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Forecasting Inflation in Chile Using State-Space and Regime-Switching Models

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Forecasting Inflation in Chile Using State-Space and Regime-Switching Models

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Abstract

The paper estimates two time-varying parameter models of Chilean inflation: a Phillips curve model and a small open economy model. Their out-of-sample forecasts are compared with those of simple Box-Jenkins models. The main findings are: forecasts that include the pre-announced inflation target as a regressor are relatively better; the Phillips curve model outperforms the small open economy model in out-of-sample forecasts; and although Box-Jenkins models outperform the two models for short-term out-of-sample forecasts, their superiority deteriorates in longer forecasts. Adding a Markov-switching process to the models does not explain much of the conditional variance of the forecast errors.

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Appendices

> *Uncertainty over model selection:*...My approach to this problem while on the Federal Reserve Board was relatively simple: Use a wide variety of models and don't ever trust any of them too much.
>
> Alan S. Blinder (1999)

I. Introduction

The Central Bank of Chile (CBCH) currently uses a variety of models to forecast inflation. One is the *Quarterly Projection Model (Modelo Trimestral de Proyección)* in two versions. The first version comprises various modules, such as the aggregate demand module and the output gap module, with some forward-looking elements. The second version is more comprehensive as it includes a well-defined supply side sector, takes into account some stock-flows relationships, and has a more disaggregated aggregate demand. The CBCH is also developing a quarterly general equilibrium model, and a real business cycle model which stresses the effects of different shocks and relates the parameters of the model to actual time series behavior. The CBCH uses VARs, filters, and leading indicators as well, and is currently exploring the possibility of developing a calibrated model for inflation projections and policy analysis.

The main objective of this paper is to add to that set of inflation-forecasting models the framework offered by state-space models, and to explore whether allowing for regime shifts seems justified by Chilean data during the period 1991–99. The state-space framework is useful not only because it provides its own forecast inflation, but also because it offers a powerful method to estimate important unobserved economic variables that are often encountered in inflation-forecasting models. Therefore, the purpose of this paper is not to enter into a horse race with other inflation-forecasting models, and thus, it only briefly compares the out-of-sample forecasts obtained using state-space models with those generated by a simple Box-Jenkins univariate time series approach. Moreover, Granger (2000) suggests that whenever there are close model specifications—as it is, for example, the case of some of the models used in the CBCH and in this research—it is optimal to find their outputs related to the purpose of the models, such as their forecasts, and pool their values.

State-space models are particularly useful for estimating relationships that might have been subject to important changes within the estimation period. In the last two decades, the Chilean economy has undergone significant structural changes that have affected the allocation of resources and its potential output growth. Those reforms have affected the output mix between tradables and nontradables, the allocation of consumption, the sources of financing of production and consumption activities, the legal framework for the allocation of leisure and work effort, and for the use of domestic and foreign capital. Similarly, the formulation and implementation of monetary policy has been changed as the country moved steadily toward an orthodox inflation targeting regime. A central feature of Chilean monetary policy in the 1990s has been the acquired autonomy of the CBCH and the pre-announcement of a 12-month point inflation target for the following calendar year starting in September of 1990. The inflation target has been attained with high precision. In September 1999, the

CBCH announced that starting in 2001, it will target CPI inflation within an inflation target band which will range between 2 and 4 percent per annum permanently. It is likely, therefore, that the arguments, and possibly the functional form of the loss function of the CBCH, have changed over time. Similarly, these developments have probably had a large effect on the functioning of markets and on the determination of inflation expectations.

The experience of other countries that have undergone reforms as important as those underwent by Chile indicates (as theory predicts) that the parameters that describe the system's dynamics and variance change. While under normal circumstances optimizing economic agents are expected to regularly revise their estimates of the coefficients of the system when new information becomes available, in cases of large structural reforms they may also have to change the set of equations describing that economic system[2]. As a result, macroeconomic policymakers in general, and central banks in particular, have found that in-sample re-fitting of traditional, fixed-parameter models to the data generating process becomes a regular exercise in rapidly changing economies. This notwithstanding, the out-of-sample forecasting ability of models tends to be poor. This has practical implications. For example, the structural instability of the models' parameters, as well as the uncertainty about the "true" model of the economy, may produce biased inflation forecasts which can lead to a breach of a central bank's inflation target.

The next section of the paper discusses the state-space framework proposed. Section III describes the data used, and tests for unit roots and breaks in the sample. Section IV presents the estimation results and the out-of-sample forecasts of the models. The last section concludes the paper.

[2] Wong (2000) finds that the responses of output and price levels to monetary shocks were quite variable in the U.S. in the sample period 1959:1–1994:12. Wong suggests the use of time-varying parameter models to analyze the variability in the effects of monetary policy on economic activity and prices; simple time-invariant linear VAR models may be misleading.

II. Models for the Forecasting of Inflation in Chile

Figure 1 displays annual inflation measured as the log difference in the average of each quarter CPI with respect to the average of the same quarter of the previous year. It is obvious that there has been a significant change in the level and in the variability of inflation over the sample period. This points to the difficulty of fitting a model of inflation in Chile during the 1990s, and thus, of forecasting inflation.

This paper starts from the premise that structural changes have altered and continue to alter the behavior of economic agents. This implies, among other things, that there will likely be instability in any econometric model that one wishes to fit to the data. The approach proposed in this paper will be, therefore, to deal with structural and regime changes by using state-space models[3]. This opens a number of possibilities with different degrees of complexity.

This paper will estimate two models of inflation for Chile. The first model is a time-varying Phillips curve model of inflation and the second model is a reduced form model of inflation in a small open economy that does inflation targeting. In turn, the first model will be estimated excluding the pre-announced official inflation target—henceforth, version one—and including the pre-announced inflation target—henceforth, version two. The time-varying Phillips curve model and the small open economy model will also be estimated allowing for a two-state Markov-switching process.

A. A Time-Varying Phillips Curve Model of Inflation

The first model of inflation is based on an expectations-augmented Phillips curve derived from Lucas' (1973) supply function in the usual manner.[4] In contrast to the standard expectations-augmented Phillips curve, the model allows the parameters to vary over time (in agreement with Lucas' (1973) well-known conclusion). In Chile, the variation of parameters over time could be interpreted as reflecting the learning process of economic agents as reforms unfolded, and the monetary policy framework approached its steady state.[5]

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[3] Appendix I briefly describes state-space models.

[4] As shown in the literature (e.g., McCallum (1989) and Turnovsky (1997)), Lucas' (1973) supply function (his equation (7)) can be transformed into a standard expectations-augmented Phillips curve.

[5] Whether the time-varying Phillips curve is consistent with the suggestion that the Phillips curve is nonlinear and asymmetric (e.g., Clark et al (1995) and Razzak (1995)) depends on the rationale given for that nonlinearity and asymmetry. For instance, the view that the non-linearity and asymmetry of the Phillips curve is mostly due to time-varying, central bank's weights on inflation and output variance, would be consistent with the rationale for the time-varying Phillips curve given in this paper. In that case, the institutional changes that made the

(continued...)

The time-varying Phillips curve is:

$$\pi_t = \mathrm{E}_{t-1}\pi_t + \beta_t(L)x_t + e_t, \qquad (1)$$

where π_t is inflation at time t; E_{t-1} is the mathematical expectation operator based on the information set available at time t-1; β_t is parameter that is allowed to vary over time; x_t is a measure of the output gap, and L is the lag operator; e_t is a stochastic process zero mean and variance σ_e^2. It is assumed that the roots of $\beta_t(L)x_t$ lie outside the unit circle.

Notice that the regressors of equation (1) are unobservable variables. To deal with that feature of the model, the strategy followed is the following. First, based on the assumption (econometrically tested below) that the inflation series has been subject to changes in its intercept as well as in its slope, the unobserved expected inflation is assumed to follow a random walk. Normally, structural shifts are best modeled as discrete shifts. However, in a context in which it is assumed that agents adjust their forecasts only when new information is received, modeling discrete changes using a random walk is a good approximation.[6] Thus,

$$\mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-2}\pi_{t-1} + \tau_t, \qquad (2)$$

where τ_t is a stochastic process zero mean and variance σ_τ^2. Second, following Clark (1987), the output gap is estimated assuming a local linear trend and an autoregressive process of order two for output behavior. The output gap model is described in Appendix II. Equations (1)–(2) can be used to calculate expected inflation as an unobserved variable, and to forecast inflation in periods t+s for s 1.

Given the significant reforms underwent by Chile (including changes to the monetary policy framework of the CBCH), the time-varying parameter β_t represents the learning process of economic agents. Therefore, the time-varying Phillips curve model captures the uncertainty introduced into the inflationary process by those changes. The time-varying Phillips curve model is also estimated allowing for another source of uncertainty, i.e., the uncertainty due to future random shocks. As explained in Appendix I, the error term e_t in equation (1) is a discrete variable S_t which evolution depends on S_{t-1} only, i.e., S_t follows an order 1 Markov process.[7] The model becomes a time-varying, Markov-Switching, model of inflation.

CBCH independent, and made price stability its primary goal, should minimize the importance of that cause of the Phillips curve's non-linearity and asymmetry over time. However, there are other rationalizations for the non-linearity and asymmetry of the Phillips curve that may be more difficult to reconcile with the rationale for the time-varying Phillips curve given in this paper.

[6] This insight is owed to Kim.

[7] See equations A1.3', A1.6, A1.7a and A1.7b.

Note that equation (1) is not identified because it is not possible to generate simultaneously an estimate of the output gap and an estimate of the time-varying parameters β_i. The alternative of estimating simultaneously the unobserved components of real GDP (i.e., a stochastic trend and a cyclical component) and a *standard expectations-augmented Phillips curve with constant β_s* was not feasible. Estimates either displayed significant serial correlation or the information matrix was singular, an indication that the model may not be identified.[8] Therefore, a two-step approach is followed by which first a series for the output gap is estimated, and then, the output gap so generated is used to estimate the time-varying parameter model (1)–(2). The price to pay for this approach is an efficiency loss due to the use of the generated regressor x_t when estimating the expectations-augmented Phillips curve with time-varying parameters.

B. A Small Open Economy Model of Inflation

An alternative to specifying a random walk process for expected inflation as in equation (2), is to substitute the set of state variables (predetermined) suggested by a structural model for expected inflation. The set of state variables is determined from a rational expectations model of a small open economy that does inflation targeting. The model is briefly described in Appendix III.[9] The inflation process can thus be represented by the following reduced form equation:

$$\pi_t = F(g_t, g_t^*, r_t^*, p_t^*, d_t, f_t, c_t) + \tau_t, \qquad (3)$$

where g_t is domestic productivity, g^*_t is the rest of the world's productivity; r^*_t is the cost of foreign financing faced by the Chilean economy (including the risk premium); p^*_t is an index of the country's terms of trade; d_t is a measure of fiscal impulse; f_t is the nominal exchange rate, c_t is the pre-announced official inflation target, and τ_t is a white noise process.

Note, however, that the estimation will be done without imposing the set of cross-equations restrictions that result from the solution of the model of Appendix III. There are simply not

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[8] Different versions of the model were tried, e.g., using different lags in the output gap, or using an ARMA representation for the output gap, or using two constants different from zero for the processes describing the cyclical parts of real GDP and inflation. During estimation of the models, difficulties were encountered either in inverting the matrix of second derivatives of the log likelihood function, or the estimation did not converge. According to Rothenberg (1971), this may indicate that a model is not locally identified, i.e., that more than one set of values for the parameters can give rise to the same value of the log likelihood function; the data cannot discriminate among the possible values.

[9] This draws on Nadal-De Simone (1999), which contains the analytical solution and simulations of a similar model using New Zealand data.

enough data to estimate all the parameters. Most importantly, the model does not provide guidance on the time-varying combinations of parameters implied by the cross-equation restrictions.

III. Data Analysis

The data used in this paper were provided by the CBCH. The data set comprises the following variables: Chilean real GDP (y_t), an index of real economic activity in partner countries (y_t*), Chilean annual inflation as measured by the CPI (π_t), annual inflation in partner countries (π_t*), the cost of financing for the Chilean economy, i.e., the three-month LIBOR rate plus a country risk premium (r_t*), an index of terms of trade (p_t*), the nominal exchange rate defined as the number of pesos exchanged for one U.S. dollar (f_t), a measure of the fiscal impulse (d_t), and the pre-announced point inflation target for the following calendar year since 1991 (c_t).[10] The data are quarterly. The sample starts in 1986:1 and finishes in 1999:3. The data used in the estimations are always in natural logarithms with the exception of interest rates. Series π_t, π_t*, r_t*, and f_t are quarter averages. The seasonal component of the series has been removed using X-11.

The variables are tested for the presence of unit roots during two different sample periods: 1986:1–1999:3 and 1990:1–1999:3. The Perron (1997) test for breaks in the inflation series for the sample period 1986:1–1999:3 is also performed. Results are presented in Tables 1a and 1b.

Given that inflation and real output seem to have a stochastic trend, the unit-root test used is the modified Dickey-Fuller t-test (DFGLS$^{\tau}$) proposed by Elliott, Rothenberg, and Stock (1996), a point-optimal invariant test which has a substantially improved power when an unknown mean or trend is present in the data. Table 1a shows that, in the period 1986:1–1999:3, the null of a unit root with a constant and a linear trend cannot be rejected for any variable.[11] Changes in all the variables are stationary. In the period 1990:1–1999:3, the null of unit root with a constant and a linear time trend cannot be rejected for any of the variables except annual inflation and the terms of trade. Changes in all the other variables are stationary.

Given the nonstationarity of inflation in the period 1986:1–1999:3, the inflation series as well as changes in it, are tested using Perron (1997) test which allows for a shift in the intercept of the trend function and/or a shift in the slope; the date of the possible change is not fixed a priori but it is endogenously determined. Table 1b shows the results for 2 models. The

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[10] Quarterly figures for the pre-announced official inflation target were calculated assuming a constant rate of decline from each yearly inflation target to the following one.

[11] The lags used in the unit-root tests are chosen using the Schwarz Information Criterion and checking that the residuals are white noise using the Box and Pierce Q statistics.

"innovational outlier model" (model 1) allows only a change in the intercept under both the null and the alternative hypothesis, and the "additive outlier model" (model 2) allows a change in both the intercept and the slope. Two methods are used to determine the break point (T_b): the first method selects as breaking point the one that minimizes the t-statistic for testing the null of unit root ($t_{\hat{\alpha}}$) while the second one minimizes the t-statistic on the parameter associated with the change in the intercept (model 1) ($t_{\hat{\theta}}$), or the change in the slope (model 2) ($t_{\hat{\gamma}}$). The lag parameter is chosen following a general-to-specific recursive procedure so that the coefficient on the last lag in an autoregression of order k is significant, and that the last coefficient in an autoregression of order greater than k is insignificant, up to a maximum order k max.

Model 1 does not reject the null hypothesis of a unit root either in the inflation series or in its changes using any of the two methods for choosing the break point T_b. The tests show a change in the intercept of inflation in 1988, and a change in the intercept of changes in inflation either in 1992, or in 1990, depending on the method chosen to estimate the break point.

Model 2 strongly rejects the null hypothesis of a unit root in the inflation series and in its change independently of the method used to choose the break point T_b. The break point in the intercept and/or slope of inflation is in 1988 while the break point in the intercept and/or slope of changes in inflation is in 1990.

Therefore, the Perron (1997) test indicates that the inflation rate is stationary in the entire sample period when allowance is made for changes in the intercept and the slope.

IV. The State-Space Representation of the Models and Results

A. Estimation of the Output Gap Series Using an Unobserved Components Model of Output

As stated above, estimation of the model (1)–(2) requires an estimate of the output gap. This is done using the entire sample period 1986:1–1999:3. The state-space representation of the estimated unobserved components model of output is:

$$y_t = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_t \\ X_t \\ X_{t-1} \\ g_t \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} T_t \\ X_t \\ X_{t-1} \\ g_t \end{bmatrix} = \begin{bmatrix} 1 \sim 0 \sim 0 \sim 1 \\ 0 \sim \theta_1 \sim \theta_2 \sim 0 \\ 0 \sim 1 \sim 0 \sim 0 \\ 0 \sim 0 \sim 0 \sim 1 \end{bmatrix} \begin{bmatrix} T_{t-1} \\ X_{t-1} \\ X_{t-2} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} h_t \\ l_t \\ 0 \\ w_t \end{bmatrix}. \tag{5}$$

Once the model is in the state-space form, it can be estimated using the Kalman filter.[12] Table 2 shows the estimated variances σ_h^2 and σ_l^2, as well as the fixed parameters of the autoregressive process of order 2 assumed for the cyclical component of real GDP, i.e., θ_1 and θ_2. The estimation was constrained such that the roots of the characteristic equation of the process $\theta(L)X_t$ lie outside the unit circle, and that the variances σ_h^2 and σ_l^2 are positive numbers. All parameters are significant at the usual significance levels. The Q-statistic tests for serial correlation as well as the Kolmogorov-Smirnov periodogram test of the standardized forecast errors and the squared of the standardized forecast errors, cannot reject the white noise null hypothesis.

Figures 2–4 show the log of real GDP and its stochastic trend component, its cyclical component, and its productivity growth component, respectively.[13] The cyclical component profile seems to match the standard description of the Chilean business cycles of the 1990s. Three points are noteworthy. First, during the 1990s, the area covered by the negative part of the cyclical component of output was larger than the area covered by the positive part of the cyclical component of output. This is consistent with the steady decline in inflation sought, and successfully obtained, by the monetary authorities during that period. Second, the cyclical component of output seems to have peaked in 1998:1, i.e., before the monetary policy tightening of the second half of 1998. Finally, average productivity growth[14] seems to have declined steadily from a quarterly average growth rate of 2.12 percent in 1992–94 to 2.05 percent in 1995–96, to 2.0 percent in 1997, and to 1.9 percent in 1998. This trend seems consistent with the view held by some observers that the potential output growth of the Chilean economy in this decade may not reach the levels of the 1990s due to the completion of one-time gains from past structural reforms. Recent policy measures to widen and deepen the domestic capital and money markets, to further liberalize the capital account, as well as to continue the unilateral trade liberalization and education reforms, may reverse that downward trend. In any case, the relevant point is that productivity growth will be shown to be an important factor in the forecasting of inflation in Chile.

B. Estimation of the Time-Varying Phillips Curve Model of Inflation

The state-space representation of the time-varying parameter Phillips curve model of inflation is:

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[12] Appendix IV contains a brief description of the Kalman filter. For a thorough description of the Kalman filter, see Hamilton (1994).

[13] The first 16 observations were used to eliminate the influence of the "wild guess" made for the nonstationary $\beta_{0|0}$ (initial values). Large values were given to the diagonal elements of the covariance matrix of β_t ($P_{0|0}$) so as to assign most of the weight in the updating equation to the new information contained in the forecast error.

[14] As mentioned in Appendix II, productivity growth includes changes in factor endowments.

$$\pi_t = \begin{bmatrix} 1 & x_t & x_{t-1} \end{bmatrix} \begin{bmatrix} E_{t-1}\pi_t \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} + e_t, \tag{6}$$

$$\begin{bmatrix} E_{t-1}\pi_t \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} = \begin{bmatrix} 1 \sim 0 \sim 0 \\ 0 \sim 1 \sim 0 \\ 0 \sim 0 \sim 1 \end{bmatrix} \begin{bmatrix} E_{t-2}\pi_{t-1} \\ \beta_{1t-1} \\ \beta_{2t-1} \end{bmatrix} + \begin{bmatrix} \tau_{0t} \\ \tau_{1t} \\ \tau_{2t} \end{bmatrix}. \tag{7}$$

The model is both locally and globally identified.[15] It seems relevant to recall, however, that identification should not be approached in a rigid manner as it is possible for an equation (or system of equations) to be identified according to the strict rules of identification but the equation may have very little predictive power if the predetermined variables in the equation have little variance. From this point of view, the relatively good forecasting performance of the models (see below) is encouraging.

Table 3 shows two sets of estimates of the time-varying Phillips curve model: the first one, excludes the pre-announced official inflation target while the second one includes it. In the second one, the dependent variable is measured as inflation deviations from the official target. The results suggest that although the estimation of the model that includes the pre-announced official inflation target displays some additional serial correlation, this version is to be preferred because it achieves a better forecasting performance than the model in terms of actual inflation without the information provided by the official pre-announced inflation target.

In both versions of the time-varying Phillips curve model, all parameters are significant at the usual confidence levels. The log-likelihood function of version two of the model is relatively higher. However, there is serial correlation in the standardized forecast errors of version two of the model. In contrast, the Q-statistic tests for serial correlation of the standardized forecast errors of version one of the model show some serial correlation at the 90 percent significance level only for Q(16) and Q(24). The Q-statistic tests for serial correlation of the squares of the standardized forecast errors show no serial correlation in any of the two versions.

Table 3 also shows the out-of-sample forecasts for 1, 2, 3, and 4 quarters and the actual values of inflation. Version two of the model achieves a considerable reduction in Theil's

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[15] Rothenberg (1971) showed that local identification at β_0 requires that the information matrix be nonsingular in a neighborhood around β_0. This criterion was used for testing for local identification of the model described by equations (1)–(2), and also of the model described by equation (3). In no case there was any difficulty in inverting the matrix of the second derivatives of the log likelihood functions. Global identification was tested working directly with the state-space representation of the model, as suggested by Burmeister et al (1986).

inequality coefficient as the value of the statistics for version two is between 19 to 64 percent of its value for version one of the model. A similar picture is offered by the root mean-square forecast percent error or the mean forecast percent error. Figure 5 shows actual and expected inflation, and Figure 6 shows actual and expected deviations from the pre-announced official inflation target, during the sample period. From both figures two observations seem important. First, the pre-announced official inflation target point was below the predicted level of inflation over two thirds of the time. Second, actual inflation was closer to the target point than to the predicted level of inflation just over half of the time.[16] While the first observation is consistent with Morandé and Schmidt-Hebbel (2000), the second is not necessarily. However, given that the framework used in this paper explicitly considers the expectations-updating process by economic agents, all results can be interpreted in the same manner the authors do, i.e., as evidence that the inflation targeting regime contributed to enhance the credibility of the CBCH and, thus, played a role in reducing inflation in Chile. The inflation target announced, and always accomplished with a great deal of accuracy, offset inflationary inertia over time.

Figures 7 and 8 show the *smoothed*[17] time-varying parameters of the contemporaneous and first lag of the output gap of version two of the model. There seems to be a tendency for the variability of the parameter values to fall over time. The absolute value of the contemporaneous parameter of the output gap also falls over time. Given that the CBCH was successful in its inflation targeting strategy over the sample period, this result should not come as a surprise because hitting the inflation target implies that the output gap variance should be reduced over time. For given private sector's expectations, inflation targeting implies inflation *forecast* targeting (Svensson, (1997)). As the implicit loss function of the independent Chilean monetary authority includes the inflation target as the primary objective of monetary policy, one of the sources for a nonzero output gap (i.e., changes in the relative weights attached by the monetary authority to output and inflation variance), and thus for inflation uncertainty, is removed. All the other forces that determine the output gap are, certainly, still operational.[18]

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[16] This is not statistically different from 50 percent.

[17] As the state vector is given a structural interpretation in model (1)–(2), it is important to form an inference about the value of the state vector based on the full sample. Therefore, the value of the contemporaneous and lagged coefficients of the output gap have been calculated based on the full set of data collected by moving through the sample backward starting with t = T-1. Therefore, the time-varying parameters have been smoothed.

[18] Given the profession's lack of agreement on the relationship between nominal and real variables, other interpretations are certainly possible. For instance, efficiency wage theories of wage determination would suggest that given a constant markup, the effect of the output gap on wages and hence on inflation would be smaller with about 3 percent average inflation in 1997 than with about 15 percent average inflation in 1992. However, the sum of the estimated output gap coefficients falls until mid-1995, to increase thereafter. As indicated by

(continued…)

C. Estimation of the Small Open Economy Model of Inflation

The state-space representation of the solution of the open economy model is:

$$\pi_t = x_t \beta_{t\,i} + e_t, \tag{8}$$

$$\beta_{t\,i} = I_k \beta_{t-1\,i} + r_{t\,i}, \tag{9}$$

where the x_t are the k-1 state variables of the open economy model, i.e., domestic productivity, foreign productivity, the real cost of financing of the Chilean economy, an index of the terms of trade, a fiscal impulse measure, the nominal exchange rate with respect to the U.S. dollar, and the pre-announced official inflation target. I_k is a k-order identity matrix.

Table 4 shows the results of the estimation of the open economy model represented by equations (8)–(9).[19] The estimated small open economy model of inflation has a somewhat higher likelihood function than version two of the time-varying Phillips curve model. Serial correlation is only present between lags 16 and 24.

All variables are highly significant at standard confidence levels with the exception of foreign productivity. The out-of-sample forecasts are less accurate than those produced by the time-varying Phillips curve model but are more stable (the Theil's inequality coefficient deteriorates less as the out-of-sample forecasting period is lengthened).

Figures 9–14 show the *smoothed* time-varying parameters of the model. There is a number of interesting observations. First, the coefficient on the cost of foreign capital is negative, and its absolute value increases until 1995. In standard models of exchange rate determination, a reduction in the cost of foreign financing is expected to increase domestic expenditure thereby pushing up inflation in non-tradable goods and in the CPI, other things equal. This is the real-sector channel of the monetary transmission mechanism. However, a reduction in the cost of foreign financing is expected to also appreciate the domestic currency, and translate into lower CPI inflation, other things equal. This is the asset-market channel of the monetary transmission mechanism. In an inflation targeting regime, in contrast, the cost of foreign financing is *negatively* correlated with domestic CPI inflation. Briefly, under inflation targeting, because liquidity is endogenous, the asset-channel part of the transmission mechanism is weakened and thus, the negative correlation between changes in the cost of

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Wong (2000) in his study for the U.S., no single theory of the monetary transmission mechanism seems capable of explaining the changing response of output and prices to monetary policy; one should rather search for a combination of economic and institutional factors.

[19] For the reasons explained in Appendix III, the model is estimated without imposing the cross-equation restrictions that result from solving the model.

foreign financing and domestic CPI inflation that result from the real-sector channel may prevail. There seems to be some indication that this happened in Chile.[20]

Second, the fiscal impulse coefficient is positive, as expected. Consistent with the relatively expansionary fiscal stance adopted after 1995, the weight of the coefficient in explaining inflation variance doubled between the end of 1996 and the mid-1998. It remained at that level thereafter.

Third, the pass-through of changes in the nominal exchange rate to domestic CPI inflation varied over time significantly. As the inflation targeting regime acquired more credibility, the value of the pass-through coefficient fell. After mid-1995, and until the last year of the sample, the pass-through coefficient was stable at about 10 percent. It seems that when annual real GDP growth fell from 8 percent in the first quarter of 1998 to 5.9 percent in the second quarter of 1998, the pass-through coefficient also started to fall. It reached a value of less than 5 percent at the end of the sample period. Collins and Nadal-De Simone (1996) show that the "pass-through coefficient" depends at a minimum, on the structure of the economy, on the nature of the shocks affecting the economy, on the composition of the CPI regimen, and on the central bank's operating procedure. One implication they draw is that pass-through coefficients are likely to be econometrically unstable across policy regimes as well as across time within the same policy regime. This econometric study seems to validate that conclusion.

Finally, judging from Figure 14, the smoothed inflation target coefficient remained quite stable after 1994. Given the framework used in this study, this seems to be compelling evidence that the credibility of the Chilean inflation target framework was well established since the mid-1990s.

D. Does it Matter to Allow for Regime-Switching in the Models of Inflation?

The squared standardized forecast errors of version two of the time-varying Phillips curve model do not seem to suggest the presence of heteroskedasticity. However, it was considered useful to explore this possibility further by estimating the time-varying Phillips curve model with a Markov-switching process for the disturbance terms. Therefore, version two of the time-varying Phillips curve model was estimated allowing for uncertainty derived not only from the economic agents' updating of the model's parameters but also for uncertainty derived from heteroskedasticity of the disturbance terms. This is the model described by equations (6)-(7) plus equations (A1.3'), (A1.6), (A1.7a) and (A1.7b) from Appendix I.

In this model, the conditional variance of the forecast error can be decomposed into conditional variance due to the unknown regression coefficients, and conditional variance due to the heteroskedasticity of the disturbance term. The first conditional variance depends on the state of the world at time t-1, S_{t-1}, while the second one depends on the state of the

[20] Nadal-De Simone (1999) confirms this point using New Zealand data.

world at time t, S_t. Figure 15 shows that during most of the sample period, the first source of conditional variance was far more important than the second source. In the period 1992:1–1998:2, the average conditional variance of inflation was 0.0025, and about 75 percent of it was accounted for by the learning process of economic agents. It is only between the third quarter of 1998 and the third quarter of 1999 that the heteroskedasticity of the disturbance term is significant; in that period, it explained about 82 percent of the 0.0048 conditional variance of inflation.

The small open economy model of inflation was also estimated allowing for Markov switching, i.e., estimating equations (8)-(9) together with equations (A1.3), (A1.6), (A1.7a) and (A1.7b). Even more forcefully than in case of the time-varying Phillips curve model, the decomposition of variance in figure 16 shows that the conditional variance of the small open economy model forecast error was mostly due to the unknown regression coefficients.

One possible explanation for these results is suggested by the presence of the pre-announced official inflation target among the regressors of the models. To the extent that the inflation target was credible, it reflected changes in the Chilean inflation process; the inflation target proxied the change in the Chilean monetary policy regime from high and variable inflation to low and stable inflation. The implication of these results is that during the sample period of this study, there is little to gain from including in the model uncertainty due to heteroskedasticity of future random shocks. However, once the inflation targeting regime is in its steady state, this is an issue that will require revisiting.

E. Estimation of Selected Time Series Models

A number of models based on Box-Jenkins techniques was also estimated. A reduced set of selected estimation results is reported in Table 5. The reported results refer to an AR(1), an AR(2), and an AR(1) transfer function model. The models are:

$$\pi_t = \phi_1 \pi_{t-1} + \varepsilon_t , \tag{10}$$

$$\pi_t = \phi_1 \pi_{t-1} + \phi_2 \pi_{t-2} + \varepsilon_t , \tag{11}$$

$$\pi_t = \phi_1 \pi_{t-1} + \theta_1 z_t + \varepsilon_t , \tag{12}$$

where π_t is measured as the deviation of annual inflation with respect to the pre-announced official inflation target in equations (10) and (11), and it is measured as annual inflation in equation (12); z_t is the pre-announced official inflation target. As before, model (12) assumes that z_t is exogenous.

The AR(1) and AR(2) models show serial correlation in the first 8 lags while the AR(1) transfer function model also shows serial correlation in the first 16 lags. The AR(1) and AR(2) models have low R^2 while the AR(1) transfer function model has a reasonable R^2 (this is obviously biased upward by the presence of serial correlation in the residuals).
Based on the Theil's inequality coefficient for the first-step forecast, the AR(1) and AR(2) models do better than version two of the time-varying Phillips curve model and the open

economy model (Table 6 has a summary of forecast results). However, the relative performance of models AR(1) and AR(2) deteriorates rapidly for subsequent forecast periods so that the time-varying Phillips curve model outperforms them. The time-series models still do better than the open economy model of inflation.

The AR(1) transfer function model does better than the time-varying Phillips curve model and the open economy model up to the third-period forecast. It does still better than the open economy model for the fourth-period forecast.

However, although the AR(1) transfer function model does better for short-run forecasts than the AR(1) and AR(2) models, it is likely that this ranking will change when the monetary policy regime enters its steady state during 2001 because the inflation target variance will become constant, i.e., its information content will fall. It will then be relatively important that the deterioration of the out-of-sample forecasts of the different models of inflation is not significant as subsequent periods are added to the forecast. On the basis of this criterion, i.e., for medium term forecasting, it seems that the time-varying Phillips curve model and the open economy model do relatively better than the AR(1) transfer function model. The open economy model forecasts actually show quite a remarkable stability in terms of bias and variance. The second version of the time-varying parameter Phillips curve model, given that it is relatively parsimonious, also does a good forecasting job.[21] However, as with the AR(1) transfer model, it remains to be seen whether the time-varying Phillips curve model will still be the best specification of the inflation process in Chile after 2000. It is possible that a richer structure becomes then necessary.

V. Conclusions

The objective of this study is to estimate and forecast inflation in Chile using a state-space framework. Two models of inflation are estimated and used for out-of-sample forecasting of Chilean inflation. The first model is a time-varying Phillips curve model estimated in two versions; version one excludes the pre-announced official inflation target point and version two includes it. The second model is a reduced form model of a small open economy that does inflation targeting. The results of those estimations are compared with those of simple Box-Jenkins specifications of the inflation process in Chile. The two models of inflation are also estimated allowing for regime changes by using a two-state Markov-switching model of order 1. The sample period comprises 1990:1–1999:3, a period which is one year short of the steady state of the monetary policy regime, i.e., a regime in which the authorities target annual CPI inflation within an inflation target band which will range between 2 percent and 4 percent on a permanent basis.

[21] For the United States, Stock and Watson (1999) have shown that a stand-alone *conventional Phillips curve* generally produces more accurate forecasts than other macroeconomic variables, including interest rates, money, and commodity prices. However, it is inferior to a generalized Phillips curve model based on measures of real activity other than unemployment such as an index based on a large number of real economic indicators.

Models that include the pre-announced official inflation target point are to be preferred to those that exclude this variable. Although including the pre-announced official inflation target introduces some serial correlation in the residuals, it also reduces forecasts errors significantly.

The out-of-sample performance of the time-varying Phillips curve model that includes the official pre-announced inflation target point is more favorable than the out-of-sample performance of small open economy model—which includes the pre-announced official inflation target point. In contrast, the statistics for the out-of-sample forecasts of the small open economy model are relatively more stable.

For the first step of the out-of-sample forecast, the Box-Jenkins models of inflation tend to do better than the time-varying Phillips curve model that includes the pre-announced official inflation target. However, their relative forecasting superiority deteriorates rapidly for forecasts further out in time.

An AR(1) transfer function model does better than both the time-varying parameter Phillips curve model and the open economy model, up to the third out-of-sample forecast. It still does better than the open economy model for the fourth out-of-sample forecast.

The addition of a Markov-switching model to the time-varying models of inflation does not improve the fit of the models. During most of the sample period, the conditional variance of the forecast error due to the unknown regression coefficients was far more important than the conditional variance due to the heteroskedasticity of the disturbance term.

A note of caution is necessary, however. It is quite likely that the ranking of models performance will change when the monetary policy regime enters its steady state in 2001. The variance of inflation will probably depend on terms of trade shocks, productivity shocks, the fiscal position, the exchange rate, and the like; the variance of the inflation target will in contrast be constant, and will thus have, ceteris paribus, no power in explaining actual inflation. It will then become important to reassess the forecasting performance of the models paying particular attention that the quality of the forecasts in the medium term does not deteriorate rapidly. In that case, it is likely that models less parsimonious than the time series models of Box-Jenkins will become necessary.

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Table 1a. Chile: Elliot, Rothenberg, and Stock Test for Unit Roots

Statistics for $\rho = 0$

	1986:1–1999:3		1990:1–1999:3	
Variables	Lags	DFGLS $^{\tau}$	Lags	DFGLS
y	3	-2.57	1	-1.53
y*	1	-1.79	1	-2.27
π	4	-1.65	3	-3.45*
r*	1	-1.62	1	-1.49
p*	1	-2.06	1	-3.07*
f	1	-1.37	1	-1.68
Δy	2	-3.10*	1	-4.02*
Δy*	1	-4.25*	1	-3.58*
$\Delta\pi$	1	-4.95*	n.a.	n.a.
Δr*	1	-6.00*	1	-5.44*
Δp*	3	-3.73*	n.a.	n.a.
Δf	1	-4.50*	1	-3.92*

All variables, except interest rates, are measured in natural logarithms. Lags are determined according to Schwarz information criterion and checking that the residuals are white noise.
The DFGLS $^{\tau}$ has a null of unit root with a constant and a linear trend. The 5 percent critical value is –2.89.

Table 1b. Chile: Perron (1997) Unit Root Test
For Annual Inflation and Changes in Annual Inflation

(1986:1–1999:3)

γ	T_b	K	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$\hat{\theta}$	$t_{\hat{\theta}}$
$Model 1: y_t = u + \theta DU_t + \beta_t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t$						
Π	1988:4	9	0.54	-4.78		
Π	1988:2	4			0.72	-4.00
$\Delta\Pi$	1992:1	9	-1.98	-5.10		
$\Delta\Pi$	1990:2	8			-0.10	-4.48
γ	T_b	K	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$\hat{\theta}$	$t_{\hat{\theta}}$
$Model 2: y_t = u + \theta DU_t + \beta t + \delta DT_t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t$						
Π	1988:3	3	0.61	-6.54*		
Π	1988:3	3			0.61	-6.54*
$\Delta\Pi$	1990:2	3	-0.32	-7.98*		
$\Delta\Pi$	1990:3	3			-0.30	-6.77*

The inflation rate is measured as the natural logarithm of the consumer price index of each quarter with respect to the same quarter of the previous year. The first T_b is the value that minimizes the t-statistic for testing $\alpha = 1(\hat{\alpha}, t_{\hat{\alpha}})$, and the second T_b is the value chosen to minimize the t-statistic on the parameter associated with model 1 or model 2. Lags are chosen following the Schwarz information criterion and checking that the residuals are white noise.

Table 2. Chile: Parameter Estimates of the Unobserved Components Model of Real GDP

(1986:1–1999:3)

Variables	Estimates	Standard Errors
σ_h	0.0119	0.0033
σ_l	0.0089	0.0035
θ_1	1.4468	0.0111
θ_2	-0.5233	0.0079
Log likelihood	95.0212	
Q-statistics for standardized forecast errors Q(8) = 9.73 Q(16) = 16.19 Q(24) = 25.05 Q(32) = 41.94		Kolmogorov-Smirnov statistic for standardized errors (K–S) Nobs = 32 Rejection limit (10%) = 0.2157 K–S = 0.0999
Q-statistics for the squares of standardized forecast errors Q(8) = 9.79 Q(16) = 16.47 Q(24) = 25.29 Q(32) = 42.45		Kolmogorov-Smirnov statistic for squares of standardized errors (K–S) Nobs = 32 Rejection limit (10% = 0.2157 K–S = 0.0991

Table 3. Chile: Parameter Estimates of the Time-Varying Parameter Model of Chilean Inflation (1990:1–1999:3)

Variables	Excluding Official Target: Estimates	Excluding Official Target: Standard Errors	Deviations from Official Target: Estimates	Deviations from Official Target: Standard Errors
σ_{e}	1.0029	(0.0035)	0.9954	(0.0018)
$\sigma_{E_{t-1}\pi_{t-1}}$	0.9893	(0.0022)	1.0080	(0.0019)
σ_{x_t}	1.0000	(0.2567)	1.0000	(0.1307)
$\sigma_{x_{t-1}}$	1.0000	(0.1482)	1.0000	(0.1232)
Log likelihood	93.4305		97.41180	

	Q-Statistics for the Standardized Forecast Errors	Q-Statistics for the Standardized Forecast Errors
	Q(8) = 12.79	Q(8) = 23.11*
	Q(16) = 26.35*	Q(16) = 33.91*
	Q(24) = 33.56*	Q(24) = 41.93*
	Q(30) = 34.46	Q(30) = 43.06*

	Q-Statistics for the Squares of Standardized Forecast Errors	Q-Statistics for the Squares of Standardized Forecast Errors
	Q(8) = 3.52	Q(8) = 11.40
	Q(16) = 10.73	Q(16) = 18.51
	Q(24) = 15.62	Q(24) = 28.88
	Q(30) = 21.19	Q(30) = 33.34

Out-of-Sample Forecasts

Periods Ahead	Inflation	Forecast	Theil's Inequality Coefficient	Forecast	Theil's Inequality Coefficient
1. 1998:4	4.57	5.34	0.078	4.71	0.015
2. 1999:1	3.94	4.79	0.087	4.57	0.051
3. 1999:2	3.68	4.28	0.084	4.15	0.054
4. 1999:3	2.93	3.97	0.098	3.44	0.058

	Root Mean-square Forecast Percent Error	Mean Forecast Percent Error	Root Mean-square Forecast Percent Error	Mean Forecast Percent Error
	0.169	0.169	0.031	0.031
	0.193	0.192	0.115	0.095
	0.184	0.183	0.119	0.106
	0.238	0.226	0.134	0.123

Table 4. Chile: Parameter Estimates of the Open Economy Time-Varying Parameter Model with Official Target of Chilean Inflation (1990:1–1999:3)

Variables	Estimates	Standard Errors
σ_e	0.9963	(0.0013)
σ_g	1.0000	(0.0507)
σ_{g^*}	19.3396	(35.9559)
σ_{r^*}	1.0000	(0.1108)
σ_{p^*}	1.0188	(0.0120)
σ_d	0.7311	(0.0825)
σ_f	1.0000	(0.0413)
σ_c	0.9942	(0.0237)
Log likelihood	98.2820	

Q-Statistics for the Standardized Forecast Errors	Q-Statistics for the Squares of the Standardized Forecast Errors
Q(8) = 12.63	Q(8) = 17.25*
Q(16) = 16.84	Q(16) = 22.98
Q(24) = 35.39*	Q(24) = 36.82*
Q(30) = 40.14	Q(30) = 38.20

Out-of-Sample Forecasts

Periods Ahead	Inflation	Forecast	Theil's Inequality Coefficient	Root Mean-Square Forecast Percent Error	Mean Forecast Percent Error
1. 1998:4	4.57	4.77	0.022	0.045	0.045
2. 1999:1	3.94	5.52	0.120	0.285	0.223
3. 1999:2	3.64	4.61	0.120	0.275	0.233
4. 1999:3	2.93	3.91	0.122	0.291	0.258

Table 5. Chile: Time Series Models of Chilean Inflation
(1990:1–1999:3)

Model	Coefficients			Serial Correlation Tests			$\overline{R}^2$	Forecast	Theil's Inequidity
	ϕ_1	ϕ_2	θ_1	Q(8)	Q(16)	Q(24)			
AR (1)	0.1349			18.13*	23.08	31.12	4.73	4.57	0.001
$y_t = \phi_1 y_{t-1} + \varepsilon_t$	(0.0610)							4.62	0.054
								4.65	0.079
								4.25	0.106
AR (2)	0.1179	0.0183		18.43*	23.03	30.97	2.07	4.56	0.001
$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$	(0.0872)	(0.0661)						4.59	0.052
								4.62	0.076
								4.21	0.103
AR (1) Intervention Model	0.1599		0.9620	18.88*	25.08*	32.62	96.07	4.35	0.024
$y_t = \phi_1 y_{t-1} + \theta_1 z_t + \varepsilon_t$	(0.0620)		(0.0189)					4.32	0.036
								4.18	0.046
								3.69	0.063

Table 6. Chile: Summary of Inflation Forecasts 1/

	TVP-Phillips Curve, Version 1	TVP-Phillips Curve, Version 2	Open Economy	AR (1)	AR (2)	AR (1) Transfer Function
Forecasts	5.34	4.71	4.77	4.57	4.56	4.35
	4.79	4.57	5.52	4.62	4.59	4.32
	4.28	4.15	4.61	4.65	4.62	4.18
	3.97	3.44	3.91	4.25	4.21	3.69
Theil's Inequality Coefficient	0.078	0.015	0.022	0.001	0.001	0.024
	0.087	0.051	0.120	0.054	0.052	0.036
	0.084	0.054	0.120	0.079	0.076	0.046
	0.098	0.058	0.122	0.106	0.103	0.063
Root Mean-Square Forecast Percent Error	0.169	0.031	0.045	0.001	0.001	0.047
	0.193	0.115	0.285	0.122	0.117	0.076
	0.184	0.119	0.275	0.182	0.176	0.100
	0.238	0.134	0.291	0.274	0.266	0.156
Mean Forecast Percent Error	0.169	0.031	0.045	0.001	-0.001	-0.047
	0.192	0.095	0.223	0.087	0.082	0.025
	0.183	0.106	0.233	0.146	0.140	0.062
	0.226	0.123	0.258	0.222	0.214	0.111

1/ In the period 1998:4-1998:3, inflation was: 4.57, 3.94, 3.68, 2.93

Figure 1. Log Differenced CPI

(Annual rates)

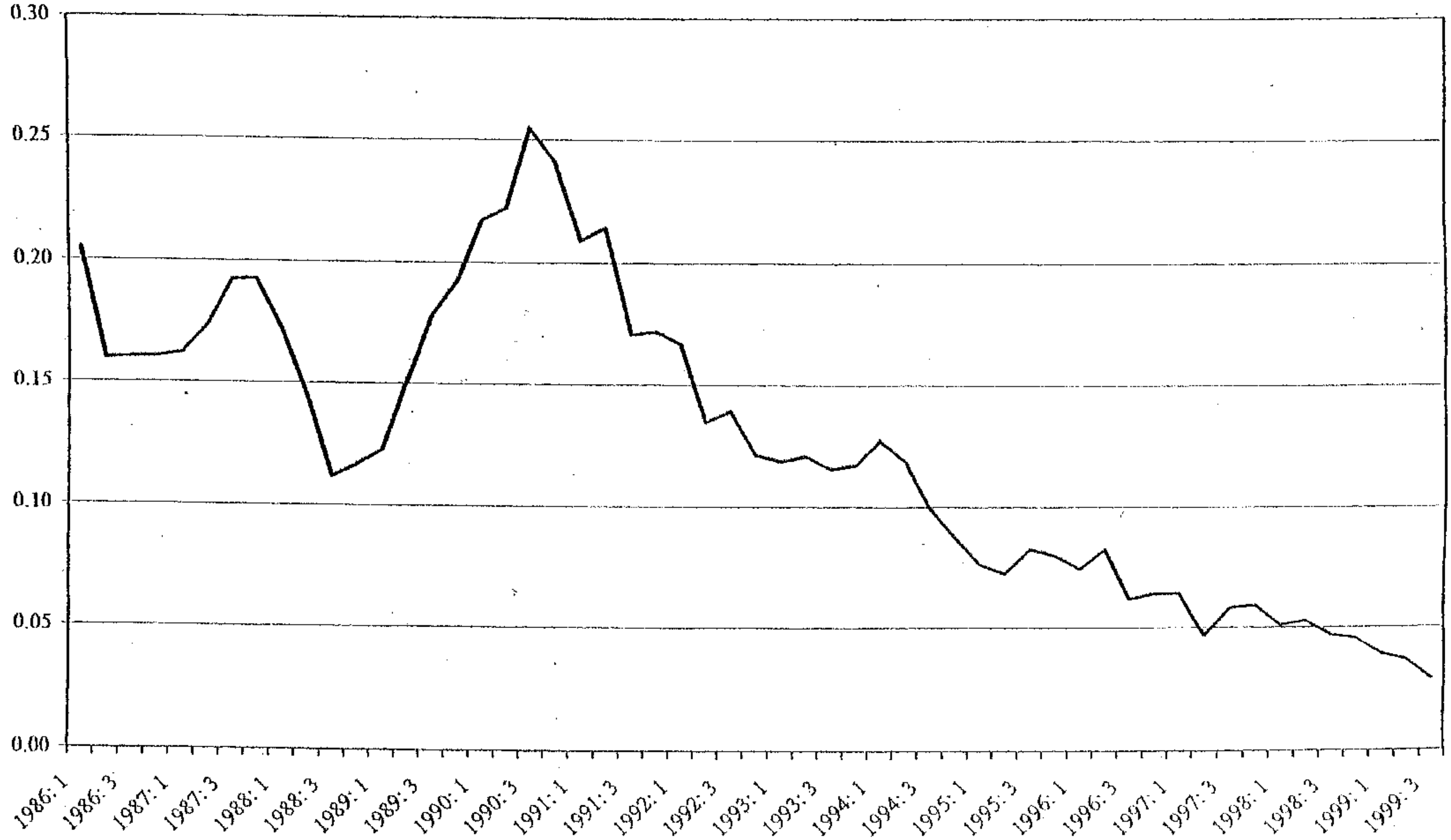


Figure 2. Chile Real GDP (-) and Its Trend Component (---)

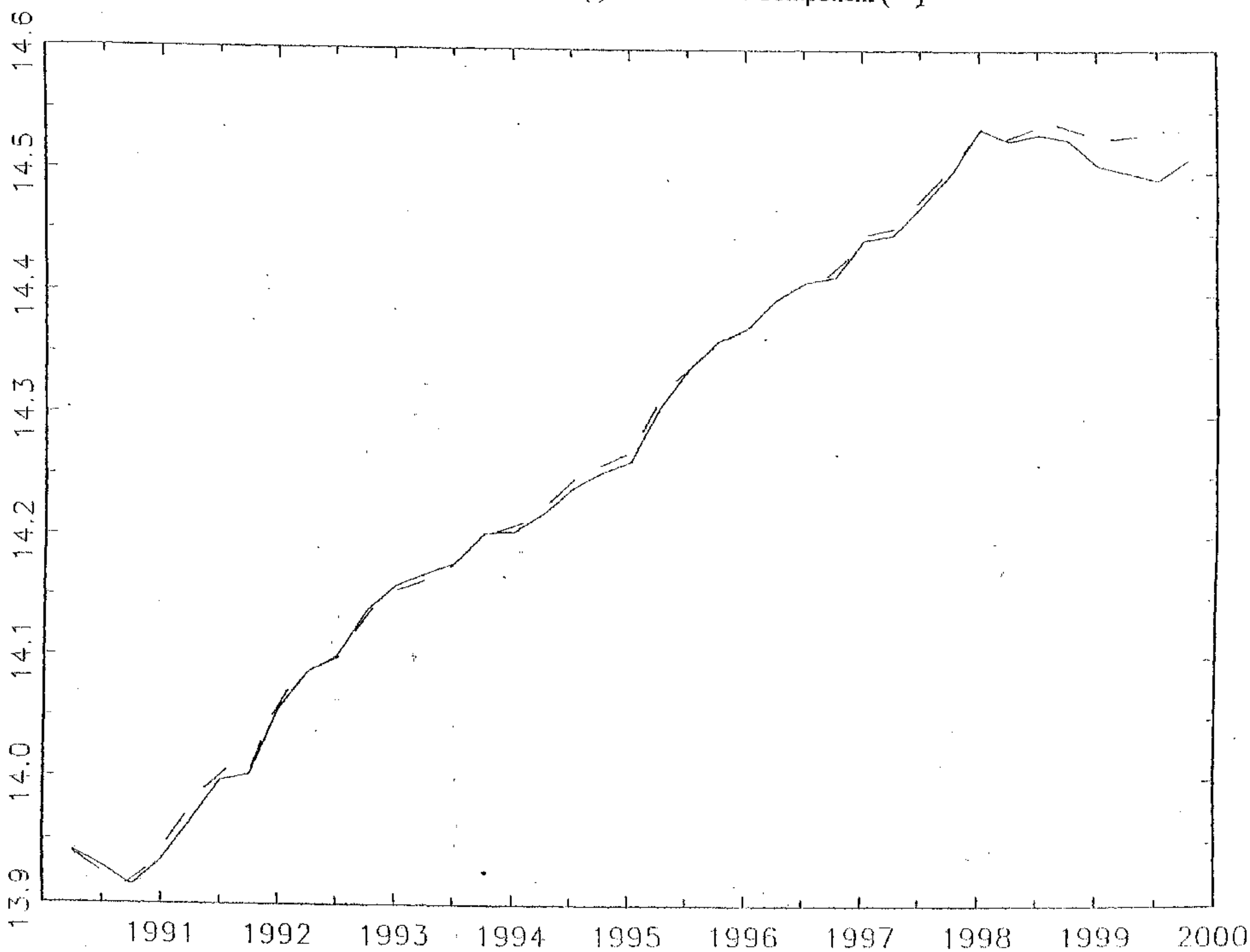


Figure 3. Chile: Cyclical Component of Real GDP

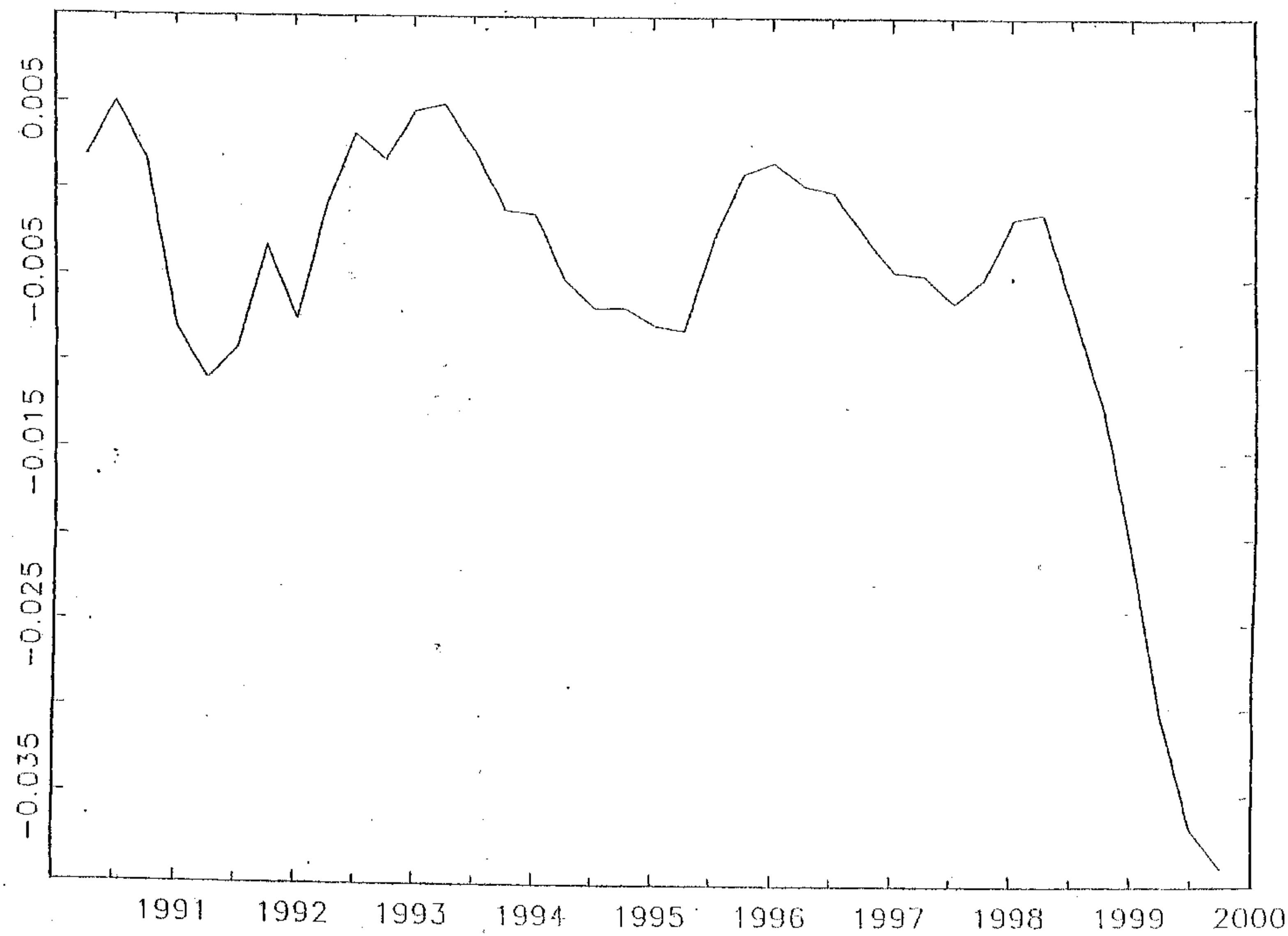


Figure 4. Chile: Productivity Growth Component of Real GDP

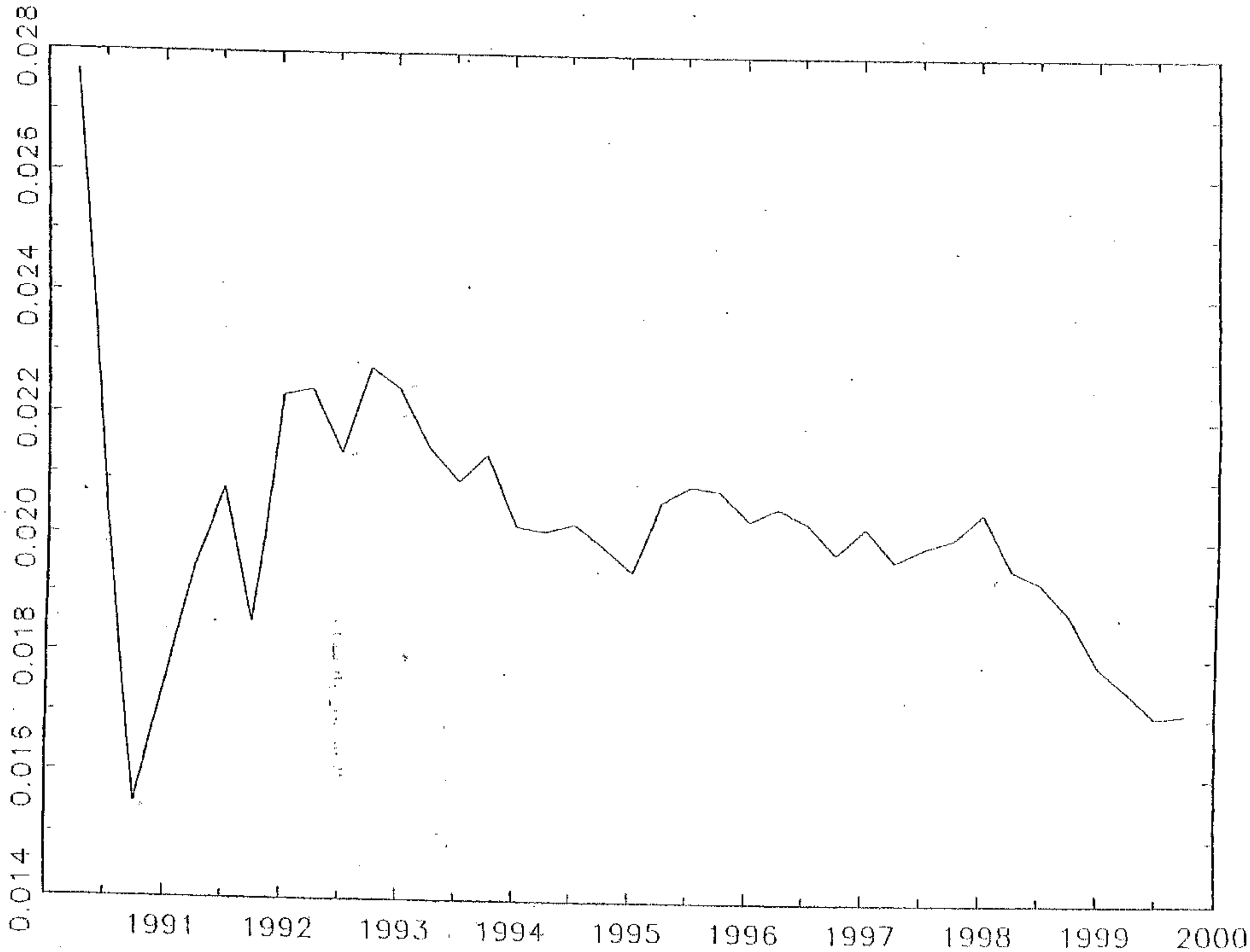


Figure 5. Expected Inflation, Inflation, and Official Inflation Target

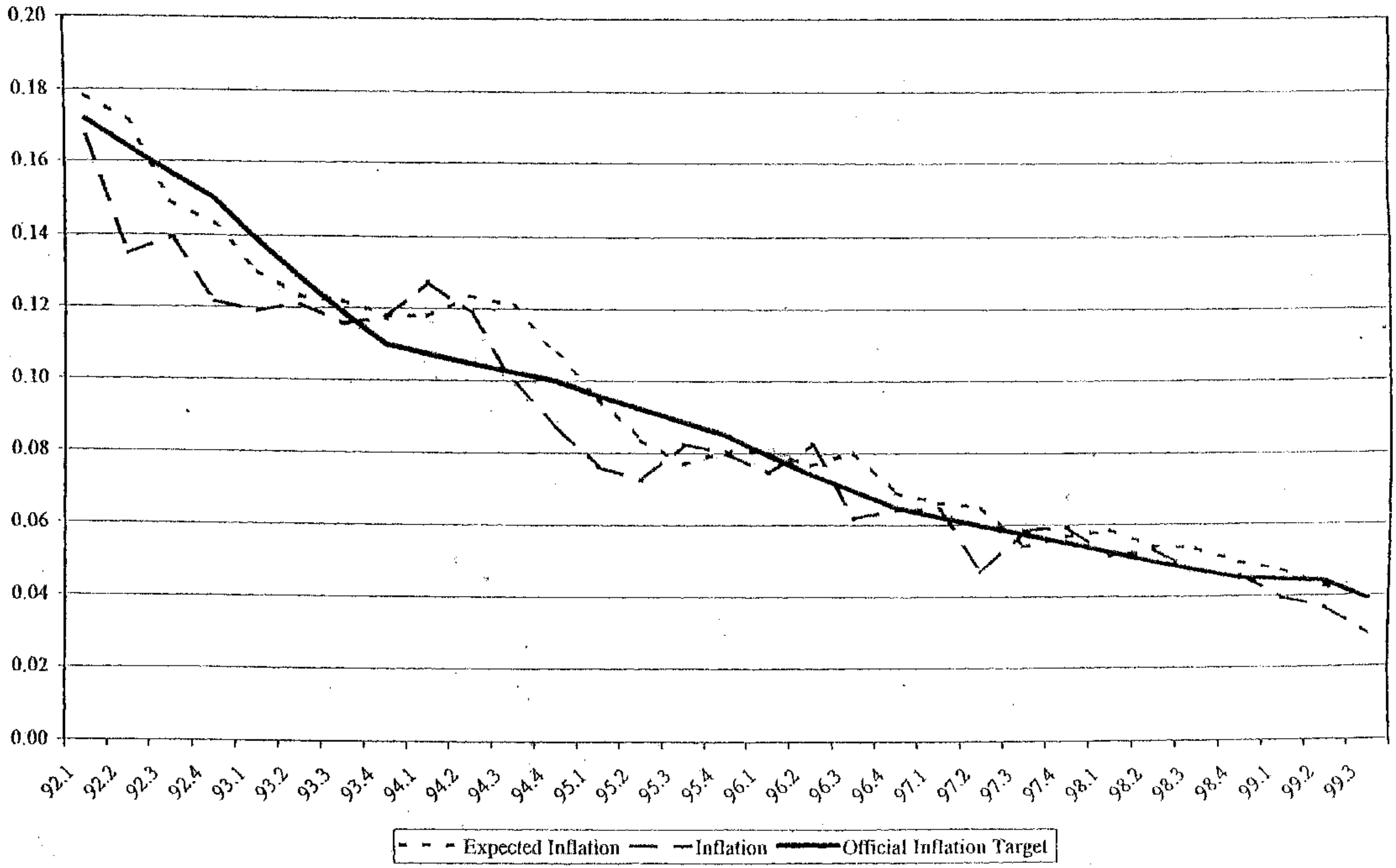


Figure 6. Expected and Actual Deviations of Inflation from the Official Inflation Target

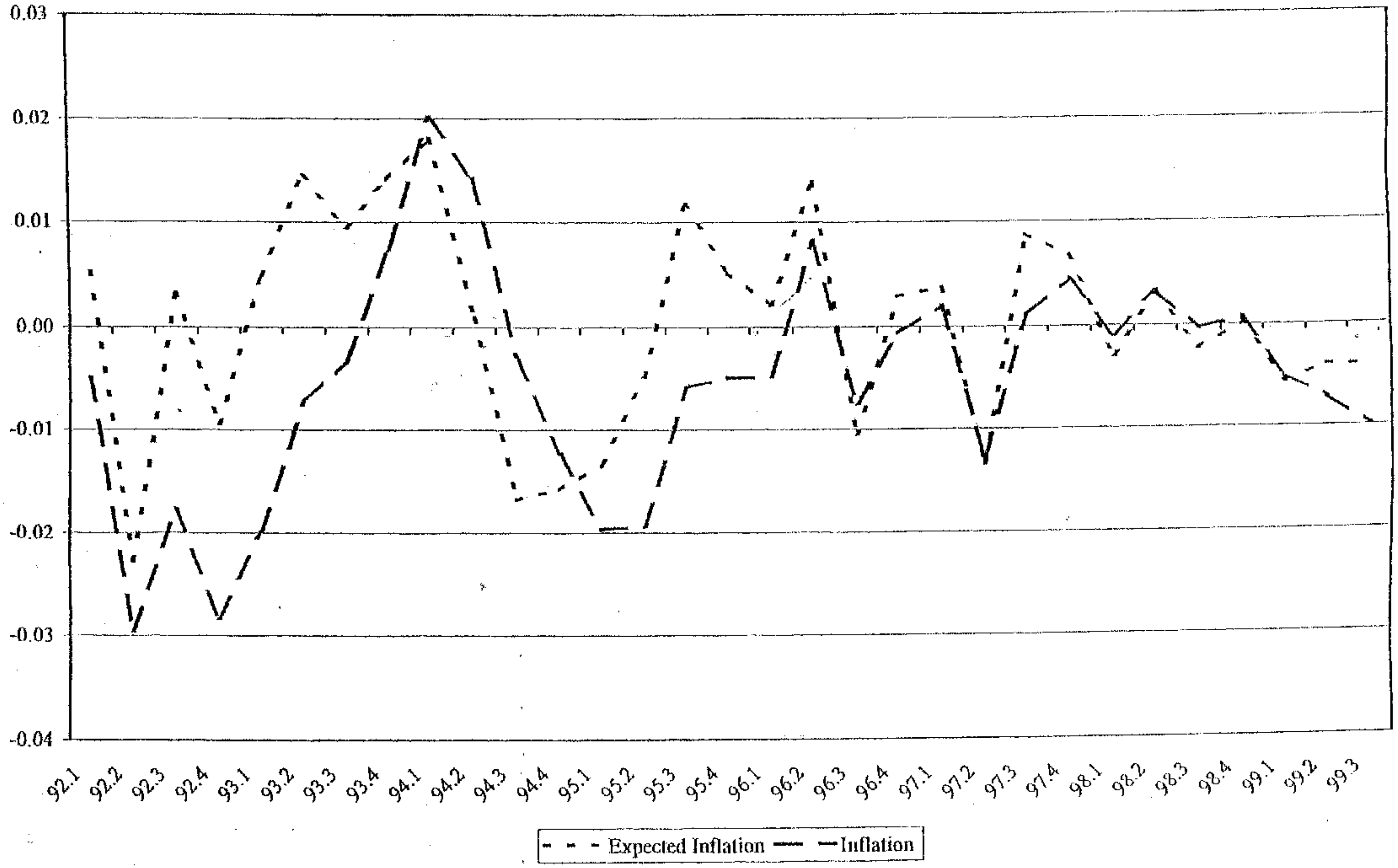


Figure 7. Chile: Smoothed Contemporaneous Output Gap Coefficient

(Model 1, Version 2)

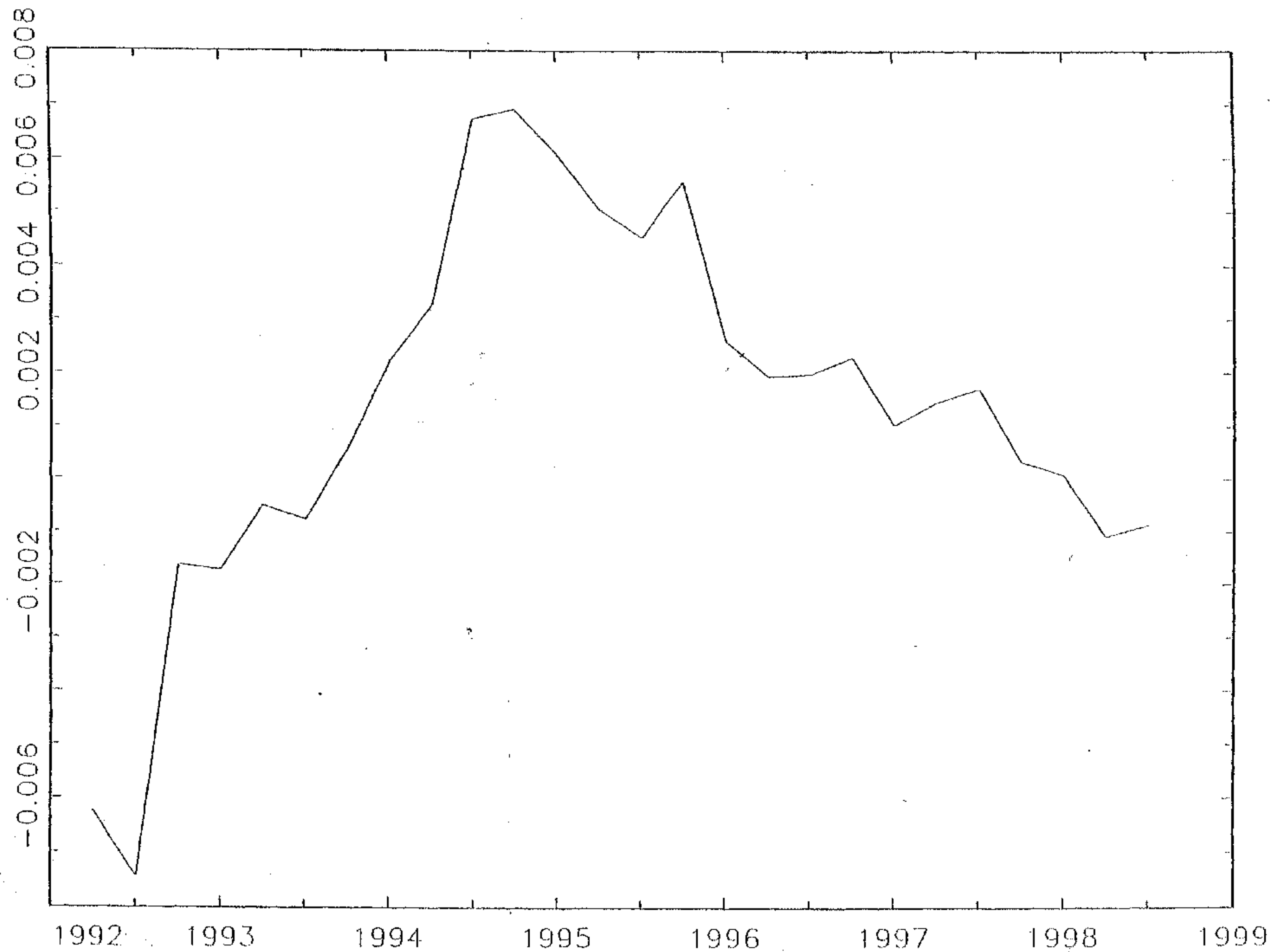


Figure 8. Chile: Smoothed Lagged Output Gap Coefficient

(Model 1, Version 2)

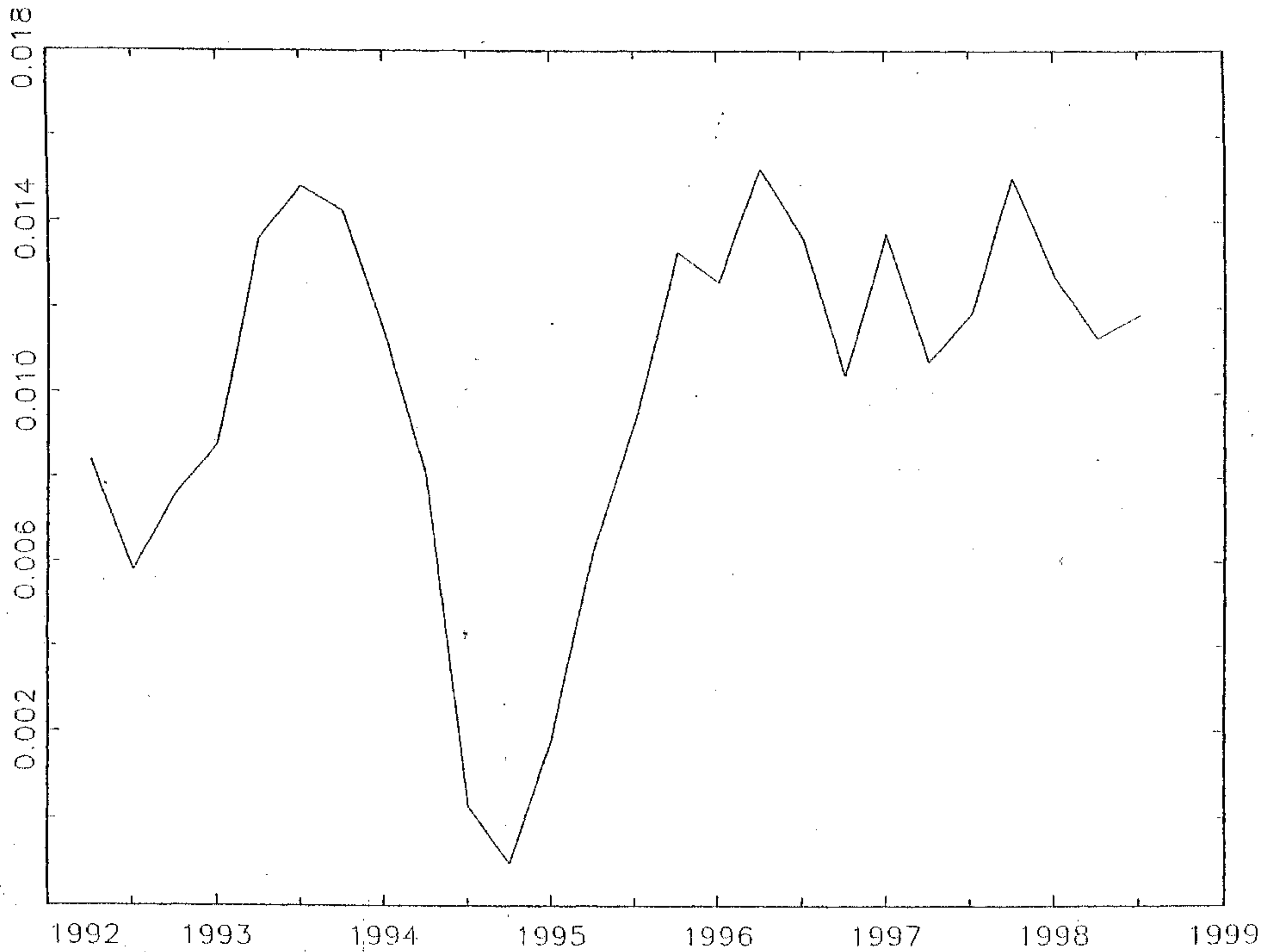


Figure 9. Chile: Smoothed Domestic Productivity Growth Coefficient

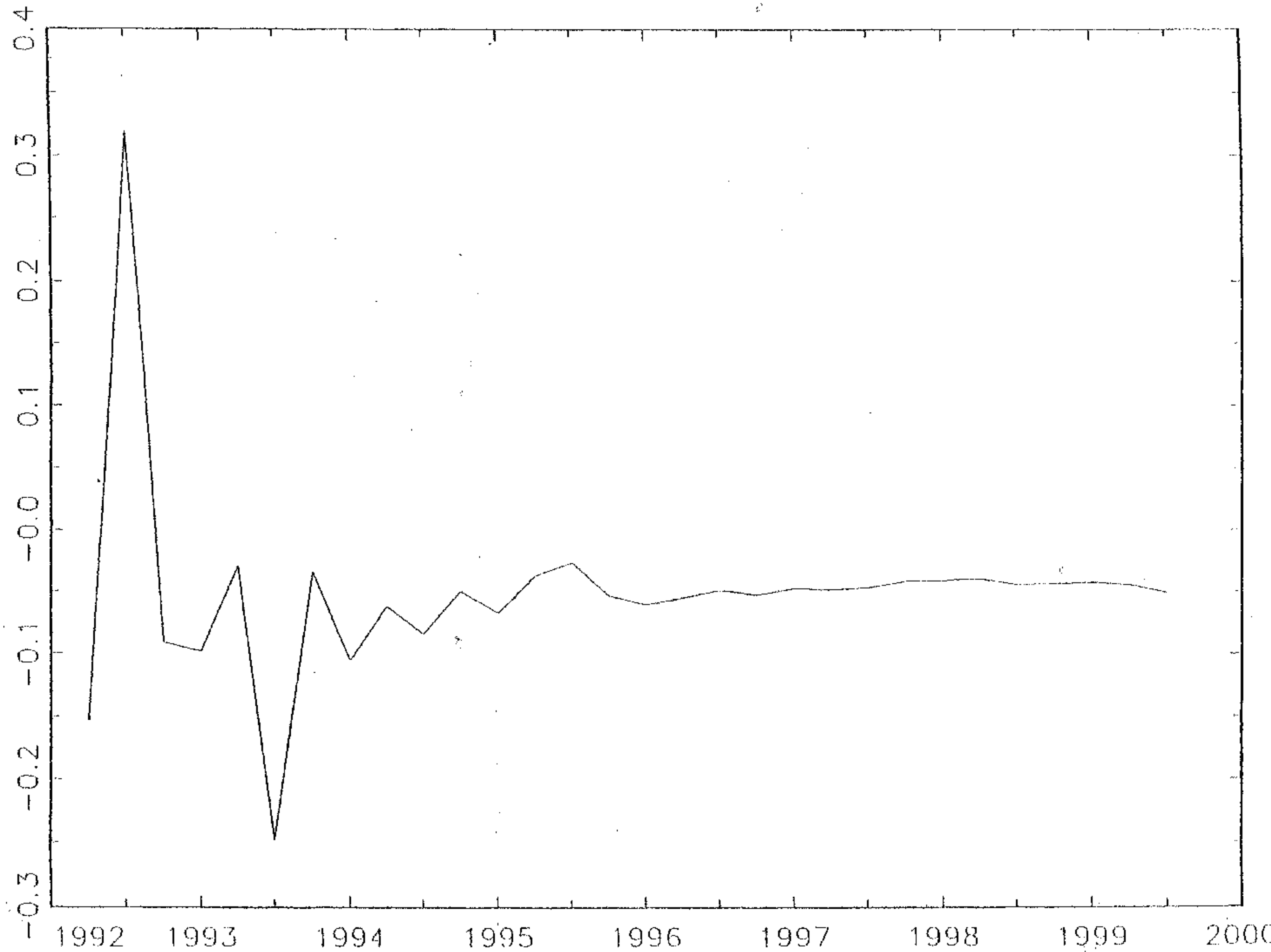


Figure 10. Chile: Smoothed Cost of Foreign Capital Coefficient

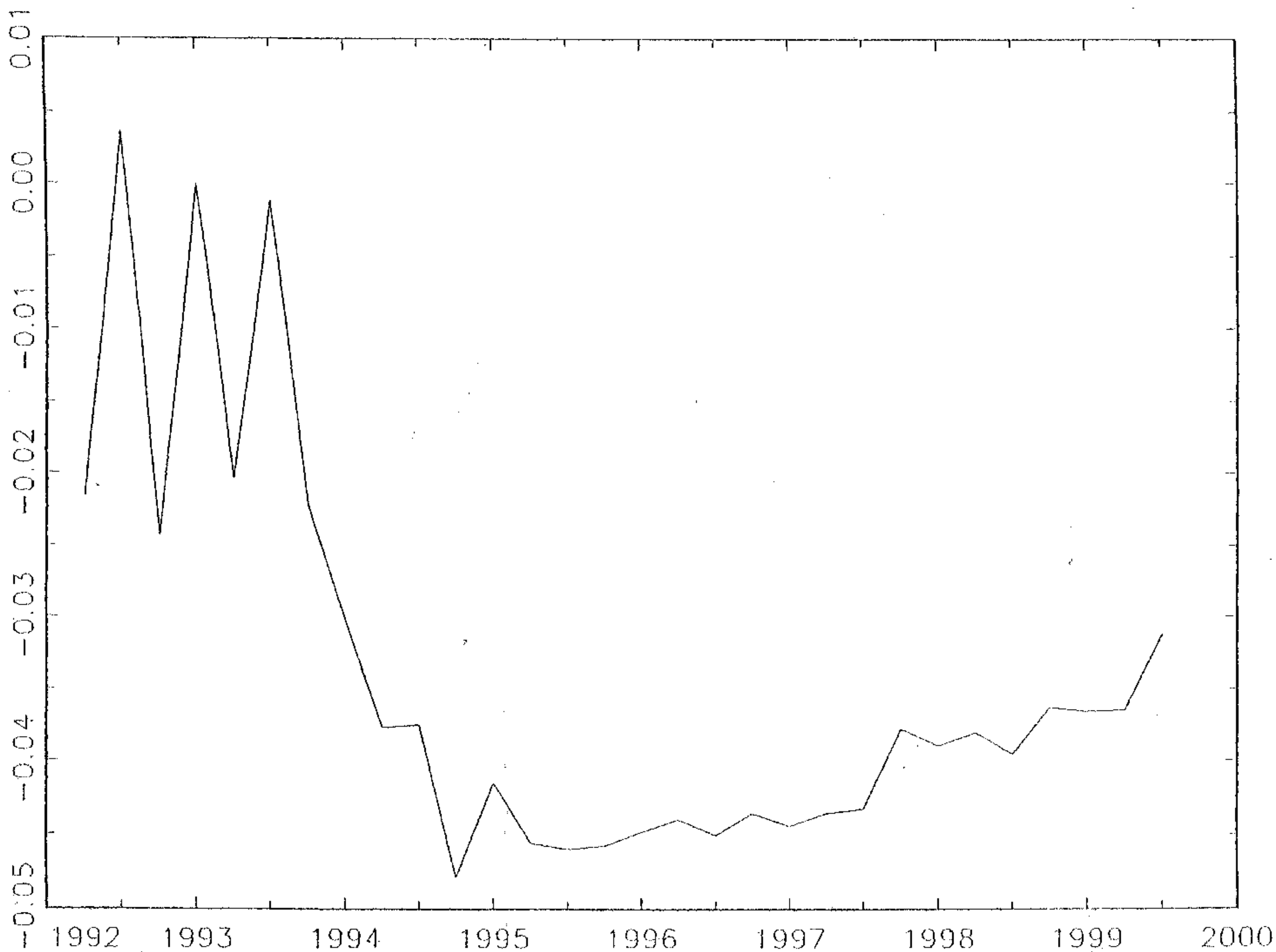


Figure 11. Chile: Smoothed Terms of Trade Coefficients

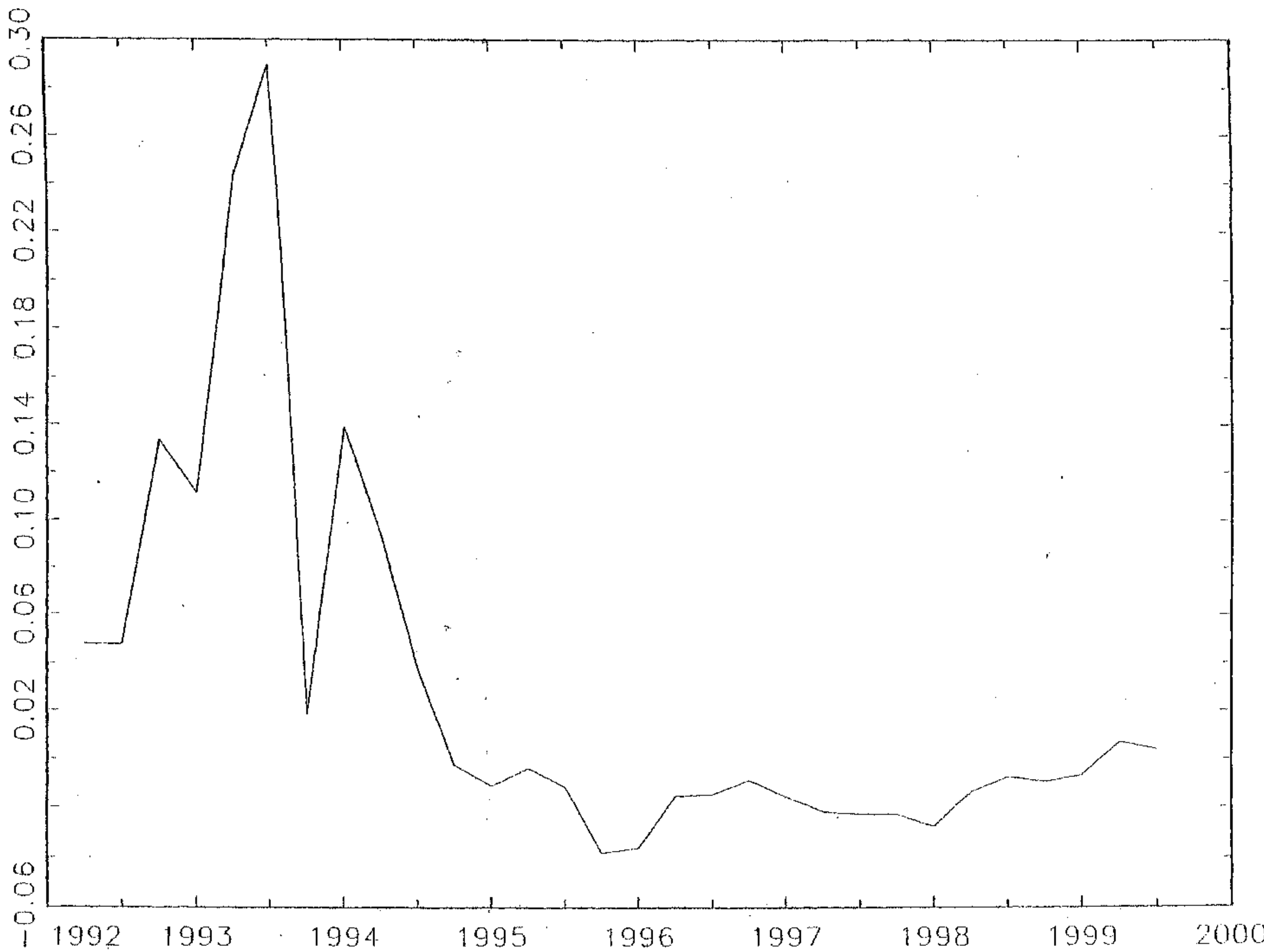


Figure 12. Chile: Smoothed Fiscal Impulse Coefficient

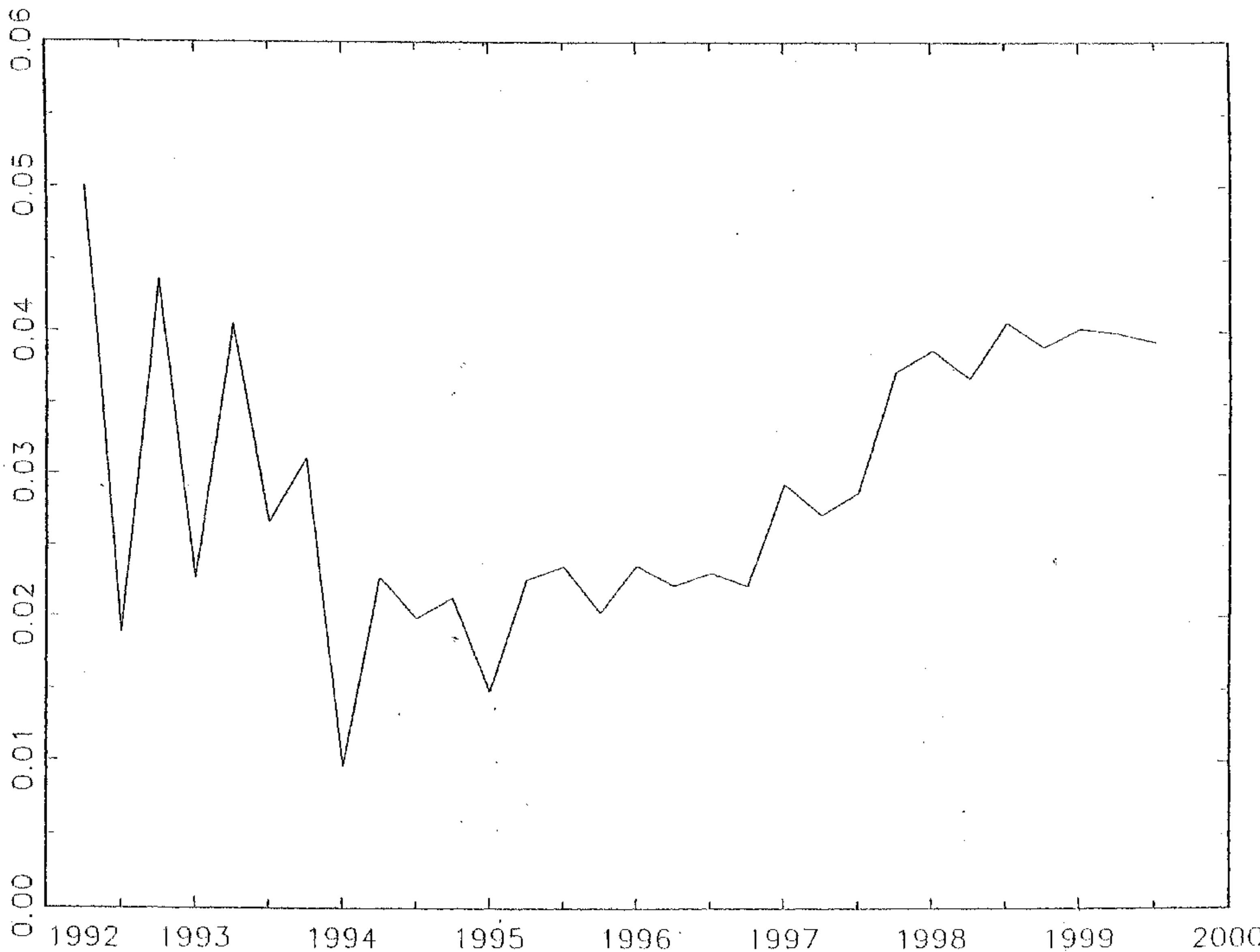


Figure 13. Chile: Smoothed Exchange Rate Coefficient

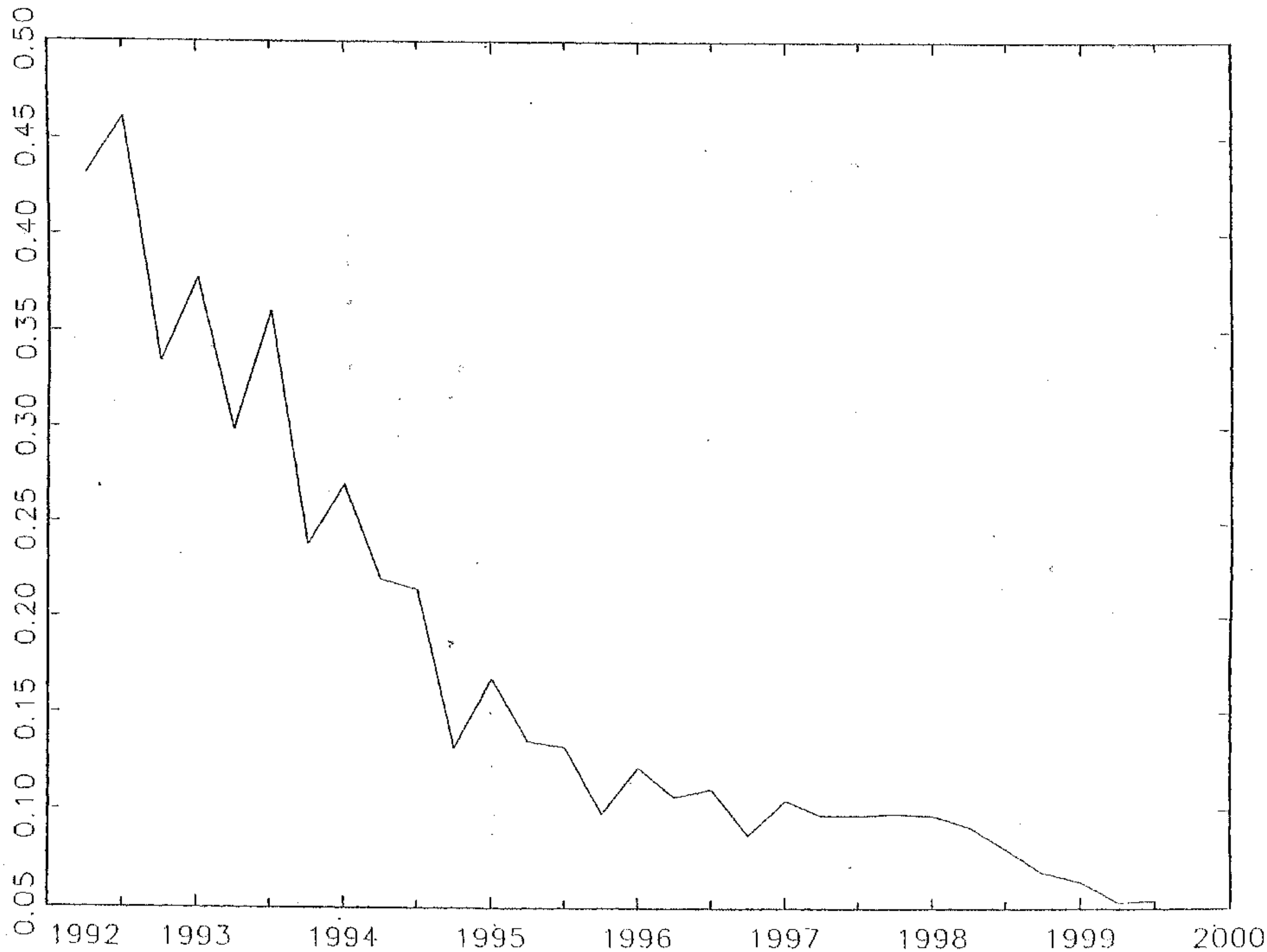


Figure 14. Chile: Smoothed Inflation Target Coefficient

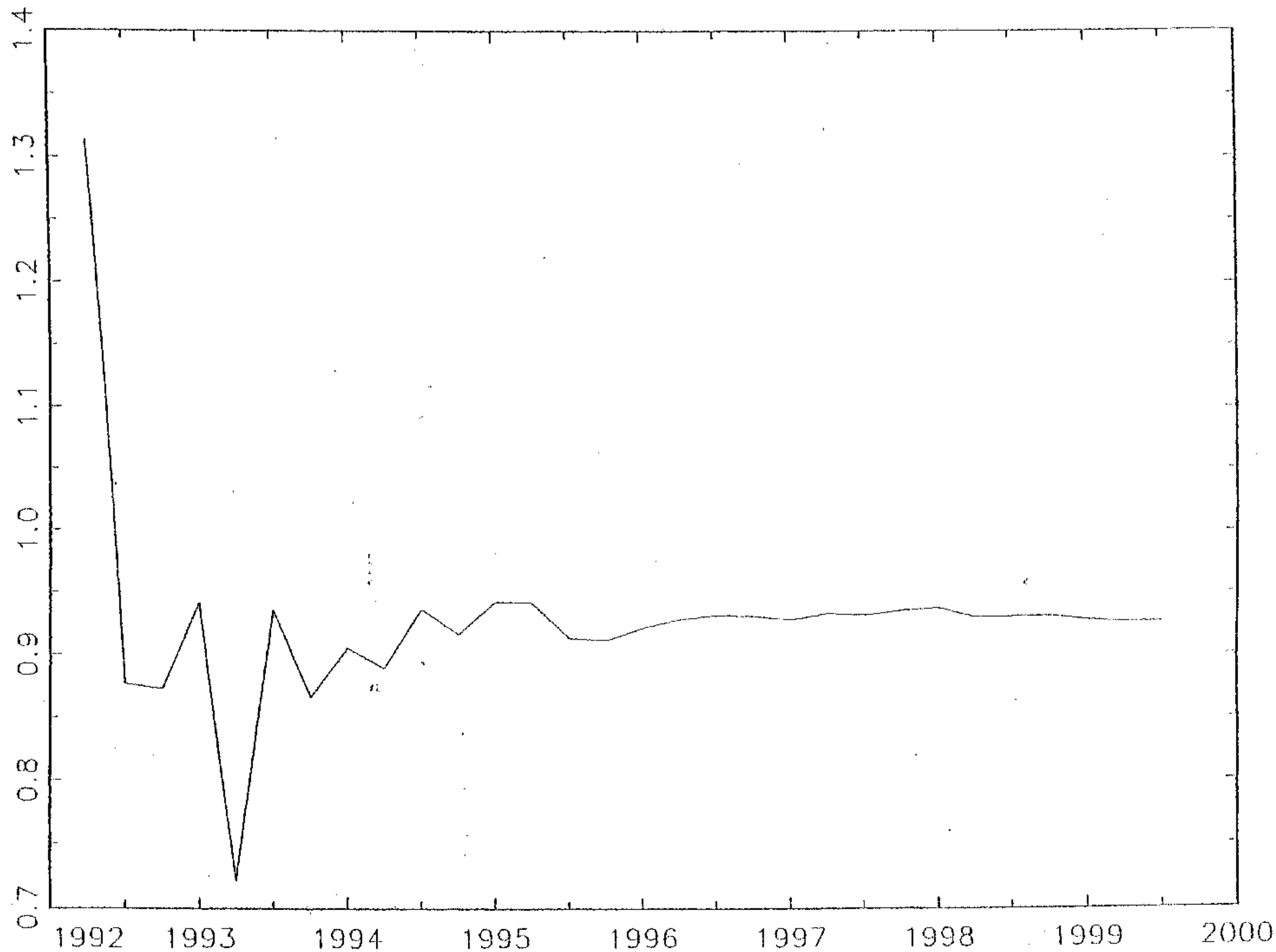


Figure 15. Time-Varying Phillips Curve Model: Decomposition of Inflation Uncertainty
(Time-Varying Parameter (...) and Heteroskeodasticity (---))

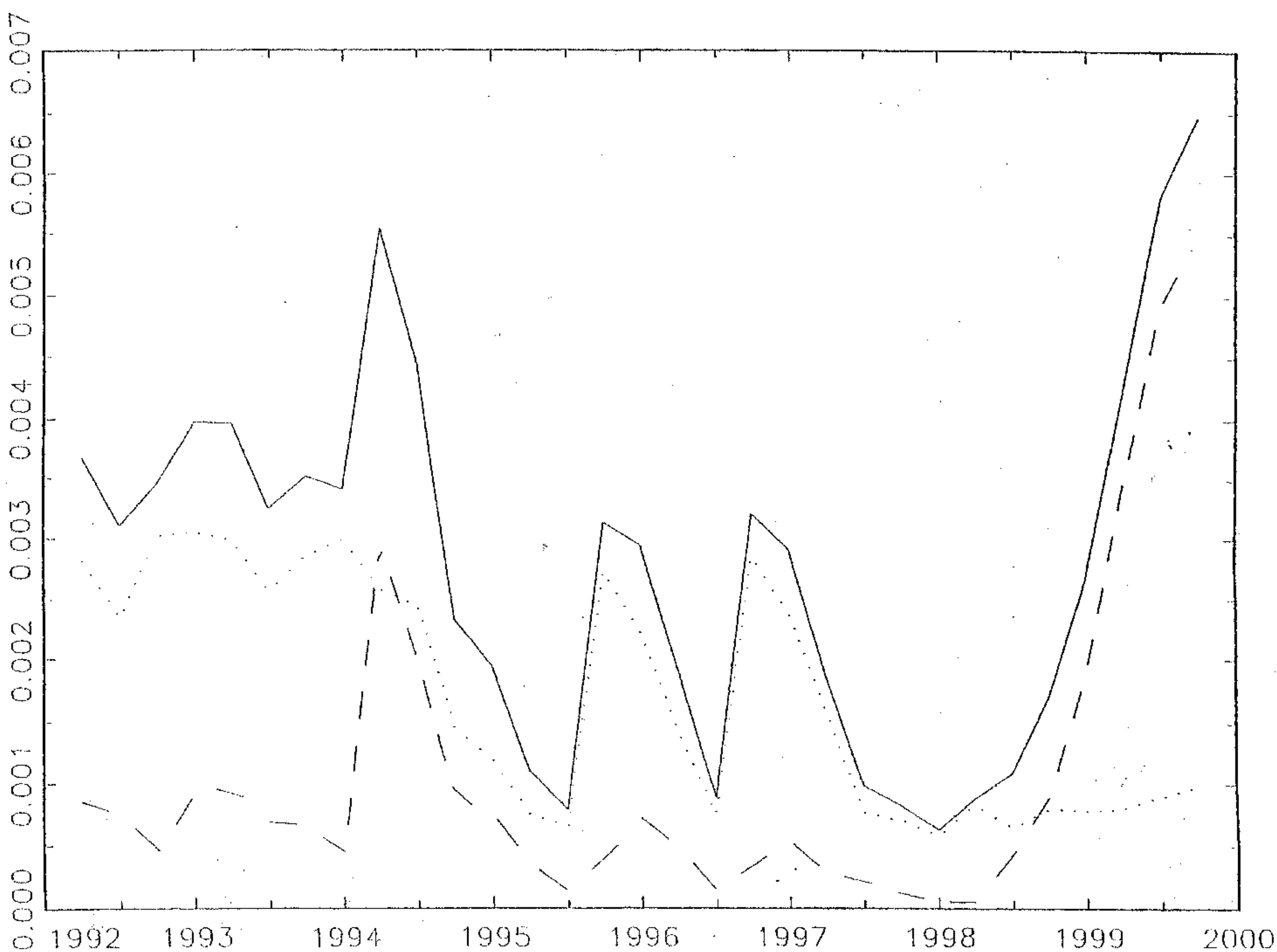
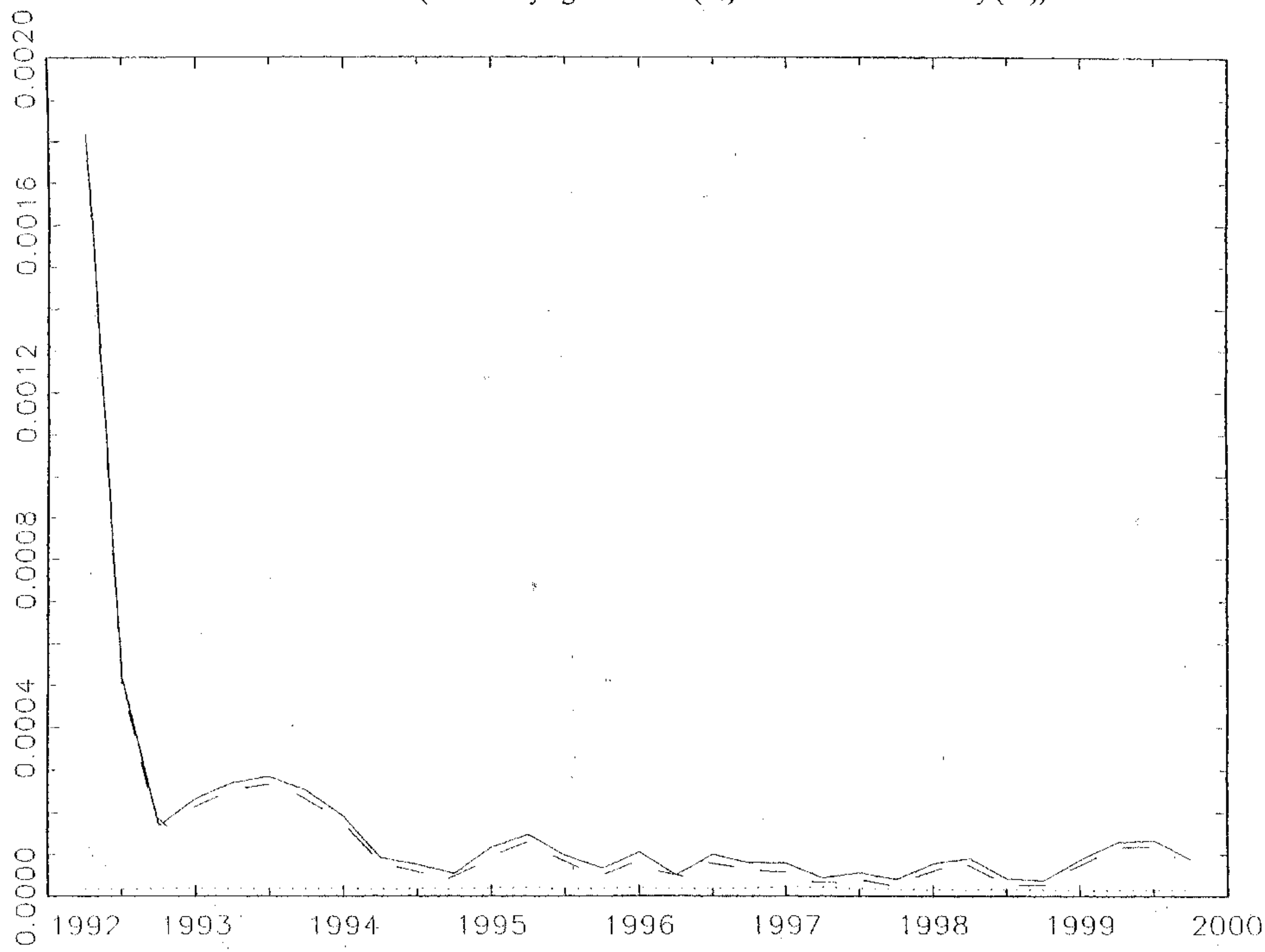


Figure 16. Small Open Economy Model: Decomposition of Inflation Uncertainty
(Time-Varying Parameter (...) and Heteroskeodasticity (---))



State-Space Models

A state space model consists of two equations: a measurement (or output) equation and a state (or transition) equation. The measurement equation relates the set of observed variables to the set of unobserved state variables. The state equation describes the dynamics of the state variables.

The following five equations describe what can be called a representative state-space model:

$$y_t = H_t \beta_t + A z_t + e_t , \tag{A1.1}$$

$$\beta_t = \mu + F \beta_{t-1} + v_t , \tag{A1.2}$$

$$e_t \approx \mathrm{N}(0, R), \tag{A1.3}$$

$$v_t \approx \mathrm{N}(0, Q), \tag{A1.4}$$

$$\mathrm{E}(e_t, v_s') = 0 , \tag{A1.5}$$

where y_t is a n x 1 vector of observed variables at time t; β_t is a k x 1 vector of unobserved state variables; H_t is an n x k matrix that links the observed y_t vector and the unobserved β_t; A is an n x r matrix of parameters; z_t is an r x 1 vector of exogenous or predetermined observed variables; μ and v_t are k x 1 vectors of constants and white noise processes, respectively; and F is an k x k matrix of parameters. Equations (A1.1) and (A1.2) are the measurement and the state equations, respectively. Equations (A1.3)–(A1.5) state that the sequences e_t and v_t follow normal processes with zero means and variances R and Q, respectively, and are uncorrelated. Note that elements of the matrix H_t can be either a set of constant parameters or data on exogenous variables. In the former case, equation (A1.1) is part of an unobserved-component model; in the latter case, equation (A1.1) is a time-varying parameter model.

State-space models have many applications. They have been used to estimate unobserved variables such as expected inflation (Burmeister *et al*, 1986), the ex ante real interest rate (García and Perron, 1996), and the common factor of major macroeconomic variables which we call business cycle (Stock and Watson, 1991). Alternatively, state-space models have been applied to the estimation of time-varying parameter models (Kim and Nelson, 1989) which give us insights as to how rational economic agents update their estimates of the coefficients of the model in a Bayesian manner when new information becomes available in a world of uncertainty, especially under changing policy regimes. Finally, state-space models have been combined with Markov-switching models of the business cycle to take into account not only that economic agents learn about the state of the economy and the coefficients of the model over time, but also to take into account that economic processes may not be symmetric, for example, over the business cycle, or that there is also uncertainty due to the presence of heteroskedasticity in random shocks (Diebold and Rudebusch, 1996).[22] In this case, changes in the conditional variance of the forecast error are viewed, in part, as a result of endogenous regime changes in the variance structure.

Equation (A1.3) of the representative state-space model (A1.1)–(A1.5) can be modified to incorporate changing uncertainty due to changes in the variance of future random shocks. By considering Markov-switching heteroskedasticity in the disturbance term e_t, it is assumed that part of the changes in the conditional variance of the forecast error result from endogenous regime changes in the variance structure.[23] A state-space model with Markov switching can be represented by substituting equation (A1.3') in the model (A1.1)–(A1.5) by the following:

$$e_t \approx \mathrm{N}\left(0, \sigma^2_{e,S_t}\right), \tag{A1.3'}$$

such that

$$\sigma^2_{e,S_t} = \sigma^2_0 + \left(\sigma^2_1 - \sigma^2_0\right)S_t, \qquad \sigma^2_1 > \sigma^2_0, \tag{A1.6}$$

$$\Pr[S_t = 1 \mid S_{t-1} = 1] = p_{11}, \tag{A1.7a}$$

$$\Pr[S_t = 0 \mid S_{t-1} = 0] = p_{00}. \tag{A1.7b}$$

The variance of the error term e_t is a discrete variable S_t which evolution depends on S_{t-1} only, i.e., S_t follows an order 1 Markov process. The process is a two-state process with transition probabilities described by equations (A1.7a) and (A1.7b).

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[22] Until recently, it was not possible to address the state-space estimation problem at the same time than the regime-switching estimation problem. However, the algorithm for approximate maximum likelihood estimation developed by Kim (1993a, 1993b, 1994) has made operational a broad class of state-space models with regime switching.

[23] A major difference between ARCH and Markov-switching heteroskedasticity is that whereas the unconditional variance is constant for the former, the unconditional variance is subject to shifts (structural changes) for the latter.

The Output Gap Model

Following Clark (1987), the output gap is estimated by modeling output as the sum of two independent unobserved components, a local linear trend and an autoregressive process of order two. Assume

$$y_t = T_t + X_t, \tag{A2.1}$$

$$T_t = T_{t-1} + g_t + h_t, \tag{A2.2}$$

$$g_t = g_{t-1} + w_t, \tag{A2.3}$$

$$X_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + l_t, \tag{A2.4}$$

$$h_t \approx \mathrm{N}\left(0, \sigma_h^2\right), \tag{A2.5}$$

$$w_t \approx \mathrm{N}\left(0, \sigma_w^2\right), \tag{A2.6}$$

$$l_t \approx \mathrm{N}\left(0, \sigma_l^2\right), \tag{A2.7}$$

where y_t is real GDP, and T_t and X_t are its stochastic trend and its cyclical components, respectively. The stochastic trend is modeled as a local linear trend with a drift term g_t included to account for changes in Chilean "productivity growth" over the sample period.[24] The local linear trend model is interesting because not only the trend T_t is described as a random walk process but also *changes in the trend* are assumed to follow a random walk plus noise process. The cyclical component of output is modeled as a stationary autoregressive process of second order.[25] The variables h_t, w_t, and l_t are independent zero-mean, constant variance processes as described in equations (A2.5)–(A2.7).

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[24] This measure of "productivity growth" includes changes in factor endowments.

[25] Lucas' model has an AR(1) for the cyclical component of output. In contrast, in this study, the data generating process of the cyclical component of output is described as an AR(2). As in Lucas, the stationarity assumption is imposed.

A Model of a Small Open Economy with Inflation Targeting

This appendix briefly discusses a linear rational expectations model of a small open economy with nominal rigidity in the goods market. The main feature of the model is that it is able to replicate under *monetary targeting* the three basic empirical regularities of the post-1971 floating exchange rate period. Those regularities are: (1) exchange rates respond more promptly to shocks than do national price levels; (2) the pure random component of exchange rate fluctuations is greater than the pure random component of national price level fluctuations and; (3) relative prices of foreign goods exhibit a great deal of persistence. The model is adapted to an *inflation targeting regime* by modeling the policy rule of the monetary authority accordingly.

Assume a small open economy that produces two goods, some of which are exported. The economy also imports and consumes foreign goods. The price of domestic goods (p_t) is determined mostly by domestic forces while the price of foreign goods (p_t^*) is determined in world markets. With all variables except interest rates in logs, the model is

$$y_t^d = a_0 - a_1 r_t + a_2 q_t + a_3 y_t^* + a_4 d_t + \upsilon_t \tag{A3.1}$$
$$r_t = r_t^* + \alpha E_t (q_{t+1} - q_t) \tag{A3.2}$$
$$q_t = e_t + p_t^* - p_t \tag{A3.3}$$
$$y_t^s = T_t + \xi (p_t - E_{t-1} \overline{p}_t) + u_t \tag{A3.4}$$
$$P_t = \alpha p_t + (1 - \alpha)(f_t + p_t^*) \tag{A3.5}$$
$$m_t - P_t = b_0 + b_1 y_t - b_2 i_t + \chi_t \tag{A3.6}$$
$$i_t = i_t^* + E_t (f_{t+1} - f_t) + \rho_t \tag{A3.7}$$
$$i_t^* = r_t^* + E_t (p_{t+1}^* - p_t^*); \tag{A3.8}$$

where y_t^d is the demand for domestic output, r_t is the domestic real interest rate, q_t is the real exchange rate, y_t^* is foreign output, d_t is a measure of fiscal impulse, υ_t is an output demand disturbance, r_t^* is the foreign real interest rate, f_t is the nominal exchange rate, p_t^* is the price of foreign output, p_t is the price of domestic goods, y_t^s is the supply of domestic output, T_t is potential output, u_t is an output supply shock, P_t is the general price level as measured by the consumer price index (CPI), α is the share of domestic goods in the CPI regimen, m_t is a broad measure of the money stock, i_t is the nominal interest rate, i_t^* is the foreign nominal interest rate, ρ_t is a measure of risk premium, and χ_t is a money demand disturbance.

Equation (A3.1) is a standard IS curve for a small open economy.[26] Equation (A3.2) posits real interest parity, consistent with the empirical evidence (Obstfeld and Rogoff, 1996).[27] The

[26] McCallum and Nelson (1997) show that the standard IS equation requires the addition of expected future income to be consistent with a fully optimizing model Given the econometric procedure used to estimate and forecast inflation in this paper, the use of the standard IS equation is innocuous.

term α is needed in equation (A3.2) so that the rates of return on domestic and foreign assets are measured in the same units. The rate of return on domestic assets, or the domestic real interest rate r_t, is measured in terms of a basket of goods that includes both foreign and domestic goods. In contrast, the foreign interest rate r_t* is measured using a basket of goods that comprises only foreign goods.[28] Equations (A3.1) and (A3.2) together imply that the current account—not explicitly modelled—is a function of q_t and y_t^*, making the current account balance consistent with equilibrium in **both** the domestic goods market and the asset market. Equation (A3.3) defines the real exchange rate. Aggregate supply behavior is represented by equation (A3.4), which embodies the "natural rate" hypothesis (see also equation A3.9 below). Equation (A3.5) defines the overall price level in terms of the prices of domestic and foreign goods. Equation (A3.6) describes the demand for money. Equation (A3.7) is uncovered interest parity and equation (A3.8) is the Fisher effect. Expectations are rational, and the information set dated at time (t) is common to market participants and to the central bank. Potential output (T_t) follows a random walk with a drift, or productivity growth term, as described in Appendix II. Similarly, foreign potential output follows a random walk with a drift, or productivity growth term. The stochastic processes assumed for domestic and foreign output allow the model to replicate the third regularity observed in the exchange rate data during the floating period.

Following McCallum (1989), "price stickiness" is introduced simply by assuming that domestic good prices are set at the value ($\overline{p}_t$) that is expected in period (t-1) to clear the market

$$p_t = E_{t-1}\,\overline{p}_t. \qquad \text{(A3.9)}$$

Therefore, p_t is the price that would prevail in the goods market if there were no unexpected shocks. The idea of (A3.9) is that market participants find it optimal to preset prices at levels that are expected to clear the market next period.[29] However, unexpected demand and supply shocks will make output realizations to be different from expected values. As a result, current period output is demand-determined, given the price preset for the current period by producers based on their information at the end of previous period.[30] This formulation allows temporary

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[27] Frankel (1991) found that during the floating exchange rate period—after the liberalization of capital account transactions—real interest rate differentials have been highly correlated with changes in the real exchange rate, or "currency risk" in his terminology.

[28] Technically, this can be seen by taking expectations in equation (A3.5), and using the expression to replace the term $E_t(P_{t+1} - P_t)$ in a Fisher effect equation such as (A3.7), but for the home country.

[29] This assumption could presumably be justified by the existence of menu costs or, more generally, by the costs of gathering and processing information (Brunner *et al.*, 1983).

[30] This implies that the analytical solution of the model will depend on last period state variables. It is consistent with McCallum and Nelson (1999) who argue that the monetary

(continued...)

rigidity of domestic good prices while permitting full adjustment of prices in later periods. As a result, the model will reflect the first two regularities found in the data during the floating period.

Equations (A3.1)–(A3.9) form the *core model*. Nadal-De Simone (1999) discusses the steady state and the stochastic solution of the core model under two monetary regimes. The first regime is monetary targeting, and assumes a Friedman's rule for the stochastic process followed by the money stock m_t. The second regime is inflation targeting.[31] Importantly, under monetary targeting, simulations show that the model does replicate the regularities observed in the behavior of exchange rates during the post-1971 floating period.

When applied to an inflation targeting economy, the core model requires the specification of the feedback function of the monetary authority. The form of this policy reaction function chosen is a standard representation suggested by McCallum (1999), but adapted to reflect the fact that in Chile the operating instrument of the CBCH is a real interest rate

$$r_t = r_{t-1} - \lambda [E_t (P_{t+1} - P_t) - c_t]. \qquad \text{(A3.11)}$$

The second term of (A3.11) measures expected deviations of the inflation rate from the target (c_t).[32] In particular, the real interest rate is adjusted upward when expected inflation exceeds the inflation target (c_t), with the reaction coefficient λ measuring the strength of the adjustment. The second term of (A3.11) corresponds to the log of the inflation forecast targeting loss function of the central bank (equation (2.10) in Svensson, 1997). Inflation targeting implies inflation *forecast* targeting.

Finally, equation (A3.6) is superfluous to the extent that m_t is not part of the right-hand side of any equation in the system. Equation (A3.6) has been kept, however, because it determines the behavior of the money stock that is necessary to support the policy reaction function (A3.11).

Drawing on Nadal-De Simone (1999), changes in domestic prices are a function of the following state variables

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authority and/or agents normally do not know the realizations of contemporaneous variables when they make their decisions. This is particularly significant in the case of potential output.

[31] The core model in Nadal-De Simone (1999) includes the fiscal impulse measure in the random shock υ_t, and makes the risk premium ρ_t constant.

[32] For analytical simplicity, only one period ahead forecast is considered. In fact, the monetary authority considers expected deviations from the inflation target over a number of periods, say for example, one to three years.

$$\pi_t = F(g_t, g_t^*, r_t^*, p_t^*, d_t, f_t, c_t) + \tau_t, \tag{A3.12}$$

where the IS shock (υ_t), the output supply shock (u_t), and the money demand shock (χ_t) are included in the error term (τ_t); the risk premium ρ_t is measured as part of the overall cost of financing faced by the Chilean economy (r_t^*). The pre-announced official inflation target (c_t) is added to the core model; it is assumed to be an exogenous stationary process.

The Kalman Filter and Estimation of β_t

Referring to the state-space representation of the model of Appendix I, the Kalman filter can be defined as a recursive procedure for computing the optimal estimate of the unobserved-state vector $\beta_t, t = 1,2,...,T,$ based on the appropriate information set, assuming that $\tilde{\mu}, F, R,$ and Q are known. It provides a minimum mean squared error estimate of β_t given the appropriate information set. Depending upon the information set used, the basic filter or smoothing are obtained. The basic filter refers to an estimate of β_t based on information available up to time *t*, and smoothing to an estimate of β_t based on all the available information in the sample through time *T*.

The notation used is the following:

ψ	the information set.
$\beta_{t\backslash t-1} = E[\beta_t \vert \psi_{t-1}]$	expectation (estimate) of β_t conditional on information up to $t-1$.
$P_{t\backslash t-1} = E\left[(\beta_t - \beta_{t\backslash t-1})(\beta_t - \beta_{t\backslash t-1})'\right]$	covariance matrix of β_t conditional on information up to $t-1$.
$\beta_{t\backslash t} = E[\beta_t \vert \psi_t]$	expectation (estimate) of β_t conditional on information up to *t*.
$P_{t\backslash t} = E\left[(\beta_t - \beta_{t\backslash t})(\beta_t - \beta_{t\backslash t})'\right]$	covariance matrix of β_t conditional on information up to *t*.
$y_{t\backslash t-1} = E[y_t \vert \psi_{t-1}] = x_t \beta_{t\backslash t-1}$	forecast of y_t given information up to time $t-1$.
$\eta_{t\backslash t-1} = y_t - y_{t\backslash t-1}$	prediction error.
$f_{t\backslash t-1} = E[\eta_{t\backslash t-1}^2]$	conditional variance of the prediction error.
$\beta_{t\backslash T} = E[(\beta_t \vert \psi_T)]$	expectation (estimate) of β_t conditional on information up to *T* (the whole sample).
$P_{t\backslash T} = E\left[(\beta_t - \beta_{t\backslash T})(\beta_t - \beta_{t\backslash T})'\right]$	covariance matrix of β_t conditional on information up to *T* (the whole sample).

Assuming that x_t is available at the beginning of time *t* and a new observation of y_t is made at the end of time *t*, the Kalman filter (basic filter) consists of the following two steps:

1. *Prediction:* At the beginning of time *t*, an optimal predictor of y_t is formed based on all the available information up to time $t-1$: $y_{t|t-1}$. To do this, $\beta_{t|t-1}$. has to be calculated.

2. *Updating:* Once y_t is realized at the end of time *t*, the prediction error can be calculated: $\eta_{t|t-1} = y_t - y_{|t-1}$. This prediction error contains new information about β_t beyond that contained in $\beta_{t|t-1}$. Thus, after observing y_t, a more accurate inference can be made of $\beta_t \cdot \beta_{t|t}$, an inference of β_t based on information up to time *t*, may be of the following form: $\beta_{t|t} = \beta_{t|t-1} + K_t \eta_{t|t-1}$, where K_t is the weight assigned to new information about β_t contained in the prediction error.

The basic Kalman filter is described by the following equations:

Prediction

$$\beta_{t\backslash t-1} = \widetilde{\mu} + F\beta_{t-1\backslash t-1}, \tag{A4.1}$$

$$P_{t\backslash t-1} = FP_{t-1\backslash t-1}F' + Q, \tag{A4.2}$$

$$\eta_{t\backslash t-1} = y_t - y_{t\backslash t-1} = y_t - H_t\,\beta_{t\backslash t-1} - Az_t, \tag{A4.3}$$

$$f_{t\backslash t-1} = H_t P_{t\backslash t-1} H_t' + R, \tag{A4.4}$$

Updating

$$\beta_{t\backslash t} = \beta_{t\backslash t-1} + K_t \eta_{t\backslash t-1}, \tag{A4.5}$$

$$P_{t\backslash t} = P_{t\backslash t-1} - K_t H_t P_{t\backslash t-1}, \tag{A4.6}$$

where $K_t = P_{t\backslash t-1} H_t' f_{t\backslash t-1}^{-1}$ is the Kalman gain.

Smoothing $(t = T-1, T-2, \ldots, 1)$

$$\beta_{t\backslash T} = \beta_{t\backslash t} + P_{t\backslash t} F' P_{t+1\backslash t}^{-1}\left(\beta_{t+1\backslash T} - F\beta_{t\backslash t} - \widetilde{\mu}\right), \tag{A4.7}$$

$$P_{t\backslash T} = P_{t\backslash t} + P_{t\backslash t} F' P_{t+1|t}^{-1}\left(P_{t+1|T} - P_{t+1|t}\right) P_{t+1|t}^{-1\,\prime} F P_{t|t}', \tag{A4.8}$$

where $\beta_{T\backslash T}$ and $P_{T\backslash T}$, the initial values for the smoothing, are obtained from the last iteration of the basic filter.

The Kalman gain is an inverse function of *R*, the variance of the measurement equation (e_t) and, given x_t, it is a direct function of the uncertainty underlying $\beta_{t\backslash t-1}$. For example,as uncertainty associated with $\beta_{t\backslash t-1}$ falls, relatively less weight is given to new information in the prediction error $\eta_{t\backslash t-1}$.