The Market Price of Risk and Macro-Financial Dynamics

Tobias Adrian, Fernando Duarte, Tara Iyer

WP/23/199
ABSTRACT: We propose the conditional volatility of GDP spanned by financial factors as a “Volatility Financial Conditions Index” (VFCI) and show it is closely tied to the market price of risk. The VFCI exhibits superior explanatory power for stock and bond risk premia compared to other FCIs. We use a variety of identification strategies and instruments to demonstrate robust causal relationships between the VFCI and macroeconomic aggregates: a tightening of financial conditions as measured by the VFCI leads to a persistent contraction of output and triggers an immediate easing of monetary policy. Conversely, contractionary monetary policy shocks cause tighter financial conditions.

JEL Classification Numbers: E32, E44, G12, C22

Keywords: Macro-Finance; Financial Conditions Index; Monetary Policy; Asset Pricing; Market Price of Risk; Consumption Volatility; Causal Identification

Author’s E-Mail Address: tadrian@imf.org
The Market Price of Risk and Macro-Financial Dynamics*

Tobias Adrian\textsuperscript{1}, Fernando Duarte\textsuperscript{2}, and Tara Iyer\textsuperscript{1}

\textsuperscript{1}International Monetary Fund
\textsuperscript{2}Brown University

September 8, 2023

Abstract

We propose the conditional volatility of GDP spanned by financial factors as a “Volatility Financial Conditions Index” (VFCI) and show it is closely tied to the market price of risk. The VFCI exhibits superior explanatory power for stock and bond risk premia compared to other FCIs. We use a variety of identification strategies and instruments to demonstrate robust causal relationships between the VFCI and macroeconomic aggregates: a tightening of financial conditions as measured by the VFCI leads to a persistent contraction of output and triggers an immediate easing of monetary policy. Conversely, contractionary monetary policy shocks cause tighter financial conditions.

Keywords: Macro-Finance, Monetary Policy, Financial Conditions, Growth-at-Risk, FCI, Market price of risk, Consumption volatility.

JEL Codes: E44, E52, G12

*The views expressed this paper are those of the authors and do not necessarily represent the views of the International Monetary Fund, its Management, or its Executive Directors. We would like to thank Miguel Acosta for sharing an updated dataset of monetary policy shocks from Nakamura and Steinsson 2018, Anna Cieslak for sharing a updated dataset of the news shocks from Cieslak and Pang 2021, and Brunnermeier et al. 2021 for making their code available. We thank Markus Brunnermeier, John Campbell, Emi Nakamura, and Harald Uhlig for helpful comments. We also thank Luu Zhang for outstanding research assistance.
1 Introduction

Financial conditions indices (FCIs) are widely used by policy makers and practitioners, and are also increasingly common in the academic literature. However, FCIs are largely empirically motivated and lack a solid link to economic theory. In this paper, we propose an FCI that is the market price of risk in the economy under general circumstances and estimate it as the conditional volatility of GDP spanned by financial factors. We call this FCI the VFCI, or Volatility-FCI.

We start with a general framework for modeling macro-financial interactions. The absence of arbitrage implies the existence of a pricing kernel that prices all assets in the economy. The volatility of the pricing kernel is generally referred to as the “market price of risk”. When a representative consumer with time separable preferences exists, the market price of risk can be measured as volatility of aggregate consumption (see Breeden (1979), Duffie and Zame 1989) and, more generically, the volatility of measures of aggregate economic activity such as GDP. When preferences are not time separable, the VFCI is a forward-looking measure of current and future expected prices of risk. This theoretical framework implies that the VFCI can be estimated as the conditional volatility of consumption or GDP that is spanned by financial factors.

Empirically, we run a linear regression with conditional heteroskedasticity where real GDP growth is the dependent variable, and lagged financial variables are the independent variables that determine the conditional mean and the conditional volatility of GDP growth. We define the VFCI to be the (log of the) predicted conditional volatility from this regression. Hence, the VFCI is the log conditional volatility of GDP growth spanned by financial factors, which, as mentioned earlier, is determined in our theoretical framework by the (current and future expected) market price of risk in the economy, motivating the interpretation of the VFCI as a measure of financial conditions. Our results are robust to using real consumption instead of GDP in the VFCI construction.

The VFCI is empirically also tightly linked to the conditional mean of GDP growth, constructed as the predicted value for the mean in the heteroskedastic regression used to construct the VFCI. Empirically, the conditional mean and conditional volatility of GDP growth are negatively correlated, generating long left tails in the unconditional distribution of GDP growth, as periods of low expected growth tend to have high volatility, creating periods of “vulnerability” in which amplification of shocks is largest precisely when growth expectations are already low. Therefore, our simple empirical approach replicates one of the main stylized facts (the negative conditional correlation between mean and volatility of GDP growth) of the literature on growth-at-risk (Adrian, Boyarchenko, and Giannone 2019, Adrian et al. 2022).

Leading examples are the habit formation model of Campbell and Cochrane 1999 and the Epstein-Zin model by Bansal and Yaron 2004.
From a theoretical point of view, it is the volatility of GDP, not the mean of GDP, that is related to the pricing of risk in the economy. Hence, we use the conditional volatility, not the conditional mean, to construct our VFCI. This is a departure from the Goldman Sachs FCI (GSFCI of Hatzis and Stehn 2018), which is estimated as conditional mean of future GDP growth.

Our estimate also deviates from the NFCI proposed by the Federal Reserve Bank of Chicago (Brave and Butters 2011). The NFCI uses a Kalman filter to extract a common component from 105 financial variables (based on Doz, Giannone, and Reichlin 2012). The NFCI is a purely statistical measure of financial conditions. In contrast, our index is derived from economic theory and thus has a more rigorous interpretation.

An alternative estimate of the price of risk in the macroeconomy is equity-implied volatility, as measured by the VIX. If the entire wealth portfolio were publicly traded, the VIX might be a good estimate of the aggregate volatility of output or other macroeconomic variables. However, it is well known that only a fraction of the overall capital stock is traded, and hence the VIX is only an imperfect measure of the market price of risk.

Of course, the VFCI, NFCI, GSFCI, and the VIX are correlated. Simple regressions show that the VFCI is better at explaining common measures of stock and bond risk premia than any of the other measures. In particular, we use a credit spread—the GZ spread—as a measure of the corporate bond risk premium (the GZ spread is by Gilchrist and Zakrajšek 2012) and Shiller’s CAPE (Shiller 2000) as metric of risk premium in stocks. In each case, the VFCI has higher significance than the alternative FCIs, and makes the alternative FCIs insignificant when included jointly. Hence, we conclude that the VFCI is the preferable metric of the price of risk in the economy from both a theoretical and an empirical perspective.

Our next contribution is to study the causal relationship between the VFCI and macroeconomic aggregates. Using a variety of identification approaches and instruments, we show that a tightening of the VFCI leads to an immediate easing of monetary policy and a persistent contraction of output. Conversely, contractionary monetary policy shocks lead to tighter financial conditions.

More specifically, we model the dynamics of the economy by a structural vector autoregression (SVAR) and identify the structural shocks by exploiting the conditional heteroskedasticity of the SVAR variables, as in Brunnermeier et al. 2021. We work with time series of quarterly frequency from 1962:Q1 to 2022:Q3 to allow for a long enough time period to capture various regime shifts in the data and use Bayesian methods to estimate the model.

In order to give economic meaning to the structural shocks identified through heteroskedasticity and gain confidence in the causal relationship between the VFCI and
macroeconomic aggregates, we use external instruments proposed in the literature to estimate the causal impact of shocks in an SVAR with instrumental variables and using local projections (LP) with instrumental variables. Based on recent results in Plagborg-Møller and Wolf 2021, the LP and SVAR models have been shown to estimate the same IRFs as long as a sufficient amount of lags are included and the entire population is modeled. However, Ramey 2016 reviews various alternative identification schemes and finds differences in the IRFs from SVARs and LPs in applications, so we show our results hold for both specifications. We also estimate a simple recursive VAR without instruments, where identification is attained through the “ordering” of the variables, that is, by assuming that certain variables respond to certain shocks contemporaneously and to other shocks with a lag. Finally, we estimate a sign-restricted SVAR that identifies the causal impact of monetary policy and VFCI shocks.

In all instances, we find robust causal, economically large, and statistically strong effects of VFCI shocks on monetary policy and GDP, and of monetary policy shocks on the VFCI. We do not find a tight link between the VFCI and inflation in either direction.

2 The VFCI as Price of Risk

Time is discrete and indexed by $t = 0, 1, 2, ...$. There exists a representative agent who values non-negative aggregate consumption streams $C = \{C_t\}_{t=0}^{\infty}$ according to a utility function $U(C)$ that is strictly increasing, concave, and differentiable. The set of feasible consumption streams is denoted by $\mathcal{C}$. The set $\mathcal{C}$ encodes all the constraints faced by the representative agent. We assume that there is a unique optimal $C^*$ in the interior of $\mathcal{C}$ that maximizes $U(C)$.

The setup just described allows for a broad range of models and economic environments including incomplete markets, non-Markovian dynamics, trading frictions, any convex and some non-convex constraints on the representative agent or on asset prices, illiquidity, partial information, real and nominal rigidities, etc. For example, the setup encompasses the habit formation model of Campbell and Cochrane 1999, the long-run risk model of Bansal and Yaron 2004, the disaster risk models of Barro 2006; Gabaix 2012, preferences with ambiguity aversion and robustness as in Hansen and Sargent 2010, and essentially all New Keynesian models that admit a representative agent.

Let the continuation value of $U$ from the point of view of time $t$ be $U_t$. We denote

\footnote{For conditions under which a representative agent of the type we assume exists, see Skiadas 2009 for an overview and Matoussi and Xing 2018 and Monoyios 2022 for very general settings. These references also show that we could further weaken our assumptions to utility functions that are not differentiable but have a well-defined “supergradient density”.}
the gradient of $U_t$ by $\nabla U_t$, and note that $\nabla U_t(C)$ is a vector known at time $t$.\footnote{Let the continuation value of $U$ from the point of view of time $t$ be $U_t$, and let the set of feasible consumption streams be $C_t \subseteq \mathcal{C}$. The time-$t$ directional derivative of $U$ at $C = \{C_s\}_{s=t}^\infty$ in the direction $h = \{h_s\}_{s=t}^\infty$ is the limit
\[
U'_t(C; h) := \lim_{\alpha \downarrow 0} \frac{U_t(C + \alpha h) - U_t(C)}{\alpha}.
\]
The time-$t$ gradient of $U$ at $C$ is a vector $\nabla U_t(C) = \{d_s\}_{s=t}^\infty$ such that
\[
U'_t(C; h) := \mathbb{E}_t \sum_{s=t}^\infty d_s h_s \quad \text{for all } h \in C_t.
\]
} To work with consumption growth $\Delta c_t = \log C_t - \log C_{t-1}$ rather than with consumption levels, we write
\[
\frac{\nabla U_t(C)}{\nabla U_{t-1}(C)} = G(\Delta c)
\]
for some function $G$ (that can be different for different $t$), where $\Delta c = \{\Delta c_t\}_{t=0}^\infty$ is the entire path of consumption growth.

The absence of arbitrage opportunities is a necessary condition for equilibrium. The Fundamental Theorem of Asset Pricing (FTAP), implies that, in equilibrium, there exists at least one stochastic discount factor $SDF = \{SDF_t\}_{t=0}^\infty$ such that
\[
1 = \mathbb{E}_t[SDF_{t+1} R_{t+1}]
\]
for all $t$ and for all returns $R_{t+1}$ that can be obtained by trading financial assets.\footnote{For a comprehensive exposition of the FTAP, including many extensions and references, see Duffie 2010. For the broad range of conditions under which the FTAP holds, see Delbaen and Schachermayer 2006 and Kabanov and Safarian 2009 for overviews, and Bálint and Schweizer 2022 for a recent formulation with particularly mild conditions.}

There exists a unique $SDF$ if, and only if, markets are complete (which we do not assume). We refer to the set of all possible cash flows that can be generated by trading financial assets as the asset span. Despite the possibility of non-uniqueness for the $SDF$, the orthogonal projection of any given $SDF$ onto the asset span is the same as the projection of any other $SDF$. Furthermore, if we denote this unique projection by $SDF^*$, then we have that
\[
1 = \mathbb{E}_t[SDF^*_{t+1} R_{t+1}]
\]
for all $t$ and for all marketed returns $R_{t+1}$, which means that $SDF^*$ is also a valid stochastic discount factor.

The FOC for optimal consumption for the representative agent is
\[
G(\Delta c) = SDF_t
\]
(1)

Under the assumptions we have made, the FOC is both necessary and sufficient for
optimality. Linearizing $G$ around $\Delta c$ gives a first-order approximation to equation (1)

$$G_t(\Delta c) \approx \bar{g} + \sum_{s=-\infty}^{\infty} g_s \mathbb{E}_t[\Delta c_s],$$

(2)

where the $\bar{g}$ and $g_s$ are linearization constants. Using (2) in (1), we get

$$\bar{g} + \sum_{s=-\infty}^{\infty} g_s \mathbb{E}_t[\Delta c_s] = SDF_t$$

(3)

If we truncate the limits of the sum to finite values, equation (3) is a linear rational expectations equation. Assuming that a solution exists and that $g_t \neq 0$, equation (3), after truncating the limits of the sum to finite values, gives

$$\Delta c_t = \mathbb{E}_t[m^{-1}(L)SDF_t]$$

where $L$ is the lag operator and $m(\cdot)$ is a polynomial. We first decompose the SDF into its expected and unexpected components

$$\Delta c_t = \mathbb{E}_{t-1}[m^{-1}(L)SDF_t] + \epsilon_t$$

where $\epsilon_t := (\mathbb{E}_t - \mathbb{E}_{t-1})[m^{-1}(L)SDF_t]$. Next, we decompose $\epsilon_t$ into a predictable and an unpredictable multiplicative components\(^5\)

$$\Delta c_t = \mathbb{E}_{t-1}[m^{-1}(L)SDF_t] + \eta_{t-1} \epsilon_t$$

(4)

where $\eta_{t-1}$ is known at $t - 1$. Last, we write $\mathbb{E}_{t-1}[m^{-1}(L)SDF_t]$ and $\log \eta_{t-1}^2$ as the sum of their orthogonal projections onto the asset span and its orthogonal complement,

$$\mathbb{E}_{t-1}[m^{-1}(L)SDF_t] = \sum_i \beta_i R_{t-1}^i + \nu_{t-1}^{sdf}$$

$$\log \eta_{t-1}^2 = \sum_i \delta_i R_{t-1}^i + \nu_{t-1}^\eta$$

where $\beta_i$ and $\delta_i$ are the projections of $\mathbb{E}_{t-1}[m^{-1}(L)SDF_t]$ and $\eta_{t-1}$ on $R_{t-1}^i$, respectively.

Using the last two equations in equation (4) gives

$$\Delta c_t = \sum_i \beta_i R_{t-1}^i + \nu_{t-1}^{sdf} + \sqrt{\exp(VFCI_{t-1})} \epsilon_t$$

(5)

$$VFCI_t = \sum_i \delta_i R_{t-1}^i + \nu_{t-1}^\eta$$

(6)

\(^5\)For any integrable process $x_t$, the decomposition $x_t = \eta_t \epsilon_t$ with $\eta_t$ a predictable integrable process and $\epsilon_t$ a martingale difference sequence always exists (see, for example, Blanchet-Scalliet and Jeanblanc 2020).
where we have defined the volatility financial conditions index VFCI by

\[ VFCI_t := \log \eta_t^2 \]

We will use equations 5 and 6 to empirically estimate the VFCI in the next section.

To see the relation between the VFCI and the price of risk, we start by writing innovations in the projected SDF as

\[(E_t - E_{t-1}) SDF_t^* = \lambda_t' \varepsilon_t^R\]

where \(\lambda_t\) is a vector of prices of risk and \(\varepsilon_t^R\) are innovations in returns scaled to have unit variance (with the corresponding scaling absorbed by the prices for risk). Then,

\[\epsilon_t = (E_t - E_{t-1})[m^{-1}(L)SDF_t^*]\]

\[= m^{-1}(L)(E_t - E_{t-1})SDF_t^*\]

\[= m^{-1}(L)(\lambda_t' \varepsilon_t^R)\]

\[= \sqrt{\exp(VFCI_{t-1})}\varepsilon_t\]

so that

\[ VFCI_t = \log \text{Var}_t \left( m^{-1}(L)[\lambda_t' \varepsilon_{t+1}^R] \right) \]

We have shown that the VFCI is a forward-looking measure of current and future expected prices of risk.

When the representative’s agent utility is a standard time-separable CRRA utility function, \(m^{-1}(L)\) is the identity operator and therefore

\[ VFCI_t = \log \|\lambda_t\|^2, \quad (7) \]

so the VFCI is, in this time separable case, the squared norm of the current vector of prices of risk.

3 VFCI Estimation

In this section, we estimate equations (5)-(6) and construct the VFCI.

3.1 Data

Table 1 shows the variables that we use to construct the VFCI. All series are at a quarterly frequency for the period 1962:Q1 to 2022:Q3, resulting in 243 observations. The starting date for the sample is determined by the earliest day for which all data
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial variables</strong></td>
<td></td>
</tr>
<tr>
<td>RET</td>
<td>Equity market return – S&amp;P500 annual returns</td>
</tr>
<tr>
<td>VOL</td>
<td>Equity market volatility – standard deviation of daily returns over the quarter</td>
</tr>
<tr>
<td>TERM</td>
<td>Term spread – 10-year minus 3-month yields on U.S. Treasuries</td>
</tr>
<tr>
<td>LIQ</td>
<td>Liquidity spread – 3-month yield on U.S. Treasuries minus effective federal funds rate</td>
</tr>
<tr>
<td>CRED</td>
<td>Credit spread – Moody’s seasoned Aaa corporate bond yield minus 10-year yield on U.S. Treasuries</td>
</tr>
<tr>
<td>DEF</td>
<td>Default spread – Moody’s Aaa minus Baa corporate bond yields</td>
</tr>
<tr>
<td><strong>Macroeconomic variables</strong></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Real gross domestic product – seasonally adjusted in bn of 2012 chained-dollars</td>
</tr>
<tr>
<td>C</td>
<td>Real aggregate consumption – seasonally adjusted real personal consumption expenditures excluding food and energy, index 2012=100</td>
</tr>
</tbody>
</table>

Table 1. **Variables used to construct the “volatility financial conditions index” or VFCI:**

Notes: The table shows the list of variables used to construct the VFCI, an estimate of the conditional volatility of real GDP growth. All time series are at a quarterly frequency for the period 1962:Q1 to 2022:Q3. S&P 500 data is from the Yahoo! Finance. The rest comes from the Federal Reserve Bank of St Louis’ FRED database.

The first two variables in the table, RET and VOL, are the returns and volatility of the S&P 500, respectively. To construct them, we use daily “close prices adjusted for splits and dividend and/or capital gain distributions” from Yahoo! Finance. RET is the percentage change between the price on the last trading day of the current quarter and the last trading day in the same quarter of the previous year. VOL is the standard deviation of daily returns over the current quarter, multiplied by $\sqrt{252}$ to annualize.

The data to construct the rest of the variables are from the Federal Reserve Bank of St. Louis’ FRED database. The term spread, TERM, is the difference between the 10-year yield on US Treasuries (FRED code DGS10) and the 3-month yield on US Treasury bills (TB3MS). The variable LIQ is a liquidity spread (TB3SMFFM), constructed as the difference between the secondary market rate on 3-month US Treasury bills (TB3MS) and the effective Federal Funds rate. The variable CRED is a credit spread (AAA10YM), the difference between Moody’s seasoned Aaa corporate bond yield and the 10-year yield on US Treasuries. The variable DEF is a default spread given by the
difference between Moody’s seasoned Aaa and Baa corporate bond yields (BAA10YM minus AAA10YM). The variables TERM, LIQ, CRED and DEF are averages of daily observations over the current quarter.

Aggregate output, Y, is seasonally adjusted real gross domestic product at a quarterly frequency (GDPC1). Aggregate consumption, C, is seasonally adjusted real personal consumption expenditures at a quarterly frequency (PCECC96).

3.2 The Asset Span

<table>
<thead>
<tr>
<th>Cumulative Variance</th>
<th>LIQ</th>
<th>CRED</th>
<th>DEF</th>
<th>TERM</th>
<th>RET</th>
<th>VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1 34.3%</td>
<td>-0.03</td>
<td>-0.49</td>
<td>-0.48</td>
<td>-0.36</td>
<td>0.34</td>
<td>-0.53</td>
</tr>
<tr>
<td>PC2 62.7%</td>
<td>0.66</td>
<td>0.33</td>
<td>-0.27</td>
<td>0.47</td>
<td>0.36</td>
<td>-0.19</td>
</tr>
<tr>
<td>PC3 76.2%</td>
<td>-0.28</td>
<td>-0.25</td>
<td>0.60</td>
<td>0.46</td>
<td>0.41</td>
<td>-0.35</td>
</tr>
<tr>
<td>PC4 88.0%</td>
<td>0.20</td>
<td>-0.33</td>
<td>-0.03</td>
<td>0.34</td>
<td>-0.76</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

Table 2. Cumulative Variance explained by PCs and PC Loadings of Variables

In Section 2, we showed that the returns \( R^i \) that appear in equations (5)-(6) are returns that together span the set of all possible cash flows that can be generated by trading financial assets, that is, the asset span.

Of course, any empirical estimate of the asset span will involve only a subset of the true asset span, since including every single traded asset in the world is infeasible. Indeed, for our purposes, the goal is not to find a large number of assets that capture as much of the asset span as possible. Instead, we seek a small number of assets that cover a subspace of the asset span that contains enough information about the price of macroeconomic risk – about \( SDF^* \) – to estimate a useful VFCL.

To this effect, we use the six financial variables listed in Table 1 as the “base assets” whose span we believe captures a large share of the price of risk that is relevant for macroeconomic dynamics. We construct an orthogonal basis for the space spanned by these six financial assets by computing their principal components (PCs). We only use the first four PCs, which together explain 88% of the variance of the underlying base assets.

Table 2 reports the cumulative variance explained by the PCs and the factor loadings for each of the base financial variables.
3.3 VFCI Estimation

Given the four PCs from the last subsection, we construct the VFCI by first estimating the following linear regression model with multiplicative heteroskedasticity:

\[
\Delta gdp_{t+1} = \theta PC_t + \varepsilon_t, \tag{8}
\]
\[
\text{Var}(\varepsilon_t^2) = \sigma_t^2 = \exp(\delta PC_t). \tag{9}
\]

In equations (8)-(9), \(\Delta gdp_{t+1} := \log(Y_{t+1}/Y_t)\) is quarterly real GDP growth between quarters \(t\) and \(t + 1\), \(PC_t := [1, PC_1, PC_2, PC_3, PC_4]\) is the quarter \(t\) vector with first component equal to 1 followed by the values of the first four principal components of the financial variables \(RET, VOL, TERM, LIQ, CRED, DEF\) constructed earlier, \(\varepsilon_t\) is a disturbance term that is independently and normally distributed with zero mean, \(\sigma_t^2\) is the variance of \(\varepsilon_t\), and \(\theta\) and \(\delta\) are \(5 \times 1\) vectors of parameters to be estimated. We refer to equation (8) as the “equation for the mean” or the “mean equation”, and to equation (9) as the “equation for the volatility” or the “volatility equation”. The equation for the mean states that real expected GDP growth is a linear function of the principal components, while the equation for the volatility states that the current logarithm of conditional volatility is also a linear function of the principal components. The choice of parametric form for the volatility guarantees that the variance is never negative. We estimate the parameters \(\theta\) and \(\delta\) using restricted maximum likelihood.

<table>
<thead>
<tr>
<th></th>
<th>Real GDP Growth</th>
<th>Real PCE Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Conditional Mean</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1</td>
<td>-0.169***</td>
<td>-0.120***</td>
</tr>
<tr>
<td>PC2</td>
<td>0.157***</td>
<td>0.109**</td>
</tr>
<tr>
<td>PC3</td>
<td>-0.208***</td>
<td>-0.184***</td>
</tr>
<tr>
<td>PC4</td>
<td>0.173***</td>
<td>0.114**</td>
</tr>
<tr>
<td>(__\text{cons})</td>
<td>0.743***</td>
<td>0.807***</td>
</tr>
<tr>
<td><strong>Panel B: Log Conditional Volatility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1</td>
<td>0.151**</td>
<td>0.204***</td>
</tr>
<tr>
<td>PC2</td>
<td>-0.499***</td>
<td>-0.405***</td>
</tr>
<tr>
<td>PC3</td>
<td>0.181*</td>
<td>0.341***</td>
</tr>
<tr>
<td>PC4</td>
<td>-0.411***</td>
<td>-0.497***</td>
</tr>
<tr>
<td>(__\text{cons})</td>
<td>-0.485***</td>
<td>-0.631***</td>
</tr>
<tr>
<td>(N)</td>
<td>242</td>
<td>242</td>
</tr>
</tbody>
</table>

\(p\)-values in parentheses
* \(p < 0.10\), ** \(p < 0.05\), *** \(p < 0.01\)

Table 3. \textbf{Heteroskedasticity Linear Regression of GDP and PCE Growth on PCs}

The second column of Table 3 shows the values of the resulting estimates for the mean equation, \(\hat{\theta}\), in Panel A, and for the volatility equation, \(\hat{\delta}\), in Panel B. The values in parenthesis are standard errors computed using a covariance matrix equal to the negative inverse.
Hessian matrix of the (restricted) log-likelihood evaluated at the estimated parameters. All estimates are significant at the 1% except for the coefficient on $PC_4$ in the mean equation, which is only significant at the 90% level. The third column of the table shows the analogous estimated parameters when the dependent variable is real consumption growth (instead of real GDP growth), with similar results.

Figure 1. The VFCI

We define the VFCI – the volatility financial conditions index – as the fitted values of the logarithm of the conditional volatility of GDP growth:

$$VFCI := \log \sqrt{\hat{\sigma}_t^2} = \hat{\delta}PC_t.$$

Figure 1 plots the time series of the VFCI, normalized to have zero mean and unit variance. Consistent with the theoretical interpretation of the VFCI as a measure of risk compensation and our interpretation as a measure of overall financial conditions, the figure shows that the VFCI has its two largest spikes during the initial phases of the Covid-19 pandemic in 2020 and the global financial crisis in 2008-2009. It also shows marked increases around other periods of financial distress, including during the tumultuous 2000-2002 period that included the burst of the dot-com bubble, the 9/11 terrorist attacks, and the corporate scandals in the early 2000’s; the 1994 bond market crisis, the “black Monday” stock market crash of 1987; the beginning of the savings and loans crisis and bank failures during the interest rate increases in the early 1980s;
and the 1973–1974 stock market crash. Furthermore, the VFCI does not increase every time there is a downturn in aggregate macroeconomic activity. For example, the VFCI does not show unusually high levels during the 1990-1991 recession. Conversely, the VFCI is not always low during booms. For example, between 1996:Q2 and 2000:Q2, year-over-year real GDP growth was above 4% every quarter, unemployment steadily declined from 5.5% to 4%, and core PCE inflation went from 3% to 2%, yet the VFCI increased consistently and showed a spike in 1998: Q3 that reflects increased financial stress during the collapse of Long-Term Capital Management and the Russian financial crisis.

Since the sign of each of the PCs used in the estimation of equations (8)-(9) is not identified, the signs of their coefficients \( \hat{\theta} \) are arbitrary. However, once the signs of the PCs (and therefore of \( \hat{\theta} \)) are fixed, the signs of the coefficients in the volatility equation, \( \hat{\delta} \), are not arbitrary. Panel B of Table 3 shows that all the coefficients \( \hat{\delta} \) are of the opposite sign of \( \hat{\theta} \). Therefore, all of the PCs induce movements of the conditional mean and the conditional volatility of GDP growth that go in opposite directions, suggesting that periods with of high expected GDP growth are usually accompanied by a low conditional volatility of GDP growth – a low VFCI.

Figure 2 shows a scatter plot of the fitted values for the mean equation, \( \Delta gdp_t = \hat{\theta} PC_t \), on the horizontal axis, against the VFCI (the fitted values for the volatility equation), on the vertical axis. The red line comes from an OLS regression on the scatter plot data. There is a tight linear relation with a negative slope. In periods of tight financial conditions – when the VFCI is high – the economy tends to be in the bottom right part of the figure. In these low-mean, high-volatility states, negative shocks translate into much larger declines in GDP than at the top left part of the figure, when volatility is low. Therefore, a high VFCI signals a greater financial amplification of shocks with a higher likelihood of sizable declines in GDP, while a low VFCI tends to be accompanied by relatively long periods of positive growth that are more resilient to shocks.\(^6\)

\(^6\)Using the national financial conditions index published by the Federal Reserve Bank of Chicago instead of the VFCI, Adrian, Boyarchenko, and Giannone 2019 find similar results using a more general estimation approach. Adrian and Duarte 2018 find that the same pattern also holds for the output gap, but not for inflation, and propose a model that rationalizes these empirical patterns by modeling shocks to financial intermediaries that, in equilibrium, affect the conditional mean and volatility of consumption and output growth via changes in the price of risk.
In this section, we review three popular financial conditions indices (FCIs), the national financial conditions index (NFCI) published by the Federal Reserve Bank of Chicago, the Goldman Sachs financial conditions index (GSFCI), and the Chicago board options exchange’s VIX index.

The NFCI is a weighted average of 105 indicators of financial activity that provides a “weekly update on US financial conditions in money markets, debt and equity markets, and the traditional and shadow banking systems” starting in January 1973. The weights are obtained through a mixed-frequency dynamic factor analysis and capture the relative importance of historical fluctuations in each of the variables (Brave and Butters 2011, Doz, Giannone, and Reichlin 2012, Aruoba, Diebold, and Scotti 2009). The NFCI is renormalized each week so that its sample mean is zero and its sample standard deviation is one, with higher values indicating tighter financial conditions. Brave and Butters 2011 find that the portion of the NFCI that cannot be predicted based on its historical dynamics is a good predictor of future economic activity.

The GSFCI is a weighted average of short-term interest rates, long-term interest rates, a trade-weighted dollar exchange rate, an index of credit spreads, and the ratio

---

7https://www.chicagofed.org/research/data/nfci/current-data
of equity prices to the 10-year average of earnings per share. It is available at a daily frequency starting January 1, 1983. The weight of each variable reflects the impact that a shock to the variable has on GDP growth over the four quarters following the shock, estimated from quarterly OLS regressions starting in 1984.\(^8\) Hence, the GSFCI is designed to be a measure of conditional GDP growth, with higher values of the GSFCI indicating tighter financial conditions and lower expected GDP growth over the following year. Most of the weight (84.7%) is given to the 10-year treasury yield and the index of credit spreads. Hatzius and Stehn 2018 show that one-hour changes in bond yields around FOMC announcements are correlated with daily changes in the GSFCI.

![Figure 3. Common FCIs (Standardized)](image)

The VIX Index is a measure of expected volatility of the US stock market over the next 30 days\(^9\). It is available daily (and at higher frequencies) since January 1, 1990. It is derived from real-time, mid-quote prices of S&P 500 Index call and put options. In contrast to realized volatility, which measures the variability of historical prices, the VIX is a forward-looking measure. The VIX is often referred to as a “fear gauge” by market participants and can also be viewed as an indicator for the price of risk of the

\(^8\)https://www.goldmansachs.com/insights/pages/case-for-financial-conditions-index.html
\(^9\)https://www.cboe.com/tradable_products/vix/
Figure 3 shows the three FCIs. While there is clear comovement, particularly in times of financial stress such as around the 2008 global financial crisis, there are also notable differences. For example, the GSFCI eased more than the two other FCIs in 2021; the VIX was more elevated around the tech bubble in the late 1990s; and the NFCI declined faster after the 1982 recession. The VFCI exhibits higher volatility during more recent recessions than alternative FCIs, but lower volatility in times of financial stress before the 2000s such as the period of recession in the early 1980s.

To further understand the relation of the VFCI with the other FCIs, we run the following linear regression:

$$ FCI_t = \alpha + \beta \delta PC_t + \varepsilon_t, $$

where $FCI$ is one of $\{NFCI, GSFCI, VIX\}$, $\alpha$ is a constant, $\beta$ is a $4 \times 1$ vector of constants, $PC_t$ are the first four principal components of the six financial variables listed in Table 1, and $\varepsilon_t$ is an error term. Table 4 shows the estimated coefficients $\hat{\beta}$ and Huber–White standard errors in parentheses. The PCs have high explanatory power for the FCIs with $R^2$s between 70 and 80 percent and all but two coefficients statistically significant at the 1% level. We view these results as evidence that the four PCAs are informative about financial conditions and that the price of risk is tightly linked to financial conditions. In addition, none of the FCIs is well explained by just one or two PCs and three or four PCs are needed to achieve the high $R^2$s shown in the table.

### Table 4. Association between FCIs and PCs

<table>
<thead>
<tr>
<th></th>
<th>NFCI</th>
<th>GSFCI</th>
<th>VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>0.152***</td>
<td>0.374***</td>
<td>3.683***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.067)</td>
<td>(0.202)</td>
</tr>
<tr>
<td>PC2</td>
<td>0.609***</td>
<td>1.109***</td>
<td>2.173***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.111)</td>
<td>(0.443)</td>
</tr>
<tr>
<td>PC3</td>
<td>-0.243***</td>
<td>-1.889***</td>
<td>-0.270</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.114)</td>
<td>(0.578)</td>
</tr>
<tr>
<td>PC4</td>
<td>0.041</td>
<td>-0.382***</td>
<td>3.221***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.111)</td>
<td>(0.381)</td>
</tr>
<tr>
<td>Num.Obs.</td>
<td>207</td>
<td>157</td>
<td>131</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.805</td>
<td>0.721</td>
<td>0.809</td>
</tr>
</tbody>
</table>

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag GZ</td>
<td>GZ</td>
<td>GZ</td>
<td>GZ</td>
<td>GZ</td>
<td>GZ</td>
</tr>
<tr>
<td></td>
<td>0.857***</td>
<td>0.908***</td>
<td>0.890***</td>
<td>0.775***</td>
<td>0.708***</td>
</tr>
<tr>
<td></td>
<td>(14.75)</td>
<td>(11.76)</td>
<td>(10.37)</td>
<td>(14.31)</td>
<td>(12.31)</td>
</tr>
<tr>
<td>VFCI</td>
<td>0.462***</td>
<td>0.359*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.61)</td>
<td>(1.96)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NFCI</td>
<td>0.077</td>
<td>0.432</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.56)</td>
<td>(1.61)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSFCI</td>
<td>0.008</td>
<td>0.045</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(0.95)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIXCLS</td>
<td>0.034***</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.92)</td>
<td>(-0.17)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>197</td>
<td>197</td>
<td>157</td>
<td>131</td>
<td>129</td>
</tr>
<tr>
<td>R²</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.82</td>
<td>0.86</td>
</tr>
</tbody>
</table>

* p < 0.1, ** p < 0.05, *** p < 0.01

Table 5. **Association between GZ spread and FCIs:** The GZ spread is a corporate bond risk premium measure of Gilchrist and Zakrajšek 2012.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag ECY</td>
<td>ECY</td>
<td>ECY</td>
<td>ECY</td>
<td>ECY</td>
<td>ECY</td>
</tr>
<tr>
<td></td>
<td>0.977***</td>
<td>0.924***</td>
<td>0.900***</td>
<td>0.942***</td>
<td>0.963***</td>
</tr>
<tr>
<td></td>
<td>(52.48)</td>
<td>(48.29)</td>
<td>(37.26)</td>
<td>(30.52)</td>
<td>(36.15)</td>
</tr>
<tr>
<td>VFCI</td>
<td>0.767***</td>
<td>-0.119</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.18)</td>
<td>(-0.70)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NFCI</td>
<td>0.265***</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.98)</td>
<td>(1.60)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSFCI</td>
<td>0.040</td>
<td>0.048</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.60)</td>
<td>(1.17)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIXCLS</td>
<td>0.030***</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.16)</td>
<td>(-0.11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>242</td>
<td>207</td>
<td>157</td>
<td>131</td>
<td>129</td>
</tr>
<tr>
<td>R²</td>
<td>0.96</td>
<td>0.95</td>
<td>0.93</td>
<td>0.92</td>
<td>0.94</td>
</tr>
</tbody>
</table>

* p < 0.1, ** p < 0.05, *** p < 0.01

Table 6. **Association between ECY spread and FCIs:** The ECY stands for the excess CAPE yield of Shiller 2000 and is a commonly used measure of the equity market CAPE equity risk premium.
5 The VFCI and Common Risk Premia

In this section, we provide empirical evidence that the VFCI is a good explanatory variable for the Gilchrist and Zakražek 2012 bond spread and the Shiller 2000 excess CAPE yield (ECY), two widely used measures of risk premia that have been proposed in the literature for corporate bonds and stocks, respectively.

Table 5 displays estimated coefficients from linear regressions of the GZ spread on its lag and different FCIs, with Huber–White standard errors in parentheses. Columns (1) through (4) show results from regressions that include a single FCI as an independent variable, while column (5) includes all four FCIs as regressors. When taken individually, the VFCI and the VIX have coefficients that are significant at the 1 percent level (columns (1) and (4)) while the NFCI (column (2)) and the GSFCI (column (3)) do not. When included together, only the VFCI remains significant.

Table 6 displays estimated coefficients and standard errors for analogous regressions that use the ECY instead of the GZ spread as a measure of risk premium. We again see that the VFCI is a good explanatory variable of risk premia, with a coefficient that is significant at the 1 percent level (column (1)). And even though the NFCI and the VIX are also good predictors individually (columns (2) and (4)), only the VFCI is significant in a regression that includes all FCIs together as predictors (column (5)).

We interpret these results not only as further evidence that the VFCI is a useful measure of risk premia and financial conditions, but also as an indication that it contains useful information not present in the other FCIs. Of course, many additional risk premium estimates have been proposed in the literature. We leave it to future research to explore further the asset pricing implications of the VFCI, including more formal cross-sectional asset pricing tests.

6 The VFCI and Macro-Financial Dynamics

In this section, we use a structural vector auto-regressive (SVAR) model to show that SVAR-identified shocks to the VFCI have large and persistent effects on output and monetary policy. Positive VFCI shocks that reflect higher risk premia and tighter financial conditions lead to quantitatively important reductions in real GDP and the Federal Funds rate, with 90 percent error bands for the corresponding impulse response functions that do not include 0 for more than 20 quarters. This means that shocks to the VFCI are an important causal source of fluctuations for macroeconomic dynamics. We also show that SVAR-identified shocks to monetary policy imply that an increase in the Federal Funds rate leads to a higher VFCI for 10 quarters, consistent with the extensive literature documenting that monetary policy shocks induce significant changes.
in risk premia across many asset classes.\footnote{See the review article Drechsler, Savov, and Schnabl 2018 and citations therein.}

The SVAR is introduced in Section 6.1. To identify the structural shocks in the SVAR, we use the identification-through-heteroskedasticity approach pioneered in economics by Rigobon 2003, described in Sections 6.2 and 6.3. The data sources and construction of variables are in Section 6.4. In Section 6.5, we estimate the model using Bayesian methods. In Sections 6.6 and 6.7, we show impulse response functions and interpret our results. Last, Section 6.8 shows that our results are robust to a number of different assumptions and specifications.

### 6.1 A Structural Vector Auto-Regression with Heteroskedasticity

The SVAR model we consider is:

\[
B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + C + \varepsilon_t
\]

(10)

where \( y_t \) is an \( n \times 1 \) vector of observed endogenous variables, \( B_0 \) is an \( n \times n \) matrix that determines the simultaneous relationships among the \( n \) variables, the \( B_j \)'s are \( n \times n \) matrices of coefficients for each lag \( j = 1, ..., p \) where \( p \) is the order of the SVAR, \( C \) is an \( n \times 1 \) vector of constants and \( \varepsilon_t \) is an \( n \times 1 \) vector of independent Student \( t \)-distributed structural shocks.

We assume that the entire sample period \( t = 1, ..., T \) is partitioned into exogenously specified subperiods indexed by \( m = 1, ..., M \), and let \( m(t) \) be the function that maps time periods to their corresponding subperiod. The variance-covariance matrix of the structural shocks, which we denote by \( \Lambda_{m(t)} \) with \( i \)th diagonal element \( \lambda_{i,m(t)} \), is allowed to be different across subperiods but remains constant within each subperiod:

\[
E[\varepsilon_t \varepsilon_t'] = \Lambda_{m(t)},
\]

(11)

with \( \Lambda_m \) a constant diagonal variance-covariance matrix for \( \varepsilon_t \) in subperiod \( m \). We impose the normalization that the cross-period structural variances average to one:

\[
\frac{1}{M} \sum_{m=1}^{M} \lambda_{i,m(t)} = 1
\]

(12)

The coefficient matrices \( B_j \) and \( C \) are fixed over the entire sample. Therefore, even though the variance of each structural shock may change across subperiods, the impact effect of shocks and the dynamic relations among the variables in \( y_t \) are time-invariant. One implication is that impulse responses have the same shape across
subperiods, although with a potentially different scale determined by the subperiod-specific variances.

### 6.2 Identification through Heteroskedasticity

The reduced form of equation (10) is:

\[ y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + D + u_t \]  

(13)

where the \(A_j\)’s are constant coefficient matrices of the same size as the \(B_j\)’s, \(D\) is a vector of constants of the same size as \(C\), and \(u_t\) is an \(n \times 1\) vector of reduced-form error terms with mean zero and variance-covariance matrix \(\Sigma_m(t)\) with \(\Sigma_m\) a constant variance-covariance matrix for regime \(m\). The structural shocks are a linear combination of the reduced-form shocks

\[ B_0 u_t = \varepsilon_t, \]  

(14)

and therefore

\[ B_0 \Sigma_{m(t)} B_0' = \Lambda_{m(t)}. \]  

(15)

If the structural shocks for each pair of the \(n\) SVAR equations (10) have variances that differ in at least one subperiod, Lanne, Lütkepohl, and Maciejowska 2010 show that equations (12) and (15) together (locally) identify the \(n^2\) parameters of \(B_0\) up to row ordering and row sign changes.

Identification “up to row ordering” means that the \(n\) estimated structural shocks cannot be automatically assigned to any one of the \(n\) equations of the SVAR, a consequence of the purely statistical nature of identification-through-heteroskedasticity. Therefore, in order to give economic meaning to the shocks, we must interpret them, which we do in Section 6.6. The benefit associated with the challenge of having to interpret the shocks is that the assumptions required for identification are, in our view, more likely to be true and less controversial than some of the assumptions required in identification schemes that require taking a stance on the economic behavior of variables that is difficult, if not impossible, to test.

### 6.3 Volatility Subperiods

Table 7 shows the volatility subperiods that we use. The most recent subperiod, which we label “Covid-19 pandemic and war in Ukraine”, starts in 2020:Q1 and ends in 2022:Q3. The rest of the subperiods are identical to those determined by Brunnermeier et al. 2021 (with the minor difference that their sample ends in 2015, so we extend their “Zero Lower Bound, Recovery from crisis” subperiod to end in 2019:Q4, after which the “Covid-19 pandemic and war in Ukraine” subperiod begins).
Although identification through heteroskedasticity requires making specific assumptions about the distribution of shocks, the behavior of volatility within subperiods, and the exact dates for the subperiods, Gouriéroux, Monfort, and Renne 2017 and Sims 2020 show that if the true distribution of structural shocks is fat-tailed and symmetric, then likelihood-based estimation assuming a $t$-distribution is consistent under more general conditions. For example, if the true volatilities are not constant within each subperiod, or if the subperiods themselves are misspecified (but retain sufficient cross-subperiod variation in the estimated variances), then identification is still achieved, coefficient estimates are still consistent, and time paths of impulse response functions still have the same shape. The cost of these types of misspecification are likely to be wider posterior density regions – making error bands seem larger than they really are – and distorted variance decompositions.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962Q1-1979Q3</td>
<td>Oil crisis and stagflation</td>
</tr>
<tr>
<td>1979Q4-1982Q4</td>
<td>Volcker disinflation</td>
</tr>
<tr>
<td>1983Q1-1989Q4</td>
<td>Major S&amp;L crisis defaults</td>
</tr>
<tr>
<td>1990Q1-2007Q4</td>
<td>Great Moderation</td>
</tr>
<tr>
<td>2008Q1-2010Q4</td>
<td>Financial crisis</td>
</tr>
<tr>
<td>2011Q1-2019Q4</td>
<td>Zero Lower Bound, Recovery from crisis</td>
</tr>
<tr>
<td>2020Q1-2022Q3</td>
<td>Covid-19 pandemic and war in Ukraine</td>
</tr>
</tbody>
</table>

Table 7. Volatility Regimes: Brunnermeier et al. 2021 provide the first six regimes.

6.4 Data Sources and Construction of Variables

Table 8 shows descriptive statistics for the variables that we use to estimate the SVAR in equation (10) As in the construction of the VFCI, all series consist of quarterly frequency observations that start in 1962:Q1 and end in 2022:Q3.

Table 8 lists the $n = 4$ variables that we include as endogenous variables in our benchmark VAR specification,

$$y_t = [\log GDP_t, \log P_t, FFR_t, VFCI_t],$$

(16)

together with some summary statistics.

All four variables are constructed with data from FRED. The series for the logarithm of real GDP, $\log GDP_t$, is constructed by taking the logarithm of the same real GDP series we use in the construction of the VFCI (FRED code GDPC1). The series for the logarithm of the price level, $\log P_t$, uses the personal consumption expenditures price
index that excludes food and energy (PCEPILFE), which is also provided by FRED directly at a quarterly frequency. The Federal Funds rate, $FFR_t$, is the average of the three monthly observations in the current quarter of the Federal Funds effective rate (FEDFUNDS). The fourth variable is the volatility financial conditions index, $VFCI_t$, that we constructed in Section 3.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Funds Rate</td>
<td>0.05</td>
<td>0.04</td>
<td>0.00</td>
<td>0.18</td>
<td>243</td>
</tr>
<tr>
<td>VFCI</td>
<td>-0.23</td>
<td>0.37</td>
<td>-0.77</td>
<td>1.52</td>
<td>243</td>
</tr>
<tr>
<td>log of Real GDP</td>
<td>9.15</td>
<td>0.50</td>
<td>8.16</td>
<td>9.91</td>
<td>243</td>
</tr>
<tr>
<td>log of Core PCE Deflator</td>
<td>4.01</td>
<td>0.62</td>
<td>2.86</td>
<td>4.82</td>
<td>243</td>
</tr>
</tbody>
</table>

Table 8. **Descriptive Statistics for the Macro-Financial Variables:** The VFCI is constructed in the previous section, the remaining data is from the FRED database of the Federal Reserve Bank of St Louis.

### 6.5 Bayesian Estimation and Model Fit

We estimate our SVAR model using the same Bayesian procedure as in Brunnermeier et al. 2021, except that we modify the parameters of the ’Minnesota prior’ placed on the reduced form coefficients $A_j$ to account for the quarterly frequency of our data (instead of monthly in Brunnermeier et al. 2021). Specifically, we use a “tightness” of 3 (instead of 5) and a “decay” of 0.5 (instead of 1). We use a Gibbs sampling method with 10,000 draws to sample from the posterior distribution of all parameters, which also provides posterior distributions for impulse response functions.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>MDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>4500.5</td>
</tr>
<tr>
<td>Gaussian</td>
<td>2553.5</td>
</tr>
</tbody>
</table>

Table 9. **Marginal Data Densities:** Differences between values are log posterior odds with equal prior weights on each model. The first row represents the main model in the paper with $t$-distributed errors, and the second row represents the same model but with Gaussian errors.

Table 9 provides the marginal data density (MDD) for the baseline model with $t$-distributed errors and compares it with the MDD for the model with Gaussian errors. The MDD is the integral of the likelihood function over the prior distribution, and

---

11We use the Minnesota prior implemented through dummy observations and the same notion of “tightness” and “decay” as in Sims and Zha 1998.
can be used as a measure of model fit in Bayesian inference. The difference between the exponential MDD values for two models reflects the log odds ratio, or the relative probability that one model fits the data better. Akin to Brunnermeier et al. 2021 who use monthly data, our quarterly frequency SVAR with $t$-distributed errors continues to capture the data better than the Gaussian errors case.

6.6 Impulse Responses and Discussion of Results

In Figure 4, we show the impulse responses of the four SVAR variables to an increase of one standard deviation in each of the four identified shocks, where the shocks are drawn from a Student-$t$ distribution with 2.5 degrees of freedom. Given the normalization in equation (12), the IRFs are “averages” across the volatility regimes\textsuperscript{12}. In this and all subsequent figures, IRFs are medians across posterior draws (the black line), with 68th and 90th percentile error bands, over 20 quarters (the dark and light shaded regions, respectively).

In the figure, we have assigned labels to the identified shocks, which, as mentioned earlier, is not provided by the purely statistical nature of the identification strategy. We now discuss the reasoning behind each of the labels together with the economic intuition and implications of the results.

The leftmost column shows impulse responses to what we interpret as a real GDP shock or, more broadly, a demand shock. There are several reasons. First, despite not having imposed any short-term zero restrictions, the GDP shock has little to no impact effect on the VFCI and the price level, essentially ruling out the interpretation that the shock under consideration is a structural shock to either of these two variables. Second, although there is some initial response of the Federal Funds rate, it is in the same direction as the response of GDP. If the shock were a Federal Funds rate shock, one would expect higher Federal Funds rate – tighter monetary policy – to be accompanied by lower GDP. Third, the pattern of a high GDP throughout, prices that rise steadily over time, and a higher Federal Funds rate, are all consistent with a demand shock in a sticky price economy in which the central bank responds with higher interest rates when it observes higher temporary output growth and higher inflation expectations. Financial conditions as measured by the VFCI show neither a contemporaneous response to the GDP shock nor a large response in any of the 20 quarters shown in the figure. While we expect that, ceteris paribus, higher economic activity would loosen financial conditions – at least eventually – we also expect (and show more explicitly later) that higher interest rates tighten financial conditions. In the impulse responses of the figure, these two opposing forces approximately offset each other, resulting in a muted response of the financial conditions.

\textsuperscript{12}Appendix A shows IRFs for each regime separately.
VFCI. If anything, financial conditions can be seen to tighten slightly at the same time that the Federal Funds rate peaks.

Figure 4. **IRFs: VFCI Shocks** Impulse responses to the VFCI structural shock in the volatility-identified SVAR model with t distributed errors over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) posterior error bands. Scaled to an “average” period with unit scale.
The second column of Figure 4 shows responses to what we label a “core PCE shock”. The main reason for this label is that the price level is the only variable that shows a large response to the shock. In addition, despite the large error bands, GDP goes down and the Federal Funds rate increases persistently, consistent with the interpretation. The VFCI is essentially unresponsive to inflation, consistent with some previous results (see footnote 6).

In the third column, Figure 4 shows responses to what we identify as the “VFCI shock”. The one standard deviation VFCI shock leads to an increase at impact of around 12 percent for the VFCI (around one-third of its standard deviation in the sample). This effect decays exponentially for around 7 quarters and then becomes essentially zero, a prototypical response to own-shocks in VARs. Real GDP decreases by around 0.4 percent for over 6 quarters, and remains persistently low, reaching close to a 0.5 percent decline over the 20 quarters plotted, with no signs of reversal. The price level declines slightly, escaping the 68 error band (but not the 90 percent band) only by the end of the 20 quarters, reaffirming the weak connection between inflation and financial conditions in this particular sample. Monetary policy responds to the VFCI shock by lowering the Federal Funds rate by around 20 basis points (in annualized terms) at the peak response that occurs around 7 quarters after the shock. The one standard deviation shock to the VFCI is therefore comparable in magnitude to the 25 basis point change in the federal fund rate that is most commonly employed by the FOMC (Federal Open Market Committee). Since the SVAR is linear, a negative shock to the VFCI would result in the same impulse responses but with signs reversed. A negative VFCI shock would lead to a response of monetary policy that can be understood as “leaning against the wind” of looser financial conditions. Overall, the delayed buildup in the responses of output, the price level and the Federal funds rate points to shocks originating in the financial sector and propagating to the real economy over time.

Compared to Brunnermeier et al. 2021, we estimate a similar response of monetary policy to financial shocks, albeit somewhat more accommodative in our case. The median response of the Federal Funds rate to the two financial stress shocks in their baseline model reaches a decrease of around 0.1 percentage points, compared to 0.20 in our model. Of course, there are many differences between the two studies that could explain the difference (including the use of different measures of financial conditions and other variables).

The right-most column of figure 4 displays responses to the shock that we label a “Federal Funds shock” or, more broadly, a monetary policy shock. The contractionary monetary policy shocks lead to a transitory yet persistent decline in output that is ill-determined. The response of the price level exhibits the “price puzzle” common to small VAR specifications. The VFCI responds strongly to the Federal Funds shock. The
median estimated response is around 5 percent upon impact, increasing to around 7.5 percent over 3 quarters before dissipating after 8 quarters. Thus, financial conditions significantly and immediately tighten in reaction to contractionary monetary policy. The tightening of financial conditions in response to monetary policy shocks that we find supports the empirical evidence in Gertler and Karadi 2015, and Brunnermeier et al. 2021, among other, who all analyze the impact of Fed Fund shocks on financial conditions measured by various types of spreads, and of the strand of the vast literature originating in Kuttner 2001, Gürkaynak, Sack, and Swanson 2004, that include Nakamura and Steinsson 2018, Caldara and Herbst 2019, Cieslak and Vissing-Jorgensen 2020, among other, who find large effects of monetary policy on risk premia.

6.7 Volatility of Structural Shocks

Table 10 reports posterior distribution medians for the variance of structural shocks in each of the different volatility subperiods. Given the normalization in equation (12), the numbers in Table 10 are volatility levels relative to the average of 1 across subperiods. One important takeaway is that the variance of the structural shocks differ substantially across volatility subperiods, supporting the identifying assumption of heteroskedasticity.

Table 10. Relative Variances: For the Four Structural Shocks across the Seven Volatility Regimes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log GDP</td>
<td>1.51</td>
<td>1.47</td>
<td>0.40</td>
<td>0.39</td>
<td>1.00</td>
<td>0.34</td>
<td>1.71</td>
</tr>
<tr>
<td>Log PCE</td>
<td>0.45</td>
<td>1.69</td>
<td>0.71</td>
<td>0.16</td>
<td>0.51</td>
<td>0.13</td>
<td>3.23</td>
</tr>
<tr>
<td>VFCI</td>
<td>0.59</td>
<td>0.54</td>
<td>1.16</td>
<td>0.79</td>
<td>1.64</td>
<td>1.13</td>
<td>1.02</td>
</tr>
<tr>
<td>Fed Funds</td>
<td>1.40</td>
<td>3.20</td>
<td>0.59</td>
<td>0.21</td>
<td>0.45</td>
<td>0.06</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The behavior of shock volatilities is also informative about the economic interpretation of shocks. VFCI shocks exhibit the largest variance during the period of the global financial crisis (2008-10), which saw a substantial tightening of financial conditions. Monetary policy shocks exhibit their highest volatility in the regimes before the early 1980s, which is in accordance with thinking that policy errors and monetary policy shocks have dampened over more recent years. Output shocks exhibit the highest volatility during 2020-22, which is not surprising given the sharp decline in US GDP after the Covid-19 shock and relatively rapid subsequent recovery. Price shocks are also the most volatile in the latest regime, reflecting the sharp uptick in inflation in the US starting in 2021 in a short interval. The Volcker disinflation period also stands out as quite volatile, which is unsurprising given the rapid changes in prices in large part caused by monetary policy.
6.8 Robustness of SVAR results

We conduct a range of robustness exercises:

- alternative specifications of stationarity by replacing GDP and PCE with stationary variables – output gap or GDP growth instead of GDP, and PCE inflation instead of the PCE price level
- alternative specifications of VFCI such as using VFCI in levels rather than logs
- alternative number of PCs instead of the baseline case of 4 PCs in the heteroskedasticity linear regression to construct VFCI
- alternative specifications of the distribution of structural shocks such as simulating draws from a normal distribution instead of the t distribution
- alternative number of posterior draws in the MCMC chain i.e. increasing the draws from the baseline of 10,000 to 1,000,000
- alternative shapes of the Minnesota prior i.e. varying the calibration from [1, 3] for the tightness and [0.3, 0.7] for the decay
- alternative time period i.e. ending just before the 2008-10 global financial crisis to mitigate the effect of unusually large structural shocks
- alternative specifications of the baseline SVAR with regime-specific IRFs where the VAR dynamics change over time
- alternative specifications of the baseline SVAR with the inclusion of a second financial variable (such as the Gilchrist and Zakrajsek (2012) bond spread or the spread of 3-month Eurodollars over 3-month Treasuries)

Appendix A shows the results of the robustness exercises and further implementation details. The core set of results regarding the magnitude, persistence, error bands, and overall shape of the impulse responses in Figure 4 remain broadly unchanged.

7 Alternative Identification Strategies

In this section, we show that four identification strategies different from the identification-through-heteroskedasticity one used in Section 6.1 give very similar impulse responses. We give a brief description of each strategy and provide more details in Appendix II:

Instrumental Variables

In the first alternative identification strategy, we estimate an SVAR model with instrumental variables (also referred to as proxy variables or external instruments) for
monetary policy, GDP and the VFCI.

We construct the monetary policy instrument by splicing two data series. Between 1969:Q1 and 1994:Q4, we use the monetary policy shock constructed in Romer and Romer 2004 using the “narrative approach”, updated by Wieland and Yang 2020\textsuperscript{13}. From 1995:Q1 to 2022:Q3, we use the monetary policy shock constructed in Nakamura and Steinsson 2018 and convert the monthly series to quarterly frequency by averaging observations over the current quarter\textsuperscript{14}. After splicing the two series, we standardize by subtracting the mean and dividing by the standard deviation.

For the GDP instrument, we use the growth news shock constructed in Cieslak and Pang 2021, available for 1983:Q1 to 2022:Q3\textsuperscript{15}. We convert to quarterly frequency by averaging the daily observations over the current quarter and then standardize by subtracting the mean and dividing by the standard deviation.

The external instrument for VFCI is constructed based on a sign-restricted VAR approach using Bayesian methods (see Uhlig 2005). The restrictions are imposed on the shape of the orthogonalized impulse response functions. The identifying assumption is that a VFCI shock reduces prices upon impact. This is not a stringent identifying assumption in light of the evidence from the volatility-identified SVAR that prices fall when there is a surprise tightening of financial conditions.

All instruments are standardized to have zero mean and unit standard deviation.

**Local Projections with Instrumental Variables**

The second alternative identification strategy we consider is the instrumental variable local-projection method proposed by Jordà, Schularick, and Taylor 2015 using the same instruments as in the previous strategy. Despite the asymptotic equivalence between VAR-based and local-projection-based IRFs, they do not always give similar results in finite samples. In addition, LP estimates can theoretically be more robust under certain conditions if a linear VAR is misspecified. Thus, we consider the use of local projections an identification strategy of its own.

**Recursive VAR**

Third, we estimate a recursive VAR in which the Federal Funds rate and the VFCI are ordered last, reflecting the assumption that they respond contemporaneously to shocks to GDP and the price level, but that GDP and the price level do not respond contemporaneously to shocks to the Federal Funds rate or to the VFCI. We show results

\textsuperscript{13}Downloaded from https://www.openicpsr.org/openicpsr/project/135741/version/V1/view on February 12, 2023. We use the series ‘resid_full’ in the file ‘RR_monetary_shock_quarterly.dta’, divided by 10.

\textsuperscript{14}We thank Emi Nakamura and Miguel Acosta for providing an updated data series.

\textsuperscript{15}We thank the authors, who provided the updated series.
with the Federal Funds rate ordered third and the VFCI ordered last, although results are essentially identical if we reverse this order.

**Sign restrictions**

As our last alternative identification strategy, we impose sign restrictions on the impact effect of monetary policy and VFCI shocks. We assume that the price level and GDP must respond negatively to a positive monetary policy shock, and that the price level must respond negatively to a positive VFCI shock.

### 7.1 A Comparison of All Identification Schemes

Figures 5 and 6 provide a comparison of all the identification schemes. In both figures, each row corresponds to a different identification scheme. The first row, labeled “Vol-SVAR”, shows impulse responses for our baseline SVAR specification in which shocks are identified by exploiting the time variation in the volatility of the endogenous variables. These impulse responses are the same impulse responses already shown in Figure 4, which we include here for ease of comparison. The second row, labeled “SVAR-IV”, uses an SVAR with instrumental variables to identify shocks. The third row, labeled “LP-IV” use local projections (rather than a VAR) and the same instrumental variables as the SVAR-IV specification. The fourth row is a recursive VAR, labeled “Cholesky”. The fifth and last row shows IRFs for the sign-restricted SVAR.

The IRFs in the heteroskedasticity-identified and the sign-restricted SVARs are estimated using Bayesian methods and correspond to medians across 10,000 draws from the posterior distribution. The other three identification schemes follow a frequentist approach. In each instance, the shock corresponds to a one standard deviation increase, and we plot the evolution of the variables over 20 quarters.

**VFCI and Monetary Policy**
Figure 5. **Comparison of Impulse Responses Across Identification Schemes: VFCI and Monetary Policy** Impulse responses to a one standard deviation increase in the Federal Funds rate and VFCI structural shocks identified through five different identification schemes over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) error bands.
Figure 5 examines the dynamic causal effects that VFCI and monetary policy have on each other. A contractionary monetary policy shock leads to an immediate tightening of financial conditions in all identification schemes (first column). Financial conditions remain tight for 5-6 quarters in the LP-IV and sign restricted models. The effect is more persistent for the volatility SVAR, SVAR-IV, and Cholesky schemes, where financial conditions remain tight for 10-15 quarters.

The peak estimated response of the VFCI to a monetary policy shock ranges between 5-10 percent, with the sign-restricted model at the top end of the range. The volatility-identified SVAR model and the SVAR-IV model produce peak responses that, at around 6-7 percent, are similar in magnitude. However, financial conditions tighten relatively quickly in the SVAR-IV model. The LP-IV and Cholesky models report peak effects that are a bit lower with an increase in the VFCI of around 5-6 percent. All models show a statistically significant tightening of financial conditions in response to monetary policy shocks within the 90 percent error bands.

In a related analysis in an SVAR-IV framework, Gertler and Karadi 2015 analyze the impact of monetary policy shocks on credit markets (represented by three types of credit spreads for three financial markets). They find that the excess bond premium increases by eight basis points on impact, the mortgage spread by two to three basis points, and the commercial paper spread by around five basis points, thus pointing toward credit costs tightening across three significant financial markets in response to a contractionary Federal Funds shock. Caldara and Herbst 2019, similar to Gertler and Karadi 2015, also primarily focus on the effects of a monetary policy shock. Using a Bayesian proxy SVAR model, they find that the excess bond premium increases by around ten basis points in response to a Federal Funds shock.

We now examine the causal impact of a shock to the VFCI. In response to a VFCI shock, we find that there is an immediate easing of monetary policy for all identification schemes (column 2). This effect is statistically significant and fairly persistent (although with some differences) in all identification schemes.

The threshold decrease in the Federal Funds rate in response to a VFCI shock ranges between 0.03-0.4 percentage points. The most accommodative monetary policy responses are found in the LP-IV and sign-restricted models.

**VFCI and GDP**

Figure 6 examines the dynamic causal effects of VFCI and GDP on each other. The response of VFCI to an output shock is not clear-cut (column 1). Financial conditions loosen upon impact in the SVAR-IV and LP-IV models. A one standard deviation surprise increase in GDP growth (around 2.4 percentage points in the sample) leads to a fall in VFCI of 25-100 percent (around 0.7-2.8 times the standard deviation of VFCI)
based on instrumenting growth. However, VFCI responds insignificantly to a structural shock to output in the heteroskedastic SVAR model.

Output clearly responds to a VFCI shock. There is a statistically significant decline in GDP and GDP growth, across all models, in response to a surprise tightening of financial conditions (column 2). In particular, the fall in output is highly significant and persistent in the heteroskedastic SVAR, where GDP decreases by close to 0.5 percent over 20 quarters. In the Cholesky scheme, GDP decreases by 0.4 percent on impact.
but reverts to the mean within 15 quarters. The delayed and persistent response of GDP to the VFCI shock in the volatility-identified SVAR potentially reflects better identification compared to the recursive scheme. In the two models that incorporate external instruments, SVAR-IV and LP-IV, GDP growth decreases on impact by around 0.02-0.2 percentage points in response to a tightening of financial conditions.

8 Robustness

In this section, we briefly discuss additional approaches to calculate the VFCI.

First, real PCE can be used instead of real GDP to calculate the VFCI. Table 3 already showed that the regression of the GDP-based VFCI and the PCE-based VFCI give rise to very similar coefficients. Here, we show graphically that the two series are virtually indistinguishable.

Another question is whether we need the computation of the PCAs. Instead of constructing the VFCI from the PCAs, one could directly run a heteroskedastic regression GDP growth on the six financial variables. It turns out that, due to the high collinearity of some of those variables, not all individual variables are statistically significant. However, the resulting VFCI is again virtually indistinguishable from our original PCA-based VFCI.

Finally, we can compute the VFCI for other countries. To illustrate, we compute the VFCI for Europe based on the underlying data of the Composite Indicator of Systemic Stress (CISS) from the ECB. The Euro Area VFCI (EA-VFCI) does look materially different from the US-VFCI, but that is to be expected. We leave it for future research to examine the VAR evidence for Europe.

\[\text{Figure 7. The GDP VFCI and PCE VFCI}\]

Another question is whether we need the computation of the PCAs. Instead of constructing the VFCI from the PCAs, one could directly run a heteroskedastic regression GDP growth on the six financial variables. It turns out that, due to the high collinearity of some of those variables, not all individual variables are statistically significant. However, the resulting VFCI is again virtually indistinguishable from our original PCA-based VFCI.

Finally, we can compute the VFCI for other countries. To illustrate, we compute the VFCI for Europe based on the underlying data of the Composite Indicator of Systemic Stress (CISS) from the ECB. The Euro Area VFCI (EA-VFCI) does look materially different from the US-VFCI, but that is to be expected. We leave it for future research to examine the VAR evidence for Europe.

9 Literature

The most closely related literature is from Brunnermeier et al. 2021, who investigate alternative financial variables in macroeconomic dynamics, and document that the corporate bond risk premia of Gilchrist and Zakrajšek 2012 and the 3-month Libor-US Treasury spread (the so-called TED spread) are significantly related to macroeconomic activity. Instead of trying alternative financial indicators as Brunnermeier et al. 2021, we estimate the price of risk in the economy from a broad cross section of financial assets, and show that this theoretically based macro-financial variable is highly significant for macroeconomic aggregates. Furthermore, we use instrumental variables to estimate robust causal relationships, in addition to the heteroskedasticity-based identification. Nevertheless, we show that including the two financial variables that Brunnermeier et al. 2021 use in our SVAR leaves the IRF of VFCI shocks on output and monetary policy, and of monetary policy shocks on the VFCI, virtually unchanged.

Other empirical strategies in this area use: (i) a small number of variables, usually
focusing on single-equation projection methods (e.g., Mian, Sufi, and Verner 2017; Jordà, Schularick, and Taylor 2015, Jordà, Schularick, and Taylor 2016, López-Salido, Stein, and Zakrajšek 2017, Krishnamurthy and Muir 2017, or (ii) binary outcomes (such as crisis/no crisis) or analysis limited to crisis periods (e.g., Schularick and Taylor 2012; Drehmann and Juselius 2014, Stock and Watson 2012, or (iii) reduced-form multi-equation specifications (e.g., Gilchrist, Yankov, and Zakrajšek 2009, Gilchrist and Zakrajšek 2012). Identification of causal effects, when present, is typically focused only on the effects of monetary policy shocks Gertler and Karadi 2015 and Caldara and Herbst 2019. In addition to Brunnermeier et al. 2021, another notable exception is Stock and Watson 2012, who use instruments external to their vector auto-regression.

Our contribution is closely related to consumption-based asset pricing and, more broadly, to the effort to understand the joint behavior of macroeconomic risk and asset prices. Consumption-based asset pricing, the idea that risk compensation is driven by the covariance of asset payoffs with consumption growth or, more broadly, marginal rates of substitution originates with the foundational contributions of Rubinstein 1976, Lucas Jr 1978, Breeden 1979, Duffie and Zame 1989. Theoretical advances have followed in many dimensions, including an understanding of existence and uniqueness of single and multi-agent equilibria, martingale methods to solve the consumption-portfolio problem, transaction costs and other frictions, dynamically complete and incomplete markets, among others (see Duffie 1991, Sundaresan 2000, Mehra 2012 and Breeden, Litzenberger, and Jia 2015 for reviews). While we do relate our estimate of the market price of risk to common risk measures of risk premia for stocks and bonds, our main goal is to study how the market price of risk interacts with macroeconomic dynamics. Other research on asset pricing using the VFCI is left for future research.

The empirical assessment of consumption-based asset pricing remains mixed. Hansen and Singleton 1982, Hansen and Singleton 1983, Mankiw and Shapiro 1986 find evidence against consumption pricing. Chen, Roll, and Ross 1986 conclude that “... the rate of change in consumption does not seem to be significantly related to asset pricing. The estimated risk premium is insignificant and has the wrong sign.” Subsequent work argues that consumption data might be noisy or poorly measured (see Campbell and Cochrane 2000). Our approach is not focused on measurement error but rather on causal identification, employing various identification strategies including instrumental variables.

One strand of asset pricing considers consumption growth mimicking-portfolios by projecting consumption growth onto the space of traded assets and creating maximally correlated portfolios. In fact, Breeden, Gibbons, and Litzenberger 1989 proves that if one would first find the maximum correlation portfolio with real consumption growth, then the CCAPM should hold where betas are measured against the returns of that
portfolio. In contrast, we focus on the the market price of risk, which is the projection of conditional consumption (or GDP) volatility onto the span of financial factors. We do not postulate a contemporaneous projection of consumption growth onto the span of financial assets. Instead, our framework implies that financial factors are predictors of consumption growth. In the end, we do find strong correlation of the VFCI with the conditional mean of consumption growth, but that is an empirical result and not an assumption in our framework.

An important point of contact with our paper in the long-run risk model of Bansal and Yaron 2004 is the presence of stochastic volatility of consumption growth. While we model the time variation in the volatility of consumption as a function of several financial factors, the long-run risk model posits an exogenous AR(1) process, which has been generalized by Bollerslev, Xu, and Zhou 2015 to a two-factor volatility structure. More generally, the literature consistently finds time variation in the volatility of consumption. For example, Ludvigson 2013 document a significant degree of stochastic volatility in aggregate consumption data. Campbell et al. 2018 derive an intertemporal CAPM with stochastic volatility. Bansal, Khatchatrian, and Yaron 2005 shows that the volatility of aggregate consumption is time varying, predicts, and is predictable by the market price–dividend ratio. A large literature estimates and models stochastic volatility of macroeconomic or financial variables going back to the ARCH-GARCH seminal contributions of Engle 1982 and Bollerslev 1986, as does the closely related stochastic volatility filtering literature (e.g., Bidder and Smith 2018, van Binsbergen and Koijen 2010). All of these approaches are fully consistent with our own approach, though none of them modeled the VFCI in the way we did.

Our asset pricing framework is at the core of a vast literature that studies macro-financial interactions. The general consumption-based theoretical setup and the rich empirical specification with macroeconomic variables, monetary policy and other identified shocks, and asset prices, can be used as a way to empirically distinguish among different transmission and amplification mechanisms. A very partial list of models with financial frictions includes Bernanke, Gertler, and Gilchrist 1996, Kiyotaki and Moore 1997, Holmström and Tirole 1998, Brunnermeier and Sannikov 2014, He and Krishnamurthy 2013. More recently, also within the consumption-based paradigm, Adrian and Duarte 2020, Bianchi, Lettau, and Ludvigson 2022, Bianchi, Ludvigson, and Ma 2022, Caballero and Simsek 2020, Caballero and Simsek 2022, and Kashyap and Stein 2022 provide a risk-centric view of macroeconomic fluctuations, emphasizing the interaction between monetary policy, asset prices, and macroeconomic fluctuations, although focusing on different frictions and mechanisms. Relative to those contributions, we emphasize the central role of the market price of risk as measured by the VFCI, as well as causal identification of macro-financial interactions using a variety of methods.
10 Conclusion

In this paper, we propose a new financial conditions index, the VFCI, derived from asset pricing theory. The VFCI is a measure of the price of risk in the economy when a representative consumer exists. In contrast to other FCIs that are mostly atheoretical, the VFCI is the first FCI to be derived from solid theoretical underpinnings. The VFCI is correlated with other leading FCIs, but has notable differences. An important one is that it exhibits better explanatory power for stock and corporate bond risk premia. The VFCI is constructed using widely available financial data, is computationally tractable, and has a relatively long time series history. The VFCI could be computed globally, thus being able to track financial conditions in real-time across countries.

We use a range of identification schemes to study the causal impact of VFCI shocks on monetary policy and output, and vice versa. Across identification strategies, the baseline conclusions remain the same: a tightening of financial conditions based on the VFCI leads to an immediate easing of monetary policy and a persistent contraction in output. Conversely, contractionary monetary policy shocks lead to a tightening of financial conditions. In contrast, output shocks do not move the VFCI much, and inflation seems to be mostly unrelated to it. These results are encouraging, as they suggest a step forward in estimating financial conditions based on economic theory, with broad applicability and uses in policymaking. Further research could compute the VFCI for additional countries, conduct asset pricing tests, and embed the VFCI into structural macro-financial models.

References


Sims, Christopher A. 2020. “Svar identification through heteroskedasticity with misspecified regimes.” *Princeton University*.


Internet Appendix

The Internet Appendix conducts a set of robustness checks to test the main predictions of the identification schemes that monetary policy shocks have a causal impact on financial conditions, and that shocks to financial conditions causally affect monetary policy and output. The sensitivity analysis includes estimating regime-specific IRFs, estimating the models with a second financial variable, checking on alternate assumptions on stationarity and the distribution of error terms, using alternate calibration of Bayesian parameters, and reporting the VFCI in exponential terms.

Appendix I: Robustness of Macro-Financial Dynamics

A Regime Specific IRFs

The baseline case reports IRFs that are the average across the seven volatility regimes, as in Figure 4. Here, we relax the assumption that the economy responds in the same way across regimes. Hence, the VAR dynamics are allowed to change along with the covariance matrix of shocks, implying that the economy reacts in different ways across time. In reporting the regime-specific IRFs here, as well as in the other robustness checks in the appendix, we calibrate the t degrees of freedom as 2.5 as in the baseline model, noting that the conclusions remain the same if this is varied. There are three main findings based on estimating the regime-specific dynamic causal effects.

First, the average causal effects remain robust to regime-specific estimation. Across all the seven regimes and regardless of times of relative calm versus crises, we find that monetary policy shocks have a significant causal effect on financial conditions. Conversely, a tightening of financial conditions leads to a significant decline in real activity and triggers an accommodative monetary policy response across all regimes. These results provide robust empirical evidence that the decision making of monetary policy makers has been causally affected by financial market activity, and vice versa, throughout recent U.S. macroeconomic history.

Second, monetary policy shocks have non-linear effects on financial conditions, with the amplification effect of shocks conditional on the extent of economic stress. For instance, the peak response of VFCI to a Fed Funds shock was above 12 percent during the Volcker Disinflation era (1979Q4-1982Q4) whereas it was less than 3 percent during two periods of relative calm –both the Great Moderation era (1990Q1-2007Q4) as well as the Zero Lower Bound era (2011Q1-2019Q4). This
could reflect the adverse impact on financial market volatility due to the unprecedented spike in the Fed Funds rate during the Volker era in a bid to bring inflation down to manageable levels. During both the Global Financial Crisis (2008Q1-2010Q4) as well as the recent Covid-19 Pandemic (2020Q1-2022Q3), the peak financial tightening was around 6 percent in response to a Fed Funds shock. Thus, compared to the two calmer time periods, financial conditions tightened by approximately double as much in response to monetary policy shocks during times of financial crises.

Third, monetary policy becomes somewhat more accommodative in response to financial tightening in times of economic crises. During the Great Moderation (1990Q1-2007Q4) and Zero Lower Bound (2011Q1-2019Q4) eras, for instance, the peak Fed Funds rate decline was below 0.2 points. During the Global Financial Crisis (2008Q1-2010Q4) and the Covid-19 pandemic (2020Q1-2022Q3) eras, however, the peak Fed Funds rate response was a bit stronger, at or above 0.2 percentage points. The results suggest that the implicit monetary policy rule in response to financial shocks has been fairly consistent over time, though the Fed has veered toward slightly greater accommodation during crises.

B Structural Shocks, Residuals, and Instruments

Figure IA.8 provides the correlation matrix of the structural shocks estimated from the heteroskedastic BVAR, and the correlation matrix of the corresponding reduced-form residuals. Figure IA.9 provides the correlation of the structural shocks estimated from the heteroskedastic BVAR, sign-restricted BVAR, and Cholesky identification schemes with the corresponding instruments used in the SVAR-IV and LP-IV models. Note that an instrument does not exist for the price level. The GDP growth instrument is also excluded as the baseline structural shocks are estimated for log real GDP.

C Forecast Error Variance Decomposition

Another way to look at the properties of the model is to estimate the forecast variance error decomposition (FEVD), or the share of each variable’s forecast variance explained by all the structural shocks. While the VFCI shock increasingly explains output and monetary policy further into the forecast horizon, it is interesting to note that the monetary policy shock explains over 40 percent of VFCI’s forecast variance after the first year, suggesting monetary policy’s strong influence on financial conditions.
D Exponential VFCI and Stationarity in the Five Identification Schemes

Figures IA.11 and IA.12 use the exponential representation of VFCI, and present the dynamic causal effects of monetary policy and VFCI on each other, and on GDP and VFCI on each other. The results corroborate those in the paper, where the baseline VFCI is reported as the log market price of risk. The magnitude of responses is not significantly different in most cases, apart from the slightly stronger response in terms of financial loosening of exponential VFCI in response to positive GDP shocks.

Figures IA.13 and IA.14 present the results across the five identification schemes in stationary terms, with the models containing inflation, GDP growth, baseline VFCI, and the Fed Funds rate. In Figure IA.13, which reports the responses of VFCI to monetary policy shocks, and the Fed Funds rate to financial conditions shocks, it is noted that the magnitude and direction of the dynamic causal effects are very similar to the models estimated in levels. In Figure IA.14, the responses of VFCI and GDP growth to shocks to each other are also similar to those in the paper. However, unlike GDP in levels which displays a persistent decrease in response to financial conditions shocks in the heteroskedastic BVAR, GDP growth drops temporarily over the next year before starting to recover.

E Second Financial Variable

This section estimates the heteroskedastic BVAR with five variables instead of four as in the paper. We seek to analyze the robustness of the dynamic causal effects with the inclusion of a second financial variable as suggested in Brunnermeier et al. 2021. In particular, we analyze a variety of cases, by including in turn separately the Excess CAPE Yield (ECY) of Shiller 2000, the Gilchrist and Zakrajšek 2012 corporate bond risk premium, the 3-month Libor-US Treasury spread (TED spread), and the NFCI of the Federal Reserve Bank of Chicago.

We find that the inclusion of the second financial variable does not change the implications of the baseline case with four variables. Including the fifth variable leads to similar responses, in terms of both significance and magnitude of the responses of monetary policy, output, and prices to VFCI shocks. VFCI also responds in a similar way to the structural shocks across all models. Therefore, the empirical results with one financial variable are found to hold robust.
F Time-Varying Parameter VAR

Figures IA.23 and IA.24 estimate VFCI shocks and responses in the time-varying parameter VAR model of Primiceri 2005 with their default priors and parameters. The model is estimated in stationary terms as the models in levels does not converge for a large set of parameter values. In general, the conclusions from the heteroskedastic BVAR remain robust in terms of the overall direction of responses of the variables to shocks, especially in 68 percent confidence bands. Tight financial conditions cause contractionary monetary policy, and vice versa.

G Additional BVAR Robustness

Gaussian Errors Assumption

Figures IA.25 and IA.26 estimate the heteroskedastic BVAR with Gaussian, instead of t-distributed, shocks. In general, the responses of monetary policy and GDP to VFCI shocks are somewhat more pronounced, as is the response of VFCI to all the structural shocks in the model. As the variance of the structural shocks differs substantially across the regimes in our dataset, the baseline model of t-distributed shocks is more efficient. However, the conclusions remain the same regardless of the assumption on the distribution so that a tightening of the VFCI leads to a persistent contraction of output and triggers an immediate easing of monetary policy. Conversely, contractionary monetary policy shocks cause tighter financial conditions.

100,000 and 1,000,000 MCMC Draws

Figures IA.27 and IA.28 estimate the heteroskedastic BVAR with 100,000 draws, while Figures IA.29 and IA.30 estimate the heteroskedastic BVAR with 1 million draws, instead of 10,000 draws as in the baseline model. The dynamic causal effects are almost identical, with slightly tighter posterior error bands in some cases. The robustness of the estimation when we increase the number of draws is useful to corroborate using 10,000 draws for the baseline models as it leads to faster estimation with the same conclusions on the impact and response of VFCI.

Pre-Global Financial Crisis and Covid-19 Crisis

Figures IA.31 and IA.32 estimate the heteroskedastic BVAR with the time period ending in 2007Q4, before the Global Financial Crisis and the Covid-19 crisis. The average causal effects remain very similar, including the peak accommodative response of monetary policy to VFCI shocks and the tightening of VFCI in response to Fed Funds rate shocks. These results are not surprising in light of the results from the
regime-specific estimation, which analyzed the causal effects specific to each time period and found that the although the responses were of different magnitudes, the conclusions were qualitatively the same. Thus, regardless of time period under consideration and whether or not crises are excluded, a VFCI shock, as a representation of tightening financial conditions, causes monetary policy to become more accommodative and real economic activity to fall. Conversely, contractionary monetary policy causes a tightening of financial conditions.

Appendix II: Details on Alternative Identification Strategies

A External Instruments

Studies in the empirical macroeconomic literature have generally used internal instruments, or shocks identified within the model, to estimate dynamic causal effects. Stock and Watson 2018 consolidate the derivation of dynamic causal effects and asymptotic theory for external instruments in LP and SVAR frameworks and find that the use of external instruments can potentially lead to more credible identification. As discussed in that paper, external instruments in macroeconometric models comprise a relatively new but promising avenue of research. We build on this literature by also using instruments for VFCI, monetary policy, and output in LP and SVAR models as alternative identification strategies to estimate dynamic causal effects.

VFCI Instrument

To implement the sign restrictions approach, we start by fitting a reduced-form Bayesian VAR on VFCI, GDP, PCE, and the Fed Funds rate, assuming that the structural shocks are distributed as $\varepsilon_t \sim (0, \sigma_\varepsilon)$. Imposing the Normal-Wishart prior on the BVAR and using an MCMC chain, a posterior distribution is formed to estimate the reduced-form coefficient and error variance matrices.

The structural shocks are then recovered using a Cholesky decomposition with resulting IRFs. At this point, an orthogonalized IRF, $\alpha$, is randomly drawn. As in Uhlig 2005, we impose a function that penalizes sign restriction violations, $\Psi(\alpha)$ for a set of constrained responses $j \in J$ and constrained periods $k \in K$, that solves the following minimization problem

$$
\min_{\alpha} \Psi(\alpha) = \sum_{j \in J} \sum_{k \in K} b(l_j) \frac{r(j, \alpha)(k)}{\sigma_j}
$$

(IA.17)
where $r(j, \alpha)(k)$ is the response of $j$ at step $k$ to $\alpha$ and $b$ is an imposed penalty. The IRFs from the Cholesky decomposition are then multiplied with $\alpha$, and the sequence of steps is repeated based on the MCMC algorithm to ultimately derive an IRF that minimizes the overall penalty function for the restricted variables.

This procedure generates an external instrument for VFCI based on the VFCI structural shock identified in the sign-restricted model. A similar sequence of steps is followed to retrieve the VFCI shock with the model in stationary terms.

**Robustness of the VFCI instrument** We derive an alternative version of the VFCI instrument that slightly deviates from the baseline case for the purposes of robustness. To do so, we use a rejection algorithm that keeps all the posterior draws that satisfy the imposed sign restrictions instead of choosing draws that minimize the penalty function. The steps to retrieve the structural shocks remain the same, but in this case, the random orthogonal IRF, $\alpha$, is estimated based on the following formula

$$\alpha = Ba$$  \hspace{1cm} (IA.18)

where $BB' = \sigma_\varepsilon$, and $a$ is an $n \times 1$ vector so that $||a|| = 1$.

**Monetary Policy Instrument**

The estimation of monetary policy shocks has been oft-explored in the literature starting from Romer and Romer 2004’s estimation of this shock through a narrative approach. Since then, the literature used various techniques to identify monetary policy shocks (Ramey 2016), such as the high-frequency identification strategy used in Nakamura and Steinsson 2018. Our instrument for the Federal Funds rate is the Romer and Romer monetary policy shock, which starts from 1969Q1 and extends until 1994Q4, interpolated with the Nakamura and Steinsson shock, which was updated and kindly shared with us, from 1995Q1 to 2022Q3.

**GDP Growth Instrument**

The external instrument related to GDP was kindly shared by the authors of Cieslak and Pang 2021 and extends from 1983Q1-2022Q3. This shock is estimated through a sign-restricted VAR approach that places identifying restrictions on the differential response of stock and bond market prices to key macroeconomic announcements. The authors identify growth news shocks among other shocks, and we use the GDP growth shock as an external instrument for GDP growth in a stationary version of our model.
B Identification through SVAR-IV

To outline the SVAR-IV identification problem, consider the following reduced-form version of equation 10 for a vector of endogenous variables, $y(t)$

$$B(L)y_t = \eta_t \quad \text{(IA.19)}$$

where the reduced-form innovations, $\eta_t$, satisfy $\eta_t \sim (0, \Sigma_\eta)$ with $E[\eta_s \eta_t'] = 0$ for $s \neq t$ and the polynomial lag operator is $B(L) = I - \sum_{k=1}^{p} B_k L^k$. The innovations, $\eta$, are related to the structural shocks, $\varepsilon$, as follows

$$\eta_t = H \varepsilon_t \quad \text{(IA.20)}$$

where $H$ is invertible. Here, in contrast to the time-varying variance assumption in the previous section, the structural shocks are distributed as $\varepsilon_t \sim (0, \sigma_\varepsilon)$.

Equations 10 and IA.19 can be written in terms of their structural moving average representations as $Y_t = \Theta(L)\varepsilon_t$ and $Y_t = C(L)\nu_t$, where $C(L) = [B(L)]^{-1}$ and $\Theta(L) = C(L)H$. Therefore, $H$ can be written as $H = C(L)^{-1}\Theta(L) = I + B_1 L + \ldots (\Theta_0 + \Theta_L + \ldots) = \Theta_0 + \text{terms in } L, L^2, \ldots$. The impact effect is $H = \Theta_0$, which implies that $\eta_t = \Theta_0 \varepsilon_t$ Stock and Watson 2018. The SVAR-IV identification problem is to identify $\Theta_0$ by finding a suitable external instrument, $Z_t$, that satisfies the following conditions

$$E\varepsilon_{1,t}z_t' = \alpha \neq 0 \quad \text{(IA.21)}$$

$$E\varepsilon_{2:n,t}z_t' = 0 \quad \text{(IA.22)}$$

Equations IA.21 and IA.22 are the instrument relevance and exogeneity conditions, meaning that the instrument must be contemporaneously correlated with the structural shock, $\varepsilon_{1,t}$, and uncorrelated with the other structural shocks.

The basic idea to estimate $\Theta_0$ is as follows, with further theory, including on the asymptotics and inference, found in Stock and Watson 2018. Suppose conditions IA.21 and IA.22 are satisfied, we are able to identify the first structural shock, $\varepsilon_{1,t}$. To recover the other structural shocks in a VAR with $n$ endogenous variables, the reduced form system in equation IA.19 is first fit to estimate the vector of innovations $\eta_t$.

All the reduced-form innovations apart from those of the first variable, $\eta_{2:n,t}$, are then regressed on $\eta_{1,t}$, using $z_t$ as an instrument. The residuals of this sequence of regressions form a vector $\kappa_{2:n,t}$. Finally, $\eta_{1,t}$ is regressed sequentially on $\eta_{2:n,t}$, using $\kappa_{2:n,t}$ as the instruments. This allows for the identification of the $\varepsilon_{2:n,t}$. Using the identified structural shocks, $\varepsilon_t$, the dynamic causal effects are estimated.\(^\text{17}\)

\(^\text{17}\)Of note is that this method is related to, but different, from the approach of using an external instrument in a recursive VAR, as in Romer and Romer 2004. As discussed in Ramey 2016, the SVAR-IV method was
The impact of the VFCI, monetary policy, and output shocks identified through external instruments in an SVAR-IV model corroborate the results from the heteroskedastic BVAR. A tightening of financial conditions caused by a positive VFCI shock, as identified by the penalty function approach, triggers an immediate easing of monetary policy and a contraction in output. The dynamic responses of both output and monetary policy, and output in particular, are somewhat less persistent compared to the heteroskedastic BVAR, but their negative responses upon impact to tight financial conditions are highly significant and similar in magnitude.

We also estimate the impact of monetary policy and growth shocks in the SVAR-IV model.\(^{18}\) A surprise increase in GDP growth leads to an immediate easing of financial conditions, but VFCI tightens in the following quarters as it reverts to the mean. The loosening of financial conditions in response to growth shock may be accorded to the plausibly better identification of these shocks using an external instrument.

### C Identification through LP-IV

The Local Projections (LP) approach Jordà 2005 has become a popular method of estimating IRFs. The LP model estimates the parameters sequentially through simple linear regressions and is computationally straightforward in practice. LP estimates can theoretically be more robust if a linear VAR is misspecified, although this is not always the case (Plagborg-Møller and Wolf 2021). The LP model can also be estimated using external instruments (Jordà, Schularick, and Taylor 2015). We use our instruments to estimate a local projections-IV (LP-IV) model for VFCI, output, inflation, and monetary policy.

SVAR and LP models were considered conceptually different in the past, but have been shown to estimate the same IRFs as long as a sufficient amount of lags are accounted for and the entire population is modeled as in Plagborg-Møller and Wolf 2021. Ramey 2016, in reviewing the literature, estimates similar models with LPs and SVARs and finds some differences, which—in light of the recent results demonstrating equivalence—could be due to assumptions, lags, samples, and so on. We take note of these previous results and given the choice of a particular sample and time period in this study, we estimate the dynamic causal effects by additionally using an LP-IV approach.

To outline the LP-IV identification problem, consider the moving average version of equation 10, which, as discussed previously, is \(Y_t = \Theta(L)\varepsilon_t\). The impulse response of \(Y_t\) at horizon \(h\) is estimated from a single regression equation as follows

---

\(^{18}\) The GDP growth shocks are estimated in a stationary VAR due to the nature of the news shock in Cieslak and Pang 2021 (it is to growth, not output)
\[ y_{i,t+h} = \Theta_{h,i} y_{1,t} + u_{i,t+h} \]  

(IA.23)

where \( u_{i,t+h}^h = \varepsilon_{t+h}, \ldots, \varepsilon_{t+1}, \varepsilon_{2:n}, \varepsilon_{t-1} \varepsilon_{t-2}, \ldots \). OLS estimation of IA.23 is not valid since \( Y_{1,t} \) is correlated with \( u_{i,t+h}^h \). However, IA.23 can be estimated if we use a suitable external instrument that satisfies the instrument relevant and exogeneity conditions, IA.21 and IA.22, along with a third condition

\[ E\varepsilon_{t+j,t} z_t' = 0, \ j \neq 0 \]  

(IA.24)

which denotes the requirement that the instrument satisfy lead-lag exogeneity. This means that \( z_t \) should be uncorrelated with historical as well as future shocks. A separate LP-IV regression is estimated for each horizon, \( h \). Also, serial correlation in the errors is modeled since the errors, \( \varepsilon_{t+h} \), are serially correlated for all \( h > 0 \) as \( \varepsilon_{t+h} \) is the moving average of the forecast errors from \( t \) to \( h \). In practice IA.23 can be estimated with control variables. The extension of LP-IV with control variables is straightforward, and discussed further in Stock and Watson 2018.

The LP-IV model is estimated both in levels and in stationary terms when estimating the causal impacts of the VFCI and monetary policy shocks, but in stationary terms with the growth shock due to the nature of the instrument. While the identified monetary policy shock has insignificant effects, the GDP growth shock leads to a tightening of the Fed Funds rate and a loosening of financial conditions upon impact.

The VFCI shock exhibits some of the same properties as in the volatility-identified BVAR and SVAR-IV models, that is, it leads to a significant easing of monetary policy and a significant contraction in output. The dynamic causal effects, as in the SVAR-IV model, are somewhat less persistent than in the heteroskedastic BVAR.

### D Identification through a Recursive VAR

We take one step back and estimate a simple recursive VAR with the ordering defined as output, prices, monetary policy, and financial conditions. VFCI is ordered last in the baseline case, but we assess the robustness of this assumption by ordering the Federal Funds rate last in an alternative specification. Financial conditions and monetary policy could be endogenous based on the empirical evidence in Cieslak, Morse, and Vissing-Jorgensen 2019 and Cieslak and Vissing-Jorgensen 2020, and we mitigate such concerns by changing the forcing variable.

While the magnitude and significance of the IRFs vary, especially with respect to output, which we attribute to a less well-defined identification scheme, the conclusion is the same. Contractionary monetary policy shocks trigger a tightening of financial conditions. Conversely, a tightening of financial conditions leads to an easing of monetary
policy and a contraction of output.

E Identification through Sign Restrictions

Sign restrictions are used on the shape of the IRFs in response to the structural shocks following the penalty function approach based on Uhlig 2005 discussed in A. To estimate the causal impact of monetary policy and VFCI shocks, we restrict the response of prices and output to be negative in identifying the monetary policy structural shock, and prices to be negative in identifying the VFCI structural shock.

As can be noted from the sign-restricted IRFs, monetary policy shocks lead to an immediate tightening of financial conditions. At the same time, VFCI shocks lead to an easing of monetary policy and decline in output. The IRFs, similar to those obtained from the LP-IV approach, are less persistent than the volatility-identified BVAR.

F Robustness of Identification through External Instruments and Sign Restrictions

We perform sensitivity analysis by perturbing the baseline set of results for the LP-IV, SVAR-IV, and sign-restricted BVAR models. The causal impact of VFCI shocks on monetary policy and output, and of monetary policy shocks on VFCI, is checked for robustness as follows

- alternative specifications of stationarity by replacing GDP and PCE with stationary variables – output gap or GDP growth instead of GDP, and PCE inflation instead of the PCE index
- alternative specifications of VFCI such as using VFCI in levels rather than logs
- alternative specifications of the external instrument for VFCI by using a rejection algorithm instead of the penalty function algorithm

The original conclusions are broadly robust to these changes.
Figure IA.1. 1962Q1-1979Q3 oil crisis and stagflation (regime 1) dynamics: VFCI Shocks and Responses Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.2. 1979Q4-1982Q4 Volcker disinflation (regime 2) dynamics: VFCI Shocks and Responses Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.3. 1983Q1-1989Q4 Major S&L crisis defaults (regime 3) dynamics: VFCI Shocks and Responses Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.4. **1990Q1-2007Q4 Great Moderation (regime 4) dynamics:** VFCI Shocks and Responses Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.5. **2008Q1-2010Q4 Financial crisis (regime 5) dynamics: VFCI Shocks and Responses** Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.6. 2011Q1-2019Q4 Zero Lower Bound, Recovery from crisis (regime 6) dynamics: VFCI Shocks and Responses Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.7.  

2020Q1-2022Q3 Covid-19 pandemic and war in Ukraine (regime 7) dynamics: VFCI Shocks and Responses

Impulse responses of VFCI to the four structural shocks, and the responses of the four variables to the VFCI structural shock, in the volatility-identified BVAR model with t distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands.
Figure IA.8. Correlation Matrices for Baseline Heteroskedastic BVAR Shocks and Reduced-Form VAR Residuals
Figure IA.9. Correlation Matrices for Structural Shocks Estimated from Identification Schemes and Corresponding Instruments
Figure IA.10. **Forecast Error Variance Decomposition** Forecast variance decompositions in the t-distributed errors model over 60 months, with 68 percent (dark blue) and 90 percent (light blue) error bands. Scaled to an “average” period with unit scale. Rows denote variables and columns denote shocks.
Figure IA.11. Exponential VFCI - Comparison of Impulse Responses Across Identification Schemes: VFCI and Monetary Policy Impulse responses to a one standard deviation increase in the Federal Funds and VFCI structural shocks identified through five different identification schemes over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) error bands.
Figure IA.12. **Exponential VFCI - Comparison of Impulse Responses Across Identification Schemes: VFCI and GDP**

Impulse responses to a one standard deviation increase in the GDP and VFCI structural shocks identified through five different identification schemes over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) error bands.
Figure IA.13. Stationary Models - Comparison of Impulse Responses Across Identification Schemes: VFCI and Monetary Policy

Impulse responses to a one standard deviation increase in the Federal Funds and VFCI structural shocks identified through five different identification schemes over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) error bands.
Figure IA.14. **Stationary Models - Comparison of Impulse Responses Across Identification Schemes: VFCI and GDP**

Impulse responses to a one standard deviation increase in the GDP and VFCI structural shocks identified through five different identification schemes over 20 quarters, with 68 percent (dark shade) and 90 percent (light shade) error bands.
Figure IA.15. **IRFs: VFCI Shocks with ECY in the Model**
Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.16. IRFs: VFCI Responses with ECY in the Model
Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.17. **IRFs: VFCI Shocks with GZ in the Model** Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.18. **IRFs: VFCI Responses with GZ in the Model**

Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.19. **IRFs: VFCI Shocks with TEDR in the Model**
Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.20. IRFs: VFCI Responses with TEDR in the Model

Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.21. **IRFs: VFCI Shocks with NFCI in the Model**
Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.22. **IRFs: VFCI Responses with NFCI in the Model**

Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.23. IRFs: VFCI Shocks in a TVP-BVAR
Impulse responses to the VFCI structural shock in a Time-Varying Parameter BVAR over 20 quarters, with 68 percent (dark grey) and 90 percent (light grey) posterior error bands.

Figure IA.24. IRFs: VFCI Responses in a TVP-BVAR
Impulse responses of VFCI to the four structural shocks in a Time-Varying Parameter BVAR over 20 quarters, with 68 percent (dark grey) and 90 percent (light grey) posterior error bands.
Figure IA.25. **IRFs: VFCI Shocks with Normal Errors** Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with Gaussian errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.26. **IRFs: VFCI Responses with Normal Errors**
Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with Gaussian errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.27. **IRFs: VFCI Shocks 100k Draws** Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.28. **IRFs: VFCI Responses 100k Draws** Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.29. IRFs: VFCI Shocks 1 Million Draws Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.30. **IRFs: VFCI Responses 1 Million Draws** Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.31. IRFs: VFCI Shocks Pre-Crises Impulse responses to the VFCI structural shock in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.
Figure IA.32. **IRFs: VFCI Responses Pre-Crises** Impulse responses of VFCI to the four structural shocks in the volatility-identified BVAR model with t-distributed errors over 20 quarters, with 68 percent (dark green) and 90 percent (light green) posterior error bands. Scaled to an “average” period with unit scale.