whims of a poorly informed public. If $x \to 0$, then the very inability of the public to punish the government ensures that the government always maximizes the public's welfare. It is $x > 0$ that brings about the connection between the informational problems of the public and the distorted incentives of the government. Indeed, going forward, it proves useful to define the term $x\left(1-\theta\right)$ as "the extent to which the government is subject to pressure from a poorly informed public".

In the baseline model, the government possesses a single binary tool, l: legislation about gambles for resurrection by the bank. The government can decide to forbid such gambles, a policy that can be interpreted as the implementation of mark-to-market accounting. This accounting practice forces the bank to immediately recognize the losses to its assets, in line with the deterioration of their market value. Instead, historical cost accounting allows the bank to continue to value assets according to their book value, unless they are sold. During a period that the bank assets lose market value, historical cost accounting buys the bank time, which in our setting translates into the possibility to gamble for resurrection.[10]

Although mark-to-market accounting here provides the government with a tool that can effectively forbid gambles for resurrection, the government's desire to veil bank insolvency from the public means that it could decide to forego on such legislation, and allow gambles to proceed. The government solves:

$$\min_{l=\{\text{Allow},\text{Forbid}\}}\left\{E\left[\min\left\{r\left(l\right)+g\left(l\right),0\right\}-x\left(l\right)\right]\right\} \tag{9}$$

[10] Since the global financial crisis, there has been an active debate on the relative merits of mark-to-market and historical cost accounting in the financial sector. This comes down to a tradeoff between, respectively, the risks from exacerbating fire sales and the risks from worsening regulatory forbearance, where our framework focuses on the latter.

where the (l) terms recognize that the bank's initial risk choice, and therefore the realization of r, depends on the government's policy, as does the occurrence of a gamble for resurrection, and the probability that the government is punished by the public.

The government's incentive to allow the bank to gamble for resurrection increases when $r < 0$ is a reality. Before the bank's return is drawn, the government sees bank insolvency, as only a possibility. Once that possibility becomes reality, the only way to avoid x is to allow the bank to gamble. Therefore, the government's legislative incentives suffer from time inconsistency.

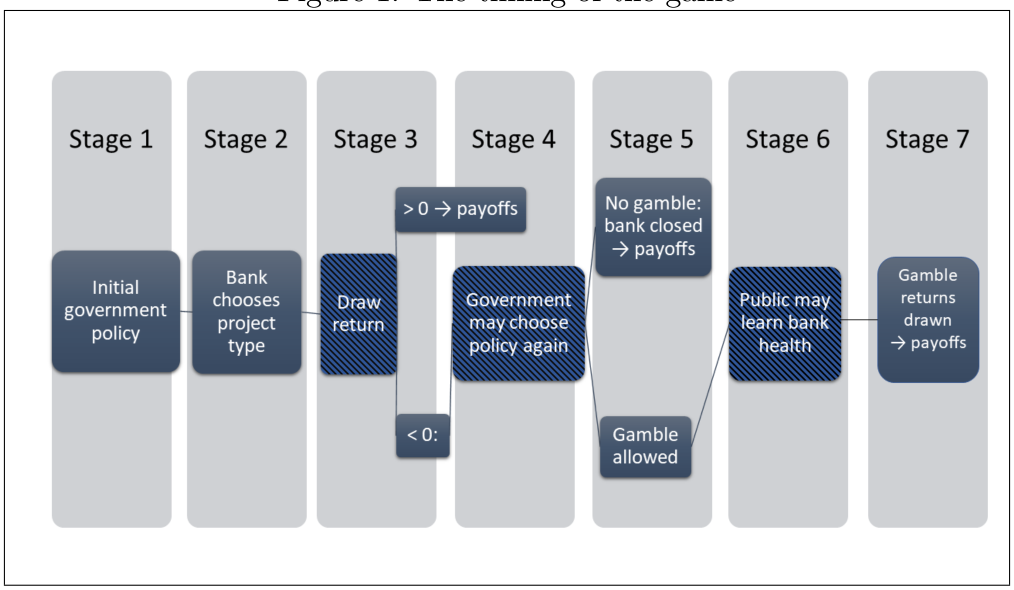
Our modeling approach parameterizes the degree to which the government's policy suffers from this time inconsistency problem. The government can legislate before the bank takes any risk, but has only a probability of being able to change its mind (i.e., rescinding mark-to-market accounting rules in favor of historical cost accounting or vice versa) after this risk has already materialized. With probability $\lambda \in (0, 1)$ the government's second decision stage (i.e., after r is drawn) exists, and with probability $(1 - \lambda)$ this stage does not exist. This can be seen as uncertainty about how quickly financial sector developments unfold. Legislating is a slow process in practice, and the government can sometimes find itself behind the curve.

A high λ thus represents agile legislation, but agility is a mixed blessing when time consistency matters. Agility can help the government at the moment that it wishes to make a change, but as seen from the government's initial decision stage, it means a less time consistent tool. In the limit case of $\lambda \to 1$ the government's initial legislation becomes completely moot, because it can change its mind later in case the bank becomes insolvent. Importantly, the bank is aware of this, and decides on its risk profile knowing that an initial decision to forbid future gambles is just cheap talk by the government. Instead, in the limit case of $\lambda \to 0$, once the government makes its initial decision, its hands are fully tied.

Legislating against future gambles reduces the bank's incentive to choose the bad project, because if that project fails, there is no second chance. Per Lemma 1 and 2, forbidding gambles raises the threshold for the bank to choose the bad project from $\underline{b}$ to $\overline{b}$. However, the government's tool to contain bank risk is not ideal. An ideal regulatory tool would target the bank's initial risk choice, ρ, in such a way that the bank would always choose $\rho = 0$, and this would overcome the tradeoffs in the model. However, a regulatory tool that can contain but not fully negate perverse incentives among banks, arguably corresponds well to the reality of financial stability policy. Alternative, but still imperfect, regulatory tools are considered in Sections 4 and 5.

2.4 Timing of the game

Figure 1: The timing of the game



The actions and the payoffs described in Sections 2.1-2.3 make up a game that consists of seven stages. Figure 1 provides an overview of these, where uncertain outcomes are represented by a striped boxes. The formal definition of the stages is:

Stage 1 The government determines l.

Stage 2 The bank chooses ρ.

Stage 3 r is drawn. If $r > 0$: the game ends with payoffs r, 0, 0 for, respectively, the bank, the public and the government. If $r < 0$: the game continues.

Stage 4 With probability λ this stage occurs, and the government can once more determine l.

Stage 5 If $l = \{\text{Forbid}\}$, then the banks shuts down, the public is repaid its deposits, with the government's deposit insurance covering the shortfall, $r < 0$, while the public imposes x on the government. If $l = \{\text{Allow}\}$, then the game continues.

Stage 6 With probability θ the public learns that $r < 0$, and imposes x on the government.

Stage 7 The bank gambles. If the gamble succeeds, then payoffs are $(r + g) > 0$, 0 for, respectively, the bank and the public, while the government's payoff is either 0 or $-x$, depending on the outcome of Stage 6. If the gamble fails, then payoffs are 0, $(r + g) < 0$, $(r + g - x)$ for, respectively, the bank, the public and the government.

3 Outcomes of the baseline model

The decision stages of the game are Stage 4, Stage 2, and Stage 1. We solve these by backward induction.

Lemma 3 *If the government gains the opportunity to legislate at Stage 4, then its policy is to forbid gambles if and only if*

$$x(1-\theta) < r + \frac{\gamma}{1-b} \tag{10}$$

Proof. On page 23. ■

From (10) we see that at Stage 4 the government will forbid gambles when $x(1-\theta)$, and that threshold is larger than zero.[11] This implies that for $x \to 0$, the government always forbids gambles at this stage. When rid of political pressure, the government purely weighs the public's well-being rather than the public's knowledge, and therefore prevents the bank from destroying expected project value with its gamble. Similarly, for $\theta \to 1$ the

[11] From (1)-(3), the lower bound on r is $\left(1 - b - \frac{1}{1-b}\right)$, and hence $r + \frac{\gamma}{1-b} > 0$ since $b < 1$ and $\gamma > 1$.

government's incentive to forbear disappears, because there is no scope to keep the public in the dark about insolvency. Next, we solve for the decisions of the bank at Stage 2 and the government at Stage 1:

Lemma 4 *The bank's Stage 2 decision depends on the government's Stage 1 decision. If at Stage 1 the government did not pass legislation against future gambles by the bank, i.e., $l = \{Allow\}$, then the bank's optimal policy is*

$$\rho = \begin{cases} b \text{ } \textit{if } b > \mathbf{b}_A \\ 0 \text{ } \textit{otherwise} \end{cases} \tag{11}$$

whereas if the government did pass such legislation and $l = \{Forbid\}$ at Stage 1 then

$$\rho = \begin{cases} b \text{ } \textit{if } b > \mathbf{b}_F \\ 0 \text{ } \textit{otherwise} \end{cases} \tag{12}$$

where $\underline{b} < \mathbf{b}_A < \mathbf{b}_F < \bar{b}$.

Proof. On page 24. ■

Lemma 5 *At Stage 1, the government's optimal policy is $l = \{Allow\}$ if*

$$b > \mathbf{b}_F \text{ } \textit{and } x(1-\theta) > \frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) \tag{13}$$

and $l = \{Forbid\}$ otherwise.

Proof. On page 25. ■

Lemmas 4 and 5 give us the elements needed to derive two key results in Propositions 1 and 2. These results highlight a complex relation between the risk taking incentives of the bank, the depth of the time consistency problem faced by the government, and the government's incentives to constrain the bank through the imposition of financial regulation.

Proposition 1 *The relationship between the bank's incentives to play on its public safety net and the government's optimal policy is nonlinear. The government always sets $l = \{Forbid\}$ at Stage 1 when either b small enough (including the limit case of $b \to 0$) or b is large enough (including the limit case of $b \to 1$).*

Proof. On page 26. ■

Ideally, a policy maker becomes more inclined to regulate when a bank's incentives to play on its limited liability rise. But the interaction between the policy maker and a poorly informed public can twist the relation between bank incentives and policy choices, as Proposition 1 shows. When b is low and the bank has little incentive to play on the option value created by public deposit insurance, the government also sees little upside to allowing the bank to gamble for resurrection in the future. The probability that the bank becomes insolvent is small and at Stage 1 the government does not place much value on the possibility that at Stage 4 it will want to shield itself from public knowledge of bank insolvency.

However, as b rises and the bank's incentives worsen, so do the government's incentives. The government becomes more amenable to retaining the future option to allow the bank to gamble, because of the increased likelihood that it will want to make use of that option and veil bank insolvency from the public.

Nevertheless, there is a limit to the government's willingness to forgo on its objective of maximizing public welfare for the sake of shielding itself from punitive action by the public. In particular, as b continues to increase, the option value that the bank derives from its limited liability rises exponentially.[12] This means that, in expected terms, the bank appropriates more and more value from the public safety net. When b is large enough, the government switches back to tough regulation, forbidding future gambles for resurrection with its Stage 1 legislation.

Proposition 2 *The time inconsistency of the government's tool, λ, interacts nonlinearly with the government's optimal policy. In (13), both $\frac{\partial \mathbf{b}_F}{\partial \lambda} < 0$ and $\frac{\partial \mathbf{b}_F}{\partial \lambda} > 0$ are possible, which*

[12] See equation (24) in the proof of Lemma 1.

means that the range of values of b for which the government allows gambles to occur, can either expand or contract when λ increases.

Proof. On page 26. ■

Proposition 2 brings in the government's time consistency problem. When $\lambda \to 0$ and the government's initial legislation can never be changed, the government's ability to influence the bank's behavior is maximized. The government can now decide whether to make the bank face the optimization problem in Lemma 1, where gambles never exist, or Lemma 2, where the option to gamble always exists. Possession of a more effective tool (lower λ) can entice the government to use it.

However, the government is not only playing against the bank; it is also playing against its future self. This game between the Stage 1 government and the Stage 4 government emanates from the evolution of the game tree. If there is a Stage 4 at all, this means that the bank has become insolvent, something that was only a possibility at Stage 1. Bank insolvency makes the government more lenient, as it hopes to hide insolvency from the public's eye. The government at Stage 1 dislikes the prospect of its more lenient future self. Therefore, a less time consistent tool can make the Stage 1 government more disposed to forbid future gambles, in an attempt to compensate in the present for the probability that it will be allowed to change its mind in the future.[13]

4 Conditioning policy on bank solvency

In the baseline model the government possesses a binary tool. How are the model's outcomes affected when the policy menu becomes more sophisticated, and the government can legislate a mapping between r and whether or not the bank is allowed to gamble for resurrection?

[13] An alternative way to look at the result of Proposition 2, is as an extension of Proposition 1. All else given, a higher λ increases the risk taking incentives of the bank. But those incentives have a nonlinear relation to the government's optimal policy, per Proposition 1. Hence, an increase in λ can lead the government's regulatory policy either way.

For instance, the government could mandate mark-to-market accounting under normal circumstances, but with a clause for times of exceptional distress, defined by a threshold of r, during which they can forgo on marking down their assets and recognizing insolvency.

In this extension, instead of a two-stage government decision process, we now model a single decision stage at which the government determines a policy mapping, $l(r)$, with

$$l(r) = \begin{cases} \{\text{Allow}\} & \text{if } r \in S \\ \{\text{Forbid}\} & \text{if } r \notin S \end{cases} \tag{14}$$

The government's task is now to maximize the objective function in (9) to its choice of S, which is the set of values of r for which the bank will be allowed to gamble. In the timing of the game shown in Section 2.4, Stage 1 now becomes: the government determines $l(r)$. Stages 2 and 3 are unchanged, Stage 4 is cut, and Stages 5, 6, and 7 remain identical but become, respectively, Stages 4, 5, and 6.

The announcement of a policy mapping can affect the incomplete information environment assumed for the public. In the baseline model, the public is unaware of the process determining bank risk, including the constraints that government regulation can impose on it. Its prior is that the bank is solvent ($r \geq 0$), unless the signal or bank closure reveal otherwise, which we believe finds a counterpart in reality, where usually the public would be oblivious to a change in accounting methods and, more generally, forms of regulatory forbearance have frequently been implemented without causing public concern or bank runs. But if the policy maker announces that a certain type of policy will only apply if the bank is insolvent, then the public could make inferences on bank solvency from observed policy actions, and forbearance ceases to work as the government intended. To address this, we now give the government an additional policy option: over desired ranges of r, it can play a mixed strategy. For instance, for values of r for which the government is indifferent about the chosen policy, a coin toss determines whether mark-to-market or historical cost accounting applies. As seen in Proposition 3, this suffices to prevent the public revelation of bank solvency based on government actions. Including elements of a mixed strategy is optimal for the government in this case, because without it, the government could not try and hide bank insolvency from the public when it wants to.

Proposition 3 *When the government is able to condition its legislation on future bank returns, its optimal policy profile becomes to set, when $r < 0$,*

$$l(r) = \begin{cases} \{Allow\} & \text{if } r < x(1-\theta) - \frac{\gamma}{1-b} \\ \{Forbid\} & \text{otherwise} \end{cases} \tag{15}$$

while the government plays a mixed strategy when $r \geq 0$.

Proof. On page 27. ■

In terms of pure strategies, (15) shows that the government only allows gambles when the bank is *sufficiently insolvent.*[14] This result brings together several features of the model. First, when the political pressure from a poorly informed public becomes small enough, the government always forbids the bank's gambles. That is, when either the public is perfectly informed and $\theta \to 1$ or when the government is indifferent to public pressure and $x \to 0$, we have that $x(1-\theta) \to 0$, in which case the condition in (15) never holds.[15]

However, when the government feels enough pressure from a poorly informed public, then the extent of insolvency affects the government's decision. If instead the bank is only barely insolvent ($r \to 0^-$) then the full upside of the gamble accrues to the bank only. The worse is the bank's position to begin with (the more negative is r), the more upside there is between r and 0. Of course, the public always bears the full burden of the downside of the gamble, which is why a government unconcerned with political pressure would never allow such a gamble. But once political pressure matters, the greater participation in the upside of the gamble is an additional sweetener, which can tip the balance.

[14] This discussion is for the $r < 0$ case. The mixed strategy for $r \geq 0$ has no effect on gambles, because the bank does not gamble when it is solvent.

[15] This is so, because the condition in (15) then becomes $r < -\frac{\gamma}{1-b}$, while the lower bound on r is $-\frac{1}{1-b}$, per (1)-(3), and this is above $-\frac{\gamma}{1-b}$.

We can also investigate (15) from a different angle. In particular, it may seem as though the variable b enters the government's optimal policy profile "appropriately". A worsening of the bad project (higher b) reduces the term $x(1-\theta)-\frac{\gamma}{1-b}$ in (15) and therefore shrinks the space of r over which the government allows gambles. That is, at first glance, it looks like worse incentives for the bank to play on its limited liability option, result in tighter financial regulation, because the government allows gambles for a smaller fraction of possible bank returns. Such an argument is incomplete, however, because it ignores the fact that r is itself a function of b. When looking from before the outset of the game (i.e., before Stage 1), and varying b, it turns out that the impact actually runs in the opposite direction: the worse is the bad project, the softer becomes financial regulation (in terms of the range of r for which gambles are allowed), as shown in Corollary 1.

Corollary 1 *Writing (15) in expected terms and rearranging, we have that the government certainly allows gambles when*

$$x(1-\theta) > E\left[r \mid r<0\right] + \frac{\gamma}{1-b} \tag{16}$$

where

$$\frac{d}{db}\left[E\left[r \mid r<0\right] + \frac{\gamma}{1-b}\right] < 0 \tag{17}$$

and therefore an increase in b expands the space of inaction on gambles for resurrection in the government's optimal policy profile in (15).

Proof. On page 27. ■

5 Targeting the bank's risk profile

Thus far, the analysis has centered on a policy tool that can affect gambles for resurrection, and thereby also indirectly impact the bank's initial risk profile choice. But what if, aside from the choice between mark-to-market and historical value accounting, the government also possessed a more direct tool to target the bank's initial risk profile? Would the government optimally use the different tools as substitutes or as complements? Would the ability to directly target the bank's project choice alter the results in Propositions 1 and 2?

In this section, in addition to regulatory policy in the form of l, the government also possesses a risk cap, $\overline{\rho}$. This can be considered a shorthand form for a risk-weighted capital requirement. That is, qualitatively, the key is to provide the government with a means to directly target the asset risk choice, ρ, and to retain tractability we choose to model this in the simplest form, with a cap on the risk that the bank is allowed to take, rather than adding the layer of bank capital and an optimization over the bank's liability side.

The imposition of the risk cap is subject to two frictions. First, there is a chance that the bank finds ways to arbitrage the regulation, for instance through financial innovation. We model this with a probability, ϕ, that the regulation is successful at binding bank risk. That is, with probability ϕ, $\overline{\rho}$ binds ρ, whereas with probability $(1-\phi)$ the bank succeeds at arbitraging the risk cap, and is free to set ρ as it pleases. Here, $\phi \to 1$ represents the case of a "perfect" regulatory tool, which gives the government full control over the bank's risk profile. Instead, the baseline model is contained in the case $\phi \to 0$, as the risk cap becomes completely ineffective and only legislation l remains as government policy.

Second, a forced adjustment of its risk profile is costly to the bank. This relates to the notion that a bank owns long term assets of limited liquidity, and therefore faces frictions associated with changing its portfolio. When the bank chooses its risk profile, it does not yet know whether it will succeed at finding opportunities to circumvent $\overline{\rho}$. If the bank chooses a value of ρ above $\overline{\rho}$ and is subsequently forced to adjust down, then the cost of doing so is $\eta\left(\rho-\overline{\rho}\right)$, where $\eta \in (0,1)$. Absent such a friction (i.e., if $\eta=0$), the bank sees no peril to ignoring the risk cap: it would first try to set its preferred risk profile, only adjusting to $\overline{\rho}$ when regulatory arbitrage fails. The timing of the game is now as follows:

Stage 1 The government chooses both the bank risk cap, $\overline{\rho}$, and the legislation on gambles, l.

Stage 2 The bank chooses ρ.

Stage 3 Regulatory arbitrage attempt: with probability ϕ, $\overline{\rho}$ successfully constrains ρ.

Stage 4 r is drawn. If $r > 0$: the game ends with payoffs $r - \eta(\rho - \overline{\rho})$, 0, 0 for, respectively, the bank, the public and the government. If $r < 0$: the game continues.

Stage 5 With probability λ this stage occurs, and the government can once more determine l.

Stage 6 If $l = \{\text{Forbid}\}$: the game ends with Stage 4 payoffs. If $l = \{\text{Allow}\}$: the game continues.

Stage 7 With probability θ the public learns that $r < 0$, and imposes x on the government.

Stage 8 Gamble: Payoffs $-\eta(\rho - \overline{\rho})^2$, $(r+g) < 0$, $(r+g-x)$ for the bank, the public and the government if the gamble fails, and $(r+g) - \eta(\rho - \overline{\rho})^2$, 0, 0 or $-x$ (from Stage 7) if it succeeds.

At Stage 1 the government now solves

$$\min_{\overline{\rho},l} \{E[\min\{r(\overline{\rho},l) + g(l), 0\} - x(\overline{\rho},l)]\} \tag{18}$$

and at Stage 2 the bank solves

$$\max_{\rho} \{E[r|\, r > 0] + E[\min\{r+g, 0\}|\, r < 0] - \phi\eta(\rho - \overline{\rho})\} \tag{19}$$

Solving by backward induction, the government's Stage 5 optimization problem is unchanged compared to (Stage 4 in) the baseline model. Therefore, Lemma 3 continues to describe optimization at this stage. For Stage 2 and Stage 1 optimization we now obtain:

Lemma 6 *At Stage 2, the bank's optimal policy is*

$$\rho = \begin{cases} \begin{cases} b \text{ if } b > \widehat{\mathbf{b}_A} \\ 0 \text{ otherwise} \end{cases} & \text{if } l = \{Allow\} \\ \\ \begin{cases} b \text{ if } b > \widehat{\mathbf{b}_F} \\ 0 \text{ otherwise} \end{cases} & \text{if } l = \{Forbid\} \end{cases} \tag{20}$$

where $\widehat{\mathbf{b}_F} > \widehat{\mathbf{b}_A}$*,* $\widehat{\mathbf{b}_A} > \mathbf{b}_A$ *and* $\widehat{\mathbf{b}_F} > \mathbf{b}_F$*. Here,* $\widehat{\mathbf{b}_A} \to \mathbf{b}_A$ *and* $\widehat{\mathbf{b}_F} \to \mathbf{b}_F$ *(recovering the baseline model) for* $\phi \to 0$*. Instead, for* $\phi \to 1$ *the bad project is never chosen (*$\widehat{\mathbf{b}_F} > \widehat{\mathbf{b}_A} > 1$*).*

Proof. On page 28. ■

Lemma 7 *At Stage 1, the government's optimal policy is* $\overline{\rho} = 0$*, combined with* $l = \{Allow\}$ *if*

$$b > \widehat{\mathbf{b}_F} \text{ and } x(1-\theta) > \frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) \tag{21}$$

and $l = \{Forbid\}$ *otherwise.*

Proof. On page 29. ■

Building on these Lemmas, we can derive the key result of this section:

Proposition 4 *As long as* ϕ *is small enough that* $\widehat{\mathbf{b}_F} < 1$*, Propositions 1 and 2 remain valid in this extension. Moreover, the government optimally uses its two policy tools,* $\overline{\rho}$ *and* l*, as complements rather than substitutes: with the risk cap in hand, the government forbids gambles using* l *over a larger range of possible values of* b *than it does without the risk cap. Instead, when* ϕ *becomes large enough, the model's unique outcome is* $\rho = 0$*.*

Proof. On page 29. ■

Proposition 4 shows that the main results of the paper go through when the government's toolset is "imperfect enough". That is, when ϕ is large enough, and the government possesses a tool that is capable of successfully targeting bank risk, because the bank finds it difficult to arbitrage and cares about adjustment cost η when arbitrage fails, then the tradeoffs identified in Propositions 1 and 2 no longer matter. However, a small enough ϕ means that the government cannot target the bank's initial risk that well, and this makes government policy sensitive to the political economy dynamics of bank insolvency, and through this also to the considerations surrounding time inconsistency that we found in the baseline model.

The continued prevalence of the time consistency problem when ϕ is small, is highlighted by the fact that the government uses its two tools as complements. A priori, when a second tool becomes available for the same target, one might expect a policy maker to optimally use the tools as substitutes, so that applying more of the one goes together with applying less of the other. But the impact of time inconsistency runs counter to this intuition. The probability that the risk cap binds, makes the bank less inclined to take risk and, even if the bank still wants to choose the bad project, it sometimes cannot do so, because its regulatory arbitrage fails. This means that at Stage 1, the government knows that bank failure happens less often. That, in turn, reduces the attraction of joining in the bank's future gamble - the need to cover up bank insolvency from the public is less likely to arise. The government therefore becomes more inclined to stop the gambles and opt for mark-to-market regulation.

Proofs

Proof of Lemma 1. From (4), the bank solves

$$\max_{\rho} \{E[\max\{r,0\}]\} \tag{22}$$

Here, we can use the uniform distribution of returns to rewrite $E[\max\{r,0\}] =$

$$\int_0^{\mu+w} r f(r)\, dr = \int_0^{1-\rho+\frac{1}{1-\rho}} r \frac{1}{\left(1-\rho+\frac{1}{1-\rho}\right)-\left(1-\rho-\frac{1}{1-\rho}\right)} dr = \frac{1-\rho}{2}\int_0^{1-\rho+\frac{1}{1-\rho}} r dr \tag{23}$$

which is always positive, because $\rho < 1$ from (3). Solving for the integral, we obtain

$$E[\max\{r,0\}] = \frac{(2+\rho(\rho-2))^2}{4(1-\rho)} \tag{24}$$

and this expression is a convex function.[16] This means that the solution to (22) can only be in the corners of $\rho = 0$ or $\rho = b$.

At $\rho = 0$, we have $E[\max\{r,0\}] = 1$. Therefore, the solution to (22) is

$$\rho = \begin{cases} b \text{ if } b > \bar{b} \\ 0 \text{ otherwise} \end{cases} \tag{25}$$

where $\bar{b}$ is the value of b for which

$$\frac{(2+b(b-2))^2}{4(1-b)} = 1 \tag{26}$$

and this gives $\bar{b} \in (0,1)$.[17] ■

[16] The minimum of the function occurs at $\rho = 0.423$ from solving $\frac{d}{d\rho}\frac{(2+\rho(\rho-2))^2}{4(1-\rho)} = 0$.
[17] The exact solution is $\bar{b} = 0.704$.

Proof of Lemma 2. Rewriting from (6) and (7) using the same approach as in the proof of Lemma 1, we obtain

$$\max_{\rho}\{E[\Pi]\} = \max_{\rho}\left\{\int_{0}^{\mu+w} rf(r)\,dr + \int_{\mu-w}^{0}\frac{1}{2}\left(r+\frac{1}{1-b}\right)f(r)\,dr\right\} \tag{27}$$

where expected profit becomes

$$E[\Pi] = \begin{cases} 1 \text{ if } \rho = 0 \\ \frac{(2+b(b-2))(4+b(b-2))}{8(1-b)} \text{ if } \rho = b \end{cases} \tag{28}$$

which is maximized when the bank sets

$$\rho = \begin{cases} b \text{ if } b > \underline{b} \\ 0 \text{ otherwise} \end{cases} \tag{29}$$

where $\underline{b}$ is the solution to (30), and this gives $\underline{b} \in \left(0, \bar{b}\right)$.[18]

$$\frac{(2+b(b-2))(4+b(b-2))}{8(1-b)} = 1 \tag{30}$$

■

Proof of Lemma 3. From (7) and (9), the government's expected payoff at Stage 4 is

$$\frac{1}{2}(0-\theta x-(1-\theta)(0)) + \frac{1}{2}\left(r-\frac{\gamma}{1-b}-x\right) = \frac{1}{2}\left(r-\frac{\gamma}{1-b}-(1+\theta)x\right) \tag{31}$$

when it allows the bank to gamble for resurrection. Instead, if the government forbids the

[18] The exact solution $\underline{b} = 0.482$, which is smaller than $\bar{b} = 0.704$.

gamble then the bank closes and the government has an immediate payoff of

$$r - x \tag{32}$$

where we recall that, conditional on being at Stage 4, it must be that $r < 0$ and therefore the expressions in both (31) and (32) are negative. The government thus chooses the option that minimizes its losses, which means it forbids the gamble when

$$r - x > \frac{1}{2}\left(r - \frac{\gamma}{1-b} - (1+\theta)\,x\right) \tag{33}$$

and this can be solved to the expression in the Lemma. ■

Proof of Lemma 4. At Stage 2, the bank solves $\max_\rho \{E[\Pi]\} =$

$$\begin{cases} \max_\rho \left\{ \begin{array}{c} \int_0^{\mu+w} rf(r)\,dr + (1-\lambda)\int_{\mu-w}^{0} \frac{1}{2}\left(r + \frac{1}{1-b}\right) f(r)\,dr \\ +\lambda \int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}} \frac{1}{2}\left(r + \frac{1}{1-b}\right) f(r)\,dr \end{array} \right\} \text{ if } l = \{\text{Allow}\} \\ \\ \max_\rho \left\{ \begin{array}{c} \int_0^{\mu+w} rf(r)\,dr + (1-\lambda)(0) \\ +\lambda \int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}} \frac{1}{2}\left(r + \frac{1}{1-b}\right) f(r)\,dr \end{array} \right\} \text{ if } l = \{\text{Forbid}\} \end{cases} \tag{34}$$

where the term $x(1-\theta) - \frac{\gamma}{1-b}$ comes from rewriting the threshold for legislative action by the government in Stage 4 from (10) to $r > x(1-\theta) - \frac{\gamma}{1-b}$. The two optimization problems in (34) are identical except for the part that multiplies $(1-\lambda)$, which is 0 when $l = \{\text{Forbid}\}$ but positive when $l = \{\text{Allow}\}$. Moreover, this upside occurs in a part of the distribution, $\int_{\mu-w}^{0} f(r)\,dr$, that can only materialize with the bad project. Therefore, the bad project is more attractive to the bank when $l = \{\text{Allow}\}$ than when $l = \{\text{Forbid}\}$ and $\mathbf{b}_A < \mathbf{b}_F$.

Applying similar reasoning, the term multiplying λ is positive and in the limit case where $\lambda \to 0$ the optimization problem given $l = \{\text{Forbid}\}$ becomes identical to that in the proof of Lemma 1. Therefore, $\mathbf{b}_F < \bar{b}$.

Lastly, in the optimization problem given $l = \{\text{Allow}\}$, the term that multiplies $(1-\lambda)$ is larger than the term that multiplies λ, because the terms inside the integrals are identical but the former is defined over $\int_{\mu-w}^{0} f(r)\,dr$ and the latter over the smaller area of $\int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}} f(r)\,dr$. In the limit case of $\lambda \to 0$, the optimization problem given $l = \{\text{Allow}\}$ becomes identical to that in the proof of Lemma 2. Hence, $\underline{b} < \mathbf{b}_A$. ■

Proof of Lemma 5. The payoff structure of the government, depends on its actions (rows) and on the bank's incentives (columns, which follow the thresholds derived in Lemma 4) as follows

	$b < \mathbf{b}_A$	$b \in (\mathbf{b}_A, \mathbf{b}_F)$	$b > \mathbf{b}_F$
Allow	0	< 0	$(1-\lambda)\int_{\mu-w}^{0}\left(\frac{1}{2}\left(r-\frac{\gamma}{1-b}\right)-x\left(\theta+\frac{1-\theta}{2}\right)\right)f(r)\,dr + Z$
Forbid	0	0	$(1-\lambda)\int_{\mu-w}^{0}(r-x)\,f(r)\,dr + Z$

$$Z = \lambda\int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}}\left(\frac{1}{2}\left(r-\frac{\gamma}{1-b}\right)-x\left(\theta+\frac{1-\theta}{2}\right)\right)f(r)\,dr + \lambda\int_{x(1-\theta)-\frac{\gamma}{1-b}}^{0}(r-x)\,f(r)\,dr$$

and therefore the only case where $l = \{\text{Allow}\}$ dominates $l = \{\text{Forbid}\}$ for the government is when $b > \mathbf{b}_F$ and

$$(1-\lambda)\int_{\mu-w}^{0}\left(\frac{1}{2}\left(r-\frac{\gamma}{1-b}\right)-x\left(\theta+\frac{1-\theta}{2}\right)\right)f(r)\,dr + Z > (1-\lambda)\int_{\mu-w}^{0}(r-x)\,f(r)\,dr + Z$$

which can be solved to

$$x(1-\theta) > \frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) \tag{35}$$

where we note that $\gamma - b > 0$ because $\gamma > 1$ and $b < 1$. ■

Proof of Proposition 1. From Lemma's 2 and 4 we know that $0 < \underline{b} < \mathbf{b}_F$ and therefore for those values of b that are below $\mathbf{b}_F$ (including the limit case $b \to 0$) Lemma 5 implies $l = \{\text{Forbid}\}$. Moreover, for the limit case $b \to 1$, the condition in (35) never holds, as the right-hand side goes to infinity, and therefore the government favors $l = \{\text{Forbid}\}$. ■

Proof of Proposition 2. From the case $l = \{\text{Forbid}\}$ in (34) we have that the bank's expected profit when choosing the bad project is

$$\int_0^{1-b+\frac{1}{1-b}} rf(r)\,dr + \lambda \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} \frac{1}{2}\left(r + \frac{1}{1-b}\right) f(r)\,dr \tag{36}$$

which given $f(r) = \frac{1-\rho}{2}$ and $\rho = b$ can be written to

$$\frac{1-b}{4}\left[2\int_0^{1-b+\frac{1}{1-b}} rdr + \lambda \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} rdr\right] + \frac{\lambda}{4} \tag{37}$$

while the bank's profit when choosing the good project is 1. Hence, $\mathbf{b}_F$ is the solution to

$$\frac{1-b}{4}\left[2\int_0^{1-b+\frac{1}{1-b}} rdr + \lambda \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} rdr\right] + \frac{\lambda}{4} = 1 \tag{38}$$

This expression does not readily lend itself to a closed-form solution to b. However, we can use the implicit function theorem on (38) to obtain an expression for $\frac{\partial \mathbf{b}_F}{\partial \lambda}$. The result is:

$$\frac{\partial \mathbf{b}_F}{\partial \lambda} = \frac{(1-b)\left(2-(2-b)b+x-bx(1-\theta)-x\theta-\gamma\right)\left(b^2-x(1-\theta)+b(x(1-\theta)-2)+\gamma\right)}{\left[\begin{array}{c} 3(\lambda-2)\left(b^4-4b^3\right)-b^2\left(40+\lambda\left(x^2(1-\theta)^2-18\right)\right) \\ +2b\left(16+\lambda\left(x^2(1-\theta)^2-6\right)\right)+\lambda\left(4-x^2(1-\theta)^2+\gamma(\gamma-2)\right)-8 \end{array}\right]}$$

and investigating $\frac{\partial \mathbf{b}_F}{\partial \lambda}$ numerically over $b, \theta, \lambda \in (0,1)$, $x > 0$ and $\gamma > 1$, both $\frac{\partial \mathbf{b}_F}{\partial \lambda} < 0$ and $\frac{\partial \mathbf{b}_F}{\partial \lambda} > 0$ are possible. For example, take $\theta = \frac{1}{2}$, $\gamma = \frac{3}{2}$, $b = \frac{1}{2}$, and $\lambda = \frac{1}{2}$. Then, $x = \frac{1}{2} \to \frac{\partial \mathbf{b}_F}{\partial \lambda} \approx -0.05$, while $x = \frac{3}{2} \to \frac{\partial \mathbf{b}_F}{\partial \lambda} \approx 0.03$. ■

Proof of Proposition 3. The government's expected payoff conditional on r is identical to (31) over $r \in S$ and to (32) over $r \notin S$. Solving $r - x > \frac{1}{2}\left(r - \frac{\gamma}{1-b} - (1+\theta)\,x\right)$ to r gives the expression in (15). Moreover, the government is a priori indifferent over policy options for $r \geq 0$, as gambles for resurrection only occur when $r < 0$. It therefore optimally uses a mixed strategy over $r \geq 0$ to ensure that its policy profile for $r < 0$ does not change priors about solvency. The optimal weights on $l = \{\text{Allow}\}$ and $l = \{\text{Forbid}\}$ in the mixed strategy can be any that satisfy $E\left[r\middle|\, l = \{\text{Allow}\}\right] \geq 0$ and $E\left[r\middle|\, l = \{\text{Forbid}\}\right] \geq 0$, which are always feasible to find given $E\left[r\right] > 0$. This means that the public's initial prior of $r \geq 0$ does not change after observing either policy action. ■

Proof of Corollary 1. The term $E\left[r\middle|\, r < 0\right]$ in (16) can be solved as

$$E\left[r\middle|\, r < 0\right] = \int_{\mu - w}^{0} r f\left(r\right) dr = \frac{1-b}{2}\left[\frac{1}{2}r^2\right]_{1-b-\frac{1}{1-b}}^{0} = -\frac{b^2\left(2-b\right)^2}{4\left(1-b\right)} \tag{39}$$

and therefore the condition in (16) can be written as

$$x\left(1-\theta\right) > \frac{4\gamma - b^2\left(2-b\right)^2}{4\left(1-b\right)} \tag{40}$$

where

$$\frac{d}{db}\frac{4\gamma - b^2\left(2-b\right)^2}{4\left(1-b\right)} = -\frac{b\left(2-b\right)\left(4+3b\left(2-b\right)\right)+4\gamma}{4\left(1-b\right)^2} < 0 \tag{41}$$

■

Proof of Lemma 6. First note that in spite of the addition of the term $-\phi\eta\left(\rho-\overline{\rho}\right)$, the bank continues to optimize over a convex function, because the convex form of (24) is not altered by a linear addition. Hence, the bank's optimization continues to boil down to a choose between $\rho = b$ and $\rho = 0$.

Assume that $\overline{\rho} = 0$, as verified in the proof of Lemma 7. Then the bank solves $\max_{\rho}\left\{E\left[\Pi\right]\right\} =$

$$\begin{cases} \max_{\rho}\left\{(1-\phi)\begin{pmatrix} \int_0^{\mu+w} rf\left(r\right)dr \\ +\left(1-\lambda\right)\int_{\mu-w}^{0}\frac{1}{2}\left(r+\frac{1}{1-b}\right)f\left(r\right)dr \\ +\lambda\int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}}\frac{1}{2}\left(r+\frac{1}{1-b}\right)f\left(r\right)dr \end{pmatrix}+\phi\left(1-\eta\rho\right)\right\} & \text{if } l=\{\text{Allow}\} \\ \\ \max_{\rho}\left\{(1-\phi)\begin{pmatrix} \int_0^{\mu+w} rf\left(r\right)dr+\left(1-\lambda\right)(0) \\ +\lambda\int_{\mu-w}^{x(1-\theta)-\frac{\gamma}{1-b}}\frac{1}{2}\left(r+\frac{1}{1-b}\right)f\left(r\right)dr \end{pmatrix}+\phi\left(1-\eta\rho\right)\right\} & \text{if } l=\{\text{Forbid}\} \end{cases} \tag{42}$$

which means that from this point, the proof is identical to the proof of Lemma 4, except that, irrespective of whether $l = \{\text{Allow}\}$ or $l = \{\text{Forbid}\}$, the term $(1-\eta\rho)$ makes $\rho = b$ less attractive (i.e., the term is $1-\eta b$ if $\rho = b$ and 1 if $\rho = 0$) than in the baseline model, which implies that $\widehat{\mathbf{b}_A} > \mathbf{b}_A$ and $\widehat{\mathbf{b}_F} > \mathbf{b}_F$.

Turning to the limit cases relative to ϕ, we note that $\widehat{\mathbf{b}_A} \to \mathbf{b}_A$ and $\widehat{\mathbf{b}_F} \to \mathbf{b}_F$ for $\phi \to 0$ follows directly from the equivalence between (42) and (34) when $\phi \to 0$. Moreover, when $\phi \to 1$, we have that the bank optimizes only the term $(1-\eta\rho)$, which implies it always chooses $\rho = 0$. More generally, as ϕ rises, so do $\widehat{\mathbf{b}_A}$ and $\widehat{\mathbf{b}_F}$ (by the above arguments), and when ϕ is large enough (close enough to 1), no $b \in (0,1)$ can be attractive enough to compensate for the adjustment cost η if arbitrage fails, implying $\widehat{\mathbf{b}_F} > \widehat{\mathbf{b}_A} > 1$.[19] ■

[19] The arguments in this proof are predicated on $\eta > 0$. For $\eta \to 0$, the bank is indifferent about ρ for the case where it becomes constrained (i.e., there is no cost to adjusting down from a higher risk profile), and therefore proceeds to optimize only over the part multiplied by $(1-\phi)$ in (42). This again recovers the baseline model (similarly to $\phi \to 0$).

Proof of Lemma 7. For l the steps of the proof are identical to those in the proof of Lemma 5. For $\overline{\rho}$: when $\overline{\rho}$ is not binding, the government is ex-post indifferent about the chosen value of $\overline{\rho}$. Instead, when $\overline{\rho}$ is binding at Stage 3 then: with $\rho = 0$, the government has a certain payoff of 0, which is the highest attainable. Hence, with ϕ probability of preferring $\overline{\rho} = 0$ and $(1 - \phi)$ probability of indifference at Stage 3, the government chooses $\overline{\rho} = 0$ at Stage 1. ■

Proof of Proposition 4. First note that $\widehat{\mathbf{b}_F} < 1 \Rightarrow \widehat{\mathbf{b}_A} < 1$, and this case is assured for low enough ϕ, from Lemma 6. For this case, the continued validity of Propositions 1 and 2 follows directly from the equivalence between Lemmas 4 and 5, and, respectively, Lemmas 6 and 7, where the only differences are the replacements of $\mathbf{b}_A$ by $\widehat{\mathbf{b}_A}$ and $\mathbf{b}_F$ by $\widehat{\mathbf{b}_F}$. The proofs of Propositions 1 and 2 would therefore be identical when replacing $\mathbf{b}_A$ by $\widehat{\mathbf{b}_A}$ and $\mathbf{b}_F$ by $\widehat{\mathbf{b}_F}$. The statement in the second sentence of Proposition 4 follows directly from (21) in comparison to (13): given $\widehat{\mathbf{b}_F} > \mathbf{b}_F$ from Lemma 6, the government chooses $l = \{\text{Allow}\}$ for a smaller subset of possible values of $b \in (0, 1)$ in Lemma 7 than in Lemma 5. Equivalently, then, the government chooses $l = \{\text{Forbid}\}$ for a larger subset of possible values of $b \in (0, 1)$ in Lemma 7 than in Lemma 5.

The above arguments are predicated on $\widehat{\mathbf{b}_A} < \widehat{\mathbf{b}_F} < 1$. If instead ϕ becomes large enough that the bad project is never chosen by the bank, $\widehat{\mathbf{b}_F} > \widehat{\mathbf{b}_A} > 1$ per Lemma 6, then $\rho = 0$ holds irrespective of government policy.[20] ■

[20] For completeness, we note that there is also a range of ϕ where $\widehat{\mathbf{b}_F} > 1 > \widehat{\mathbf{b}_A}$. In this case, the government always sets $l = \{\text{Forbid}\}$, per (21).

Additional analysis: Bank value transfers to the public

In the baseline model, the bank's activities do not generate any direct value for the public. We now consider that the bank makes a fixed value transfer of v to the public. This can represent an interest payment or an investment in the provision of banking services, which is costly to the bank but creates value for the public. All the stages and payoffs of the game described in 2.4 are now adjusted to include this payment. With bank transfers, Stage 1, 2 and 4 in the game depicted in 2.4 remain the same, while the other stages become, respectively:

Stage 3 r is drawn. If $r > v$: the game ends with payoffs $r - v$, v, v for, respectively, the bank, the public and the government. If $r < v$: the game continues.

Stage 5 If $l = \{\text{Forbid}\}$ then the banks shuts down, the public is repaid its deposits, with the government's deposit insurance covering the shortfall, $r < v$, while the public imposes x on the government. Payoffs are 0, r, $r - x$ for, respectively, the bank, the public and the government. If $l = \{\text{Allow}\}$ then the game continues.

Stage 6 With probability θ the public learns $r < v$ and imposes cost x on the government.

Stage 7 The bank gambles. If the gamble succeeds, then payoffs are $(r + g) - v$, v for, respectively, the bank and the public, while the government's payoff is either v or $v - x$, depending on the outcome of Stage 6. If the gamble fails, then payoffs are 0, $(r + g) < 0$, $(r + g - x)$ for, respectively, the bank, the public and the government.

A larger v means that the bank is more valuable to the public, and this enters the government's objective function. But this is a level effect: it always applies and therefore does not alter the government's incentive to legislate. A larger v also means a greater likelihood that the bank becomes insolvent, and the prospect of insolvency raises the government's concern about political repercussions x. Therefore, a bank that transfers more value to the public finds itself subjected to more stringent regulatory bounds (in the sense of the parameter thresholds in (43)), as shown in Proposition 5. A larger value transfer is thus doubly positive for the public, because the public directly obtains v and the government becomes more inclined to legislate against harmful gambles for resurrection.

Proposition 5 *With bank value transfers, the government's optimal policy at Stage 1 is to forbid gambles if and only if*

$$b > \mathbf{b}_F \text{ and } x\left(1-\theta\right) > \frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) - v \tag{43}$$

where $\mathbf{b}_F$ *decreases as* v *rises, and* $\frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) - v$ *also decreases as* v *rises.*

Proof. Following same steps as in Lemma 3, if Stage 4 occurs, then the government forbids gambles if and only if

$$r - x > \frac{1}{2}\left(r + v - \frac{\gamma}{1-b} - \left(1+\theta\right)x\right) \Leftrightarrow x\left(1-\theta\right) < r - v + \frac{\gamma}{1-b}$$

while nothing changes for bank behavior as shown in Lemma 4, so that at Stage 1 the payoff structure of the government is as follows

	$b < \mathbf{b}_A$	$b \in (\mathbf{b}_A, \mathbf{b}_F)$	$b > \mathbf{b}_F$
Allow	0	< 0	$(1-\lambda)\int_{\mu-w}^{0}\left(\frac{1}{2}\left(r+v-\frac{\gamma}{1-b}\right) - x\left(\theta + \frac{1-\theta}{2}\right)\right)f\left(r\right)dr + Z$
Forbid	0	0	$(1-\lambda)\int_{\mu-w}^{0}\left(r-x\right)f\left(r\right)dr + Z$

where the term for Z is different than in Lemma 5, but this does not affect the rest of the derivation since Z drops out in the comparison below. The only case where $l = \{\text{Allow}\}$ dominates $l = \{\text{Forbid}\}$ for the government is when $b > \mathbf{b}_F$ and

$$(1-\lambda)\int_{\mu-w}^{0}\left(\frac{1}{2}\left(r+v-\frac{\gamma}{1-b}\right) - x\left(\theta + \frac{1-\theta}{2}\right)\right)f\left(r\right)dr + Z > (1-\lambda)\int_{\mu-w}^{0}\left(r-x\right)f\left(r\right)dr + Z$$

which can be solved to

$$x(1-\theta) > \frac{1}{1-b}\left(\gamma - b + \frac{1}{2}b^2\right) - v \tag{44}$$

and this term obviously decreases as v increases, which concludes this segment of the proof.

It remains to show that $\frac{\partial \mathbf{b}_F}{\partial v} < 0$. For this, we return to the proof of Proposition 2. In the baseline model, $\mathbf{b}_F$ is the solution to (38) whereas with value transfers, $\mathbf{b}_F$ is the solution to

$$\frac{1-b}{4}\left[2\int_0^{1-b+\frac{1}{1-b}} (r-v)\,dr + \lambda \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} (r-v)\,dr\right] + \frac{\lambda}{4} = 1 - v \tag{45}$$

which can be written to

$$\frac{1-b}{4}\begin{bmatrix} 2\int_0^{1-b+\frac{1}{1-b}} rdr \\ +\lambda \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} rdr \end{bmatrix} + \frac{\lambda}{4} = 1 - v\begin{bmatrix} 1 - \frac{1-b}{2}\int_0^{1-b+\frac{1}{1-b}} dr \\ -\frac{\lambda}{4}\int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} dr \end{bmatrix} \tag{46}$$

where we know that the term multiplying v is positive because: $\frac{1-b}{2} < \frac{1}{2}$ and $\frac{\lambda}{4} < \frac{1}{2}$ and $\int_0^{1-b+\frac{1}{1-b}} dr + \int_{1-b-\frac{1}{1-b}}^{x(1-\theta)-\frac{\gamma}{1-b}} dr < 1$. Hence, compared to the baseline model, the addition of v can be expressed as no change to the left-hand side of the equation and a reduction of the right-hand side. From Lemma's 1 and 2 we know that at the threshold value of b above which the bank chooses the bad project, the bank's profit must be an increasing function of b. That is, at $b = \mathbf{b}_F$ the left-hand side of (46) is an increasing function of b. Since v shifts down the right-hand side of equation (46) as compared to the baseline model, this equation can only hold if $\mathbf{b}_F$ is lower as well. ■