

IMF Working Paper

A Dynamic General Equilibrium Framework of Investment with Financing Constraint

Danyang Xie and Chi-Wa Yuen

IMF Working Paper

IMF Institute

A Dynamic General Equilibrium Framework of Investment with Financing Constraint

Prepared by Danyang Xie and Chi-Wa Yuen¹

Authorized for distribution by Reza Vaez-Zadeh

February 2002

Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

In this paper, we provide a dynamic general equilibrium framework with an explicit investment-financing constraint. The constraint is intended as a reduced form to capture the balance sheet effects, which have been widely regarded as an important determinant of financial crises. We derive a link between the value of the firm and the social welfare and we find that the value of the firm can be greater with than without the constraint. Our model also sheds light on how the effects of productivity shocks and bubbles may be amplified by the financing constraint.

JEL Classification Numbers: C61; D92

Keywords: investment constraint; value of the firm

Authors' E-Mail Addresses: dxie@imf.org and cwyuen@hku.hk

¹ International Monetary Fund and University of Hong Kong, respectively. We thank the Hong Kong Research Grants Councils for financial support. We appreciate comments from Andrew Feltenstein, Rodney Ramcharan, and Reza Vaez-Zadeh.

	Page
I. Introduction	3
II. The Model	4
A. The Firm's Value Maximization Problem.....	4
B. The Consumer's Utility Maximization Problem.....	5
C. Equilibrium Firm Value and Consumer Utility.....	5
D. Policy Functions and Numerical Algorithms	6
E. Financing Constraint	7
III. A Numerical Example.....	9
IV. Discussion and Possible Extensions.....	10
Figures	
1. Investment Function and Financing Constraint	12
2. Consumption Paths.....	13
3. Capital Paths and Real Interest Rate Paths	14
4. Interest Rate Functions.....	15
5. Change in Productivity.....	16
6. Bubble and Crash	17
Appendix I.....	18
References	20

I. INTRODUCTION

At the very beginning of the Asian financial crisis (AFC), most people took it as yet another currency crisis and many viewed it to belong to the second generation (self-fulfilling) type *à la* Obstfeld (1996) rather than first generation (fundamental) type *a la* Krugman (1979). As the crisis unfolded, however, it became obvious that, unlike exchange rate crises, the AFC was more related to banking and financial problems in the process of financing business investment. Since then, quite a few theories (so-called ‘third generation’ models) have been proposed to understand its sources—moral hazard or guaranteed bailouts (Krugman 1998), financial fragility (Chang and Velasco 2000), and balance sheet effects (Krugman 1999).²

As Krugman (2001) concludes, balance sheet effects are now believed to be the most crucial element behind the AFC. In particular, if firms are highly leveraged with debt denominated in foreign currency, then anything that triggers a massive capital outflow will result in a depreciation of the domestic currency and thus an increase in the firms’ debt burden. As a consequence, net worth of the firms will be reduced, limiting its ability to borrow to finance its new investment. The resulting investment and output collapse will validate the capital flight and make the crisis self-fulfilling.

Despite its general acceptance by the profession as an important determinant of financial crises, the balance sheet effect has been studied mostly in models with complicated banking structure and multiple types of agents. For studies of firms’ balance sheet effect on business cycle, see Carlstrom and Fuerst 1997 and Bernanke, Gertler, and Gilchrist 1999. For a growth analysis that incorporates banks’ balance sheet effect, see Chakraborty and Ray 2001. The balance sheet effect has also been embedded in the study of the bank capital channel of monetary policy (see Van den Heuvel 2001, Kashyap and Stein 1995, Chami and Cosimano 2001). A related set of papers that emphasize the role of durable assets as collateral include Kiyotaki and Moore 1997 and Chen 2001.

In this paper, we provide a dynamic general equilibrium framework with an infinitely long-lived representative agent. We impose an explicit investment-financing constraint that is intended as a reduced form to capture the balance sheet effects. At the expense of less microfoundation, our approach has the advantage of simplicity. We think of our contribution as similar to that of MIUF that complements the CIA and the OLG models of money with more microfoundation. The lasting influence of MIUF is clearly seen in its wide adoption in the recent open economy literature (Obstfeld and Rogoff 1996). It is certainly our hope to see a future adaptation of our investment-financing constraint to an RBC model, but as a first step, we focus on a continuous time and deterministic setting.

In this setup, we derive a link between the value of the firm and the social welfare and we find that the value of the firm can be greater with than without the constraint. Our model also sheds light on how the effects of productivity shocks and bubbles may be amplified by the financing constraint.

The organization of the paper is as follows. Sections 2.1-2.4 lay out the model and characterize solutions to the firm’s value maximization and the consumer’s utility maximization

²See Schneider and Tornell (2000) for an attempt to synthesize some of these effects.

problems without the financing constraint. The constraint is introduced in Section 2.5, and numerical solutions reported in Section 3. Section 4 discusses implications from the model and possible extensions.

II. THE MODEL

Consider an infinite horizon economy where capital is the only factor of production. The representative household is endowed with some initial stock of capital, k_0 . Using this capital stock, the household sets up a representative firm to produce output and to invest in new capital. The firm's output net of investment will be distributed back to the household to support its consumption.

A. The Firm's Value Maximization Problem

At any time t , the firm uses capital k_t to produce output $f(k_t)$ and invests an amount $\dot{k} + \delta k$ (where δ is the depreciation rate). Its problem is to choose \dot{k} to maximize the present value of output net of investment, i.e.,

$$V^o(k_0) = \max \int_0^{\infty} e^{-\int_0^t r_s ds} [f(k) - \delta k - z] dt$$

subject to : $\dot{k} = z,$
 k_0 given.

where z is net investment. The superscript O stands for original, emphasizing the situation without an investment-financing constraint. Implicitly, we are assuming that the firm borrows funds from banks at a competitive interest rate r_t to finance its investment. A more explicit discussion about the role of the banking sector in this model economy is contained in the Appendix.

The first order conditions of this problem imply the familiar interest rate expression as follows:³

$$r_t = f'(k) - \delta. \quad (1)$$

³The first order conditions are given by

$$e^{-\int_0^t r_s ds} = \lambda,$$

and

$$\dot{\lambda} = -e^{-\int_0^t r_s ds} [f'(k) - \delta],$$

where λ is the multiplier associated with $\dot{k} = z$. The interest rate relation can be obtained by taking time derivative of the former and equating the resulting expression to the latter.

B. The Consumer's Utility Maximization Problem

The consumer's problem is simply to choose consumption, c , to maximize his utility subject to the budget constraint that the present value of his consumption cannot exceed the value of the firm he owns, i.e.,

$$U^o(k_0) = \max \int_0^{\infty} e^{-\rho t} \left(\frac{c^{1-\sigma} - 1}{1-\sigma} \right) dt$$

subject to: $\int_0^{\infty} e^{-\int_0^t r_s ds} c dt \leq V(k_0).$

Implicit in the budget constraint is the assumption that the household is the supplier of loanable funds (via the bank at the competitive interest rate r) to help finance the firm's investment. (See Appendix for details.)

The first order condition⁴ implies that

$$\frac{\dot{c}}{c} = \frac{r_t - \rho}{\sigma} \quad (2)$$

C. Equilibrium Firm Value and Consumer Utility

In equilibrium,

$$\dot{k} = f(k) - \delta k - c \quad (3)$$

Substituting \dot{k} from (3) and r_s from (2) into the firm's value function, we have

$$V^o(k_0) = c_0^\sigma \int_0^{\infty} e^{-\rho t} c^{1-\sigma} dt, \quad (4)$$

which turns out to be equal to

$$V^o(k_0) = c_0^\sigma \left[(1-\sigma)U^o(k_0) + \frac{1}{\rho} \right].$$

To our knowledge, this is the first time that an explicit link is established between the value of the representative firm and the welfare of representative agent.

When $f(k) = Ak^\sigma$, $c(k) = [\rho + (1-\sigma)\delta]k/\sigma$ (see Xie 1991). We can show (see Appendix) that

$$U^o(k_0) = \left(\frac{1}{1-\sigma} \right) \left\{ \frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho} \right) A}{\left[\frac{\rho+(1-\sigma)\delta}{\sigma} \right]^\sigma} - \frac{1}{\rho} \right\}.$$

⁴It is given by

$$c^{-\sigma} e^{-\rho t} = \mu e^{-\int_0^t r_s ds},$$

where μ is the multiplier associated with the budget constraint.

Hence,

$$V^o(k_0) = k_0 + \left(\frac{1 - \sigma}{\rho} \right) Ak_0^\sigma.$$

D. Policy Functions and Numerical Algorithms

For more general production functions, say, $f(k) = Ak^\alpha$ where $\alpha \neq \sigma$, there is no analytical solution. But we can still derive the differential equations governing the policy function, $c^o(k)$, and the firm's value function, $V^o(k)$.

The differential equation governing the policy function $c^o(k)$ can be obtained by substituting (3) and (1) into (2):

$$\frac{c^{o'}(k)}{c^o(k)} [Ak^\alpha - \delta k - c^o(k)] = \frac{\alpha Ak^{\alpha-1} - \delta - \rho}{\sigma}.$$

From (4), we can differentiate the firm's value function

$$V^o(k_\tau) = c_\tau^\sigma \int_\tau^\infty e^{-\rho(t-\tau)} c^{1-\sigma} dt$$

with respect to τ to obtain

$$\begin{aligned} V^{o'}(k)\dot{k} &= \sigma c^{\sigma-1} \dot{c} \int_\tau^\infty e^{-\rho(t-\tau)} c^{1-\sigma} dt + c^\sigma \left[-c^{1-\sigma} + \rho \int_\tau^\infty e^{-\rho(t-\tau)} c^{1-\sigma} dt \right] \\ &= rV^o(k) - c^o(k) \end{aligned} \quad (5)$$

Substituting (3) and (1) into (5) and rearranging terms, we get

$$V^{o'}(k) = \frac{(\alpha Ak^{\alpha-1} - \delta) V^o(k) - c^o(k)}{Ak^\alpha - \delta k - c^o(k)}$$

To compute the solutions numerically, we need to shoot back from the steady state capital stock, k^* , where k^* is obtained by combining (1) and (2) and solving $\alpha Ak^{\alpha-1} - \delta = \rho$,

$$k^* = \left(\frac{\alpha A}{\rho + \delta} \right)^{1/(1-\alpha)}.$$

Throughout the paper, we assume that k^* is greater than k_0 .

At the steady state (with $\dot{k} = 0$), consumption is given by

$$c^o(k^*) = Ak^{*\alpha} - \delta k^*$$

For backward shooting purpose, we need to compute $c^{o'}(k^*)$. Applying the *L'Hopital* rule,

$$\begin{aligned} c^{o'}(k^*) &= \left\{ \frac{\alpha Ak^{\alpha-1} - \delta - \rho}{\sigma [Ak^\alpha - \delta k - c^o(k)]} \right\} (Ak^\alpha - \delta k) \Big|_{k \rightarrow k^*} \\ &= \left\{ \frac{\alpha(\alpha - 1)Ak^{*\alpha-2}}{\sigma [\rho - c^{o'}(k^*)]} \right\} (Ak^{*\alpha} - \delta k^*). \end{aligned}$$

The above is a quadratic equation in $c^o(k^*)$, which can be solved to yield the following solution:

$$c^o(k^*) = \frac{\sigma\rho + \sqrt{\sigma^2\rho^2 + 4\sigma\alpha(1-\alpha)Ak^{*\alpha-2}(Ak^{*\alpha} - \delta k^*)}}{2\sigma}, \quad (6)$$

where the negative root has been ruled out by the assumptions of free disposal and no satiation. Using $c^o(k^*)$, we can shoot backward from $c^o(k^*)$ to obtain $c^o(k)$.

As for the firm's value function, note that along the steady state path with $k = k^*$ and $r = \rho$, $V^o(k^*) = (Ak^{*\alpha} - \delta k^*)/\rho$. Again, we use L'Hopital rule to compute $V^{o'}(k^*)$:

$$\begin{aligned} V^{o'}(k^*) &= \left. \frac{(\alpha Ak^{\alpha-1} - \delta)V^o(k) - c^o(k)}{Ak^\alpha - \delta k - c^o(k)} \right|_{k \rightarrow k^*} \\ &= \frac{[\alpha(\alpha-1)Ak^{*\alpha-2}]V^o(k^*) + (\alpha Ak^{*\alpha-1} - \delta)V^{o'}(k^*) - c^{o'}(k^*)}{[\alpha Ak^{*\alpha-1} - \delta - c^{o'}(k^*)]} \\ &= \frac{[\alpha(\alpha-1)Ak^{*\alpha-2}](Ak^{*\alpha} - \delta k^*)/\rho + \rho V^{o'}(k^*) - c^{o'}(k^*)}{[\rho - c^{o'}(k^*)]}, \end{aligned}$$

which implies that

$$V^{o'}(k^*) = 1 + \frac{[\alpha(1-\alpha)Ak^{*\alpha-2}](Ak^{*\alpha} - \delta k^*)}{\rho c^{o'}(k^*)}. \quad (7)$$

Again given $V^{o'}(k^*)$, we can shoot backward from $V^o(k^*)$ to obtain $V^o(k)$.

Lastly, we can compute $I^o(k)$ as follows:

$$\begin{aligned} I^o(k) &= \dot{k} + \delta k \\ &= Ak^\alpha - c^o(k), \end{aligned}$$

which is smooth and hump-shaped. We will see in the next section that an introduction of a financing constraint will lead to a kinked investment function.

E. Financing Constraint

In this paper, we examine the case where the representative firm's investment is limited by its ability to obtain financing. We assume that there is an implicit, competitive banking sector that provides loans (at the real interest rate r_t) to finance the firm's investment no greater than some fraction of its net present value, namely,

$$\dot{k} + \delta k_t \leq \gamma V(k_t) \text{ for any } t.$$

There could be many reasons why the firm may not be able to borrow any amount bigger than its fundamental value—especially, capital market imperfections such as default possibilities and asymmetric information problems. (See, e.g., Bernanke, Gertler, and Gilchrist 1999.) This financing constraint can be viewed as a reduced form representation of these imperfections that we do not explicitly model in this paper.

Intuitively, there exists a critical value for k , k^c , which solves $I(k) = \gamma V(k)$, such that the investment constraint is binding only when $k_t < k^c$. In the special case where $\alpha = \sigma$, we can solve $I(k) = \gamma V(k)$ explicitly to yield,

$$k^c = \begin{cases} \left\{ \frac{[1 - \frac{\gamma(1-\sigma)]A}{\gamma + \frac{\rho + (1-\sigma)\delta}{\sigma}}}{0} \right\}^{1/(1-\sigma)} & \text{for } \gamma < \frac{\rho}{1-\sigma} \\ 0 & \text{for } \gamma \geq \frac{\rho}{1-\sigma} \end{cases} .$$

In the presence of the financing constraint, the firm's problem becomes:

$$V(k_0) = \max \int_0^{\infty} e^{-\int_0^t r_s ds} (Ak^\alpha - \delta k - z) dt$$

subject to : $\dot{k} = z$,
 $\delta k + z \leq \gamma V(k)$,
 $k(0) = k_0$ given.

The most difficult part of this problem is to ensure that the value of the firm as perceived by the bank when imposing the financing constraint is consistent with the firm's actual value.

The first order conditions are given by:⁵

$$e^{-\int_0^t r_s ds} = \lambda - \theta,$$

and

$$\dot{\lambda} = -e^{-\int_0^t r_s ds} (\alpha Ak^{\alpha-1} - \delta) - \theta[\gamma V'(k) - \delta],$$

where λ and θ are the multipliers associated with the $\dot{k} = z$ and financing constraints respectively and θ satisfies the following complementary slackness condition:

$$\theta [\gamma V(k) - \delta k - \dot{k}] = 0.$$

The consumer's problem remains the same as before. Therefore, (2) still holds and so also will (4) and (5).

When $k \geq k^c$, the financing constraint is not binding so that $\theta = 0$ and the policy and value functions are the same as in the unconstrained case described in the previous subsections, with $c(k) = c^o(k)$, $V(k) = V^o(k)$, etc.

In what follows, let us focus on the case where the constraint is binding, i.e., $k < k^c$ and $\theta > 0$. Is there a differential equation similar to (6) that governs $c(k)$? From (3) and the binding constraint, we have

$$Ak^\alpha - c(k) = \gamma V(k).$$

Differentiating this with respect to t and using (5) and (2), we obtain

⁵Taking derivative of the first condition with respect to t and combining the resulting expression with the second condition, we get

$$r_t [\lambda - \theta] = [\lambda - \theta] [\alpha Ak^{\alpha-1} - \delta] + \theta [\gamma V'(k) - \delta] + \dot{\theta}$$

$$[\alpha Ak^{\alpha-1} - c'(k)] \dot{k} = \left[\rho + \sigma \frac{c'(k)}{c(k)} \dot{k} \right] [Ak^\alpha - c(k)] - \gamma c(k),$$

which implies that

$$c'(k) = \frac{\alpha Ak^{\alpha-1} c(k)}{[(1-\sigma)c(k) + \sigma Ak^\alpha]} + \frac{(\gamma + \rho)c^2(k) - \rho c(k) Ak^\alpha}{[(1-\sigma)c(k) + \sigma Ak^\alpha] [Ak^\alpha - \delta k - c(k)]}$$

We can compute $c(k)$ by backward shooting starting from k^c and $c(k^c) = A(k^c)^\alpha - \gamma V^o(k^c)$.

As for $V(k)$, it can be found from the financing constraint simply as

$$V(k) = [Ak^\alpha - c(k)] / \gamma.$$

The fact that we make use of (5) in our derivation of $c'(k)$ above ensures that this $V(k)$ is the same as the $V(k)$ in the financing constraint.

III. A NUMERICAL EXAMPLE

We surmise that the firm will invest at a slower rate and probably earn a lower net present value $V(k)$ with than without the financing constraint. In the absence of explicit analytical solutions, we shall resort to numerical simulations to better understand the economic effects of this constraint.

In our numerical solutions, we assume the following benchmark parameter values: $\alpha = 0.36$, $\sigma = 0.5$, $\gamma = 0.015$, $\rho = 0.03$, $\delta = 0.1$, $A = 12$, and $k_0 = 20$. We first compute the policy functions $c^o(k)$, $V^o(k)$, and $I^o(k)$ in the absence of the financing constraint by shooting backward from k^* to k_0 . Then, we use $I^o(k) = \gamma V^o(k)$ to solve for the critical value k^c .⁶ The corresponding functions $c(k)$, $V(k)$, and $I(k)$ in the presence of the investment constraint can be obtained by shooting backward from k^c to k_0 for $k \in [k_0, k^c]$ (when the constraint is binding) and combining it with $c^o(k)$, $V^o(k)$, and $I^o(k)$ for $k \in [k^c, k^*]$ (when the constraint is non-binding). The graphs for $\gamma V(k)$ with and without the financing constraint as well as the investment function $I^o(k)$ are displayed in Figure 1.

Not surprisingly, $I(k) < I^o(k)$ and, since contemporaneous output is unaffected by changes in investment, $c(k) > c^o(k)$ for $k < k^c$ (see Figure 2, panel 1). It is, however, surprising to find that $V(k) > V^o(k)$. In order to understand this, it is necessary to also compute the consumption path over time because the equilibrium value of the firm is simply the present value of equilibrium consumption in our model (without the labor-leisure choice; see budget constraint of the representative consumer).

⁶The time T required for $k(T) = k^c$ can be solved from the following differential equation:

$$\begin{aligned} \dot{k} &= Ak^\alpha - \delta k - c(k) \\ k(0) &= k_0 \text{ given,} \\ k(T) &= k^c. \end{aligned}$$

From the time path of consumption (Figure 2, panel 2), we see that while consumption under the financing constraint exceeds its unconstrained counterpart, it grows at a slower rate and is soon surpassed by the latter.⁷ As a result, consumer utility is lowered by the constraint, i.e., $U(k_0) < U^o(k_0)$. This may give the impression that $V(k) < V^o(k)$. However, the firm's value also depends on the effect of discounting. We thus have to consider how the financing constraint affects the behavior of interest rate over time. As shown in Figure 3, panel 2, initially interest rate is significantly lower with than without the investment constraint. The constraint induces a jump in the interest rate from 4% up to 5% at the time when the capital stock hits its critical value and gradually converges to its steady state value (3%) thereafter. The discount rate at time t (given by $\int_0^t r_s ds$), represented by the area under the interest rate paths from 0 to t , will, at any rate, be smaller with than without the constraint despite the interest rate jump. It turns out that this discounting effect dominates the consumption growth effect to make $V(k) > V^o(k)$ under the set of parameter values we have chosen.

The equilibrium relation between the firm's value and consumer utility, $V(k_0) = c_0^\sigma \left[(1 - \sigma)U(k_0) + \frac{1}{\rho} \right]$, holds irrespective of the financing constraint. In terms of this relation, whether $\sigma > 1$ or $\sigma < 1$, it is possible that $V(k_0) > V^o(k_0)$ while $U(k_0) < U^o(k_0)$ provided that c_0 is sufficiently larger with than without the constraint. When $\sigma \rightarrow 0$, however, $V(k_0)$ and $U(k_0)$ will be positively correlated and will both be lowered by the constraint.

The interest rate behavior under the financing constraint may suggest a partial resolution to the Lucas (1990) puzzle why capital doesn't flow from rich to poor countries. In particular, the interest rate functions as portrayed in Figure 4 indicate that while a ten-fold difference in capital stocks between rich and poor countries (say, $k = 20$ versus $k = 200$) could induce a more than 13-fold difference in their interest rates ($r(20) = 0.535$ versus $r(200) = 0.0455$) in the absence of the constraint, the interest rate gap will be significantly reduced to 4-fold ($r(20) = 0.188$ versus $r(200) = 0.0455$) under the constraint. This may sound tautological that the presence of financing constraint reduces interest rate differential across countries. In fact, it could be given empirical content if one could calibrate parameter γ to obtain a quantitative measure of the reduction in interest rate differential. The remaining differential can then be attributed to other factors such as political risk, institutional and trade barriers.

IV. DISCUSSION AND POSSIBLE EXTENSIONS

Our simple model can easily be extended to include labor as an additional input in the firm's production technology and the labor-leisure choice in the consumer's utility maximization problem. This extension would allow us to examine the effect of the financing constraint on employment as well — especially when the constraint does not apply just to investment-financing, but also to hiring workers and footing their wage bills. In the presence of this more severe

⁷Observe that while the “constrained” consumption *function* lies everywhere above its “unconstrained” counterpart, the same is not true for the consumption *paths*. This is because capital (of which consumption is a function) will grow more slowly with than without the constraint. The same logic applies to comparisons between policy functions of other variables and their corresponding time paths.

constraint, employment and output could both be adversely affected so that consumption may not even surge at the beginning despite the fall in investment.

This model is cast in a deterministic framework and therefore the following arguments on its potential applications to cases with uncertainty are only suggestive. Nevertheless, we list them here for discussions.

- An increase in A will shift both the $\gamma V(k)$ and $I(k)$ schedules upward. The impact on investment is not a monotonic function of the capital stock. As shown in Figure 5 based on our numerical computations, the impact on investment is hump-shaped. This suggests that in the emerging countries, broadly interpreted as countries with the size of capital between that of the less-developed countries and the developed, investment is more responsive to productivity shocks A than in the rest of the world.
- Bubbles could help relax the financing constraint and speed up investment when they arise. But when burst, they make the constraint more stringent and they restrict investment more severely. The magnitude of the bust in investment depends on the level of the capital stock. The closer is the capital stock to the steady state, the milder is the bust in investment. This implies that bubbles in developed countries have less adverse impact in investment than those in the developing countries when they burst. The reader can use Figure 6 to run these thought experiments. To be sure, a logically consistent model of bubbles would require an explicit probability specification of the magnitude of bubbles and of the timing of their crash.
- Implications for government regulations: higher accounting standards and more transparency may make γ bigger and help relax the financing constraint.

For future research, we can examine how an imposition of the financing constraint may help us better calibrate an RBC model to explain business downturns or crises and analyze whether and to what extent business cycles are asymmetric.

Figure 1: Investment Function and Financing Constraint

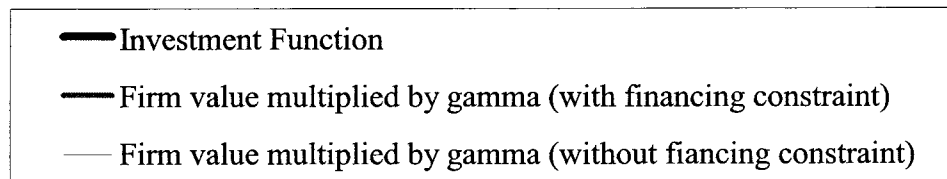
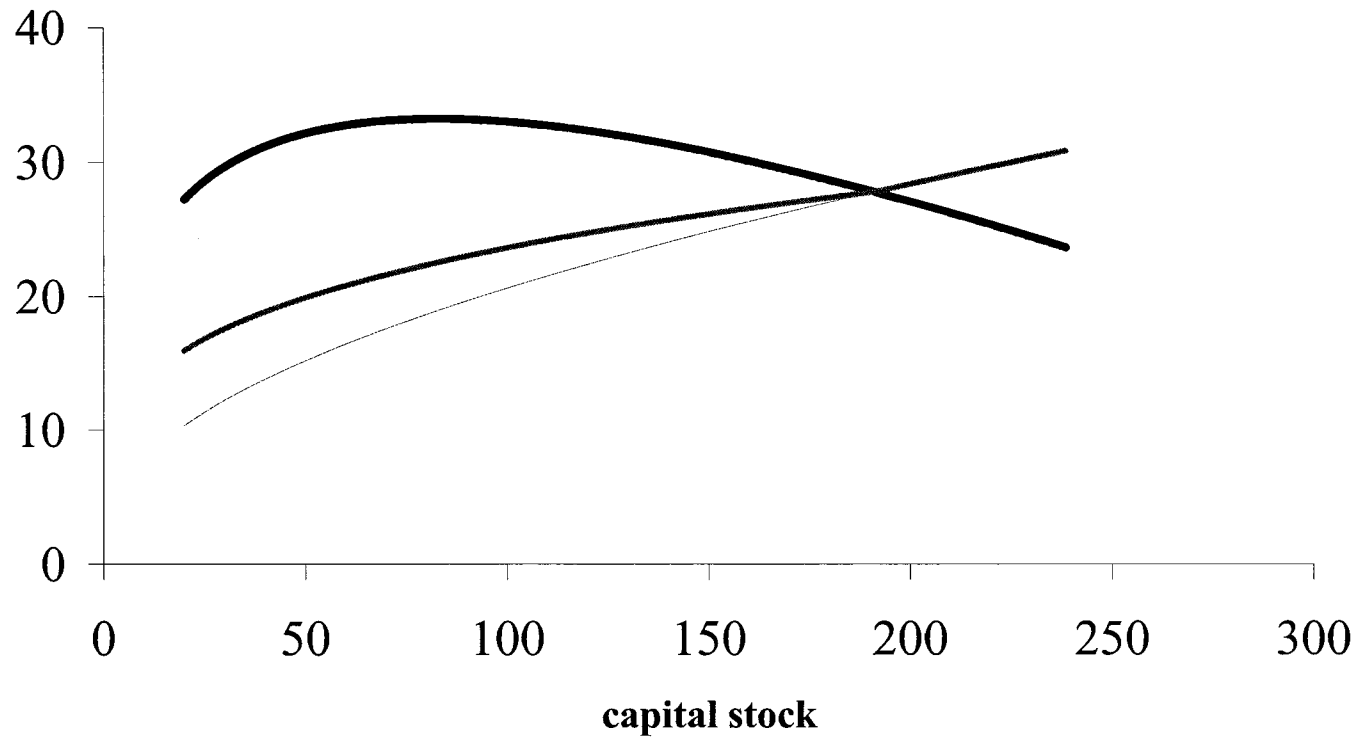
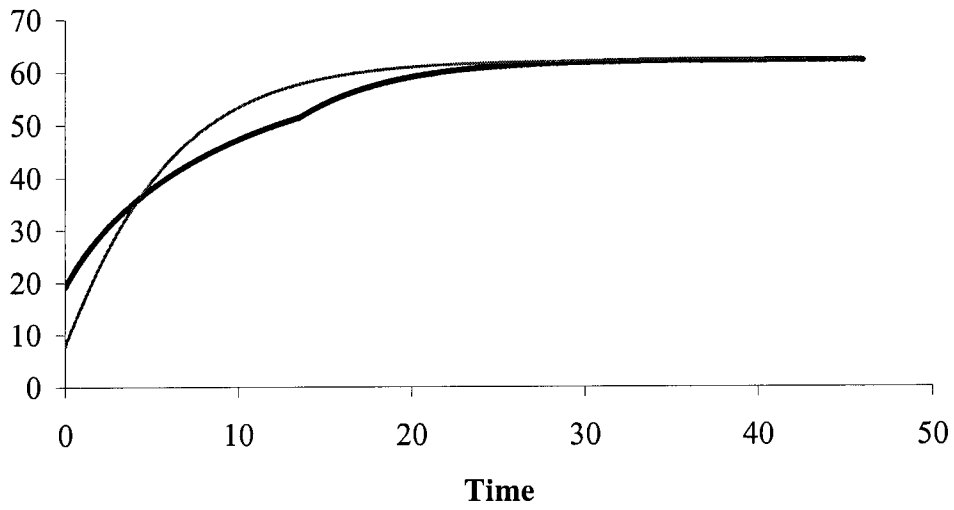
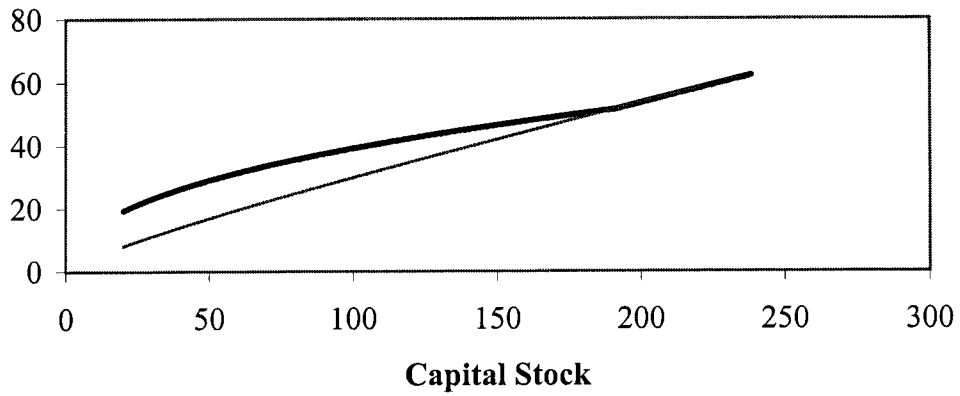


Figure 2
Consumption Paths



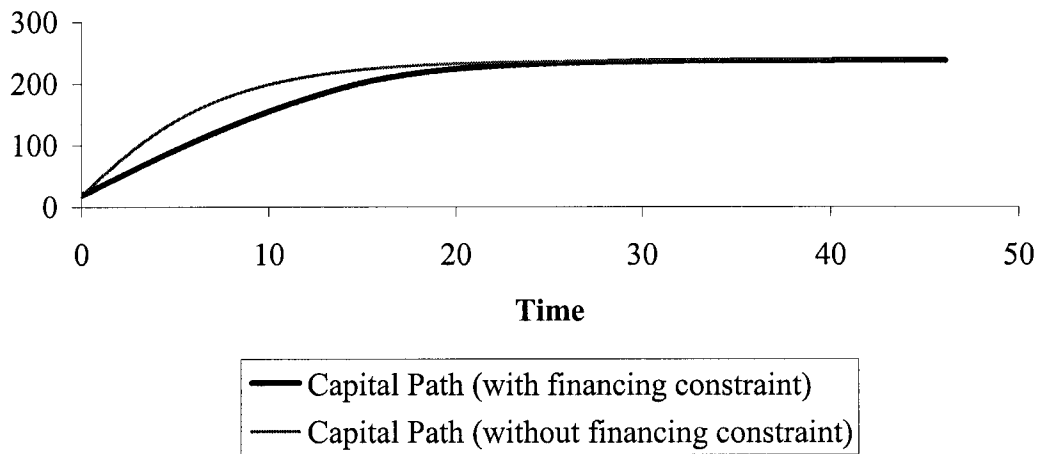
Consumption Functions



— Consumption function (with financing constraint)
— Consumption function (without financing constraint)

Figure 3

Capital Paths



Real Interest Rate Paths

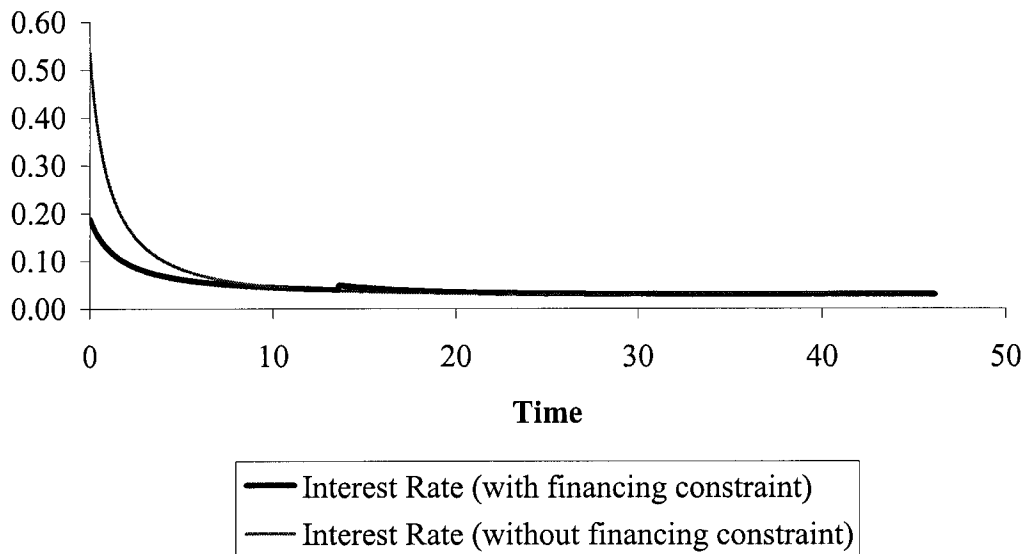
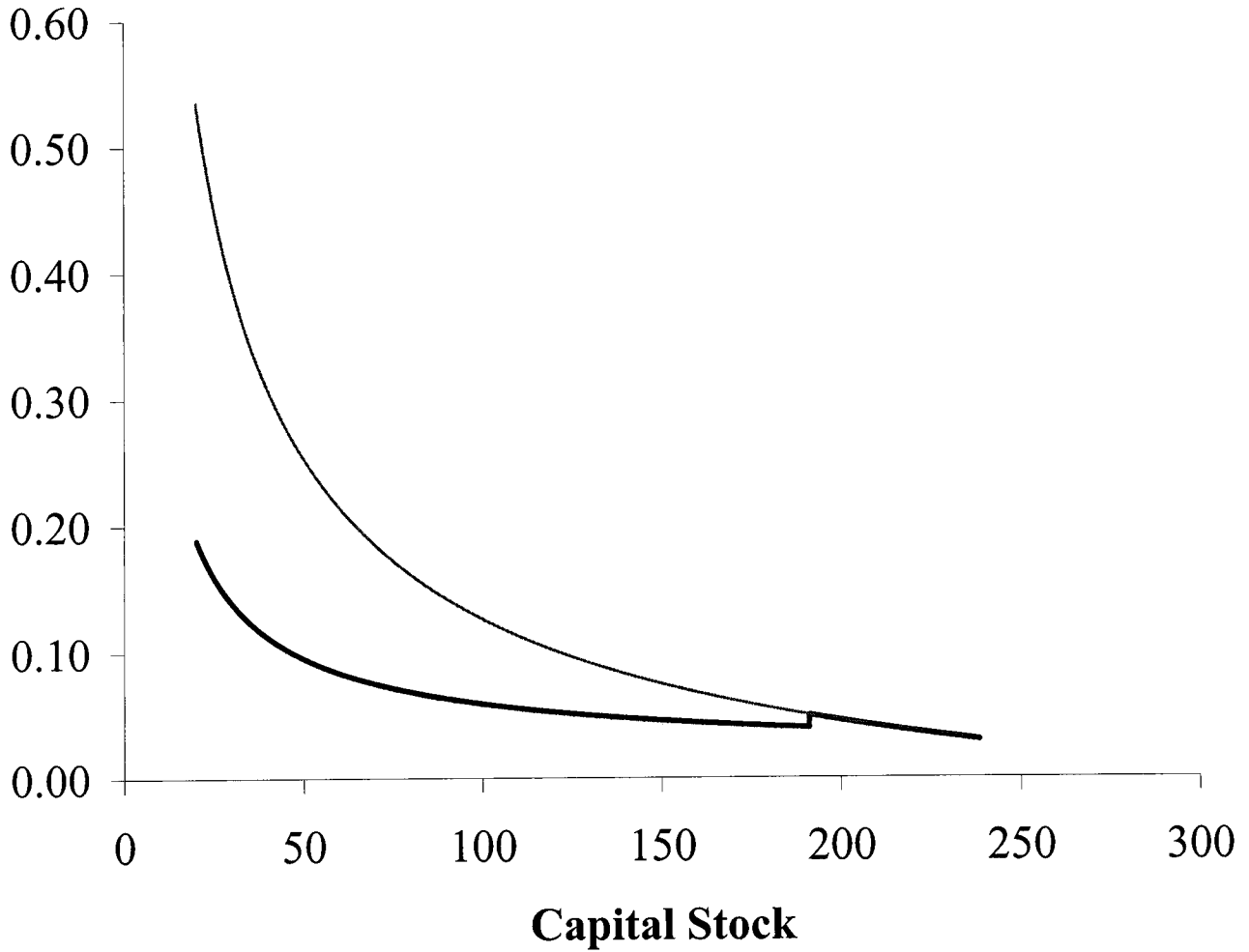
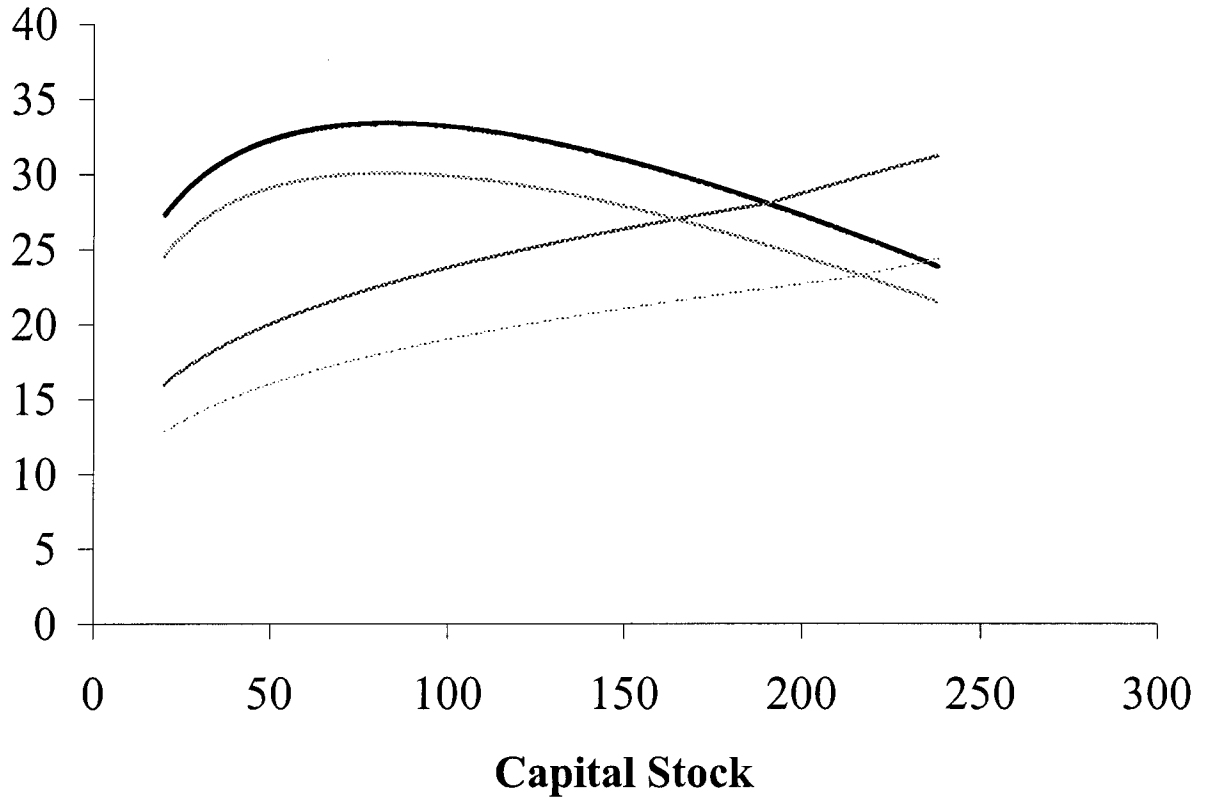


Figure 4: Interest Rate Functions



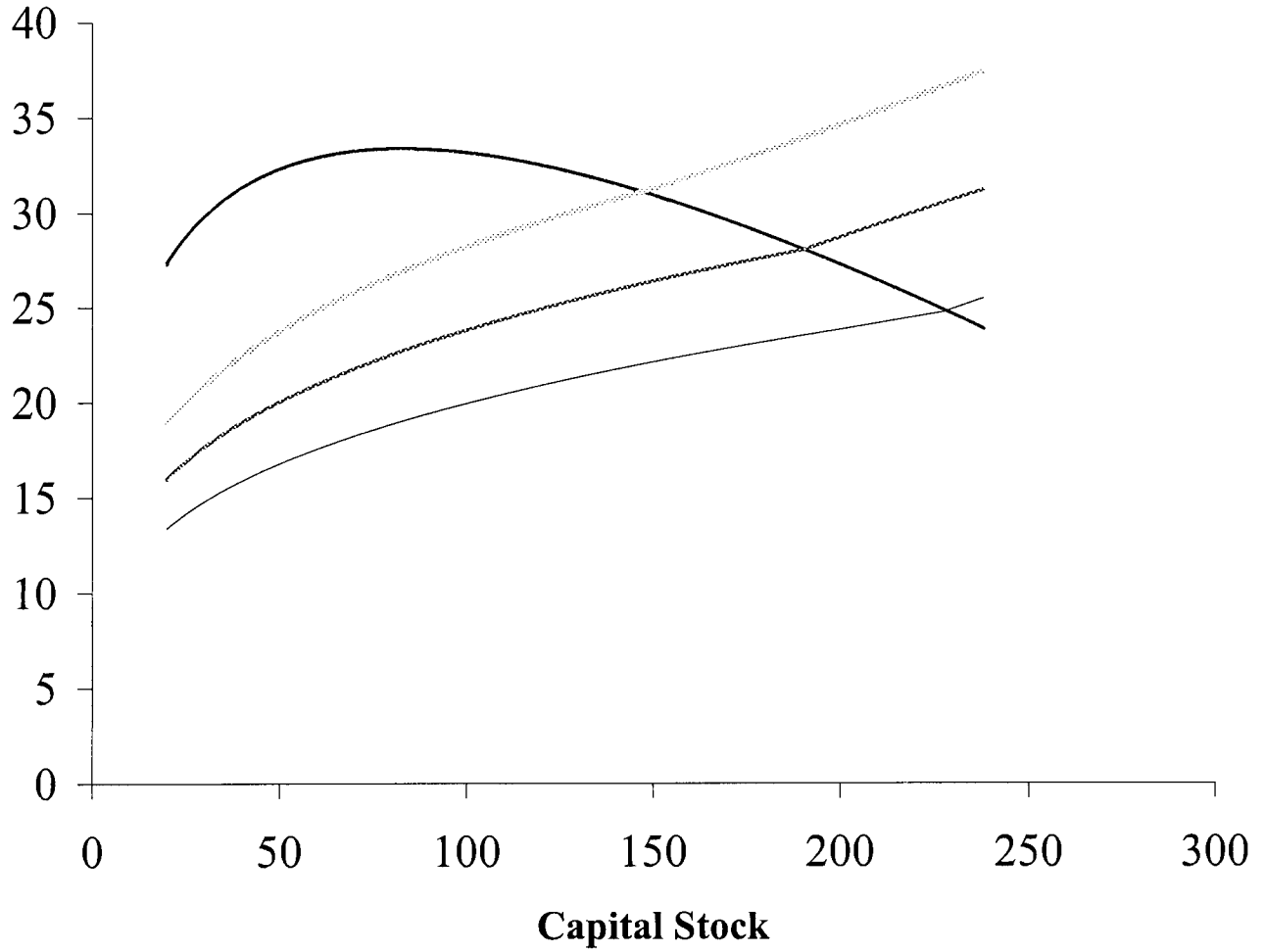
— Interest Rate Function (with financing constraint)
— Interest Rate Function (without financing constraint)

Figure 5: Change in Productivity



- Investment function (high A)
- Value of the firm multiplied by gamma (high A)
- Investment function (low A)
- Value of the firm multiplied by gamma (low A)

Figure 6: Bubble and Crash



— Investment function Value of the firm Bubble — Crash

A. A more detailed description of the banking sector in discrete time

At time 0, the household uses its initial capital k_0 to purchase shares of the firm and thus becomes its owner. With k_0 , the firm produces $f(k_0)$, which it pays to the household as dividends. It then borrows I_0 from the bank at the competitive interest rate r_0 to finance its investment. When time 1 comes around, the capital stock grows to $k_1 (= I_0 + (1 - \delta)k_0)$, yielding output $f(k_1)$. After repaying principal and interest to the bank, the residual $f(k_1) - I_0(1 + r_0)$ is paid out to the household. A new loan is then raised to finance investment I_1 at interest rate r_1 . At time 2, the capital stock $k_2 (= I_1 + (1 - \delta)k_1)$ generates output $f(k_2)$ and dividend $f(k_2) - I_1(1 + r_1)$. So on and so forth.

Therefore, value of the firm equals the present value of the net cash flow, i.e.,

$$\begin{aligned} V(k_0) &= f(k_0) + \left(\frac{1}{1+r_0}\right) [f(k_1) - I_0(1+r_0)] \\ &\quad + \left(\frac{1}{1+r_0}\right) \left(\frac{1}{1+r_1}\right) [f(k_2) - I_1(1+r_1)] + \dots \\ &= [f(k_0) - I_0] + \left(\frac{1}{1+r_0}\right) [f(k_1) - I_1] \\ &\quad + \left(\frac{1}{1+r_0}\right) \left(\frac{1}{1+r_1}\right) [f(k_2) - I_2] + \dots \end{aligned}$$

Regarding the household, she receives $f(k_t) - I_{t-1}(1 + r_{t-1})$ from the firm as its shareholder and $I_{t-1}(1 + r_{t-1})$ from the firm as its debt-holder, consumes $c_t = f(k_t) - S_t$, and deposits her savings S_t with the bank.

In eq. of loans by the household (S_t) equals demand by loans by the firm (I_t), so that $c_t = f(k_t) - I_t$ and the present value of consumption simply equals the firm's value.

B. Derivation of $U(k_0)$ when $\alpha = \sigma$

Given $c = [\rho + (1 - \sigma)\delta] k / \sigma$,

$$\begin{aligned} U(k_0) &= \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c^{1-\sigma} dt - \frac{1}{(1-\sigma)\rho} \\ &= \frac{1}{1-\sigma} \left[\frac{\rho + (1-\sigma)\delta}{\sigma} \right]^{1-\sigma} \int_0^\infty e^{-\rho t} k^{1-\sigma} dt - \frac{1}{(1-\sigma)\rho}, \end{aligned}$$

where, with $\dot{k}/k = [\sigma A k^{\sigma-1} - (\rho + \delta)] / \sigma$,

$$\begin{aligned} \int_0^\infty e^{-\rho t} k^{1-\sigma} dt &= -\frac{e^{-\rho t}}{\rho} k^{1-\sigma} \Big|_0^\infty + \int_0^\infty \frac{e^{-\rho t}}{\rho} (1-\sigma) k^{-\sigma} \dot{k} dt \\ &= \frac{1}{\rho} k_0^{1-\sigma} + \int_0^\infty \frac{e^{-\rho t}}{\rho} (1-\sigma) \left[\frac{\sigma A - (\rho + \delta) k^{1-\sigma}}{\sigma} \right] dt \\ &= \frac{1}{\rho} k_0^{1-\sigma} + \frac{(1-\sigma)A}{\rho^2} - \frac{(1-\sigma)(\rho + \delta)}{\rho\sigma} \int_0^\infty e^{-\rho t} k^{1-\sigma} dt \end{aligned}$$

Hence,

$$\int_0^{\infty} e^{-\rho t} k^{1-\sigma} dt = \frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho}\right) A}{\frac{\rho+(1-\sigma)\delta}{\sigma}},$$

implying

$$U(k_0) = \left(\frac{1}{1-\sigma}\right) \left\{ \frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho}\right) A}{\left[\frac{\rho+(1-\sigma)\delta}{\sigma}\right]^{\sigma}} - \frac{1}{\rho} \right\},$$

and

$$V(k_0) = k_0 + \left(\frac{1-\sigma}{\rho}\right) A k_0^{\sigma}.$$

References

- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, Volume 1, North-Holland.
- Carlstrom, Charles and Timothy Fuerst, 1997, "Agency Cost, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, Vol. 87, pp. 893–910.
- Chakraborty, Shankha and Tridip Ray, 2001, "Bank-Based Versus Market-Based Financial Systems: A Growth-Theoretic Analysis," mimeo, Hong Kong University of Science and Technology.
- Chami, Ralph and Thomas F. Cosimano, 2001, "Monetary Policy with a Touch of Basel," IMF Working Paper 01/151 (Washington: International Monetary Fund).
- Chang, Roberto, and Andres Velasco, 2000, "Financial Fragility and Exchange Rate Regimes," *Journal of Economic Theory*, Vol. 92, pp. 1–34.
- Chen, Nan-Kuang, 2001, "Bank Net Worth, Asset Prices and Economic Activity," *Journal of Monetary Economics*, Vol. 48, pp. 415–436.
- Kashyap, Anil K., and Jeremy C. Stein, 1995, "The Impact of Monetary Policy on Bank Balance Sheets," *Carnegie-Rochester Conference Series on Public Policy*, Vol 42, pp. 151–195.
- Kiyotaki, Nobuhiro and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*, Vol. 105, pp. 211–248.
- Krugman, Paul, 2001, "Crises: The Next Generation," paper presented at the Razin Conference, Tel Aviv University, March 25–26.
- _____, 1999, "Balance Sheet Effects, the Transfer Problem, and Financial Crises," *International Tax and Public Finance*, No. 6, pp. 459–472.
- _____, 1998, "Bubble, Boom, Crash: Theoretical Notes on Asia's Crisis," mimeo, January.
- _____, 1979, "A Model of Balance of Payment Crises," *Journal of Money, Credit, and Banking*, Vol. 11, pp. 311–325.
- Lucas, Robert E., Jr., 1990, "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review Papers and Proceedings*, Vol. 80, pp. 92–96.

Obstfeld, Maurice, 1996, "Models of Currency Crises with Self-fulfilling Features," *European Economic Review*, Vol. 40, pp. 1037–1048.

Obstfeld, Maurice and Kenneth Rogoff, 1996, "Foundations of International Macroeconomics," The MIT Press.

Schneider, Martin, and Aaron Tornell, 2000, "Balance Sheet Effects, Bailout Guarantees, and Financial Crises," Working Paper, UCLA, November.

Van den Heuvel, Skander J., 2001, "The Bank Capital Channel of Monetary Policy," mimeo, University of Pennsylvania.