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# Bailout and Conglomeration<sup>1</sup>

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#### **Abstract**

The paper suggests that when firms differ stochastically in their productivity, a bank may find it optimal not to bail out the failed nonconglomerate firms at all, but to bail out conglomerates fully. Expectation of such bailout policy may encourage risk-averse firms to join a conglomerate to minimize the risk of liquidation. Furthermore, in case of private information, bad firms follow good firms' decision on conglomeration to hide their type. Finally, the paper discusses the impact of conglomeration on the debt-equity ratio and the expansion of existing conglomerates through mergers and acquisitions.

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### I. INTRODUCTION

In this paper, I present a model of conglomeration based on the inter-relationship between bailout and conglomeration. This paper focuses on how the presence of conglomerates affects the bank's information set and hence its optimal bailout (or liquidation) decision, and how the banks' bailout decision, in turn, affects the firms' optimal decision on joining a conglomerate or not. In particular, conglomeration is seen as a device that is designed to maximize the chance of bailout (or minimize the risk of liquidation) in case of default.

The recent financial crisis in East Asia reignited the vast public and academic interest in the issue of bailout. In the countries hit by the crisis, a question has been repeatedly raised which firms should be bailed out and which firms should be liquidated among the distressed. In particular, given that conglomerates have been liquidated much less than non-conglomerates, the important questions that arise, are "should conglomerates be treated differently from non-conglomerates in the bank's bailout decision?", and "doesn't the bank's preferential treatment on conglomerates induce some moral hazard problem of the firms?" <sup>2</sup>

The issue of bankruptcy, liquidation and bailout has been addressed by law and economics scholars in various context (see, e.g., Bulow and Shoven (1978), White (1980), Bolton and Scharfstein (1990, 1996), Hart and Moore (1994, 1998), Aghion, Hart and Moore (1992) and Povel (1996)). For example, Bulow and Shoven (1978) present a model with three asymmetrical claimants on the assets (stockholders, bondholders, and bank lenders), and show how maturity and priority structure of the firm's debt affects the bankruptcy choice of the bank lender. In a model of incomplete financial contract, Bolton and Scharfstein (1990) analyze the optimal debt contract in an environment where investors' commitment to liquidate firms whose performance is poor, can mitigate managers' incentives to divert resources to themselves but encourage rivals' incentives to prey. Povel (1996) sets up a model where earlier rescues of non-financial firms in financial distress are much cheaper, and it may pay for the creditors to be forgiving in bankruptcy since it will encourage the managers having private information to reveal difficulties as early as possible rather than to gamble for

For example, in Korea, the bankruptcy of the big Korean conglomerates (so called "chaebol"s) has been very rare for the last three decades. Even in 1997, the year of the worst crisis in its recent history, none of the big five conglomerates has failed, and only several medium-size conglomerates, which were not considered "too big to fail", failed and were not bailed-out by the government. Hahm, Ferri and Bongini (1998), using a sample of 500 Korean corporations, recently suggested that liquidity constraints that could lead to bankruptcy are more stringent for non-conglomerates than for conglomerate firms.

resurrection. In the existing literature, however, the issue of conglomeration and bailout has not been closely examined.<sup>3</sup>

To address this issue, the paper presents a two-period heterogenous agent model of lending, default and conglomeration. There are risk-averse firms which stochastically differ in productivity: good firms and bad firms. Good firms receive a positive productivity shock with higher probability than bad firms, though the size of a positive shock is the same regardless of whether they belong to good or bad firms. Because of the stochastic heterogeneity, the bank cannot verify whether a particular firm is good or bad. This assumption reflects the fact that in reality, it is often very difficult to distinguish the good from the bad firms and to judge whether a defaulting firm is good or bad, particularly in developing countries where the banking system's capacity to assess the profitability of firms is less sophisticated.<sup>4</sup>

The information set, however, is non-empty. First, the underlying probability distributions are common knowledge. All participants in the economy know the probability distribution of productivity for good and bad firms, together with the proportion of good and bad firms in the initial period. In addition, information set in the second period expands compared to the first period (as the productivity shock of the first period is revealed). In the second period when shocks are already observed, the bank still cannot tell whether a particular firm is good or bad, but can make an inference on the fraction of good and bad firms in the distressed, as well as in the non-distressed sample. Exploiting the new information generated by the revelation of the shock, banks make their decisions on reallocation of capital across firms in the second period. A key aspect of reallocation decision is liquidation and bailout, since liquidation implies reallocating capital that was used by failed firms to non-failed firms. The bank's bailout decision affects the quality composition of high productivity and low productivity firms, and hence the total output and welfare in the second period.

Another distinctive feature of the model is that individual firms are allowed to join a conglomerate, whose main function is to cross-guarantee debt payments.<sup>5</sup> Cross

<sup>&</sup>lt;sup>3</sup> In addition, our model focuses on the impact of liquidation on quality composition, while most existing literature emphasizes the role of liquidation in mitigating agency problems.

<sup>&</sup>lt;sup>4</sup> Our model builds partly on the recent incomplete contracting theory of debt (see Hart (1995) for an overview of this literature). Hence, the relation between the borrower and lender is regarded not only as a contract which specifies repayment amounts but also the conditions under which control passes from the borrower to the lender. However, our model deviates from the literature by focusing on the quality composition of heterogenous firms, and conglomeration.

<sup>&</sup>lt;sup>5</sup> The model highlights the key financial reason for forming a conglomerate, that is, cross guarantees of debt payments. Of course, there also can be technological reasons for conglomeration, including benefits from vertical integration that reduces transaction costs, and from pooling of industry-specific risks by diversification.

guarantees allow failed members of a conglomerate to raise financing with backing from non-failed members.<sup>6</sup> While the cross guarantee may raise the chance of payment failure for an individual firm, it also raises the chance of bailout and hence the expected utility of risk-averse entrepreneurs.

Several important conclusions follow from the model presented in this paper. First, the bank's liquidation of failed firms rather than bailout can be used as a screening device to select good firms among the firms that stochastically differ in productivities. Although the bank may not definitely tell whether each defaulting firm is good or bad, the revelation of productivity shock allows the bank to infer the probability that a defaulting firm is good or bad, given the underlying probability distribution of good and bad firms. Based on such signal extraction, the bank can make bailout decision, which depends on the benefits and costs of liquidation (or costs and benefits of bailout). Liquidation is costly as far as reallocation of a firm's capital to another firm requires some adjustment costs. On the other hand, liquidation is beneficial because it prevents some of bad firms from operating alongside good firms, and therefore raises capital productivity and the future output. If the chance of bad firms among the failed is much higher than among the non-failed, and hence the benefit is larger, the optimal policy will be a "no-bailout" (or "full liquidation").

Second, the presence of conglomerates, based on mutual debt payment guarantees, may change the bank's optimal bailout decision from a zero-bailout to a full-bailout. A conglomerate is formed as a congregation of both bad and good firms, and hence the chance of bad firms among a failed conglomerate will be the same as that of the original sample. Given the difficulty of probabilistic distinction between good and bad firms in the presence of conglomerates, the failure of conglomerates may not provide sufficient signal to the bank: the bank finds the conglomerate, even failed, not sufficiently bad (in terms of quality composition) to be liquidated. Therefore, the optimal bailout will be a "full-bailout" for conglomerates.

<sup>6</sup> The model assumes that there is no profit-sharing, and therefore each member of conglomerate gets its own profit.

<sup>7</sup> If there is no uncertainty on the types of the firms and hence the bank can distinguish good from bad firms, the optimal policy would be simple: bail out the good firms, let the bad firms go bankrupt.

<sup>&</sup>lt;sup>8</sup> The existing literature in the Schumpeterian tradition, has usually focused on the cleansing effects of recessions (see Caballero and Hammour (1994)). In our model, however, revelation of shock itself, regardless of positive or negative shock, has cleansing effects on the economy because they eliminate inefficient firms as far as bad firms have higher probability of failure. In addition, the bailout policy can magnify the cleansing effect in our model. In a similar setting, Izvorski and Kim (1999) explore how business cycles affect the optimal bailout decision, and the cleansing effects of recessions can be amplified by the liquidation decision.

Third, given the bank's optimal bailout policy, the risk-averse firms with rational expectation may prefer joining a conglomerate to use it as a noise signaling device, even though it lowers the expected profit. Because firms are aware that there would be a full bailout in case of conglomeration, this possibility creates a moral hazard problem. By joining a conglomerate as gathering of good and bad firms, individual firms can dilute the information that can be extracted by the bank from failure of firms after revelation of shocks. The information dilution or noise signaling strategy allows the firms to avoid liquidation, by reducing the incentive of the banks to liquidate them. Then it also follows that from welfare standpoint, the equilibrium with conglomeration can be sub-optimal.<sup>9</sup>

Fourth, there will be no equilibrium along which the two types of the firms can be separated in their behavior on conglomeration, even in case of private information where the type of a firm is known only to the firm itself. The reason is clear. If bad firms distinguish themselves from good ones (for example, by forming a conglomerate, while good firms do not), the bank will exploit the signal and liquidate all the failed bad firms. Hence, it is optimal for bad firms to conceal what type they belong to by following good firms' decision on conglomeration.

Fifth, the model suggests that conglomerates tend to have higher debt-equity ratio than non-conglomerates. The reason is that equity can function as a buffer stock to repay the required amount to the bank in case of failure and hence to eliminate the chance of liquidation, as conglomeration does. Then the entrepreneurs with higher wealth may find it optimal to raise equity rather than to join a conglomeration.<sup>10</sup>

Finally, a variant model shows how conglomerates get bigger through mergers and acquisitions. Suppose that failed firms can sell themselves to conglomerates after productivity shock is revealed but before repayment to the bank is made. If the entrepreneur does not sell the failed firm, she would receive nothing in the second period given the bank's policy of full liquidation on failed unaffiliated firms. But if she sells it to a conglomerate, the failed firm will be fully bailed out under the umbrella of the conglomerate, and hence have a positive expected profits. As far as a bargaining guarantees positive fractions of the expected surplus to each of the conglomerate and the failed firm, both parties have incentive to engage in merger and acquisition.

<sup>9</sup> This model focuses on adverse effects of conglomeration on the economy, through cross debt guarantees. However, this does not mean that conglomeration is always bad, since conglomeration may have some favorable effects on the economy, for example, through externality of good entrepreneurship, by risk diversification or through vertical integration. <sup>10</sup>Based on regression analysis using 10,064 Korean firms during 1981-96, Lee and Lee (1998) suggest that the debt ratio of conglomerates is higher than that of independent firms even after controlling major determinants of debt ratio such as firm size, growth rate, and the rate of return.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 and Section 4 examines the bank's optimal bailout policy in case of non-conglomeration and conglomeration, respectively. Section 5 analyses the firms' optimal decision on conglomeration, and Section 6 examines the asymmetric information case. Section 7 discusses the debt ratio, and Section 8 examine mergers and acquisitions by existing conglomerates. Section 9 concludes the paper.

#### II. THE BASIC MODEL

The model economy has two types of players, firms and banks, and lasts two periods t = 1, 2.

## A. Firms and Conglomeration

The economy consists of a continuum of risk-averse entrepreneurs of measure one, each of whom owns a firm but has no wealth of his own in the initial period. Firms produce homogenous output using capital as the only input given a linear technology:

$$y_t = A_t k_t \tag{1}$$

where  $k_t$  is the amount of capital invested,  $A_t$  is the contemporaneous firm-specific productivity shock. For simplicity, assume that output can be reinvested and hence transformed into capital without any cost, but capital is irreversible so that it cannot be transformed into consumption goods. In addition, assume that capital does not depreciate at all.

Firms come in two varieties, good (high average productivity) and bad (low average productivity) firms depending on the probability distribution of productivity shock. The idiosyncratic shock affecting good firms, that is independently distributed across periods and agents, is given by:

$$A_t = \begin{cases} A & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$
 (2)

while the shock affecting bad firms is:

$$A_t = \begin{cases} A & \text{with probability } \delta, \ \delta < \pi \\ 0 & \text{with probability } 1 - \delta \end{cases}$$
 (3)

That is, good firms get the positive shock A > 0 with a higher probability,  $\pi$ . This is the crucial difference between good and bad firms: good firms are, a priori or on average, more productive than bad firms, guaranteed by the restriction  $\pi > \delta$ . Suppose that at the beginning, the x fraction of the firms are good, and therefore the ratio of good to bad firms in the initial sample is x/(1-x).

<sup>&</sup>lt;sup>11</sup>Section 7 studies the case where entrepreneurs have positive wealth at the beginning.

The parameters  $(\pi, \delta, x)$  are common knowledge and hence known to anybody in the economy at the beginning of the first period. In the benchmark model, however, the type of each individual firm is not known (and hence not verifiable) to the bank and to the entrepreneur of the firm herself as well, at the beginning of the first period when the debt contract is made (while Section 6 studies the case where this assumption is relaxed). The incomplete information persists also in the second period.

The risk-averse entrepreneurs could form a conglomerate in the form of cross debt payment guarantees. More specifically, member firms of a conglomerate make a contract of the cross guarantees, according to which the member firms having a good productivity shock are obliged to raise financing to pay the debt of the members having a bad productivity shock.

There are costs of enforcing the cross-guarantee contract. The enforcement cost differs depending on the composition of the members of a conglomerate. There are n subgroups (that we may call "families", or "communities") of equally populated entrepreneurs. Each entrepreneur belongs to a subgroup indexed by j=1,2...n. For simplicity, I assume that the contract made among entrepreneurs belonging to the same family is enforceable without costs except for the n-th subgroup. For the n-th subgroup, the enforcement cost is infinity, and therefore the contract is not enforceable within the subgroup. For the contract made among those who belong to different subgroups (for example, between an entrepreneur from j=1 and another from j=2), the enforcement cost is also infinity. This restricts the size of a conglomerate within the size of a subgroup or a family. For simplicity, we assume there is no profit-sharing among the members, and therefore each member of a conglomerate gets its own profit.

We assume that all the agents in the economy (entrepreneurs of firms and the bank) have the identical utility function that depends only on the consumption of the final period (the second period here),  $u(c_2)$ . Each entrepreneur has only one income source, entrepreneurial profits. Thus, each entrepreneur seeks to maximize the expected utility derived from the profits of his own firm in the second period. An important optimization decision of the firms is made on whether they join a conglomerate or not, at the beginning of the first period (after capital is distributed across the entrepreneurs by banks).<sup>13</sup>

A firm's liquidation and transfer of its capital to a new firm incurs costs. More specifically, the liquidation value for capital is given by L = lk where l(<1) is a

<sup>&</sup>lt;sup>12</sup>Such a difference in enforcement costs across agents may explain why a large portion of conglomerates, particularly in Asia are family-owned or family-based.

 $<sup>^{13}</sup>$ We may also assume that production of each firm is affected by the default of the other firms in a conglomerate. For example, assume A=0 if any of the other member firm goes bankrupt. This assumption suggests that the bankruptcy of a firm has strong externality on the other firms in the same conglomerate.

constant capturing the liquidation value of capital relative to its original value.<sup>14</sup> Since l is less than one, letting failed firms go bankrupt is costly.<sup>15</sup>

### B. Banks and the Debt Contract

The economy has a representative bank interacting with heterogenous firms.<sup>16</sup> The representative bank seeks to maximize the revenue from lending at the end of the second period. It makes loans to the firms according to a debt contract analyzed in more detail below.

The bank does not own any production technology, so it has to lend money to entrepreneurs. At the beginning of the first period, the bank lends the same amount k to all of them. The reason is that in the very beginning the bank cannot distinguish the good from the bad firms.

The debt contract specifies the payment to be made from the output  $y_1$  at the end of the first period (denoted by d,  $0 \le d$ )<sup>17</sup> and the condition that if firms are not able to pay d, they are declared in default.<sup>18</sup> In case of default, control rights (particularly on physical capital k) pass to the bank. Then the bank may bail out the failed firms by lending money d to them so that they can meet the repayment requirement by paying the same amount of money (d) to the bank.<sup>19</sup> Instead, the bank may liquidate them and acquire L = lk. The bank decides on the fraction f of the non-conglomerate firms it would bail-out and, correspondingly, the fraction (1 - f) of

<sup>14</sup>For simplicity, I assume l is constant. But I may assume that the liquidation value can be affected by various aspects of debt structure such as the number of creditors, the allocation of securities interests and voting covenants as in Bolton and Scharfstein (1996).

<sup>15</sup>An alternative explanation is that if a firm is liquidated at the end of the first period, its assets are sold to another firm. In order for a new firm to use the capital that used to be installed in the old firm, an adjustment cost in the amount (k-L) is borne.

<sup>16</sup>Whether we assume a risk-averse or a risk-neutral bank does not affect any result in this model where the bank can fully diversify risks.

<sup>17</sup>We may instead assume that the firms can make payments from selling physical capital as well by redefining d as  $d \ge k$ , which does not alter the results of the paper.

<sup>18</sup>Bolton and Scharfstein (1996) call this type of defaults that arise when the firm does not have the cash to make debt payments as *liquidity* defaults, and distinguish it from the *strategic* defaults that occur because managers want to divert cash to themselves. To focus on the effect of liquidation on quality composition of capital, I do not explicitly introduce the strategic defaults.

<sup>19</sup>Given the representative bank framework where the bank from which a failed firm borrows the bailout money is identical to the one to which it repays, this type of bailing out is equivalent to rolling over, or not requiring the failed firms to pay d to the bank. If we explicitly introduce a large number of identical banks, bailing out a firm may not imply rolling over the debt to the firm. But this modification does not change the results of the paper.

the firm it would liquidate. It also decides on the fraction  $f^C$  of the failed conglomerate it would bail-out.

In addition, the debt contract recognizes that the bank has access to a technology (court system, reputational punishment, etc.) that allows it to claim a fraction  $\phi < 1$  of the second period output produced by any firm (regardless of conglomeration). That is, the debt contract gives rise to cash flows equal to d at the end of the first period if the firm repays, (1 - f)L in liquidation revenue if the firm defaults at the end of the first period, and a fraction  $\phi$  of the second period output.

Therefore, the debt contract can be written as  $(k, d, f, f^C, \phi)$ . Notice that the debt contract explicitly allows for default, liquidation, bail-out (forgiveness), as well as incomplete recovery in the second period.

# C. Timing of Events

The exact timing of events is as follows. The model economy has two periods, t=1,2. Each period go through the following sequence of events. At the very beginning, the bank decides the allocation of capital across firms. At the second stage, firms decide on whether to join a conglomerate or not, and then engage in production activity using capital borrowed from the bank, which is followed by the third stage where idiosyncratic productivity shock is revealed. At the final stage, based on production outcome affected by productivity shock, repayment is carried out.

In particular, at the beginning of the second period, the bank decides its capital allocation decision, the key of which is bailout decision, whether to forgive failed firms (and by doing so, to let them continue to use capital), or not to forgive them (hence to transfer the capital to non-failed firms). Another important decision concerns whether to join a conglomerate or not. The conglomeration decision is made by the firms in the first period, after the bank lent them money.

#### III. LIQUIDATION AS A SCREENING DEVICE

We now examine the bank's optimal bailout decision made at the beginning of the second period. This section focuses on the case where there is no conglomerate, and illustrate how non-bailout (or liquidation) can be used as a screening device among heterogenous firms.

At the beginning of each period, the bank decides capital allocation among different firms. The decision in the first period is trivial. At the beginning of the first period, the bank lends the same amount k to all of them, because the bank cannot distinguish the good from the bad firms.

However, the decision in the second period is not trivial. After firms engage in production with the initially allocated capital k, the first-period idiosyncratic productivity shock is revealed. Then there emerge two groups of firms - those with a

realization of the idiosyncratic shock equal to A > 0 and those with a zero realization of the shock. The firms in the first group produce identical output of  $y_1 = Ak > 0$ , while the firms in the second group produce nothing. Note that both groups of firms include high and low productivity firms, and in a rational expectations equilibrium the relative size of these firms are well-known as shown below.

At the end of the first period, based on the productivity shock, repayment is carried out. If the output of a firm in the first period is not sufficient to cover the due payment on the loan, d, the firm is declared in default and its control rights are passed to the bank. To ensure that firms that produce non-zero output in the second period cannot go bankrupt, the condition that Ak > d is assumed.

In the beginning of the second period, the bank makes a decision on whether to either bail out the failed firms or liquidate them. This bailout decision is a key decision on capital allocation for the second period. Note that if the bank could distinguish between the good and the bad failed firms, it would bail out only the good ones and let the bad ones fail. But this is not possible, since there are only observations, repayment or not. Thus, there persists incomplete information even at the beginning of the second period when bailing-out and liquidation should occur (as at the beginning of the first period when lending occurs); the type of each firm cannot be verified even after its first period output is observed.

To analyze the decision of the bank at the beginning of the second period, denote the number of failed and surviving non-conglomerate firms at the end of the first period by  $N_f$  and  $N_s$ , respectively. Since  $x\pi$  good firms and  $(1-x)\delta$  bad firms survive at the end of the first period, the measures of the surviving and the failed firms at the end of the first period are given by:

$$N_s = x\pi + (1-x)\delta \tag{4}$$

$$N_f = 1 - N_s = x(1 - \pi) + (1 - x)(1 - \delta)$$
(5)

Therefore, in the sample of failed firms at the end of the first period there are  $x(1-\pi)$  good firms and  $(1-x)(1-\delta)$  bad firms, a total of  $N_f$ .

Note that the restriction  $\pi > \delta$  guarantees that the fraction of surviving good firms to total surviving firms at the end of the first period will be higher than the fraction of failed good firms to total failed firms:

$$\frac{x\pi}{x\pi + (1-x)\delta} > \frac{x(1-\pi)}{x(1-\pi) + (1-x)(1-\delta)} \tag{6}$$

Notice that the expression on the left side of (6) denotes the conditional probability that a firm which does not fail at the end of the first period is good, while the expression on the right denotes the conditional probability that a firm which has failed is good. This expression suggests that due to the revelation of shock in the first period,

the bank can partially separate two groups whose expected productivity differ, and therefore may have better capital allocation in the second period.

How does the bank determine which firms to bail out and which to liquidate? Recall that the bank cannot determine which of the failed firms are good and which are bad. So it cannot make its bail-out decision firm by firm, but as a fraction of the aggregate. At the beginning of the second period, the bank decides on the fraction f of failed firms which it would bail out, though which firms are bailed out is randomly determined. Therefore, the bank bails out  $fx(1-\pi)$  good firms and  $f(1-x)(1-\delta)$ bad firms.

This bailout affects the capital allocation. The bank lends money d to failed but bailed-out firms. With the bailout money (whose total is  $fdN_f$ ), the firms will then repay the same amount of money to the bank so that they can meet the repayment requirement. Hence there will be no change in the bank's position of money that is induced by bailout.<sup>20</sup> However, the bank's capital position will increase because of the repayments of non-failed firms and the liquidation of some failed firms. Total revenue from repayments of surviving firms amounts to:  $(1 - N_f)d$ , while the revenue from liquidation is:  $(1-f)N_fL = (1-f)N_flk$ . Therefore, total cash flow from liquidation and repayment is given by:  $\Phi = (1 - N_f)d + (1 - f)N_f lk$ . Therefore, after receiving repayments and making liquidations, the bank has an additional capital in the amount  $\Phi$  to lend to firms for their operations in the second period.

Assume that the bank lends all of the capital to firms for production in the second period. In addition, suppose that each firm that has defaulted but has been bailed out does not receive new capital, while each firm that has not defaulted in the first period receives additional capital  $k_a$  per firm at the beginning of the second period.<sup>21</sup> Then, the amount of capital that a bailed-out firm has in the second period, denoted by  $k_2^B$ , is given by  $k_2^B = k$ 

which is the same as in the first period.

But the capital of a firm that has not failed is given by:

$$k_2^S = ((A+1)k - d) + \left[d + (1-f)lk\frac{N_f}{1 - N_f}\right]$$
(8)

(7)

where the first item in the right side, ((A+1)k-d), represents the residual after repayment per firm (recall that output can be transformed into capital and that there is

<sup>&</sup>lt;sup>20</sup>Note that bailout here is equivalent to not requiring failed firms to pay d to the bank. So there is no new resources required for bailout.

<sup>&</sup>lt;sup>21</sup>We can show that under certain conditions, this capital allocation rule is the result of the bank's optimal decision on the allocation of new capital across surviving firms and bailed-out firms.

no depreciation of capital), and the second item captures the total additional capital given to each surviving firm, which must be equal to total amount obtained from liquidation and repayment divided by the measure of the surviving firms,  $(1 - N_f)^{.22}$ 

Then how is the expected second-period total output of the firms that have been bailed out,  $EY^B$ , determined? Note that the bank bails out  $fx(1-\pi)$  good firms and  $f(1-x)(1-\delta)$  bad firms, and that the  $\pi$  fraction of the former and  $\delta$  fraction of the latter group will have a positive productivity shock in the second period. Using this, we have

$$EY^{B} = f \left\{ x(1-\pi)\pi + (1-x)(1-\delta)\delta \right\} Ak$$

$$\equiv fN_{f,s}Ak \tag{9}$$

where  $N_{f,s}$  represents the measure of the firms that have an adverse shock for t = 1, but a favorable shock for t = 2. That is,

$$N_{f,s} \equiv x(1-\pi)\pi + (1-x)(1-\delta)\delta$$
 (10)

In addition, note that the expected output is equal to the actual output  $(Y^B)$  because of the law of large numbers:  $EY^B = Y^B$ .

Similarly, the expected second-period total output of the firms that have not defaulted at the end of the first period (consisting of  $x\pi$  good firms and  $(1-x)\delta$  bad firms) is derived as:

$$EY^{S} = \{(x\pi)\pi + ((1-x)\delta)\delta\} Ak_{2}^{S}$$

$$= \{x\pi^{2} + (1-x)\delta^{2}\} A\left((A+1)k + (1-f)lk\frac{N_{f}}{1-N_{f}}\right)$$

$$\equiv N_{s,s}A\left((A+1)k + (1-f)lk\frac{N_{f}}{1-N_{f}}\right)$$
(11)

where  $N_{s,s}$  represents the measure of the firms that have positive shocks for both t=1 and t=2. That is,

$$N_{s,s} \equiv x\pi^2 + (1-x)\delta^2 \tag{12}$$

Note that from the standpoint of the bank, firm-specific idiosyncratic shocks can be fully diversified. Hence the risk averse bank does not need to have concerns about the variability of the revenue. Because of the law of large numbers, the expected output is equal to the actual output:  $EY^B = Y^B$  and  $EY^S = Y^S$ , where  $Y^B$  and  $Y^S$  represent the actual total output for the bailed-out firms and the surviving firms, respectively.

 $<sup>^{22}</sup>$ At the end of the second period, the bank could get k from each firm. But the capital will be useless at that time, given our simplifying assumption that it cannot be transformed into consumption goods. The relaxation of this assumption, however, does not alter the main qualitative results of the paper.

Given the expectation, the bank's optimization problem at the beginning of the second period is reduced to maximizing the expected revenue at the end of the second period:

 $\max_{f} V \equiv \max_{f} \left\{ \phi \left( Y^{B} + Y^{S} \right) \right\} \tag{13}$ 

Recall that  $\phi$  is the fraction of the second period output that the bank can claim (by some technology) and, therefore, value.

The derivative of the value function with respect to f is:

$$\frac{dV}{df} = \left[N_{f,s} - N_{s,s}l \frac{N_f}{1 - N_f}\right] \phi Ak \tag{14}$$

Given the linear technology assumption, we have bang-bang solutions. The optimal ratio of bailout, denoted by  $f^*$ , is decided to be  $f^* = 0$  or  $f^* = 1$ . In particular, if  $N_{f,s} < N_{s,s} l \frac{N_f}{1-N_f}$ , the derivative is negative, and therefore we have  $f^* = 0$ .

So we can establish the following proposition.

Proposition 1 If  $N_{f,s} < N_{s,s} l \frac{N_f}{1-N_f}$ , the bank's optimal policy is zero-bailout  $(f^* = 0)$ .

The intuition behind the proposition is straightforward. The bank's liquidation can be used as a device sorting between good and bad firms. The revelation of productivity shock in the first period allows the bank to partially separate between good and bad firms, since the portion of bad firms among the failed firms is higher (as shown in eq. (6)). Using the new information, the bank can raise the expected productivity of capital by reallocating capital from the failed to the non-failed firms. However, the reallocation of capital through liquidation (including the transfer of capital to non-failed firms) incurs costs (1-l) for a unit of capital. Hence, the bank's decision depends on the relative size of the benefit due to allocating capital to the good firms and the costs associated with capital transfer. But if the former is larger, the optimal policy can be zero-bailout. In particular, as the difference between the chance of a positive shock for bad firms  $(\delta)$  and good firms  $(\pi)$  gets larger (so that the firms are more heterogenous), the benefit from liquidating failed firms becomes larger  $(\delta)$  and therefore, the chance of no-bailout policy also increases.

<sup>&</sup>lt;sup>23</sup>In this case, the difference between the conditional probability of good firms among surviving firms and that among failed firms (as in eq. (6)) also gets larger, and so does the ratio  $\frac{N_f}{1-N_f}$ .

<sup>&</sup>lt;sup>24</sup>For example, in a case where  $\delta = 0$  and l is close to one, the derivative is reduced to  $\frac{dV}{df} = -(1-x)\pi\phi Ak < 0$ , which is negative. Hence the optimal solution is  $f^* = 0$ .

#### IV. CONGLOMERATES AND OPTIMAL BAILOUT

The previous section examined the optimal bailout policy in the case where each of the entrepreneurs makes her own independent firm, and does not join any conglomerate. In the situation, the bank's optimal bailout policy included zero bailout. This section considers the case where some individual entrepreneurs or firms join a conglomerate.

Suppose that at the beginning of the first period, the  $\tau$  fraction of the firms join conglomerates, and  $(1-\tau)$  firms do not.<sup>25</sup> As assumed before, the key function of conglomerates is to provide member firms cross debt payment guarantees. Note that the cross guarantees may raise the probability of payment failure of all the firms in the conglomerate, like tied ships in chain that are under fire. In particular, assume that d is set to satisfy the following condition:

$$[x\pi + (1-x)\delta]Ak < d \tag{15}$$

The left side is the expected output at the end of the first period, which is the actual per firm output of a conglomerate, which consists of a large number of firms (because of the law of large numbers). The right side represents a required repayment per firm. The condition suggests that total output of non-failed firms in a conglomerate is not enough to meet the repayment requirement for both failed and non-failed firms in a conglomerate.<sup>26</sup> Therefore, it is certain that the conglomerate fails to repay the pre-specified per firm amount d, given mutual debt payment guarantees.<sup>27</sup> In addition, similar to the previous section, assume that no new capital is provided to any firms belonging to a defaulting conglomerate.

In this case, at the end of the first period, conglomerates will default for sure, so the size of total defaulting firms attached to conglomerates will be  $\tau$ . Among non-conglomerate individual firms, the  $N_f$  fraction of the firms will fail, so the total number of failed non-conglomerates will be  $(1-\tau)N_f$ . Further, the size of defaulting

<sup>&</sup>lt;sup>25</sup>How the fraction  $\tau$  and the size of each conglomerate are determined in the model will be discussed in the next section.

<sup>&</sup>lt;sup>26</sup>If the bank sets d such that  $[x\pi + (1-x)\delta]Ak > d$ , conglomerates will not go bankrupt in any circumstance, which is equivalent to an automatic full bailout, and therefore d cannot work as a monitoring device. To derive a nontrivial optimal bailout policy on conglomerates, the bank needs to set d as in (15), so that the bailout is not automatic. In addition, to be an effective monitoring device for nonconglomerate firms, d should be set at a level lower than Ak. Therefore, d can be set any levels that satisfy  $[x\pi + (1-x)\delta]Ak < d < Ak$ .

<sup>&</sup>lt;sup>27</sup>In the absence of profit-sharing (that we assume), risk-averse firms cannot diversify their idiosyncratic risks.

individual firms will be raised from  $N_f$  in the case of no conglomeration to  $\tau + (1 - \tau)N_f$  (>  $N_f$ ) in the case of conglomeration.

In this situation, what will be the optimal bailout policy? At the beginning of the second period, the bank will decide its optimal bailout policy, more specifically, the bailout rate for non-conglomerates (f) and the rate for conglomerates  $(f^C)$ . The bailout rates affect the expected output at the end of the second period that the bank seeks to maximize.

The expected revenue of the bank depends on three elements: the expected second-period total output of non-conglomerate firms that have been bailed out, that of the conglomerate that have been bailed out, and that of the firms that have not defaulted at the end of the first period. The expected second-period total output of non-conglomerate firms that have been bailed out is:

$$EY^{B} = f(1-\tau)[x(1-\pi)\pi + (1-x)(1-\delta)\delta]Ak$$
  
=  $f(1-\tau)N_{f,s}Ak$  (16)

To calculate the expected second-period total output of the conglomerates that have been bailed out, note that the member firms that had a positive shock in the first period have capital (A+1)k, while the members that had an adverse shock have capital k. Using this, the expected output is:

$$EY^{C} = f^{C}\tau[[x\pi^{2} + (1-x)\delta^{2}]A(A+1)k + [x(1-\pi)\pi + (1-x)(1-\delta)\delta]Ak]$$
  
=  $f^{C}\tau[N_{s,s}(A+1) + N_{f,s}]Ak$  (17)

Finally, consider the expected second-period total output of the firms that have not defaulted at the end of the first period. Note that given condition (16), all of these firms are non-conglomerates. In the second period, each of the non-failed (and non-conglomerate) firms has four different sources of capital: capital carried over from the first period, k, reinvestment of output, Ak, capital reallocated from liquidated non-conglomerate firms,  $(1-f)lk\frac{N_f(1-\tau)}{(1-N_f)(1-\tau)}$ , and that from conglomerated firms,  $(1-f^C)lk\frac{\tau M}{(1-\tau)(1-N_f)}$  where  $M = [x\pi + (1-x)\delta](A+1) + [x(1-\pi) + (1-x)(1-\delta)] = (1-N_f)(A+1) + N_f$ . Using

this, we have

$$EY^{S} = (1-\tau)[x\pi^{2} + (1-x)\delta^{2}]A$$

$$[(A+1)k + (1-f)lk\frac{N_{f}(1-\tau)}{(1-N_{f})(1-\tau)} + (1-f^{C})lk\frac{\tau M}{(1-\tau)(1-N_{f})}]$$

$$= (1-\tau)N_{s,s}A[(A+1)k + (1-f)lk\frac{N_{f}}{1-N_{f}} + (1-f^{C})lk\frac{\tau M}{(1-\tau)(1-N_{f})}]$$
(18)

At the beginning of the second period, the bank seeks to maximize the expected revenue, which is the sum of  $EY^B$ ,  $EY^C$ , and  $EY^S$ , taking  $\tau$  as given:

$$\max_{f,f^C} V \equiv \max_{f,f^C} \left\{ \phi \left( EY^B + EY^C + EY^S \right) \right\} \tag{19}$$

The derivative of the value function with respect to f is:

$$\frac{dV}{df} = (1 - \tau)[N_{f,s} - N_{s,s}l\frac{N_f}{1 - N_f}]\phi Ak$$
 (20)

The derivative of the value function with respect to  $f^C$  is:

$$\frac{dV}{df^{C}} = \tau [N_{s,s}(A+1) + N_{f,s} - N_{s,s}lM\frac{1}{1 - N_{f}}]\phi Ak$$

$$= \tau [N_{s,s}(A+1) + N_{f,s} - N_{s,s}l[(A+1) + \frac{N_{f}}{1 - N_{f}}]]\phi Ak$$

$$= \tau [N_{s,s}(A+1)(1-l) + N_{f,s} - N_{s,s}l\frac{N_{f}}{1 - N_{f}}]\phi Ak \tag{21}$$

The sign of the derivatives depends on the parameter values. Assume that the parameters satisfies the following condition:

Condition (i)

$$N_{s,s}l\frac{N_f}{1 - N_f} - N_{s,s}(A+1)(1-l) < N_{f,s} < N_{s,s}l\frac{N_f}{1 - N_f}$$
(22)

Then, it can be easily shown that  $\frac{dV}{df} < 0$ , but  $\frac{dV}{df^C} > 0$ , which suggests that the bank's optimal bailout policy is zero-bailout for non-conglomerate firms but full bailout for conglomerates. The above discussion can be summarized by the following proposition.

Proposition 2 If it holds  $N_{s,s}l\frac{N_f}{1-N_f} - N_{s,s}(A+1)(1-l) < N_{f,s} < N_{s,s}l\frac{N_f}{1-N_f}$ , the bank's optimal policy is zero-bailout for non-conglomerates  $(f^*=0)$ , but full-bailout for conglomerates  $(f^{C^*}=1)$ .

The proposition suggests that the presence of conglomerates based on mutual debt payment guarantees, may drastically change the bank's optimal bailout policy. The key factor behind the result is the role of conglomeration in diluting information. In the absence of conglomerates, a revelation of productivity shocks may provide new

information that is substantial enough to allow the bank to find more beneficial to liquidate failed firms (and hence to reallocate capital from the failed to the non-failed). However, in the presence of conglomerates, a revelation of shocks does not provide new information that allows the bank to better distinguish the good from the bad firms, since conglomeration as gathering of both good and bad firms dilutes the information that can be obtained from payment failures of firms.

Recall that due to mutual debt payment guarantees conglomeration may raise the chance of payment failure to one. However, conglomeration reduces the risk of loosing capital through liquidation to zero, as far as conglomerates receive full bailout after payment failure.<sup>28</sup> <sup>29</sup>

#### V. OPTIMAL CHOICE OF CONGLOMERATION

So far we have looked at the bank's optimal bailout decision at the beginning of the second period, assuming the firms' decision on conglomeration (and hence  $\tau$ ) are given from the previous period. Now, we go backward to the first period, and explore the individual firms' decision on whether to join a conglomerate or not.

Suppose that at the beginning of the first period, risk-averse entrepreneurs who seek to maximize the expected utility choose whether to join a conglomerate or not:

$$\max_{\omega} [\omega E u(c_2^C) + (1 - \omega) E u(c_2^{NC})]$$
 (23)

where E is the expectation operator,  $\omega$  is the index number which is 1 in case where the firm chooses to join a conglomerate, and 0 in case where it chooses not, and  $c_2^C$  and  $c_2^{NC}$  are consumption at the end of the second period in case of joining a conglomerate and of remaining a non-conglomerate, respectively. Further assume that the utility function takes the standard CRRA form:

<sup>28</sup>We may instead consider the case where the firms belonging to a conglomerate divide total second period profits of the conglomerate equally across the member firms. In this case, the expected profit is the same as  $EY^C$  in equation (17), and hence the bank's optimal bailout policy does not alter.

<sup>&</sup>lt;sup>29</sup>Further, the above discussion provides an explanation of "too big to fail." Suppose that a conglomeration which is composed of m members. Define  $f^*(m)$  as a bailout ratio for conglomerates with m member firms. In an extreme case where m is equal to 1 (the case of independent firms), it is equal to  $f^*(1) = 0$  as shown above. In another polar case where m goes to infinity (as in the case of conglomerates discussed above), it is determined as  $f^*(\infty) = 0$ . We can also easily show that it hold that for any m' > m,  $f^*(m') \geq f^*(m)$ , that is, the optimal bailout rate is non-decreasing with the size of a conglomerate. This suggests that the bank tends to bail out larger firms more than smaller ones.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{24}$$

Since the sole source of income for entrepreneurs is profit and the profit is  $(1-\phi)$  fraction of output in the second period, the objective function of entrepreneurs can be rewritten as:

$$\max_{\omega} \left[ \omega Eu(\Pi_2^C) + (1 - \omega) Eu(\Pi_2^{NC}) \right] \tag{25}$$

where  $\Pi_2^C$  and  $\Pi_2^{NC}$  are profits made at the end of the second period in case of joining a conglomerate and of not doing so, respectively.

Note that in this maximization problem, entrepreneurs with rational expectation perfectly foretell what will be the optimal bailout policy of the bank in the second period. Assume that condition (i) holds. So the entrepreneurs predict that the bank will fully bail out failed conglomerates ( $f^{C*} = 1$ ), but fully liquidate failed non-conglomerated firms ( $f^* = 0$ ).

Recall that the cross payment guarantee contract is enforceable without any cost within any subgroup indexed by j=1,2...n-1, but is not enforceable due to infinite enforcement costs for the n-th subgroup. Hence entrepreneurs belonging to the n-th subgroup cannot make an effective cross guarantee contract with any others, even when they choose to join a conglomerate. The entrepreneurs have only to remain as non-conglomerates, which will be fully liquidated in case of having an adverse shock. Entrepreneurs who belong to any of subgroups j=1,2...n-1, however, can make an enforceable cross guarantee contract within the same subgroup and hence effectively form a conglomerate, if they choose to.

The risk-averse entrepreneurs belonging to j = 1, 2...n - 1, who seek to maximize the expected utility in the second period, compare both expected profits and risks of zero profits (due to liquidation) between the cases of joining a conglomerate and of not doing it. We now examine how the expected profit is determined in the two cases, respectively.

Consider the case when a firm does not join a conglomerate. In this case, the entrepreneur, who does not know which type she belongs, expects to have positive output consecutively in both the first and the second period with probability  $\pi^2$  if it is a good firm (whose probability is x) and with  $\delta^2$  if it is a bad firm (whose probability is 1-x). Given zero bailout for non-conglomerates, the firm does not expect positive output in the second period except the above cases of positive output in both periods. It also expects the capital in the beginning of the second period to be

 $k_2^S=(A+1)k+lk\frac{N_f}{1-N_f}$ , which reflects the transfer of capital from liquidated firms. Using these, the expected output in the second period is given by:<sup>30</sup>

$$EY^{NC} = [x\pi^2 + (1-x)\delta^2]A\left((A+1)k + lk\frac{N_f}{1-N_f}\right)$$
 (26)

which yields the expected profit

$$E\Pi_2^{NC} = (1 - \phi)N_{s,s}A\left((A+1)k + lk\frac{N_f}{1 - N_f}\right).$$
 (27)

In case of joining a conglomerate, the entrepreneur expects a positive output in the second period even when it has an adverse shock in the first period. The chance of having an adverse shock for t=1 and a positive shock for t=2 is  $\pi(1-\pi)$  if it is a good firm and  $\delta(1-\delta)$  if it is a bad firm. Using this, the expected output of a firm in the second period is given by

$$EY^{C} = [x\pi^{2} + (1-x)\delta^{2}]A(A+1)k + [x(1-\pi)\pi + (1-x)(1-\delta)\delta]Ak$$
 (28)

and therefore the expected profit is given by:

$$E\Pi_2^C = (1 - \phi)[N_{s,s}(A+1) + N_{f,s}]Ak$$
 (29)

It then can be easily shown that given condition (i), it holds

$$E\Pi_2^C < E\Pi_2^{NC} \tag{30}$$

which tells us that the expected profit is lower in the case of joining a conglomerate.

Now consider the risk of liquidation and subsequent zero profits. The chance of lowest profit (zero profit) in the case of non-conglomeration is given by

$$N_{f,s} + N_{f,f} + N_{s,f} = 1 - N_{s,s} = 1 - x\pi^2 - (1 - x)\delta^2,$$
(31)

while the chance in the case of conglomeration is

$$N_{f,f} + N_{s,f} = 1 - N_{s,s} - N_{f,s} = 1 - x\pi^2 - (1-x)\delta^2 - x(1-\pi)\pi - (1-x)(1-\delta)\delta.$$
 (32)

So the chance of zero profit is reduced drastically by  $N_{f,s}$  in the case of conglomeration.<sup>32</sup>

<sup>&</sup>lt;sup>30</sup>The equation holds for any  $\tau < 1$ , which is ensured by our assumption that for the n-th subgroup, the contract of mutual debt payment guarantees is not enforceable at all. In case where  $\tau = 1$ , the expected profit would be given by  $E\Pi_2^{NC} = (1 - \phi)N_{s,s}A(A+1)k$ , which is less than  $E\Pi_2^C$ .

<sup>31</sup>Note that  $EY^{NC}$  here is the same as  $EY^S$  in eq. (18) with  $f^{C*} = 1$  and  $f^* = 0$ .

<sup>&</sup>lt;sup>31</sup>Note that  $EY^{NC}$  here is the same as  $EY^{S}$  in eq. (18) with  $f^{C*} = 1$  and  $f^{*} = 0$ . <sup>32</sup>Alternatively, the chance of positive profit increases substantially in the case of conglomeration, though the highest profit a firm can get  $((1 - \phi)A(A + 1)k)$  is lower than non-conglomeration case (which is  $(1 - \phi)A[(A + 1)k + lk\frac{N_f}{1 - N_f}]$ ).

Whether or not the benefit of conglomeration from reducing the risk of liquidation outweighs the loss of expected profits critically depends on the risk aversion. If the risk aversion is very large (e.g.,  $\sigma$  goes to infinity), the entrepreneurs would be extremely concerned about eliminating the risk of liquidation, and hence prefer joining a conglomerate. In another polar case where  $\sigma$  is close to zero (risk-neutral), they would not care about the risk of liquidation and hence prefer remaining a non-conglomerate which provides higher expected profit compared to the case of conglomeration. The above discussion then establishes the following proposition.<sup>33</sup>

**Proposition 3** There exists a threshold level of risk aversion  $(\sigma^*)$  for entrepreneurs belonging to j = 1, 2...n-1, above which their optimal choice of conglomeration is  $\omega^* = 1$ .

This proposition suggests that risk-averse entrepreneurs may prefer joining a conglomerate even in case where conglomeration lowers the expected profit. The reason is that from the standpoint of the firms, joining a conglomerate is like having an insurance. Conglomeration, by working as a noise signaling strategy that dilutes information, changes the bank's optimal policy from zero-bailout to full-bailout. Given expectation on such behavior of the bank's bailout policy, entrepreneurs have incentive to form a conglomerate to reduce the risk of liquidation (and hence the risk of losing capital).<sup>34</sup>

Now consider the size of each conglomerate and the fraction of conglomerates in equilibrium. Given our assumption on the costs of enforcing the cross debt payment contract in section 2, any entrepreneur in the second period will find it optimal not to pay the debt of the failed firms which belong to the same conglomerate but different

<sup>&</sup>lt;sup>33</sup>To have a formal proof of this proposition, we can define  $\Delta EU(\sigma) = E[\frac{(c^C)^{1-\sigma}}{1-\sigma}] - E[\frac{(c^NC)^{1-\sigma}}{1-\sigma}]$ . Then we can show that  $\Delta EU(0) = E[c^C] - E[c^{NC}] = E\Pi_2^C - E\Pi_2^{NC} < 0$ . In addition,  $\Delta EU(\infty) = \lim_{\sigma \to \infty} [\frac{((A+1)Ak)^{1-\sigma}}{1-\sigma}N_{s,s} + \frac{(Ak)^{1-\sigma}}{1-\sigma}N_{f,s} - \frac{(Ak_2^S)^{1-\sigma}}{1-\sigma}N_{s,s}] = N_{f,s} > 0$ . Further, by showing that  $\frac{d\Delta EU(\sigma)}{d\sigma} > 0$ , we can establish the proposition. The threshold level can be derived from  $E[\frac{(c^C)^{1-\sigma}}{1-\sigma}] = E[\frac{(c^{NC})^{1-\sigma}}{1-\sigma}]$ .

<sup>&</sup>lt;sup>34</sup>If we allow for profit-sharing among the member firms in a conglomerate (in addition to mutual debt payment guarantees), the result can be even strengthened. Consider the case where firms in a conglomerate divide the second period profits equally across the members. In this case, the variability of profits can be further reduced while the expected profit is the same as  $EY^C$  in equation (28) (hence the bank's full bailout policy does not change).

subgroup.<sup>35</sup> Anticipating this, the entrepreneurs belonging to j=1,2...n-1 form a conglomerate only with those belonging to the same subgroup. So the measure of each conglomerate does not exceed the size of a subgroup, 1/n, though given the simplifying assumption that the enforcement cost is zero or infinity, the size of each conglomerate is indeterminate.<sup>36</sup> The addition, the fraction of the firms that join to any conglomerate is determined as  $\tau = \frac{n-1}{n}$ .

Further, note that the equilibrium with conglomerates (derived under condition (i)) is suboptimal. It is obvious from a comparison of the expected output (which is the same as the actual output) between the cases of conglomeration and non-conglomeration (eq. (26) and eq. (28)). The equilibrium deviates from the first-best solution, because entrepreneurs cannot hedge against the risk of liquidation without conglomeration, while the society as a whole (or the bank) can fully hedge the risk by aggregating idiosyncratic shocks.

If in the beginning of the first period the bank announces that it will not bail out any conglomerate and individual firms believe the announcement, the firms would not join a conglomerate and consequently efficiency would be enhanced. In our model, however, the threat of zero bailout for conglomerates is not credible. In the second period, the bank will find it optimal to fully bailout any conglomerate, though it has already announced its policy of no bailout for conglomerates in the first period.

#### VI. PRIVATE INFORMATION AND INFORMATION DILUTION

So far we have examined the bank's bailout policy in the case of symmetric information where nobody knows which type one belongs to. Now we consider the case

<sup>&</sup>lt;sup>35</sup>We implicitly assume that the legal system is not used to strengthen the enforcement of the cross debt payment guarantees among entrepreneurs that come from different subgroups. A reason is that the society or the government is less likely to pursue such an enforcement by law even when it can, because the cross guarantees and subsequent conglomeration could be suboptimal.

<sup>&</sup>lt;sup>36</sup>If a smooth convex cost function is instead assumed, the size of each conglomerate is uniquely determined.

<sup>&</sup>lt;sup>37</sup>We may change the assumption on the enforceability so that in equilibrium there should be only one conglomerate to which all the firms in an economy belong. In this case also, the full bailout for the conglomerate is obvious. Because of the cross guarantees, if the bank liquidates some firms in the conglomerate, the other firms will also go bankrupt. Without bailout, no firms cannot engage in production activities, and hence the second period output will be zero. With full bailout, however, there will be a positive output in the second period. Therefore, given conglomeration, the optimal solution in this case is full bailout.

of asymmetric information where each entrepreneur knows her type, while others do not.<sup>38</sup>

In this case of private information, firms may have different behavior in joining a conglomerate depending on their type as far as the bank maintains the policy of full bailout for conglomerates. In this case, however, the bank may accordingly change its optimal bailout policy so that such a separation may not be an equilibrium outcome. To see this, suppose that the entrepreneurs expect the bank to fully bailout failed conglomerates, and examine the possible responses of the two types of the firms belonging to j = 1, 2, ...n - 1.

Consider the bad type of entrepreneurs. Using that the chance they belong to the good type is x = 0, they calculate the expected profits in cases of remaining a non-conglomerate and joining a conglomerate which will be fully bailed out as:

$$E\Pi_2^{NC} = (1 - \phi)\delta^2 A \left( (A + 1)k + lk \frac{N_f}{1 - N_f} \right)$$
 (33)

and

$$E\Pi_2^C = (1 - \phi)[\delta^2(A + 1) + \delta(1 - \delta)]Ak$$
 (34)

which implies

$$\Delta\Pi_{2}^{Bad} \equiv E\Pi_{2}^{C} - E\Pi_{2}^{NC} = (1 - \phi)Ak\delta[(1 - \delta) - \delta l \frac{N_{f}}{1 - N_{f}}]$$
 (35)

We can show that for  $l \in [0,1]$  it always holds that  $\Delta \Pi_2^{Bad} > 0$ , that is, for bad firms, the expected profit is always larger in case of conglomeration than non-conglomeration. Hence if the bank fully bails out failed conglomerate, the bad firms would join a conglomerate ( $w^* = 1$ ).

For good firms, the expected profits are given by

$$E\Pi_2^{NC} = (1 - \phi)\pi^2 A \left( (A + 1)k + lk \frac{N_f}{1 - N_f} \right)$$
 (36)

and

$$E\Pi_2^C = (1 - \phi)[\pi^2(A + 1) + \pi(1 - \pi)]Ak$$
 (37)

<sup>&</sup>lt;sup>38</sup>In the case of complete information where everybody knows which type each of entrepreneurs belongs to, the optimal bailout policy is obvious: liquidate bad ones and bail out good ones.

which implies

$$\Delta\Pi_2^{Good} \equiv E\Pi_2^C - E\Pi_2^{NC} = (1 - \phi)Ak\pi[(1 - \pi) - \pi l \frac{N_f}{1 - N_f}]$$
 (38)

Given  $\pi > \delta$ , we can easily show that there exists a range of parameters for which it holds that  $\Delta\Pi_2^{Good} < 0$ , that is, for good firms, the expected profit is smaller in case of conglomeration than non-conglomeration. The range is:  $\frac{1-N_f}{1-N_f+lN_f} < \pi$ . So depending on risk aversion parameter, good firms may prefer remaining an independent firm or joining a conglomerate. Denote the threshold level of the risk aversion parameter as  $\sigma^{**}$ , and suppose that  $\sigma < \sigma^{**}$ .

Then if the bank fully bails out the failed conglomerate, good firms' optimal choice of conglomeration would be  $\omega^* = 0$  while bad firms' choice would be  $\omega^* = 1$ . As a result, private information would separate two types of firms in their behavior of conglomeration, because for bad firms with high chance of failing, it would be more beneficial to eliminate the chance of liquidation by joining a conglomerate, while for good firms, it would be more costly.

However, such a separation between the two types cannot be an equilibrium outcome, since in this case, the bank's optimal policy over conglomerates is no longer a full-bailout. Given information set including parameter values, the bank can infer that if some of the firms belonging to j = 1, 2...n - 1 have joined a conglomerate while others have not, the firms who have joined conglomerates must be bad firms. Due to the signaling effect, the bank would liquidate all the failed conglomerates at the beginning of the second period.

Bad firms, at the beginning of the first period, also foretell this change in the bank's bailout policy, and hence they do not join any conglomerate. Suppose they join a conglomerate. Then, the chance of failure and subsequent liquidation is 100 percent, so their expected profit becomes zero  $(E\Pi_2^C=0)$ . In case of not joining a conglomerate, however, the chance of having a positive output is positive, and hence they have a positive expected profit  $(E\Pi_2^C>0)$ .

Now consider the case where  $\sigma > \sigma^{**}$  so that the good firms find it optimal to join conglomerates. In this case, bad firms will also join conglomerates because in this case joining a conglomerate is not a signal of bad firms and hence the bank will fully bail out failed conglomerates. Therefore, bad firms will always follow the behavior of good firms on conglomeration, while good firms are not affected by bad firms' decision on conglomeration.

**Proposition 4** In case of private information, there exists a threshold risk aversion  $\sigma^{**}$ , below which the entrepreneurs' optimal choice of conglomeration is  $\omega^* = 0$  for both good and bad firms, and above which  $\omega^* = 1$  for both types.

The proposition suggests that private information cannot separate two types of firms in their behavior of conglomeration. The reason is that it is optimal for bad firms not to send any signal that allows the bank to distinguish themselves from good ones. It is the information dilution motive that derives firms not to join a conglomerate in this case.

### VII. CONGLOMERATION AND DEBT RATIO

In this section, we discuss how conglomeration is related to debt-equity ratio. In the previous sections, we assumed that entrepreneurs do not have initial wealth so that the debt-equity ratio of a firm is trivially determined to be infinity. This section allows for entrepreneurs to have initial wealth that can be paid in as equity, and examines how differences in initial wealth affects entrepreneurs' optimal decision on conglomeration and debt-equity ratio.

Suppose that at the beginning of the first period, a half of entrepreneurs are endowed with a large initial wealth,  $w = w^H$ , while another half with small initial wealth,  $w = w^L < w^H$ . For simplicity, assume  $w^H$  is equal to d (so large enough to pay back d), while  $w^L$  is zero.

The initial wealth can be paid in as equity in the beginning of the first period, and used as a buffer stock to pay back d to the bank in case of failure at the end of the first period. So the equity can work as a substitute for joining a conglomerate in reducing the risk of liquidation. For simplicity, assume equity cannot be invested into physical capital for production. In addition, the initial wealth can be stored without depreciation until the end of the second period, and then used for consumption. The bank only observes whether each firm repays d or not, so it does not know whether repayment d comes from  $w^H$  or the production  $y_1$ .

The wealth distribution is independent of the productivity distribution, so that a high equity of a firm does not provide any signal on its productivity.<sup>39</sup> Further suppose that risk aversion is such that  $\sigma > \sigma *$ . Except those described above, the model in this section is the same as the benchmark model with no private information. In particular, suppose that the bank's optimal bailout policy in the second period can be characterized by zero bailout for failed non-conglomerate firms, full bailout for

<sup>&</sup>lt;sup>39</sup>Note that the bank does not have any incentive to allocate more capital to the entrepreneurs with high initial wealth in the current setup where the bank's main concern is the productivity of the firms.

conglomerates and transfer of capital to the surviving non-conglomerate firms, as in the benchmark model.<sup>40</sup>

In this situation, any of entrepreneurs belonging to the n-th subgroup do not join a conglomerate. But a half of them pay  $w^H$  in as equity to eliminate the chance of liquidation. Among the remaining half,  $N_f$  fraction will have an adverse shock and then be liquidated, while  $1 - N_f$  fraction will have a positive shock in the first period.

For entrepreneurs belonging to j=1,2...n-1, the optimal decision on conglomeration differ depending on their initial wealth. We first consider those who have no initial wealth. These entrepreneurs have two options. First, they may remain as a non-conglomerate. In this case, however, they cannot paid in equity that can be used to raise the chance of bailout in case of failure, since  $w^L=0 < d$ . So the expected profit and the chance of zero profit are the same as in the benchmark case (i.e., eq. (27) and eq. (31), respectively). Second, they may join a conglomerate. Joining a conglomerate provides an insurance against the risk of liquidation, and the expected profit and the chance of zero profit are determined as in the benchmark case (i.e., eq. (29) and eq. (32), respectively). Then given that  $\sigma > \sigma *$ , the risk-averse entrepreneurs with no initial wealth find it optimal to join a conglomerate. Therefore, the conglomerate firms may not have any equity, which implies that the debt-equity ratio of conglomerates is infinity.

Now consider entrepreneurs with large wealth in the initial period  $(w^H = d)$ . They might join a conglomerate. Given that joining a conglomerate guarantees a full bailout, the expected profit and the chance of zero profit are determined as in the benchmark case (eq. (29) and eq. (32)). Their expected utility is also affected by their wealth as  $EU = \frac{(\Pi_2^C + w^H)^{1-\sigma}}{1-\sigma}$ . Alternatively, they may remain as a non-conglomerate firm. By holding  $w^H$  as a buffer stock to pay d, they can eliminate the chance that they are liquidated even without joining a conglomerate. In case of having a bad shock, it pays the required repayment d using  $w^H$ , and hence they will have the same chance of zero profit (given by eq. (32)). Further, because they do not default, they receive d back together with new additional capital loan which is transferred from failed non-conglomerates (amounting to  $lk \frac{N_f/2}{(1-N_f)/2+1/2n+z}$ , where  $N_f/2$  represents the measure of the failed among those with no initial wealth,  $(1-N_f)/2$  captures the surviving ones with no initial wealth, 1/2n is the firms belonging to the n-th subgroup firms with large wealth, and z is a measure of the wealthier entrepreneurs who belong to j=1,2...n-1 and remain as non-conglomerates). Therefore, the expected profit in this case is determined as

$$E\Pi_2^{NC} = (1 - \phi)[N_{s,s}A\left((A+1)k + lk\frac{N_f/2}{(1 - N_f)/2 + 1/2n + z}\right) + N_{f,s}Ak]$$
(39)

<sup>&</sup>lt;sup>40</sup>The condition to generate such an optimal policy here is not exactly the same as the benchmark case (i.e., condition (i)), but can be generated in a similar way.

which is greater than the case of joining a conglomeration (eq. (29)), because the expected profit when remaining as an independent firm is raised by capital transfers which is not allowed in case of joining a conglomerate. Therefore, the entrepreneurs with large initial wealth, who can have enough equity that work as a buffer stock against the risk of liquidation, find it optimal not to join a conglomerate. As a result, non-conglomerate firms will have equity of  $w^H$  and debt of k, and hence a lower debt-equity ratio,  $k/w^H$ .

The above discussion suggests that entrepreneurs with larger initial wealth are likely to remain as a non-conglomerate and use their wealth as equity to reduce the chance of liquidation, which leads to a lower debt-equity ratio. In contrast, entrepreneurs with smaller initial wealth tend to join a conglomerate, which induces a higher debt-equity ratio.

## VIII. MERGER AND ACQUISITION

In this section, I briefly address the issue of existing conglomerates' rapid expansion through acquisitions of non-conglomerate firms, which is often viewed as a key feature of conglomerates. In the basic model of Sections 2-5, firms' decision on conglomeration is made only once (at the beginning of the first period). Therefore the basic model explains how individual firms form a conglomerate, but it does not explain how existing conglomerates acquire unaffiliated firms.

To deal with this issue, I introduce another variant of the basic model. In particular, assume that firms including conglomerates can buy others or sell themselves after productivity shock is revealed but before repayment to the bank is made. Suppose that a shock was revealed at time t=1, and as a result, there emerged failed firms that cannot repay d. Without selling the failed firm (as assumed in the basic model), the entrepreneur of the failed firm would receive nothing in the second period given the bank's policy of full liquidation on failed individual firms. In contrast, if she can earn a positive expected remuneration by selling its failed firm to some others (as assumed in this variant of the basic model), she has a strong incentive to sell the firm rather than being liquidated by the bank.

The conglomerates also have incentives to buy failed firms. If a failed firm is acquired by a conglomerate, then the failed firm will be fully bailed out under the umbrella of the conglomerate though it cannot repay d. Therefore, the firm that failed but was bought by the conglomerate will be able to operate in the second period. As far as the conglomerate can make a positive net expected profit from the newly acquired firm after subtracting what it pays to the original entrepreneur of the firm, it has a incentive to buy the firm.

To discuss this more formally, let S denote the expected surplus to the conglomerate and the failed firm that is generated by a conglomerate's acquisition of a failed firm. For simplicity, assume that a member firm in the conglomerate that had a

positive productivity shock in the first period runs the newly acquired and previously unaffiliated firm, but that the net profit earned from this newly acquired firm is equally distributed among the members. Then the expected surplus can be easily calculated as the expected profit of the firm that was failed but bought by the conglomerate. As the management is replaced by the entrepreneur of the firm having had positive shock in the first period, the expected profit is given by

$$S = (1 - \phi)N_{s,s}Ak > 0 \tag{40}$$

which suggests that a conglomerate's acquisition of failed individual firms generates positive expected profits, and consequently positive expected surplus.

Suppose that the bargaining between the conglomerate and the failed individual firm determines the division of the surplus in the second period between the share for conglomerate,  $\chi S$ , and that for failed firms,  $(1-\chi)S$ . As far as  $\chi$  is positive but less than one, both the conglomerate and the failed firm have incentives to engage in this trading.

Note that the conglomerate's acquisition of individual firms may improve the quality composition of conglomerates, since the entrepreneur of the firm changes from the one having had an adverse shock to a new one having received a positive shock in the first period. Hence, the bank's optimal bailout policy for the conglomerate can remain as a "full bailout".

## IX. CONCLUSION

This paper presents a model of bailout and conglomeration, in which firms stochastically differ in their productivity. The model suggests that in a situation where the type of each firm is only stochastically known, the bank makes signal extraction from a revelation of productivity shock to infer the probability of bad firms among the failed, and uses liquidation as a screening device to select good firms. It also shows that joining a conglomerate, based on mutual debt payment guarantees, may drastically change the bank's bailout (or liquidation) decision. For non-conglomerate firms, the optimal bailout policy can be a "no-bailout" (or full liquidation), particularly when the chance of bad firms in the failed sample is sufficiently large. By contrast, for the firms in a conglomerate which is a congregation of good and bad firms in the original sample, the optimal bailout can be a "full-bailout" since the bank may find the conglomerate, even failed, not sufficiently bad in terms of quality composition to be liquidated. The paper also shows that with an expectation on such bank bailout policy, risk-averse firms may want to join a conglomerate to use it as a noise-signaling or information-diluting device that helps to eliminate the risk of liquidation, though expected profits is reduced. Further, it shows that in case of private information where each firm knows only its own type, it may be optimal for bad firms to follow good firms' decision on conglomeration, and, by doing so, not to reveal which type they belong to. Finally, it discusses the effect of conglomeration on debt-equity ratio, and how existing conglomerates expand through acquisitions of non-conglomerate firms.

#### REFERENCES

- Aghion, Philippe, Oliver Hart, and John Moor, 1992. The Economics of Bankruptcy Reform. Journal of Law, Economics, Organization 8, 523-546.
- Bolton, Patrick and David Scharfstein, 1990. A Theory of Predation Based on Agency Problems in Financial Contracting. American Economic Review 80, 94-106.
- —, 1996. Optimal Debt Structure and the Number of Creditor. Journal of Political Economy 104, 1-25.
- Bulow Jeremy I.,. and John B. Shoven, 1978. The Bankruptcy Decision. Bell Journal of Economics 9, 437-456.
- Caballero, Ricardo and Mohamad Hammour, 1994. The Cleansing Effect of Recessions. American Economic Review 84, 1350-68.
- Hahm, Hongjoo, Giovanni Ferri and Paola Bongini, 1998. Corporate bankruptcy in Korea: Only the Strong Survive?. mimeo, World Bank.
- Hart, Oliver, 1995. Firms, Contracts, and Financial Structure. Oxford University Press.
- Hart, Oliver and John Moore, 1994. A Theory of Debt Based on the Inalienability of Human Capital. Quarterly Journal of Economics 109, 841-79.
- —, 1998. Default and Renegotiation: A Dynamic Model of Debt. Quarterly Journal of Economics 113, 1-41.
- Izvorski, Ivailo and Se-Jik Kim, 1999. Default and Optimal Bailout Policy. mimeo, IMF.
- Lee, Jong-Wha and Youngsoo Lee, 1998. Corporate Debt Structure in Korea: A Comparison between Conglomerates and Non-conglomerates during 1981-96. mimeo, Korea University.
- Povel, Paul, 1996. Optimal Soft' or Tough' Bankruptcy Procedures. LSE Financial Markets Group Discussion Paper No.240.
- White, Michelle J., 1980. Public Policy toward Bankruptcy: Me-First and Other Priority Rules. Bell Journal of Economics 11, 550-564.