

Demographic Change in Asia: The Impact
on Optimal National Saving, Investment,
and the Current Account

Serge Besanger, Ross S. Guest and Ian McDonald

IMF Working Paper

IMF Institute

**Demographic Change in Asia:
The Impact on Optimal National Saving, Investment, and the Current Account**

Prepared by Serge Besanger¹, Ross S. Guest, and Ian McDonald

Authorized for distribution by Donal J. Donovan

June 2000

Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

This paper calculates the levels of optimal national saving, investment, and the current account balance for five Asian economies—Hong Kong SAR, Japan, Singapore, Malaysia, and the Philippines—for the period 1997–2050 using a simulation approach. These calculations show the sensitivity of results to changing demographic structures on employment participation, labor productivity, and consumption demands. In particular, the simulations reveal that variations in prospective demographic change across economies cause considerable variations in the patterns of optimal national saving rates.

JEL Classification Numbers: E17, E21, J11, O21

Keywords: Current Account Balance, Savings, Investments, Aging, Demography

Authors' E-Mail Addresses: serge.besanger@freesbee.fr
r.guest@mailbox.gu.edu.au
Imcdonal@cupid.ecom.unimelb.edu.au

¹ Prior international consultant, IMF-Singapore Regional Training Institute (STI); Griffith University; and University of Melbourne, respectively. The authors would like to thank Donal Donovan, Françoise Le Gall, Joshua Greene, and Peter Montiel for their very helpful comments and encouragements. The views expressed in this paper are those of the authors and do not reflect the position of the IMF.

Contents	Page
I. Introduction.....	3
II. The Opportunities and Challenges of Demographic Change	5
III. Modeling the Socially Optimal Level of National Saving	10
IV. Data and Calibration.....	13
V. Simulations of Optimal Saving.....	16
VI. Conclusions.....	19
References	21
Tables	
1. Productivity Weights for 1997	6
2. Consumption Weight by Age Group.....	8
3. Values of Parameters and Exogenous Variables.....	15
4. Actual and Optimal National Saving Rates (Percent of GDP)	17
Charts	
1. Support Ratios	24
2. Optimal Saving Ratios.....	25
3. Optimal Current Account Balances (Ratios)	26
4. The Effect of Population Aging on Optimal Saving Ratios	27
5. The Effect of Alternative Consumption Weights on Optimal Saving Ratios.....	28
Appendices	
I. Calculation of Productivity Weights	29
II. The Equations of the Model.....	31
III. Sensitivity Analysis.....	34

I. INTRODUCTION

Should economies boost national saving in response to their aging populations? The Organization for Economic Cooperation and Development (OECD 1996) has expressed concern over the aging of populations throughout the world and called for higher saving now.² A judgment on the most desirable level of saving can be based on the notion that saving is a means to redistribute consumption over time. A high level of saving shifts consumption into the future. A low level of saving shifts consumption toward the present. The optimal rate of saving balances consumption now against consumption in the future in a way that maximizes the total utility enjoyed by all people—that is, by those living now and those living in the future.

A major influence on the optimal pattern of consumption over time—and thus on the optimal saving rate—is changing demographic structures. For example, an economy with an aging population faces the challenge of satisfying the consumption demands of a future population with relatively low productive potential. The optimal saving rate has to take into account the need to balance these future demands against the consumption demands of the current population (Besanger, Guest, and McDonald, 1999).

This paper applies the concept of the optimal saving rate for economies with aging populations to five Asian economies—Hong Kong SAR, Japan, Malaysia, the Philippines, and Singapore—for the period 1997–2050. The optimal levels of saving are simulated in a model of the small open economy. The simulations calculate the levels of optimal national saving, investment, and the current account balance for these five Asian economies. Allowance is made for the effects of changing demographic structures on employment

² Not all economists share this concern. In the case of the United States, Cutler and others (1990) have argued that there is no need to increase national saving, for three reasons. First, a smaller workforce has lower capital requirements. Second, the burden imposed by a high proportion of old people is partly offset by the benefits of having a smaller proportion of young people. Third, a significant part of the rest of the world (as measured by GDP) is aging faster than the United States. Thus, the returns to investment are lower in the rest of the world, leading to exports of capital to the United States. As a result, the United States may have lower saving requirements, and higher saving may not be necessary for at least some small countries (Guest and McDonald, 1999a, 1999b; Yashiro and Oishi, 1997). For Australia, for example, Guest and McDonald argue that the current saving rate will deliver an increasing level of consumption per person with no rundown in wealth—even when the aging population is factored in. They also argue that the projected increases in government outlays due to the aging population will not put undue pressure on the Australian economy. Even for Japan, which is aging more rapidly than most countries, Yashiro and Oishi argue that lower saving and investment can be largely offset by policies to stimulate technical change and extend working lives.

participation, labor productivity, and consumption demands. The calculations are based on a model of the socially optimal level of saving for a small open economy developed in previous work (see Appendix B; Guest & McDonald, 1998a, 1999a). The simulations show that variations in prospective demographic changes across the five economies cause considerable variations in the patterns of optimal rates of national saving, reinforcing the idea that evaluation of an economy's saving performance should take these changes into account.

This paper is an exercise in normative, rather than positive economics, in that it attempts to calculate the socially optimal outcome—that is, the outcome that maximizes economic welfare. In a similar study, Cutler and others (1990) sought to calculate the socially optimal rate of national saving for the United States for the period 1990–2070. In the case of Japan, Noguchi (1990) and the Economic Planning Agency (1991) (reported in Yashiro and Oishi, 1997) have calculated socially optimal rates of saving. Less closely related to this paper are two studies based on an overlapping generations model (OLG). Auerbach and Kotlikoff (1992) and Miles (1999) use the OLG model to project the impact of demographic change on the rate of saving.³ Even less closely related are the econometric models of Masson and Tryon (1990) and Yashiro and Oishi (1997), which use equations fitted to data rather than to utility and production functions. These analyses are primarily exercises in positive economics rather than in normative economics, although the estimating equations are derived from privately optimizing behavior. The conclusion of this paper compares the results of these papers with the results of this simulation.

The outcomes this paper terms “socially optimal” are driven by two important principles. These are the principle of optimal consumption smoothing over time and the principle of optimal distribution of wealth between domestic capital and overseas assets. Additional aspects may need to be taken into account in calculating optimal outcomes. First, governments may wish to avoid excessive foreign debt, for example. High foreign debt, especially short-term foreign debt, is a potential cause of currency crises, even if the underlying fundamentals of saving and investment are sound (see, for example, Radelet and Sachs 1998). Governments may also want to increase growth rates—an objective that lies outside the mainstream of optimal saving theory, although some economies pursue it. These limitations to our definition of social optimality must be kept in mind in interpreting the paper's results.

The paper is structured as follows. Section II analyzes the impact of demographic change on employment potential and consumption demands for the five Asian economies. Section III outlines a representative agent model of optimal national saving in a small open economy. Section IV describes the data and calculations required that are required to apply this model

³ In the Auerbach-Kotlikoff model the utility function reflects overlapping generations. Decisionmaking on consumption and labor supply is privately optimal, but, because it is influenced by explicit incorporation of tax rates, it is not necessarily socially optimal.

empirically. Section V explains the results of the simulations of socially optimal saving for the five Asian economies, and Section VI presents some conclusions.

II. THE OPPORTUNITIES AND CHALLENGES OF DEMOGRAPHIC CHANGE

Labor productivity and consumption demands vary across age and gender groups in the population. In this study, we attempt to allow for these variations. Employment and population are measured by indexes that account for the different labor productivities and consumption demands of different age groups. In addition, for labor productivities, the effect of gender is included.

To allow for variations in labor productivity across age groups, we constructed measures of aggregate employment and assigned a productivity weight to each age and gender group. The productivity weights were calculated using the simplifying assumption that the age-by-gender distribution of earnings reflects the age distribution of labor productivity. Data on the age-by-gender distribution of earnings were obtained for Hong Kong SAR, Japan, and Singapore but were not available for Malaysia or the Philippines. For the latter economies, we used the mean weights calculated from the data for Singapore and Hong Kong SAR. Japan was not included, because we judged that the weights for Singapore and Hong Kong SAR would be better indicators of weights for Malaysia and the Philippines. For all the years in our projections, we used the weights based on 1997 data. The weights for each economy are shown in Table 1 (see Appendix A for calculations of the productivity weights).

The employment projections were constructed by multiplying the projected population figures for each of 22 age/gender groups by the employment/population (L/N) ratios of 1997. The L/N ratios were calculated from data provided by government agencies of each economy. This method maintains the employment-to-population ratios for each age/gender group at their 1997 level throughout the projection period. Population projections by age/gender group are given in United Nations (1999).⁴

⁴ The populations for each economy, prepared by the Population Division of the United Nations Secretariat, are based on population census data and take into account the birth and mortality rates of males and females and international migration rates.

Table 1. Productivity Weights for 1997

		15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65+
Hong Kong SAR	Male	0.753	1.158	1.158	1.517	1.517	1.506	1.506	1.158	1.158	0.869	0.869
	Female	0.753	1.042	1.042	1.518	1.158	0.811	0.811	0.695	0.695	0.579	0.579
Japan	Male	0.640	0.849	1.167	1.167	1.501	1.501	1.698	1.698	1.381	1.381	1.043
	Female	0.555	0.724	0.869	0.869	0.915	0.915	0.885	0.885	0.807	0.807	0.773
Malaysia	Male	0.613	0.816	1.114	1.423	1.457	1.452	1.431	1.257	1.063	0.918	0.816
	Female	0.681	0.826	0.971	1.093	1.092	0.893	0.841	0.784	0.648	0.590	0.826
Philippines	Male	0.613	0.816	1.114	1.423	1.497	1.452	1.431	1.257	1.063	0.918	0.816
	Female	0.681	0.826	0.971	1.093	1.092	0.893	0.841	0.784	0.648	0.590	0.826
Singapore	Male	0.439	0.439	0.991	1.232	1.294	1.297	1.257	1.257	0.897	0.897	0.897
	Female	0.752	0.752	1.112	1.269	1.269	1.205	1.078	1.078	0.743	0.743	0.743

The demand for private and public consumption varies across age groups. For example, demand for education is relatively high among young people, while demand for health services is relatively low. Among older people, demand for education is much lower and for health services much higher. Private consumption demand is also lower for young people than for older groups. To capture these relative demands for consumption, we constructed population figures weighted by the relative consumption demands of different age groups (no differences by gender were assumed). These constructed population figures give an indication of how the changing age structure of the population will influence the aggregate demand for consumption over the projected period. In order to construct the consumption weights, we needed data on both private and public expenditure by age group. These data were not readily available for the Asian economies under consideration. Consequently, we used two alternative sets of consumption weights, one based on Australian data (see Guest and McDonald, 1999a, 1999b),⁵ and the other based on U.S. data (see Cutler and others, 1990) (Table 2). The U.S. weights place a higher value on the consumption demands of older people than do the Australian weights.⁶ Multiplying the consumption weights in Table 2 by the population in each age group and then summing yields an aggregate population measured in consumption units.

⁵ Olekalns (1999) suggests that the consumption weights are very similar across industrial economies. Adopting Australian weights as proxies for Hong Kong SAR, Japan, and Singapore is thus defensible. We used the same weights for Malaysia and the Philippines as for the other three economies.

⁶ For Australia, the consumption weights were calculated as follows. For private consumption levels, we used the household expenditure survey for 1993–94 (ABS Cat No. 6530.0). Following the equivalence scale approach in Bradbury and Saunders (1990), we assumed that a young person consumes half the private consumption of a working-age adult. Thus, the weights for private consumption for young, working-age adults, and older adults are 0.5, 1, and 0.75, respectively. For government-provided consumption, we used the detailed Australian survey of state social expenditure per person by age group for education and health (CDC 1990). This survey divided the population into nine age groups. Combining private consumption expenditures with government consumption expenditures yielded the consumption weights for the nine age groups given in Table 2 (for further details, see Guest and McDonald, 1999a). This method follows the method Cutler and others (1990) use for the United States. The main reason why Cutler and others calculate a higher weight for older people is that they assume a weight of 1 for the private consumption of older people, while our weight is 0.75. (They give no reason for using the heavier weight.) The estimates of Schultz and Borowski (1991) for the United States imply a weight for the private consumption of older people of 0.85. In our view, the 1.0 weight places too much value on the consumption demands of older people, even for the United States.

Table 2. Consumption Weights by Age Group

	0-15	16-24	25-39	40-49	50-59	60-64	65-69	70-74	75 +
Weights (A)	0.68	0.89	1.00	0.98	1.00	1.05	0.87	0.95	1.19
Weights (U)	0.72	0.72	1.00	1.00	1.00	1.00	1.27	1.27	1.27

Note: A = weights based on Australian data, U = weights based on U.S. data.

A summary statistic of the demographic structure of an economy that captures the information on labor productivity and consumption demands described above is the support ratio, or the ratio of employment to total population.⁷ Employment is an important determinant of an economy's output, and total population is an important determinant of consumption demands. A decrease in the support ratio presents the challenge of meeting increased consumption demands with a constrained level of output. An increase in the support ratio provides an opportunity to create extra wealth that will enhance consumption in periods when the support ratio is lower. Using both employment (to allow for the different labor productivities across age/gender groups) and population (to allow for the different consumption demands of different age groups), as described above, yields a more useful measure of the support ratio.

Chart 1 plots the support ratios (employment to population) for the five Asian economies, adjusting employment for productivity weights and population for consumption weights (weights A). Chart 1 also shows support ratios for the alternative consumption weights. An increase (decrease) in the support ratio implies a decreased (increased) burden of dependents. Thus, a downward trend implies increased pressure on resources. In both cases, the chart shows a considerable difference between the support ratios of Malaysia and the Philippines and those of Hong Kong SAR, Japan, and Singapore. For the former economies, the support ratios describe a "hump," with the peak at 2025 for Malaysia and 2035 for the Philippines. This curve suggests that the populations in these two economies will become younger in the near term, so that the burden of supporting dependents declines until the years 2025 and 2035, respectively. Thereafter, population aging begins, as reflected in the decreasing support ratios.

For Hong Kong SAR, Japan, and Singapore, population aging will occur sooner. Hong Kong SAR's support ratio increases very slightly from 1997 until 2005 and then declines

⁷ Cutler and others (1990) use the support ratio. We prefer this concept to the commonly used dependency ratio, because its two components are direct inputs into our calculation of optimal saving. (The dependency ratio is the ratio of people of nonworking age to people of working age.)

rapidly—more rapidly, in fact, than in any of the other economies, including Japan. The main reason for this pattern is that the employment-to-population ratio for older people in Hong Kong SAR is lower than it is in the other economies, so that the aging population has a more marked impact on the support ratio. Singapore's support ratio declines after 2000, and Japan's has been declining since 1997. In other words, Japan's dependency burden increases throughout the projection period. However, it is not increasing as fast as Hong Kong SAR's nor, for part of the projection period (2015–30), as rapidly as Singapore's. This disparity occurs in part because the employment-to-population ratio for older people is a little higher in Japan than in Singapore—and considerably higher than in Hong Kong SAR—and in part because Japan's population began aging earlier and will tend to level out sooner.

The consumption weights based on U.S. data yield lower support ratios for each of the five economies (see Chart 1). This lower support ratio is a result of the higher weight attached to the consumption demands of older people. The pattern of the support ratios is similar for the two sets of weights, however.

The pattern of support ratios is important in driving the pattern of optimal saving. The results for optimal saving are discussed below, but the essential intuitive argument can be summarized as follows. It is optimal, in the sense of yielding greater utility, for economies to smooth consumption over time.⁸ Temporary changes in the support ratios cause temporary changes in output and wealth and are assumed to be perfectly foreseen by an economic agency. Hence they have relatively small but permanent effects on optimal consumption. For example, the upward trend in Malaysia's support ratio does not call for an increase in consumption. Similarly, the downward trend does not call for a decrease in consumption. Rather, these changes are anticipated, and their effects on consumption are smoothed. The implication is that population aging in the future, which reduces output and wealth, affects both current and future consumption—a trend reflected in an increase in current saving to provide for future consumption. Optimal saving patterns tend to follow the pattern of the support ratio, so that an upward trend followed by a downward trend in the support ratio will be mirrored in the optimal saving ratio.

The projections of the support ratios in Chart 1 assume that consumption weights, productivity weights, and employment population ratios will not change over time. The projections ignore the possibility that insofar as a decline in the support ratio places pressure on resources, there will be some adjustments to expenditure patterns, productivity, or employment-population ratios. Pressure on resources and the improved health of older people can be expected to increase employment-population ratios for the elderly.⁹ However, trends in health expenditures

⁸ We can loosely speak of economies (rather than agents) smoothing consumption, in the sense that our economies are assumed to consist of a collection of representative agents that differ only in terms of their relative labor productivity and consumption needs.

⁹ Guest and McDonald (1999b) investigate the impact on the support ratio and optimal saving for Australia of increasing the retirement age.

in the United States suggest that our measures of the support ratio underestimate the prospective burden of the aged. Arguably, health expenditures may increase faster than labor productivity if recent trends continue. Hurd (1997) points out that for the United States health expenditures per person (private plus government) increased 4–5 percent annually during 1970–90. Health expenditures for the aged in our five Asian economies are expected to increase faster than labor productivity, so that the support ratios in Chart 1 do indeed underestimate the prospective burden of the aged.¹⁰

III. MODELING THE SOCIALLY OPTIMAL LEVEL OF NATIONAL SAVING

Our simulations of the optimal level and composition of saving are based on the model of optimal saving used in Guest and McDonald (1998a).¹¹ The equations of the model are in Appendix B. The model is based on the standard open economy model with many periods (see, for example, Obstfeld and Rogoff, 1996). However, we have modified the standard model to make simulating actual economies easier (including using a positive rate of technical progress and a putty-clay vintage production function) and to allow for a changing demographic structure.¹²

The model is of a small open economy. A social planner is assumed to maximize a social welfare function. This function is the sum of the utility levels generated from consumption by a concave utility function for a representative consumer up to h periods in the future and the level of wealth at the end of the h periods. The utility function has a constant elasticity form and is additive over time periods. The rate of time preference is assumed to be constant over the planning horizon. The representative consumer is a “composite” individual who reflects the relative demands for private and government consumption of the three age groups, as discussed in Section II. The specific form of the social welfare function is:

¹⁰ Cutler and others (1990) and Yashiro and Oishi (1997) argue that a shortage of labor will enhance the rate of increase of productivity growth. They cite OECD cross-country evidence to support this claim (OECD 1996).

¹¹ The model in this paper differs in two respects from the model in Guest and McDonald (1998a). First, the interest rate in this model is assumed to be constant over the time horizon. Second, we replace an approximation used in the specification of the vintage production function with an exact specification in this paper. This latter change has an insignificant effect on the calculations of optimal levels.

¹² In the Obstfeld and Rogoff (1996) section cited in the text, some of the output (called government spending) is assumed not to add to consumption, investment, or the accumulation of overseas assets. Our model does not include such a category of expenditure.

$$V(C_1, C_2, \dots, C_h, W_h) = \left\{ \sum_{j=1}^h N_j \left(\frac{C_j}{P_j} \right)^{1-\beta} \frac{(1+\rho)^{1-j}}{(1-\beta)} \right\} + N_h \omega \left(\frac{W_h}{N_h} \right)^{1-\psi} \frac{(1+\rho)^{1-h}}{(1-\psi)} \quad (1)$$

where:

- C = aggregate consumption,
- N = the number of people in natural units,
- P = the number of people in effective units, which are found by summing the consumption –demands–weighted number of natural units in each age group,
- ρ = the rate of time preference,
- β = the reciprocal of the elasticity of intertemporal substitution,
- ψ = the reciprocal of the elasticity of the elasticity of substitution between terminal consumption and terminal wealth, and
- ω = the weight on terminal wealth.

The economy produces one type of output. The production function is putty-clay.¹³ The capital-labor ratio of new capital is chosen from a range of possibilities described by a Cobb-Douglas production function. Once installed, capital has a fixed capital-labor ratio and depreciates physically at a fixed exogenously determined rate. The age at which capital is scrapped is determined optimally. The capital is operated by an exogenously determined number of “representative” employees who reflect the relative labor productivities of the age/gender groups, as discussed in Section II. The production function is given by

$$Y_j = \sum_{k=1}^T (1 - \delta)^{k-1} A_{j-k} I_{j-k}^\alpha k_{j-k}^{1-\alpha} \quad (2)$$

¹³ In the putty-clay or vintage production function, technical progress is embodied in new investment rather than “falling” on all capital goods. Thus, technical progress does not change the productivity of existing vintages of capital goods. Apart from the extra realism of capturing the embodied nature of technical progress, the vintage specification has the advantage of introducing a more gradual adjustment of the capital stock to shocks than would a production function in which capital is homogenous. (Of course, with homogenous capital gradual adjustment can be introduced by assuming a cost of adjusting capital.) In a steady state, the vintage production function can be represented by a production function with homogenous capital.

where:

- $-Y_j$ = aggregate output at time j ,
- I_i = the number of capital goods of vintage i ,
- L_i = the number of efficiency units of labor operating capital of vintage i ,
- δ = the rate at which existing capital goods depreciate, and
- T = the age of the oldest capital good in use.

Over the operating life of the function, I_i and l_i remain constant, but their productivity declines at rate δ .

The economy faces an exogenously determined world rate of interest. The value of the rate of time preference, ρ , is chosen so that the asymptotic growth rates of consumption and output are equal. In a model with zero technical progress, this restriction is the familiar $\rho=r$. In the case of positive technical progress and the Cobb-Douglas production function, the corresponding restriction is

$$\rho = \frac{(1+r)}{(1+a)^{\frac{\beta}{1-\alpha}}} - 1 \quad (3)$$

Although some regard setting the rate of time preference by condition (3) as unacceptable (because it appears to relate the value of one exogenous variable to the values of other exogenous variables), we argue that in the current context it is a reasonable procedure. As Guest and McDonald (1998b) show, if the rate of time preference is determined endogenously by using Uzawa preferences (Uzawa, 1968), then it will tend to the value determined by equation (1). For other discussions of the use of this condition, see Blanchard and Fischer (1989) and Barro and Sala-i-Martin (1995). Most importantly for the interpretation of the results of this paper, our practice for determining the rate of time preference implies that with a positive trend growth of labor productivity, consumption per person will increase over time. Thus the ethical position embodied in this choice of the rate of time preference implies that people in the future will be better off than people are today. Another way of putting this notion is that the simulations in this paper treat the interests of people in the future generously—some may say too generously.

We define wealth as the sum of capital stock and overseas assets. The terminal wealth condition is implied by the first order condition with respect to wealth. Parameter values are chosen so that terminal wealth is positive. Thus, the simulations do not “eat up” wealth.

The pattern of consumption per consumption unit (C/P) over time generated by the optimal plans of the model reflects two mechanisms. These mechanisms are a relative-efficiency (Bentham) mechanism and a consumption-smoothing (Modigliani) mechanism. First, because older people are less efficient at turning consumption into utility, the optimal plan has a tendency to shift consumption toward periods with a smaller proportion of aged people. This tendency is apparent from the positive relation between the optimal growth of C/P and the ratio

of population growth in natural units to population growth in consumption units. The optimal growth of C/P is equal to $[(1+n^N)/(1+n^P)]^{1/\beta}(1+\alpha)^{1/(1-\alpha)-1}$, where n^N is the rate of population growth in natural units and n^P is the rate of growth of the population in consumption units. In an aging society, this ratio of population growth rates is low and so tends to cause slower growth in C/P . The implication is that consumption shifts toward the present when, in this example, the number of older, inefficiently consuming people is comparatively small. However, as calculations with the formula above for the optimal growth of C/P show, with realistic values of β (0.9 or more), the effect of aging is quite small, largely because the effect is offset by the second mechanism (the consumption-smoothing mechanism). A higher β implies a stronger preference for consumption smoothing.

It is reassuring that for the parameter values used in this paper, the relative efficiency effect is dominated by the consumption-smoothing effect. Many analysts (see in particular Sen, 1973) would find it ethically indefensible to reduce the consumption levels of older people because these consumers are less efficient.

IV. DATA AND CALIBRATION

The set of parameter values used for all the simulations is listed in Table 1. The planning horizon, h , is chosen to be long enough so that the path of optimal saving to output, S/Y for the period up to 2050, is sufficiently close to the path that would obtain for an infinite horizon. The criterion for "sufficiently close" is a further extension of the horizon that would change the value of S/Y in the year 2050 by a lower level of tolerance (specified as 0.1 percentage points). The resulting value of h is 200 years.¹⁴ The values of α , the elasticity of output with respect to capital; β , the reciprocal of the reciprocal of the elasticity of intertemporal substitution; ψ , the reciprocal of the elasticity of the elasticity of substitution between terminal consumption and terminal wealth; and δ , the rate of depreciation, are based on typical empirical estimates. In particular, the values of α , δ , and r are the same as those used by Barro and Sala-i-Martin (1995). The value of β (=2.0) is consistent with estimates in Skinner (1985). The value of m , the proportion of debt to be repaid in each year, is set at 0.15 to approximate a 10-year loan. For each economy, ω is set to generate a terminal value of wealth to consumption equal to the exogenously given initial value of wealth to consumption for that economy.

The value of the rate of technical progress, a , is based on two considerations. First, estimates of historical rates of technical progress for Asian economies from a range of studies (see Dowling

¹⁴ This value, and the corresponding assumption that the optimal saving path is one that obtains over an infinite horizon, has important implications for the results of the simulation. A much shorter time horizon, reflecting the assumption that regional economies would converge more quickly to U.S. per capita GDP, would generate higher estimates of optimal saving. Optimal saving will also be higher the more constrained is each economy's access to world capital markets.

and Summers, 1998, for a summary of results) vary considerably, depending on the economy, the data set, the production function, and the estimation method. An approximate mean annual rate of technical progress from these studies is 0.02. The other consideration is that some degree of convergence toward the rates of technical progress of industrial economies can be expected over a time period as long as the 200-year planning horizon used in this paper. Historical rates of technical progress of industrial economies also vary widely. Barro and Sala-i-Martin (1995) choose a benchmark value of 0.014 as representative of results from their estimates.¹⁵ For our simulations, we set the rate of technical progress for a particular economy at a constant rate, which ensures that the level of GDP per capita for the economy converges to that of the United States at the end of the 200-year horizon.^{16,17} The results are given in Table 3.

An initial value for 1997 must be determined for each economy for the efficiency parameter A in the production function. After 1997 the values of A grow at the rate of technical progress. The value of A for 1997 is determined as follows.

From the production function:

$$A_{1997} = \frac{Y_{1998} - (1-\delta)Y_{1997}}{I_{1997}^{\alpha} (L_{1998} - (1-\delta)L_{1997})^{1-\alpha}} \quad (4)$$

¹⁵ The value of 0.014 is implied by their benchmark value for the growth rate of labor productivity (0.02)—which, given their benchmark value for capital elasticity (0.3) and a Cobb-Douglas production function, yields a value for the rate of technical progress of 0.014.

¹⁶ The equation for the rate of technical progress(a) for economy i is : $a^i = \left(\frac{y^{US}}{y^i} \right)^{\frac{1}{200}} (1.01) - 1$

where y is labor productivity and 0.01 is a projected rate of labor productivity growth for the United States, based on an average of historical estimates (see Barro and Sala-i-Martin, 1995, for an example). Here employment growth and population growth are assumed to be equal.

¹⁷ One could model the convergence of the rate of technical progress over time from 0.02 to 0.014. However, with our utility function, a decreasing rate of technical progress causes consumption to shift forward in time in order to spread the high output levels in the future to the present, yielding very low optimal saving ratios. This scenario can be avoided, perhaps by replacing the additive preferences in this paper with Uzawa preferences (see Guest and McDonald, 1998b). But we leave this question for future research.

where:

- Y = GDP,
- L = aggregate employment, and
- I = the optimal level of investment.

We determine A_{1997} by setting $Y_{1998} = 1 + G Y_{1997}$, where G is the projected one-year growth rate of actual output for the particular economy. We set this rate at 0.03, which is thought to be a realistic representative rate for the economies chosen. Using (4), we get A_{1997} as a function of exogenous variables.

Section II discussed the projections of population and employment to 2050. Population and employment growth rates for each economy are expected to approach zero by the year 2100 (according to a partial adjustment mechanism). In the case of Hong Kong SAR, Japan, and Singapore, the projected population growth rates approach zero from below. For Malaysia and the Philippines, the growth rates approach zero from above, since the growth rates of these economies are positive until 2050. A growth rate of zero for all economies beyond 2100 is justified, based in part on the notion that negative population growth rates are undesirable in the long term and in part on the uncertainty of projections beyond 2050.

Table 3. Values of Parameters and Exogenous Variables

H , the planning horizon	200 years
α , the partial elasticity of output with respect to capital	0.3
β , the reciprocal of the elasticity of intertemporal substitution	2.00
ψ , the reciprocal of the elasticity of substitution between W_h and C_h	2.00
δ , the depreciation rate	0.05
m , the proportion of debt to be repaid in each year	0.15
R , interest rate	0.06

	Philippines	Malaysia	Hong Kong SAR	Singapore	Japan
ρ , rate of time pref.	0.0149	0.0271	0.0358	0.0376	0.0383
α , rate of tech. prog.	0.0153	0.0111	0.0073	0.0081	0.0075
$\omega^{1/\beta}$, terminal wealth to consumption ratio	1.7400	3.0500	2.2400	5.1400	8.0700

V. SIMULATIONS OF OPTIMAL SAVING

In this section we report the simulations of optimal saving for the five economies, using the model and parameter values described above. Although all simulations assume a 200-year horizon, we focus on the results for the period leading up to 2050. We ignore the post-2050 period as being too far in the future to be of any interest.¹⁸ It should also be emphasized that the simulations protect the initial level of wealth, in that the terminal condition for all simulations requires a wealth-consumption ratio equal to the initial wealth-consumption ratio. Thus, the simulations do not run down wealth over the 200-year horizon. Chart 2 presents the levels of optimal national saving calculated by the model we have discussed. In discussing these results, we first compare the levels of optimal national saving in 1996–97 with the actual level of national saving and then examine the pattern of optimal national saving over the period 1997 to 2050.

Table 4 compares optimal national saving for 1996-97 with the actual rate for 1996 and the actual level before the Asian crisis. It seems wise to treat the crisis as an aberration and thus to regard the precrisis numbers as “normal” rates of saving to which the economies will return. For Hong Kong SAR, Japan, and the Philippines, the actual rates of saving are close to the optimal rates (the rate for the Philippines is the furthest from optimal). Malaysia and Singapore appear to be saving at rates well in excess of optimal levels.¹⁹

The patterns of optimal saving ratios for the five economies during the period 1997–2050 shown in Chart 2 reflect the opportunities and challenges created by the changing demographic structure in these economies. These changes are summarized by the patterns of the support ratios shown in Chart 1.²⁰ The behavior of these economies allows us to distinguish three effects, all of them aspects of consumption smoothing.

¹⁸ Chart 2 shows that the optimal saving ratios for each of the five economies tend to become similar rate close to the year 2050. This trend reflects the similarity in the assumptions made in the projections about population growth rates for the post-2050 period.

¹⁹ Optimal savings rates for various countries also correspond to calculated values for optimal current account balances. In the case of Malaysia, where an “excessive” savings rate coexisted with a substantial current account deficit, reducing the degree of oversaving would require either a corresponding decline in investment or the acceptance of an even larger current account deficit covered by net capital inflows or reserve draw-downs.

²⁰ The changing demographic structure drives the variations in the optimal saving rates, as the discussion of the effects of assuming away changes in demographic structure makes clear.

Table 4. Actual and Optimal National Saving Rates, 1996–97 (Percent of GDP)

Economy	Optimal national saving 1996–97	Actual national saving 1996	Degree of oversaving (Actual minus optimal)
Philippines	23.2	18.3	-4.9
Japan	29.6	30.4	0.8
Hong Kong SAR	28.4	31.1	2.7
Malaysia	25.0	37.8	12.8
Singapore	30.9	45.4	14.5

First, when the support ratio begins to increase significantly—that is, when pressure from dependents begins to decrease significantly—initially the opportunity exists to increase consumption. At this point the optimal consumption rate rises and thus the optimal saving rate falls, as Malaysia and the Philippines demonstrate. In the early part of the projection period, these two economies face significant increases in the support ratio. These increases take place over almost 30 years for Malaysia (peaking in 2025) and 40 years for the Philippines (peaking in 2035). These changes are the driving force behind the pattern of optimal saving, which shows a decrease for the first 8 years of the projection period for Malaysia and for the first 13 years for the Philippines.

Second, an increase in the support ratio followed by a decline induces the hump-shaped pattern of optimal saving seen in Chart 1. This pattern reflects two trends. The first is the opportunity to increase wealth when the support ratio is high and increasing by using output produced by a relatively large and increasing employment level. The second is the challenge of meeting increasing consumption demands when the support ratio is declining. By saving a relatively larger proportion of the GDP produced during the years the support ratio peaks, economies can augment consumption in the ensuing years of a declining support ratio. This effect is shown in the increasing rate of optimal saving for Malaysia after 2005 and for the Philippines after 2010, both of which are driven by the anticipation of a peak in support ratios in 2025 and 2035, respectively, and the subsequent decline. For Hong Kong SAR this effect is apparent early on. Hong Kong SAR exhibits an increasing support ratio after 1997 that peaks in 2005 and then declines. Its optimal rate of saving rises to a peak in 2010 and then falls.

Third, a decreasing support ratio induces a decreasing rate of optimal saving, representing the challenge of meeting increasing consumption demands relative to employment and output levels. The optimal response is to shift consumption toward the future—that is, for the rate of saving to decrease. For Japan, this effect is quite strong. The Japanese support ratio decreases

fairly rapidly during the first 20 years of the projection period, but then begins falling more slowly. The rate of optimal saving also shows a relatively rapid decrease.

The optimal rate of saving for Singapore in 1997–2035 reflects both the second and third effects. Singapore's support ratio begins decreasing at the start of the projection period and falls slowly until around 2007. Following 2007, the support ratio declines relatively rapidly, flattening out after 2035. Driven by these trends, the optimal rate of saving for Singapore remains fairly constant until 2007 and then falls rapidly for 28 years. Of all the economies, consumption smoothing has the largest effect on Singapore's optimal saving ratio.

The patterns for the optimal current account balances (CABs) reflect the patterns of the optimal saving rates (Chart 3).²¹ In particular, the peaks in the optimal CABs occur at or near the peaks in the optimal saving rates. Thus, more of the intertemporal variation in the optimal rates of saving flows into the rate of accumulation of foreign assets than into the rate of accumulation of domestic capital. This effect illustrates how a small open economy can make full use of the international capital market. For Hong Kong SAR, Japan, and Singapore, the socially optimal current account surpluses appear very large in some years (greater than 15 percent of GDP). The desirability of such large current account surpluses is open to debate. To the extent that our assumptions underestimate the productivity of domestic investment or overestimate the return from overseas assets, the CABs produced by the model are biased upward relative to the true socially optimal level.

The paper's calculations of optimal saving are based on a simple model and a particular set of parameter values. Are there reasons for thinking that either the model's specifications or the choice of parameter settings biases the results in any direction? First, as pointed out above, the choice of the rate of time preference arguably treats future generations rather generously, in that the trend is for consumption per person to grow with labor productivity. Thus, on this count our calculations of optimal rates of national saving cannot be considered too low—in fact, they could be considered too high. Second, the model ignores any adjustment costs incurred in shifting resources to the export and import-substitution sectors. Insofar as there are costs associated with shifting these resources, the optimal plan would require smaller fluctuations in the rate of saving. The model also ignores the effect of a habitual level of consumption on people's utility. As people get used to a particular level of consumption, they tend to favor minor rather than major changes in consumption levels. Because we ignore adjustment costs

²¹ The optimal CABs in Chart 3 show a number of sharp increases, which reflect discontinuities in the patterns of optimal investment levels that are not shown in the charts. The results from the population projections are for five-year periods, so population growth rates change by a discrete amount at five-year intervals, causing increases in investment. Smoothing out population growth rates would introduce an extra degree of arbitrariness into the assumptions, because population growth rates can be smoothed in many ways. Furthermore, our main focus is saving, and in this context the increases in investment are automatically smoothed by the optimizing process. On balance, it seems best to keep these shifts in the investment levels and to ignore them when interpreting the CABs.

and habitual levels of consumption, our calculations of the optimal rates of national saving and CABs may have a greater variation than the true optimal levels. It remains to be investigated how more complicated functional forms might affect the optimal outcomes.

The hypothesis that demographic changes in the populations of the five economies can have a major impact on the pattern of the optimal rates of saving can be illustrated with projections of optimal saving rates that assume an unchanging demographic structure. The projections in Chart 4 assume that for each economy, the aggregate population is the same as it is in the population projections used above. But the projections ignore the age structure. Thus, the aggregate employment/population ratio is constant at its 1997 value throughout the projection period. Furthermore, Chart 4 assumes that labor productivity and consumption demands are independent of the age structure of the population. The resulting patterns of optimal saving are very similar across the five economies and show little intertemporal variation. With the exception of the general level of saving, there is almost no difference in the saving patterns, a result that supports the view that demographic change has an important impact on the level of optimal saving.

Using the consumption weights based on U.S. data, optimal saving rates are generally higher than rates calculated using Australian weights, but the patterns are similar over the period 1997–2050 (Chart 5). Appendix C presents further tests of the sensitivity of our calculations to changes in parameter values. These simulations show that the level—and especially the shape—of the optimal saving profile of the base case is fairly robust with respect to alternative parameter values. This result increases our confidence in the reliability of our calculations of optimal national saving.

VI. CONCLUSIONS

This paper has calculated the optimal rates of saving for the Philippines, Malaysia, Singapore, Hong Kong SAR, and Japan for the period 1997–2050, using a model of the optimal rate of saving in a small open economy. For 1997, the first year in the projections, the rates of optimal national saving in Japan, Hong Kong SAR, and the Philippines are fairly close to the actual rates in 1996, just before the Asian currency crisis. For Malaysia and Singapore, the calculations suggest a considerable degree of excessive saving in 1996. The pattern of the rate of optimal saving over the period varies considerably across economies. The optimal pattern for Japan would be a decrease in the rate of saving for a 20-year period beginning in 1997. For Singapore, the optimal pattern is a decline in the rate of saving over a 30-year period beginning later than Japan. The Philippines and Malaysia show quite different patterns. The optimal rate of saving in those two economies follows a U-shape after 1997, with troughs occurring in 2005 for Malaysia and 2010 for the Philippines. Following those troughs, the rate of optimal saving increases for 28 years in the case of Malaysia and for 40 years in the case of the Philippines. For Hong Kong SAR, the optimal saving rate increases beginning in 1997, peaks in 2010, and decreases for the next 35 years.

The differences in the patterns of optimal rates of saving are driven by the differences in the changes in demographic structures. The optimal response for economies that expect a

significant improvement in their support ratios (employment to population) is to allow the rate of national saving to decrease.²² Malaysia and the Philippines will be in this position for the next 15 to 20 years. Economies where support ratios have already begun to decrease, such as Japan, should allow their saving rates to fall as their support ratios decline. But economies that expect their support ratios to begin decreasing some 10 years or so into the future (Hong Kong SAR and Singapore, for example) should have a high or even an increasing rate of saving in the intervening period. Once the decline begins, the savings rate should also fall.

Our results can be compared with the studies referred to in the introduction. Cutler and others (1990), using an infinitely lived agent framework, conclude that the socially optimal rate of saving for the United States is hump-shaped, peaking in 2010.²³ This pattern is in keeping with the behavior of our model. For Japan, Noguchi (1990) and the Economic Planning Agency (1991) calculate a decreasing rate of optimal saving for the early part of the 21st century. This calculation is similar to our projection of the optimal rate of saving for Japan. Other studies have used the OLG models of Auerbach and Kotlikoff (1992) and Miles (1999). For the United States, Auerbach and Kotlikoff (1992) use a simulation beginning in 1960 to calculate a hump in the savings rate that peaks in 1985. Miles (1999) predicts, in a simulation beginning in 1961, no hump in the rate of saving for the United Kingdom. Instead, the saving rate decreases, levels out between 1980 and 2005, and then begins decreasing again. Miles calculates a similar pattern for continental Europe. Given that both the United Kingdom and Europe have a support ratio that decreases after 1960 (OECD 1996), the socially optimal rate of saving using our model can be expected to decrease following a pattern similar.

Finally, some studies using econometric models are not closely related to ours, as they are based on equations fitted to data rather than to specific functional forms for utility and production functions. For the United States, the econometric study of Masson and Tryon (1990) predicts an increase in the savings rate until 2015, followed by a decrease. For Japan, both Masson and Tryon (1990) and Yashiro and Oishi (1997) predict a decrease in the savings rate over the first quarter of the 21st century that is similar to our results.

Looking at CABs, Cutler and others (1990) project a deficit for the United States until 2005, followed by 15 years of surplus. Masson and Tyron (1990) also project a hump-shaped CAB deficit for the United States, but theirs peaks in 2015. This pattern is in line with the outcome of our model, in particular the result for Hong Kong SAR's CAB. Of the five economies considered here, Hong Kong SAR has the support ratio that most closely resembles the United States'. For Japan, a hump in the projected CAB appears in Noguchi (1990), peaking between 2000 and 2010. This paper projects a decreasing current account surplus throughout the period, consistent with the findings of both Masson and Tyron (1990) and Yashiro and Oishi (1997).

²² The ratio of employment to population.

²³ This is from their simulation with an open economy, Cutler and others (1990, Figure 14, p. 37).

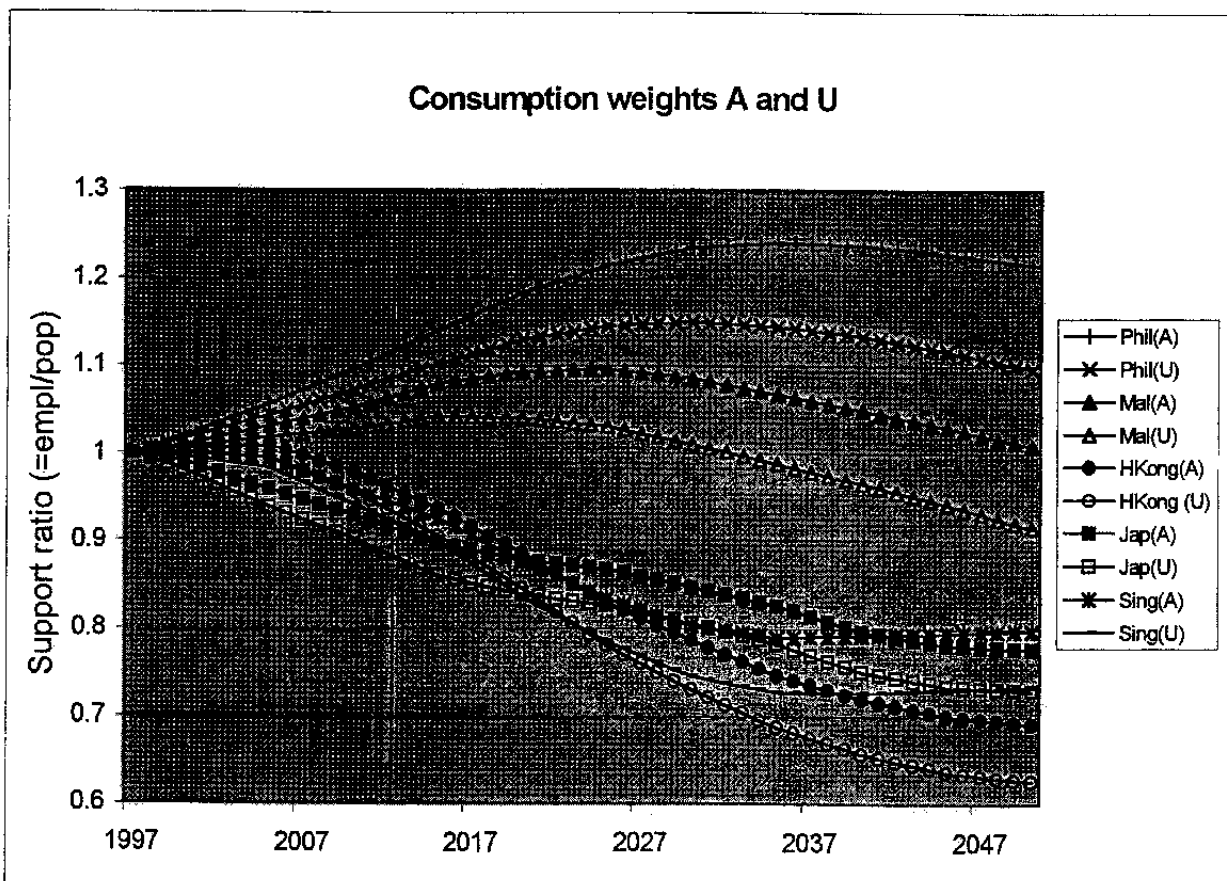
References

- Auerbach, A.J., and Kotlikoff, L.J., (1992), "The Impact of the Demographic Transition on Capital Formation", *Scandinavian Journal of Economics*, 94, pp.281-301.
- Barro, R.J. and Sala-i-Martin, X. (1995), "*Economic Growth*", McGraw-Hill, New York.
- Besanger, S. Guest, R. and McDonald, I., "Is Asia Saving Too Much?" Discussion Paper DPW10.99, Asia Pacific Strategic Forum, 1999.
- Blanchard, O.J., and Fischer, S., (1989), "*Lectures in Macroeconomics*", MIT Press, Cambridge, Ma.
- Bradbury, B. and Saunders, P. (1990), "How Reliable are Estimates of Poverty in Australia? Some Sensitivity Tests for the Period 1981-82 to 1985-86", *Australian Economic Papers*, December, 154-81.
- Census and Statistics Department (1999), Hong Kong SAR Special Administrative Region.
- Commonwealth Department of Community Services and Health* (1990) "The Impact of Population Ageing on Commonwealth and State Social Outlays 1987-88", Policy Development Division, Canberra.
- Cutler, D.M., Poterba, J.M., Sheiner, L.M. and Summers, L.H. (1990) "An Aging Society: Opportunity or Challenge?" *Brookings Papers on Economic Activity*, (1), pp.1-74.
- Dowling, M., and Summers, P.M., (1998) "Total Factor Productivity and Economic Growth – Issues for Asia", Department of Economics, University of Melbourne.
- Economic Planning Agency, (1991) *2010 Nen no Sangyo Keiza* (Simulation of the Industry and Economy in 2010), Tokyo, Government Printing Office.
- Guest, R.S. and McDonald, I.M., (1998a), "The Socially Optimal Level of Saving in Australia, 1960-61 to 1994-95", *Australian Economic Papers*, Volume 37, No 3, pp. 213-35.
- Guest, R.S. and McDonald, I.M., (1998b), "Simulating Optimal Consumption Paths in a Small Open Economy with Uzawa Preferences", Research Paper No 639, Department of Economics, University of Melbourne.
- Guest, R.S. and McDonald, I.M., (1999a), "Ageing, Immigration and Optimal Saving in Australia", Department of Economics, University of Melbourne.
- Guest, R.S. and McDonald, I.M., (1999b), "The Effect of Later Retirement on Optimal National Saving in Australia", Department of Economics, University of Melbourne.

- Hurd, M.D. (1997) "Effects of Demographic Trends in the US" in Hurd, M.D. and Yashiro, N. *The Economic Effects of Aging in the United States and Japan*, University of Chicago Press, NBER, Chicago.
- Masson, P.R., and Tryon, R.W., (1990) "Macroeconomic Effects of Population Ageing in Industrial Countries" *IMF Staff Papers*, 37, pp.453-85.
- Miles, D., (1999) "Modelling the impact of demographic change on the economy" *Economic Journal*, 109, pp.1-36.
- Ministry of Manpower Planning (1999), Singapore.
- Noguchi, Y., (1990), "Jinko Kozo to Chochiko/Toshi: Kakkoku Hikaku ni yoru Bunseki" (The Age Structure of the Population and Saving/Investment: An Analysis Based on Cross-Country Comparisons), *Financial Review* (Institute of Fiscal and Monetary Policy, Ministry of Finance) 17, pp.39-50.
- OECD (1996) *Ageing in OECD Countries: A Critical Policy Challenge* Social Policy Studies No 20 OECD, Paris.
- Olekalns, N., (1999) "What a Drag it is Getting Old. Ageing and Government Expenditure in the OECD", Department of Economics, University of Melbourne.
- Radelet, S. and Sachs, J.D., (1998) "The East Asian Financial Crisis: Diagnosis, Remedies, Prospects", *Brookings Papers on Economic Activity*, 1, pp.1-74.
- Schultz, J., and Borowski, A. (1991) "Estimating the Economic "Burden" of the Aged in the United States" in *The Economics of Population Aging: The Greying of Australia, Japan and the United States*, Westport, Connecticut, PB Greenwood, Auburn House.
- Sen, A. (1973) *On Economic Inequality* Clarendon Press, Oxford.
- Skinner, J. (1985), "Variable Lifespan and the Intertemporal Elasticity of Consumption", *Review of Economics and Statistics*, 67 (4), 616-23.
- Statistical yearbook of Japan 1998*, Japan Statistical Association.
- United Nations (1999), "The Sex and Age Distributions of the World Populations." Revision, United Nations, New York.
- Uzawa, H. (1968), "Time Preference, the Consumption Function and Optimal Asset Holdings" in John N. Wolfe, ed., *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh University Press, Edinburgh, pp. 485-504.

Yashiro, N. and Oishi, A.S. (1997), "Population Aging and the Savings-Investment Balance in Japan" in Hurd, M.D., and Yashiro, N.(eds) *The Economic Effects of Aging in the United States and Japan*, University of Chicago Press for the NBER, Chicago.

Chart 1. Support Ratios

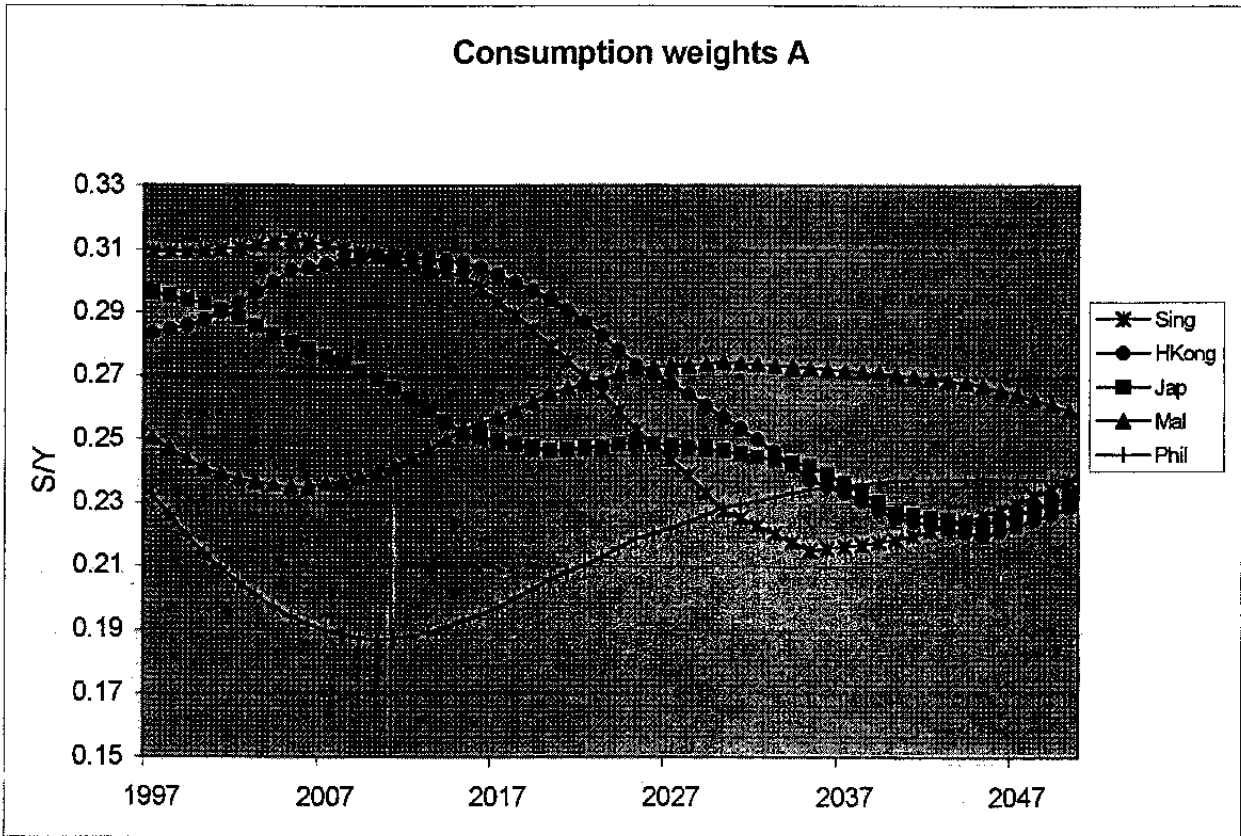


Note: A = weights based on Australian data.

U = weights based on U.S. data.

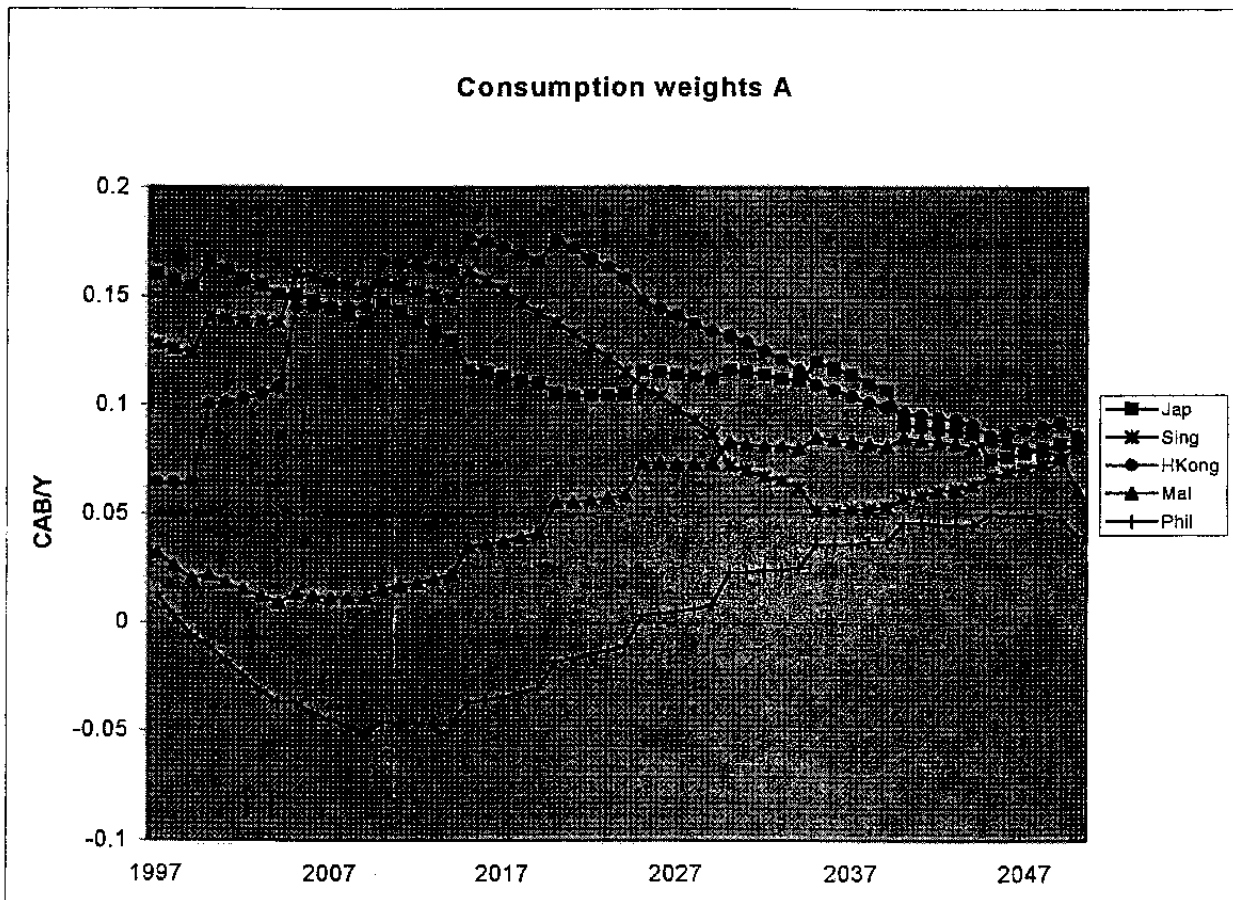
Source: Officials and IMF staff estimates.

Chart 2. Optimal Saving Ratios (S/Y)



Note: A = weights based on Australian data.
Source: Officials and IMF staff estimates.

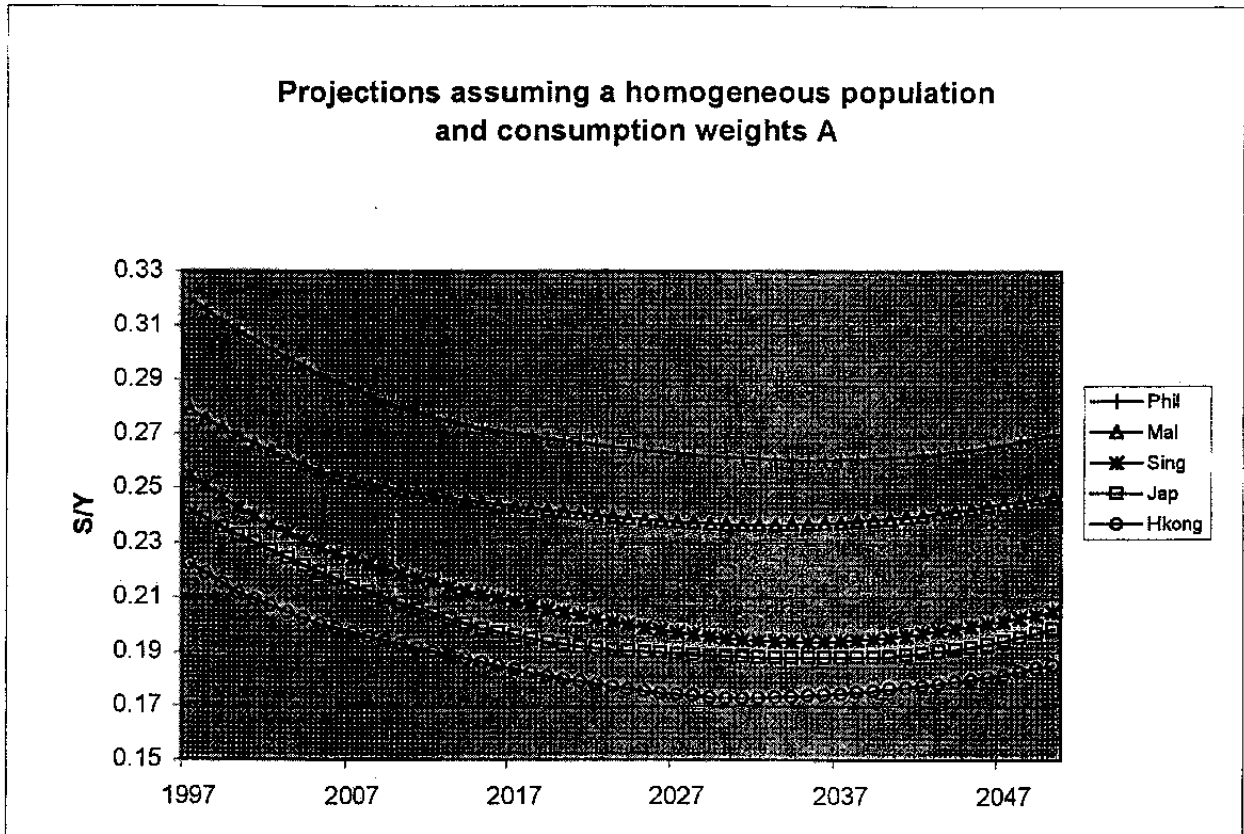
Chart 3. Optimal Current Balances (Ratios)



Note: A = weights based on Australian data.

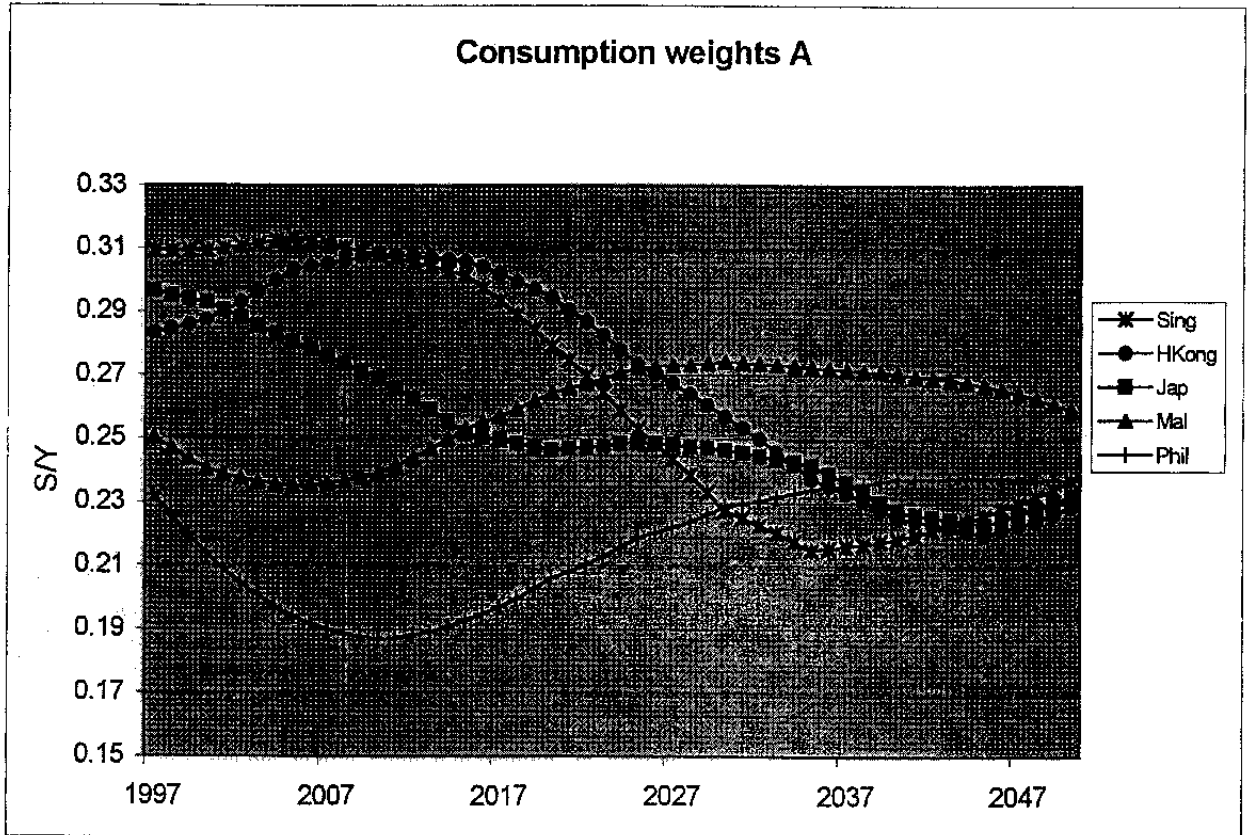
Source: Officials and IMF staff estimates.

Chart 4. The Effect of Population Aging On
Optimal Saving Ratios



Note: A = weights based on Australian data.
Source: Officials and IMF staff estimates.

Chart 5. The Effect of Population Aging
on Optimal Saving Ratios (Alternative Weights)



Note: A = weights based on Australian data
Source: Officials and IMF staff estimates

Calculation of Productivity Weights (Table 1)

This Appendix explains in more detail the calculation of the productivity weights given in Table 1. We obtained data on the age-by-gender distribution of earnings for Hong Kong SAR, Japan, and Singapore but were not able to obtain these data for Malaysia or the Philippines. For the latter economies we used the mean weights for each age group for Singapore and Hong Kong SAR (we did not include Japan in this calculation, as explained in the text).

The data sources were as follows: Singapore — the Ministry of Manpower Planning (1999); Hong Kong SAR – the Census and Statistics Department (1999), Hong Kong SAR; Japan – the Statistical Yearbook of Japan (1998).

Table A1. Singapore: Mean Monthly Income of Employed Persons, by Age Group
(1997 Singapore \$)

	15-24	25-29	30-34	35-39	40-44	45-54	55 & +
Male	994.99	2,244.96	2,790.64	2,931.98	2,937.70	2,847.19	2,031.15
Female	1,278.54	1,889.97	2,157.63	2,156.24	2,048.85	1,831.61	1,262.24

Table A2. Hong Kong SAR: Median Monthly Earnings of
Employed Persons by Age Group
(1997 Hong Kong SAR \$)

	15-19	20-29	30-39	40-49	50-59	60 & +
Male	6,500	10,000	13,100	13,000	10,000	7,500
Female	6,500	9,000	10,000	7,000	6,000	5,000

Table A3. Japan: Mean Monthly Contract Earnings of
Full-Time Employees by Age Group
(1996 Yen in thousands)

Mean earnings	15-24	20-24	25-34	35-44	45-54	55-64	65 & +
Male	170.5	229.0	311.0	400.0	452.5	368.0	278.0
Female	148.0	193.0	231.5	244.0	236.0	215.0	206.0

There is some lack of correspondence in the data for the three economies. For Hong Kong SAR the figures are the median earnings rather than the mean. The age groups do not correspond exactly. To address this problem we divided age groups into finer subgroups by assuming that earnings were distributed evenly across ages within the group. For example, for the 15–24 age group for Singapore we assumed that earnings of the 15–19 group were the same as those of the 20–24 age group. We acknowledge the limitation of this simplifying assumption. One odd feature in the data is that males in the 15–24 age group in Singapore earn less, on average, than their female counterparts. They also earn a lower proportion of average earnings across all age groups than their counterparts in Hong Kong SAR.

The Equations of the Model

This appendix describes the model used to calculate the optimal levels of saving in the text. The model is based on the following maximization problem.

$$\begin{aligned}
 & \text{Maximise} \quad \Gamma = V(C_1, C_2, \dots, C_h, W_h) \\
 & + \sum_{j=1}^h \lambda_j \left[Y_j + B_j - I_j - C_j - D_0 (1-m)^{j-1} (m+r_0) - \sum_{k=1}^{j-1} B_k (1-m)^{j-k-1} (m+r_k) \right] \\
 & + \sum_{j=2}^h \psi_j \left\{ \left[\sum_{k=1}^{T_j} (1-\delta)^{k-1} A_{j-k} F(I_{j-k}, l_{j+1-k}) \right] - Y_j \right\} + \\
 & + \xi_j \sum_{j=2}^h \left[L_j - \sum_{k=1}^{T_j} (1-\delta)^{k-1} l_{j+1-k} \right] \\
 & + \varphi \left\{ \left[\sum_{k=1}^{T_h} (1-\delta)^{k-1} I_{h-k} \right] - D_0 (1-m)^h - \left[\sum_{k=1}^h B_k (1-m)^{h-k} \right] - W_h \right\}
 \end{aligned}$$

by choice of

- (C_1, C_2, \dots, C_h)
- $(I_1, I_2, \dots, I_{h-1})$
- (B_1, B_2, \dots, B_h)
- (Y_2, Y_3, \dots, Y_h)
- (l_2, l_3, \dots, l_h)
- (T_2, T_3, \dots, T_h)
- (W_h)

where:

- Subscript = time period
- C = aggregate consumption
- W = wealth = capital stock plus overseas assets
- B = borrowing from overseas
- I = aggregate investment
- D = overseas debt
- M = proportion of debt repaid in each period
- R = world interest rate
- δ = rate of depreciation
- l = number of workers employed on a vintage of capital during its first period of use
- t = age of oldest capital good in use
- h = terminal period of the maximisation problem

$\lambda_j, j = 1, \dots, h, \psi_j, j = 2, \dots, h, \xi_j, j = 2, \dots, h,$ and φ are Lagrange multipliers.

The first constraint is the international budget constraint; the second is the production function. The third is the employment constraint, and the fourth constraint is the determination of terminal wealth.

It is assumed that $(h - T_h) > 0$. T_j is the age of the oldest capital good in use at time j . T_j^+ is the (expected) age of scrapping of a capital good installed (built) at time j . The production function $F(\cdot)$ has constant returns to scale. The summation signs go from low (after the equal sign) to high. They apply only if the high sign is an integer equal to or greater than the low sign.

The exogenous variables are:

$$(I_{2-T_2}, \dots, I_1)$$

$$(I_{3-T_2}, \dots, I_1)$$

$$(Y_1)$$

$$(D_0)$$

The production function is a vintage putty-clay form. At the time of the investment decision the technique of production can be chosen from a Cobb-Douglas function

$$A_{j-1} I_{j-1}^\alpha I_j^{1-\alpha}$$

where l_j is labor employed on capital of vintage j . If, as in the simulations in this paper, the rate of interest and the rate of technical progress i.e. the rate of increase in A , are assumed constant, then the optimal age of the oldest capital good in use is constant. Then the production function can be written

$$Y_j = \sum_{k=1}^T \left\{ (1-\delta)^{k-1} A_{j-k} I_{j-k}^\alpha \left[\frac{L_{j+1-k} - (1-\delta)L_{j-k}}{1 - (1-\delta)^{T+1} (1+n)^{-T}} \right]^{1-\alpha} \right\}$$

With these functional forms the first order conditions imply

$$(1) \quad \Lambda_j = (1+r)^{h-j} \quad \text{for } j = 1, \dots, h$$

$$(2) \quad T = \frac{\ln[(1-\alpha)(1-\delta)]}{\ln(1-\delta) - [\alpha/(1-\alpha)]} \quad \text{for } j = 2, \dots, h$$

$$(3) \quad E_j = \frac{\Lambda_j}{\sum_{k=1}^h \Lambda_{j+k} (1-\delta)^{k-1}} \quad \text{for } j = 2, \dots, h-T-1$$

$$(4) \quad E_j = \frac{\Lambda_j - (1-\delta)^{h-j-1}}{\sum_{k=1}^h \Lambda_{j+k} (1-\delta)^{k-1}} \quad \text{for } j = h-T, \dots, h$$

$$(5) \quad 1_j = \frac{L_j - (1-\delta)L_{j-1}}{1 - (1-\delta)^{T+1} (1+n)^{-T}} \quad \text{for } j = 1, \dots, h$$

$$(6) \quad I_j = \left[\frac{\alpha A_j}{E_j} \right]^{\frac{1}{1-\alpha}} 1_{j+1} \quad \text{for } j = 2, \dots, h-1$$

$$(7) \quad Y_j = \sum_{k=1}^T \left\{ (1-\delta)^{k-1} A_{j-k} I_{j-k}^\alpha \left[\frac{L_{j+1+k} - (1-\delta)L_{j-k}}{1 - (1-\delta)^{T+1} (1+n)^{-T}} \right]^{1-\alpha} \right\} \quad \text{for } j = 2, \dots, h$$

$$(8) \quad \Lambda_j = \frac{N_j}{N_h} \left(\frac{P_h}{P_j} \right)^{1-\beta} (1+\rho)^{h-j} \left(\frac{C_h}{C_j} \right)^\beta \quad \text{for } j = 1, \dots, h$$

The calculations in the paper are made from these equations.

Sensitivity Analysis

In this appendix, we consider the sensitivity of the projections of optimal rates of saving to different demographic projections and to changes in values of the exogenous parameters β (the reciprocal of the elasticity of substitution) and a (the rate of technical progress).

Sensitivity to Demographic Projections

In addition to the demographic projections used in the text, the United Nations (1999) provides "high" and "low" variant demographic projections. The impact of these variations on the central projection for each of the five economies is shown in Figures C1.1 to C1.5. High projections tend to decrease the support ratio, and low projections tend to increase it. For all five economies the pattern of the support ratios over time is unaffected by the variations in the demographic projection. However, the impact does vary across economies. For Hong Kong SAR, Japan, and Singapore the high and low variants have a relatively small impact, but the effect for Malaysia and Philippines is relatively large.

Sensitivity to the Elasticity of Substitution, β , and the Rate of Technical Progress, a

Changes in β have a negligible impact on the profile of optimal rate of saving. This is because the model used imposes a condition that in the long run the growth rate of consumption equals the growth rate of output. For a long planning horizon (200 years in our simulations), this condition is required to prevent accumulation or decumulation of foreign assets to an extent that would be inconsistent with the perfect world capital market assumption of the model. Satisfying this condition in turn requires another condition, that a change in the elasticity of substitution, β , is offset by a compensating change in the rate of time preference, ρ , such that the growth rate of consumption remains unchanged and equal to the long run growth rate of output.

We considered two alternative values of the rate of technical progress, a , which were plus or minus half a percentage point from the base case. These alternative values of a made a considerable difference to the catch-up year: this is the year average labour productivity in that economy would catch up to the U.S (assuming that labour productivity in the U.S. grows at its base case rate of 1.0% p.a). These catch-up years are shown in Table C1. For example, for Singapore, the case of $a=0.013$ implies that the catch-up year is 2046. (The catch-up year in the base case for all economies was assumed to be 2197). Note that for Hong Kong SAR, Japan, Singapore, and Malaysia, the lower rate of technical progress implies that labor productivity will never catch up with the US.

Figure C1.1 Support Ratios Under Alternative Demographic Projections – Hong Kong SAR

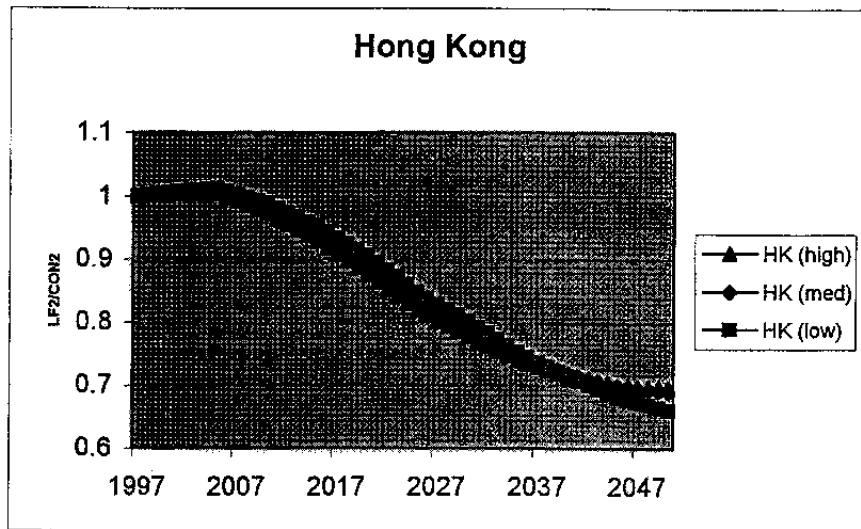


Figure C1.2 Support Ratios Under Alternative Demographic Projections – Japan

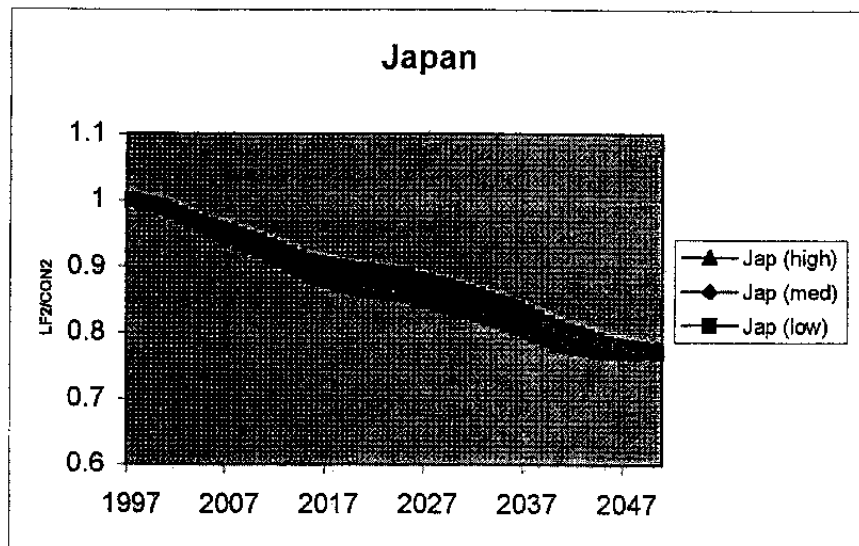


Figure C1.3 Support Ratios Under Alternative Demographic Projections – Malaysia

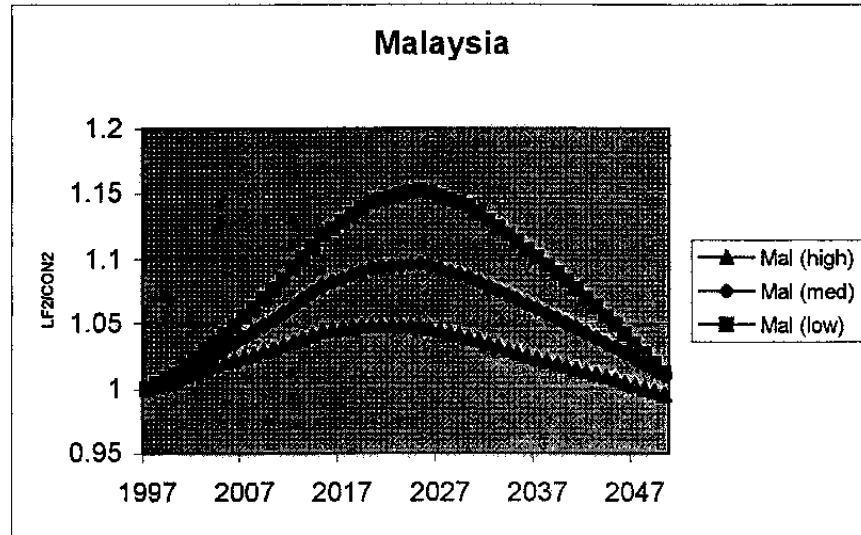


Figure C1.4 Support Ratios Under Alternative Demographic Projections – Singapore

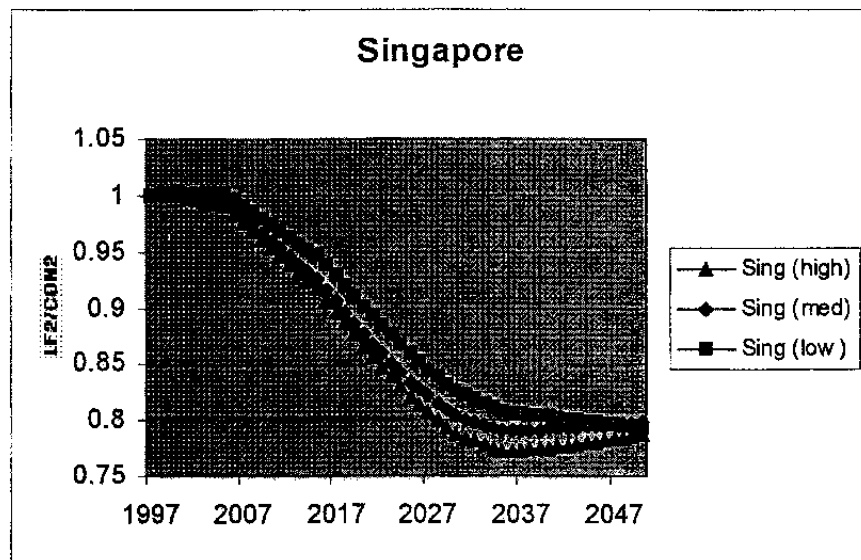
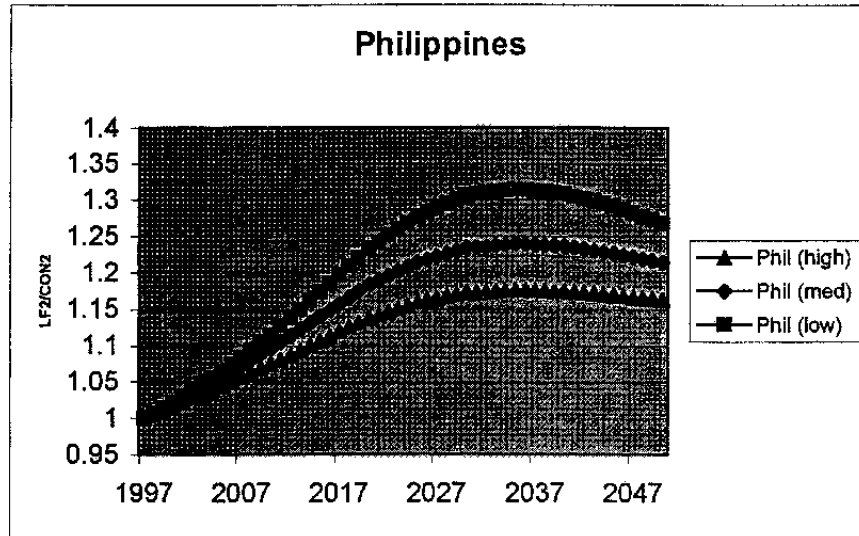


Figure C1.5 Support Ratios Under Alternative Demographic Projections – Philippines



The differences between economies in turn imply different responses of optimal saving. These are shown in Figures C2.1 to C2.5 below. It can be seen that for each of the five economies, the pattern over time or profile of the optimal rate of saving is similar for all variants of the population projections. The largest impact on optimal rates of saving are for the Malaysia and Philippines, reflecting the relatively large variation of the support ratio for these economies.

Figure C2.1 Optimal National Saving Ratios Under Alternative Demographic Projections – Hong Kong SAR

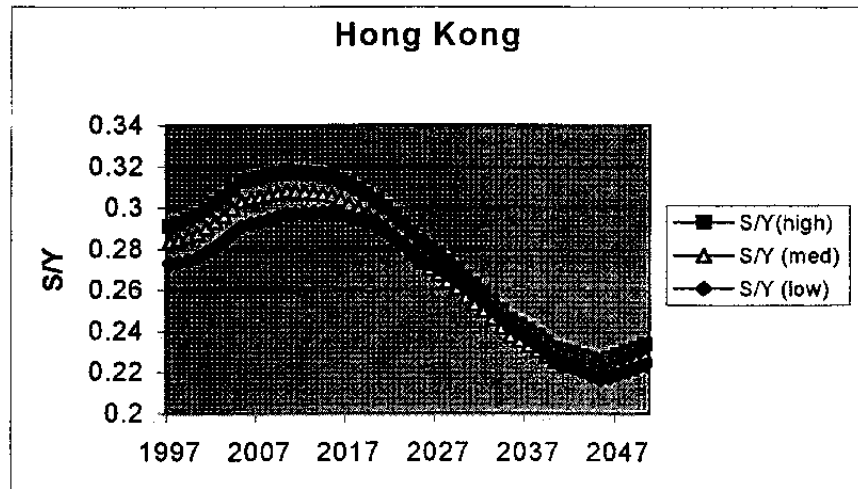


Figure C2.2 Optimal National Saving Ratios Under Alternative Demographic Projections – Japan

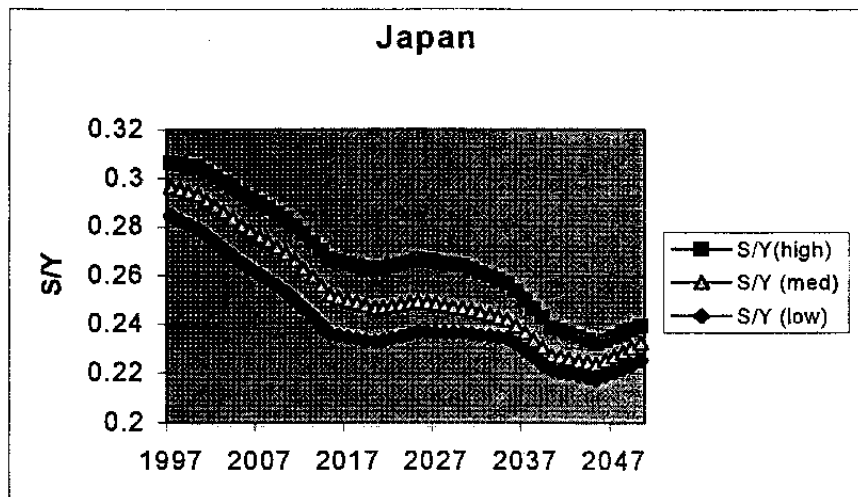


Figure C2.3 Optimal National Saving Ratios Under Alternative Demographic Projections – Singapore

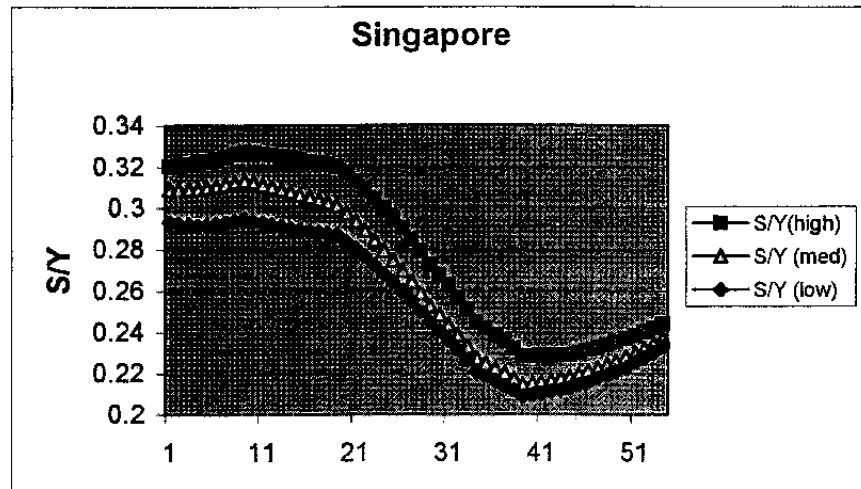


Figure C2.4 Optimal National Saving Ratios Under Alternative Demographic Projections – Malaysia

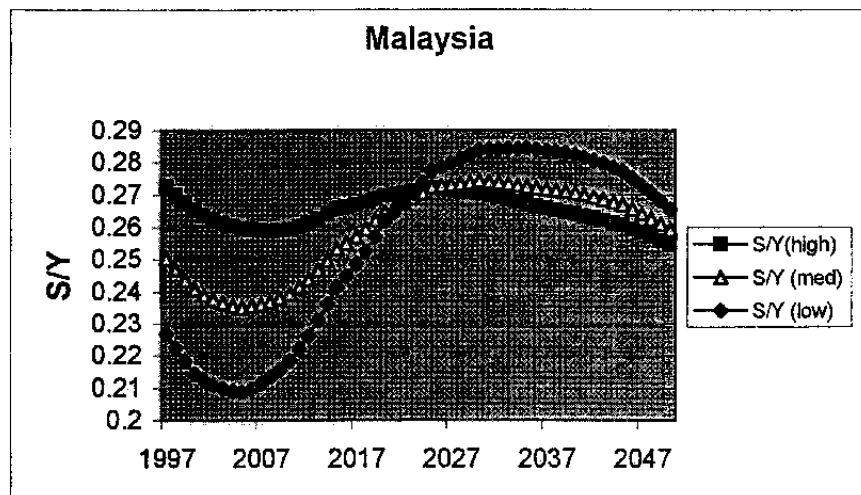


Figure C2.5 Optimal National Saving Ratios Under Alternative Demographic Projections – Philippines

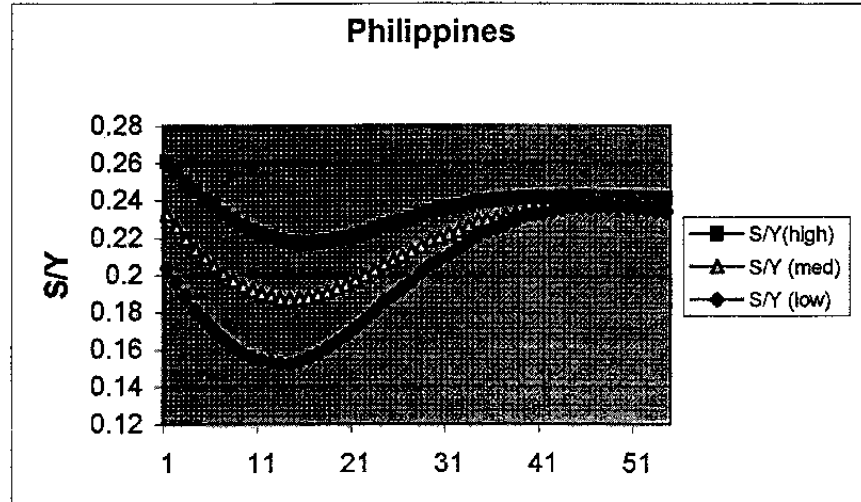
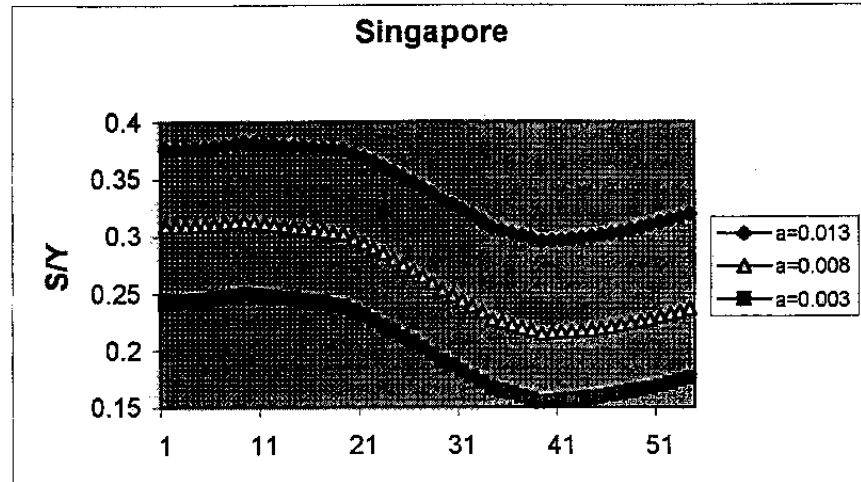


Table C1. Sensitivity of the Catch-Up Year and Optimal National Saving to Technical Progress, α

Country	α	catch-up year from base 1997	average S/Y deviation % of GDP
Hong Kong SAR	0.003	never	-6.3
	0.013	2012	8.1
Japan	0.003	never	-6.6
	0.013	2023	8.4
Singapore	0.003	never	-6.2
	0.013	2046	7.6
Malaysia	0.007	never	-5.1
	0.017	2105	6.1
Philippines	0.007	2339	-4.5
	0.017	2135	5.3

The alternative values of α tend to raise or lower the profile of the optimal rate of saving without changing the shape of that profile. Figure C3 shows as an example the case of Singapore.

Figure C3. Optimal National Saving Ratios Under Alternative Rates Of Technical Progress – Singapore



Because for all five economies changing the value of a does not change the pattern of optimal saving, only the level, we report only summary indicators for optimal national saving shown in the final column of Table C1. For example, for Singapore, the average deviation over the period 1997 to 2050 of the optimal rate of national saving if the rate of technical progress is 0.013 is 7.6 percent of GDP greater than the base case. It can be seen from Table C1 that the deviation in the rate of optimal national saving caused by the alternative rates of technical progress is fairly uniform across all five economies. A decrease (increase) of 0.5 percentage points in the rate of technical progress will cause the average optimal rate of national saving to decrease (increase) by between 4.5 and 6.6 (5.3 and 8.4) percent of GDP.