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The Market Price of Risk and Macro-Financial Dynamics

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The Market Price of Risk and Macro-Financial Dynamics*

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Abstract

We construct the Volatility Financial Conditions Index (VFCI) as the component of the market price of risk spanned by financial assets. We write a no-arbitrage model with general preferences to show how to estimate the VFCI from the conditional volatility of GDP, even when markets are incomplete. Empirically, the VFCI has greater predictive power than other FCIs for equity, Treasury, and corporate bond risk premia. Across multiple identification strategies, a VFCI shock that tightens financial conditions causes a persistent decline in output and an immediate monetary policy easing. Conversely, contractionary monetary policy shocks cause financial conditions to tighten.

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I Introduction

Financial conditions indices (FCIs) are widely used by policymakers, practitioners, and, increasingly, academics. Despite their popularity, most existing FCIs are empirically driven and lack rigorous theoretical foundations. We develop a Volatility Financial Conditions Index (VFCI) derived from a model general enough to accommodate incomplete markets, non-time-separable preferences, and various frictions. We define the VFCI as the projection of the market price of risk—the conditional volatility of the stochastic discount factor—onto the span of financial asset returns. We use 1962Q1-2022Q3 quarterly data to estimate the VFCI as the fitted conditional log volatility from a heteroskedastic linear regression of GDP growth on lagged asset returns, with the log variance of GDP growth also linear in the same returns.

The resulting index predicts risk premia of equities, Treasuries, and corporate bonds, empirically corroborating its theoretical role. Narrative evidence and comparisons with established FCIs show that the VFCI also captures broader financial conditions. We then link the VFCI to macroeconomic outcomes. Conditioning on the VFCI generates a negative correlation between the mean and volatility of GDP, a core stylized fact in the "growth-at-risk" literature. Finally, a structural vector autoregression (SVAR) identified through heteroskedasticity reveals that VFCI shocks corresponding to exogenous tightening of financial conditions cause persistent output contractions and monetary policy easing, whereas contractionary monetary policy shocks tighten financial conditions.

Our starting point is a model of macro-financial interactions. The absence of arbitrage—a necessary condition for equilibrium—implies the existence of a stochastic discount factor (SDF) that prices all assets in the economy (Harrison and Kreps (1979)). We assume a representative agent whose utility depends on aggregate consumption but otherwise maintain full generality. The representative agent's first-order condition links aggregate consumption to the SDF.

Under complete markets and constant-relative-risk-aversion utility, the market price of risk is fully spanned by returns and proportional to the conditional volatility of consumption growth. With general utility and incomplete markets, the market price of risk can lie outside the asset span and can depend on the entire covariance structure of consumption growth. To handle this general case, we treat the first-order condition of the representative agent as a system of rational expectations equations. We linearize and solve the system to show that current-period consumption growth is, to first order, a linear function of past, present, and future expected values of the SDF. We decompose this function in two stages. First, we separate components known in the current period from innovations. Second, we decompose each of these two components into parts spanned and not spanned by asset returns. This decomposition rigorously justifies the heteroskedastic linear regression strategy that we use to estimate the VFCI.

In our baseline specification, we use the first four principal components of a larger cross-section of asset returns rather than the returns themselves, and real GDP instead of real consumption to make the VFCI more directly comparable to other FCIs and to results in the growth-at-risk literature. Nevertheless, we show that using individual asset returns or consumption produces a nearly identical VFCI that generates the same results in the identified SVAR.

Our empirical estimation reproduces the key empirical regularity from the growth-at-risk literature that the conditional mean and conditional volatility of GDP growth are strongly negatively correlated (Adrian, Boyarchenko, and Giannone (2019), Adrian et al. (2022)). In that literature, the correlation emerges when measures of financial conditions are added as conditioning variables on top of macroeconomic variables. Periods with tight financial conditions tend to have low expected growth and high expected volatility, representing states of "vulnerability," in which shocks are amplified precisely when marginal utility is high. We generate the same strong negative correlation by conditioning solely on the VFCI, without requiring any macroeconomic variables.

The VFCI correlates with, but is distinct from, other widely used FCIs. The Goldman Sachs Financial Conditions Index (GSFCI, Hatzius and Stehn (2018)) is a linear combination of financial variables that estimates the conditional mean of GDP growth. Due to the negative correlation between the conditional mean and conditional volatility of GDP growth, the GSFCI and the VFCI naturally covary. From a theoretical perspective, it is the volatility of the SDF, not its mean, that determines risk pricing. The volatility of GDP growth should then capture the price of risk better than its mean. Consistent with this intuition, the two indices diverge when the macroeconomic outlook is very positive but financial stress is high. For example, the VFCI shows a substantial tightening of financial conditions in the second half of 1998, when Russia defaulted on its debt and Long-Term Capital Management collapsed, while the GSFCI, more influenced by strong output growth and low inflation, indicated an easing.

Our estimate also differs from the National Financial Conditions Index, or NFCI, developed by the Federal Reserve Bank of Chicago (Brave and Butters (2011)). The NFCI uses dynamic factor analysis to extract a common component from 105 financial variables (based on Doz, Giannone, and Reichlin (2012)). The NFCI is purely statistical. Because our index is theory-based, it is likely less prone to overfitting and misspecification and more structurally interpretable.

Another commonly used proxy for the price of risk is the VIX, a measure of implied volatility of the S&P 500 over the next 30 days. Although the VIX is undoubtedly informative, it is based solely on the stock market and only available since 1990. The VFCI is estimated starting in 1962 and uses a wider set of financial assets.

Other prominent FCIs include the Federal Reserve's FCI-G, which aggregates seven financial variables weighted by coefficients of impulse response functions of GDP growth with respect to each of the financial variables' shocks (Ajello et al. 2023), the ECB's Composite Indicator of Systemic Stress (CISS), which aggregates market-specific stress using portfolio theory (Holló, Kremer, and Lo Duca 2012), and the BIS's index constructed using a dynamic factor model with a broad data set of financial variables (Lombardi, Manea, and Schrimpf 2025).

In simple linear regressions, the VFCI outperforms other FCIs at predicting the risk premia of equities (using the S&P 500 excess CAPE yield of Shiller (2000)), Treasuries (using the ten-year term premium of Adrian, Crump, and Moench (2013)), and corporate bonds (using the GZ spread of Gilchrist and Zakrajšek (2012)). Furthermore, when all FCIs are included jointly, the VFCI drives out the significance of the other indices.

The VFCI and macroeconomic aggregates are causally related. We estimate an SVAR identified through heteroskedasticity as in Brunnermeier et al. (2021), which rests on the assumption that the relative volatilities of macroeconomic variables change over time. Our 1962Q1-2022Q3 quarterly sample is long enough to include multiple policy regimes and financial episodes, making the identifying assumption particularly plausible. In our view, the identifying assumption is also more easily testable than those of other identification strategies, some of which are not testable at all. Nonetheless, we show that our results are robust to using local projections and alternative strategies that include proxy variables, different recursive orderings, and sign restrictions.

Our results hold across extensive robustness tests. The VFCI does not substantially change when we: use consumption instead of GDP, replace principal components with the individual asset returns used to construct them, vary the number of principal components, include lags of GDP growth, or control for innovations in future expected SDFs. SVAR impulse responses are similar whether we: use growth rates instead of levels or log levels, exponentiate the VFCI to measure volatility rather than log volatility, increase posterior draws to one million, vary prior distributions, or include one or both of the GZ and TED spreads in the estimation. Estimating the VFCI and SVAR jointly also leaves all conclusions unchanged.

We present a review of the related literature in Section IX and Section X concludes.

II The VFCI as Price of Risk

This section presents our model, derives the VFCI, and shows how to estimate it empirically. We start with complete markets and constant-relative-risk-aversion (CRRA) utility, then continue with incomplete markets and general preferences.

II.A The Price of Risk and the VFCI

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. We assume the absence of arbitrage opportunities, which is a necessary condition for equilibrium. The Fundamental Theorem of Asset Pricing then guarantees the existence of a stochastic discount factor $SDF = \{SDF_t\}_{t=0}^{\infty}$ such that, for any traded asset i with return $R_{i,t+1}$,

$$1 = \mathbb{E}_t[SDF_{t+1} R_{i,t+1}]. \tag{1}$$

The market price of risk η_t is, by definition, the conditional volatility of the SDF

$$\eta_t := \sqrt{Var_t(SDF_{t+1})}.$$

We write η_t as the sum of two orthogonal components, $\eta_t = \eta_t^* + \eta_t^{\perp}$, where we use a superscript $*$ to denote the projection of a variable onto the asset span—the space spanned by returns $R_{i,t}$—and a superscript $\perp$ to denote the projection onto the orthogonal complement of the asset span. We define the VFCI as the logarithm of the market price of risk spanned by returns, standardized to have zero mean and unit variance:

$$VFCI_t := \frac{\log \eta_t^* - \mathbb{E}[\log \eta_t^*]}{\sqrt{Var(\log \eta_t^*)}}. \tag{2}$$

II.B Complete Markets and CRRA Preferences

A representative agent facing complete markets values nonnegative aggregate consumption streams $C = \{C_t\}_{t=0}^{\infty}$ according to:

$$U(C) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$ and $\beta \in (0, 1)$. Since markets are complete, the SDF is uniquely determined by returns through equation (1). The representative agent takes asset prices, and hence the SDF, as given.

The first-order condition (FOC) for optimal consumption is

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} = SDF_{t+1}, \tag{3}$$

for all t and all states of nature. Unlike many other representative agent models in which the SDF is *defined* by equation (3), we instead define the SDF by equation (1). In our model, (3) is not an identity. It is an optimality condition that equalizes marginal rates of substitution determined by $U(C)$ to marginal rates of transformations given by the variable SDF.

Rewriting the FOC in terms of $\Delta c_{t+1} := \log(C_{t+1}/C_t)$ and linearizing around $\Delta c_{t+1} = 0$ gives

$$\beta(1 - \gamma \Delta c_{t+1}) = SDF_{t+1}. \tag{4}$$

The right-hand side is already linear, requiring no approximation.

We now decompose SDF_{t+1} into its expected and unexpected parts

$$SDF_{t+1} = \mathbb{E}_t SDF_{t+1} + \nu_{t+1},$$

where $\nu_{t+1} := SDF_{t+1} - \mathbb{E}_t SDF_{t+1}$. Substituting into equation (4) and solving for Δc_{t+1},

$$\Delta c_{t+1} = \frac{1}{\gamma} - \frac{1}{\gamma\beta} \mathbb{E}_t[SDF_{t+1}] - \frac{1}{\gamma\beta} \nu_{t+1}.$$

Market completeness implies that any variable known at time t can be expressed as a linear combination of returns $R_{i,t}$. Therefore, constants $\tilde{\theta}_0$ and θ_i exist such that

$$-\frac{1}{\gamma\beta} \mathbb{E}_t SDF_{t+1} = \tilde{\theta}_0 + \sum_i \theta_i R_{i,t},$$

where the index i runs over all traded assets. Defining $\theta_0 := 1/\gamma + \tilde{\theta}_0$ and $\varepsilon_{t+1} := -\nu_{t+1}/(\gamma\beta)$, equation (4) becomes

$$\Delta c_{t+1} = \theta_0 + \sum_i \theta_i R_{i,t} + \varepsilon_{t+1}. \tag{5}$$

To identify the market price of risk, we start from

$$Var_t(\varepsilon_{t+1}) = \frac{1}{\gamma^2 \beta^2} Var_t(\nu_{t+1}) = \frac{1}{\beta^2 \gamma^2} \eta_t^2. \tag{6}$$

None of the variables in equation (6) are directly observed. We relate $Var_t(\varepsilon_{t+1})$ to observables using that, again due to complete markets, there are constants δ_0 and δ_i such that

$$\log Var_t(\varepsilon_{t+1}) = \delta_0 + \sum_i \delta_i R_{i,t}, \tag{7}$$

We focus on the logarithm of the variance rather than its level to automatically ensure it is nonnegative during estimation.

Equations (5)-(7) have the form of a linear regression with multiplicative conditional heteroskedasticity. This is not a stochastic volatility model; equation (7) contains no shock, making the log variance a deterministic function of $R_{i,t}$. The error term ε_{t+1} in equation (5) has zero mean

$$\mathbb{E}_t[\varepsilon_{t+1}] = -\frac{1}{\gamma\beta}\mathbb{E}_t[\nu_{t+1}] = -\frac{1}{\gamma\beta}\mathbb{E}_t[SDF_{t+1} - \mathbb{E}_t SDF_{t+1}] = 0,$$

and, by the law of iterated expectations, is uncorrelated with the regressors R_{it}

$$\mathbb{E}[R_{i,t}\varepsilon_{t+1}] = \mathbb{E}[\mathbb{E}_t[R_{i,t}\varepsilon_{t+1}]] = \mathbb{E}[R_{i,t}\mathbb{E}_t[\varepsilon_{t+1}]] = 0.$$

We can therefore get consistent estimates $\hat{\theta}_0$, $\hat{\theta}_i$, $\hat{\delta}_0$, and $\hat{\delta}_i$ by standard econometric methods, such as maximum likelihood or Harvey's (1976) two-step generalized least-squares. The only data needed are time series of consumption growth and returns.

Combining (6) and (7) gives

$$\log \eta_t = \log(\beta\gamma) + \frac{1}{2}\delta_0 + \frac{1}{2}\sum_i \delta_i R_{i,t}, \tag{8}$$

which shows that knowing δ_0 and δ_i identifies the log market price of risk $\log \eta_t$ up to an unknown constant level shift of $\log(\beta\gamma)$. With complete markets, the market price of risk and its projection onto the asset span coincide. Then, the definition of the VFCI in (2) and equation (8) imply that

$$VFCI_t = \frac{\sum_i \delta_i (R_{i,t} - \mathbb{E}[R_{i,t}])}{\sqrt{Var(\sum_i \delta_i R_{i,t})}}. \tag{9}$$

Replacing δ_i with $\hat{\delta}_i$ gives an empirical estimate for $VFCI_t$; the estimate $\hat{\delta}_0$ is not used. By defining the VFCI as a variable normalized to have mean zero we have bypassed the need to estimate the unknown constant $\log(\beta\gamma)$.

Equations (5), (6), (8) and (9) show that the log conditional volatility of consumption growth and log market price of risk are affine in the VFCI:

$$\log Var_t(\Delta c_{t+1}) = \varphi_0 + \varphi_{VFCI} VFCI_t, \tag{10}$$

$$\log \eta_t = \frac{1}{2}\varphi_0 + \log(\beta\gamma) + \frac{1}{2}\varphi_{VFCI} VFCI_t, \tag{11}$$

where

$$\varphi_0 := \delta_0 + \sum_i \delta_i \mathbb{E}[R_{i,t}], \qquad \varphi_{VFCI} := \sqrt{Var\big(\sum_i \delta_i R_{i,t}\big)}.$$

The VFCI captures all time variation in both the conditional volatility of consumption growth and the price of risk, but not their level. The level of η_t cannot be recovered because $\log(\beta\gamma)$ remains unidentified. On the other hand, $\log Var_t(\Delta c_{t+1})$ can be completely recovered from $VFCI_t$ by constructing estimates for φ_0 and φ_{VFCI} using $\hat{\delta}_0$ and $\hat{\delta}_i$.

Equation (10) also reveals that variation in the VFCI is equivalent to heteroskedasticity in consumption growth conditional on returns. Consumption growth could still be *un*conditionally homoskedastic, and even conditionally homoskedastic with respect to a different information set. Equation (10) is not an assumption imposed on the data generating process, but an equilibrium condition between two endogenous variables.[1]

II.C General Framework

We now extend the framework to incomplete markets and general utility. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ be a complete filtered probability space. Stochastic processes are in the space of square-integrable random variables. A representative agent values nonnegative aggregate consumption streams $C = \{C_t\}_{t=-\infty}^{\infty}$ according to a utility function $U(C)$ that is strictly increasing, concave, and differentiable. We treat the pre-sample history $\{C_t\}_{t<0}$ as fixed, non-stochastic initial data. A set $\mathcal{C}$ of feasible consumption streams encodes all the constraints faced by the representative agent. We assume U admits a unique maximizer in the interior of $\mathcal{C}$. A risk-free asset is traded, but markets are otherwise allowed to be arbitrarily incomplete.

The FOC for consumption is

$$\frac{\partial U/\partial C_{t+1}}{\partial U/\partial C_t} = SDF_{t+1}, \tag{12}$$

where the partial derivatives are evaluated at the optimum and, in general, can depend on

[1]Though we neither define nor assume equilibrium (here or later), we use "equilibrium condition" because in any equilibrium—however defined—no-arbitrage and the representative agent's first-order condition (3) must hold, which together with complete markets imply equation (10).

the entire sequence C. Under the assumptions we have made, this FOC is both necessary and sufficient for optimality.

Our framework accommodates a broad range of models and economic environments including non-Markovian dynamics, trading frictions, convex and some non-convex constraints on the representative agent or asset prices, illiquidity, partial information, and real and nominal rigidities. Models covered by our framework include the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the disaster risk models of Barro (2006) and Gabaix (2012), preferences with ambiguity aversion and robustness as in Hansen and Sargent (2010). Requiring a representative agent is our most restrictive assumption; models without one lie outside the scope of our framework.[2]

A linear approximation of equation (12) with respect to $\{\Delta c_t\}_{t=0}^{\infty}$ around zero is[3]

$$\bar{g} + \sum_{s=1}^{\infty} g_{-s}\Delta c_{t+1-s} + g_0\Delta c_{t+1} + \sum_{s=1}^{\infty} g_s\mathbb{E}_t[\Delta c_{t+1+s}] + \sum_{s=1}^{\infty} k_s\mathbb{E}_{t+1}[\Delta c_{t+1+s}] = SDF_{t+1}, \quad (13)$$

where $\bar{g}$, g_s, and k_s are linearization coefficients.[4]

Treating SDF_{t+1} as an exogenous disturbance, equation (13) is a linear rational expectations equation in $\{\Delta c_t\}_{t=0}^{\infty}$. We assume that a solution exists and is unique. Al-Sadoon (2024) shows the solution can be written as

$$\Delta c_{t+1} = \bar{h} + \sum_{s=1}^{\infty} h_{-s}SDF_{t+1-s} + h_0 SDF_{t+1} + \sum_{s=1}^{\infty} h_s\mathbb{E}_{t+1}[SDF_{t+1+s}], \quad (14)$$

where $\bar{h}$ and h_s are constants that can be found as functions of $\bar{g}$, g_s and k_s.

We now rewrite equation (14) as a heteroskedastic linear regression that allows us to estimate the VFCI using only observed variables. Denote the right-hand side of equation (14) by z_{t+1}. First, we decompose z_{t+1} as the sum of its unconditional expectation, updates

[2]For a review of conditions for existence of a representative agent, see Skiadas (2009). We do not pursue the most general version of our framework to avoid technical arguments and lengthy notation. For example, Mostovyi and Siorpaes (2025) and Monoyios (2022) show how to accommodate utility functions that are non-differentiable (with well-defined supergradient densities) and that allow for more complex forms of state-dependence. The vector autoregression in Section VIII.A further extends the framework to utilities that depend on other endogenous variables (e.g., labor). This extension allows the framework to cover many New Keynesian and other dynamic stochastic general equilibrium models that admit a representative agent.

[3]Linearization can be done around *any* constant consumption path, permitting affine approximations that, as Lopez, Lopez-Salido, and Vazquez-Grande (2018) show, perform comparably to global solution methods with recursive preferences, time-varying disaster risk, and Campbell-Cochrane habits.

[4]The coefficients g_s and k_s depend on s but not on calendar time t. While this property holds for virtually all standard preferences (e.g., habits, recursive utility), it does restrict the class of admissible utility functions. Specifically, we require that the cross-derivative $\partial^2 U/(\partial C_{\tau_1}\partial C_{\tau_2})$ depends on time only through the difference $\tau_2 - \tau_1$. See Appendix A for the complete set of assumptions and a formal derivation of the linearization within a general normed function space.

in the time-t conditional expectation relative to time 0, and news between t and $t+1$:

$$z_{t+1} = \underbrace{\mathbb{E}_0 z_{t+1}}_{\substack{\text{Unconditional} \\ \text{expectation}}} + \underbrace{\mathbb{E}_t z_{t+1} - \mathbb{E}_0 z_{t+1}}_{\substack{\text{Update to } \mathbb{E}_t z_{t+1} \\ \text{relative to } \mathbb{E}_0 z_{t+1}}} + \underbrace{z_{t+1} - \mathbb{E}_t z_{t+1}}_{\substack{\text{News between} \\ t \text{ and } t+1}} . \tag{15}$$

Then, we orthogonally decompose

$$\mathbb{E}_t z_{t+1} - \mathbb{E}_0 z_{t+1} = \underbrace{(\mathbb{E}_t z_{t+1} - \mathbb{E}_0 z_{t+1})^*}_{\substack{\text{Spanned by} \\ \text{traded assets}}} + \underbrace{(\mathbb{E}_t z_{t+1} - \mathbb{E}_0 z_{t+1})^\perp}_{\substack{\text{Orthogonal} \\ \text{to asset span}}}, \tag{16}$$

When markets are incomplete, the term orthogonal to the asset span can be non-zero. The component in the asset span can be written as a linear combination of traded returns, that is, there are constants $\tilde{\theta}_0$ and θ_i such that

$$(\mathbb{E}_t z_{t+1} - \mathbb{E}_0 z_{t+1})^* = \tilde{\theta}_0 + \sum_i \theta_i R_{i,t}. \tag{17}$$

Using equations (15), (16), and (17) in (14), we get

$$\Delta c_{t+1} = \theta_0 + \sum_i \theta_i R_{i,t} + \sum_{s=1}^{\infty} h_s (\mathbb{E}_{t+1} - \mathbb{E}_t) SDF_{t+1+s} + \varepsilon_{t+1}, \tag{18}$$

where we define $\theta_0 := \mathbb{E}_0[z_{t+1}] + \tilde{\theta}_0$ and $\varepsilon_{t+1} := [(\mathbb{E}_t - \mathbb{E}_0) z_{t+1}]^\perp + h_0 \nu_{t+1}$, and recall that, as in the last subsection, $\nu_{t+1} := SDF_{t+1} - \mathbb{E}_t SDF_{t+1}$ is the unexpected component of the SDF.

Terms of the form $(\mathbb{E}_{t+1} - \mathbb{E}_t) SDF_{t+1+s}$ in equation (18) are unobserved. We approximate them by a quadratic function of bond yields and term premia.[5] Term premia are also unobserved, but can be estimated with any arbitrage-free term-structure model. Making a given arbitrage-free term-structure model consistent with our framework imposes precisely one restriction: the projection of SDF_{t+1} onto the space spanned by a constant and bond returns, denoted SDF_{t+1}^{bond}, must equal the term structure model's SDF. This restriction

[5] Appendix A.7 derives the quadratic approximation. It also shows that the variance of the approximation error is theoretically bounded above by a term of fourth-order in one-quarter-maturity yield innovations. Using term premia from Adrian, Crump, and Moench (2013), this upper bound equals 0.1 basis points, showing that the approximation error is negligible.

implies a lower bound on the market price of risk

$$\eta_t = \mathrm{Var}_t(SDF_{t+1}) \geq \mathrm{Var}_t(SDF_{t+1}^{bond}),$$

since any part of SDF orthogonal to SDF^{bond} increases $\mathrm{Var}_t(SDF_{t+1})$ but not $\mathrm{Var}_t(SDF_{t+1}^{bond})$. Apart from this lower bound, η_t remains unrestricted by the use of the abitrage-free term structure model.

Let $B_{t+1}^{(s)}$ be the quadratic function that approximates $(\mathbb{E}_{t+1} - \mathbb{E}_t)SDF_{t+1+s}$ using observed yields and term premia from the no-arbitrage term structure model. Equation (18) can then be written in terms of the observables Δc_{t+1}, $R_{i,t}$, $B_{t+1}^{(s)}$, unknown parameters to be estimated (θ_0, θ_i, h_s), and a residual ε_{t+1}:

$$\Delta c_{t+1} = \theta_0 + \sum_i \theta_i R_{i,t} + \sum_{s=1}^{\infty} h_s B_{t+1}^{(s)} + \varepsilon_{t+1}. \tag{19}$$

To relate the market price of risk η_t to observables, we apply the decompositions in equations (15) and (16) to $\log \varepsilon_{t+1}^2$, to get

$$\log \varepsilon_{t+1}^2 = \mathbb{E}_0 \log \varepsilon_{t+1}^2 + u_t^* + \xi_{t+1}, \tag{20}$$

where $u_t^* := \left[(\mathbb{E}_t - \mathbb{E}_0) \log \varepsilon_{t+1}^2\right]^*$, $u_t^\perp := \left[(\mathbb{E}_t - \mathbb{E}_0) \log \varepsilon_{t+1}^2\right]^\perp$, and $\xi_{t+1} := u_t^\perp + \log \varepsilon_{t+1}^2 - \mathbb{E}_t \log \varepsilon_{t+1}^2$.

Since u_t^* is in the asset span, there are constants $\tilde{\delta}_0$ and δ_i such that

$$u_t^* = \tilde{\delta}_0 + \sum_i \delta_i R_{i,t}. \tag{21}$$

Using (21) in (20) gives

$$\log \varepsilon_{t+1}^2 = \delta_0 + \sum_i \delta_i R_{i,t} + \xi_{t+1}, \tag{22}$$

where $\delta_0 := \mathbb{E}_0 \log \varepsilon_{t+1}^2 + \tilde{\delta}_0$. Equation (22), being just a decomposition, is an identity. We have derived it without making any additional assumptions and without restricting any of the data generating processes of the model's variables.

Using the definition of ε_{t+1} and that $[(\mathbb{E}_t - \mathbb{E}_0)z_{t+1}]^\perp$ is known at time t, we have

$$Var_t(\varepsilon_{t+1}) = Var_t([(\mathbb{E}_t - \mathbb{E}_0)z_{t+1}]^\perp + h_0 \nu_{t+1}) = h_0^2 Var_t(\nu_{t+1}) = h_0^2 \eta_t^2.$$

Combining with equation (22) and using the definition of the VFCI from equation (2) shows that the log market price of risk is affine in the VFCI:

$$\log \eta_t = \phi_0 + \phi_{VFCI} VFCI_t + e_t, \tag{23}$$

where

$$\phi_0 := \frac{1}{2} \sum_i \delta_i \mathbb{E}[R_{i,t}] + \frac{1}{2}(\delta_0 - \log(h_0^2)), \qquad \phi_{VFCI} := \frac{1}{2} \sqrt{Var\big(\sum_i \delta_i R_{i,t}\big)},$$
$$e_t := \frac{1}{2} \log \mathbb{E}_t[\exp(\xi_{t+1})].$$

Just as in the complete-markets CRRA case, the level of $\log \eta_t$ is not recoverable since h_0 is not identified. Unlike the complete-markets CRRA case, $VFCI_t$ is now a noisy proxy for $\log \eta_t$. Section B.4 documents that estimates for the unprojected $\log \varepsilon_{t+1}^2$ and the unspanned ξ_{t+1} have little causal impact on macroeconomic variables in the identified SVAR. These findings suggest that it is precisely the projection of the market price of risk onto the span of traded assets—the VFCI—that teases out the signal from $\log \varepsilon_{t+1}^2$ that is relevant for macro-financial interactions, and that the part of $\log \varepsilon_{t+1}^2$ not accounted for by the VFCI is of little importance for macroeconomic dynamics.

Estimation Strategy

Equations (19) and (22) have the form of a linear regression with multiplicative heteroskedasticity. It has two new features compared to the simpler case of complete markets and CRRA utility: the mean equation (19) includes regressors $B_{t+1}^{(s)}$ and the variance equation (22) includes the error term ξ_{t+1}, which makes volatility stochastic.

Estimating this system presents a potential endogeneity problem. The regressors $B_{t+1}^{(s)}$ may be correlated with the error term ε_{t+1}. This endogeneity can happen because news arriving between t and $t+1$ can simultaneously affect both the SDF innovation ν_{t+1} (a component of ε_{t+1}) and revisions to expected future SDFs (which make up $B_{t+1}^{(s)}$). While returns $R_{i,t}$ are pre-determined at time t and thus orthogonal to the shocks, the endogeneity of $B_{t+1}^{(s)}$ can lead to biased and inconsistent estimates.

This endogeneity, however, does not materially affect our results. Figure B.1 in Appendix B.1.2 confirms that estimation with and without the $B_{t+1}^{(s)}$ terms produce nearly identical VFCI series. Figure B.5 in Appendix B.2.3 confirms that the impulse response functions in Section VI—one of our main results—are also robust to using a VFCI constructed with and without $B_{t+1}^{(s)}$.

Given this evidence, we drop $B_{t+1}^{(s)}$ from our baseline specification and estimate (19) and (22) consistently using maximum likelihood and use the resulting $\hat{\delta}_i$ to construct the VFCI using equation (9).

II.D Examples

We apply the framework to two utilities that are not time-separable. First, consider internal habit formation preferences with period utility given by $\beta^t \left(C_t - \alpha C_{t-1}\right)^{1-\gamma}/(1-\gamma)$, where $0 < \alpha, \beta < 1$. The linearized FOC is

$$SDF_{t+1} = \beta + \frac{\beta\gamma}{(1-\alpha)(1-\alpha\beta)}\Big[\alpha\Delta c_t + ((1-\alpha)\alpha\beta - 1)\Delta c_{t+1} + \alpha\beta\big(\mathbb{E}_{t+1}\Delta c_{t+2} - \mathbb{E}_t\Delta c_{t+1}\big)\Big].$$

Treating this FOC as a rational expectations equation and solving gives

$$\Delta c_{t+1} = \frac{1}{\gamma} - \frac{(1-\alpha)(1-\alpha\beta)}{(1-\alpha^2\beta)\beta\gamma}\left(\sum_{s=1}^{\infty}\alpha^s SDF_{t+1-s} + SDF_{t+1} + \sum_{s=1}^{\infty}(\alpha\beta)^s\mathbb{E}_{t+1}[SDF_{t+1+s}]\right).$$

Next, consider Epstein-Zin preferences with discount rate $0 < \beta < 1$, coefficient of relative risk aversion $\gamma > 0$ and elasticity of intertemporal substitution $\psi > 0$. The linearized FOC is

$$SDF_{t+1} = \beta - \frac{\beta}{\psi}\Delta c_{t+1} - \beta\left(\gamma - \frac{1}{\psi}\right)\sum_{k=1}^{\infty}\beta^k(\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta c_{t+1+k}],$$

with solution

$$\Delta c_{t+1} = -\frac{\psi}{\beta}(SDF_{t+1} - \beta) + \frac{\psi^2}{\beta}\left(\gamma - \frac{1}{\psi}\right)\sum_{k=1}^{\infty}\beta^k(\mathbb{E}_{t+1} - \mathbb{E}_t)[SDF_{t+1+k}]$$

Appendix A provides details of the derivations and generalizes to multi-period habits.

III VFCI Estimation

III.A Data

Table 1 shows the variables we use to construct the VFCI. All series are publicly available and free, in quarterly frequency from 1962Q1 to 2022Q3, totaling 243 observations. The sample start date is the earliest for which all series available.

The variable *Ret* represents S&P 500 returns, calculated as the percentage price change

Table 1

Variables used to construct the volatility financial conditions index (VFCI)

Panel A: financial variables	
Variable	Description
Ret	S&P 500 returns
Vol	S&P 500 volatility
Term	10-year minus 3-month Treasury yields
Liq	3-month Treasury secondary market rate minus effective federal funds rate
Cred	Aaa corporate bond yield minus 10-year Treasury yield
Def	Aaa minus Baa corporate bond yield
Panel B: macroeconomic variables	
Variable	Description
GDP	Real GDP
C	Real aggregate consumption

Notes: All series are quarterly from 1962Q1 to 2022Q3 (243 observations). S&P 500 returns, *Ret*, are percentage price changes between the last day of the current quarter and the last day of the same quarter of the previous year, obtained from Yahoo Finance. S&P 500 volatility, *Vol*, is the annualized standard deviation of daily S&P 500 returns in the current quarter, also from from Yahoo Finance. All other variables are from FRED. Corporate bond yields are Moody's Seasoned Bond Yields. Real GDP, *Y*, is seasonally adjusted real gross domestic product. Aggregate consumption, *C*, is seasonally adjusted real personal consumption expenditures.

between the last trading day of the current quarter and of the same quarter a year earlier, obtained from Yahoo Finance.[6] *Vol* is the annualized standard deviation of daily S&P 500 returns in the current quarter.

The rest of the variables are constructed using data from the FRED database of the Federal Reserve Bank of St. Louis. The term spread *Term* is the difference between the 10-year and the 3-month Treasury yields (FRED code GS10 minus FRED code TB3MS). The liquidity spread *Liq* is the difference between the secondary market 3-month Treasury rate and the effective federal funds rate (TB3SMFFM). The credit spread *Cred* is the difference between Moody's Aaa corporate bond yield and the 10-year Treasury yield (AAA10YM). The default spread *Def* is the difference between Moody's Aaa and Baa corporate bond yields (BAA10YM minus AAA10YM). *Term*, *Liq*, *Cred*, and *Def* are quarterly averages of monthly observations.

GDP is seasonally adjusted real gross domestic product (GDPC1). Aggregate consumption C is seasonally adjusted real personal consumption expenditures (PCECC96).

[6] We use the Yahoo Finance series because it is free; it is identical to the one in the Center for Research in Security Prices (CRSP).

III.B The Asset Span

TABLE 2

Properties of principal components of financial variables

	Loadings						
	Ret	Vol	Term	Liq	Cred	Def	Cumulative Variance
PC1	0.33	-0.55	-0.35	-0.02	-0.50	-0.47	34.1%
PC2	0.38	-0.21	0.48	0.65	0.33	-0.23	62.8%
PC3	0.52	-0.27	0.34	-0.32	-0.19	0.63	77.4%
PC4	-0.65	-0.37	0.50	0.13	-0.41	0.04	88.3%

Notes: *PC1–PC4* are the first four principal components of the financial variables listed in Panel A of Table 1. Row i of the "Loadings" column shows the weight of the i^{th} principal component on each of the financial variables. Row i of the "Cumulative variance" column gives the share of the total variance explained jointly by the first i principal components.

Section II introduced the asset span, the space spanned by returns R_i of traded assets. Any empirical estimate necessarily involves only a subset of the true asset span, since including every traded asset is infeasible. Our goal is not to maximize coverage of the asset span, but rather to identify a small number of assets whose span contains sufficient information about the price of risk to estimate a VFCI useful for understanding macro-financial interactions.

To this effect, we use the six financial variables listed in Table 1 as base assets. We choose these variables because they are common components of FCIs that are publicly available for our entire sample. We then compute their principal components (PCs) to form an orthogonal basis for the subspace that these assets span. We retain the first four PCs, which jointly explain 88% of the total variance of the base assets. Table 2 reports the cumulative variance explained by each successive PC and the factor loadings for each base asset.

Using PCs instead of the base assets offers three advantages. First, it allows us to use a subset of PCs without generating omitted variable bias. Because the PCs are orthogonal by construction, omitting the fifth and sixth PCs does not bias the coefficients on the first four. In contrast, using a collection of individual base assets that omits some relevant ones would bias the regression coefficients if the included assets are correlated with the omitted ones, as is typically the case.

Second, the PC approach ensures that our results are less sensitive to the specific choice of base assets. As long as our initial set of assets is broad enough to span the relevant risk factors, the first few PCs are stable. Adding more variables or replacing several of our base assets with common alternatives leaves the first four PCs, and our resulting VFCI, largely unchanged.

Third, using PCs allows us measure the dimension of the subspace relevant for macro-financial interactions. Our baseline results in Section VI show that a four-dimensional subspace is sufficient to identify large causal effects of VFCI shocks on the macroeconomy. A key contribution of our paper is to show that this relevant subspace is low-dimensional and can be spanned effectively using a small set of standard, publicly available financial variables.

We choose to retain four PCs because they explain a large fraction of the variance (88%) while keeping the model parsimonious. Our main results are robust to this choice; Appendices B.1.2 and B.2 show that using three, five, or six PCs (which is equivalent to using all individual base assets), produces a VFCI and causal effects on the macroeconomy that are virtually identical to the baseline specification. The appendices also show that a rich enough set of assets must be used. For example, a VFCI constructed using only stock market returns fails to increase during the onset of the Covid-19 pandemic and generates a more muted response of macroeconomic variables to financial conditions, especially for monetary policy.

III.C VFCI Estimation

To construct the VFCI, we first estimate the following linear regression model with multiplicative heteroskedasticity:

$$\Delta gdp_{t+1} = \theta PC_t + \varepsilon_{t+1}, \tag{24}$$

$$Var_t(\varepsilon_{t+1}) = \exp(2\delta PC_t), \tag{25}$$

where $\Delta gdp_{t+1} := \log(GDP_{t+1}/GDP_t)$ is quarterly real GDP growth, PC_t is the column vector $[1, \mathrm{PC1}_t, \ldots, \mathrm{PC4}_t]'$ with first component equal to 1 followed by the four principal components of financial variables constructed earlier, ε_{t+1} is a zero-mean error term, and θ and δ are 1×5 row vectors of parameters to be estimated. Equations (24) and (25) model, respectively, the first and second conditional moments of real GDP growth as linear functions of the principal components.

We estimate the model via restricted maximum likelihood assuming multiplicative errors in the variance equation $\log(\varepsilon^2_{t+1}) = 2\delta PC_t + \xi_{t+1}$, as in the model.[7] The GDP column in Table 3 shows estimates of $\hat{\theta}$ in Panel A and $\hat{\delta}$ in Panel B. Numbers in parentheses are t-statistics computed using the negative inverse Hessian matrix of the (restricted) log-likelihood as the covariance matrix. The consumption column uses consumption instead of

[7]We use restricted rather than standard maximum likelihood because it is more robust to small-sample bias, especially for the variance component.

TABLE 3

Means and volatilities of GDP and consumption growth rates spanned by financial assets

	Panel A: Conditional Mean		Panel B: Log Conditional Volatility	
	GDP	C	GDP	C
PC1	0.19***	0.15***	-0.22***	-0.17**
	(4.11)	(3.93)	(-3.05)	(-2.44)
PC2	0.16***	0.10***	-0.34***	-0.27***
	(3.35)	(2.65)	(-3.36)	(-2.71)
PC3	0.14**	0.16***	-0.26***	-0.30***
	(2.50)	(3.44)	(-2.86)	(-2.99)
PC4	0.11*	0.07	-0.34***	-0.73***
	(1.82)	(1.29)	(-3.38)	(-7.21)

Notes: This table shows restricted maximum likelihood estimates of the model

$$\Delta y_{t+1} = \theta PC_t + \varepsilon_{t+1}, \quad Var_t(\varepsilon_{t+1}) = \exp(2\delta PC_t),$$

where Δy_{t+1} is either real quarter-over-quarter GDP growth (columns labeled *GDP*) or real quarter-over-quarter aggregate consumption growth (columns labeled *C*); PC_t is a vector with an intercept and the first four principal components of the financial variables in Table 1; ε_{t+1} is a zero-mean error term; θ and δ are vectors of parameters to be estimated.

Panel A shows estimates $\hat{\theta}$ and Panel B shows estimates $\hat{\delta}$ for each of the principal components (the estimate for the intercept is not displayed).

Sample: 1962Q1-2022Q3 (243 quarterly observations).

Numbers in parenthesis are t-statistics computed using the negative inverse Hessian matrix of the restricted log-likelihood as covariance matrix.

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

GDP in equation (24). Table 3 indicates most estimates are significant at the 1% level.

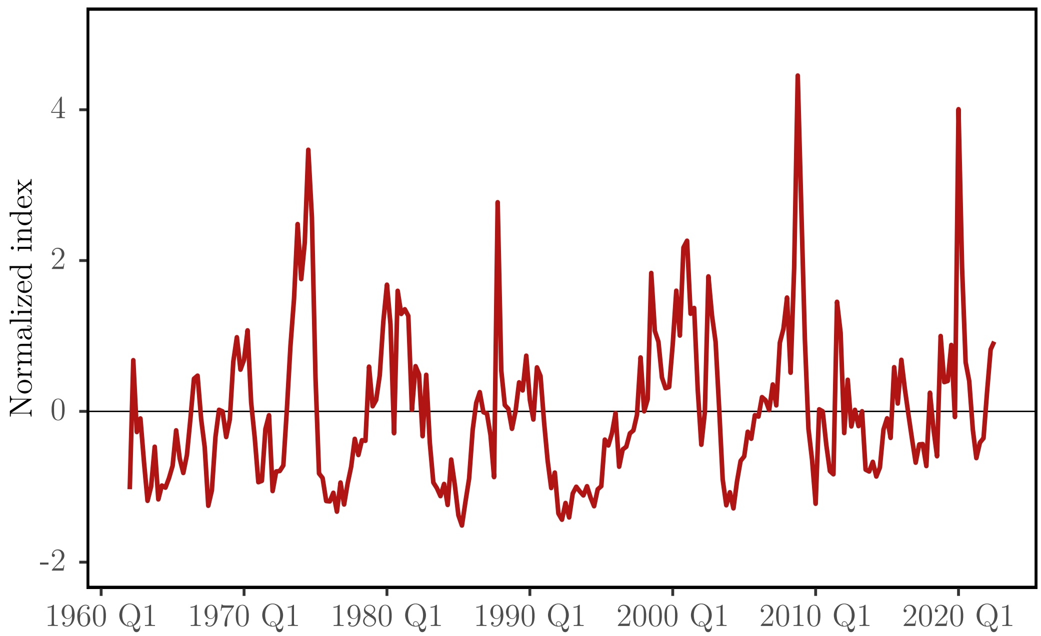
The VFCI—the volatility financial conditions index—is, by definition, the fitted value of the logarithm of the conditional volatility of GDP growth, $\hat{\delta}PC_t$, standardized to have zero mean and unit variance.

III.D VFCI Time Series

Figure 1 plots the time series of the VFCI. The figure shows that the behavior of the VFCI is consistent with the theoretical characterization of the VFCI as a measure of risk compensation and, more broadly, of overall financial conditions. The two largest spikes occur during the global financial crisis in 2008-2009 and the onset of the Covid-19 pandemic in 2020. The VFCI also shows marked increases around other periods of financial distress: the tumultuous 2000-2002 period that included the burst of the dot-com bubble, the 9/11 terrorist attacks, and the corporate scandals of the early 2000's; the 1994 bond market crisis; the "black Monday" stock market crash of 1987; the savings and loans crisis of the early

FIGURE 1

The volatility financial conditions index (VFCI)



Notes: We construct the volatility financial conditions index (VFCI) from

$$\Delta gdp_{t+1} = \theta PC_t + \varepsilon_{t+1}, \quad Var_t(\varepsilon_{t+1}) = \exp(2\delta PC_t),$$

where Δgdp_{t+1} is real quarter-over-quarter GDP growth, PC_t is a vector with an intercept and the first four principal components of the financial variables in Table 1, ε_{t+1} is a zero-mean error, and θ and δ are parameter vectors. We estimate parameters by restricted maximum likelihood and define the VFCI as $\hat{\delta}PC_t$ standardized to have zero mean and unit variance.
Sample: 1962Q1-2022Q3 (243 quarterly observations).

1980s; and the 40% stock market decline in 1973–1974. Furthermore, financial conditions and real aggregate activity are, even if tightly related, not one and the same. The VFCI, being a measure of financial conditions, is thus related to real economic activity but not a pure reflection of it. Indeed, the VFCI can be low during macroeconomic downturns and high during booms. For example, the VFCI does not show unusually high levels during the 1990-1991 recession. Conversely, the VFCI increased steadily between 1996Q2 and 2000Q2 and spiked in 1998Q3 during the collapse of Long-Term Capital Management and the Russian financial crisis, while real economic performance was superb, with year-over-year real GDP growth above 4% every quarter, and unemployment and core inflation declining from 5.5% to 4% and from 3% to 2%, respectively.

Since the sign of each of the PCs used in the estimation of equations (24)-(25) is not identified, the signs of the coefficients $\hat{\theta}$ are arbitrary. We select the signs of the PCs so that the coefficients of $\hat{\theta}$ in Panel A of Table 3 are all positive. Once the signs of the PCs (and therefore of the elements of $\hat{\theta}$) are chosen, the signs of the coefficients $\hat{\delta}$ in the volatility equation are determined by the data and are *not* arbitrary. Comparing the signs of the coefficients $\hat{\delta}$ in Panel A and $\hat{\theta}$ in Panel B shows that all of the PCs induce movements of the conditional mean and the conditional volatility of GDP growth and consumption growth that go in opposite directions. This means that periods with high expected GDP growth are usually accompanied by a low conditional volatility of GDP growth—a low VFCI.

Figure 2 shows a scatter plot of the VFCI against the fitted values for equation (24), $\hat{\theta}PC_t$. There is a tight linear relation with a negative slope, shown by the OLS line in red. In periods of tight financial conditions—when the VFCI is high—the economy tends to be in the bottom right part of the figure. In these low-mean, high-volatility states, negative shocks of a given magnitude translate into much larger declines in GDP than at the top left part of the figure, when volatility is low. Therefore, a high VFCI indicates greater financial amplification of shocks with a higher likelihood of sizable declines in GDP. In contrast, a low VFCI tends to be accompanied by relatively long periods of positive growth that are more resilient to shocks.[8]

IV Comparison With Other Financial Conditions Indices

In this section, we review three popular financial conditions indices (FCIs): the national financial conditions index (NFCI) published by the Federal Reserve Bank of Chicago, the Goldman Sachs financial conditions index (GSFCI), and the VIX index from the Chicago Board Options Exchange.

The NFCI is a weighted average of 105 indicators of financial activity that provides a "weekly update on US financial conditions in money markets, debt and equity markets, and the traditional and shadow banking systems" starting in January 1973 (Brave and Butters (2011)).[9] The weights are obtained through a mixed-frequency dynamic factor analysis and capture the relative importance of historical fluctuations in each of the variables. The NFCI is renormalized each week so that its sample mean and variance are zero and one, respectively, with higher values indicating tighter financial conditions.

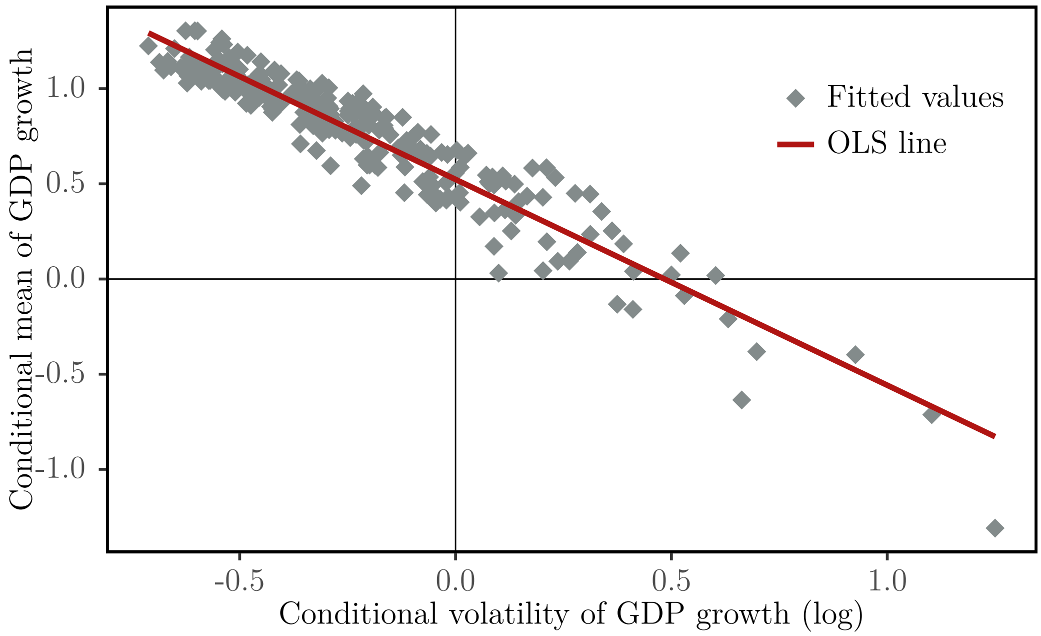
The GSFCI is a weighted average of short-term interest rates, long-term interest rates, a

[8]Using the national financial conditions index published by the Federal Reserve Bank of Chicago instead of the VFCI, Adrian, Boyarchenko, and Giannone (2019) find similar results using a more general estimation approach. Adrian and Duarte (2018) find that the same pattern also holds for the output gap, but not for inflation, and propose a model that rationalizes these empirical patterns by modeling shocks to financial intermediaries that, in equilibrium, affect the conditional mean and volatility of consumption and output growth via changes in the price of risk.

[9]https://www.chicagofed.org/research/data/nfci/current-data

Figure 2

Conditional mean and volatility of GDP growth



Notes: Each gray diamond plots one quarter's observation, with the conditional log volatility of GDP growth on the horizontal axis and the conditional mean of GDP growth on the vertical axis. We construct both from

$$\Delta gdp_{t+1} = \theta PC_t + \varepsilon_{t+1}, \quad Var_t(\varepsilon_{t+1}) = \exp(2\delta PC_t),$$

where Δgdp_{t+1} is real quarter-over-quarter GDP growth, PC_t is a vector with an intercept and the first four principal components of the financial variables in Table 1, ε_{t+1} is a zero-mean error, and θ and δ are parameter vectors. We use restricted maximum likelihood to get estimates $\hat{\delta}$ and $\hat{\theta}$. The conditional log volatility of GDP growth is measured by the $VFCI_t$, constructed as $\hat{\delta}PC_t$ standardized to have zero mean and unit variance. The conditional mean of GDP growth is constructed as $\hat{\theta}PC_t$. The red line shows the OLS fit from regressing $\hat{\theta}PC_t$ on a constant and $VFCI_t$.
Sample: 1962Q1-2022Q3 (243 quarterly observations).

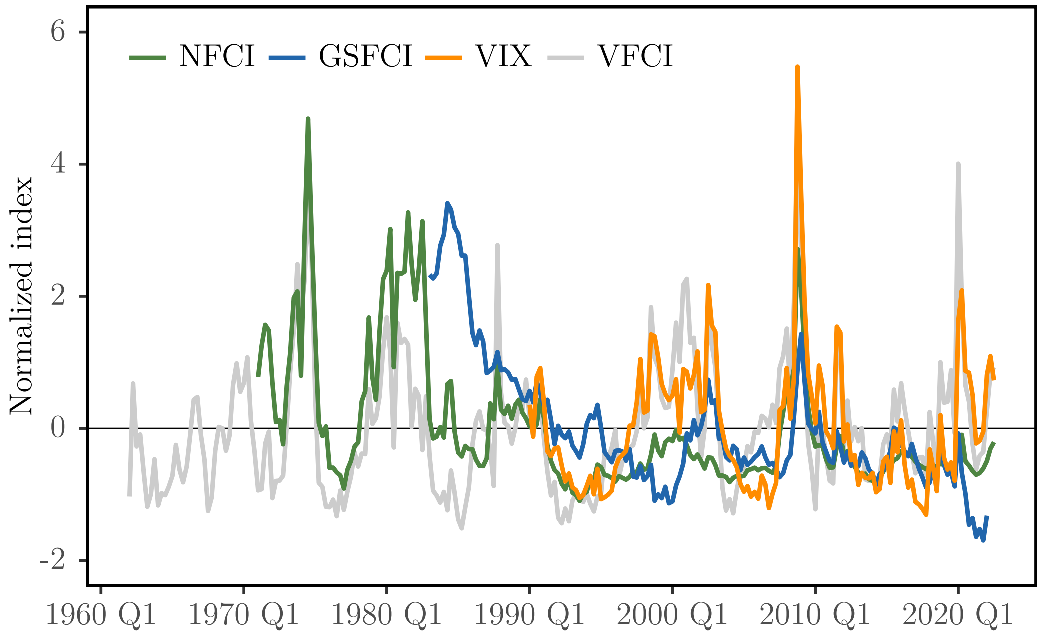
trade-weighted dollar exchange rate, an index of credit spreads, and the ratio of aggregate stock market prices to the 10-year average of earnings per share. It is available on a daily basis starting January 1, 1983. The weight of each variable reflects the impact that a shock to the variable has on GDP growth over the four quarters following the shock, estimated from quarterly OLS regressions starting in 1984.[10] Hence, the GSFCI is designed to be a measure of conditional GDP growth, with higher values of the GSFCI indicating tighter financial conditions and lower expected GDP growth over the following year. The 10-year Treasury yield and the index of credit spreads account for most of the index, with a sum of their weights equal to 84.7%.

The VIX Index is a measure of expected volatility of the aggregate US stock market over

[10] https://www.goldmansachs.com/insights/pages/case-for-financial-conditions-index.html

Figure 3

Comparison of different financial conditions indices (FCIs)



Notes: The green line plots *NFCI*, the Federal Reserve Bank of Chicago's national financial conditions index; the blue line plots *GSFCI*, the Goldman Sachs financial conditions index; the orange line plots *VIX*, the VIX index from the Chicago Board Options Exchange; the gray line plots *VFCI*, the volatility financial conditions index constructed in Section III. All series are standardized to have zero mean and unit variance.

the next 30 days.[11] It is available daily (and at higher frequencies) starting in January 1, 1990. It is derived from real-time, mid-quote prices of S&P 500 Index call and put options. The VIX is often referred to as a "fear gauge" by market participants and can also be viewed as an indicator of the price of risk of the aggregate stock market.

Figure 3 shows the four FCIs. While there is clear comovement, particularly in times of financial stress such as around the 2008 global financial crisis, there are also notable differences. For example, the GSFCI eased more than the other FCIs in 2021; the VIX and the VFCI were more elevated around the tech bubble in the late 1990s; and the NFCI declined faster after the 1982 recession. The VFCI exhibits higher volatility during more recent recessions than alternative FCIs, but lower volatility in times of financial stress before the 2000s such as the period of recession in the early 1980s.

To better understand the relationship of the VFCI with the other FCIs, we run the following linear regression for each FCI:

$$FCI_t = \alpha + \beta PC_t + \varepsilon_t,$$

[11] https://www.cboe.com/tradable_products/vix/

Table 4

Regressions of financial conditions indices on principal components of financial variables

	VFCI	NFCI	GSFCI	VIX
PC1	−0.11***	−0.16***	−0.33***	−3.73***
PC2	−0.17***	−0.61***	−1.07***	−2.06***
PC3	−0.13***	0.30***	1.72***	0.82
PC4	−0.17***	0.00	0.92***	−2.92***
N	243	207	157	131
R^2	1.00	0.81	0.73	0.80

Notes: This table shows OLS estimates $\hat{\beta}$ for regressions

$$FCI_t = \alpha + \beta PC_t + \varepsilon_t,$$

where FCI_t is a financial conditions index, PC_t is a vector with the first four principal components of the financial variables in Table 1, ε_t is a zero-mean error, and α and β are parameters to be estimated. Each column corresponds to a single regression that uses a different FCI_t: column *VFCI* uses the volatility financial conditions index constructed in Section III, column *NFCI* uses the Federal Reserve Bank of Chicago's national financial conditions index, column *GSFCI* uses the Goldman Sachs financial conditions index, and column *VIX* uses the VIX index from the Chicago Board Options Exchange. All series are quarterly and use the longest sample available for each FCI.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

where *FCI* is one of {*VFCI*, *NFCI*, *GSFCI*, *VIX*}, the intercept α and the 1×4 vector β are parameters to be estimated, PC_t is a column vector with the first four principal components of the six financial variables listed in Table 1, and ε_t is an error term. Table 4 shows the estimated coefficients $\hat{\beta}$. The PCs have high explanatory power for all the FCIs, with R^2s between 70 and 80 percent and all but two coefficients statistically significant at the 1% level. We view these results as evidence that the four PCs are informative about financial conditions and that the price of risk is tightly connected to financial conditions. In addition, none of the FCIs are well explained by just one or two PCs. Three or four PCs are needed to achieve the high R^2s shown in the table. Increasing the number of principal components beyond four adds little additional information.[12]

V The VFCI and Common Risk Premia

In this section, we provide evidence that the VFCI predicts risk premia of equities, Treasuries, and corporate bonds, and that it does so better than the other FCIs discussed in Section IV. We use the S&P 500 excess CAPE yield (ECY) of Shiller (2000) as a measure of risk premia for equities, the 10-year term premium (TP) of Adrian, Crump, and Moench (2013)

[12] See Table B.1 in the appendix.

for Treasuries, and the GZ spread of Gilchrist, Yankov, and Zakrajšek (2009) for corporate bonds. The three risk premia measures are in units of annualized percentage points and all FCIs are standardized to have zero mean and unit variance.

Panel A of Table 5 displays estimated coefficients from linear regressions of ECY on its lag and the different FCIs we consider. Numbers in parentheses are *t*-statistics computed using Newey and West (1987) standard errors with 4 lags. Columns (1) through (4) show results from regressions that include only one FCI as regressor, while column (5) includes all four FCIs as regressors. Column (1) shows that the VFCI has a coefficient of 0.13 that is significant at the 1% level when it is the only FCI included as predictor. A one-standard deviation increase in the VFCI, which represents tighter financial conditions, predicts an increase of 13 basis points in next quarter's (annualized) equity risk premium. The magnitude of the coefficient means that the VFCI predicts 22% of one-quarter-ahead ECY variation not explained by lags of ECY itself, a sizable effect. Columns (2) shows that the NFCI has a coefficient significant at the 5% level, while columns (3) and (4) show small and insignificant coefficients for GSFCI and VIX. Column (5) shows that when all four FCIs are included as predictors, the coefficient on the VFCI is the only significant one and now accounts for 31% of the one-period-ahead variance of ECY not already accounted for by the current level of ECY itself.

Panels B and C show analogous results for Treasury and corporate bond risk premia, respectively. In both panels, the VFCI has a coefficient that is significant at the 1% level when included individually in column (1) and jointly with the rest of the FCIs in column (5). As individual predictors, the other FCIs have coefficients that are relatively small in magnitude and not always significant. None of the FCIs have significant coefficients when included jointly with the VFCI. In terms of magnitude, the VFCI explains 26% and 52% of the variance in Treasury and bond premia not explained by lags of the risk premium measures themselves, respectively, when the VFCI is the only FCI included in the regression. When the VFCI is included jointly with the other FCIs, the numbers increase to 48% and 60%.

We conclude that the VFCI predicts risk premia for three major asset classes when used by itself, and the only FCI with predictive power when considered jointly with the other FCI.

Table 5

Financial conditions indices' ability to predict risk premia

Panel A: Stocks

	Excess PE Ratio (ECY)				
	(1)	(2)	(3)	(4)	(5)
Lag ECY	0.97***	0.93***	0.91***	0.94***	0.94***
	(45.1)	(31.5)	(30.9)	(27.8)	(32.1)
Lag VFCI	0.13***				0.18**
	(2.8)				(2.5)
Lag NFCI		0.19**			0.04
		(2.4)			(0.2)
Lag GSFCI			0.05		0.10
			(1.1)		(0.9)
Lag VIX				0.05	-0.12
				(1.1)	(-1.3)
N	242	206	157	130	129
R^2	0.95	0.95	0.93	0.90	0.91

Panel B: Treasuries

	Term Premium (TP)				
	(1)	(2)	(3)	(4)	(5)
Lag TP	0.98***	0.96***	0.95***	0.96***	1.02***
	(68.4)	(55.9)	(36.2)	(48.3)	(33.0)
Lag VFCI	0.08***				0.15***
	(4.9)				(2.9)
Lag NFCI		0.07***			0.05
		(3.7)			(0.9)
Lag GSFCI			0.03		-0.03
			(0.8)		(-0.4)
Lag VIX				0.06***	-0.08
				(2.9)	(-1.4)
N	242	206	157	130	129
R^2	0.95	0.94	0.94	0.94	0.94

Table 5 Continued

Panel C: Corporate Bonds

	Credit Spread (GZ)				
	(1)	(2)	(3)	(4)	(5)
Lag GZ	0.82***	0.90***	0.89***	0.68***	0.60***
	(14.0)	(13.7)	(12.8)	(6.4)	(4.4)
Lag VFCI	0.21***				0.24***
	(2.9)				(4.7)
Lag NFCI		0.08			0.31
		(1.6)			(1.3)
Lag GSFCI			-0.00		0.11
			(-0.1)		(1.0)
Lag VIX				0.33**	0.06
				(2.6)	(0.8)
N	197	197	157	130	129
R^2	0.86	0.82	0.79	0.85	0.89

Notes: This table shows OLS estimates $\hat{b}$ and $\hat{c}$ for regressions $RP_t = a + bRP_{t-1} + cFCI_{t-1} + \varepsilon_t$, where RP_t is a measure of risk premia, FCI_{t-1} is the first lag of either a financial conditions index or a vector of financial condition indices, ε_t is a zero-mean error, and a, b and c are parameters to be estimated. As measures of risk premia RP_t, Panel A uses ECY, the excess cyclically adjusted PE ratio of Shiller (2000), which captures equity premia of the aggregate stock market; Panel B uses uses TP, the 10-year Treasury term premium of Adrian, Crump, and Moench (2013), which captures risk premia in government bonds; and Panel C GZ, the corporate bond credit spread index of Gilchrist and Zakrajšek (2012), which captures risk premia in corporate bonds. The three risk premia measures are in units of annualized percentage points and all FCIs are standardized to have zero mean and unit variance.

Numbers in parenthesis are t-statistics, computed using Newey-West standard errors with 4 lags. All series are quarterly.

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

VI The VFCI and Macro-Financial Dynamics

In this section, we use a structural vector auto-regressive model (SVAR) to show that identified structural shocks to the VFCI have large and persistent effects on output and monetary policy. Positive one-standard-deviation VFCI shocks that reflect higher risk premia and tighter financial conditions lead to economically large reductions in real GDP and the federal funds rate, with corresponding 90 percent error bands that do not include zero for more than 20 quarters. We interpret this result as evidence that shocks to the VFCI are an important causal source of fluctuations for macroeconomic variables. We also show that a positive one-standard-deviation identified shock to the federal funds rate leads to a higher VFCI for ten quarters, consistent with the extensive literature documenting that monetary policy shocks induce significant changes in risk premia across many asset classes.[13]

VI.A The Structural Vector Auto-Regression

The SVAR is

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + C + \varepsilon_t, \tag{26}$$

where y_t is an $n \times 1$ vector of observed endogenous variables, B_0 is an $n \times n$ constant matrix that determines the simultaneous relationships among the n variables, B_j are $n \times n$ constant matrices of coefficients for each lag $j = 1, \ldots, p$, C is an $n \times 1$ vector of constants and ε_t is an $n \times 1$ vector of mean zero structural shocks that are independent across equations and time.

The reduced form of equation (26) is

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + D + u_t, \tag{27}$$

where A_j are constant coefficient matrices of the same size as the corresponding B_j, D is a vector of constants of the same size as C, and u_t is an $n \times 1$ vector of mean-zero reduced-form residuals.

Comparing equations (26) and (27) shows that the structural shocks ε_t are a linear combination of the reduced-form residuals u_t:

$$\varepsilon_t = B_0 u_t. \tag{28}$$

VI.B VAR Variables

The $n = 4$ endogenous variables in our VAR specification are

$$y_t = [\mathit{logGDP}_t,\ \mathit{logP}_t,\ \mathit{fedfunds}_t,\ \mathit{VFCI}_t],$$

where *logGDP* is the logarithm of real GDP, *logP* is the logarithm the aggregate price level, *fedfunds* is the federal funds rate, and *VFCI* is the volatility financial conditions index.

VAR series are quarterly from 1962Q1 to 2022Q3, matching the sample used in the construction of the VFCI. All variables other than *VFCI* are constructed using FRED data. For *logY*, we use the same real GDP series that we use in the construction of the VFCI (FRED code GDPC1). For *logP*, we use the personal consumption expenditures price index that excludes food and energy (PCEPILFE). The variable *fedfunds* is the average over the

[13]See the review article Drechsler, Savov, and Schnabl (2018) and citations therein.

current quarter of the monthly-frequency federal funds effective rate (FEDFUNDS). The variable *VFCI* is constructed as in Section III. Table 6 shows summary statistics.

TABLE 6
Summary statistics for VAR variables

	Mean	SD	Min	Max
logGDP	9.23	0.51	8.23	10.00
logP	3.93	0.62	2.79	4.75
fedfunds	4.87	3.72	0.06	17.78
VFCI	0.00	1.00	-1.51	4.45

Notes: All series are quarterly from 1962Q1 to 2022Q3 (243 observations). Variable *logGDP* is the logarithm of real GDP; *logP* is the logarithm of the price level measured by the personal consumption expenditures price index that excludes food and energy; *fedfunds* is the federal funds rate; *VFCI* is the volatility financial conditions index constructed in Section III.C. The variables *logGDP*, *logP*, and *fedfunds* are from FRED.

VI.C Identification Through Heteroskedasticity

We partition the entire sample period $t = 1, \ldots, T$ into M exogenously specified subperiods indexed by $m = 1, \ldots, M$, and let $m(t)$ be the function that maps time periods to their corresponding subperiod. We denote the diagonal variance-covariance matrix of the structural shocks by $\Lambda_{m(t)}$ and its i^{th} diagonal element by $\lambda_{i,m(t)}$. We assume $\Lambda_{m(t)}$ is constant within each subperiod but allow it to be different across subperiods. We denote by Λ_m the constant diagonal variance-covariance matrix for subperiod m, and its diagonal elements by $\lambda_{i,m}$. We impose the normalization that the variance of each structural shock averages to one across subperiods:

$$\frac{1}{M}\sum_{m=1}^{M}\lambda_{i,m} = 1. \tag{29}$$

By equation (28), the variance-covariance matrix of the reduced form residuals u_t, which we denote by $\Sigma_{m(t)}$, is related to $\Lambda_{m(t)}$ by

$$B_0 \Sigma_{m(t)} B_0' = \Lambda_{m(t)}. \tag{30}$$

Since B_0 is constant, $\Sigma_{m(t)}$ is constant across subperiods, just as $\Lambda_{m(t)}$.

When the variances of each pair of structural shocks in the SVAR differ for at least one subperiod, the normalization conditions in (29) together with the covariance restrictions in (30) locally identify the n^2 parameters of B_0 up to row ordering and row sign changes.[14]

[14]See, for example, Lanne, Lütkepohl, and Maciejowska (2010).

Identification "up to row ordering" means that even though each of the n estimated structural shocks is the structural shock to one of the n VAR variables, there is no identification regarding which shock corresponds to which variable, a consequence of the purely statistical nature of identification through heteroskedasticity. In Section VI.E, we assign each identified structural shock to one of the VAR variables using impulse response functions and economic intuition.

The first two columns of Table 7 show the volatility subperiods that we use. The most recent subperiod, which we label "Covid-19 pandemic," starts in 2020Q1 and ends in 2022Q3. In addition to the pandemic, this subperiod has high inflation and includes the war in Ukraine. The earliest subperiod, which we label "Lowflation," covers 1962Q1 to 1969Q4 and covers a period of high economic growth before the economic difficulties of the 1970s. The rest of the subperiods are identical to those determined by Brunnermeier et al. (2021).

Table 7

Volatility subperiods used for identification through heteroskedasticity

		Relative Variance			
Subperiod	Description	logGDP	logP	fedfunds	VFCI
1962Q1-1969Q4	Lowflation	1.13	0.17	0.38	0.62
1970Q1-1979Q3	Oil crisis, stagflation	1.67	1.08	2.13	0.62
1979Q4-1982Q4	Volcker disinflation	1.65	1.47	3.02	0.88
1983Q1-1989Q4	S&L crisis defaults	0.42	0.70	0.74	0.91
1990Q1-2007Q4	Great Moderation	0.46	0.14	0.29	0.83
2008Q1-2010Q4	Financial crisis	0.80	0.75	0.46	1.65
2011Q1-2019Q4	ZLB, post-crisis recovery	0.33	0.14	0.07	1.33
2020Q1-2022Q3	Covid-19 pandemic	1.53	3.01	0.58	1.10

Notes: We take the six volatility subperiods from Brunnermeier et al. (2021) and add a 2020Q1-2022Q3 subperiod labeled "Covid-19 pandemic," and a 1962Q1-1969Q4 subperiod labeled "Lowflation." The third column, labeled "Relative Variance," shows the variance of each of the VAR variables in each subperiod relative to the average variance across subperiods, which we normalize to 1. The reported relative variances are posterior distribution medians from our Bayesian estimation procedure in Section VI.D.

The last column of Table 7, labeled "Relative Variance," reports posterior distribution medians for the variance of structural shocks in each of the different volatility subperiods. Given the normalization in equation (29), the numbers in Table 7 are volatility levels relative to the average of 1 across subperiods. One important takeaway is that the variance of the structural shocks differ substantially across volatility subperiods, supporting the identifying assumption of heteroskedasticity.

We assume that the structural shocks, ε_t, are drawn from a Student-t distribution. This assumption provides a better fit to the data than a Gaussian distribution primarily because the fat tails of the Student-t can accommodate large, isolated shocks. For instance, the federal funds rate increased by approximately 600 basis points in 1980:Q4 (a 6.8 standard deviation event), and the VFCI increased by more than four standard deviations in 2020:Q1. Such events are highly improbable under a normality assumption.

Table 8 reports a measure of fit for alternative VAR specifications, the log marginal data densities (MDD). The MDD for our specification with Student-t shocks is in the first row, whereas the identical model with Gaussian shocks is in the third row. The difference in MDD of 189 (=1,513−1,324) is substantial; since the exponentiated difference in MDD can be interpreted as a Bayes factor, this result implies that if the two specifications have equal prior probabilities, the posterior probability of the Gaussian model is effectively zero. Beyond model fit, Appendix B.2.6 reports that Mardia and Shapiro-Wilk tests reject normality. The MDD values in Table 8 also favor our baseline specification over homoskedasticity (second row, MDD = 1,393) and time-varying VAR coefficients (fourth row, MDD = 958).

Table 8
Marginal Data Densities (MDD) for Different VAR Specifications

	Variation across subperiods in...	Distribution	MDD
(baseline)	Structural shock variances	t	1,513
	Nothing	Gaussian	1,393
	Structural shock variances	Gaussian	1,324
	VAR coefficients A_j	Gaussian	958

Notes: Taking differences of the reported MDD values gives the log posterior odds, assuming equal prior weights on each model. Larger MDD values indicate a more preferred specification. The first row is for our baseline specification that has different variances of the structural shocks across subperiods, t-distributed errors, and constant VAR coefficients A_j across subperiods. The second row is for a model with Gaussian shocks, and shock variances and VAR coefficients that are constant across subperiods. The third row also has Gaussian shocks and constant VAR coefficients, but allows for subperiod-specific shock variances. The fourth row fits a separate unrestricted reduced-form model to each subperiod using Gaussian shocks, thus allowing for time variation in both the shock variances and the VAR coefficients.

Using t-distributed shocks provides two additional benefits. First, they provide stronger identification than Gaussian shocks.[15] Second, they deliver robust inference under misspecification.[16]

[15] See, for example, Karlsson, Mazur, and Nguyen (2023), Lanne and Luoto (2020), and Lanne, Meitz, and Saikkonen (2017).

[16] Gouriéroux, Monfort, and Renne (2017) and Sims (2020) show that likelihood-based estimation with t-distributed shocks remains consistent when the true shocks are fat-tailed and symmetric. If volatilities vary within subperiods or subperiod boundaries are misspecified, identification and consistency of coefficient estimates are preserved provided sufficient cross-subperiod variation exists. The impulse response shapes

VI.D Bayesian Estimation

We estimate the SVAR using the same Bayesian procedure as in Brunnermeier et al. (2021), except that we modify the parameters of the Minnesota prior placed on the reduced form coefficients A_j by using a "tightness" of 3 (instead of 5) and a "decay" of 0.5 (instead of 1) to account for the quarterly frequency of our data (rather than the monthly frequency in Brunnermeier et al. (2021)).[17] We use a Gibbs sampling method with 10,000 draws to sample from the posterior distribution of all parameters, which also provides posterior distributions for impulse response functions.

VI.E Impulse Responses and Interpretation of Shocks

Figure 4, shows impulse responses of the four SVAR variables to an increase of one standard deviation in each of the four identified shocks, where the shocks are drawn from a Student-t distribution with 2.5 degrees of freedom, which is the median value across posterior draws. In this and all subsequent IRF figures, IRFs are also medians across posterior draws (the black line), with 68 percent (darker shade) and 90 percent (lighter shade) highest posterior density regions displayed as error bands. Given the normalization in equation (29), the IRFs are "averages" across the volatility subperiods. Since B_0, B_j and C are constant (the same for all subperiods), the impact effect of shocks and the dynamic relations among the variables in y_t are time-invariant; impulse responses have the same shape across subperiods (and the same shape as the averages in Figure 4), although with a potentially different scale determined by the subperiod-specific variances of shocks from Table 7.

In Figure 4, we have assigned labels to the identified shocks, which, as mentioned earlier, is not provided by the purely statistical nature of the identification strategy. We now discuss the reasoning behind the labeling together with the economic intuition and implications of the results.

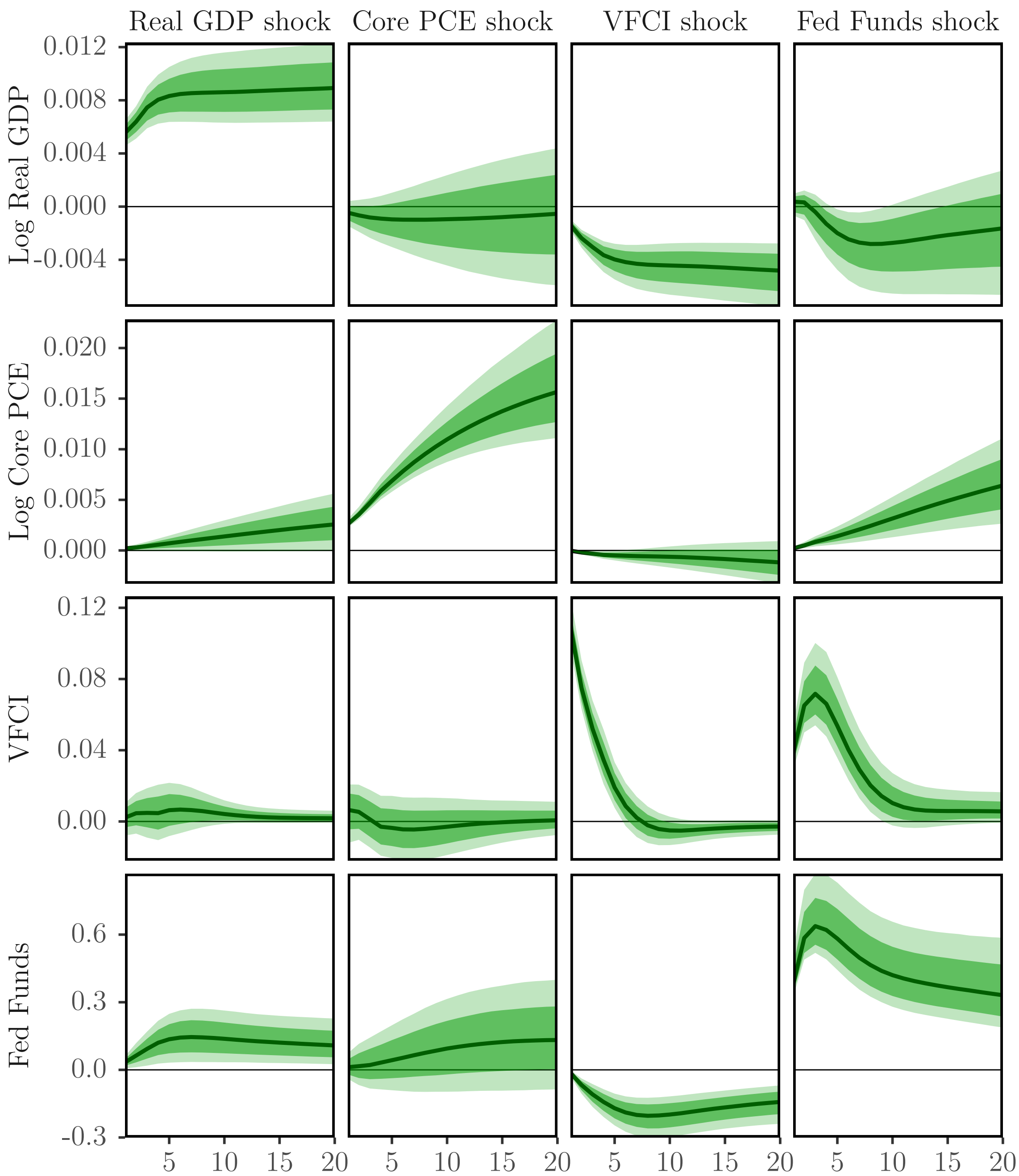
The leftmost column shows impulse responses to what we interpret as a real GDP shock or, more broadly, a demand shock. There are several reasons. First, despite not having imposed any short-term zero restrictions, the GDP shock has little effect on the VFCI and the price level on impact, against the interpretation of this shock as a VFCI or price-level shock. Second, although there is some initial response of the federal funds rate, it is in the same direction as the response of GDP. If the shock were a federal funds rate shock, one would expect higher federal funds rate—tighter monetary policy—to be accompanied

remain correct, though posterior credible sets are wider and variance decompositions may be potentially distorted.

[17] We use the Minnesota prior implemented through dummy observations and the same notion of "tightness" and "decay" as in Sims and Zha (1998).

Figure 4

Impulse response functions with heteroskedasticity-identified shocks



Notes: Impulse response functions (IRF) using structural shocks identified through heteroskedasticity with 68 percent (dark shade) and 90 percent (light shade) posterior uncertainty regions. IRF scaled to an average subperiod (variance of shocks is the average across subperiods). VAR estimated using quarterly data from 1962Q1 to 2022Q3.

by lower GDP. Third, the pattern of a high GDP throughout, prices that rise steadily over time, and a higher federal funds rate, are all consistent with a demand shock in a sticky price economy in which the central bank responds with higher interest rates when it observes a positive shock to output and inflation. Financial conditions as measured by the VFCI show neither a contemporaneous response to the GDP shock nor a large response in any of the 20 quarters shown in the figure. While we expect that, ceteris paribus, higher economic activity would loosen financial conditions—at least eventually—we also expect (and show more explicitly later) that higher interest rates tighten financial conditions. In the impulse responses of the figure, these two opposing forces approximately offset each other, resulting in a muted response of the VFCI. If anything, financial conditions can be seen to tighten slightly at the same time that the federal funds rate peaks.

The second column of Figure 4 shows responses to what we label an inflation shock. The main reason for this label is that the price level is the only variable that shows a large response to the shock. In addition, despite the large error bands, GDP goes down and the federal funds rate increases persistently, consistent with the interpretation and the reasoning in the last paragraph. The VFCI is essentially unresponsive to inflation, consistent with previous literature (see footnote 8).

The third column of Figure 4 shows responses to what we label a VFCI shock. This shock leads to positive response of the VFCI at impact that decays exponentially for around 7 quarters and then becomes essentially zero, a prototypical response to own-shocks for autoregressive VAR variables. Real GDP decreases by around 0.4 percent for over 6 quarters, and remains persistently low, reaching close to a 0.5 percent decline over the 20 quarters plotted, with no signs of reversal. The price level declines slightly, escaping the 68 error band (but not the 90 percent band) only by the end of the 20 quarters, reaffirming the weak connection between inflation and financial conditions in our sample. Monetary policy responds to the VFCI shock by lowering the federal funds rate by around 20 basis points (in annualized terms) at the peak of the IRF that occurs around 7 quarters after the shock. The one standard deviation shock to the VFCI is therefore comparable in magnitude to a 25 basis point change in the federal fund rate. Since the SVAR is linear, a negative shock to the VFCI would result in the same impulse responses but with signs reversed. A negative VFCI shock would lead to a response of monetary policy that can be understood as "leaning against the wind" of looser financial conditions. Overall, the delayed buildup in the responses of output, the price level and the Federal funds rate points to a sluggish propagation of financial conditions to the real economy.

The right-most column of Figure 4 displays responses to the shock that we label a federal funds shock or, more broadly, a monetary policy shock. The contractionary monetary policy shocks lead to a transitory yet persistent decline in output. The response of the price level exhibits the "price puzzle" common to small VAR specifications. The VFCI responds strongly to the federal funds shock. The median estimated response is around 5 percent upon impact, increasing to around 7.5 percent over 3 quarters before dissipating after 8 quarters.

By construction, the VFCI is a measure of the price of risk. In Section IV, we argued it was also a financial conditions index. In both of these roles, the response of the VFCI to monetary policy shocks is consistent with empirical evidence in the literature. A higher price of risk in response to tighter monetary policy is one of the main takeaways of the vast literature originating in Kuttner (2001) and Gürkaynak, Sack, and Swanson (2004), with more recent studies in Nakamura and Steinsson (2018), Caldara and Herbst (2019), Cieslak and Vissing-Jorgensen (2020), among other. The tightening of financial conditions after a positive federal funds rate shock is documented in Gertler and Karadi (2015), Brunnermeier et al. (2021), and others.

VII Alternative Identification Strategies

In this section, we show that the impulse response functions obtained using identification through heteroskedasticity are very similar to those obtained using four other plausible identification strategies. We give a brief description of each strategy and provide more details in Appendix B.3.

Instrumental Variables

Our first alternative identification strategy is to use instrumental variables (also referred to as proxy variables, and as external instruments) for shocks to GDP, the federal funds rate, and the VFCI.

For the GDP instrument, we use the growth news shock constructed in Cieslak and Pang (2021), available between 1983Q1 and 2022Q3.[18] We convert the instrument to quarterly frequency by averaging the daily observations over the current quarter.

For the federal funds rate instrument, we splice two data series. Between 1969Q1 and 1994Q4, we use the monetary policy shock constructed in Romer and Romer (2004) using the "narrative approach," updated by Wieland and Yang (2020).[19] From 1995Q1 to 2022Q3, we use the monetary policy shock constructed in Nakamura and Steinsson (2018) and convert the

[18] We thank the authors, who provided the updated series.

[19] Downloaded from https://www.openicpsr.org/openicpsr/project/135741/version/V1/view on February 12, 2023. We use the series 'resid_full' in the file 'RR_monetary_shock_quarterly.dta', divided by 10.

monthly series to quarterly frequency by averaging observations over the current quarter.[20]

The external instrument for the VFCI shock treats the VFCI as an exogenous process. This implies that all reduced form innovations to the VFCI are structural VFCI shocks. While this is a strong assumption, we find comparing the results to our baseline informative.

Local Projections with Instrumental Variables

Our second alternative identification strategy is the instrumental variable local-projection (LP) method proposed by Jordà, Schularick, and Taylor (2015) using the same instruments as in the previous strategy. Despite the asymptotic equivalence between VAR-based and local-projection-based IRFs (Plagborg-Møller and Wolf (2021)), they can differ in finite samples. Whether VAR or LP should be preferred depends on the setting and the researcher's goals. For example, VAR and LP have different bias-variance trade-offs (Li, Plagborg-Møller, and Wolf (2024)) and different robustness properties (Montiel Olea and Plagborg-Møller (2021)).

Recursive VAR

Third, we estimate a recursive VAR in which the federal funds rate and the VFCI are ordered last, reflecting the assumption that they respond contemporaneously to shocks to GDP and the price level, but that GDP and the price level do not respond contemporaneously to shocks to the federal funds rate or to the VFCI. We show results with the federal funds rate ordered third and the VFCI ordered last, although results are essentially identical if we reverse this order.

Sign restrictions

As our last alternative identification strategy, we impose sign restrictions on the impact effect of monetary policy, real GDP, and VFCI shocks. We assume that the price level and GDP must respond negatively to a positive monetary policy shock, that the federal funds rate responds positively to a real GDP shock, and that real GDP and the federal funds must respond negatively to a VFCI shock.

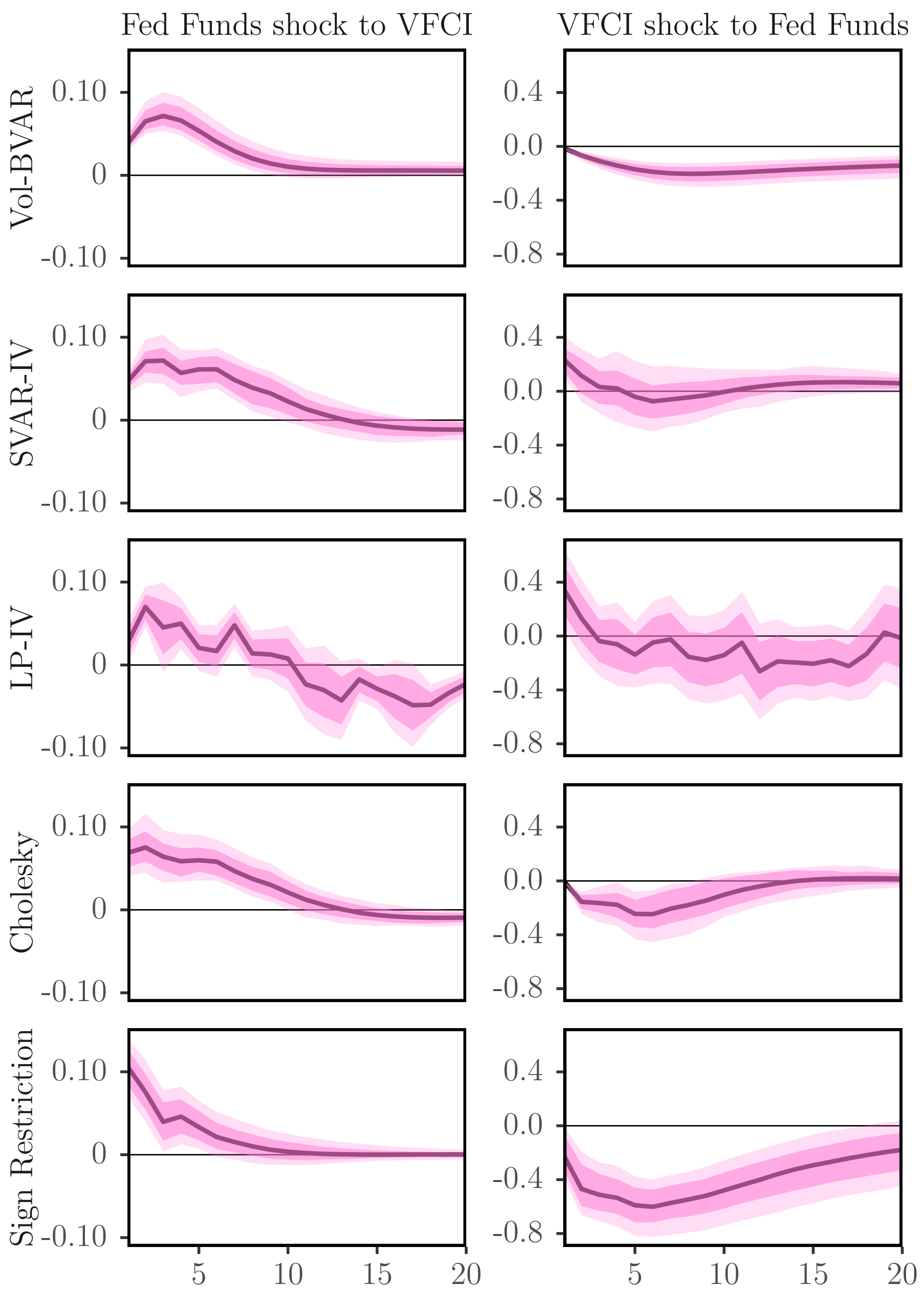
VII.A A Comparison of All Identification strategies

Figures 5 and 6 compare IRFs for all the identification strategies. Figure 5 focuses on the interactions between the VFCI and monetary policy while Figure 6 looks at VFCI and GDP. In both figures, each row corresponds to a different identification strategy. The first

[20] We thank Emi Nakamura and Miguel Acosta for providing an updated data series.

Figure 5

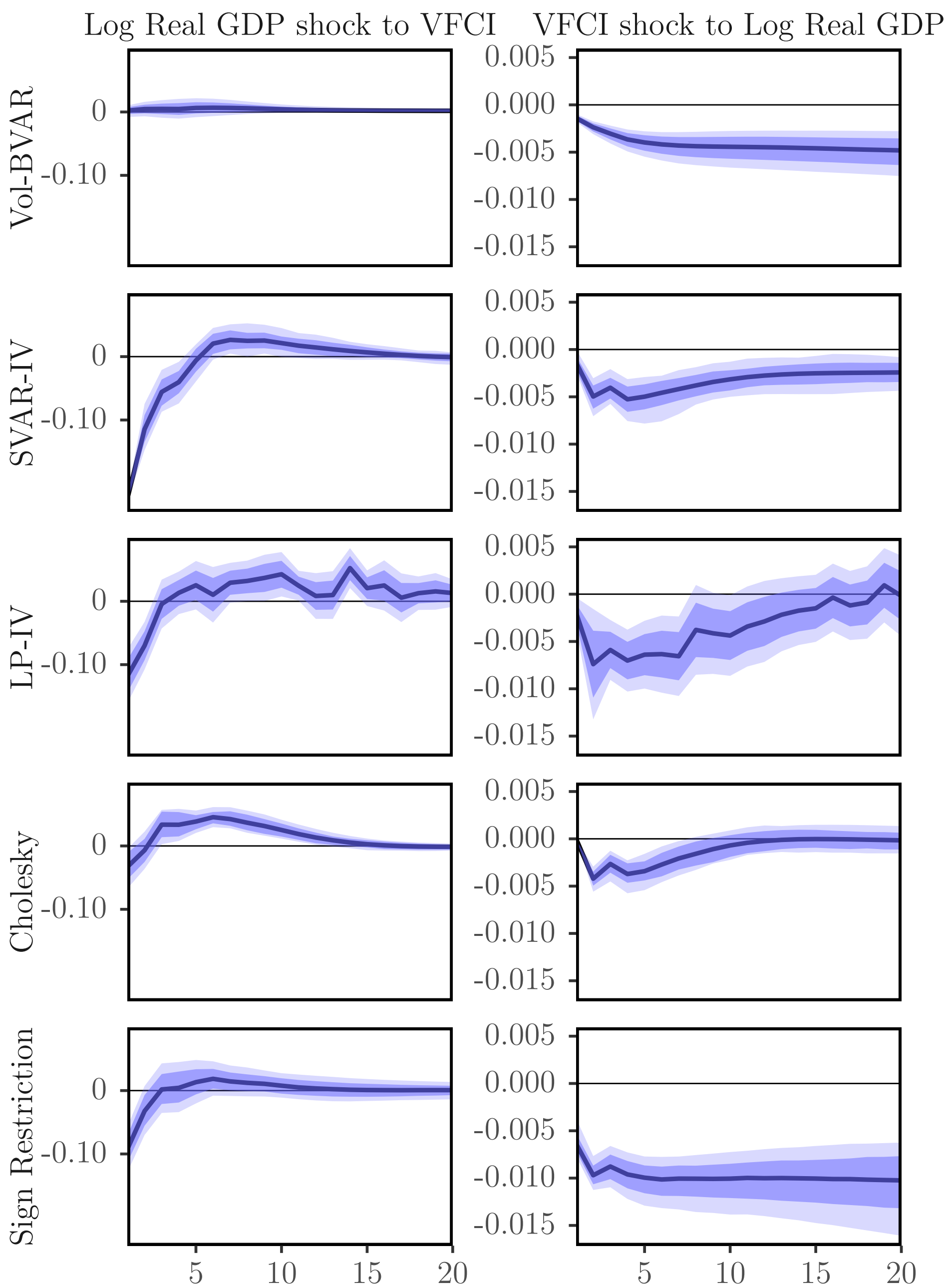
Comparison of IRFs across identification strategies: VFCI and monetary policy



Notes: IRF of the VFCI to a federal funds rate shock (left column) and of the federal funds rate to a VFCI shock (right column) for five different identification strategies (across rows), with 68 percent (dark shade) and 90 percent (light shade) posterior uncertainty regions.

Figure 6

Comparison of IRFs across identification strategies: VFCI and output



Notes: IRF of the VFCI to a GDP shock (left column) and of GDP to a VFCI shock (right column) for five different identification strategies (across rows), with 68 percent (dark shade) and 90 percent (light shade) posterior uncertainty regions.

row, labeled "Vol-SVAR," shows impulse responses for our baseline SVAR specification in which shocks are identified by heteroskedasticity. These impulse responses are the same impulse responses already shown in Figure 4, which we include here for ease of comparison. The second row, labeled "SVAR-IV," uses instrumental variables to identify shocks. The third row, labeled "LP-IV" uses local projections and the same instruments as the SVAR-IV specification. The fourth row, labeled "Cholesky," shows IRF for the recursive VAR. The fifth row shows IRFs for the sign-restricted SVAR.

The IRFs in the heteroskedasticity-identified and the sign-restricted SVARs are estimated using Bayesian methods and correspond to medians across 10,000 draws from the posterior distribution. The other three identification strategies are estimated using frequentist methods. In each instance, the shock corresponds to a one standard deviation increase, and we plot the evolution of the variables over 20 quarters.

VIII Robustness

We assess robustness through several variations in our model and estimation procedures.

Appendix B.1 shows that the VFCI remains materially unchanged when we: replace GDP with consumption; use all financial variables from Table 1 rather than their first four principal components; use three or five principal components instead of four; include SDF news; or add four lags of log GDP.

Appendix B.2 shows that the impulse responses in Figure 4 remain robust when we: use the alternative VFCI constructions from Appendix B.1; use stationary variables; replace VFCI with exp(VFCI); expand MCMC draws from 10,000 to 1,000,000; or vary Minnesota prior parameters (tightness of 1, 2, or 5 instead of 3; decay of 0.3 or 0.7 instead of 0.5).

Appendix B.2.4 confirms that adding a second financial variable (GZ spread, excess CAPE yield, 3-month Libor-Treasury spread, or NFCI) leaves VFCI shock dynamics unchanged. Following Brunnermeier et al. (2021), Appendix B.2.5 estimates a BVAR with VFCI, GZ spread, and TED rate, showing that the VFCI shock remains unchanged and drives GZ spread movements.

VIII.A Estimating the VFCI within the VAR

Until now, we have first estimated the VFCI and then included it as an additional variable in the VAR. However, the VAR implies its own path for the volatility of GDP growth which may or may not be consistent with the one estimated in the construction of the VFCI. In Appendix B.2.7, we show that estimating the VFCI using the implied volatility of GDP growth from the same VAR that we use in Section VI generates essentially the same VFCI and impulse responses. Estimating the VFCI in this multi-variate setting also generalizes the theoretical framework from Section II by allowing the utility function of the representative agent to depend on variables other than consumption (as anticipated in footnote 2).

IX Related Literature

Our empirical methodology is based on Brunnermeier et al. (2021), who also use a heteroskedasticity identified SVAR to link financial shocks to the macroeconomy. While they use the GZ and TED spreads, we use the VFCI. The VFCI contains information beyond these spreads; adding them to our SVAR leaves impulse responses unchanged (Appendix B.2).

The VFCI is rooted in consumption-based asset pricing. Rather than attempting to recover the entire SDF—a task that is notoriously difficult (Jackwerth (2000) and Chabi-Yo, Garcia, and Renault (2008)) and requires high-dimensional data such as the full cross-section of options (Aït-Sahalia and Lo (2000) and Rosenberg and Engle (2002))—we project the SDF's conditional volatility onto a small set of traded assets. The resulting implementation is transparent and uses standard methods and data. Although this parsimony suffices for the questions we ask, the omitted components may matter for more demanding asset pricing applications, which we leave to future research.

Our work relates to the ARCH/GARCH approach, which models volatility statistically as a function of its own history, without connecting to economically motivated state variables (Engle (1982), Bollerslev (1986)). The VFCI estimation method is closest to a GARCH-X, but with external variables that arise from theory and enter not only the conditional variance, but also the conditional mean. Volatility that depends on endogenous financial conditions also departs from modern leading consumption-based asset pricing models, where volatility dynamics are typically autonomous (e.g., Campbell and Cochrane (1999), Bansal and Yaron (2004), and Gabaix (2012)).

Based on the definition in Jurado, Ludvigson, and Ng (2015), our VFCI is a measure of uncertainty for output and consumption, connecting to the broader literature spurred by Bloom (2009) on macroeconomic uncertainty, its relation to the business cycle (Bloom et al. 2018; Ludvigson, Ma, and Ng 2021) and the construction of indices (Baker, Bloom, and Davis 2016; Jurado, Ludvigson, and Ng 2015; Ahir, Bloom, and Furceri 2022).

Our findings share the risk-centric view of macro-financial interactions among risk, policy, and the macroeconomy (Bianchi, Lettau, and Ludvigson (2022), Bianchi, Ludvigson, and Ma (2022), Caballero and Simsek (2020), Caballero and Simsek (2022), Caballero, Caravello, and Simsek (2024), Kashyap and Stein (2023)). We contribute by introducing the VFCI as a new theoretically-grounded measure of the market price of risk, identifying its causal effects, and documenting its two-way interaction with monetary policy.

Adrian and Duarte (2018) provides a specific microfoundation that is consistent with our theoretical model and rationalizes our empirical results. In their New Keynesian model with frictions modeled as value-at-risk constraints on intermediaries, shocks to the market price of risk generate a negative mean-volatility trade-off for output and have causal effects on output and monetary policy, but not inflation, providing an explicit general equilibrium mechanism for our findings.

X Conclusion

In this paper, we propose a new financial conditions index, the VFCI, derived from asset pricing theory. The VFCI is a measure of the price of risk in the economy when a representative consumer exists. The VFCI is derived from solid theoretical underpinnings. The VFCI is correlated with other leading FCIs, but has notable differences. An important one is that it exhibits better predictive power for stock, Treasury, and corporate bond risk premia. The VFCI is constructed using widely available financial data, is computationally tractable, and has a relatively long time series history. The VFCI could be computed globally, thus being able to track financial conditions in real-time across countries.

We use a range of identification strategies to study the causal impact of VFCI shocks on monetary policy and output, and vice versa. Across identification strategies, the baseline conclusions remain the same: a tightening of financial conditions based on the VFCI leads to an immediate easing of monetary policy and a persistent contraction in output. Conversely, contractionary monetary policy shocks lead to a tightening of financial conditions. In contrast, output shocks do not move the VFCI much, and inflation seems to be mostly unrelated to it. These results are encouraging, as they suggest a step forward in estimating financial conditions based on economic theory, with broad applicability and uses in policy-making. Further research could compute the VFCI for additional countries, conduct asset pricing tests, and embed the VFCI into structural macro-financial models.

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Supplemental Appendix (Internet Appendix)

A Theoretical Framework

A.1 Setting

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in\mathbb{Z}}, \mathbb{P})$ be a filtered probability space. Consider strictly positive consumption paths $C = \{C_s\}_{s\in\mathbb{Z}}$ with $C_s > 0$. For each t, the continuation value $U_t(C)$ and the SDF $\mathcal{M}_t(C)$ are $\mathcal{F}_t$-measurable functionals of the entire path C. Let $\bar{C} > 0$ be a given constant. We refer to the path $\{C_s \mid C_s = \bar{C}$ for all $s \in \mathbb{Z}\}$ as the constant path $\bar{C}$.

The consumption log-deviation from the constant path and one-period log-growth are:

$$g_s := \log C_s - \log \bar{C}, \qquad \Delta c_s := g_s - g_{s-1}.$$

We equip sequences with the one-sided exponentially weighted L^2 norm,

$$\|x\| := \Big(\mathbb{E}\Big[\sum_{t\in\mathbb{Z}} \beta^{|t|}|x_t|^2\Big]\Big)^{1/2},$$

for a fixed $\beta \in (0, 1)$. All sequences considered are assumed to belong to the space defined by this norm and to have pre-sample histories (values for periods $t < 0$) that are fixed and non-stochastic. All $o(\cdot)$ remainders below are with respect to this norm.

We use the following assumptions:

A1. Differentiability of U: The continuation value U_t is twice continuously Fréchet differentiable in an open neighborhood of the constant path $\bar{C}$.

A2. Conditional Riesz representations for DU_t: The Fréchet derivative of U_t at $\bar{C}$, denoted by $DU_t[\bar{C}]$, admits a conditional Riesz representation with $\mathcal{F}_{\max\{t,s\}}$-measurable kernel $G_{t,s}(C)$ such that

$$DU_t[\bar{C}] \cdot h = \sum_{s\in\mathbb{Z}} \mathbb{E}_t[G_{t,s}(\bar{C})h_s]$$

for all $h \in L^2$.

A3. Conditional Riesz representations for $D\mathcal{M}_{t+1}$: The Fréchet derivative of $\mathcal{M}_{t+1}$ at $\bar{C}$, denoted by $D\mathcal{M}_{t+1}(C)$, admits a conditional Riesz representation with $\mathcal{F}_{\max\{t+1,s\}}$-

measurable kernels $K_{t+1,s}(\bar{C})$ and $Z_{t+1,s}(\bar{C})$ such that

$$D\mathcal{M}_{t+1}[\bar{C}] \cdot h = \sum_{s\in\mathbb{Z}} \mathbb{E}_{t+1}[K_{t+1,s}(\bar{C})h_s] - \sum_{s\in\mathbb{Z}} \mathbb{E}_t[Z_{t+1,s}(\bar{C})h_s]$$

for all $h \in L^2$.

A4. The kernels $K_{t+1,s}(\bar{C})$ and $Z_{t+1,s}(\bar{C})$ are deterministic, absolutely summable, and depend on (t, s) only through the difference $s - (t + 1)$. We can therefore write them as:

$$K_{t+1,s}(\bar{C}) = K_{s-(t+1)},$$
$$Z_{t+1,s}(\bar{C}) = Z_{s-(t+1)}.$$

A5. Level of C determined by its growth rate: The consumption process satisfies $g_{t-m} \to 0$ as $m \to \infty$ in L^2 (which implies $g_t = \sum_{j=0}^{\infty} \Delta c_{t-j}$).

A.2 Representation of Linearized SDF

Linearization of the SDF at $\bar{C}$. A first-order expansion of C_s around $\bar{C}$ gives

$$C_s - \bar{C} = \bar{C}g_s + o(\|g\|). \tag{A.31}$$

Let $\Lambda^0 := \mathcal{M}_{t+1}(\bar{C})$. Using the representation from Assumption A3, the deterministic and time-homogeneous kernels from Assumption A4, and equation (A.31), we have:

$$\mathcal{M}_{t+1}(C) = \Lambda^0 + \bar{C}\left(\sum_{s\in\mathbb{Z}} K_{s-(t+1)}\,\mathbb{E}_{t+1}[g_s] - \sum_{s\in\mathbb{Z}} Z_{s-(t+1)}\,\mathbb{E}_t[g_s]\right) + o(\|g\|).$$

Re-indexing each sum with $p = s - (t + 1)$ gives the linearized SDF:

$$\mathcal{M}_{t+1}(C) = \Lambda^0 + \bar{C}\left(\sum_{p\in\mathbb{Z}} K_p\,\mathbb{E}_{t+1}[g_{t+1+p}] - \sum_{p\in\mathbb{Z}} Z_p\,\mathbb{E}_t[g_{t+1+p}]\right) + o(\|g\|). \tag{A.32}$$

Representation with deviations from $\bar{C}$. Substituting $g_s = \sum_{j=0}^{\infty} \Delta c_{s-j}$ and swapping sums (justified by absolute summability of $\{K_p\}$ and $\{Z_p\}$ in Assumption A4) gives

$$\mathcal{M}_{t+1} - \Lambda^0 = \bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} K_p \Big) \mathbb{E}_{t+1}[\Delta c_{t+1-k}] - \bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} Z_p \Big) \mathbb{E}_t[\Delta c_{t+1-k}] + o(\|\Delta c\|). \tag{A.33}$$

We have substituted the remainder $o(\|g\|)$ for $o(\|\Delta c\|)$. The two are equivalent because the lag operator L is a contraction under the assumed norm and the linear operator $\Delta c \mapsto g = (I - L)^{-1} \Delta c$ is bounded.

Representation with one-period growth rates and news. Define the news term for any horizon $m \in \mathbb{Z}$ by

$$N_{t+1,m} := \mathbb{E}_{t+1}[\Delta c_{t+1+m}] - \mathbb{E}_t[\Delta c_{t+1+m}].$$

For any past growth rate where $m < 0$, the variable Δc_{t+1+m} is $\mathcal{F}_t$-measurable, which implies $N_{t+1,m} = 0$. We use the decomposition $\mathbb{E}_{t+1}[\Delta c_{t+1-k}] = \mathbb{E}_t[\Delta c_{t+1-k}] + N_{t+1,-k}$ in the first sum or (A.33):

$$\begin{aligned} \mathcal{M}_{t+1} - \Lambda^0 = \bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} K_p \Big) \Big(\mathbb{E}_t[\Delta c_{t+1-k}] + N_{t+1,-k} \Big) \\ - \bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} Z_p \Big) \mathbb{E}_t[\Delta c_{t+1-k}] + o(\|\Delta c\|). \end{aligned}$$

Grouping terms with the same conditional expectation operator gives

$$\bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} (K_p - Z_p) \Big) \mathbb{E}_t[\Delta c_{t+1-k}] + \bar{C} \sum_{k \in \mathbb{Z}} \Big(\sum_{p=-k}^{\infty} K_p \Big) N_{t+1,-k} + o(\|\Delta c\|).$$

This leads to the representation

$$\mathcal{M}_{t+1}(C) = \alpha + \sum_{j=1}^{\infty} \kappa_j \Delta c_{t+1-j} + \sum_{k=0}^{\infty} \xi_k \, \mathbb{E}_t[\Delta c_{t+1+k}] + \sum_{k=0}^{\infty} \zeta_k N_{t+1,k} + o(\|\Delta c\|), \tag{A.34}$$

with coefficients

$$\begin{aligned}
\alpha &= \Lambda^0, \\
\kappa_j &= \bar{C} \sum_{p=-j}^{\infty} (K_p - Z_p) && \text{for } j \geq 1, \\
\xi_k &= \bar{C} \sum_{p=k}^{\infty} (K_p - Z_p) && \text{for } k \geq 0, \\
\zeta_k &= \bar{C} \sum_{p=k}^{\infty} K_p && \text{for } k \geq 0.
\end{aligned}$$

A.3 Linearization of Utility with Internal Habit Formation

Consider

$$U_t(C) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, H_s), \qquad H_s = \sum_{j=1}^{\infty} \alpha_j C_{s-j},$$

with $0 < \beta < 1$ and $\sum_{j=1}^{\infty} |\alpha_j| < \infty$. The coefficients α_j are deterministic and independent of C. Habits are internal, so consumption that enters H_s is chosen optimally together with C_s.

First-Order Condition. For $r \geq t$,

$$\frac{\partial U_t(C)}{\partial C_r} = \mathbb{E}_t \Big[\beta^{r-t} u_C(C_r, H_r) + \sum_{k=1}^{\infty} \beta^{r+k-t} \alpha_k u_H(C_{r+k}, H_{r+k}) \Big].$$

In particular,

$$M_t(C) := \frac{\partial U_t(C)}{\partial C_t} = u_C(C_t, H_t) + \sum_{j=1}^{\infty} \beta^j \alpha_j \mathbb{E}_t \big[u_H(C_{t+j}, H_{t+j}) \big],$$

and the first-order condition of the representative agent is

$$\mathcal{M}_{t+1}(C) = \beta \frac{M_{t+1}(C)}{M_t(C)}. \tag{A.35}$$

Linearization. All derivatives of u below are evaluated at $(\bar{C}, \bar{H})$ and are denoted with subscripts (u_C, u_H, u_{CC}, and so on). We linearize around the constant path $\bar{C}$. Define

$$A := \sum_{j=1}^{\infty} \alpha_j, \qquad \bar{H} = A\bar{C}, \qquad \bar{M} := u_C + \sum_{j=1}^{\infty} \beta^j \alpha_j u_H.$$

The deviation of marginal utility from its level at $\bar{C}$ is

$$M_t - \bar{M} \approx \sum_{k=-\infty}^{\infty} \frac{\partial M_t}{\partial C_k}\bigg|_{C=\bar{C}} g_k = \frac{1}{\bar{C}} \sum_{k=-\infty}^{\infty} \phi_{t-k} g_k,$$

where the coefficients ϕ_j are defined below. Substituting the expression for g_k gives

$$M_t - \bar{M} \approx \sum_{k=-\infty}^{\infty} \phi_{t-k} \left(\sum_{s=-\infty}^{k} \mathbb{E}_t[\Delta c_s] \right).$$

Swapping the order of summation,

$$M_t - \bar{M} \approx \sum_{s=-\infty}^{\infty} \left(\sum_{k=s}^{\infty} \phi_{t-k} \right) \mathbb{E}_t[\Delta c_s] = \sum_{s=-\infty}^{\infty} \left(\sum_{i=-\infty}^{t-s} \phi_i \right) \mathbb{E}_t[\Delta c_s].$$

Defining the cumulative impact coefficients $\Xi_j = \sum_{i=-\infty}^{j} \phi_i$, we have

$$M_t - \bar{M} \approx \sum_{s=-\infty}^{\infty} \Xi_{t-s} \mathbb{E}_t[\Delta c_s], \qquad M_{t+1} - \bar{M} \approx \sum_{s=-\infty}^{\infty} \Xi_{t+1-s} \mathbb{E}_{t+1}[\Delta c_s].$$

The linearized SDF is $\mathcal{M}_{t+1}(C) \approx \beta(1 + (M_{t+1} - M_t)/\bar{M})$ and therefore

$$M_{t+1} - M_t \approx \sum_{s=-\infty}^{\infty} \Xi_{t+1-s} \mathbb{E}_{t+1}[\Delta c_s] - \sum_{s=-\infty}^{\infty} \Xi_{t-s} \mathbb{E}_t[\Delta c_s].$$

Using the identity $\Xi_{t+1-s} = \Xi_{t-s} + \phi_{t+1-s}$, we substitute into the first sum and group terms:

$$\begin{aligned} M_{t+1} - M_t &\approx \sum_{s=-\infty}^{\infty} (\Xi_{t-s} + \phi_{t+1-s}) \mathbb{E}_{t+1}[\Delta c_s] - \sum_{s=-\infty}^{\infty} \Xi_{t-s} \mathbb{E}_t[\Delta c_s] \\ &= \sum_{s=-\infty}^{\infty} \phi_{t+1-s} \mathbb{E}_{t+1}[\Delta c_s] + \sum_{s=-\infty}^{\infty} \Xi_{t-s} (\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta c_s]. \end{aligned}$$

Using $\mathcal{M}_{t+1}(C) = SDF_{t+1}$, substituting into the FOC in equation (A.35) and re-indexing the sums ($j = t+1-s$ in the first sum, $j = t-s$ in the second) gives

$$SDF_{t+1} \approx \beta \left[1 + \frac{1}{\bar{M}} \sum_{j=-\infty}^{\infty} \phi_j \mathbb{E}_{t+1}[\Delta c_{t+1-j}] + \frac{1}{\bar{M}} \sum_{j=-\infty}^{\infty} \Xi_j (\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta c_{t-j}]\right] \tag{A.36}$$

where

$$\begin{aligned}
\bar{M} &= u_C + u_H \sum_{k=1}^{\infty} \beta^k \alpha_k, \\
\phi_0 &= \bar{C}\left(u_{CC} + u_{HH} \sum_{k=1}^{\infty} \beta^k \alpha_k^2\right), \\
\phi_j &= \bar{C}\left(u_{CH}\alpha_j + u_{HH} \sum_{k=1}^{\infty} \beta^k \alpha_k \alpha_{k+j}\right), \quad \text{for } j > 0, \\
\phi_{-m} &= \bar{C}\left(\beta^m \alpha_m u_{CH} + u_{HH} \sum_{k=m+1}^{\infty} \beta^k \alpha_k \alpha_{k-m}\right), \quad \text{for } m > 0, \\
\Xi_j &= \sum_{i=-\infty}^{j} \phi_i.
\end{aligned}$$

Equation (A.36) can be equivalently written as

$$\begin{aligned}
SDF_{t+1} = \beta - \bar{C}\Gamma_C \Delta c_{t+1} - \bar{C}\Gamma_H \sum_{j=1}^{\infty} \alpha_j \Delta c_{t+1-j} \\
- \bar{C}\mu_1 \mathbb{E}_t \Delta c_{t+1} - \bar{C} \sum_{s=2}^{\infty} \mu_s \mathbb{E}_t \Delta c_{t+s} + \frac{\beta}{\bar{M}} \mathcal{N}_{t+1},
\end{aligned}$$

where

$$\begin{aligned}
\mu_1 &= \Gamma_{HH} \sum_{h=2}^{\infty} \beta^{h-2} (\alpha_{h-1} - \beta\alpha_h) S_{h-1}, \\
\mu_s &= \Gamma_H \beta^{s-2} \alpha_{s-1} + \Gamma_{HH} \sum_{h=s+1}^{\infty} \beta^{h-2} (\alpha_{h-1} - \beta\alpha_h) S_{h-s}, \quad \text{for } s \geq 2, \\
S_m &= \sum_{k=1}^{m} \alpha_k, \\
\Gamma_C &= -\beta u_{CC}/\bar{M}, \quad \Gamma_H = -\beta u_{CH}/\bar{M}, \quad \Gamma_{HH} = -\beta u_{HH}/\bar{M},
\end{aligned}$$

$$Q_{t+h} = \bar{C} \sum_{r=1}^{h} \Big[u_{HC} + u_{HH} \sum_{m=1}^{h-r} \alpha_m \Big] \Delta c_{t+r},$$

$$\mathcal{N}_{t+1} = \sum_{h=2}^{\infty} \beta^{h-1} \alpha_{h-1} (\mathbb{E}_{t+1} - \mathbb{E}_t) Q_{t+h}.$$

A.3.1 One-Period Internal Habits with CRRA Surplus Consumption

Let $u(C,H) = (C-H)^{1-\gamma}/(1-\gamma)$, $\alpha_1 = \alpha$, and $\alpha_j = 0$ for $j > 1$. Then

$$SDF_{t+1} = \beta + \frac{\beta\gamma}{(1-\alpha)(1-\alpha\beta)} \Big[\alpha \Delta c_t + ((1-\alpha)\alpha\beta - 1)\Delta c_{t+1} + \alpha\beta \big(\mathbb{E}_{t+1} \Delta c_{t+2} - \mathbb{E}_t \Delta c_{t+1} \big) \Big].$$

A.4 Linearization of Epstein-Zin Utility

The Epstein-Zin utility functional is defined recursively by

$$U_t = [(1-\beta) C_t^{\rho} + \beta S_t^{\rho}]^{1/\rho}, \quad S_t = \left(\mathbb{E}_t [U_{t+1}^{1-\gamma}] \right)^{1/(1-\gamma)},$$

where $\beta \in (0,1)$, $\gamma > 0$ (risk aversion), $\psi > 0$ (EIS), and $\rho := 1 - 1/\psi$.

First-Order Condition. The FOC of the representative agent is

$$\mathcal{M}_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left(\frac{U_{t+1}}{S_t} \right)^{1/\psi - \gamma}.$$

Utility Kernel $G_{t,s}$. First, compute one-step derivatives

$$\frac{\partial U_k}{\partial C_k} = (1-\beta) C_k^{\rho-1} U_k^{1-\rho}.$$

Then,

$$DU_k = \mathbb{E}_k \left[K_{k+1}^{\text{chain}} DU_{k+1} \right], \quad K_{k+1}^{\text{chain}} = \beta S_k^{\rho+\gamma-1} U_k^{1-\rho} U_{k+1}^{-\gamma}.$$

By recursive application of the chain rule:

$$G_{t,s}(C) = \left(\prod_{k=t+1}^{s} K_k^{\text{chain}} \right) \frac{\partial U_s}{\partial C_s}.$$

SDF Kernels. We start log-linearized FOC

$$D \log \mathcal{M}_{t+1} = -\frac{1}{\psi} \Delta c_{t+1} - \theta \sum_{j=1}^{\infty} \beta^j (\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta c_{t+1+j}],$$

where $\theta := \gamma - 1/\psi$.

Since to first order we have $D\mathcal{M}_{t+1} = \beta D \log \mathcal{M}_{t+1}$, expanding and collecting terms gives

$$K_p = \begin{cases} -\frac{\beta^{p+1}\theta}{\bar{C}}(1-\beta) & p \geq 1 \\ \frac{1}{\bar{C}}(-\frac{\beta}{\psi} + \beta^2\theta) & p = 0 \\ 0 & p < 0 \end{cases},$$

and

$$Z_p = \begin{cases} -\frac{\beta^{p+1}\theta}{\bar{C}}(1-\beta) & p \geq 1 \\ -\frac{1}{\bar{C}}(\frac{\beta}{\psi} - \beta^2\theta) & p = 0 \\ -\frac{\beta}{\psi\bar{C}} & p = -1 \\ 0 & p < -1 \end{cases}.$$

Representation of Linearized FOC. Using the above, the FOC is

$$\mathcal{M}_{t+1}(C) = \alpha + \sum_{j=1}^{\infty} \kappa_j \Delta c_{t+1-j} + \sum_{k=0}^{\infty} \xi_k \mathbb{E}_t[\Delta c_{t+1+k}] + \sum_{k=0}^{\infty} \zeta_k N_{t+1,k} + o(\|\Delta c\|),$$

where $\alpha = \beta$, $\kappa_j := \bar{C}\sum_{p=-j}^{\infty}(K_p - Z_p)$, $\xi_k := \bar{C}\sum_{p=k}^{\infty}(K_p - Z_p)$, and $\zeta_k := \bar{C}\sum_{p=k}^{\infty} K_p$ or, more explicitly,

$$\begin{aligned} \kappa_j &= 0 && \text{for all } j \geq 1, \\ \xi_0 &= -\frac{\beta}{\psi}, \\ \xi_k &= 0 && \text{for } k \geq 1, \\ \zeta_0 &= -\frac{\beta}{\psi}, \\ \zeta_k &= -\beta^{k+1}\left(\gamma - \frac{1}{\psi}\right) && \text{for } k \geq 1. \end{aligned}$$

The last expression can also be re-written as

$$\mathcal{M}_{t+1}(C) = \beta - \frac{\beta}{\psi}\Delta c_{t+1} - \beta\left(\gamma - \frac{1}{\psi}\right)\sum_{k=1}^{\infty} \beta^k (\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta c_{t+1+k}] + o(\|\Delta c\|).$$

A.5 Solving the FOC Rational Expectations Equation

Rewrite the FOC in equation (A.34) as

$$SDF_{t+1} = \bar{g} + \sum_{s=1}^{\infty} g_{-s}\Delta c_{t+1-s} + g_0 \Delta c_{t+1} + \sum_{s=1}^{\infty} g_s \mathbb{E}_t[\Delta c_{t+1+s}] + \sum_{s=1}^{\infty} k_s \mathbb{E}_{t+1}[\Delta c_{t+1+s}],$$

where SDF_{t+1} is an exogenous stochastic discount factor with arbitrary mean and autocovariance structure. The constant coefficients $\bar{g}$, g_s, k_s are functions of $\alpha, \kappa_j, \zeta_k$.

The solution to the rational expectations equation (A.5):

1. Exists if the coefficient system (g_s, k_s) admits a canonical Wiener–Hopf factorization,
2. Is unique if $B(z)$ has no zeros on the unit circle.

The solution, if it exists, is measurable with respect to the filtration $\sigma\{SDF_s, \Delta c_s : s \leq t\}$.

A.6 Bond Prices, Yields, Returns and Term Premium

Let $P_t^{(n)}$ be the price of a zero-coupon bond with maturity n periods, with corresponding log-price $p_t^{(n)} := \log P_t^{(n)}$ and $P_t^{(0)} := 1$ by convention. The no-arbitrage price of a one-period bond is

$$P_t^{(1)} = \mathbb{E}_t SDF_{t+1}, \tag{A.37}$$

where, as in the main body of the paper, SDF_{t+1} is the one-period stochastic discount factor to discount payoffs from date $t+1$ back to t. Only the case $n = 1$ is required below; we therefore omit the general multi-period pricing relation.

The continuously compounded yield $y_t^{(n)}$ is

$$y_t^{(n)} := -\frac{1}{n} p_t^{(n)}, \tag{A.38}$$

with the convention $y_t^{(0)} := 0$. The term premium is

$$\mathrm{TP}_t^{(n)} := y_t^{(n)} - \frac{1}{n} \sum_{j=0}^{n-1} \mathbb{E}_t y_{t+j}^{(1)}, \tag{A.39}$$

where the summation is taken to be zero if the upper limit is below the lower limit.

A.7 SDF News

In this section, we derive an empirically estimable approximation of SDF news using bond yields and term premia, prove that it is unbiased, and demonstrate that its approximation error is small.

A.7.1 Approximation of SDF News

Define the innovations operator

$$\Delta_{t+1}(\cdot) := \mathbb{E}_{t+1}[\cdot] - \mathbb{E}_t[\cdot].$$

We note that $\mathbb{E}_t\Delta_{t+1}(\cdot) = 0$ by the law of iterated expectations.

For $i = 1, \ldots$, news to future stochastic discount factors are

$$\Delta_{t+1}SDF_{t+1+i} = \mathbb{E}_{t+1}SDF_{t+1+i} - \mathbb{E}_t SDF_{t+1+i}.$$

Using the law of iterated expectations and (A.37),

$$\begin{aligned}
\Delta_{t+1}SDF_{t+1+i} &= \mathbb{E}_{t+1}SDF_{t+1+i} - \mathbb{E}_t SDF_{t+1+i} \\
&= \mathbb{E}_{t+1}[\mathbb{E}_{t+i}SDF_{t+1+i}] - \mathbb{E}_t[\mathbb{E}_{t+i}SDF_{t+1+i}] \\
&= \mathbb{E}_{t+1}P^{(1)}_{t+i} - \mathbb{E}_t P^{(1)}_{t+i} \\
&= \Delta_{t+1}P^{(1)}_{t+i} \\
&= \Delta_{t+1}e^{p^{(1)}_{t+i}}.
\end{aligned}$$

Thus, SDF news are news about a future bond price with one period left to maturity.

Consider the approximation

$$\begin{aligned}
\Delta_{t+1}SDF_{t+1+i} &= \Delta_{t+1}e^{p^{(1)}_{t+i}} \\
&\approx e^{\mathbb{E}_t p^{(1)}_{t+i}} \left\{ \Delta_{t+1}p^{(1)}_{t+i} + \tfrac{1}{2}(\Delta_{t+1}p^{(1)}_{t+i})^2 \right\} - B_i,
\end{aligned} \tag{A.40}$$

where B_i is the constant

$$B_i := \frac{1}{2}\mathbb{E}\Big[e^{\mathbb{E}_t p^{(1)}_{t+i}} \big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\Big]. \tag{A.41}$$

A.7.2 Bond Price News

Multiplying (A.39) by n gives

$$ny_t^{(n)} = \sum_{j=0}^{n-1} \mathbb{E}_t y_{t+j}^{(1)} + n\mathrm{TP}_t^{(n)}. \tag{A.42}$$

Setting $n = i + 1$ in (A.42) gives

$$(i+1)y_t^{(i+1)} = \sum_{j=0}^{i} \mathbb{E}_t y_{t+j}^{(1)} + (i+1)\mathrm{TP}_t^{(i+1)},$$

while $n = i$ gives

$$iy_t^{(i)} = \sum_{j=0}^{i-1} \mathbb{E}_t y_{t+j}^{(1)} + i\mathrm{TP}_t^{(i)}.$$

Subtracting the second equation from the first and solving for the conditional expectation,

$$\mathbb{E}_t y_{t+i}^{(1)} = (i+1)y_t^{(i+1)} - iy_t^{(i)} - (i+1)\mathrm{TP}_t^{(i+1)} + i\mathrm{TP}_t^{(i)}. \tag{A.43}$$

Replacing t by $t+1$ and i by $i-1$ gives

$$\mathbb{E}_{t+1} y_{t+i}^{(1)} = iy_{t+1}^{(i)} - (i-1)y_{t+1}^{(i-1)} - i\mathrm{TP}_{t+1}^{(i)} + (i-1)\mathrm{TP}_{t+1}^{(i-1)}. \tag{A.44}$$

Subtracting (A.43) from (A.44),

$$\begin{aligned}\Delta_{t+1} y_{t+i}^{(1)} = &-(i-1)\big(y_{t+1}^{(i-1)} - \mathrm{TP}_{t+1}^{(i-1)}\big)\\ &+ i\big(y_{t+1}^{(i)} - \mathrm{TP}_{t+1}^{(i)} + y_t^{(i)} - \mathrm{TP}_t^{(i)}\big)\\ &- (i+1)\big(y_t^{(i+1)} - \mathrm{TP}_t^{(i+1)}\big).\end{aligned}$$

Using (A.38) gives news to the log-price $p_{t+i}^{(1)}$:

$$\begin{aligned}\Delta_{t+1} p_{t+i}^{(1)} = &(i-1)\big(y_{t+1}^{(i-1)} - \mathrm{TP}_{t+1}^{(i-1)}\big)\\ &- i\big(y_{t+1}^{(i)} - \mathrm{TP}_{t+1}^{(i)} + y_t^{(i)} - \mathrm{TP}_t^{(i)}\big)\\ &+ (i+1)\big(y_t^{(i+1)} - \mathrm{TP}_t^{(i+1)}\big).\end{aligned} \tag{A.45}$$

A.7.3 Empirical Measure of SDF News

Let

$$\hat{n}_{i,t} := iy_t^{(i)} - (i+1)y_t^{(i+1)} + (i+1)\text{TP}_t^{(i+1)} - i\text{TP}_t^{(i)}, \tag{A.46}$$

$$\hat{n}_{i-1,t+1} := (i-1)y_{t+1}^{(i-1)} - iy_{t+1}^{(i)} + i\text{TP}_{t+1}^{(i)} - (i-1)\text{TP}_{t+1}^{(i-1)}, \tag{A.47}$$

$$\hat{N}_{i,t+1} := \hat{n}_{i-1,t+1} - \hat{n}_{i,t}, \tag{A.48}$$

$$\hat{B}_i := \frac{1}{2}\frac{1}{T-i}\sum_{t=1}^{T-i} e^{\hat{n}_{i,t}}\hat{N}_{i,t+1}^2, \tag{A.49}$$

where $T > 0$ is the number of observations ($T > i$ is needed to implement the above estimators). By (A.43)-(A.44), $\hat{n}_{i,t}$ and $\hat{n}_{i-1,t+1}$ are estimators for $\mathbb{E}_t p_{t+i}^{(1)}$ and $\mathbb{E}_{t+1} p_{t+i}^{(1)}$, respectively. By (A.41) and (A.45), $\hat{N}_{i,t+1}$ and $\hat{B}_i$ are estimators for $\Delta_{t+1} p_{t+i}^{(1)}$ and B_i, respectively.

The approximate SDF news from equation (A.40) can then be empirically estimated by:

$$\Delta_{t+1} SDF_{t+1+i} \approx e^{\hat{n}_{i,t}}\left(\hat{N}_{i,t+1} + \tfrac{1}{2}\hat{N}_{i,t+1}^2\right) - \hat{B}_i.$$

A.7.4 Approximation of Bond Price News

In the remainder of Section A.7, we show that the approximation (A.40) is unbiased, has an error whose variance is theoretically guaranteed to be bounded above by a fourth-order term, and that a consistent empirical estimate of this fourth-order upper bound is below 3×10^{-5} basis points for all maturities.

A second-order Taylor expansion of $e^{p_{t+i}^{(1)}}$ around $\mathbb{E}_t p_{t+i}^{(1)}$ gives

$$e^{p_{t+i}^{(1)}} = e^{\mathbb{E}_t p_{t+i}^{(1)}}\left[1 + \left(p_{t+i}^{(1)} - \mathbb{E}_t p_{t+i}^{(1)}\right) + \tfrac{1}{2}\left(p_{t+i}^{(1)} - \mathbb{E}_t p_{t+i}^{(1)}\right)^2\right] + R_{3,t+i}, \tag{A.50}$$

where, for some ξ_{t+i} between $p_{t+i}^{(1)}$ and $\mathbb{E}_t p_{t+i}^{(1)}$,

$$R_{3,t+i} = \frac{e^{\xi_{t+i}}}{3!}\left(p_{t+i}^{(1)} - \mathbb{E}_t p_{t+i}^{(1)}\right)^3$$

is of third order:

$$\mathbb{E}_t R_{3,t+i} = \mathcal{O}\left(\mathbb{E}_t \left|p_{t+i}^{(1)} - \mathbb{E}_t p_{t+i}^{(1)}\right|^3\right).$$

Applying $\mathbb{E}_t$ to (A.50),

$$\mathbb{E}_t e^{p_{t+i}^{(1)}} = e^{\mathbb{E}_t p_{t+i}^{(1)}}\left(1 + \tfrac{1}{2}\operatorname{Var}_t\left(p_{t+i}^{(1)}\right)\right) + \mathbb{E}_t R_{3,t+i}. \tag{A.51}$$

Applying $\mathbb{E}_{t+1}$ to (A.50) and subtracting (A.51),

$$\Delta_{t+1}e^{p^{(1)}_{t+i}} = e^{\mathbb{E}_t p^{(1)}_{t+i}}\Big[\Delta_{t+1}p^{(1)}_{t+i} + \tfrac{1}{2}\mathbb{E}_{t+1}\Big[\big(p^{(1)}_{t+i} - \mathbb{E}_t p^{(1)}_{t+i}\big)^2\Big] - \tfrac{1}{2}\operatorname{Var}_t\big(p^{(1)}_{t+i}\big)\Big] + \Delta_{t+1}R_{3,t+i}. \tag{A.52}$$

A.7.5 Mean of Approximation Error

The approximation error in (A.40) is

$$\text{error}_{i,t+1} := \Delta_{t+1}e^{p^{(1)}_{t+i}} - e^{\mathbb{E}_t p^{(1)}_{t+i}}\Big[\Delta_{t+1}p^{(1)}_{t+i} + \tfrac{1}{2}\big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\Big] + B_i. \tag{A.53}$$

Using (A.52), the error can be written as

$$\text{error}_{i,t+1} = \tfrac{1}{2}e^{\mathbb{E}_t p^{(1)}_{t+i}}\left[\mathbb{E}_{t+1}\Big[\big(p^{(1)}_{t+i} - \mathbb{E}_t p^{(1)}_{t+i}\big)^2\Big] - \operatorname{Var}_t\big(p^{(1)}_{t+i}\big) - \big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\right] + B_i + \Delta_{t+1}R_{3,t+i}. \tag{A.54}$$

Taking $\mathbb{E}_t$,

$$\mathbb{E}_t\text{error}_{i,t+1} = -\tfrac{1}{2}e^{\mathbb{E}_t p^{(1)}_{t+i}}\mathbb{E}_t\left[\big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\right] + B_i$$

Taking $\mathbb{E}$ and using the law of iterated expectations

$$\begin{aligned}
\mathbb{E}\text{error}_{i,t+1} &= \mathbb{E}\mathbb{E}_t\text{error}_{i,t+1} \\
&= -\tfrac{1}{2}\mathbb{E}\left\{e^{\mathbb{E}_t p^{(1)}_{t+i}}\mathbb{E}_t\left[\big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\right]\right\} + B_i \\
&= -\tfrac{1}{2}\mathbb{E}\left[e^{\mathbb{E}_t p^{(1)}_{t+i}}\big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\right] + \frac{1}{2}\mathbb{E}\Big[e^{\mathbb{E}_t p^{(1)}_{t+i}}\big(\Delta_{t+1}p^{(1)}_{t+i}\big)^2\Big] \\
&= 0.
\end{aligned}$$

Thus, the approximation is unbiased (the constant B_i is defined to make the error zero on average).

A.7.6 Bound on Variance of Approximation Error

Start with

$$p^{(1)}_{t+i} - \mathbb{E}_t p^{(1)}_{t+i} = \big(p^{(1)}_{t+i} - \mathbb{E}_{t+1}p^{(1)}_{t+i}\big) + \Delta_{t+1}p^{(1)}_{t+i}.$$

Square and apply $\mathbb{E}_{t+1}$

$$\mathbb{E}_{t+1}\big[(p^{(1)}_{t+i} - \mathbb{E}_t p^{(1)}_{t+i})^2\big] = \mathbb{E}_{t+1}\big[\big(p^{(1)}_{t+i} - \mathbb{E}_{t+1}p^{(1)}_{t+i}\big)^2\big]$$

$$
\begin{aligned}
&+\mathbb{E}_{t+1}\big[(\Delta_{t+1}p_{t+i}^{(1)})^2\big]\\
&+2\mathbb{E}_{t+1}\big[\big(p_{t+i}^{(1)}-\mathbb{E}_{t+1}p_{t+i}^{(1)}\big)\Delta_{t+1}p_{t+i}^{(1)}\big]\\
=&\ \mathbb{E}_{t+1}\big[\big(p_{t+i}^{(1)}-\mathbb{E}_{t+1}p_{t+i}^{(1)}\big)^2\big]+(\Delta_{t+1}p_{t+i}^{(1)})^2,
\end{aligned}
$$

where the second line uses that $\Delta_{t+1}p_{t+i}^{(1)}$ is $\mathcal{F}_{t+1}$-measurable and $\mathbb{E}_{t+1}[p_{t+i}^{(1)}-\mathbb{E}_{t+1}p_{t+i}^{(1)}]=0$. Subtract $(\Delta_{t+1}p_{t+i}^{(1)})^2$ from both sides and apply Var_t and $\mathbb{E}$:

$$
\begin{aligned}
\mathbb{E}\Big[\mathrm{Var}_t\Big(\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-(\Delta_{t+1}p_{t+i}^{(1)})^2\Big)\Big]&=\mathbb{E}\Big[\mathrm{Var}_t\Big(\mathbb{E}_{t+1}\big[\big(p_{t+i}^{(1)}-\mathbb{E}_{t+1}p_{t+i}^{(1)}\big)^2\big]\Big)\Big]\\
&=\mathbb{E}\Big[\mathrm{Var}_t\Big(\mathrm{Var}_{t+1}\big(p_{t+i}^{(1)}\big)\Big)\Big].
\end{aligned}
\tag{A.55}
$$

Multiply (A.54) by $e^{-\mathbb{E}_tp_{t+i}^{(1)}}$ and apply Var_t

$$
\begin{aligned}
\mathrm{Var}_t(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\mathrm{error}_{i,t+1})&=\mathrm{Var}_t\Big(\tfrac{1}{2}\Big[\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-\mathrm{Var}_t\big(p_{t+i}^{(1)}\big)-(\Delta_{t+1}p_{t+i}^{(1)})^2\Big]\\
&\quad+e^{-\mathbb{E}_tp_{t+i}^{(1)}}B_i+e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&=\mathrm{Var}_t\Big(\tfrac{1}{2}\Big[\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-(\Delta_{t+1}p_{t+i}^{(1)})^2\Big]+e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&=\tfrac{1}{4}\mathrm{Var}_t\Big(\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-(\Delta_{t+1}p_{t+i}^{(1)})^2\Big)\\
&\quad+\mathrm{Var}_t\Big(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&\quad+\mathrm{Cov}_t\Big(\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-(\Delta_{t+1}p_{t+i}^{(1)})^2,e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&=\tfrac{1}{4}\mathrm{Var}_t\Big(\mathbb{E}_{t+1}\big[(p_{t+i}^{(1)}-\mathbb{E}_tp_{t+i}^{(1)})^2\big]-(\Delta_{t+1}p_{t+i}^{(1)})^2\Big)\\
&\quad+\mathrm{Var}_t\Big(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&\quad+\mathrm{Cov}_t\Big(\mathrm{Var}_{t+1}(p_{t+i}^{(1)}),e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big).
\end{aligned}
$$

Apply $\mathbb{E}$, and use (A.55)

$$
\begin{aligned}
\mathbb{E}\,\mathrm{Var}_t(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\mathrm{error}_{i,t+1})&=\tfrac{1}{4}\mathbb{E}\Big[\mathrm{Var}_t\Big(\mathrm{Var}_{t+1}\big(p_{t+i}^{(1)}\big)\Big)\Big]+\mathbb{E}\,\mathrm{Var}_t\Big(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big)\\
&\quad+\mathbb{E}\,\mathrm{Cov}_t\Big(\mathrm{Var}_{t+1}(p_{t+i}^{(1)}),e^{-\mathbb{E}_tp_{t+i}^{(1)}}\Delta_{t+1}R_{3,t+i}\Big).
\end{aligned}
\tag{A.56}
$$

Using (A.56), we have

$$
\begin{aligned}
\mathrm{Var}(\mathrm{error}_{i,t+1})&=\mathbb{E}\,\mathrm{Var}_t(\mathrm{error}_{i,t+1})+\mathrm{Var}(\mathbb{E}_t\mathrm{error}_{i,t+1})\\
&=\mathbb{E}\,\mathrm{Var}_t(\mathrm{error}_{i,t+1})\\
&\le\Big(\sup_t e^{2\mathbb{E}_tp_{t+i}^{(1)}}\Big)\mathbb{E}\Big[\mathrm{Var}_t(e^{-\mathbb{E}_tp_{t+i}^{(1)}}\mathrm{error}_{i,t+1})\Big]
\end{aligned}
$$

$$= \Big(\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}\Big)$$
$$\times \Big(\tfrac{1}{4}\mathbb{E}\Big[\operatorname{Var}_t\big(\operatorname{Var}_{t+1}(p^{(1)}_{t+i})\big)\Big] + R_{5,i}\Big),$$

where

$$\begin{aligned} R_{5,i} := \mathbb{E}\Big[&\operatorname{Var}_t\Big(e^{-\mathbb{E}_t p^{(1)}_{t+i}}\Delta_{t+1}R_{3,t+i}\Big) \\ &+ \operatorname{Cov}_t\Big(\operatorname{Var}_{t+1}(p^{(1)}_{t+i}), e^{-\mathbb{E}_t p^{(1)}_{t+i}}\Delta_{t+1}R_{3,t+i}\Big)\Big]. \end{aligned}$$

contains terms of order five or higher. Continuing with the bound,

$$\begin{aligned} \operatorname{Var}(\text{error}_{i,t+1}) &= \Big(\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}\Big) \\ &\quad\times \Big(\tfrac{1}{4}\mathbb{E}\Big[\operatorname{Var}_t\big(\operatorname{Var}_{t+1}(p^{(1)}_{t+i})\big)\Big] + R_{5,i}\Big) \\ &\leq \Big(\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}\Big) \\ &\quad\times \Big(\tfrac{1}{4}\mathbb{E}\Big[\mathbb{E}_t\Big[\big(\operatorname{Var}_{t+1}(p^{(1)}_{t+i})\big)^2\Big]\Big] + R_{5,i}\Big) \\ &\leq \Big(\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}\Big) \\ &\quad\times \Big(\tfrac{1}{4}\mathbb{E}\Big[\mathbb{E}_t\Big[\mathbb{E}_{t+1}\Big[\big(p^{(1)}_{t+i} - \mathbb{E}_{t+1}p^{(1)}_{t+i}\big)^4\Big]\Big]\Big] + R_{5,i}\Big) \\ &= \Big(\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}\Big) \\ &\quad\times \Big(\tfrac{1}{4}\mathbb{E}\Big[\big(p^{(1)}_{t+i} - \mathbb{E}_{t+1}p^{(1)}_{t+i}\big)^4\Big] + R_{5,i}\Big), \end{aligned}$$

where the second line uses that $\operatorname{Var}_t(X) \leq \mathbb{E}_t[X^2]$ for any random variable X, the third line follows by Jensen's inequality, and the last line by the law of iterated expectations. It follows that the variance of the approximation error is bounded above by a fourth-order term.

A.7.7 Empirical Bound

Ignoring terms in $R_{5,i}$ that are of order five and higher, the empirical upper bound for $\operatorname{Var}(\text{error}_{i,t+1})$ can be constructed as

$$\operatorname{Var}(\text{error}_{i,t+1}) \leq \tfrac{1}{4}\widehat{c}_i\widehat{k}_i,$$

where

$$\widehat{k}_i := \frac{1}{T-i}\sum_{t=1}^{T-i}(-y^{(1)}_{t+i} - \hat{n}_{i-1,t+1})^4, \tag{A.57}$$

$$\widehat{c}_i := \max_{1\le t\le T} e^{2\hat{n}_{i,t}}, \tag{A.58}$$

with $\hat{n}_{i,t}$, $\hat{n}_{i-1,t+1}$ given by (A.46)-(A.47) and where T is the number of observations. The variable $\widehat{k}_i$ is an estimator for $\mathbb{E}\left[\left(p^{(1)}_{t+i} - \mathbb{E}_{t+1}p^{(1)}_{t+i}\right)^4\right]$, and $\widehat{c}_i$ is an estimator for $\sup_t e^{2\mathbb{E}_t p^{(1)}_{t+i}}$.

The estimators converge in probability to their population counterparts under standard regularity conditions (e.g., covariance stationarity, α-mixing with geometric decay, and finite moments of order $4+\delta$ for some $\delta > 0$).

B Robustness of Empirical Results

B.1 VFCI Estimation

B.1.1 Financial Condition Indexes and the Asset Span

In Table B.1 we show the r-squared values from regressing n number of principal components of financial variables on each of the financial conditions indexes: NFCI, GSFCI, and VIX. It is clear that more than one principal component is needed to span any of the indexes, though the VIX has a large r-squared with just one. Only the GSFCI remains unspanned with two principal components. After reaching four principal components, it seems that each additional principal component adds little additional information. This analysis informs our decision to use four principal components for our baseline estimation of the VFCI, though we show that it is robust to choosing three or five.

B.1.2 VFCI Variations

First, real PCE can be used instead of real GDP to calculate the VFCI. Table 3 already showed that the regression of the GDP-based VFCI and the PCE-based VFCI give rise to very similar coefficients. In Figure B.1, we show graphically that the two series are virtually indistinguishable.

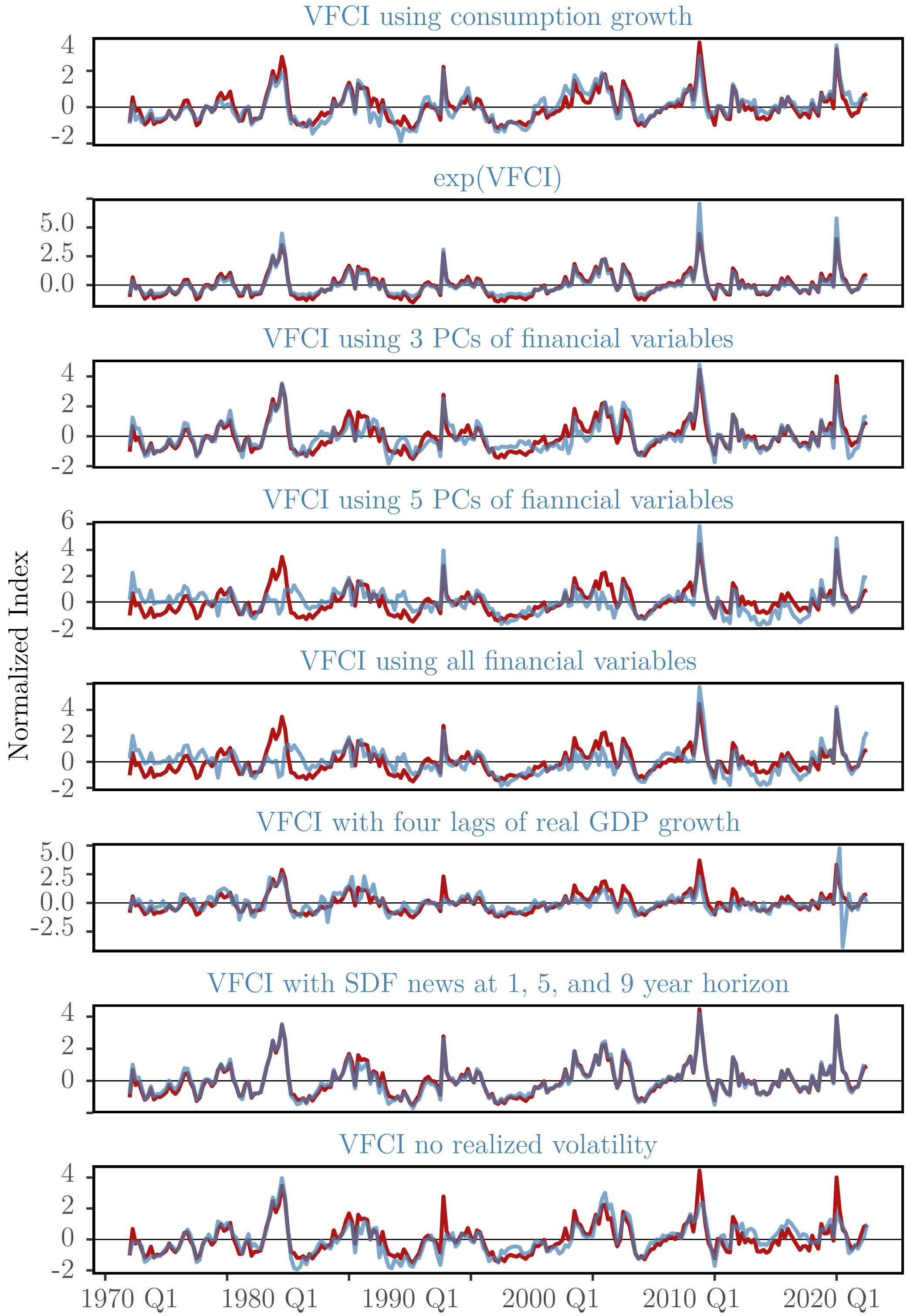
The VFCI we estimate is a measure of log-volatility. In Figure B.1 we show $\exp(VFCI)$, a measure of volatility, also rescaled to be mean zero with a standard deviation of one. This transformation preserves the shape of the VFCI but exacerbates the peaks.

As we show in Table B.1, we could capture the variation of financial assets with three or five principal components instead of our baseline of four. In Figure B.1 we show that using three or five principal components does not materially change the VFCI. When using five, we see less of a peak during the 1970s, but otherwise a very similar series.

Another question is whether we need the computation of the PCAs. Instead of construct-

FIGURE B.1

VFCI and VFCI Variations



Notes: Each red line plots the *VFCI*, the volatility financial conditions index, reproduced from Figure 1. The light blue lines show variations of the VFCI estimated for robustness. All series are scaled to be mean zero with a standard deviation of one.

Table B.1

R-squared of regressions of financial conditions indices on principal components of financial variables

Number of PCs	VFCI	NFCI	GSFCI	VIX
1	0.23	0.04	0.05	0.63
2	0.68	0.73	0.11	0.71
3	0.83	0.81	0.61	0.71
4	1.00	0.81	0.73	0.80
5	1.00	0.83	0.74	0.82
6	1.00	0.83	0.75	0.85

Notes: This table shows the R^2 of OLS estimates $\hat{\beta}$ for regressions

$$FCI_t = \alpha + \beta PC_t + \varepsilon_t,$$

where FCI_t is a financial conditions index, PC_t is a vector with the first n principal components of the financial variables in Table 1; ε_t is a zero-mean, i.i.d. disturbance; α and β are parameters to be estimated. Each column corresponds to regressions that use a different FCI_t measures— *NFCI* uses the Federal Reserve Bank of Chicago's national financial conditions index, *GSFCI* uses the Goldman Sachs financial conditions index, and *VIX* uses the VIX index from the Chicago Board Options Exchange. All series are quarterly and use the longest sample available for each FCI.

ing the VFCI from the PCAs, we run a heteroskedastic regression of GDP growth on the six financial variables. The resulting VFCI is again virtually indistinguishable from our original PCA-based VFCI, with a smaller peak for the VFCI during the 1970s.

Some models give rise to an estimation of the VFCI that controls for lags of the dependent variable (either real GDP or PCE growth). We construct an additional set of VFCIs that include four lags of the dependent variable (real GDP growth), in both the mean and volatility equation. The resulting series, shown in Figure B.1, is similar to the baseline VFCI. A notable difference is during COVID-19 crisis, where including the lags in the mean equation causes an overshooting and rebound during the crisis.

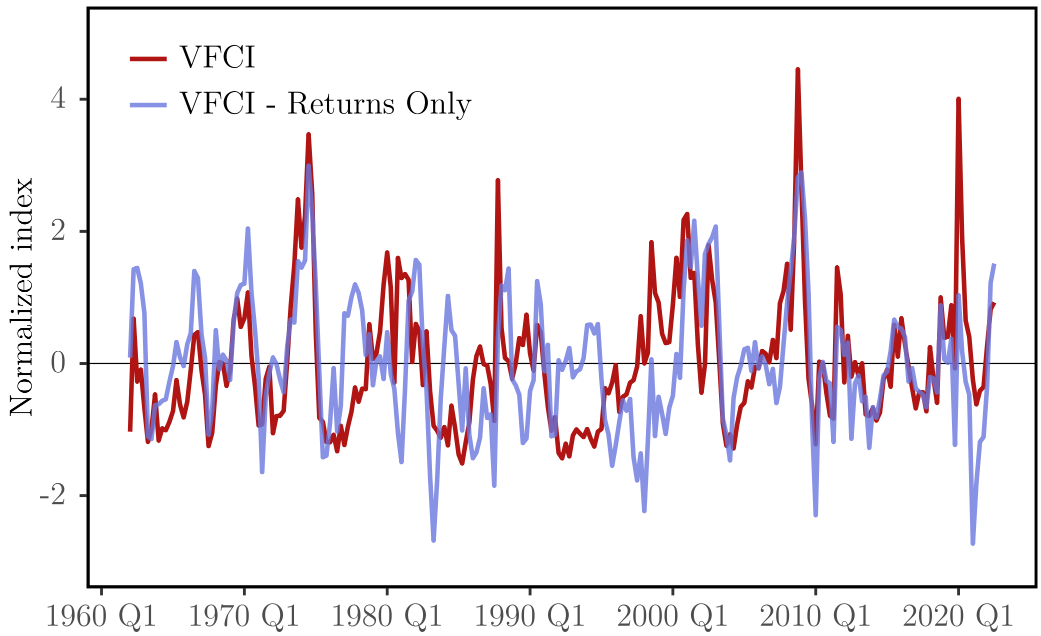
Some models give rise to an estimation of the VFCI that controls for yields at various maturities. We construct an additional VFCI that includes the one, five, and ten year U.S. Treasury yields in the estimation of the mean. The resulting series, shown in Figure B.1, is nearly identical to the baseline VFCI.

We use six financial variables in estimating the VFCI which are described in Table 1. The only variable that is not traded is realized volatility, which is instead constructed from returns. We remove realized volatility and use the first four principal components of the remaining five financial variables to estimate a new VFCI, shown in figure B.1. This robustness check shows that we estimate a similar VFCI when using only traded financial assets.

In Figure B.2 we consider a VFCI constructed using only one financial asset—equity returns. This is more comparable to an index like the VIX, which is based only on equities. The VFCI constructed from returns is noticeably different than the baseline VFCI, particularly during the 1990s and the COVID-19 period.

FIGURE B.2

VFCI and VFCI estimated with only equity returns



Notes: The red line plots the *VFCI*, the volatility financial conditions index, reproduced from Figure 1. The purple line shows a VFCI estimated using equity returns only. All series are scaled to be mean zero with a standard deviation of one.

B.2 Robustness of Macro-Financial Dynamics

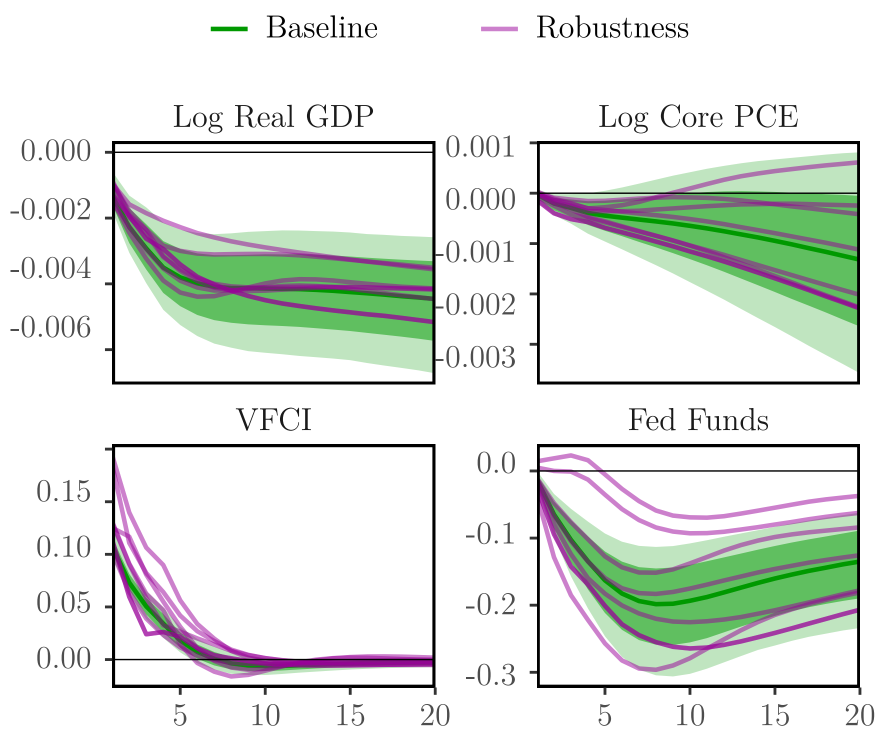
This section reports robustness checks for the Bayesian VAR with heteroskedastic regimes.

B.2.1 VFCI Variations

We begin by using alternate definitions of the VFCI (which are shown in Figure B.1) to estimate our four variable BVAR, where the variables are log real GDP, log core PCE, the federal funds rate, and the alternate VFCI. The BVARs are identified using the same heteroskedastic regimes as in the baseline case.

Figure B.3

IRFs of VFCI Shock: VFCI Variations Robustness



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The purple lines show multiple robustness BVARs using alternative definitions of the VFCI.

The IRFs from a VFCI shock using these alternate BVARs are shown in Figure B.3. Each robustness line (shown in purple) represents one BVAR using one of the alternate definitions of the VFCI. In the background, we show the VFCI shock using the baseline VFCI definition along with the 68% and 95% confidence intervals (in dark and light green shade, respectfully). As we can see from the figure, these alternate VFCI specifications identify a similar VFCI shock to the baseline VFCI. The only two specifications of note use are (1) the VFCI using five principal components and (2) the VFCI using all financial variables. These two robustness specifications have a milder response of the federal funds rate, which is well outside the posterior error bands of the baseline VFCI.

The two VFCI variations with slightly attenuated responses of the federal funds rate are the VFCI using five principal components of the financial variables and the VFCI using all six of the financial variables. These two variations of the VFCI are very similar to each other (see Figure B.1) and have a lower measure of the VFCI during the 1970s compared to the baseline series.

B.2.2 VFCI estimated with only equity returns

We consider the identified VFCI IRFs from a VFCI that is constructed using only equity returns (as opposed to six financial variables) in Figure B.4. While the responses of the VFCI, real GDP, and prices are similar, we see that there is almost no response of the federal funds rate using this alternative VFCI. We prefer our baseline VFCI as a better measure of financial conditions.

B.2.3 BVAR Robustness

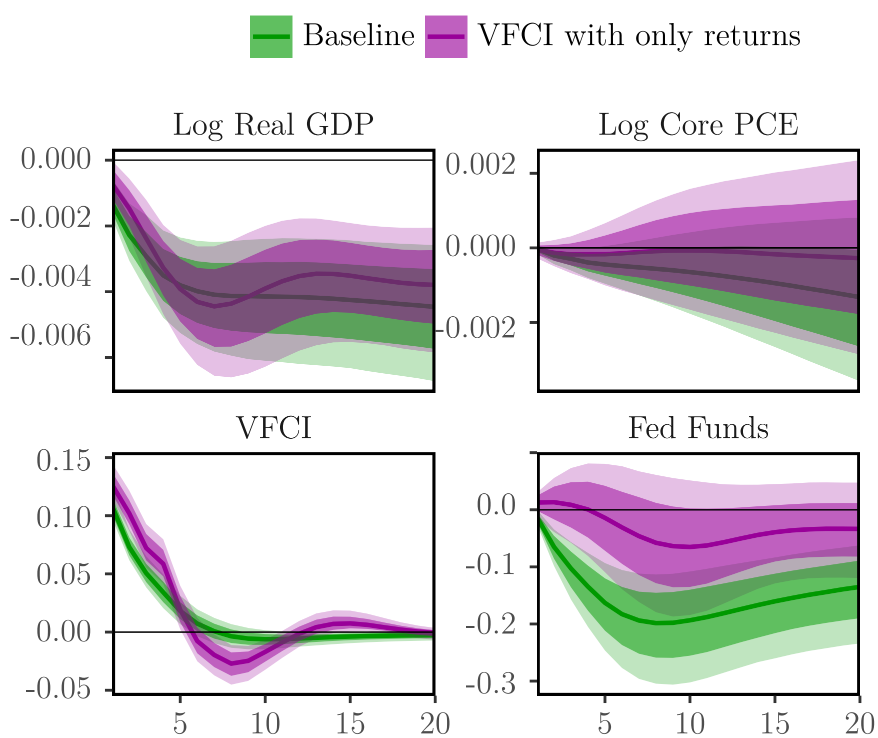
Next we consider robustness checks to the BVAR. We hold constant the baseline BVAR variables: log real GDP, log core PCE, federal funds rate, and the baseline VFCI and instead vary other aspects of the BVAR setup and construction. Figure B.5 shows the IRFs for a VFCI shock with each line representing a different robustness check. The baseline VFCI IRFs are shown in green with the 68% and 95% confidence intervals (shown in dark and light green, respectfully).

We estimate the heteroskedastic BVAR with the time period ending in 2007Q4, before the Global Financial Crisis and the Covid-19 crisis (as opposed to 2022Q3 in the baseline). The average causal effects remain very similar, though the response of log real GDP is attenuated in this sample. Thus, regardless of time period under consideration and whether or not crises are excluded, a VFCI shock, as a representation of tightening financial conditions, causes monetary policy to become more accommodative and real economic activity to fall.

We estimate the heteroskedastic BVAR with stationary variables: real GDP growth, core PCE inflation, federal funds rate, and the baseline VFCI. The IRFs from this model are shown in B.5, with the responses of real GDP growth and core PCE inflation transformed to cummalative IRFs. This allows us to compare the results to the baseline model which has those variables in levels. As seen in the figure, the stationary BVAR IRFs are remarkably similar to our baseline estimation.

Figure B.4

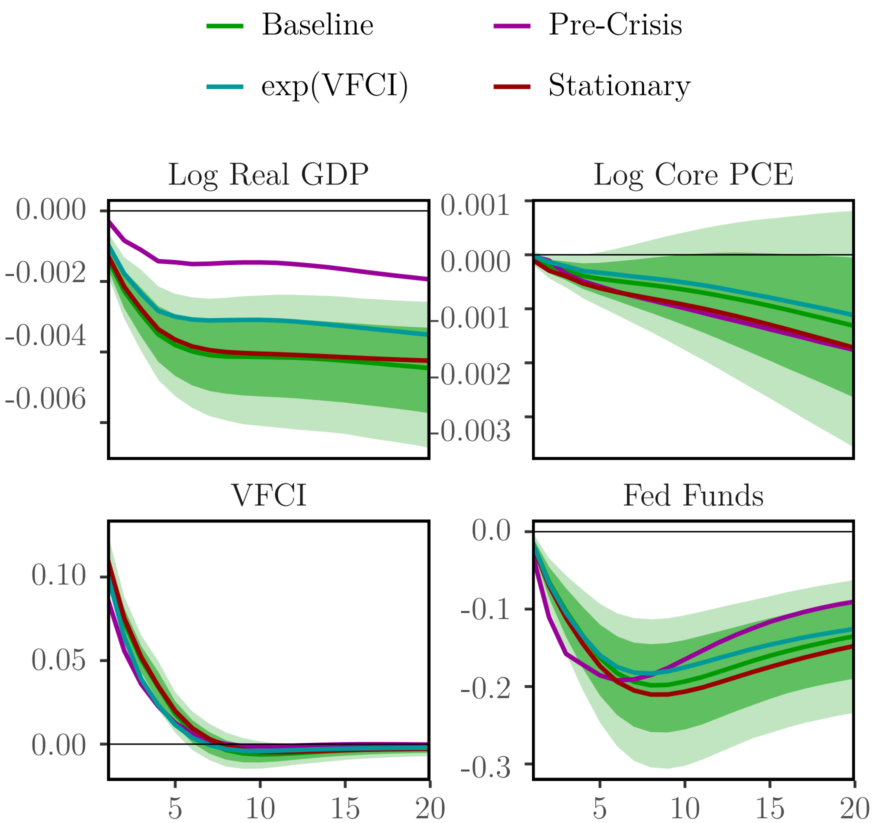
IRFs of VFCI Shock: VFCI with only equity returns



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The purple line shows the impulse responses to the VFCI structural shock in a volatility-identified BVAR model, with the VFCI constructed using only equity returns.

Figure B.5

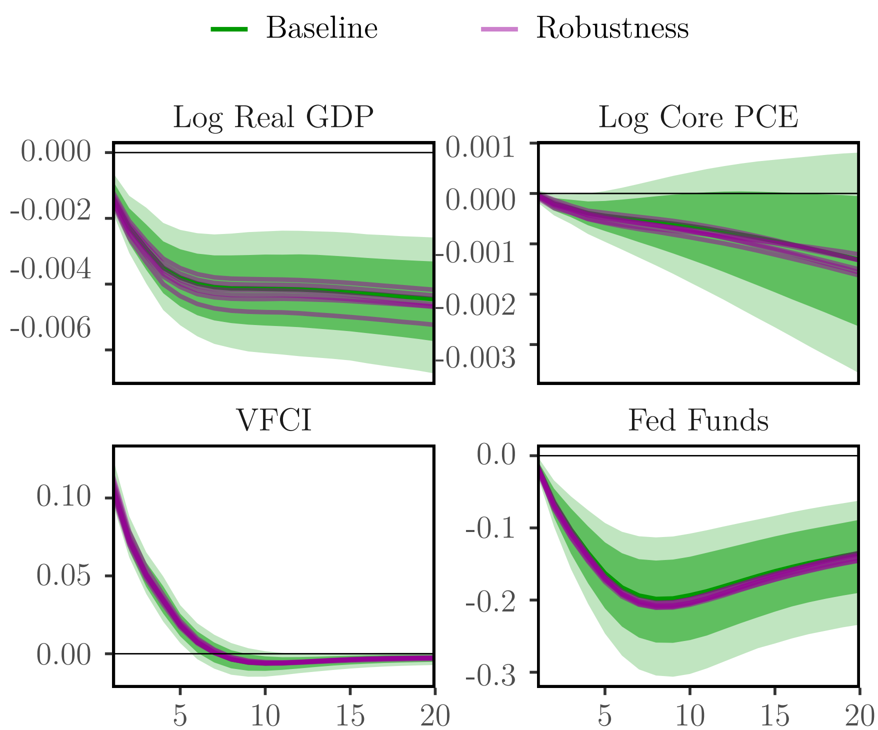
IRFs of VFCI Shock: Other Robustness Checks



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The other lines show various robustness checks of the BVAR.

The Bayesian VAR estimation requires a Minnesota prior with two parameters: "tightness" and "decay". Our baseline uses values of 3 and 0.5, respectfully. For robustness, we try values of 1, 2, and 5 for the tightness parameter, and values of 0.3 and 0.7 for the decay parameter. In addition, we estimate the heteroskedastic BVAR with 1,000,000 draws instead of 10,000 draws as in the baseline model. The robustness of the estimation when we increase the number of draws is useful to corroborate using 10,000 draws for the baseline models as it leads to faster estimation with the same conclusions on the impact and response of VFCI.

FIGURE B.6
IRFs of VFCI Shock: Bayesian Parameter Robustness



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The purple lines show multiple robustness BVARs using alternative parameters for the BVAR.

All of the results of varying these Bayesian VAR parameters are shown in Figure B.6. The dynamic causal effects of all variations to the Bayesian VAR parameters are almost identical.

B.2.4 Second Financial Variable

This section estimates the heteroskedastic BVAR with five variables instead of four as in the paper. We seek to analyze the robustness of the dynamic causal effects with the inclusion of a second financial variable as suggested in Brunnermeier et al. 2021. In particular, we analyze a variety of cases, by including in turn separately the Excess CAPE Yield (ECY) of Shiller 2000, the Gilchrist and Zakrajšek 2012 (GZ) corporate bond risk premium, and the 3-month Libor-US Treasury spread (TED spread), and the NFCI of the Federal Reserve Bank of Chicago.

We find that the inclusion of the second financial variable does not change the implications of the baseline case with four variables. Including the fifth variable leads to similar magnitude of the responses of monetary policy, output, and prices to VFCI shocks. Therefore, the empirical results with one financial variable are found to hold robust.

B.2.5 Three Financial Variables

We construct a BVAR that includes the VFCI, the GZ spread, and the TED rate to test any dynamics between the different financial variables.

We look at the dynamics of the VFCI shock when both the TED rate and GZ spread are included. Here we see that the VFCI shock recreates the baseline results, with a prolonged negative reaction from real GDP and the federal funds rate. We do see a larger negative response from core PCE, in contrast to the insignificant result in the baseline. We also notice that a VFCI shock does create a positive response in the GZ spread, but delayed by a few quarters. However, a VFCI shock has no impact on the TED rate.

Next, consider the estimated responses within this BVAR of a GZ shock when both the VFCI and TED rate are included. The GZ shock has little impact on the VFCI, real GDP, or the federal funds rate, controlling for the other shocks in the BVAR.

The TED rate shock, by contrast, has some long run impact on real GDP and the federal funds rate. It also does not have any impact on the VFCI or the GZ spread.

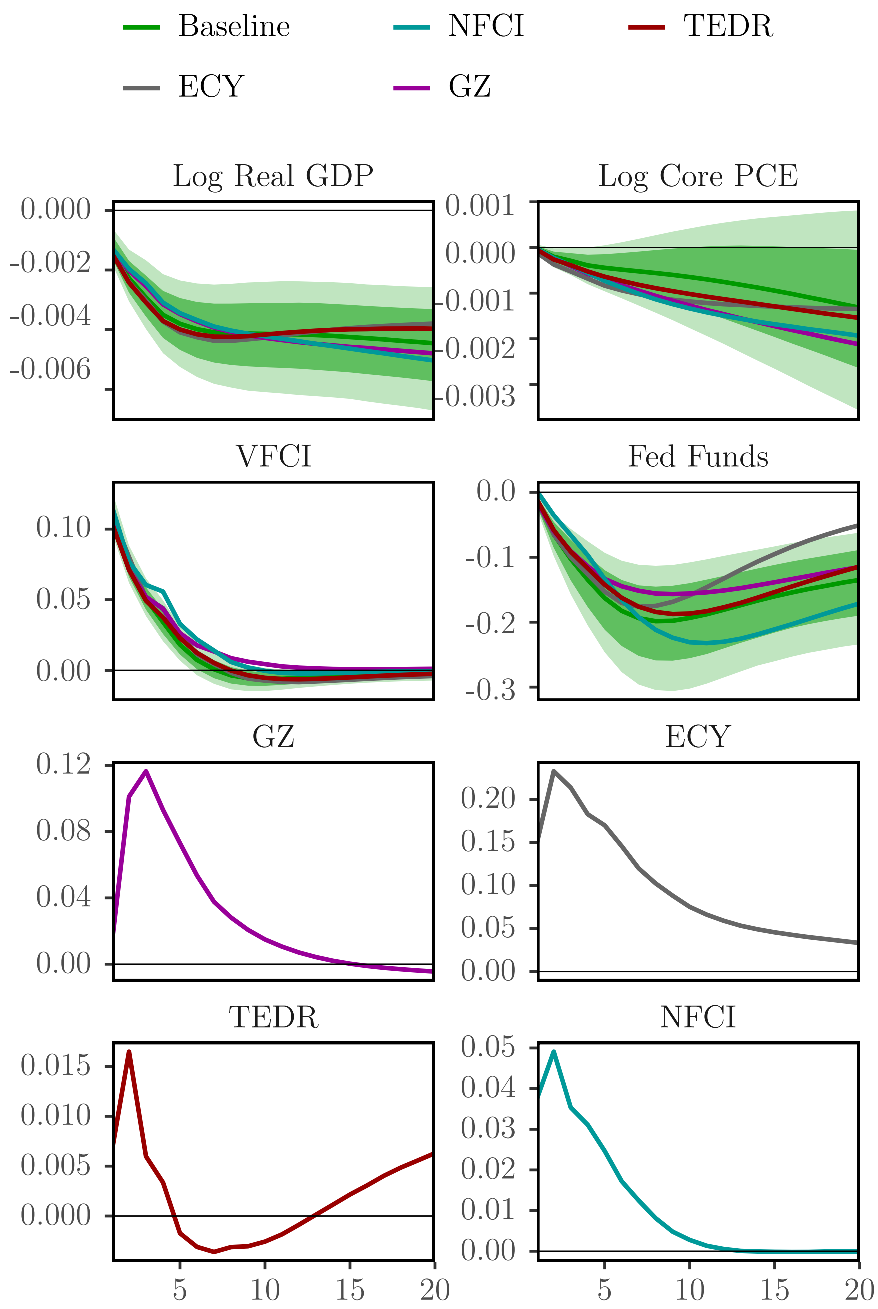
B.2.6 Gaussian Errors

In this section we consider the robustness of our assumption in the baseline heteroskedastic BVAR to use a t-distribution for the residuals rather than a Gaussian distribution. We visually inspect the distribution of the BVAR residuals with a qq-plot, run statistical tests for normality, and finally present IRFs of a BVAR estimated using a gaussian prior for the residuals.

We check the distributions of the VAR residuals using qq-plots (quantile-quantile plots), which is a graphical method to compare two probality distributions by comparing their quantiles. In this case, we compare the actual distribution of each of the VAR residuals to a theoretically normal distribution. In Figure B.9, normally distributed data would fall along

Figure B.7

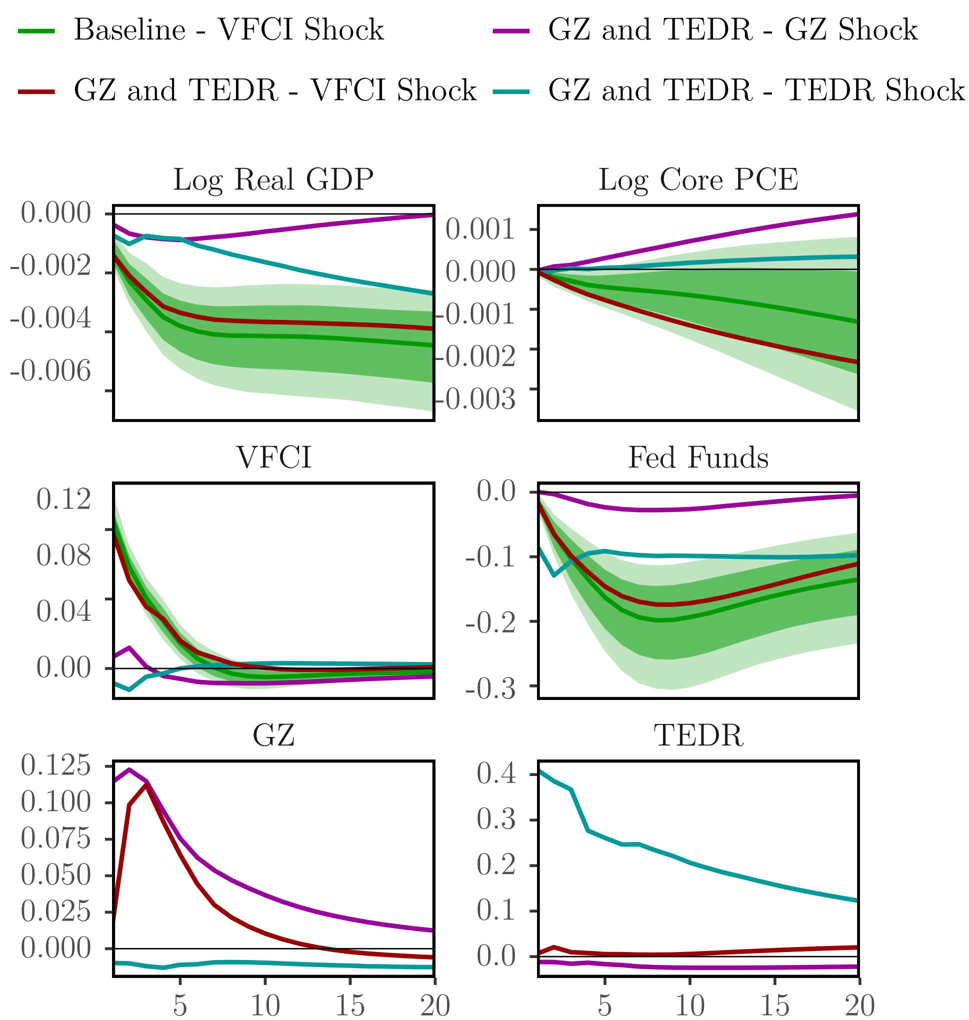
IRFs of VFCI Shock: Second Financial Variable



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The other lines show impulse responses to the VFCI structural shock in a five variable volatility-identified BVAR model where the additional variable is one of: GZ spread, Excess CAPE Yield, TED rate, or the NFCI.

Figure B.8

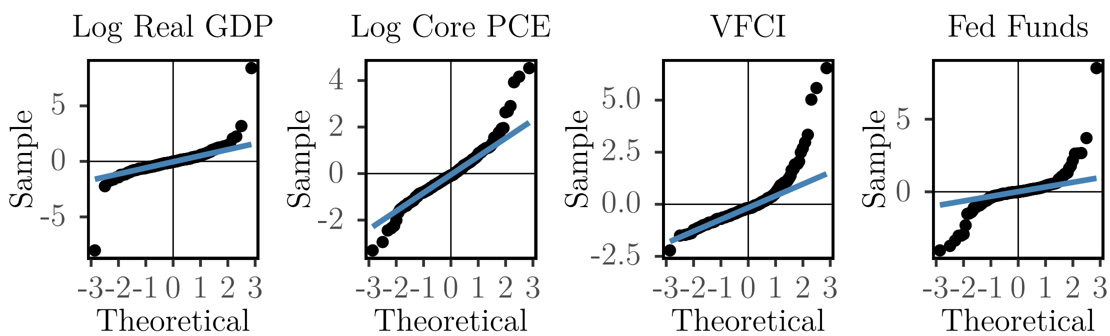
IRFs of Various Financial Shocks: Three Financial Variables



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The other lines show impulse responses to either the VFCI, GZ, or TED rate shock in the volatility-identified BVAR model with six variables (GZ spread and TED rate included with four variables in the baseline).

the 45 degree line. Instead, each of the VAR residuals have strong nonlinear patterns in their sample distributions, implying non-normality.

FIGURE B.9
Q-Q Plot of VAR Residuals



Notes: The theoretical distribution is a standard normal. The sample data comes from the BVAR residuals. The 45 degree line shows where the sample data would fall if it was normally distributed, while the data points show where the actual observed sample quantiles fall.

In addition, we perform statistical tests (Mardia and Shapiro-Wilk) on the residuals of the BVAR to check if the residuals are normally distributed. The results of these tests are reported in Table B.2. All tests reject with a high level of significance that the residuals are normally distributed. The Mardia tests for kurtosis and skewness reject that kurtosis and skewness are not present in the data. If either test was significant, we would reject normality; in this case both are significant. The Shaprio-Wilk test of normality also rejects the null hypothesis for all of the variables.

TABLE B.2
Univariate Normality Tests of the VAR Residuals

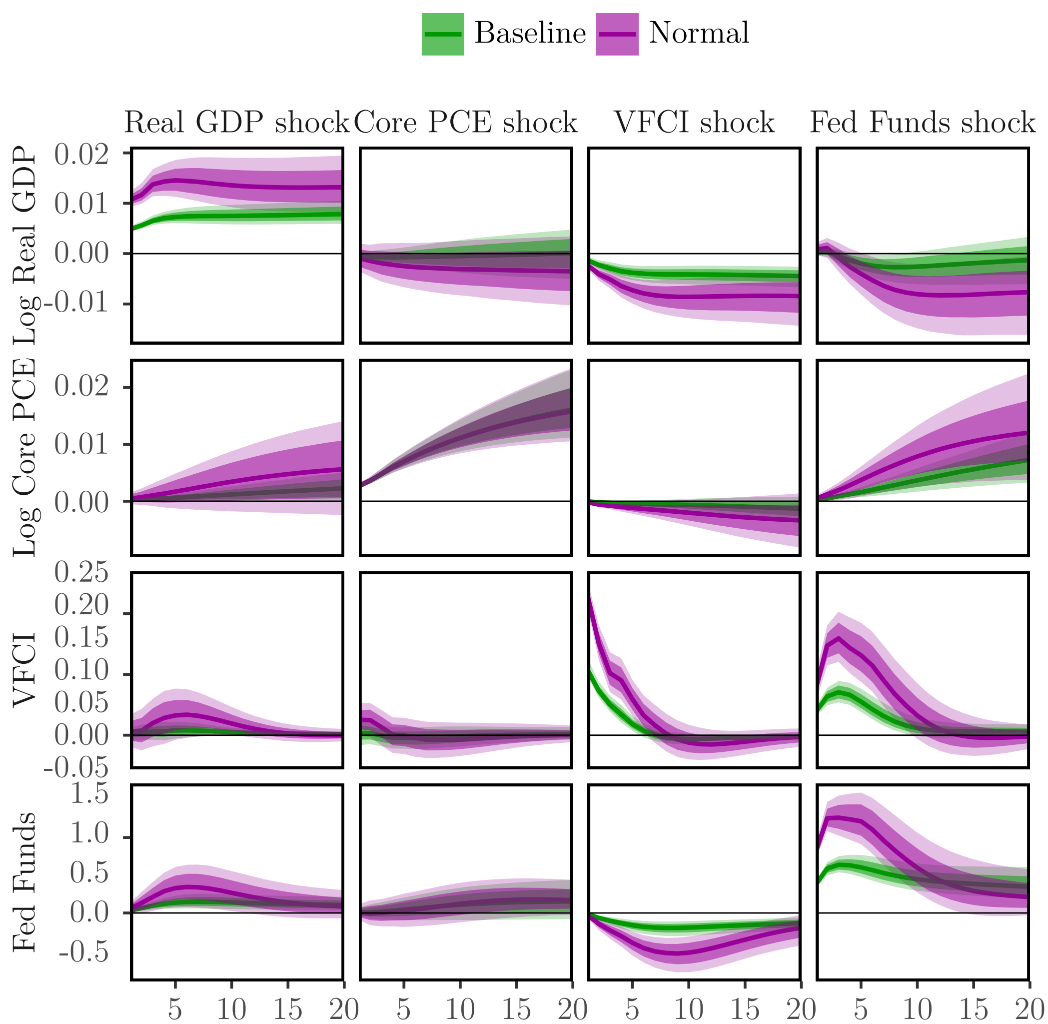
	Mardia				Shapiro-Wilk	
	Kurtosis		Skewness		Normality	
Variable	Statistic	p-value	Statistic	p-value	Statistic	p-value
Log Real GDP	116	0	9	0.0021	0.68	0
Log Core PCE	11	0	25	0	0.94	0
VFCI	42	0	332	0	0.76	0
Fed Funds	82	0	184	0	0.68	0

Notes: Small p-values indicate non-normality. The Mardia tests the kurtosis and skewness of the data and rejects normality if either is significant. The Shapiro-Wilk tests if the sample is from a normally distributed population and rejects normality if the statistic is significant.

Figure B.10 estimates the heteroskedastic BVAR with Gaussian, instead of t-distributed, shocks. In general, the responses of monetary policy and GDP to VFCI shocks are somewhat more pronounced, as is the response of VFCI to all the structural shocks in the model. As the variance of the structural shocks differs substantially across the subperiods in our dataset, the baseline model of t-distributed shocks is more efficient. However, the conclusions remain the same regardless of the assumption on the distribution so that a tightening of the VFCI leads to a persistent contraction of output and triggers an immediate easing of monetary policy. Conversely, contractionary monetary policy shocks cause tighter financial conditions.

FIGURE B.10

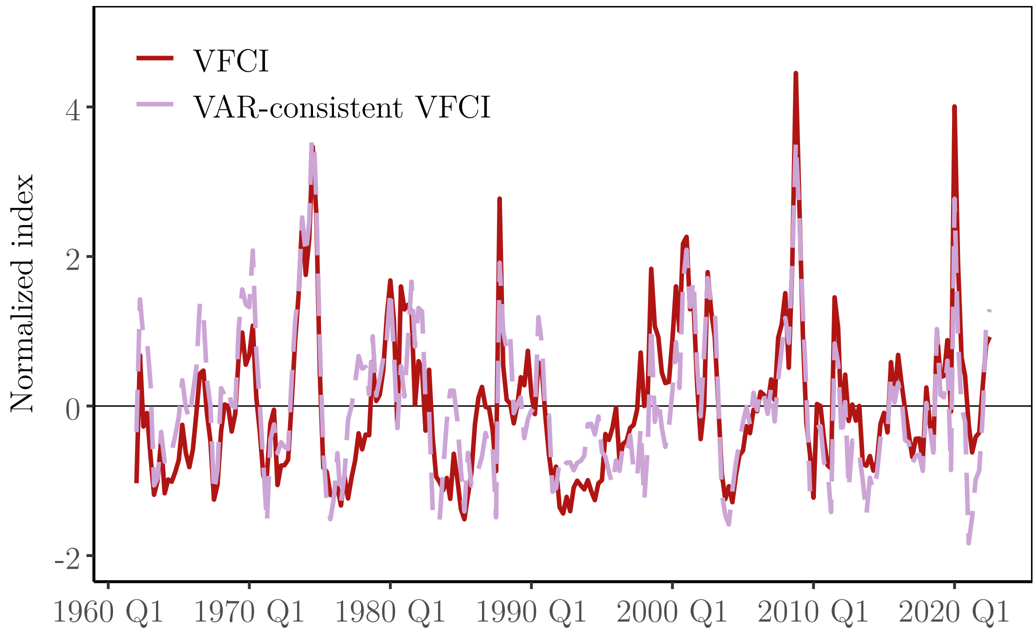
IRFs of VFCI Shock: Normal Errors



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The purple line shows impulse responses to the VFCI shock in the volatility-identified BVAR model using a Gaussian distribution for the residuals (with 68 and 90 percent posterior bands).

Figure B.11

VFCI and VAR-consistent VFCI



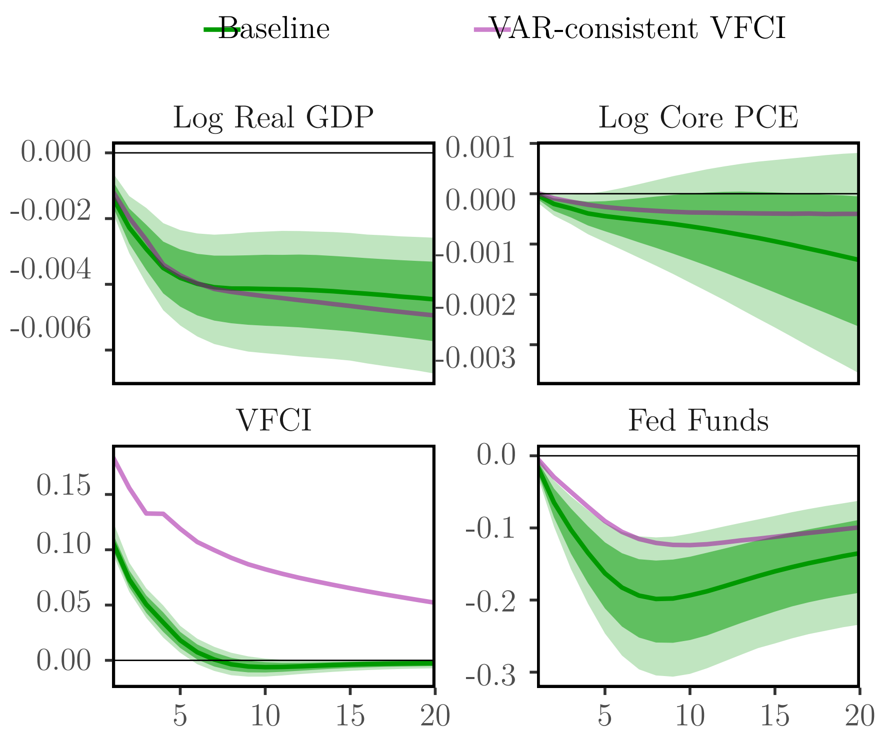
Notes: The red line plots the *VFCI*, the volatility financial conditions index, reproduced from Figure 1. The dotted purple line plots the *VFCI* consistent with a VAR (log real GDP, log personal consumption expenditure price index, and the federal funds rate) with four lags.

B.2.7 Estimating the VFCI within the VAR

In VIII.A, we referemced an alternative VFCI estimated consistently with the BVAR, which we labeled "VAR-consistent" VFCI. This "VAR-consistent" VFCI is constructed using the VAR reduced form residuals of log GDP as ε_{t+1} in equation 25. In other words, we replace the first step (equation 24) of a two-step estimation of heteroskedasticity with our reduced form VAR. The second step remains unchanged, using the same principal components of financial variables to estimate the conditional volatility. Figure B.11 shows this VAR-consistent VFCI is almost the same as the baseline VFCI. In Figure B.12, we show the impulse responses of a shock to this VAR-consistent VFCI identified with heteroskedastic regimes in a BVAR. While the impulse of the VFCI shock is much larger, the impact on the other three variables in the BVAR are similar to the baseline VFCI.

FIGURE B.12

IRFs of VFCI Shock: Internal VAR VFCI



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The purple line shows impulse responses to the VFCI shock in the volatility-identified BVAR model using the internal VAR VFCI.

B.3 Alternative Identification Strategies

B.3.1 External Instruments

Studies in the empirical macroeconomic literature have generally used internal instruments, or shocks identified within the model, to estimate dynamic causal effects. Stock and Watson 2018 consolidate the derivation of dynamic causal effects and asymptotic theory for external instruments in LP and SVAR frameworks and find that the use of external instruments can potentially lead to more credible identification. As discussed in that paper, external instruments in macroeconometric models comprise a relatively new but promising avenue of research. We build on this literature by also using instruments for monetary policy and output in LP and SVAR models as alternative identification strategies to estimate dynamic causal effects.

VFCI Instrument For the VFCI instrument we treat the VFCI as an exogenous process. When estimating the SVAR-IV, this means we can simply use a Cholesky decomposition with the VFCI ordered first to recover the identified shock. When estimating the LP-IV, we use the VFCI time series as the instrument.

Monetary Policy Instrument The estimation of monetary policy shocks has been oft-explored in the literature starting from Romer and Romer 2004's estimation of this shock through a narrative approach. Since then, the literature used various techniques to identify monetary policy shocks (Ramey 2016), such as the high-frequency identification strategy used in Nakamura and Steinsson 2018. Our instrument for the Federal Funds rate is the Romer and Romer monetary policy shock, which starts from 1969Q1 and extends until 1994Q4, interpolated with the Nakamura and Steinsson shock, which was updated and kindly shared with us, from 1995Q1 to 2022Q3.

GDP Growth Instrument The external instrument related to GDP was kindly shared by the authors of Cieslak and Pang 2021 and extends from 1983Q1-2022Q3. This shock is estimated through a sign-restricted VAR approach that places identifying restrictions on the differential response of stock and bond market prices to key macroeconomic announcements. The authors identify growth news shocks among other shocks, and we use the GDP growth shock as an external instrument for GDP growth in a stationary version of our model.

B.3.2 Identification through SVAR-IV

To outline the SVAR-IV identification problem, consider the following reduced-form version of equation 26 for a vector of endogenous variables, $y(t)$

$$B(L)y_t = \eta_t \tag{B.59}$$

where the reduced-form innovations, η_t, satisfy $\eta_t \sim (0, \Sigma_\eta)$ with $E[\eta_s \eta_t'] = 0$ for $s \neq t$ and the polynomial lag operator is $B(L) = I - \sum_{k=1}^{p} B_k L^k$. The innovations, η, are related to the structural shocks, ε, as follows

$$\eta_t = H\varepsilon_t \tag{B.60}$$

where H is invertible. Here, in contrast to the time-varying variance assumption in the previous section, the structural shocks are distributed as $\varepsilon_t \sim (0, \sigma_\varepsilon)$.

Equations 26 and B.59 can be written in terms of their structural moving average representations as $Y_t = \Theta(L)\varepsilon_t$ and $Y_t = C(L)v_t$, where $C(L) = [B(L)]^{-1}$ and $\Theta(L) = C(L)H$. Therefore, H can be written as $H = C(L)^{-1}\Theta(L) = I + B_1 L + ...)(\Theta_0 + \Theta_L + ...) = \Theta_o$+terms in $L, L^2, ...$ The impact effect is $H = \Theta_0$, which implies that $\eta_t = \Theta_0 \varepsilon_t$ Stock and Watson 2018. The SVAR-IV identification problem is to identify Θ_0 by finding a suitable external instrument, Z_t, that satisfies the following conditions

$$E\varepsilon_{1,t} z_t' = \alpha \neq 0 \tag{B.61}$$

$$E\varepsilon_{2:n,t} z_t' = 0 \tag{B.62}$$

Equations B.61 and B.62 are the instrument relevance and exogeneity conditions, meaning that the instrument must be contemporaneously correlated with the structural shock, $\varepsilon_{1,t}$, and uncorrelated with the other structural shocks.

The basic idea to estimate Θ_0 is as follows, with further theory, including on the asymptotics and inference, found in Stock and Watson 2018. Suppose conditions B.61 and B.62 are satisfied, we are able to identify the first structural shock, $\varepsilon_{1,t}$. To recover the other structural shocks in a VAR with n endogenous variables, the reduced form system in equation B.59 is first fit to estimate the vector of innovations η_t.

All the reduced-form innovations apart from those of the first variable, $\eta_{2:n,t}$, are then regressed on $\eta_{1,t}$, using z_t as an instrument. The residuals of this sequence of regressions form a vector $\kappa_{2:n,t}$. Finally, $\eta_{1,t}$ is regressed sequentially on $\eta_{2:n,t}$, using $\kappa_{2:n,t}$ as the instruments. This allows for the identification of the $\varepsilon_{2:n,t}$. Using the identified structural shocks, ε_t, the dynamic causal effects are estimated.[21]

The impact of the VFCI, monetary policy, and output shocks identified through external instruments in an SVAR-IV model corroborate the results from the heteroskedastic BVAR. A tightening of financial conditions caused by a positive VFCI shock, as identified by the penalty function approach, triggers an immediate easing of monetary policy and a contraction in output. The dynamic responses of both output and monetary policy, and output in particular, are somewhat less persistent compared to the heteroskedastic BVAR, but their negative responses upon impact to tight financial conditions are highly significant and similar in magnitude.

We also estimate the impact of monetary policy and growth shocks in the SVAR-IV model. A surprise increase in real GDP leads to an immediate easing of financial conditions, but VFCI tightens in the following quarters as it reverts to the mean. The loosening of financial conditions in response to growth shock may be accorded to the plausibly better identification of these shocks using an external instrument.

B.3.3 Identification through LP-IV

The Local Projections (LP) approach Jordà 2005 has become a popular method of estimating IRFs. The LP model estimates the parameters sequentially through simple linear regressions and is computationally straightforward in practice. LP estimates can theoretically be more robust if a linear VAR is misspecified, although this is not always the case (Plagborg-Møller and Wolf 2021). The LP model can also be estimated using external instruments (Jordà, Schularick, and Taylor 2015). We use our instruments to estimate a local projections-IV (LP-IV) model for VFCI, output, inflation, and monetary policy.

SVAR and LP models were considered conceptually different in the past, but have been shown to estimate the same IRFs as long as a sufficient amount of lags are accounted for and the entire population is modeled as in Plagborg-Møller and Wolf 2021. Ramey 2016, in reviewing the literature, estimates similar models with LPs and SVARs and finds some differences, which—in light of the recent results demonstrating equivalence— could be due to assumptions, lags, samples, and so on. We take note of these previous results and given the choice of a particular sample and time period in this study, we estimate the dynamic causal effects by additionally using an LP-IV approach.

To outline the LP-IV identification problem, consider the moving average version of equation 26, which, as discussed previously, is $Y_t = \Theta(L)\varepsilon_t$. The impulse response of Y_i at

[21] Of note is that this method is related to, but different, from the approach of using an external instrument in a recursive VAR, as in Romer and Romer 2004. As discussed in Ramey 2016, the SVAR-IV method was developed as an alternative way to use external instruments in a VAR framework.

horizon h is estimated from a single regression equation as follows

$$y_{i,t+h} = \Theta_{h,i1} y_{1,t} + u^h_{i,t+h} \tag{B.63}$$

where $u^h_{i,t+h}= \varepsilon_{t+h}, ..., \varepsilon_{t+1}, \varepsilon_{2:n,}, \varepsilon_{t-1}\varepsilon_{t-2,...}$. OLS estimation of B.63 is not valid since $Y_{1,t}$ is correlated with $u^h_{i,t+h}$. However, B.63 can be estimated if we use a suitable external instrument that satisfies the instrument relevant and exogeneity conditions, B.61 and B.62, along with a third condition

$$E\varepsilon_{t+j,t} z'_t = 0, j \neq 0 \tag{B.64}$$

which denotes the requirement that the instrument satisfy lead-lag exogeneity. This means that z_t should be uncorrelated with historical as well as future shocks.

A separate LP-IV regression is estimated for each horizon, h. Also, serial correlation in the errors is modeled since the errors, ε_{t+h}, are serially correlated for all $h > 0$ as ε_{t+h} is the moving average of the forecast errors from t to h. In practice B.63 can be estimated with control variables. The extension of LP-IV with control variables is straightforward, and discussed further in Stock and Watson 2018.

While the identified monetary policy shock has insignificant effects, the GDP growth shock leads to a tightening of the Fed Funds rate and a loosening of financial conditions upon impact.

The VFCI shock exhibits some of the same properties as in the volatility-identified BVAR and SVAR-IV models, that is, it leads to a significant easing of monetary policy and a significant contraction in output. The dynamic causal effects, as in the SVAR-IV model, are somewhat less persistent than in the heteroskedastic BVAR.

B.3.4 Identification through a Recursive VAR

We take one step back and estimate a simple recursive VAR with the ordering defined as output, prices, monetary policy, and financial conditions. VFCI is ordered last in the baseline case, but we assess the robustness of this assumption by ordering the Federal Funds rate last in an alternative specification. Financial conditions and monetary policy could be endogenous based on the empirical evidence in Cieslak, Morse, and Vissing-Jorgensen 2019 and Cieslak and Vissing-Jorgensen 2020, and we mitigate such concerns by changing the forcing variable.

B.3.5 Identification through Sign Restrictions

Sign restrictions are used on the shape of the IRFs in response to the structural shocks following the penalty function approach based on Uhlig 2005. We impose three sets of sign restrictions to identify three shocks. Each shock is identified from its own model, meaning that the sign restrictions are not exclusionary from one another.

We restrict the response of prices and output to be negative in identifying the monetary policy structural shock. For a real GDP shock, we restrict the response of the federal funds rate to be positive. To identify a VFCI shock, we restrict the response of real GDP and the federal funds rate to be negative. Any variable not listed above is unrestricted and can take any sign in response to the identified shock. In addition, the sign restriction imposes nothing on the magnitudes of the IRFs identified, only on the immediate sign on impact.

B.4 Total and Residual Volatility

The VFCI is estimated according to equation (25) and defined to be $\hat{\delta}PC_t$. This is a measure of the conditional log volatility of real GDP growth. In this section, we consider two robustness checks to our identified macro-financial dynamics by investigating the residual log volatility $\hat{\upsilon}_{t+1}$ and total log volatility $\hat{\delta}PC_t + \hat{\xi}_{t+1}$ from equation (25).

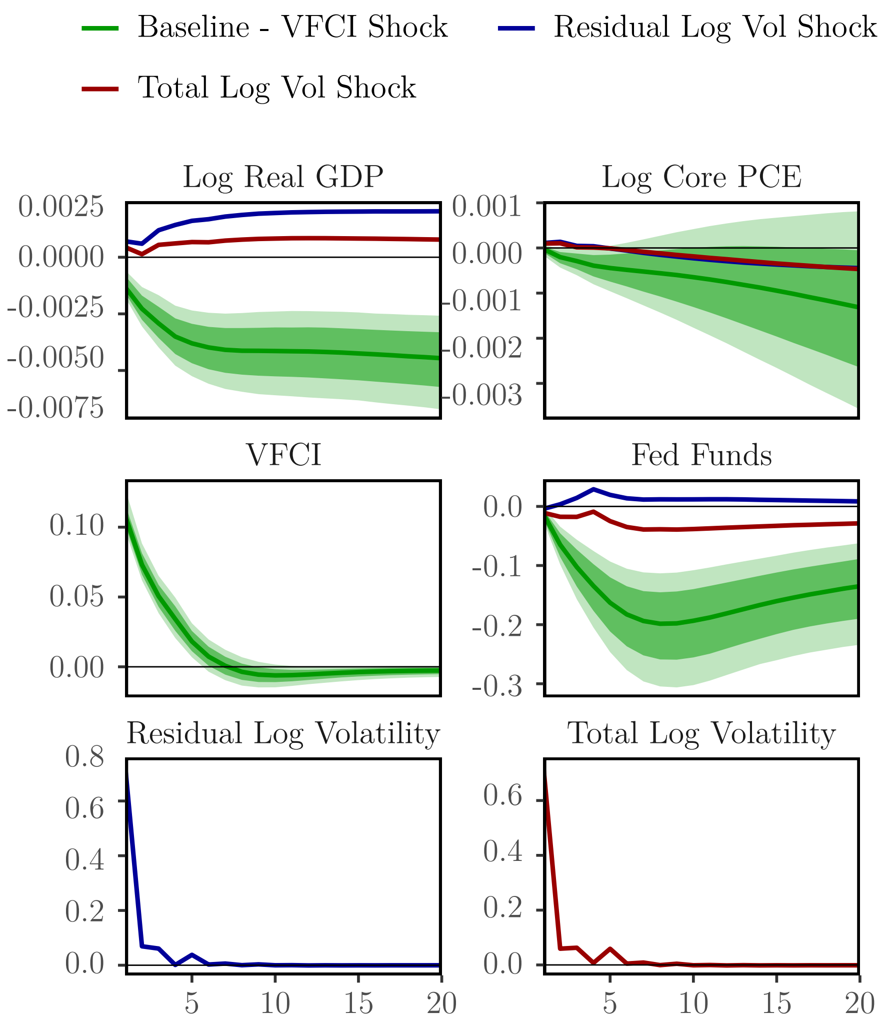
We then re-estimate the heteroskedastic BVAR using either of these alternative log volatility measures instead of the VFCI.

In Figure B.13, we can see that the IRFs computed using the residual log volatility instead of the VFCI are quite different from those estimated using the VFCI. They show a slightly positive response in real GDP, and no response in the federal funds rate or the price level, showing that the VFCI—the projection of the log market price of risk onto the asset span—is the only part of the total log volatility that has economically meaningful effects on macroeconomic dynamics.

Figure B.13 also shows that the IRFs computed using the total log volatility are also quite different from those estimated using the VFCI. They show little to no impact on macroeconomic dynamics, which shows that conditioning the total log volatility on returns—projecting onto the asset span—is an important step in the construction of the VFCI as it extracts from the total log volatility a signal that is not immediately available in the total log volatility.

Figure B.13

IRFs of Log Volatility Shocks: VFCI, Residual, and Total



Notes: The green line shows impulse responses to the VFCI structural shock in the volatility-identified BVAR model with 68 percent (dark green) and 90 percent (light green) posterior error bands. The red line shows impulse responses to the Total Log Volatility Shock in a volatility-identified BVAR model estimated with total log volatility replacing VFCI. Likewise, the blue line shows the impulse responses to the residual log volatility shock in a volatility-identified BVAR model estimated with the residual log volatility replacing the VFCI.

PUBLICATIONS

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