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Costly Increases in Public Debt when r < g

Yongquan Cao, Vitor Gaspar, and Adrian Peralta-Alva

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Prepared by Yongquan Cao, Vitor Gaspar, and Adrian Peralta-Alva*

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ABSTRACT: This paper quantifies the costs of a permanent increase in debt to GDP. We employ a deterministic, overlapping generations model with two assets and no risk of default. The two assets are public debt and private (productive) capital. We assume that the return on private capital equals the interest rate on public debt plus an exogenously given spread. Employing a analytical version of the model we show an example in which a permanent rise in the public debt ratio leads to a significant reduction in steady-state GDP even as r<g. Following McGrattan and Prescott (2017) we consider a calibrated model of the US economy including a rich set of features of national accounts, fixed assets, distribution of household incomes and demographics. The intuition (and even the orders of magnitude) from the simple analytical model carries over to this richer environment: the increase in the debt ratio, from 60 to 120 percent of GDP, is associated with a reduction in the capital stock of about 15 percent and a reduction in steady state GDP of about 8 percent.

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Author's E-Mail Address:	YCao2@imf.org, APeralta-Alva@imf.org and VGaspar@imf.org

WORKING PAPERS

Costly Increases in Public Debt when r < g

Prepared by Yongquan Cao, Vitor Gaspar, and Adrian Peralta-Alva [1]

I.   Introduction

In 2020, a curious anomaly was apparent in global financial markets: despite rising and historically high debt-to-GDP ratios, sovereign bond yields were declining to very low levels (even, in some cases, turning negative in real terms). This led to the coincidence of record levels of debt with very low debt servicing costs. Olivier Blanchard characterizes this landscape as 'unusual' in his book Fiscal Policy under Low Interest Rates, where he extensively explores the associated facts and policy implications. While one might be tempted to dismiss these low-interest rates as a historical aberration—especially in light of recent inflationary pressures and high nominal rates—such a dismissal may be premature. Prior to the COVID-19 pandemic, both real and nominal neutral interest rates had been on a declining trajectory for decades. Various structural factors, from sluggish economic growth and demographic changes, as outlined by the IMF (2023), to a global scarcity of safe assets, as discussed by Caballero et al. (2017), contributed to this enduring trend. Many of these factors are expected to remain operating going forward. Hence, as inflation reduces, low interest rates are likely to return.[1]

The negative differential between interest rates and growth has led scholars like Blanchard (2023) and Blanchard (2019) to argue that elevated levels of public debt may incur minimal fiscal and social costs. The focus of our research is to rigorously investigate the conditions under which public debt does impose significant costs, setting aside any consideration of the potentially productive uses of such debt.

The limitations imposed by resource scarcity and finite production capacity apply to all sectors, including governmental activities. These constraints become particularly apparent when we explicitly consider the endogeneity of the condition $r - g < 0$. Both the real interest rate (r) and the economic growth rate (g) are not independent of economic policy. For the purposes of this study, we operate under the assumption that marginal product of capital m is greater than growth g, treating the spread between m and r as exogenously determined.

Our paper addresses two critical questions. First, do increasing levels of public debt result in economic costs within standard overlapping generations models when $m > g > r$—that is, when the economy is dynamically efficient despite $r - g < 0$? Our analysis confirms that surging public debt indeed crowds out private capital, thus suppressing long-term output. Second, we quantify the magnitude of these detrimental effects.

It is well understood that high debt levels expose countries to multiple equilibria that include attacks on sovereign debt (see for example Cole and Kehoe (2000) and Aguiar and Amador (2021), for a comprehensive survey on sovereign debt and default issues). We will abstract away from these considerations. Our paper focuses on crowding out. We build a standard overlapping generations model of a closed economy whereby higher debt may crowd out private capital. A key feature of our model is that it includes two assets. The rate of return on private capital will be determined in equilibrium. Government debt is assumed to

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[1]Other, more temporary factors, might have also payed a role in keeping interest rates low, including quantitative easing policies implemented by major central banks. Determining the overall quantitative effects of the interplay of all these forces is beyond the scope of this paper.

yield a return equal to that of private capital minus an exogenously determined convenience yield. This convenience yield represents the reward for holding public debt. Aguiar et al. (2022), show a simple way to generate a wedge between private capital and government debt. They also show that crowding out is a critical force to be dealt with in such framework. Our focus is not on explaining the convenience yield but rather on understanding its implications.

The exercise we have in mind to address our research questions is straightforward. We depart from an economy on a balanced growth path, and focus on parameterizations that allow for an equilibrium with $r-g < 0$. Unexpectedly, debt to GDP starts a gradual increase until it eventually stabilizes at a new, higher, level. No further shocks are expected to happen thereafter. We study the impact of the higher level of debt to GDP on key macroeconomic aggregates. In a simplified version of the model that we can solve analytically, we can illustrate the crowding out mechanism in steady state and quantify its impact. The crowding out mechanism operates irrespective of whether $r-g$ is positive or negative. We then consider a version of the model suitable for quantitative analysis (with realistic life-spans and fiscal policies) using a calibration corresponding to the United States. In this case we examine both, the transitional and steady-state impacts of higher debt to GDP ratios.

Our analysis reveals that, over a period where interest rates were low, and expected to remain low, a permanent increase in debt from 60 to 120 percent of GDP, equivalent to the experience of the United States over the last 20 years, can incur substantial costs. In the long term, we observe a reduction in GDP by around 8% (with the analytic model yielding a similiar result). In the transition phase of the quantitative model, GDP exhibits a monotonous decline, while interest rates rise, even though $r-g$ remains negative. The primary mechanism behind these results is the decline of about 15% in the stock of private capital. This occurs because, in contrast to small changes in saving, individuals need to allocate a larger share of their wealth to government bonds rather than private capital. In equilibrium, the capital stock decreases, leading to a drop in output and an increase in the return on capital. Notably, a higher return rate for private capital also drives up the interest rate on government debt. Additionally, it's worth highlighting that the elasticity of the interest rate on government bonds in response to changes in the debt-to-GDP ratio, as determined by our calibrated model, falls within the range of empirical estimates found in the existing literature.

We conduct a final robustness check by extending the model to account for the fact that about 30 percent of the debt is held by foreign investors in recent years. Under these conditions, the qualitative findings remain consistent, affirming our earlier conclusions. The quantitative impacts are reduced, resulting in a permanent decline of 4 percent in GDP.

The literature on the consequences of public debt is large. Blanchard (2019) illustrates that the interest rate minus growth differential has been low. If the future is like the past, he argues, it is possible that debt carries no fiscal cost. Our analysis shows that higher debt may change the future relative to the past, with increasing interest rates. He also uses an overlapping generations model and shows that the differences between the interest rate on government debt, the marginal product of capital, and the growth rate of the economy matter for determining whether high debt is welfare improving or welfare reducing. Our model is

similar and finds this is indeed the case. Under the calibrations we consider, higher debts can nevertheless have significant negative impacts on the economy.

Our work builds on Reis (2021) study, which also explores the macroeconomic impact of rising public debt. Reis focuses on the government's fiscal constraints in scenarios where the interest rate on government debt is lower than economic growth. His insights on resource scarcity parallel our own discussion on $m > g > r$ conditions and the crowding-out of private capital. Like Reis, we emphasize the intrinsic fiscal constraints that governments face, but we extend the analysis by calibrating our models to U.S. economic conditions to quantify the long-term GDP effects of higher public debt levels.

It is important to delineate the focus and constraints of this research. The study is constructed to examine the economic consequences of increasing levels of public debt, specifically emphasizing the associated costs. We intentionally abstain from analyzing potential beneficial effects of public spending, such as investments in infrastructure, education, or climate transition. Moreover, our model does not incorporate forces that could generate large fiscal multipliers. This methodological choice is designed to isolate the costs associated with increasing levels of public debt. As such, the paper should not be considered an exhaustive assessment of real-world fiscal policies, including those of the United States, as a comprehensive evaluation would necessitate a model that accounts for both the advantages and disadvantages of public debt.

A paper with mechanisms intimately linked to ours is Aguiar et al. (2022). The authors work with an incomplete markets model in the spirit of Bewley-Huggett-Aiyagari whereby households are facing idiosyncratic risk and hold precautionary savings in the form of private capital and government debt. They show that the welfare effect of increasing government debt depends crucially on the aggregate savings schedule. Indeed, when the government issues bonds the interest rate can increase, reducing private capital. These are also the mechanisms at work in our analysis.

Lastly, Acharya and Dogra (2022) present a study that aligns closely with the questions explored in our research. They utilize an overlapping generations model, much like ours, but extend their framework to include nominal rigidities and idiosyncratic capital income risk. Importantly, their model features an endogenous convenience yield, in contrast to the exogenous yield considered in our work. This endogenization allows for a nuanced analysis of welfare implications. The authors posit that an increased supply of 'safe assets,' primarily in the form of government debt, could restore the efficacy of conventional monetary policy in economies constrained by the zero lower bound (ZLB). Notably, Acharya and Dogra observe that, even in models with endogenous convenience yields, crowding out effects mirror our findings: they are costly and lead to diminished investments in physical capital. These observations hold true especially in economies where the safe interest rate can assume negative values without compromising dynamic efficiency. While their study offers valuable theoretical insights, it remains largely qualitative. Our research is instead calibrated meticulously to reflect U.S. economic conditions and policies, and has as objective the quantification of the repercussions on GDP of escalating levels of public debt.

II. A simplified model

A. Economic Environment

Time is discrete and runs to infinity. Each individual lives for N periods. Hence, individuals born at time t live for dates $t, t+1, ..., t+N-1$. Preferences over consumption , can be represented by the time separable utility function

$$\sum_{j=0}^{N-1} \beta^j u(c_{t+j}^t),$$

where function $u : \mathbb{R}_+ \to \mathbb{R}$ satisfies standard assumptions, and $0 < \beta$ is the discount factor.

We assume exponential population growth at constant rate $n > 0$, and an exogenously given population size at period 0, L_0.

$$L_t = (1+n)^t L_0$$

Production is undertaken by competitive firms. The aggregate production function is F and satisfies constant return to scale and other standard assumptions

$$Y_t = F(K_t, L_t).$$

Let $k = K/L$, so that $f(k) = \frac{F(K,L)}{L} = F(k, 1)$. Going forward, capital letters will denote aggregates while small caps will denote per capita values. Factor markets are assumed competitive. Individuals can only work during $N-1$ periods, and supply one unit of labor inelastically earning the wage rate w_t.

To keep the model simple, we do not endogenize individuals' asset allocation decisions. Instead, we follow Braun and Joines (2014) and assume that individuals save in a fund that allocates an exogenous fraction $1 - \phi_t$ of savings to government debt and a fraction ϕ_t to firms' capital (In equilibrium, the value of ϕ_t will turn out to be consistent with the levels of debt and private capital stock). Hence, the gross rate of return to saving is a weighted average of the rental rate of capital i_t^k and the rate paid by government bonds i_t^b so that

$$1 + i_t^k = f'(k_t) + 1 - \delta$$
$$R_t = \phi_t(1 + i_t^k) + (1 - \phi_t)(1 + i_t^b)$$

We will assume $1 + i_t^b = (1 + i_t^k - \gamma)$, with γ convenience yield.

Finally, there is a government with an initial debt level B_0 that finances a predetermined stream of expenditures $\{G_t\}$, $G_t > 0$ for all $t = 0, 1, 2, ...$ Government expenditures do not directly affect the return function of any private agent of this economy. The government can tax households using only consumption taxes, τ_t [2].

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[2] A comprehensive and realistic set of direct and indirect taxes will be introduced in the quantitative version of the model

B.   Intertemporal optimization

The optimization problem of an individual born at time t, is

$$\max \sum_{j=0}^{N-1} \beta^j u(c_{t+j}^t)$$

subject to:

$$(1 + \tau_{t+j})c_{t+j}^t + s_{t+j}^t = R_{t+j}s_{t+j-1}^t + w_t l_{t+j}^t$$
$$l_{t+j}^t = 1 \, j = 0...N - 2, l_{t+N-1}^t = 0$$
$$s_{t-1}^t = 0$$

Firms will generate zero profit so, Wage $w = f(k_t) - f'(k_t)k_t = (1 - \alpha)f(k_t)$. The interest rate received by households and the marginal product of capital are related in the following way (given the assumptions on portfolio holdings)

$$\begin{aligned} R &= \phi(1 + i^k) + (1 - \phi)(1 + i^k - \gamma) \\ &= f'(k^*) - \gamma(1 - \phi). \end{aligned}$$

C.   Equilibrium

Total savings in the economy are

$$S_t = \sum_{i=0}^{N-1} \mu_{i,t} s_t^{t-i}$$

where $\mu_{i,t}$ are the population weights in period t of all individuals who save (the weighted sum of the savings of individuals alive with age $0...N - 1$). Market clearing for assets (assuming in this case full depreciation) requires

(1) $$K_{t+1} + B_{t+1} = S_t.$$

Consistency of expected and actual returns requires

$$\phi_t = \frac{K_t}{K_t + B_t}.$$

The law of motion for debt is the standard one

$$B_{t+1} = G_t + (1 + i_t^b)B_t - \tau_t C_t$$

or

$$b_{t+1} = \frac{g_t + (1 + i_t^b)b_t - \tau_t c_t}{1 + n}$$

An equilibrium, for a given sequence of government debt, expenditure, and taxes, is a sequence of prices (interest rates and wages), and allocations (consumptions and savings for all individuals) such that individuals solve their optimization problems, taking prices as given, and markets clear.

D. Comparative Statistics (steady state)

We now study the long-run impact of increasing public debt. For simplicity, we assume individuals live for two periods and preferences can be represented by the log utility $u = log(c)$. A steady state is an equilibrium where per capita variables are constant over time. Exogenous growth can be easily introduced but does not change any of the conclusions (one would focus on a balanced growth path where all per capita variables grows at an exogenously given constant rate).

Notice that since the only tax is the consumption tax, and in a steady state such tax is constant over time, taxation does not affect the intertemporal Euler equation of individual of generation t, namely

$$\frac{1}{w-s} = \beta R \frac{1}{Rs}$$

solving for savings yields

(2)
$$s = \frac{\beta w}{1+\beta}.$$

For asset markets to clear

$$(1+n)k^* + (1+n)b^* = s = \frac{\beta w}{1+\beta},$$

must hold. The following equation characterizes the steady state of the model

(3)
$$(1+n)k^* + (1+n)b^* = \frac{(1-\alpha)\beta f(k^*)}{1+\beta}.$$

This model is as a generalization (to the case of two assets) of standard OLG analyses in models with production. Its graphical representation is in Figure 1, and it is the counterpart of Figure 9.2 in McCandles and Wallace, 1991. As is apparent from the diagram, the steady state capital with zero debt, k^{max} is the maximal attainable. Higher levels of debt reduce steady state capital. Hence, the crowding out mechanism holds regardless of the value of $i^b - g$.

Having established the basics of our analytical framework and its implications for the steady-state equilibrium, we now proceed to a more rigorous investigation via a series of lemmas. These lemmas extend the model's insights, providing a more nuanced understanding of the dynamics at play. When $r - g < 0$ there is a primary deficit that is compatible with a stationary debt to GDP ratio. We are interested in the relation between such a sustainable primary deficit and the steady state public debt ratio. When the debt ratio is zero the primary deficit compatible with a constant debt ratio is zero as well (even with $r - g < 0$). But increasing the debt ratio from zero initially increases the sustainable primary deficit as well. But increasing debt ratios push interest rates up so that the primary deficit reaches a maximum sustainable level. Subsequently, it declines. When $r - g = 0$, in steady state, the

Figure 1. Characterization of equilibrium in the analytical model
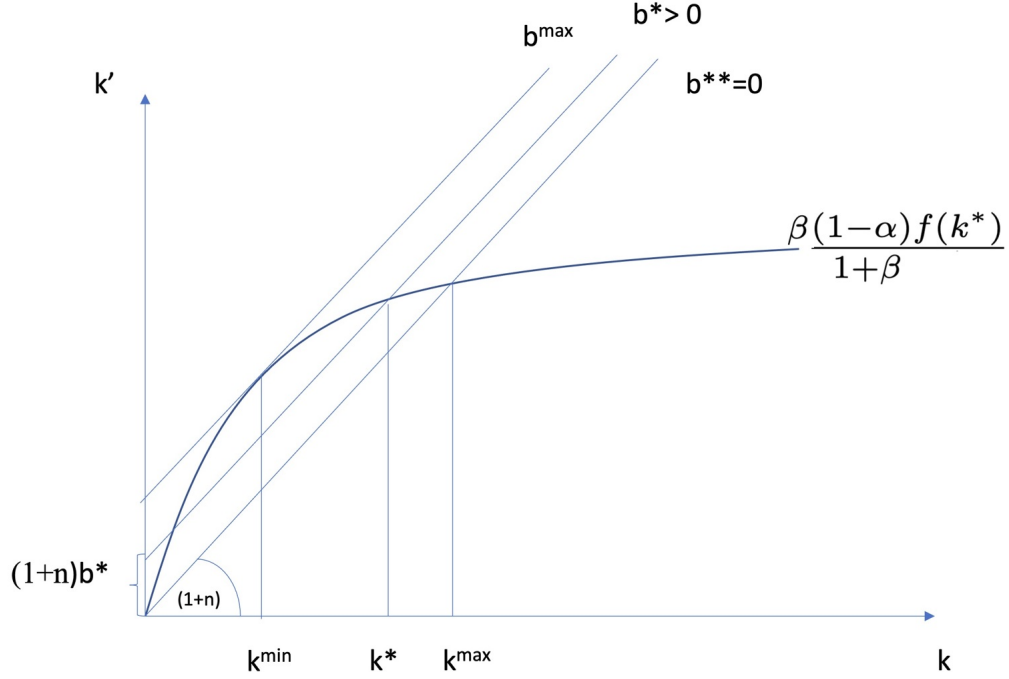
sustainable primary deficit is zero as well. Subsequently, for even larger debt ratios $r-g$ turns positive and constant public debt ratios require primary surpluses. Therefore, the relation between the debt ratio and the primary deficit looks like an inverted U-curve (very much like a Laffer curve – see Figure 2). The difference is that on the horizontal axis there is the debt ratio instead of the tax rate. The debt ratio can be interpreted as analogous to a tax base.

Lemma 1. At the stable steady state, government bond always crowds out private capital.

Proof. The stability of the steady state requires that $\frac{\alpha\beta(1-\alpha)}{(1+\beta)}k^{\alpha-1} \le 1+n$. According to 3, under standard calibration (i.e., $\alpha, \beta \in (0,1), n > -1$),

$$(4) \quad \frac{\partial b^*}{\partial k^*} = -1 + \frac{\alpha\beta(1-\alpha)}{(1+n)(1+\beta)}k^{*\alpha-1} = \frac{1}{1+n}\left(-1 + \frac{\alpha\beta(1-\alpha)}{(1+n)(1+\beta)}k^{*\alpha-1}\right) < 0.$$

So, whenever government bond increases, private capital decreases. □

The conclusion of the analysis this far, is that increasing government deficits is costly because it will crowd out private investment. This will happen even when $r-g$ is negative. Further, there exists a maximum amount of per capita debt compatible with the existence of equilibrium.

Lemma 2. At the stable steady state, there is a maximum amount of debt, b_{max}, and a minimum amount of capital, k_{min} the economy can support.

$$(5) \qquad k_{min} = \left(\frac{(1+n)(1+\beta)}{\alpha\beta(1-\alpha)}\right)^{\frac{1}{\alpha-1}}$$

$$(6) \qquad b_{max} = -k_{min} + \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}k_{min}^{\alpha}$$

Proof. According to 3, we know $\frac{db^*}{dk^*} = -1 + \frac{\alpha\beta(1-\alpha)}{(1+n)(1+\beta)}k^{\alpha-1}$, which is a decreasing function in k^*. Therefore, by setting $\frac{db^*}{dk^*}$ to zero, we get the maximum of the debt that the economy can support. At the stable steady state, there is a negative relationship between private capital and government bond. So, the corresponding level of capital is the minumum amount of capital, k_{min}, the economy can support. Therefore, $k_{min} = \left(\frac{(1+n)(1+\beta)}{\alpha\beta(1-\alpha)}\right)^{\frac{1}{\alpha-1}}$ and $b_{max} = -k_{min} + \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}k_{min}^{\alpha}$. $\qquad\square$

The government budget constraint at the steady state is the following:

$$(1+n)b^* = g^* + (1+i^b)b^* - \tau C^*$$
$$pd^* = g^* - \tau C^* = (1+n)b^* - (1+i^b)b^* = -(r-g)b^*$$

where pd is the primary deficit.

Lemma 3. When $r - g < 0$, there is a maximum of primary deficit the economy can support at the stable steady state.

Proof. At the steady state, primary deficit, pd, can be characterized as follows:

$$(7) \qquad pd = -(r-g)b = -(\alpha k^{\alpha-1} - (1+n) - \gamma)b$$

So the first derivative of primary deficit over primary capital is

$$(8) \qquad \frac{\partial pd}{\partial k} = -(\alpha k^{\alpha-1} - (1+n) - \gamma)\frac{\partial b}{\partial k} - (\alpha(\alpha-1)k^{\alpha-2})b$$

when $b = b_{max}$ and $k = k_{min}$, $\frac{\partial b}{\partial k} = 0$. So,

$$(9) \qquad \left.\frac{\partial pd}{\partial k}\right|_{b=b_{max},k=k_{min}} = -(\alpha(\alpha-1)k_{min}^{\alpha-2})b_{max} > 0.$$

When $b = 0$ and $r - g < 0$,

$$(10) \qquad \left.\frac{\partial pd}{\partial k}\right|_{b=0} = -(\alpha k^{\alpha-1} - n - \gamma)\frac{\partial b}{\partial k} < 0$$

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The second derivative of primary deficit over primary capital is

$$
\begin{aligned}
\frac{\partial^2 pd}{\partial k^2} &= -(\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k} - (\alpha(\alpha-1)(\alpha-2)k^{\alpha-3})b \\
&\quad -(\alpha k^{\alpha-1} - (1+n) - \gamma)\frac{\partial^2 b}{\partial k^2} - (\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k} < 0
\end{aligned}
$$

(11)

Therefore, as k increases and b decreases, the first order derivative turn from positive to negative. So, over $b \in (0, b_{max})$, pd is hump shaped function in b and there is a maximum of pd which is determined by setting 8 to zero. □

The intuition is that, even though $r - g$ is negative, government debt can crowd out private capital and raise the interest rate. As a result, the $r - g$ becomes less negative. Consequently, the sustainable deficit in the economy follows is reminiscent of Laffer curve, as a function of government debt.[3]

In the context of grounding this two-period model to reality and illustrating the possible quantitative impact of increasing government debt, we assume a period equals about 40 years. We assume a zero percent growth rate for population, a standard discount factor $\beta = 0.987^{40}$, and a Cobb-Douglas production function with a standard value for the capital share $\alpha = 0.484$. The convenience yield is set to 4% annually to guarantee $r - g < 0$. The maximum debt to GDP the model economy can support is 396 percent of GDP ($b_{max} = 396$). Figure 2, illustrates some of the results in Lemmas 1-3 for the parameters we employ for the two period model. As Lemma 3 shows, the maximum deficit is a inverted-U shape as a function of debt to GDP, with a maximum at 7 percent of GDP ($pd_{max} = 7$).

These illustrative steady state calculations where 212 percent of GDP for public debt can be sustained together with a 7 percent for the maximum sustainable deficit are an order of magnitude higher than reasonable. One reason for such large numbers is that the model abstracts from risk of default and the associated potential instability in bond markets. More important, the fact that a maximum sustainable deficit exists should not be interpreted in any way as a desirable objective to attain. Indeed our analysis shows that the cost of such high deficits and debts in terms of foregone investment and economic potential are very high.

With all the adequate caveats now in place, we calculate the steady state implications of increasing debt to GDP ratio to 120 percent of GDP (closer to recent values for the US) from 60 percent of GDP. In this case, we find the economy is still on the increasing part of the hump shape curve relating deficits and debt, and the economy can finance a 2.3 percent higher deficit. This also translates into a very substantial 8.0 percent decline in output. The output declines further by 13.3 percent if debt to GDP increases from 120 percent to 212 percent when maximum sustainable deficit is achieved. The large negative effect on GDP happens independently of whether $i^b - g$ is negative or positive.

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[3]Aguiar et al. (2022) and Mian et al. (2021) find inverted U curves for the deficit vs debt to GDP relation.

Figure 2. Results in the Analytical Model
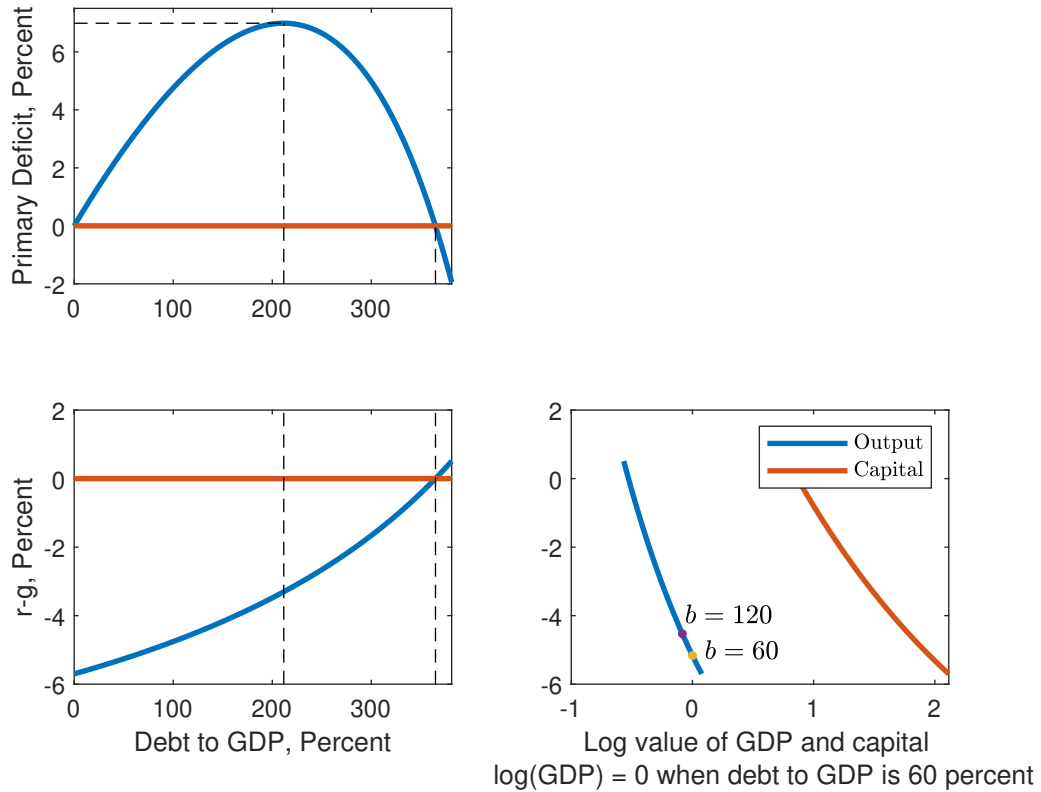
III.   The quantitative model

The model considered here is based on McGrattan and Prescott (2017), which includes a rich set of features in the taxation system (for both, individuals and different types of businesses) that are key to match the U.S. data. As in the analytical version of the model, we introduce a convenience yield for government debt. To do so, we assume the interest rate on government bonds equals the interest rate on private assets minus an exogenously given convenience yield.

Households have an OLG structure with measure n_t^1 arriving as working-age households at the beginning of date t. The year since entry into the workforce is called age and is denoted by j. The measure of age j households at date t is n_t^j The maximum possible age is J. The probability of an age $j < J$ household of any type at date t surviving to age $j+1$ is $\sigma_t^j > 0$. The n_t^1 are parameters that define the income population at date t, which evolves as follows:

$$n_{t+1}^1 = (1 + \eta_t)n_t^1$$

with $n_0^1 = 1$, where η_t is the growth rate of households entering the workforce.

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A. State Vector

To simplify notation, we use recursive competitive equilibrium language. All stocks are beginning-of-period stocks. The variables that define the aggregate state vector s are as follows:

- (i) $t = 0, 1, 2, \ldots$, is the time period.

- (ii) $\{a^j, n^j\}$ are the assets a^j (net worth) of an age j, and n^j is the measure of these households.

- (iii) B is the government debt owned by the private sector.

- (iv) K_{T1} and K_{T2} are aggregate tangible capital stocks for two business sectors (described below).

- (v) K_{I1} and K_{I2} are aggregate intangible capital stocks for two business sectors.

B. Portfolios and the return of assets

Individuals are assumed to save in shares of ownership of an asset constituted by a fraction ϕ_t of government debt and a fraction $1 - \phi_t$ of claims to the flows from private firms' capital, as is also assumed in Braun and Joines (2015) and Kitao (2015) Hence, the rate of return of the only financial asset available is a weighted average of the returns of government debt and private capital. As the capital account is assumed to be closed, total asset holdings in this composite financial asset must equal government debt and private capital for equilibrium in financial markets to hold. Hence, given the exogenous stock of government debt and its interest rate, fraction ϕ_t will be computed as part of equilibrium, to achieve consistency with the assumed fractions of assets in the portfolio (rational expectations).

C. Prices and Policy

The relevant equilibrium price sequences for the households are government debt and private capital interest rates $\{i_t^b, i_t^k\}$ and wage rates $\{w_t\}$.

Policy specifies the following sequences:

- (i) Tax rates $\tau = \{\tau_t^c, \tau_{1t}^d, \tau_{2t}^d, \tau_{1t}^\pi\}$, where c denotes consumption, d distributions from businesses to their owners, and π profits. Sector 2 businesses are not subject to the corporate profit tax and must distribute all their profits to their owners.

- (ii) Net tax schedules $\{T_t^w(\cdot), T_t^r(\cdot)\}$ for those with positive labor income (e.g., workers) and no labor income (e.g., retirees).

- (iii) Government debt $\{B_t\}$.

- (iv) Pure public good consumption $\{G_t\}$.

D.   The Household's Problem

The value function of a household of age $j \in \{1, 2, \ldots, J\}$ satisfies

$$v_j(a, s) = \max_{a', c, \ell \geq 0} \{u(c, \ell) + \beta \sigma_t^j v_{j+1}(a', s')\}$$

subject to

$$(1 + \tau_{ct})c + a'\sigma_t^j \leq \left(\phi_t(1 + i^b) + (1 - \phi_t)(1 + i^k)\right) a + y_t - T_t^j(y_t)$$
$$y_t = w_t \ell \epsilon^k$$
$$s' = F(s).$$

Symbol ℓ denotes the labor services of a household. Households with $j > J_R$ are retired and their ℓ's are zero. The net tax schedule for retirees $(j > J_R)$ is $T^j(y) = T^r(0)$ and is equal to the (negative) transfers to retirees since they have no labor income. The net tax schedule for workers $(j \leq J_R)$ is $T^j(y) = T^w(y)$ and is equal to their total taxes on labor income less any transfers. The prime denotes the next period value of a variable and $v_{J+1} = 0$. Savings are in the form of an annuity that makes payments to members of a cohort in their retirement years conditional on them being alive. Effectively, the return on savings depends on the survival probability as well as the interest rate. Aggregate labor supply L is

$$L = \sum_j n^j \ell^j \epsilon^j.$$

The equilibrium law of motion of the aggregate state variable, F, is taken as given by the private agents.

E.   Technology

The U.S. tax code has an explicit treatment of taxation based on legal status of incorporation. To match this fact, the model considers one sector is subject to the corporate income tax and produces intermediate good Y_{1t}, while the other is the non-corporate sector, which also produces intermediate good Y_{2t}. The aggregate production function of the composite final good is

$$Y_t = Y_{1t}^{\theta_1} Y_{2t}^{\theta_2},$$

where the exponents are positive and sum to 1.

The aggregate sectoral production function is Cobb-Douglas with inputs of tangible capital K_{iTt}, intangible capital K_{iIt}, and labor L_{it}:

$$Y_{it} = K_{iTt}^{\theta_{iT}} K_{iIt}^{\theta_{iI}} (\Omega_t L_{it})^{1 - \theta_{iT} - \theta_{iI}}$$

for $i = 1, 2$. The labor-augmenting technical level at date t in both sectors is Ω_t, which grows at rate γ, so

$$\Omega_{t+1} = (1 + \gamma)\Omega_t.$$

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Capital stocks depreciate at a constant rate, so

$$K_{iT,t+1} = (1 - \delta_{iT})K_{iTt} + X_{iTt}$$
$$K_{iI,t+1} = (1 - \delta_{iI})K_{iIt} + X_{iIt}$$

for $i = 1, 2$, where T and I denote tangible and intangible, respectively, and X is investment. Depreciation rates are denoted as δ and are indexed by sector and capital type. The resource balance constraint is

$$Y_t = C_t + X_{Tt} + X_{It} + G_t,$$

where $X_{Tt} = \sum_i X_{iTt}$ and $X_{It} = \sum_i X_{iIt}$.

F. Government Budget Constraints

Some notation must be set up before the law of motion for government debts can be specified. The prices of the intermediate good relative to the final good are p_{1t} and p_{2t}. The accounting profits of Schedule C corporations are given by

$$\Pi_{1t} = p_{1t}Y_{1t} - w_t L_{1t} - X_{1It} - \delta_{1T}K_{1Tt},$$

and distributions to the corporations' owners are

$$D_{1t} = (1 - \tau_{1t}^\pi)\Pi_{1t} - K_{1T,t+1} + K_{1Tt}.$$

Other business distributions to their owners are

$$D_{2t} = \Pi_{2t} = p_{2t}Y_{2t} - w_t L_{2t} - X_{2It} - \delta_{2T}K_{2Tt}.$$

We can now specify the law of motion of government debt:

$$B_{t+1} = B_t + i_t^b B_t + G_t - \sum_j n_t^j T_t^j (w_t \ell_t^j \epsilon^j) - \tau_t^c C_t - \tau_{1t}^\pi \Pi_{1t} - \tau_{1t}^d D_{1t} - \tau_{2t}^d D_{2t}.$$

Thus, next period's debt is this period's debt plus interest on this period's debt, plus public consumption, minus tax revenues (net of transfers). Taxes are levied on labor income and consumption, on profits of Schedule C corporations, on distributions of Schedule C corporations to their owners, and on distributions of other business firms to their owners

G. Equilibrium Conditions

The equilibrium conditions are as follows:

- (i) Labor, capital, and goods markets clear at each point in time.

- (ii) The household policy functions $\{a' = f_j(s)\}_j$ imply the aggregate law of motion $s' = F(s)$.

IV.   Calibration

We follow the calibration strategy of McGrattan and Prescott (2017), which is consistent with both the national accounts and fixed assets reported by the BEA from 2000 to 2010, as well as the distribution of individual and household incomes reported in the CPS.

A.   Parameters Based on Macro Data

For the baseline economy, we assume a zero percent growth rate of the population, a work life of 45 years, and survival probabilities calibrated to match the 2010 life tables in Bell and Miller (2005). The preference parameters are chosen such that the model's labor input and labor share align with CPS data from the United States. Assuming logarithmic preferences given by the function:

$$u(c, \ell) = \log c + \alpha \log(1 - \ell),$$

we set α to 1.185, which implies a fraction of work time of approximately 0.3. The growth rate of labor-augmenting technology is set at 2 percent, consistent with the trend growth in the United States.

The choice of tangible capital shares (θ_{1T}, θ_{2T}) and tangible depreciation rates (δ_{1T}, δ_{2T}) ensures alignment between the model's investments and fixed assets with the tangible investments and stocks reported by the BEA and Flow of Funds. The two sectors have tangible capital shares of $\theta_{1T} = 0.182$ and $\theta_{2T} = 0.502$. The annual depreciation rates, $\delta_{1T} = 0.051$ and $\delta_{2T} = 0.015$, are selected to generate investment rates consistent with U.S. data. The intangible capital shares and depreciation rates, denoted as θ_{1I}, θ_{2I}, δ_{1I}, and δ_{2I}, are set to be consistent with Table 1. In addition, we set $\beta = 0.987$ to ensure that the model's predicted division of income into labor and capital aligns with that of the U.S. national accounts.

We set the level of government consumption as a percentage of GDP to 0.08 in the initial steady state, denoted as $\phi_{G0} = 0.08$. This value is consistent with the average government deficits as a percent of GDP over the period 2001-2005. The initial debt parameter, ϕ_{Bt}, is set equal to the ratio of U.S. government debt to GDP in 2004, resulting in $\phi_{B0} = 0.6$.

Our quantitative analysis consists of increasing government consumption unexpectedly. We assume that the government uses debt to finance the additional government spending. The return on government bonds is lower than the return on private assets due to the convenience yield ($CY = i^k - i^b$). The convenience yield is calibrated to be 4.0 percent, resulting in an interest rate on government bonds of 0.79 percent. This interest rate broadly aligns with the real interest rate in 2005, which was estimated to be around 0.93 percent.[4]

The capital tax rates are assumed to be the same for all asset holders. However, there are two categories of businesses that are subject to different taxation: Schedule C corporations and all other businesses. Schedule C corporations are subject to the corporate income tax. The effective corporate income tax rate, denoted as τ_1^π, is approximately 33 percent over the period 2000–2010[5]. In addition to the corporate income tax, investors in these corporations

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[4]Average monthly difference between Long-Term Government Bond Yields and CPI inflation in 2005.

[5]This figure is based on the calculation in McGrattan and Prescott (2017)

also pay an additional tax on distributions, denoted as τ_1^d. This additional tax rate is 14.4 percent and is applied to distributions in the form of dividends and share buybacks. For other businesses, the tax rate on distributions, denoted as τ_2^d, is 38.2 percent.

Table 1—Marcoeconomic Aggregates (Model vs Data), Averages Relative to Adjusted GNP, 2000–2010

Category	Data	Model
Total Adjusted Income	1.000	1.000
Labor Income	0.585	0.584
Capital Income	0.415	0.416
Consumption	0.745	0.714
Tangible investment	0.211	0.208
Government Deficit*	0.019	0.018
Tangible Capital	4.117	4.045
Intangible Capital	1.700	1.700

Note: Government deficit is calculated using the average of 2001-2005

Table 2—Parameters of the Economy Calibrated to U.S. Aggregate Data

Names	Value
Demographic parameters	
Growth rate of population (η)	0
Work life in years	45
Number of workers per retiree	3.93
Preference parameters	
Disutility of leisure (α)	1.185
Discount factor (β)	0.987
Technology parameters	
Growth rate of technology (γ)	2
Income share, Schedule C corporations (θ_1)	0.500
Capital shares	
Tangible capital, Schedule C (θ_{1T})	0.182
Intangible capital, Schedule C (θ_{1I})	0.190
Tangible capital, other business (θ_{2T})	0.502
Intangible capital, other business (θ_{2I})	0.095
Depreciation rates	
Tangible capital, Schedule C (δ_{1T})	0.050
Intangible capital, Schedule C (δ_{1I})	0.050
Tangible capital, other business (δ_{2T})	0.015
Intangible capital, other business (δ_{2I})	0.050
Initial Spending and debt shares	
Government spending (ϕ_G)	0.080
Government debt (ϕ_B)	0.600
Capital tax rates	
Profits, Schedule C corporations (τ_1^π)	0.330
Distributions, Schedule C corporations (τ_1^d)	0.144
Distributions, other business (τ_2^d)	0.382

B. Parameters Based on Micro Data

The next step involves setting the parameters that are relevant to the model's predicted distributions of income and net taxes. In our model, we assume the existence of one representative family for one age cohort. To calibrate the model, we use labor efficiency units based on productivities derived from CPS family data, which captures the life-cycle income profile. It is important to note that households will not join the model economy until they reach at least 15 years of age.

To parameterize the initial net tax schedules, $T_0^j(\cdot)$, we use piecewise net tax schedule in McGrattan and Prescott (2017). We assume that all workers face the same schedule, $T_0^j(y) = T^w(y)$, $j \leq J_R$. Similarly, all retirees face the same schedule, $T_0^j(y) = T^r(0)$, $j > J_R$,

which does not vary with labor income because retirees are assumed to have no labor income. $T^w(y)$ on each AGI income interval $[\underline{y}_i, \bar{y}_i]$, $i = 1, \ldots, I$, are calibrated as follows:

Table 3—Current and future labor income net tax schedules, $T_w^\infty(y) = \alpha_i + \beta_i y$.

Earnings Over	α_i	β_i
0	-11,762	0.059
5,132	-12,819	0.246
11,664	-13,518	0.264
17,418	-14,365	0.293
23,718	-15,211	0.316
29,692	-15,971	0.332
36,351	-17,000	0.349
45,274	-18,503	0.367
58,274	-20,359	0.382
74,560	-22,880	0.396
106,007	-28,810	0.409
191,264	-45,792	0.409

Note: Earnings and net tax function intercepts are reported in 2004 dollars.

The last parameter to be set is the consumption tax, which is determined residually in order to balance the government's budget. In our baseline parameterization, this requires setting the tax rate to 8.2 percent.

V. The impact of a permanent increase in public debt

In this section, we quantify the impact of the long-term rise in public debt in the US. We achieve the gradual increase in the debt-to-GDP ratio by implementing a single policy change: an annual increase in government expenditures[6] by 3.5 percent over a span of 20 years, while maintaining constant tax rates. To finance the resulting deficits, the government issues debt. After 20 years, government spending is readjusted to the initial steady state, and the debt-to-GDP ratio is stabilized using a consumption tax. It is important to emphasize that all other factors, particularly demographics, are held constant in order to isolate the effects of the increased debt-to-GDP ratio. The resulting debt path closely resembles the long-term trend observed in US data, as depicted in the figure 3.

The model analysis reveals that the increase in government debt has detrimental effects on the economy. It leads to a crowding out of capital (see figure 4), resulting in a decline of capital by 9 percent over the initial 20-year period and 15 percent in the long run. However,

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[6]Aguiar et al. (2022) have noted that increasing revenue through debt issuance can be utilized to subsidize capital, mitigate crowding-out effects and support a pareto improvement when $r - g$ is negative. Whether this use of government resources can be also achieved in our model is a quantitative question (depending on the savings elasticity, and also on the relative size of debt and capital to GDP). It is beyond the scope of the present paper, as our focus is exclusively on the costs of debt.

the return on capital (MPK) experiences a slight increase over time, rising by 0.2 percentage points in the first 20 years and 0.4 percentage points in the long run. [7]

Furthermore, the increase in government debt has adverse consequences for GDP (see figure 5), which experiences a decline of over 4 percent in the initial 20-year period and over 8 percent in the long run. In contrast, labor supply remains relatively stable throughout this period.

$i^b - g$, which represents the return on capital net of GDP growth and convenience yield, initially increases over the first 20 years due to high interest rate and the negative GDP growth (see figure 6). However, it gradually declines towards the new steady state as GDP growth returns from negative to zero. Importantly, it is noteworthy that the $i^b - g$ value remains negative throughout the analyzed period. These results indicate that policymakers should exercise caution and carefully evaluate the potential costs and benefits associated with increasing government debt.

Figure 3. Debt Path over 2004-2022
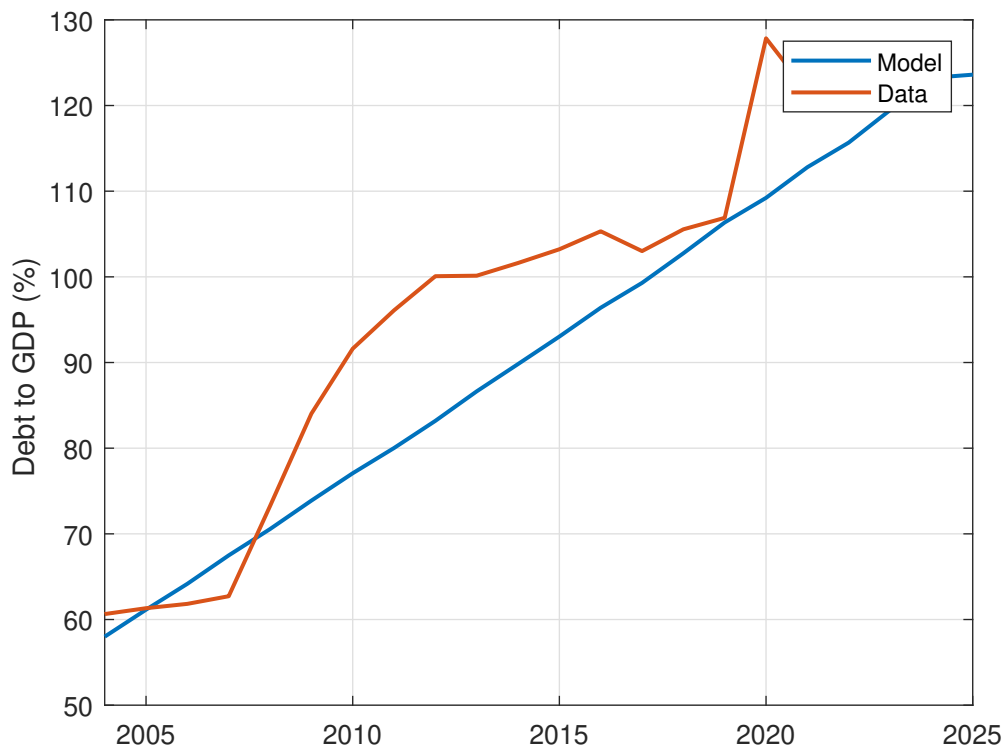
Note: Data refers to the total public debt as percent of gross domestic product from the FRED, 2004-2022.

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[7]Rachel and Summers (2019) suggest that a debt-GDP ratio increase of 1 percentage point of GDP raises interest rates by 3-4 basis points, which is larger than the results obtained from our simulations. One possible explanation for this discrepancy could be the erosion of the convenience yield. We investigate this further later.

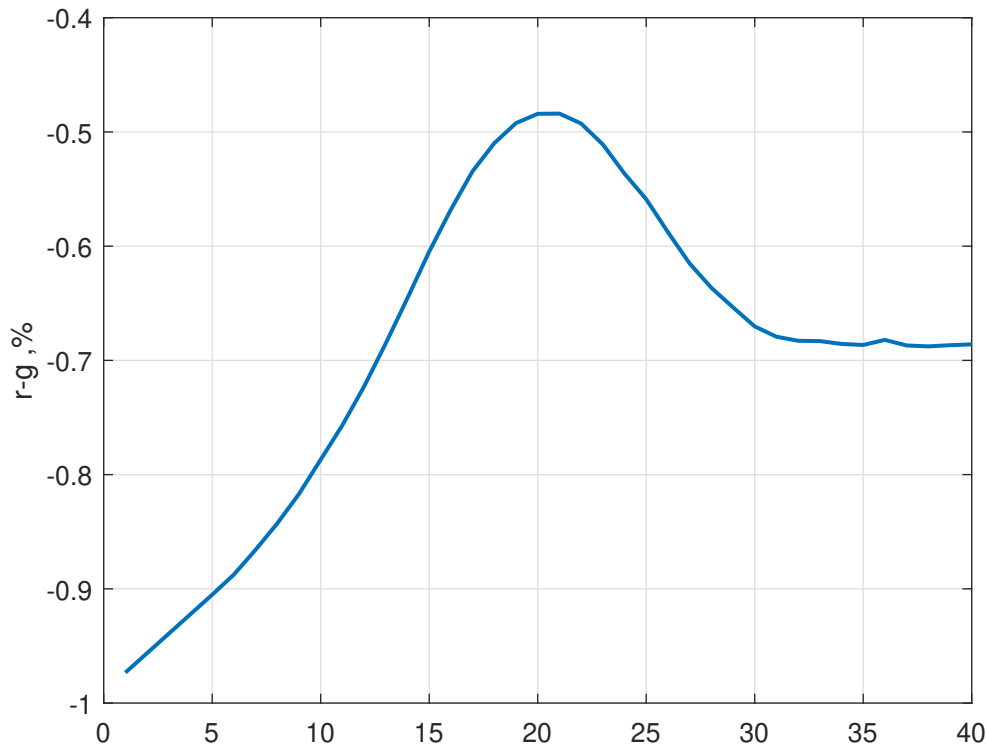Figure 4. Capital Stock over Time
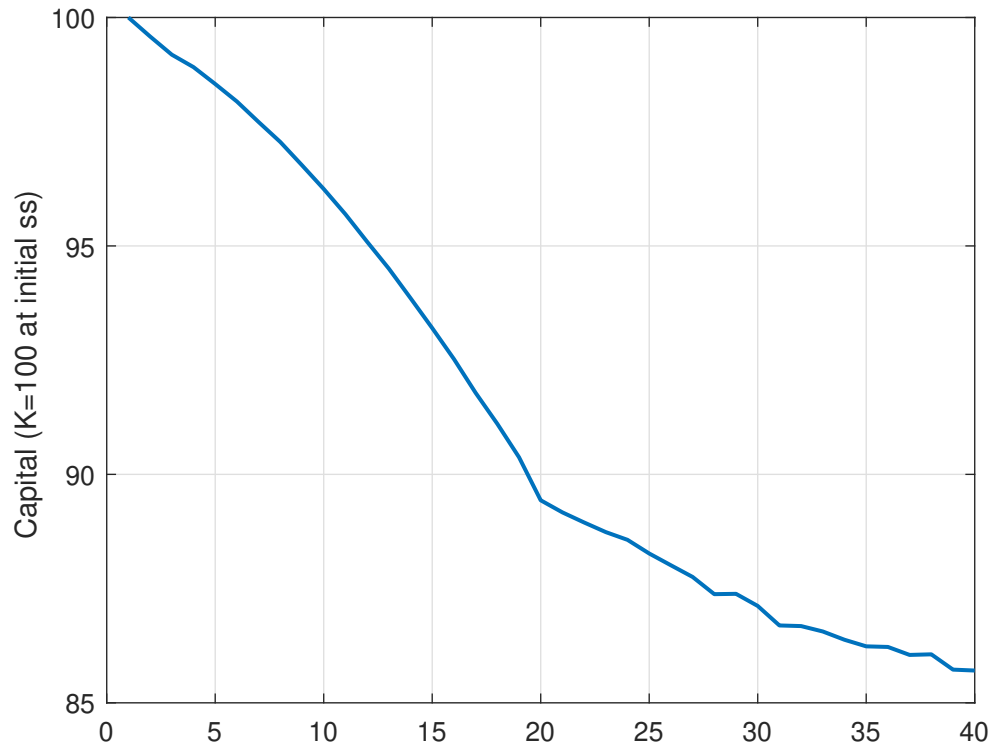
Note: We set the initial steady state capital as 100. Capital consists of both tangible and intangible capital.

Figure 5. Gross Domestic Product over time
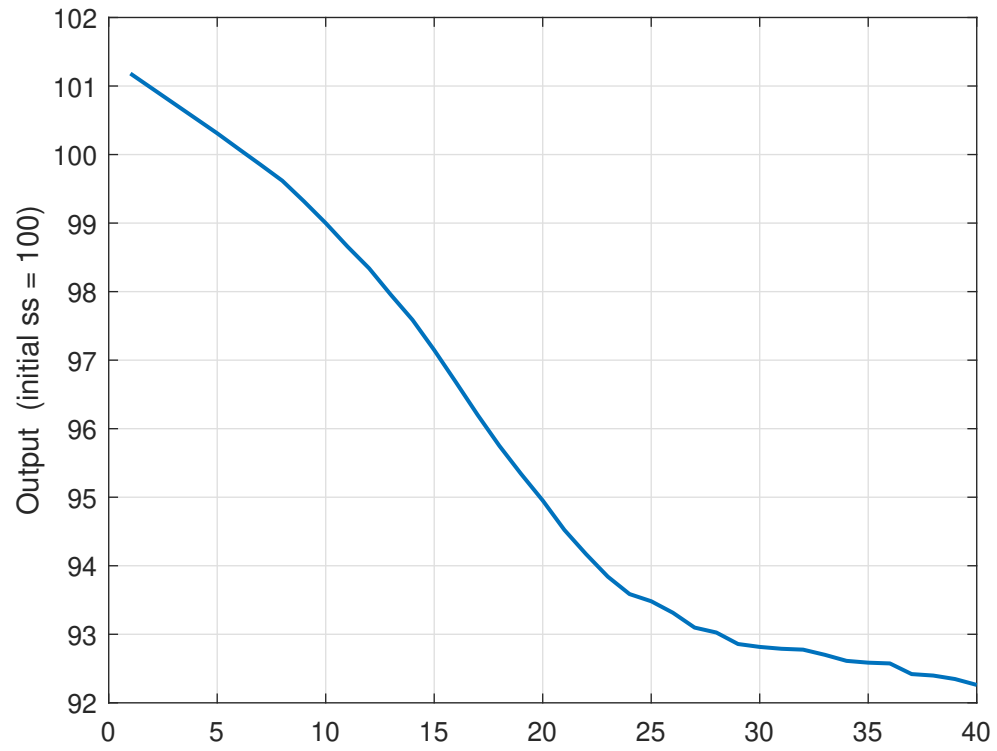
Note: We set the initial steady state GDP as 100.

Figure 6. $i^b - g$, interest and growth differentials over time

Note: $i^b - g$ represents the return on capital net of GDP growth and convenience yield.

VI. Alternative specifications

In this section, we explore alternative specifications to test the robustness and extend the applicability of our main results. The first subsection investigates the impact of foreign ownership of U.S. public debt, offering a more nuanced view of how global demand for safe assets can affect domestic economic outcomes. The second subsection addresses the crucial policy question of financing mechanisms for public spending. Specifically, we compare the long-term effects on GDP between debt financing and tax financing. Both exercises aim to expand the scope and relevance of our findings while preserving the fundamental insights of our original model.

A. Foreigners also hold public debt

To assess the robustness of our quantitative findings, we conducted an alternative simulation that assumes 30 percent of U.S. government debt is held by foreign investors, as has been the case in recent years (see Figure 7). Under these modified conditions, the consequences for GDP are consistent with our initial findings but exhibit a smaller magnitude (see Figure 8). Specifically, GDP declines by about 4 percent in the initial 20-year period, and by slightly

more over the long run. The crowding-out effect is mitigated because a portion of newly-issued government debt is absorbed by foreign investors, reducing the domestic impact.

Figure 7. Federal Debt Held by Foreign and International Investors
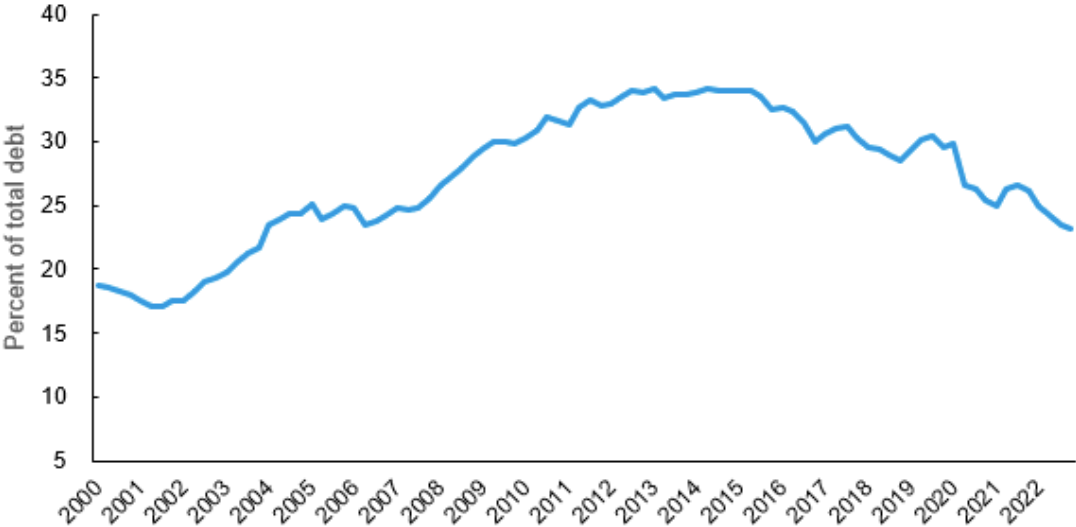
Source: Federal Reserve Economic Data by the Federal Reserve Bank of St. Louis.

It is essential to recognize that this robustness check is not a substitute for a more comprehensive model capable of accurately capturing the global demand and supply dynamics of safe assets. However, the theoretical insights gleaned from our simpler model underscore the importance of the elasticity of demand for safe assets as a key variable in quantifying the crowding-out effect. Notably, the elasticity of interest rates on government debt in response to changes in the debt-to-GDP ratio within our model aligns well with empirical estimates in existing literature, particularly falling within the lower bound region. This correspondence lends credence to the robustness of our main findings.

Figure 8. Gross Domestic Product over Time
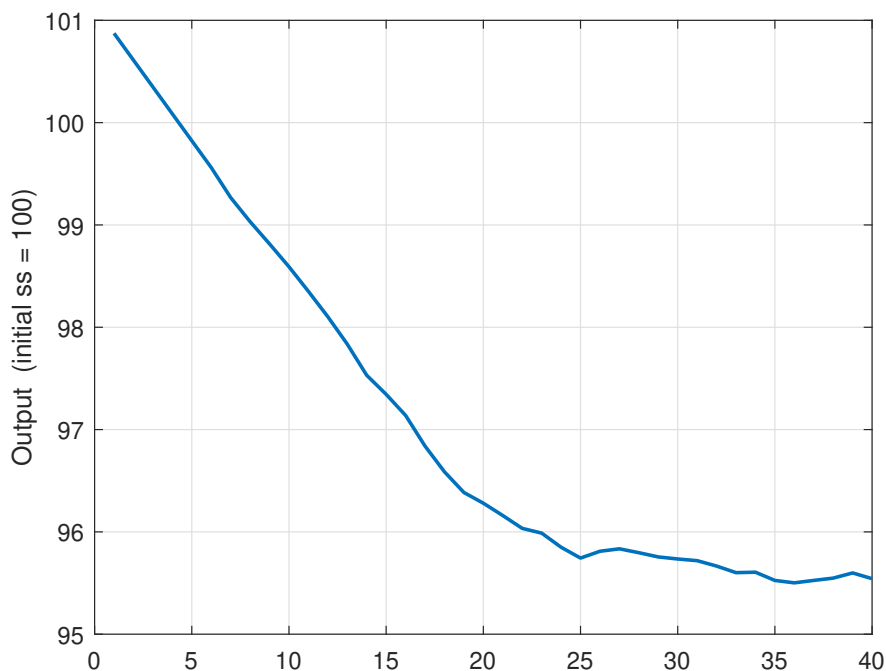
Note: We set the initial steady-state GDP as 100.

B. Tax financing vs Debt Financing

Our analysis centers on the costs of public debt and abstracts from the possible benefits of public spending. Still, it is interesting to consider, given the spending associated to our baseline experiments, whether debt financing or tax financing would be preferred. The preferred way to finance spending would be that resuling in the least negative impact on GDP. Our quantitative model is rich in detail, including progressive taxation. So, to address this question one has to take a stand on what tax to use. We opt for using consumption taxes. The reason is that in steady state, consumption taxes would not be distorting directly any marginal decisions. And consumption taxes are also known to have lower macro impact than others (e.g. increasing labor taxes). We thus take the same path of government spending of our baseline quantitative experiment and contrast the effects of GDP if instead of increasing debt the government adjusted consumption taxes as needed to keep debt to GDP at is original steady state level.

The analysis suggests financing through consumption taxation while keeping public debt to GDP stable would have small effects on GDP, in contrast to our baseline experiment based on debt financing and debt stabilizing only in the distant future.

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VII. Conclusion

This paper examined the macroeconomic costs of increasing levels of public debt within a general equilibrium overlapping generations framework, abstaining from considerations of default and uncertainty. Our focus has been on the crowding-out mechanism, captured in a model that incorporates two assets. Public debt is posited to pay an interest rate that equals the net return on private capital minus an exogenously determined convenience yield.

An essential aspect of our model lies in the feasibility constraints that all agents, including the government, must adhere to. This adds another layer of complexity to the discussion around the sustainability of public debt. The endogeneity of both the real interest rate (r) and the economic growth rate (g) plays a pivotal role in our analysis. These variables are influenced by policy decisions, and their interaction with elevated public debt levels warrants more scrutiny.

By using a calibrated version of our model tailored to the United States, we uncover that the quantitative impact of increasing public debt can be highly substantial. In particular, the increase in public debt from 60 to 120 percent of GDP over the last 20 years could lead to a permanent reduction in long-term GDP by about 8 percentage points.

While our findings suggest that the fiscal costs of public debt may appear manageable in a low-interest-rate environment, the macroeconomic costs are notably significant. Our study calls for a nuanced understanding of public debt dynamics, especially in light of the endogenous nature of key economic indicators.

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