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Holding International Reserves in an Era of High Capital Mobility

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Holding International Reserves in an Era of High Capital Mobility

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Abstract

Why do countries hold so much international reserves? Global reserve holdings (excluding gold) were equivalent to 17 weeks of imports at the end of 1999. That is almost double what they were at the end of 1960 and about 20 percent higher than they were at the start of the 1990s. In this paper we study countries' reserve holdings in light of both the increased financial volatility experienced in the last decade and diminished adherence to fixed exchange rates. We find that buffer-stock reserve models work about as well in the modern floating-rate period as they did during the Bretton Woods regime. During both periods, however, the models' fundamentals explain only a small portion (10-15 percent) of reserves volatility.

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I. Introduction

Last September, as calls grew louder for European Central Bank (ECB) intervention to reverse the euro's fall, The Economist ran an article with the lead: "If central banks are so reluctant to intervene in foreign-exchange markets, why do they still hold so many reserves?"[2] At the time, the ECB and the euro area's national central banks together held about $226 billion of foreign-exchange reserves, not counting their sizeable gold holdings.[3]

The question is being asked more often these days. The Economist suggested that the idea of having lots of reserves is partly a hangover from the Bretton Woods system, when central banks were obligated to defend their parities against the dollar through intervention and so needed a lot of reserves. Yet, as the Economist noted, Bretton Woods broke down thirty years ago and today many fewer countries peg their exchange rates. Indeed, the currency and financial crises of the 1990s have led some observers to conclude that in a world of high capital mobility, fixed exchange rates such as the European exchange-rate mechanism or the East Asian pegs before 1997-98 cannot work for long.[4] As a result, more countries have shifted to floating exchange rates. With less need to hold reserves to defend currency values, one would expect global reserve holdings to decline. But just the opposite has occurred—world reserve holdings are at record levels.

Figure 1 shows that global reserve holdings (excluding gold) were equivalent to 17 weeks of imports at the end of 1999. That is almost double what they were at the end of 1960 and about 20 percent higher than they were at the start of the 1990s. Figure 2 shows that when measured as a share of global income, reserve holdings have trended upwards also. At the end of 1999, reserves were about 6 percent of global GDP, 3.5 times what they were at the end of 1960 and 50 percent higher than in1990.[5]

Calvo and Reinhart (2000) believe that reserve holdings are high for some countries because their announced shift to greater exchange-rate flexibility is an illusion. Examining 39 countries during the January 1970-November 1999 period, they find that self-identified floaters and managed floaters look more like peggers (in terms of the probability that the monthly percentage change in their nominal exchange rate will fall within a narrow band). Thus some countries may hold large stocks of reserves because they still manage their exchange rates. Hausman, Panizza and Stein (2000) suggest that countries unable to borrow

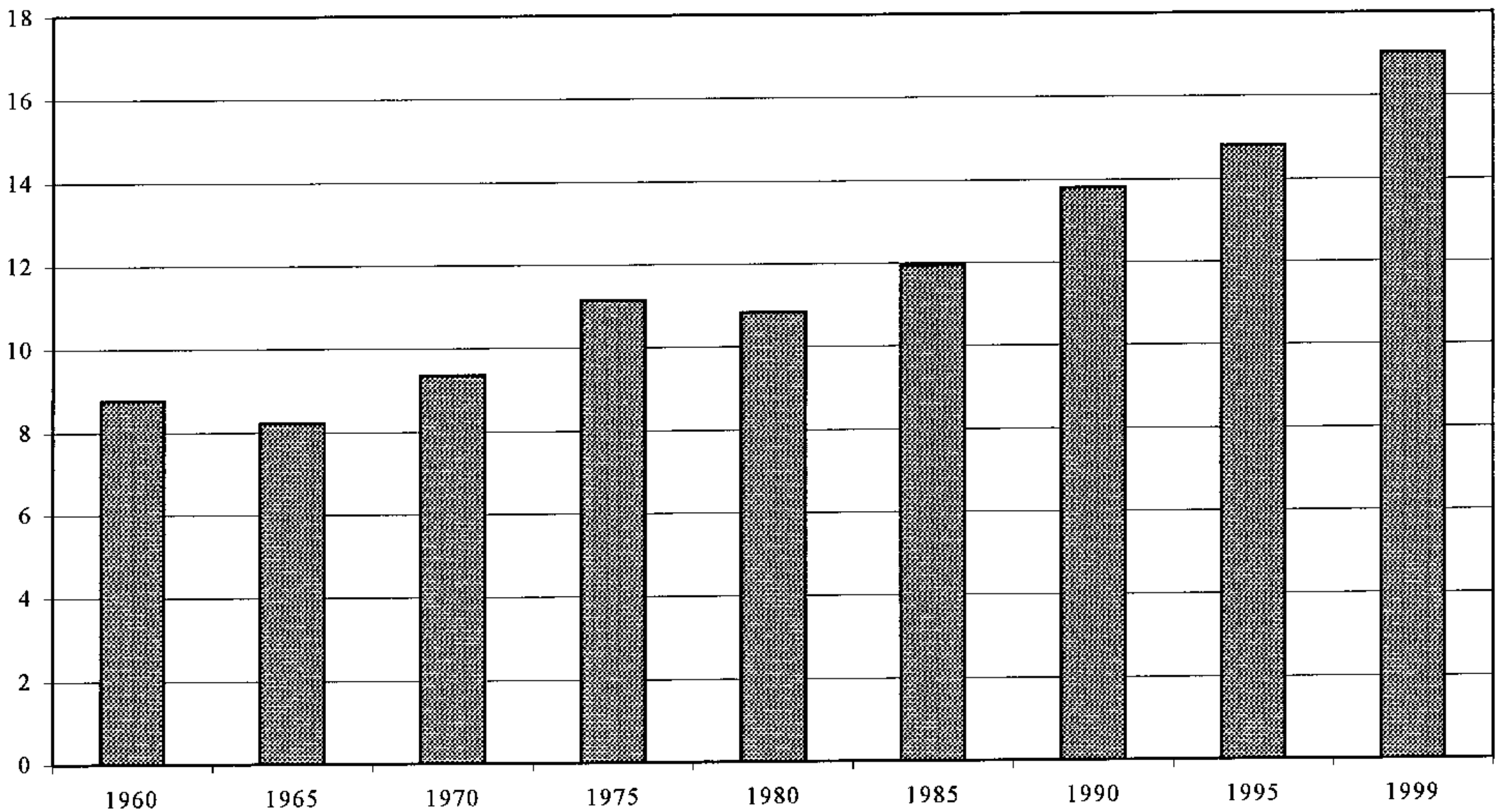
[2] *Economist*, September 23-29, 2000, p. 89.

[3] *Economist*, September 23-29, 2000, p. 89.

[4] For example, see Fischer (2001).

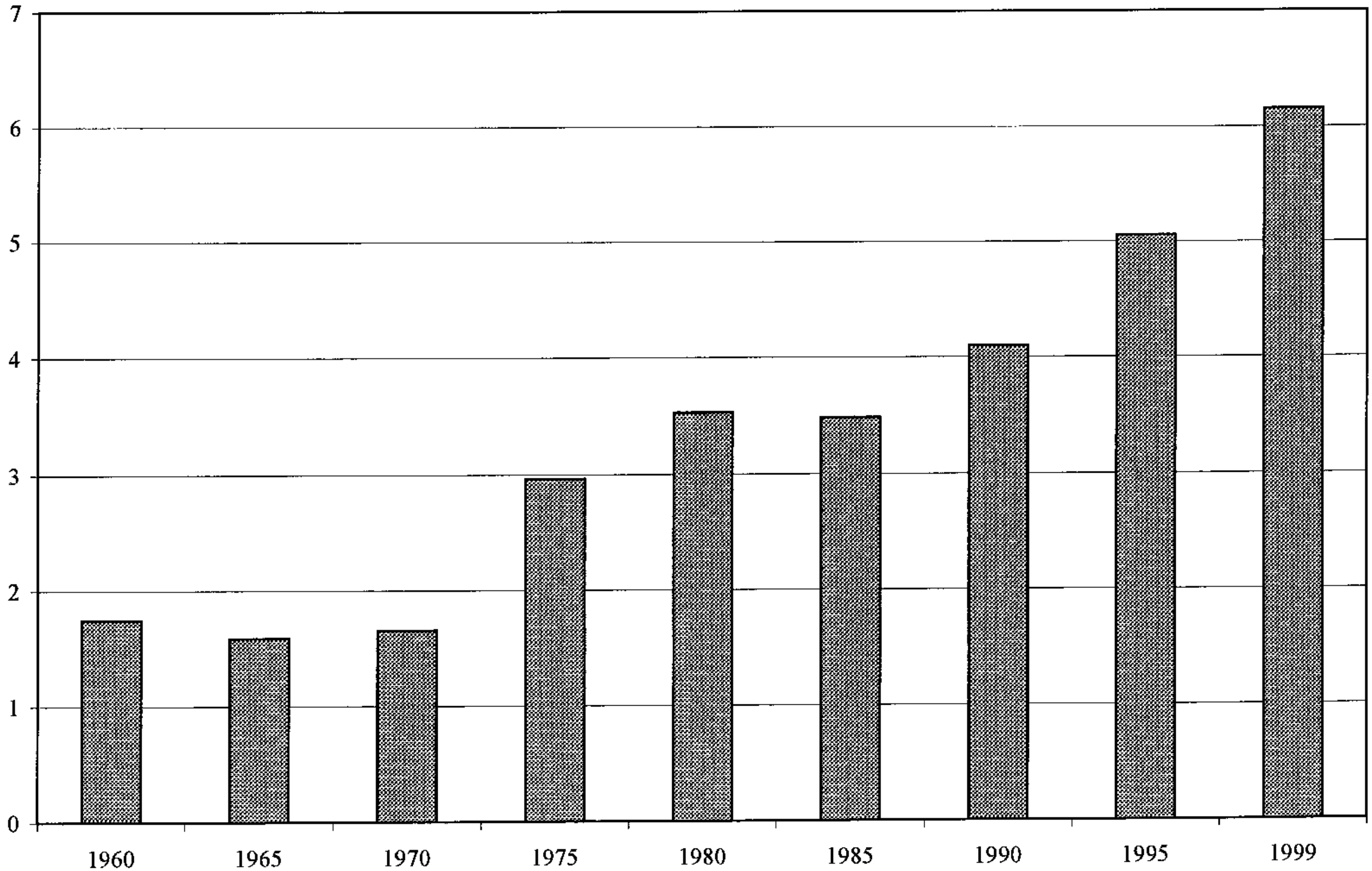
[5] The countries in the global sample vary over time. Using a 44-country sample for which there is continuous data, and which account for 57 percent of global reserve holdings today, we uncover the same pattern of increased reserve holdings over time. In Section II, we discuss how these trends are modified when measured reserves include gold holdings.

Figure 1. Global Reserves Excluding Gold
Numbers of Weeks' Import Cover



Source: IMF

Figure 2. Global Reserves Excluding Gold
As Percentage of World GDP



Source: International Monetary Fund.
Note: World GDP unavailable for 1960s; 1960 and 1965 ratios use sample of countries accounting for over 90% of global reserve holdings in 1960s.

internationally in their own currency may seek reduced exchange-rate flexibility in order to limit the damage from currency mismatches in their liabilities. That exchange-rate policy requires a potentially large stockpile of reserves.

Countries actually operating floating exchange rates might still hold reserves so they can intervene in the foreign-exchange market on occasion to influence the value of their currencies. But since most studies suggest that intervention, at least the sterilized kind that neutralizes the effects of foreign-exchange market intervention on the money supply, rarely works, and since intervention occurs so infrequently in practice, there is still the puzzle of why floaters hold so many reserves. On the surface, holding lots of reserves seems like a costly practice.[6]

In the end, the Economist was able to offer only one reason other than exchange-rate management for why a rational central bank might want to hold a large stockpile of reserves. The reason is to have "a safety cushion in times of war, a trade embargo or a banking crisis".[7] In most developed countries, the need for such a large safety net seems unjustified, especially if they can borrow foreign currency in the world capital market when needed. For other countries, however, holding large reserves as a safety cushion may be understandable.

Obviously, there could be any number of reasons why central banks continue to hold substantial quantities of reserves. It is necessary to take a systematic look at the issue. This is not the first time that researchers have turned their attention to the subject of central bank reserve holdings. In the mid-1960s, the debate about needed reforms of the Bretton Woods system led researchers to ask whether reserve levels were "adequate" and were distributed optimally across countries. In the late 1970s and early 1980s, researchers were interested in whether the "demand for reserves" had substantially changed after the demise of Bretton Woods. They were also curious about whether developed and developing countries differed in their "demand for reserves." Eventually attention was directed away from reserve holdings by the widespread assumption that international reserves would be stable—and probably low—in an era of increased exchange-rate flexibility and very high capital mobility.

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[6] As the *Economist* says: "It is rather as though a household with lots of cash sitting idle in a low-interest bank account was at the same time paying a much higher interest rate on its debts. It would make more sense to repay some of that debt." (*Economist*, September 23-29, 2000, p. 90.) See also Flood and Jeanne (2000). However, once the capital gains on foreign-currency reserves arising from exchange-rate changes are taken into account, the opportunity cost of holding reserves is negligible.

[7] The *Economist*, (2000), p. 90.

Now is a good time to revisit this issue. The last decade of the 20^{th} century has strengthened three trends in the international economy that could potentially have an important influence on reserve holdings. The first is increasing capital mobility, as more economies liberalize their financial markets and dismantle capital controls. The second is the increasing frequency and intensity of currency and financial crises, with a number of countries facing speculative attacks on their fixed exchange rates or panicked foreign creditors worried about possible defaults. The third trend is the increasing number of countries reporting a switch to flexible exchange rates. How have these trends affected central bank reserve holdings? Are the determinants of international reserve holdings in a world of high capital mobility different from the earlier era?

Our paper is organized as follows. In Section II, we examine some stylized facts about global and country-specific reserve holdings. In Section III, we discuss the buffer stock model of reserve holdings introduced by Frenkel and Jovanovic (1981). This model says that central banks choose an optimal level of reserves to balance the macroeconomic adjustment costs incurred in the absence of reserves with the opportunity cost of holding reserves. Reserve holdings turn out to be a stable function of just a few variables—the adjustment cost, the opportunity cost and reserve volatility. We study the empirical application of the buffer stock (inventory) model when we attempt to replicate it and then extend it using more recent data.

In Section IV, we argue that earlier methods for taking the buffer stock model to data may have been misguided. In Section V, we propose a different empirical approach for testing the buffer stock model that is consistent with the current experience of high capital mobility and periodic currency crises. It is also consistent with the view that reserve movements are an endogenous response to central bank and private sector behavior. We provide a new measure of the volatility that affects the central bank decision to hold reserves. This volatility measure captures the increasingly important phenomenon of nominal (financial) uncertainty.

In Section VI, we test the buffer stock model of optimal reserve holdings using our new measure of volatility. We also make use of market-determined interest rates that have appeared in the aftermath of financial liberalizations to construct a measure of the opportunity cost of holding reserves. In Section VII, we consider some factors not identified specifically by the buffer stock model that might affect adjustment costs, such as the country's degree of exchange-rate flexibility and its financial and real-side openness.

Our empirical work shows that the performance of the buffer stock model in explaining reserve holdings in the 1990s is mixed. The buffer stock model's prediction that international reserve holdings increase with higher volatility is quite robust. That said, most of the variation in reserve holdings is "explained" by country-specific adjustment costs (fixed effects) that have, to date, not been made explicit. The remaining challenge is to explain these cross-country variations in reserve holdings. Section VIII concludes.

II. A Descriptive Look at Reserve Holdings

We now take a look at international reserve holding patterns over time, both globally and for specific country groups. Before we do so, however, we need to address some measurement issues.

The first measurement issue concerns the definition of reserves. In an oft-cited article, Robert Heller (1966) wrote that international reserves must possess two qualities. First, "they must be acceptable at all times to foreign economic units for payment of financial obligations." Second, "their value, expressed in foreign units of account, should be known with certainty." (Heller (1966), pp. 296-97). Using Heller's definition, the four types of assets that qualify are official holdings of gold, special drawing rights (SDRs), convertible foreign exchange, and the unconditional drawing rights with the IMF (the country's reserve position in the Fund).[8]

Some economists in the 1960s (e.g., Kenen and Yudin (1965)) puzzled over the merits of adjusting this measure of reserve assets to account for public and private liabilities towards foreigners. They concluded that there were no acceptable criteria for including or excluding the many types of these liabilities. For practical reasons, then, almost all studies thereafter concentrated on gross foreign reserve assets.

Even though gold holdings were a significant share of monetary authorities' reserve assets under the Bretton Woods system, they have declined in relative importance since then.[9] Probably for that reason, most empirical studies assessing the determinants of financial crises in the 1980s and 1990s have used a reserves measure that excludes gold. Yet if we are to examine patterns of reserve holdings over the last fifty years, it is important that gold be included. Even though official gold holdings accounted for less than 3 percent of international reserve holdings at the end of 1999, in 1960 they comprised about two-thirds of reserve holdings.[10]

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[8] The reserve position in the Fund comprises the reserve tranche position and has the characteristics of a reserve asset. Technically, the reserve tranche position "arises from the payment of part of a member's subscription in reserve assets and the Fund's net use of the member's currency. Normally, a member's reserve tranche position is equal to its quota less the adjusted Fund holdings of its currency, less subscriptions receivable, less the balances held in the administrative accounts of the Fund." (IMF, International Financial Statistics, June 2001, p. xiv and other issues.)

[9] The Second Amendment to the Articles of Agreement of the International Monetary Fund, which came into effect in 1978, eliminated the special role of gold.

[10] IMF, International Financial Statistics.

With gold included in the reserves measure, one must decide how to value it. Under the Bretton Woods system, official gold holdings were valued at $35 an ounce. Eventually the SDR replaced the U.S. dollar as the conversion rate. We follow current IMF convention and value official gold holdings at SDR 35 per ounce.[11]

In our examination of data, we shall define the monetary authority's international reserve holdings in the standard way—as the sum of gold (valued at SDR 35 per ounce), SDRs, foreign exchange, and reserve position in the Fund. We shall denominate them in end-of-period billions of U.S. dollars

Finally, individual countries' reserve holdings cannot be compared or traced through time unless they are scaled in some way to reflect differences in countries' size. One possibility is to scale reserves by GNP, and we shall do so in a number of the time-series comparisons below. In our later empirical work we will investigate several scaling methods.

It is worth noting, however, that in the early post-war period, the most widely used scale variable was imports. The reserves-to-imports ratio was thought to be targeted by countries as part of their reserve-management policies. The rationale for this scaling variable was never fully justified. Grubel (1971), in his survey of the early reserves literature, suggested that the choice was influenced by the quantity theory of money. Since private persons and governments required cash to even out receipts and payments, and in the case of the private sector the volume of receipts and payments was measured by national income, the analogous measure for governments was thought to be imports. Even though the analogy was imperfect, even at the time, the reserves-to-imports ratio is still used today in popular discussions of reserve "adequacy."

The notion of "reserve adequacy" changed with the onset of currency and financial crises in the 1990s.[12] Calvo (1996) suggested that a country's vulnerability to crisis should be measured, in part, by the size of its money supply, defined broadly, relative to its reserve holdings, since broad money reflects a country's potential exposure to the withdrawal of assets. In that case, broad money, or M2, would be an appropriate scaling variable. Empirical crisis-prediction models have shown that the ratio of short-term foreign-currency debt in relation to reserves was an important determinant of a country's vulnerability to financial crisis in the 1990s.[13] That would suggest reserves scaled by short-term foreign-currency debt

[11] For an alternative way of valuing gold using the London average second fixing rate, see Bussiere and Mulder (1999).

[12] The post-1980 speculative attack literature has emphasized the point that "reserve adequacy" is different for different policy packages. See e.g., Flood and Marion (1999).

[13] See, for example, Radelet and Sachs (1998).

should be an important variable to monitor over time. The idea of scaling reserves by some foreign liability measure brings us full circle to the debates in the 1960s about whether to report reserves in gross terms or in net terms that are free of claims on them.

We now turn to the data. Figures 1 and 2 in the previous section showed that global international reserves (excluding gold) have been increasing since 1960, whether they are measured in terms of weeks of import cover or as a share of income. We now examine the trend of international reserve holdings over time using a more complete measure of reserves that includes gold holdings.

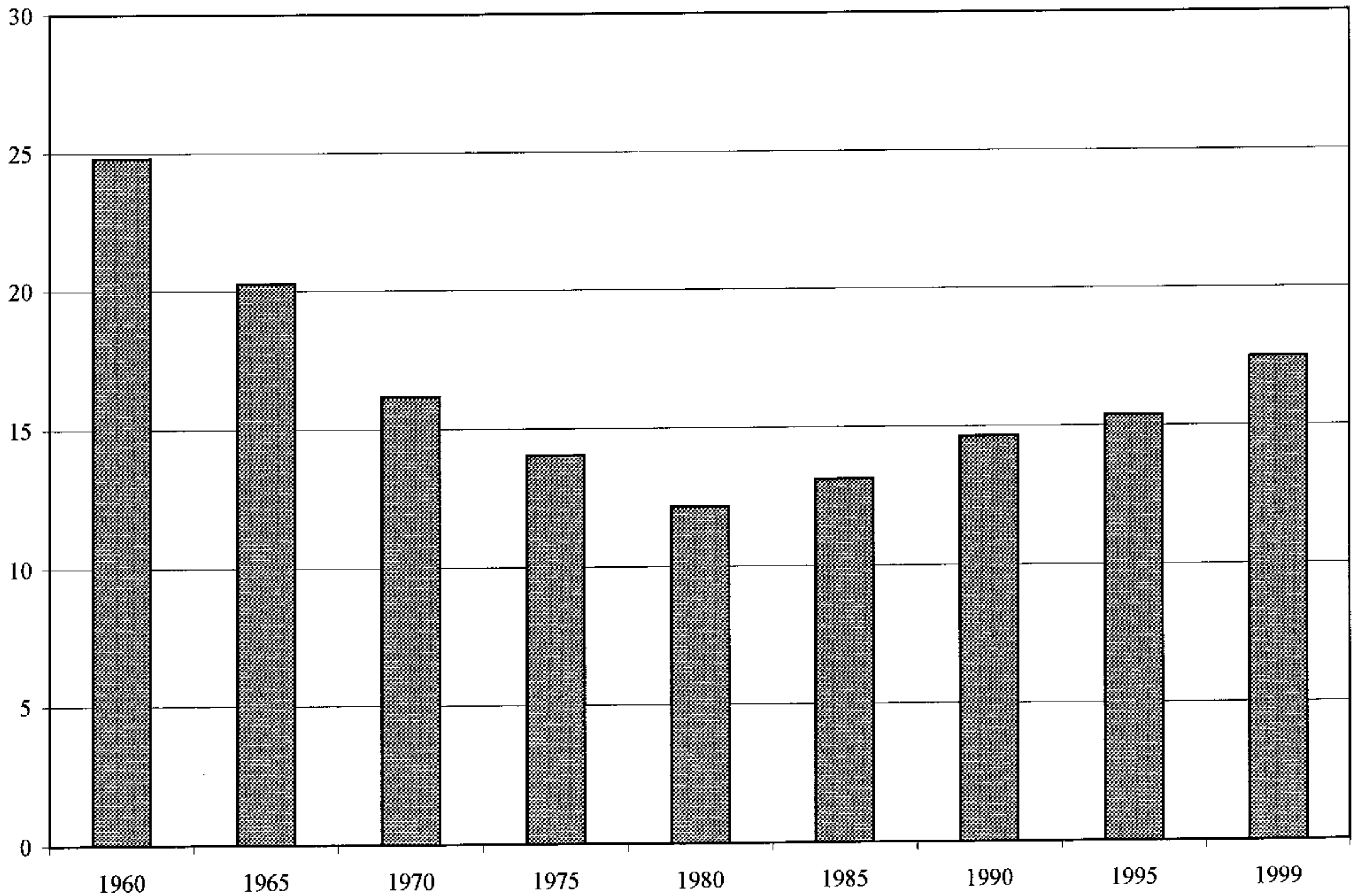
Figures 3 and 4 reveal that when gold is included in the reserves measure, global reserve holdings in terms of weeks of import cover or as a share of world GDP exhibit a different pattern. In terms of weeks of import cover (Figure 3), international reserve holdings have actually fallen over time. They covered 24.8 weeks of imports at the end of 1960 and only 17.5 weeks of imports at the end of 1999. The decline has not been smooth. From their peak in 1960, reserves in terms of weeks of import cover fell by more than half over the next twenty years, reaching their lowest point in 1982, the first year of the international debt crisis. Then they began to climb again. Between 1982 and 1999, reserve holdings in terms of weeks of import cover increased by 54 percent, from 11.4 weeks of import cover at the end of 1982 to 17.5 weeks at the end of 1999.

Figure 4 shows that reserves were 6.3 percent of global income at the end of 1999, about 1 percent higher than at the end of 1960. As in the previous figure, reserve holdings have exhibited a U-shaped pattern over the past 40 years.

A problem with the data pictured in Figures 1-4 is that the global totals come from an unbalanced sample. A few countries, such as China, are not included in the early years but are in the sample for the last twenty years. Although scaling probably minimizes the distortions created by an unbalanced sample, we nevertheless want to examine the same data for a balanced sample of countries

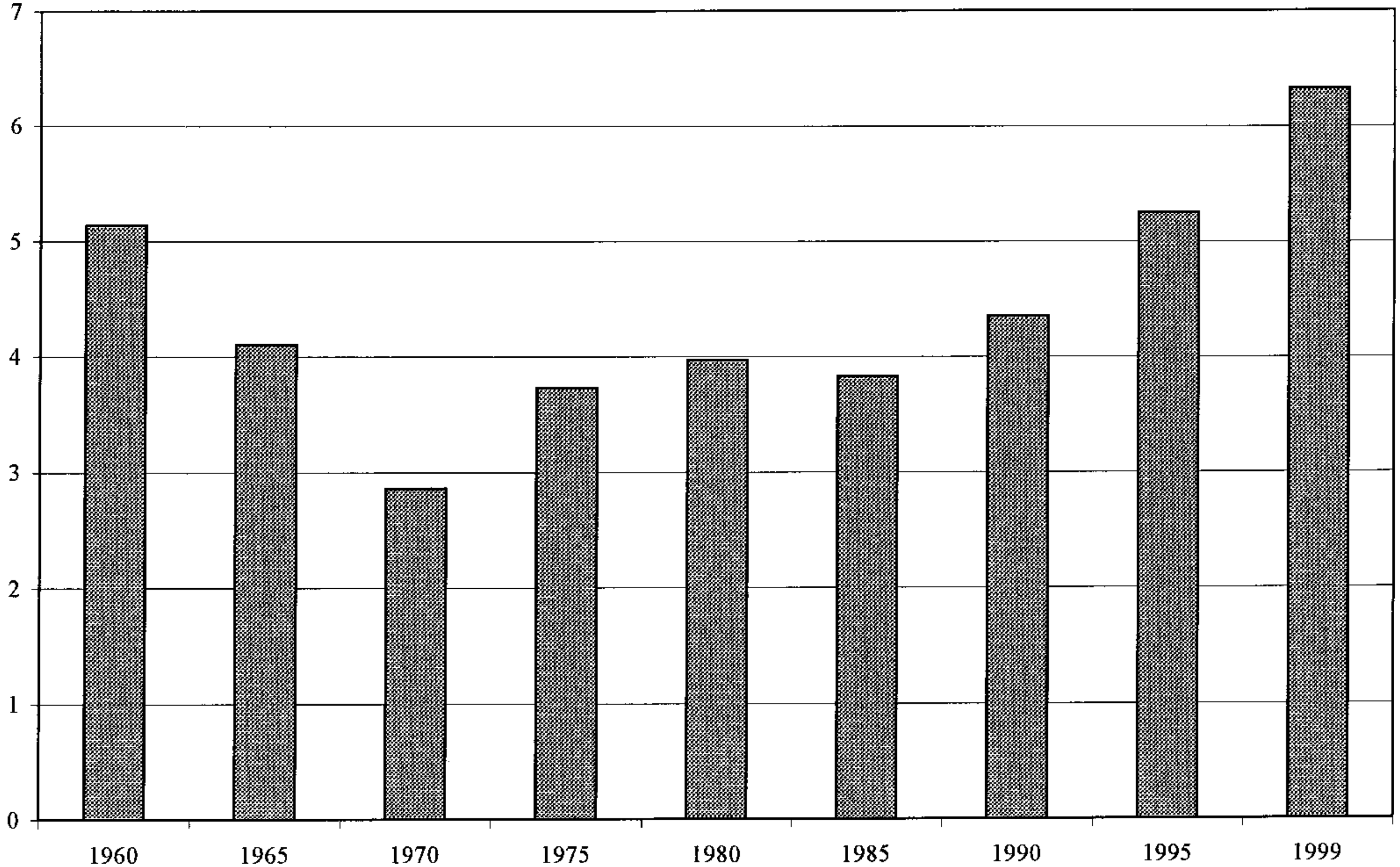
Figure 5 illustrates reserve holdings for 44 countries between 1960 and 1998 for which we have continuous data. These 44 countries accounted for 57 percent of global reserve holdings at the end of the 1990s. To facilitate comparison with our earlier figures, reserves are measured both excluding and including gold and reserves are scaled both by weeks of import cover and by GNP.

Figure 3. Global Reserves Including Gold
Numbers of Weeks' Import Cover



Source: IMF

Figure 4. Global Reserves Including Gold
As percentage of World GDP



Source: IMF.

Notes: World GDP unavailable for 1960s; 1960 and 1965 ratios use sample of countries accounting for over 90% of global reserve holdings in 1960s.

The patterns in reserve holdings are the same for the 44-country sample and the global sample. Reserves excluding gold show a marked increase over the last forty years, whether scaled by weeks of import cover or by income. When gold is included in the reserve measure, the decline in reserve holdings after1960 is eventually reversed. In both the global and 44-country samples, reserves, including gold, as a share of weeks of import cover are lower now than they were in 1960. As a share of income, they are closer to where they were in 1960.

Figure 6 shows data on reserve holdings over time for a number of specific countries. Reserves are measured with gold and scaled by GNP. The time period is more extensive. It runs from 1948 through 1998. The figures do not reveal any clear pattern across countries. For the developing countries, however, there does seem to be an increase in reserves as a share of national income over the last twenty years. With some notable exceptions, reserve holdings in developed countries have not fallen much over the same interval.

Table 1 shows reserve holdings for a set of 56 countries, 22 developed and 34 developing, over various decades and includes information on a group of emerging markets. In the table, reserves (including gold) are scaled by GNP, by imports and by M2. The table confirms the picture that emerges from the earlier figures. Using our more complete measure of reserves, which includes gold, we find that reserve holdings for developed countries, when scaled by GNP, imports or M2, have not changed appreciably over the last 30 years. In all cases, their reserve holdings are lower in the 1990s than in the Bretton Woods period of 1948-70.

For developing countries, reserve holdings have increased over the last three decades, whether scaled by income or imports, but have not changed much when scaled by M2. Compared with the Bretton Woods period, developing countries held more reserves as a share of income or import cover in the 1990s. The growth in reserve holdings over the last two decades has been particularly strong.

The trends for emerging markets are similar to those of developing countries, only more dramatic. Between the 1980s and 1990s, reserve holdings as a share of income have more than doubled. Reserves as a share of both imports and M2 have increased by almost 60 percent.

In the appendix, we discuss reserve patterns for some interesting case studies, namely Taiwan Province of China, South Korea, and China.

Table 1. Reserves Including Gold by Country Category

	Category	Period Average 1948-70	1971-80	1981-90	1991-99
Reserves/GNP (percentage)	All	8.2	7.6	6.6	10.5
	Developed	9.3	7.3	7.0	7.7
	Developing	7.3	7.8	6.3	12.4
	Emerging	5.5	5.8	6.0	14.9
Reserves in Weeks Import Cover	All	22.5	17.9	15.4	20.6
	Developed	23.0	16.1	13.9	16.6
	Developing	22.1	19.2	16.4	23.3
	Emerging	19.7	19.7	17.6	27.6
Reserves/M2 (percentage)	All	31.3	20.5	15.2	21.2
	Developed	17.8	12.0	11.7	11.3
	Developing	37.7	25.1	17.1	26.6
	Emerging	28.7	23.5	19.1	29.9

Source: International Monetary Fund.

Note: The "all" category includes data for 56 countries. The "all" category is separated into 22 "developed" countries and 34 "developing" using the IMF Classification in 1979. Developed countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, and United Kingdom. Developing countries are: Argentina, Brazil, Chile, China, Colombia, Costa Rica, Dominican Republic, Egypt, El Salvador, Ghana, Guatemala, Honduras, Hong Kong SAR, India, Indonesia, Israel, Jamaica, Jordan, Korea, Malaysia, Mexico, Myanmar, Nicaragua, Pakistan, Panama, Paraguay, Peru, Philippines, Sudan, Thailand, Tunisia, Turkey, and Sri Lanka. The separate category of "emerging" markets in the 1990s is made up of the following 10 countries: Brazil, Chile, Colombia, Hong Kong SAR, Korea, Mexico, Peru, Philippines, South Africa, Thailand. Coverage for developing and emerging variable.

Figure 5. Reserves Holding in 44 Countries

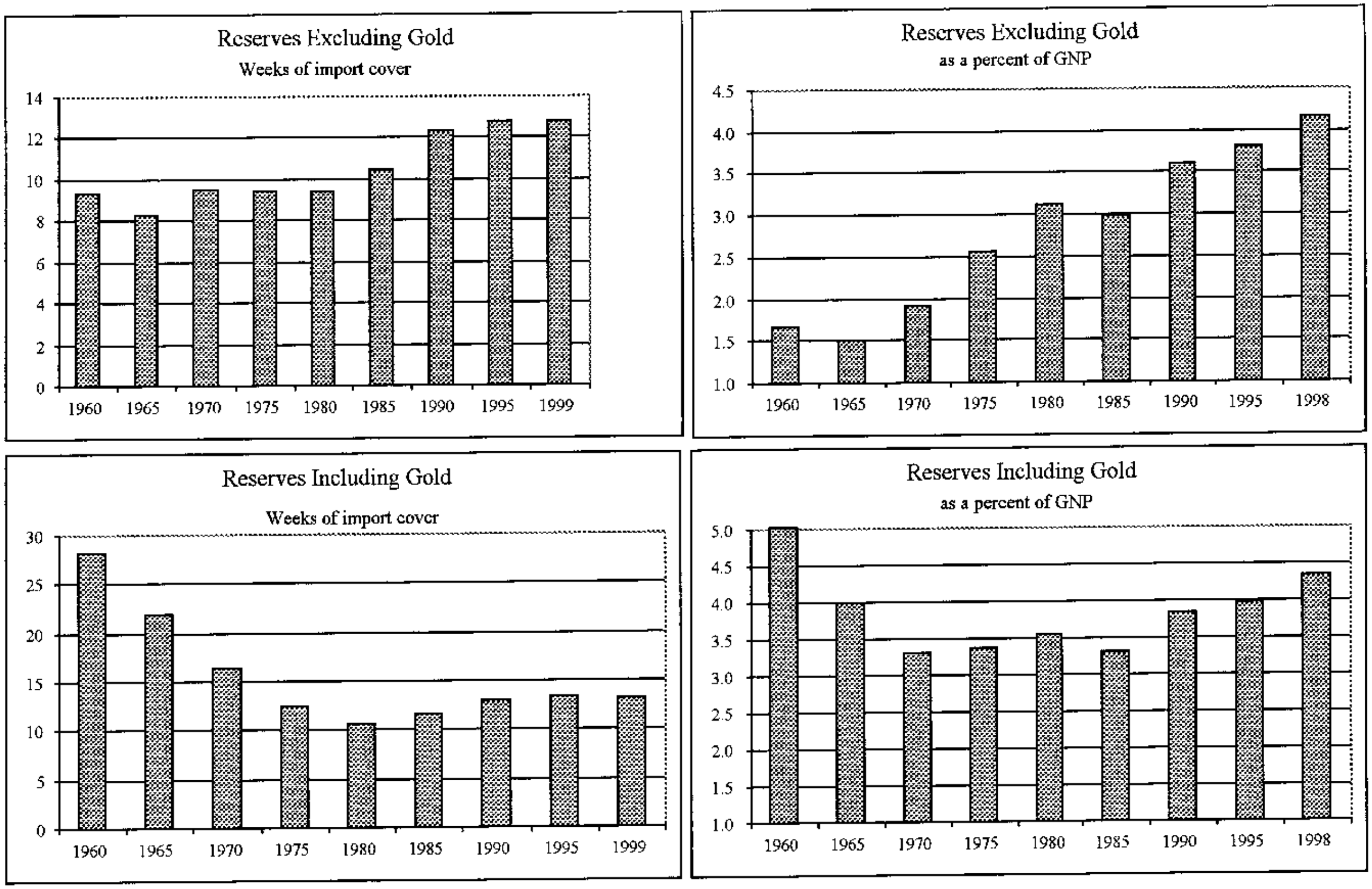


Figure 6. Reserves/GNP in International Countries

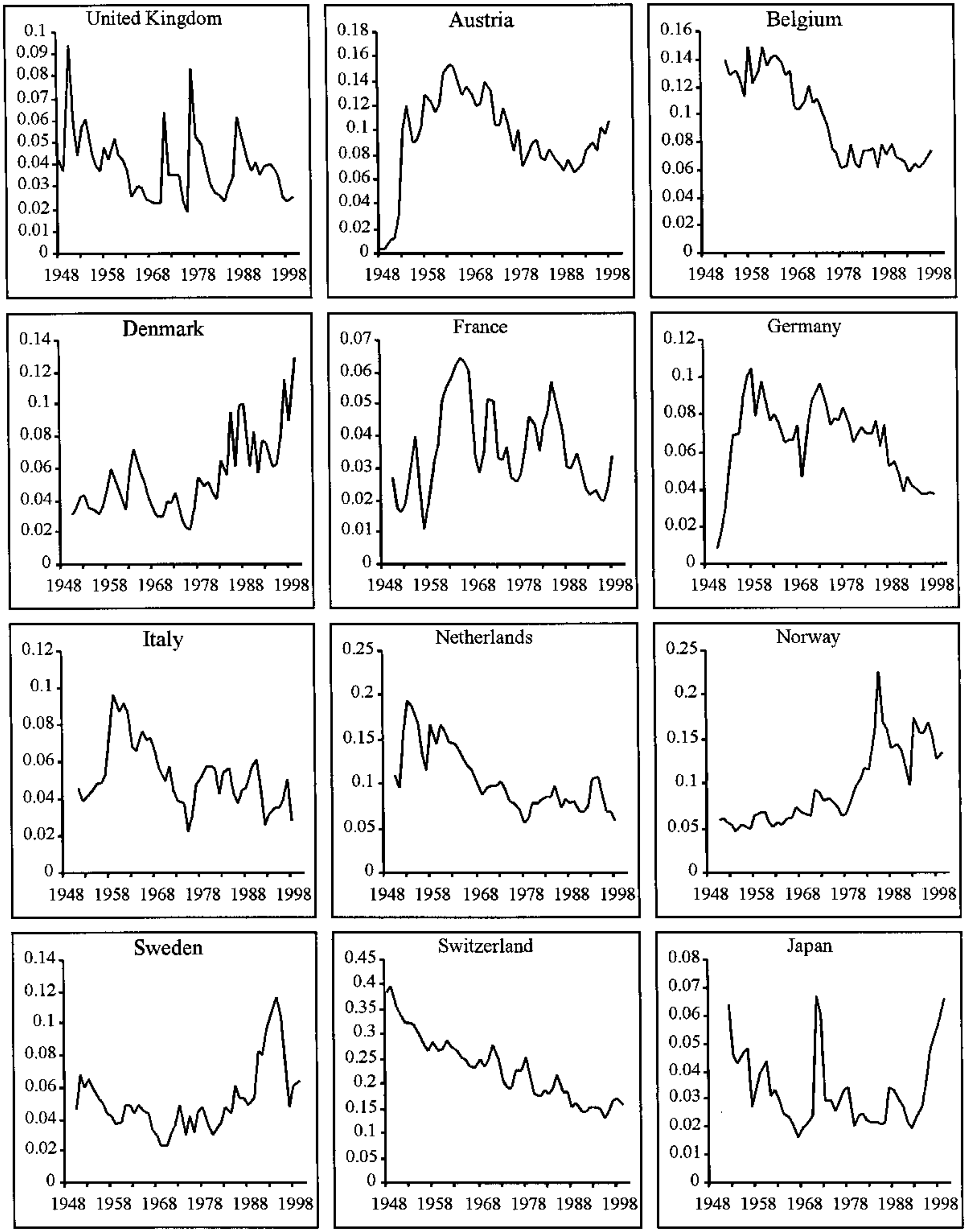


Figure 6 (Continued). Reserves/GNP in International Countries

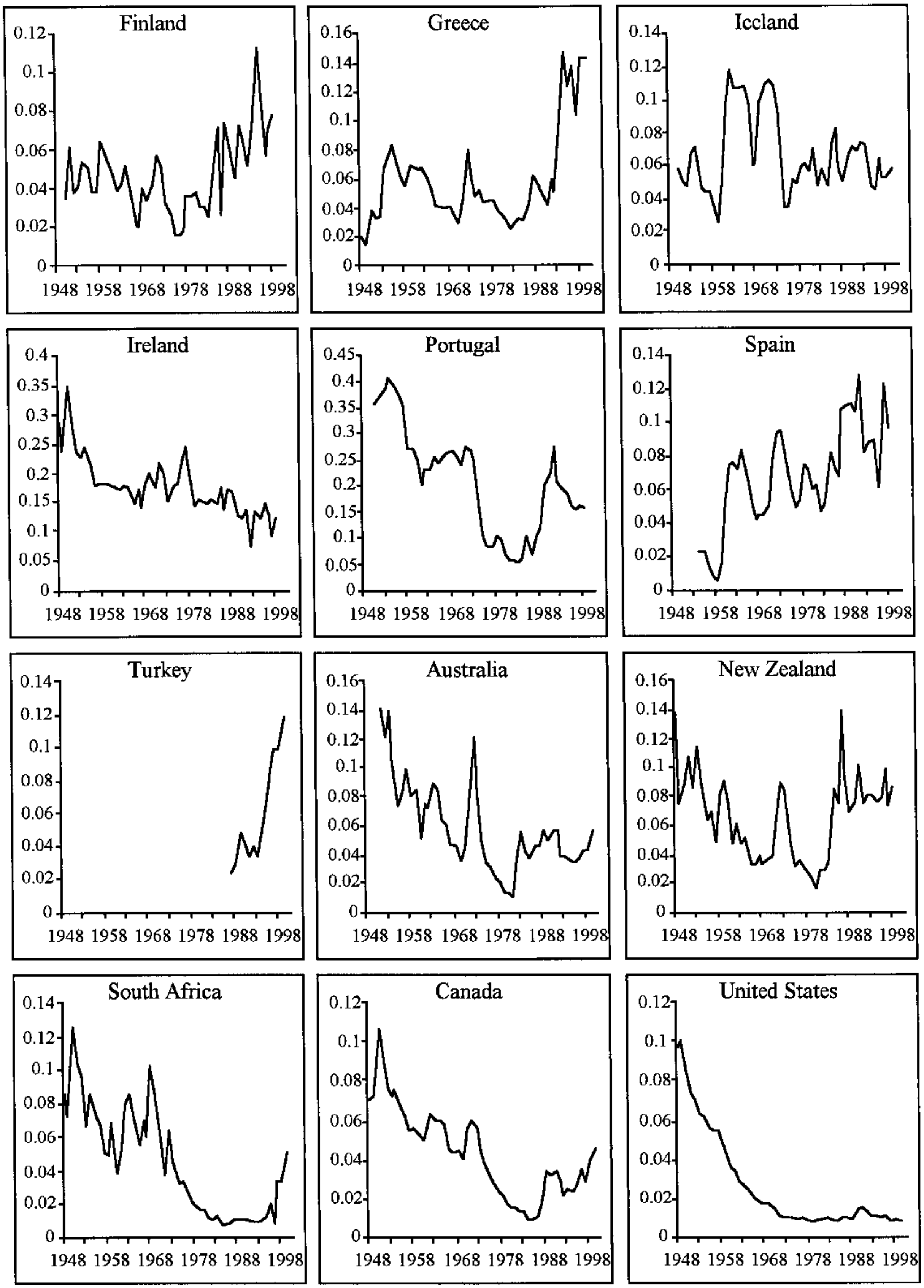


Figure 6 (Continued). Reserves/GNP in International Countries

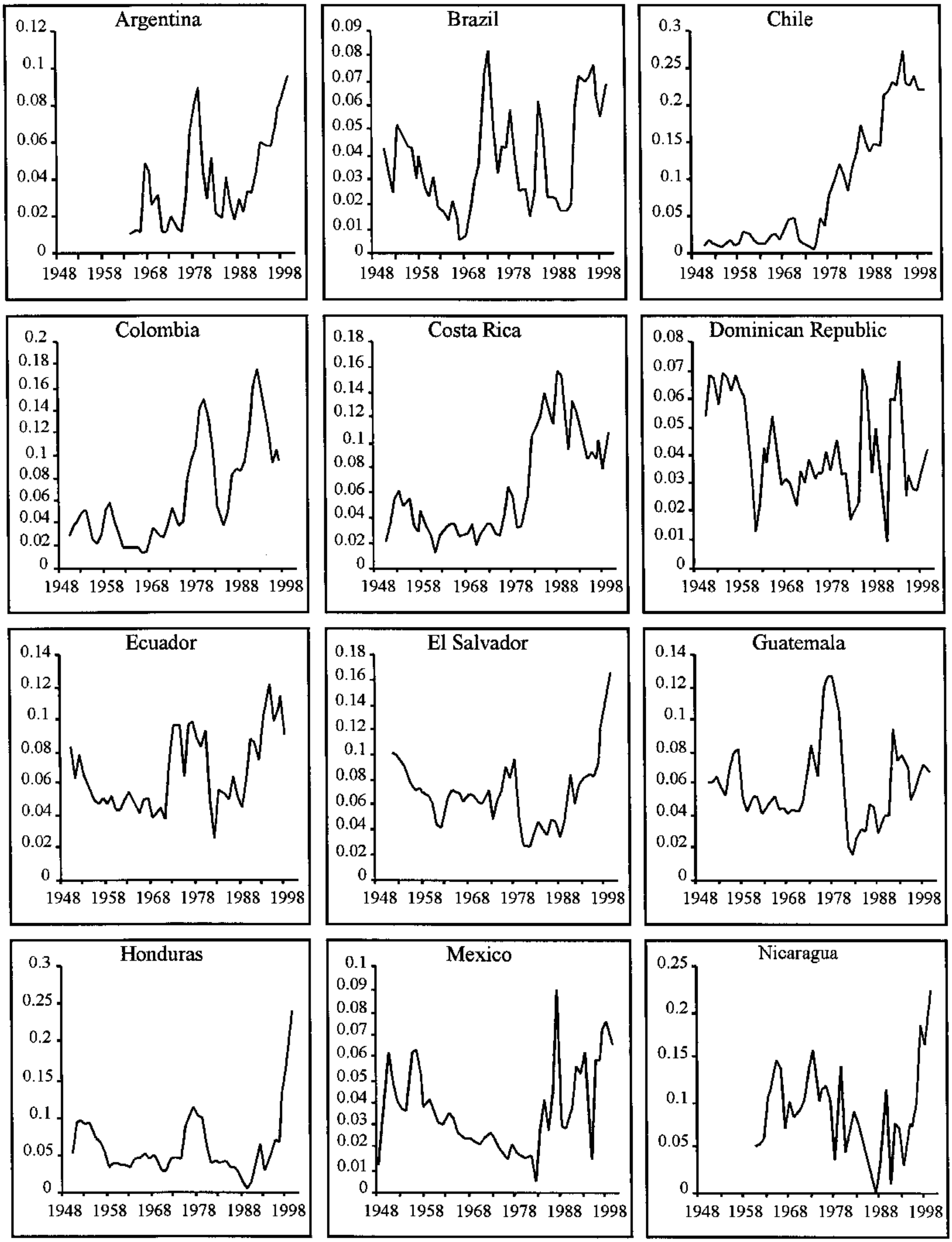


Figure 6 (Continued). Reserves/GNP in International Countries

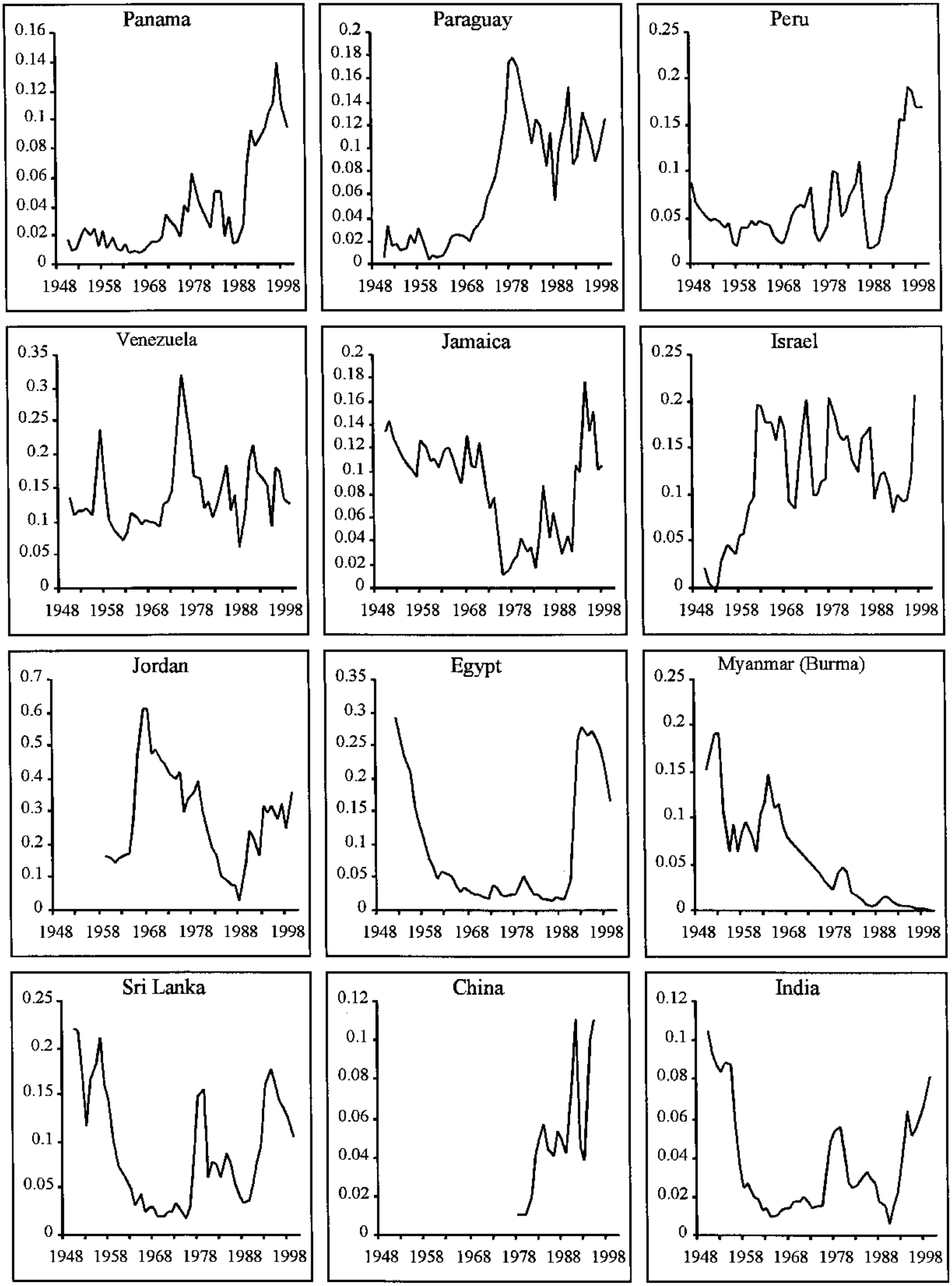
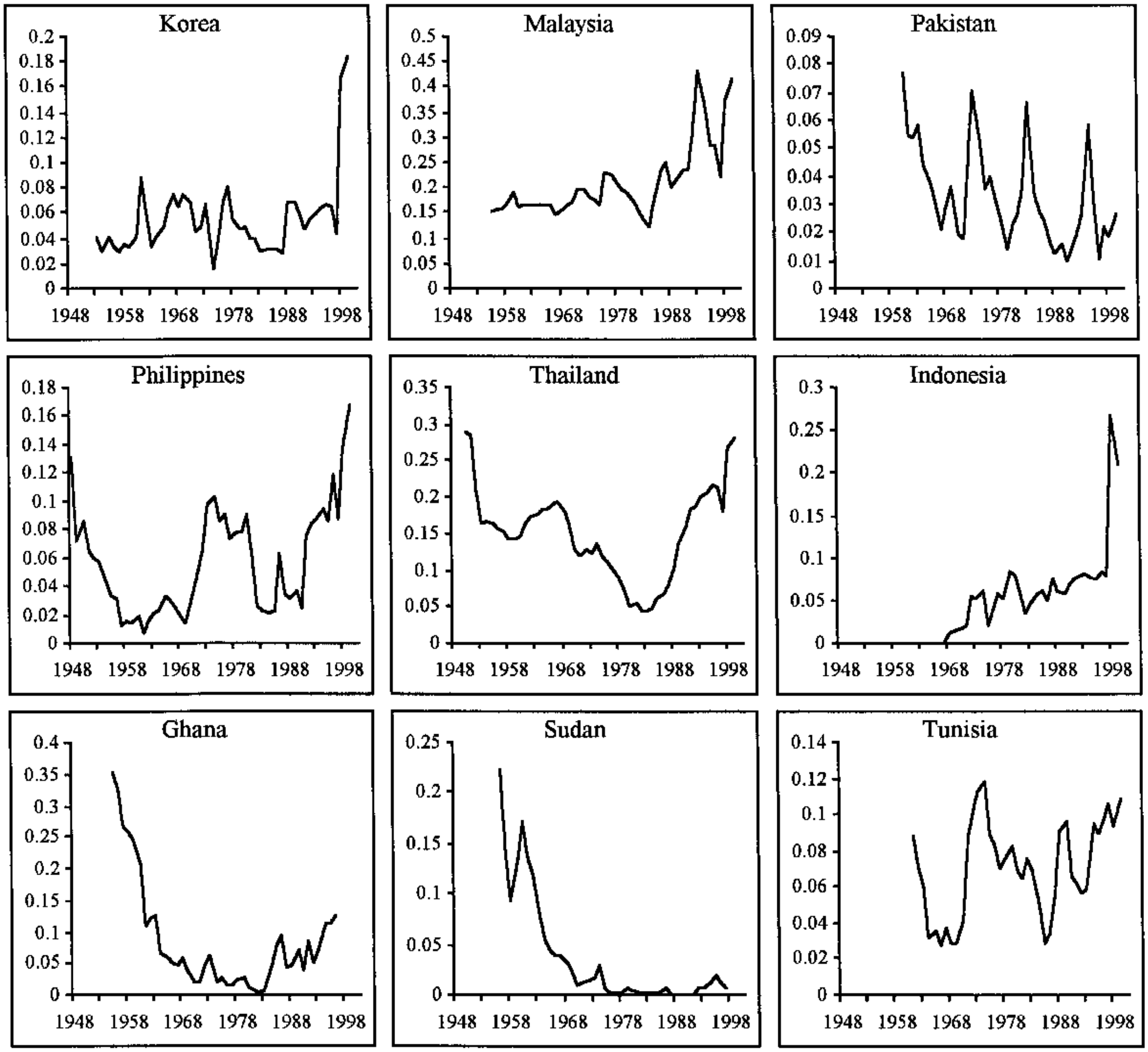


Figure 6 (Concluded). Reserves/GNP in International Countries



III. The Buffer Stock Model

The buffer stock, or inventory, model has been remarkably successful in explaining international reserve holdings in the post-World War II period. The model postulates that the reserve authority will choose an initial level of reserve holdings that minimizes its total expected costs. The model identifies two costs incurred by the reserve authority. The first is the opportunity cost of holding reserves. The second is the adjustment cost that is incurred whenever reserves reach some lower bound. The adjustment cost is interpreted generally to be the output or welfare forgone by having to take other, costly, policy measures to generate the external payments surplus necessary for reserve accumulation.

The two costs are interrelated since a higher stock of reserves reduces the probability of having to adjust and thus reduces the expected cost of adjustment, but this benefit comes at the cost of higher forgone earnings. Optimal reserve management involves finding the cost-minimizing level of reserves to acquire once reserves have reached their lower bound. Recall that the basic idea in inventory management models is to optimize the trade-off between flow holding costs and fixed restocking costs.

Miller and Orr (1966) were the first to model desired money holdings in a stochastic inventory-theoretic framework.[14] Frenkel and Jovanovic(1981) applied this inventory-theoretic approach to international reserve management.

Frenkel and Jovanovic (FJ) hypothesized that reserve movements between the occasional restockings are generated by an exogenous Wiener process, where the incremental change in reserves in a small time interval is distributed normally.[15] Frenkel and Jovanovic also assumed that the deterministic part of the incremental change in reserves is a negative drift while the stochastic part is without drift. They set the lower bound for reserves at zero.

In the special case of no reserve drift between stock adjustments, a second-order Taylor-series approximation of optimal reserve holdings yields the following equation for reserves:

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[14] The basic (nonstochastic) model of the demand for money was developed by Baumol (1952) and Tobin (1956).

[15] Hamada and Ueda (1977) treated reserves as a random-walk process between restocking periods. A random walk process is the discrete-time analogue to the continuous time Wiener process. Kenen and Yudin (1965) also specified reserves as a random walk process. In addition, they suggested that central bank reserve holdings are sensitive to the volatility in the balance of payments rather than the absolute size of the gap between international payments and receipts, as in Heller (1966). Claassen (1965) used an inventory model of international reserve choice to clarify the nature of restocking costs.

$$R_0 = \sqrt{\frac{C\sigma}{r^{0.5}}} \tag{1}$$

where R_0 is the optimal starting level for international reserves after restocking, C is a country-specific nominal constant capturing the fixed cost of adjustment, σ is the standard deviation of the Wiener increment in the reserves time-series process operating between stock adjustments and r is the opportunity cost of holding reserves.

Equation (1) shows that, in the buffer stock model, optimal reserve holdings increase with the volatility of reserves (σ). Higher volatility means that reserves hit their lower bound more frequently. The reserve authority is therefore willing to restock a larger amount of reserves and tolerate greater opportunity costs in order to incur the adjustment cost less frequently. Equation (1) also shows that a bigger adjustment cost increases optimal reserve holdings while a higher opportunity cost reduces them.

Equation (1) is expressed in the familiar square-root form. More useful for empirical work is the log transformation:

$$\ln R_0 = c_0 + 0.5 \ln \sigma - 0.25 \ln r \ . \tag{1'}$$

In their empirical work, FJ turned equation (1') into an estimating equation that is amazingly successful. A key step in taking equation (1') to data is the additional assumption that observed reserves, R_t, are proportional to optimal reserves up to an error term that is uncorrelated with σ and r. Hence, $R_0 = BR_t\, e^{-u}$. The estimating equation becomes:[16]

$$\ln R = b_0 + b_1 \ln \sigma + b_2 \ln r + u \tag{2}$$

In equation (2), international reserves are defined as the sum of gold, Special Drawing Rights, foreign exchange and reserve position at the IMF. FJ defined R in nominal terms. They generated σ by computing for each year the standard deviation over the previous 15 years of the trend-adjusted annual changes in the stock of international reserves. To obtain a variability measure that was free of scale, FJ divided the standard deviation by the value of imports. The opportunity cost of holding reserves, r, was approximated by a country's government bond yield. The constant b_0 is interpreted to be country specific and regime specific. It is nominally denominated and incorporates country-specific adjustment costs and the possibly country-specific proportionality factor, B.

[16] In data, of course, $R_0 = BR_t\, e^{-u}$ need not hold. We return to this point below.

In their work, FJ, and later Frenkel (1983), estimated equations exactly and closely related to equation (2) using ordinary least squares on various cross-section and panel data sets. Before we begin our effort to replicate, update and modify their work, we remind the reader of one equation estimated in FJ across 22 developed countries over the 1971-75 period:[17]

$$\ln R = b_0^i + 0.505\ln\sigma - 0.279\ln r \qquad (3)$$
$$\qquad\qquad (0.110) \qquad (0.149)$$

$$R^2 = .97,\ \mathrm{n} = 110,\ \mathrm{S.E.} = 0.234$$

where (OLS) standard errors are in parentheses and the country-specific constant terms range between 3.42 and 6.78. By the standards we are used to in international macroeconomics, the above equation is nothing short of miraculous. The estimated elasticities of reserve holdings with respect to σ and r are very close to the predictions of the theoretical model given by equation (1').

We now try to replicate the FJ regression in equation (3) using revised data from the IMF's International Financial Statistics. Our replication will not be exact because of a scaling issue. Many early estimates of reserve holdings included a scale variable such as income or imports as a separate regressor. From equation (1), we know that under the null, when we scale R_t by (say) Y_t, then we scale the right-hand side such that $R_t/Y_t = \sqrt{\frac{(C_t/Y_t)(\sigma_t/Y_t)}{r^{0.5}}}$.[18] We therefore scale the dependent variable as well as the volatility measure and (implicitly) the constant term, rather than adding a separate scaling regressor.

We reestimate equation (2) trying several different scaling variables. Table 2 reports results using four of them: None, Real (price level), GNP, and (nominal) Imports. All of the equations are estimated with and without country fixed effects and they are estimated over three different periods, 1971-75 (as in FJ), 1976-97, and 1971-97.[19]

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[17] Countries included in the regression are listed at the bottom of Table 2. All data used by FJ were taken from the IMF's *International Financial Statistics*.

[18] The reader should note that our specification now sets $C=C_t$, which we implicitly hypothesize to be proportional to whichever scaling variable we use. Recall that C has nominal units in the FJ model, so it makes sense that it be proportional to a nominal quantity.

[19] Frenkel (1980), using a similar reserve-holding equation, found a structural break around the demise of Bretton Woods in 1971-73. Lizondo and Mathieson (1987) found another structural break around the 1982 international debt crisis. We ignore these breaks.

Table 2. Replication and Extension of Frenkel and Jovanovic (1981) Study

Scaling	Sample Period	No Fixed Effects							Fixed Effects						
		Reserve Vol.		Interest Rate		R-sq.	R-sq. adj.	Std. Error	Reserve Vol.		Interest Rate		R-sq.	R-sq. adj.	Std. Error
None	1971-75	0.9486	***	-0.8127	***	0.8259	0.8226	0.5548	0.2674	***	-0.2573	**	0.9721	0.9646	0.2478
		(0.0709)		(0.2400)					(0.0889)		(0.1302)				
	1976-97	0.9565	***	-0.2149	***	0.8800	0.8795	0.5129	0.9093	***	0.0462		0.9338	0.9304	0.3898
		(0.0311)		(0.0773)					(0.0439)		(0.1004)				
	1971-97	0.9194	***	-0.2979	***	0.8782	0.8778	0.5286	0.8106	***	-0.0775		0.9362	0.9336	0.3896
		(0.0261)		(0.0713)					(0.0242)		(0.0767)				
Real	1971-75	0.9553	***	-0.8272	***	0.8250	0.8217	0.5557	0.1787	*	-0.5056	***	0.9725	0.9651	0.2459
		(0.0719)		(0.2396)					(0.0965)		(0.1233)				
	1976-97	0.9509	***	-0.2179	***	0.8658	0.8652	0.5126	0.7837	***	0.0051		0.9286	0.9250	0.3824
		(0.0319)		(0.0763)					(0.0590)		(0.0977)				
	1971-97	0.9192	***	-0.3022	***	0.8538	0.8533	0.5314	0.6624	***	-0.1158		0.9279	0.9250	0.3800
		(0.0295)		(0.0724)					(0.0388)		(0.0745)				
GNP	1971-75	0.5877	***	-0.9102	***	0.3649	0.3531	0.5233	0.0736		-1.0815	***	0.8770	0.8441	0.2569
		(0.1781)		(0.3003)					(0.1303)		(0.1408)				
	1976-97	0.8394	***	-0.1797	***	0.4673	0.4651	0.5083	0.4398	***	0.0262		0.7739	0.7625	0.3387
		(0.0715)		(0.0618)					(0.0548)		(0.0806)				
	1971-97	0.7793	***	-0.2498	***	0.4155	0.4135	0.5280	0.4187	***	-0.1013		0.7416	0.7312	0.3575
		(0.0685)		(0.0644)					(0.0509)		(0.0666)				
Imports	1971-75	0.5592	***	-0.9741	***	0.4714	0.4615	0.4926	0.2159		-1.5422	***	0.8535	0.8143	0.2892
		(0.0939)		(0.2485)					(0.1604)		(0.1449)				
	1976-97	0.5919	***	-0.2170	***	0.3595	0.3568	0.4700	0.4855	***	-0.0487		0.6683	0.6517	0.3458
		(0.0695)		(0.0593)					(0.0592)		(0.0717)				
	1971-97	0.5750	***	-0.2873	***	0.3566	0.3544	0.4908	0.5154	***	-0.1843	***	0.6347	0.6199	0.3766
		(0.0602)		(0.0610)					(0.0511)		(0.0673)				

Note: Standard errors (heteroskedasticity- and autocorrelation consistent via GMM) are reported in parentheses. *, ** and *** denote significance at the 10 percent, 5 percent and 1 percent level, respectively. Countries included are: Austria, Australia, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, South Africa, Sweden, Switzerland, Turkey, and United Kingdom. All panels are balanced (except for one missing interest rate observation for Iceland in 1993).

We interpret our replication results in Table 2 as consistent with those of FJ; our approach differs from theirs primarily in that we use different scaling methods and we calculate GMM-style standard errors for estimated coefficients. In our replication, reserve volatility still positively influences reserve holdings, with an elasticity of 0.5 or more in almost all of the regressions. The interest rate has a negative and significant coefficient in all regressions with no fixed effects, but with fixed effects, this coefficient is neither reliably negative nor significant in the later sample periods. Finally we note from comparing the adjusted R^2's of the fixed-effects regressions with those of the no-fixed-effects regressions that the non-constant explanatory variables are consistently picking up 15-50 percent of the variation of the dependent variables. Further, with one exception, the point estimates of R^2 rise between the earlier and later sample periods for the no-fixed-effects regressions. The R^2 estimates fall for the later sample periods once fixed effects are included.

The FJ-style reserve equations have, in our view, held up well when confronted with new data. We have some reasons to be unsure about our interpretation of these results, however.

IV. Rethinking the Empirical Implementation of the Buffer Stock Model

Under the FJ null hypothesis, international reserves follow a Wiener process until they hit the lower boundary—zero. Then, in a one-step adjustment, reserves jump back up to their optimal restocking level and commence once again to follow the Wiener process.

In empirical applications, FJ constructed a measure of reserve volatility using a 15-year rolling standard deviation, with no attempt to separate typical incremental volatility (the Wiener increments) from the relatively large upward restocking adjustments that take place from time to time under the null hypothesis.[20] Indeed, there is nothing in widely used reserve data sources (e.g., IFS) that allows researchers to separate private-sector-induced reserve increments from the reserve-management authorities' adjustments. Including the big upward adjustments imparts possible positive skewness to the reserve increments measure (under the null) and leads to a potential upward bias in the estimated coefficient on volatility. We provide a simple example of this bias in the technical appendix. The potential upward bias is an important issue. The primary research objective of the buffer-stock models is to estimate the extent to which volatility matters for reserve holdings.[21]

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[20] Other researchers used different measures for the rolling reserve-increment standard deviation, see, e.g. Lizondo and Mathieson (1987).

[21] FJ acknowledged the problem associated with reserve-increment measurements. In a footnote, they wrote: "Since we do not know the exact time at which actual adjustment took place, it is possible that our proxy of σ is biased upwards, since it may reflect in part reserve changes that are associated with a recent adjustment. (FJ, 1981, p. 510).

Skewness induced by periodic adjustments by the reserve authority is potentially serious, but it is only part of the problem. The data that FJ studied were generated largely during an era of relatively low capital mobility, with few speculative attacks in the data and no models of speculative attacks in the literature. The speculative attack models of Salant and Henderson (1978) and Krugman (1979) were very new when FJ wrote. The models, however, were prophetic for reserve movements after 1980. Large modern-day reserve increments seem about as likely to be generated by speculative attacks (big downward shocks) as by occasional macro-based restocking of reserve inventories.

We thus confront two difficulties. First, currency and financial crises generate large negative reserve increments before reserves hit their lower bound, violating the assumption that reserves follow a Wiener process up until they hit the barrier. Crises impart negative skewness to the reserve increment measure. Second, empirical methods that rely on multi-period rolling averages to calculate reserve volatility will capture the large positive reserve increments that characterize reserve restocking. Including the restocking increments imparts positive skewness to the reserve increment measure. If we, as researchers, were really lucky, the two types of big reserve shocks would just cancel out and we could proceed to update FJ's work for the 1980s and 1990s, blithely ignoring these skewness issues.

As a check on the data to see if we did, indeed, get lucky, we measured the skewness of monthly reserve increments using Pearsons SK statistic for 68 individual countries over different time periods. We found that skewness is idiosyncratic by country and by time period. During the FJ period of 1971-75, 36 percent of the countries showed significant positive skewness and 11 percent showed negative skewness. In the 1982-97 period those percentages became 41 percent and 21 percent, respectively.[22]

We interpret these data as telling us that the reserve increment process is complicated. In statistical terms, the process is apparently a mixture of typical increments—possibly distributed conditionally normally—plus some sort of endogenous jump process. The jumps down are associated with speculative attacks on reserve stocks and the jumps up represent macro-policy changes that induce reserve accumulation.

Since the 1971-75 period examined by FJ was dominated by positive skewness in the reserve increments, that skewness imparts a positive bias to the coefficient on reserve volatility. Moreover, extending the window used to construct the volatility measure can compound this bias.

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[22] These data are available from the authors. We also examined annual reserve-increment skewness. As expected, skewness is less dramatic statistically when increments are constructed from more aggregated annual data.

For example, using the unscaled FJ regression, when three years is chosen as the window for constructing the volatility measure, the coefficient on reserve volatility is insignificantly different from zero. As the window is extended, the coefficient becomes increasingly positive and more significant. For the FJ regression scaled by GNP, a similar pattern emerges. These results are consistent with an interpretation that attributes the positive correlation between reserve holdings and their volatility to positive skewness in the reserve increment data.

We re-estimated the FJ regression for the sample period 1971-75, controlling for the degree of skewness with an interactive dummy on the volatility coefficient that varies according to whether the country had negative skewness, positive skewness, or no skewness in its reserve increments during the period. While the results are consistent with the view that skewness affects the estimated coefficient on volatility, they are not very reliable because of the small number of countries in each of the skewness categories.

Once we begin to account for endogenous and discrete reserve jumps (from attacks and policy changes), the statistical process governing the evolution of reserves becomes quite different from the process assumed by FJ and the FJ derivation of optimal behavior would not apply. Fortunately, we can (in theory) sidestep the statistical issues associated with reserve increment distributions by shifting attention to movements of a variable invented after FJ wrote, the shadow exchange rate.

The shadow exchange rate is the exchange rate that would be determined in the foreign exchange market if foreign exchange reserves were exhausted and the exchange rate were allowed to float freely. Let $\tilde{S}$ be the shadow exchange rate defined as the price of foreign exchange (reserves) and suppose that $\bar{S}$ is the nonfree-floating exchange rate with reserves above their lower boundary and some foreign-exchange intervention using the reserve stock. Regardless of the specific policy governing $\bar{S}$ (e.g. perhaps $\bar{S}$ is constant), if $\tilde{S}$ were to drift above $\bar{S}$ then speculators seeking capital gains would try to purchase the remaining reserves devoted to the current policy in a speculative attack and force a policy adjustment.[23] Thus, regardless of the value of reserves or other variables, the probability of reserves hitting their lower bound is identical to the probability of $\tilde{S}$ hitting $\bar{S}$ from below.

[23] In an attack, speculators purchase reserves at $\bar{S}$ with the hope of selling them for a capital gain at a higher $\tilde{S}$. Speculative attacks of this nature were introduced by Salant and Henderson (1978) and applied to the foreign exchange market by Krugman (1979) and Flood and Garber (1984). The shadow rate-related attack mechanism is adapted easily to later "generation" speculative-attack models, see, e.g., Flood and Marion (1999).

In the next section, we make a case for replacing FJ's reserve-increment volatility measure with our own model-specific volatility measure. Since reserve increments are endogenous to optimal reserve management, to speculative attacks and to many aspects of private behavior, we end up replacing the volatility of these increments that are endogenous under the null with an equivalent volatility measure that is a function of variables that logically may be maintained to be exogenous under the null. We identify economic fundamentals that drive the shadow exchange rate to its upper bound at the same instant that reserves are driven to their lower bound. Our volatility measure is then the volatility of these fundamentals.

V. THE FJ INVENTORY MODEL AND THE SHADOW EXCHANGE RATE

By assuming that reserve increments follow a Wiener process, FJ avail themselves of many well-known results. They proceed formally by assuming that reserves have just been reset at their optimal point and the restocking costs have just been paid. The problem for the reserve authority is then to estimate future costs so as to balance appropriately the fixed costs of restocking against the holding costs of reserve inventories. Future costs can be separated into two components, (i) the holding costs incurred up to the next optimal restocking, and (ii) the costs incurred following the next restocking decision.

To calculate the expected present value of the holding costs up to the next restocking, the reserve authority must determine the probability that as of time t reserves have not hit the lower boundary since time 0, when they were reset at their optimal level. The authority must also determine the expected value of reserves at time t conditional on reserves not having passed through the lower boundary between time 0 and time t.

The expected present value of costs following the decision to restock is just the fixed adjustment cost of the next restocking plus all future holding and restocking costs. Its calculation requires knowledge of the probability that at time t reserves pass through the lower boundary after starting at their optimal level at time 0.

The analogy between the reserve process and the shadow exchange-rate process is straightforward. The probability that as of time t reserves have not hit the lower boundary since time 0, when they were set optimally at R_0, is also the probability that $\tilde{S}$ has not hit $\overline{S}$ in that same time interval. The probability that R passes the lower boundary at t after starting at R_0 it is also the probability that $\tilde{S}$ hits $\overline{S}$ at t when reserves start at R_0. Finally, since the expected value of reserves conditional on their not having hit the lower boundary depends only on R_0 and the distribution of the reserve increments, it also depends on R_0 and the distribution of the (maintained exogenous) fundamentals that influence the shadow rate.

In the technical appendix, we present a model that identifies the (exogenous) economic fundamentals determining the shadow exchange rate. We can then use the distribution of these fundamentals to construct a new volatility measure.

VI. A Test of the Buffer Stock Model Using a New Volatility Measure

In this section we estimate several versions of the reserve-holdings equation derived from the buffer stock model using our new volatility measure. In none of our work do we model time-series processes for the error terms of our estimating equations or specify partial-adjustment models. Instead, we report GMM-style standard errors that are robust to heteroskedasticity and serial correlation.

The model we estimate involves variations of the following equation:

$$\ln(R_t / X_t) = \beta_0^i + \beta_1 \ln(\sigma_t / X_t) + \beta_2 \ln(i_t) + u_t \tag{4}$$

where R is the level of reserves

X is a scale variable taking on one of five values: unity or "None;" the price level or "Real;" GNP; the nominal value of imports or "Imports," and $M2$.

β_0^i is the coefficient on country i's fixed-effects dummy,

σ is volatility. It is measured as the standard deviation of the previous two years of monthly shocks to the shadow-rate fundamentals' time series process, which is assumed to be a random walk with drift, $i = (1+r)/(1+r^*)$ where r and r^* are domestic (no *) and U.S. (*) money market, Treasury bill, deposit, lending or government bond rates.

Although in the previous section we discussed the rationale for the fundamentals volatility measure in equation (4), we have not carefully discussed the opportunity-cost regressor. We now turn our attention to it.

In the early literature, the opportunity cost variable (proxied by the own-government bond rate in the FJ estimation of developed countries' reserve holdings) was difficult to measure exactly and was generally not a significant variable. Consequently, it was often left out of estimating equations for reserves. For developing countries, interest rates were government controlled rather than market determined, so an opportunity cost measure based on interest rates was not meaningful. Kenen and Yudin (1965) suggested that the opportunity cost of holding reserves was not lending them for capital formation, so the opportunity cost should be measured by the marginal product of capital. (The interest earned on reserves was ignored since it was likely to be small and stable.) Since in theory GNP is inversely related to the marginal product of capital, they suggested that a per capita income measure could proxy for the opportunity cost.[24] Edwards (1985), studying the problem 20 years later, was able to obtain interest-rate data for 17 developing countries that borrowed in the Eurocurrency market. He used the difference between the interest rate faced by these countries in the Eurocurrency market and LIBOR as his measure of opportunity cost and found that it had a significant, negative effect on reserve holdings, just as the inventory model would predict.

In the 1990s, many emerging markets liberalized their economies and moved to market-determined interest rates. We take advantage of the interest-rate data available from this decade so that we can calculate our opportunity cost measure.[25] Our measure is the difference between domestic and U.S. interest rates on government bonds (or Treasury bills, money market, or lending/deposits).

We now proceed to estimate reserve holdings using our new volatility and opportunity cost measures. We use panel data for 36 developed and developing countries over the 1988-97 period. Our purpose is to uncover whether reserve holdings over that ten-year period are sensitive to volatility in the macroeconomic environment; that is, the volatility in the fundamentals driving the shadow exchange rate. We also wish to learn whether reserve holdings vary negatively with their opportunity cost, measured as a standard interest-rate differential. We report our results in Table 3. We also include in the table results using the FJ reserve volatility measure (now based on the previous two years of monthly shocks to the reserves process.)

That table is divided into ten parts, corresponding to each of our five scaling methods with and without country fixed effects. For explanation, let us turn first to the top, left-hand part of the table—scaling is "None" for both the Dependent Variable and the volatility measure. The first two rows give results using the "new" fundamentals volatility measure. The next two rows show results using the "FJ" reserve volatility measure. Recall that the FJ measure contains skewed increments that interact with measurement error in the dependent variable to impart a positive bias to the estimated coefficient on volatility. Estimation in the top left-hand section is without fixed effects.

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[24] The results using the income proxy were disappointing. In later studies, GNP was used as a scaling variable or to represent the transactions demand for money by a central bank. In some studies, an income term was added as part of a specification of disequilibrium in the money market that would affect the slow adjustment of reserves to their desired level.

[25] Interest is earned only on the nongold holdings of central bank foreign-exchange reserves. It will not matter too much that we do not decompose each country's reserves into gold (which earns no interest) and foreign-exchange reserves. Nor will it matter much that we do not specify the currency composition of foreign-currency reserves. That is because the interest rates on the United States, European and Japanese short-term government bonds, which are the preferred instruments for official foreign-exchange holdings, are small and stable and dominated by movements in own-government bond yields.

Table 3. Reserve Regressions With New and Old Volatility Measures

		No Fixed Effects					Fixed Effects			
Scaling	σ	σ	i	R-sq.	R-sq. adj.	Std. Err.	σ	i	R-sq.	R-sq. adj.
None	"new"	0.1665 ***	0.2189	0.1345	0.1296	1.2099	0.1723 ***	-0.1421 *	0.9142	0.9042
		(0.0374)	(0.1855)				(0.0344)	(0.0855)		
		0.1284 ***		0.0951	0.0925	1.2936	0.1565 ***		0.9129	0.9031
		(0.0455)					(0.0318)			
	"FJ"	1.0128 ***	-0.1429 **	0.7437	0.7423	0.6556	0.4417 ***	-0.2936 ***	0.9167	0.9071
		(0.0576)	(0.0708)				(0.0680)	(0.0493)		
		1.0118 **		0.7402	0.7395	0.6575	0.4482 ***		0.9161	0.9068
		(1.1841)					(0.0665)			
Real	"new"	0.1624 ***	0.2217	0.1298	0.1249	1.1984	0.1417 ***	-0.1574 **	0.9269	0.9185
		(0.0373)	(0.1812)				(0.0264)	(0.0744)		
		0.1241 ***		0.0903	0.0877	1.2215	0.1278 ***		0.9261	0.9177
		(0.0456)					(0.0247)			
	"FJ"	1.0080 ***	-0.1432 **	0.7374	0.7359	0.6556	0.3261 ***	-0.2956 ***	0.9247	0.9161
		(0.0581)	(0.0710)				(0.0615)	(0.0460)		
		1.0076 **		0.7341	0.7334	0.6576	0.3328 ***		0.9243	0.9158
		(1.1450)					(0.0607)			
GNP	"new"	0.0812 ***	-0.0055	0.1056	0.1006	0.7311	0.1058 ***	-0.0634	0.8560	0.8393
		(0.0208)	(0.0749)				(0.0185)	(0.0474)		
		0.0607 ***		0.0594	0.0568	0.7408	0.0791 ***		0.8456	0.8283
		(0.0194)					(0.0191)			
	"FJ"	0.6186 ***	-0.1648 ***	0.4291	0.4259	0.5816	0.1703 ***	-0.1899 ***	0.8440	0.8260
		(0.0550)	(0.0625)				(0.0435)	(0.0407)		
		0.6161 ***		0.4129	0.4113	0.5829	0.1696 ***		0.8397	0.8218
		(0.0547)					(0.0433)			
Imports	"new"	0.0854 ***	0.3784 ***	0.1572	0.1524	0.6192	0.0908 ***	0.0134	0.8228	0.8023
		(0.0188)	(0.0669)				(0.0203)	(0.0496)		
		0.0399		0.0284	0.0256	0.7034	0.0545 **		0.8250	0.8053
		(0.0314)					(0.0220)			
	"FJ"	0.5071 ***	0.0306	0.3043	0.3004	0.5603	0.1701 ***	-0.1030 ***	0.8152	0.7940
		(0.0566)	(0.0617)				(0.0372)	(0.0369)		
		0.5701 ***		0.3329	0.3311	0.5804	0.1816 ***		0.8291	0.8101
		(0.0619)					(0.0389)			
M2	"new"	0.1310 ***	0.2893 ***	0.1986	0.1941	0.7620	0.1239 ***	-0.0672 *	0.8845	0.8711
		(0.0234)	(0.0751)				(0.0203)	(0.0395)		
		0.0916 ***		0.1123	0.1098	0.8125	0.0982 ***		0.8822	0.8690
		(0.0295)					(0.0176)			
	"FJ"	0.6605 ***	-0.0941 *	0.5464	0.5438	0.5711	0.2162 ***	-0.2128 ***	0.8696	0.8546
		(0.0443)	(0.0511)				(0.0414)	(0.0312)		
		0.6687 ***		0.5484	0.5471	0.5772	0.2148 ***		0.8730	0.8589
		(0.0442)					(0.0412)			

Note: Standard errors (heteroskedasticity- and autocorrelation-consistent via GMM) are reported in parentheses. *, ** and *** denote significance at the 10 percent, 5 percent and 1percent level, respectively. Countries included are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Costa Rica, Denmark, Finland, France, Germany, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Pakistan, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, Thailand, United Kingdom, and Venezuela. Sample period 1988-97; all panels are balanced (exception: Belgium starts only in 1992) Durbin-Watson statistics are in the range of 0.288 - 0.693 (No Fixed Effects) and 1.088 - 1.369 (Fixed Effects) Whenever i is missing as a regressor, 0.25(r-r*) has been added to the dependent variable.

In the first line of that section, the estimated coefficient on $\ln\sigma$ is 0.1665 and is highly significant. The coefficient on $\ln i$ is 0.2189 but is not significant. These variables produce an adjusted R^2 of .1296. Since it is difficult to find instruments for a plausibly endogenous opportunity cost, and the coefficient on the opportunity cost is not significant, we also try constraining the coefficient to be –0.25 as the theory suggests, then adding the constrained opportunity cost to the dependent variable. The results of that experiment are reported in the second row of the top left section and consequently indicate no estimated coefficient for the opportunity cost variable. The coefficient on the new volatility measure continues to be highly significant, although its size is reduced by about one standard deviation.

In the same section, note that for the unscaled FJ regressions, the coefficient on the reserve volatility measure is highly significant, although much larger than theory would suggest, and the coefficient on the opportunity cost is significant and has the expected negative sign. Using the old reserve volatility measure obviously inflates the adjusted R^2. Since the equations in the upper left-hand part of the table involve a dependent variable that is trending in our sample, we prefer to jump to another section of the table before further interpreting our results.

In the third section of Table 3, where reserves and volatility are scaled by GNP, we see that our new volatility measure is again highly significant with and without fixed effects and whether or not the opportunity cost coefficient is constrained. The estimated coefficient on the opportunity cost is negative but always insignificant when left unconstrained.[26] The other sections of the table reveal that the coefficient on volatility is highly significant regardless of scaling or the addition of fixed effects.

As we did with the FJ regressions, we investigated the panel further according to a higher moment measure and looked at some alternative volatility windows. For FJ we categorized countries according to skewness. For fundamentals volatility, the appropriate analog is coskewness.[27] We divided the sample on the basis of coskewness into three equal-sized parts and then estimated the regression scaled by GNP with three separate coefficients on fundamentals volatility corresponding to our three-way coskewness breakdown. The three coefficients were always significant at the 1 percentage level and equality of the coefficients could not be rejected at the 10 percent level.

[26] In all our estimations, we have used the interest rate differential, ln((1+i)/(1+i*)), as the opportunity cost of reserve holding. We also estimated reserve-holding equations with the realized excess return, ln((1+i)*S[t]/(1+i*)S[t+1]), as the opportunity cost variable. This variable replacement did not improve the fit of the estimated equations nor did it solve the problem of finding useful instrumental variables for the opportunity cost. Counting capital gains from devaluation on the books of the reserve-management authority also sets up odd incentives.

[27] See the technical appendix for elaboration.

We also experimented with different "volatility window" sizes. When we varied the window for calculating reserve volatility, we obtained results consistent with the FJ equation being subject to skewness bias. In Table 3, the fundamentals volatility is constructed from a window containing the most recent two years of monthly fundamentals innovations. As an additional check to see if our results might be due to coskewness, we re-estimated variously-scaled versions of equation (4) using volatility windows of 12, 24, 36 and 48 months for constructing our volatility measure. We found that the estimated coefficients on the volatility measure were very similar numerically and statistically significant regardless of window size. Our finding is consistent with no effect from coskewness.

We also tested how well the fixed-effects version of the estimating equation (4) forecasted out-of-sample. Using available data to construct fundamentals volatility and the opportunity-cost measure for 1998 and 1999, we were able to compute predicted reserve holdings for some countries and for some versions of scaled reserves in 1998 and 1999. We then compared these predicted values to actual reserve holdings over the same period. We found that the equation did reasonably well at forecasting on average, but it tended to underestimate reserve holdings in 1998 and 1999 for emerging markets such as Israel, Mexico, and South Korea, and overestimate reserve holdings for several industrialized countries, such as Canada, and for Brazil, an emerging market that ran into difficulties in the late 1990s.[28]

Summarizing our results, we see from Table 3 that when equation (4) is estimated on data for the1990s, volatility always has a positive and highly significant effect on reserve holdings. This result holds regardless of scaling and whether or not fixed effects are added. When we constrain the coefficient on the opportunity cost to be consistent with the null, volatility is still highly significant in 9 out of 10 runs.

When we started our investigation of the FJ buffer-stock reserve equations, we suspected the well-known results to be too good and due, perhaps, to a statistical anomaly. We have been unable to overturn them, however. International reserve holdings increase with volatility, even in a world of high capital mobility. The prediction of the buffer stock model that says reserve holdings should decline with increasing opportunity costs does not hold up as well—it never has. The coefficient on the opportunity cost measure is not reliably negative and significant.

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[28] Disyatat and Mathieson (2001) estimated a FJ-type reserve-holding equation for 15 Asian and Latin American countries and found that the financial crises in 1997-98 produced no clear structural break. They did find that reserve holdings were more sensitive to reserve volatility after the Asian crisis.

Comparing the adjusted R^2 of equation (4) with and without fixed effects, we note that the fixed effects pick up about 75 percent of the cross-country variation in reserve holdings. That leaves 25 percent to be explained and we pick up almost one-half of that with our volatility and opportunity cost measures. That said, the volatility and opportunity cost measures together explain only about 10-15 percent of the variation in reserve holdings. By most standards, their explanatory power is low. Yet it is comparable to the ability of empirical models to explain movements in nominal exchange rates.

Unfortunately, the buffer stock model does not provide guidance about the nature of the adjustment costs captured by the fixed effects. To say that the fixed effects "explain" about 75 percent of the variation in cross-country reserve holdings does not advance our understanding very much. In the next section, we explore some research directions in an attempt to learn more about these fixed effects.

VII. Extensions

A country's exchange-rate policy is generally thought to affect its reserve-holding behavior. It has long been assumed that countries with fixed or heavily-managed exchange rates must be prepared to intervene in the foreign-exchange market and so will hold more reserves than countries with more flexible exchange-rate policies. Frenkel (1974, 1980) and Edwards (1983) found some supporting evidence for this view.

To take account of a country's degree of exchange-rate flexibility, we add a control to our estimating equation (4). The control is the standard deviation of the innovation to the percentage change in the nominal effective exchange rate. Since this volatility measure is already in percentage terms, it is not scaled. The idea behind using this control is that the greater the degree of exchange-rate flexibility, the lower the adjustment cost if reserves should hit their lower bound. Consequently, we would expect greater exchange-rate flexibility to be associated with lower reserve holdings.

A country's openness may also affect its reserve-holding behavior. In the early literature, Heller (1966) reasoned that in the absence of reserves, any temporary deficit in the balance of payments would have to be corrected via a reduction in aggregate expenditures. The required change would be smaller, the higher the propensity to import. He concluded that an increased propensity to import, by reducing the adjustment cost, would be negatively related to reserve holdings. Frenkel (1983) and others interpreted the propensity to import as measuring the economy's openness and hence its vulnerability to external shocks. Since a more open economy could face more frequent adjustment costs, greater openness would be associated with higher reserve holdings. In that case, reserve holdings should be positively related to the import propensity. In empirical work, researchers substituted the average propensity to import for the marginal propensity because of data limitations and often found it to be positively correlated with reserves.

The currency and financial crises of the 1990s raise the possibility that a country's openness on the financial side as well as its openness on the real side might affect its vulnerability to a crisis and the frequency with which it faces adjustment costs. In addition, openness may influence the size of those costs. If the adjustment cost is interpreted to be the output lost during a crisis, then a country with greater financial and real-side openness may face a steeper output decline.[29] To the extent that financial and real-side openness increase both the size and frequency of the adjustment cost, they should be positively correlated with reserve holdings.

To take account of openness in our empirical work, we add a control for real-side openness, measured as the ratio of exports plus imports to GNP, and a control for financial-side openness, measured as the ratio of gross capital flows to GNP.

The results of adding only the effective exchange-rate volatility control are disappointing. The coefficient on exchange-rate volatility is negative and significant when the estimating equation is scaled by "none," "real" or "GNP," supporting the view that countries with more flexible exchange rate hold fewer reserves. However, the coefficient is not significant when the scaling is "imports" or "M2." More importantly, exchange-rate volatility never has any significant effect explaining reserve holdings over and above the explanatory power of country fixed effects. The adjusted R^2 with and without exchange-rate volatility are nearly identical.

When both openness measures as well as exchange-rate volatility are added as controls, the results are more promising. When scaling is "none," "real" or "GNP," each of the openness measures is positive and significant and exchange-rate volatility is negative and significant.[30] Moreover, adding the three controls triples or quadruples the explanatory power of the buffer stock model without fixed effects.

An example of the regression results appears below, where scaling is GNP (X=GNP), ERV is nominal effective exchange-rate volatility, FOP is financial openness and ROP is real-side openness:

[29] Ben-Bassat and Gottlieb (1992) found a positive relationship between the average propensity to import and the output cost of an external default for a sample of 14 default cases over the 1960-82 period.

[30] The results are less satisfactory when scaling is "imports" or "M2." There is also the question of whether the controls are completely exogenous.

$$\ln(R_t / X_t) = \beta_0^i + 0.0969 \ln(\sigma_t / X_t) + 0.0048 \ln(i_t)$$
$$\qquad\qquad (.0134) \qquad\qquad (.0387)$$

$$-1.7283\, ERV_t + 0.1059 \ln(FOP_t) + 0.5335 \ln(ROP_t) + u_t$$
$$(.4418) \qquad (.0375) \qquad\qquad (.1667) \qquad\qquad (5)$$

$$\overline{R}^2 = .88 \quad n=333 \quad S.E. = .2712$$

GMM-type standard errors are in parentheses and there are country-specific constants. Even accounting for the fixed effects, the explanatory power of the regression is enhanced by the three controls. Without the fixed effects, the explanatory power quadruples, with fundamentals volatility, opportunity cost and the three control variables together explaining 42 percent of the cross-country variation in reserve holdings.

Country-specific characteristics probably account for differences in adjustment costs facing countries that run out of reserves. Our results suggest that financial and real-side openness as well as exchange-rate volatility are sensible candidates for helping to explain these differences.

VIII. Conclusion

Three points should be emphasized. First, the buffer stock model of international reserve holding works about as well in the era of high capital mobility as it did when capital was less mobile. Its prediction that increased volatility significantly increases reserve holdings is very robust. While the model works well statistically, it explains very little about countries' reserve holdings—only about 10-15 percent. Most of the "explanation" in our regressions is due to country-specific fixed effects.

Second, country characteristics that might logically affect the cost of adjustment in the event of depleted reserves can improve the explanatory power of the buffer stock model. We have found that effective exchange-rate stability and a country's financial and real-side openness, together with volatility and opportunity-cost elements, can explain about 40 percent of the variation in countries' reserve holdings.

Third, empirical studies of optimal reserve holdings are hampered by the fact that the researcher does not observe optimal holdings, only actual holdings. Consequently, measurement error in the variable to be explained—optimal reserve holdings--can interact with the constructed volatility measure to generate a misleading correlation between reserve holdings and volatility. A key prediction of the buffer stock model is that uncertainty influences optimal reserve holdings. Greater capital mobility in the 1990s, while beneficial in many respects, may have increased uncertainty in the international economy, in part by increasing the vulnerability of some countries to financial crises. It is important for researchers to test whether increased uncertainty helps explain increased reserve holdings. But they must do so in a way that keeps statistical biases to a minimum.

A Case Study: Taiwan Province of China

Taiwan Province of China has the fourth largest international reserve holdings in the world, just behind Japan, China and Hong Kong SAR.[31] Most observers attribute Taiwan Province of China's large reserve stockpile to persistent current-account surpluses.[32] Taiwan Province of China has recorded a current-account surplus every year in the last two decades.

The dramatic build-up in Taiwan Province of China's reserves occurred in the mid-1980s, when Taiwan Province of China's current-account surplus as a share of GNP rose to 14.8 percent in 1985, 21.4 percent in 1986 and 18.3 percent in 1987. Capital transactions were strictly controlled at the time, and although the country had officially adopted a flexible exchange rate in 1979, exchange-rate movements were modest. In 1986 and 1987, the authorities intervened heavily to prevent a drastic appreciation of the currency in response to the current-account surpluses. Because of this intervention, reserves doubled in 1986, from US$22.7 billion to US$46.5 billion, and they increased by two-thirds in 1987, to US$77 billion. (Despite this intervention, the New Taiwan dollar appreciated in nominal terms by over 35 percent against the U.S. dollar.) Since the undervalued domestic currency was contributing to the current-account surplus, exchange-rate policy was changed and the New Taiwan dollar was allowed to float. Since 1988, the central bank has intervened infrequently in the foreign-exchange market. It has done so when political tensions escalate, as they did in1991, 1994, 1999, and 2000, and each of those years saw reserve *increases* of 10 percent or more.

Taiwan Province of China has taken steps to liberalize capital movements, but there are still many restrictions, particularly on capital inflows. For instance, non-institutional foreign investors were allowed to invest in its stock markets starting only in 1996. On the eve of the Asian crisis (June, 1997), Taiwan Province of China's private sector was a net foreign creditor, with foreign assets of US$35.5 billion and foreign debts of US$30 billion. Public foreign debt was negligible. Consequently, Taiwan Privince of China did not face the adverse balance-sheet problems of its Asian neighbors. During the initial months of the Asian financial crisis, Taiwan Province of China intervened in the market to prevent its currency from depreciating, but it ceased intervention in mid-October of 1997. Its reserve holdings fell by 5 percent that year. Since then they have resumed their upward climb. At the end of 1999, total reserves were equal to 36 percent of its GNP and accounted for 50 weeks of import cover.

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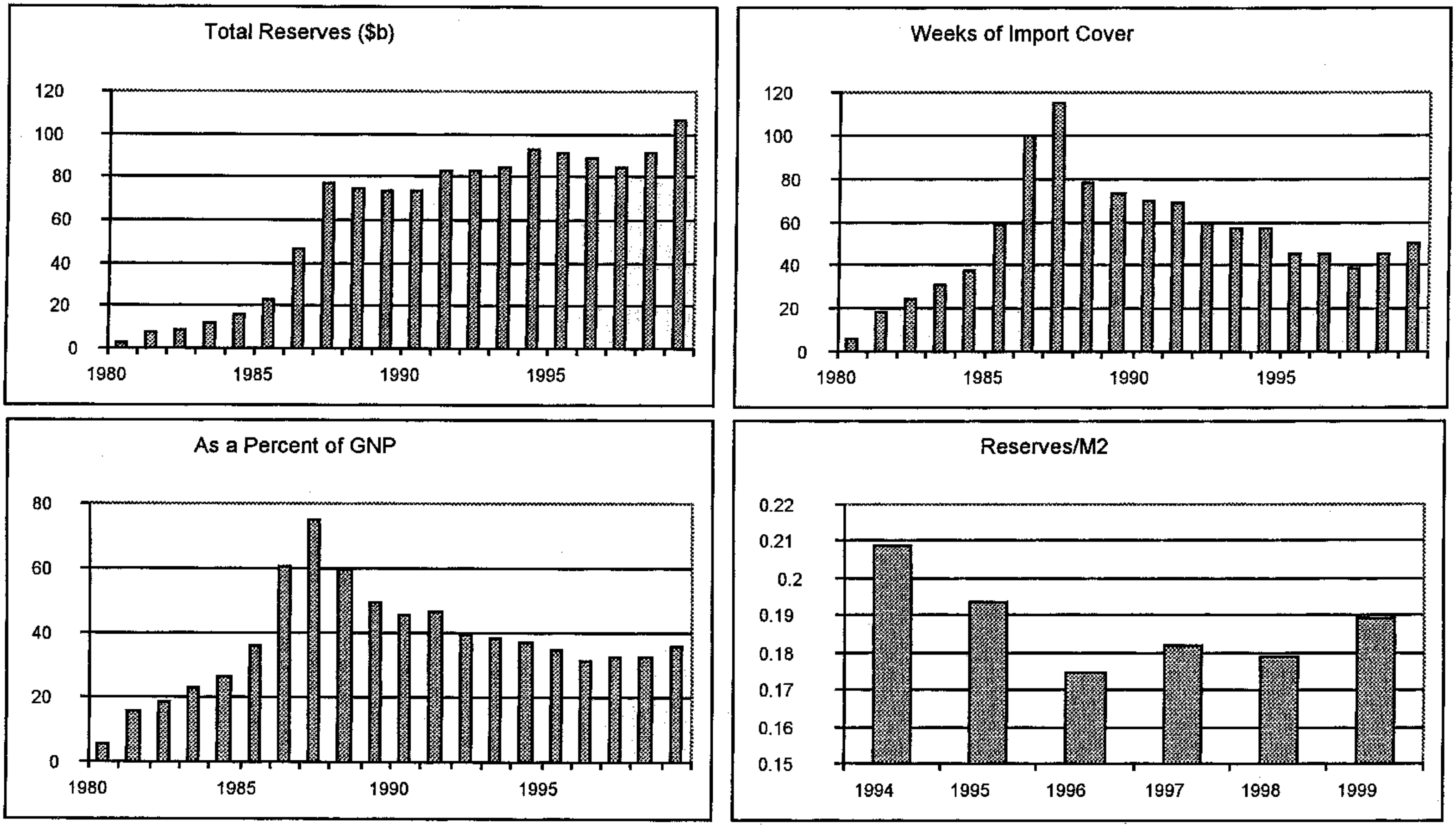
[31] Data used in the case studies are from the IMF's International Financial Statistics database and from national central bank and government statistics publications.

[32] For example, see Kuo and Liu (1998).

Figure 7:Taiwan Province of China's Reserve Holdings



Sources: Central Bank of China; Directorate-General of Budget, Accounting and Statistics, Republic of China, "Monthly Statistics of the Republic of China."

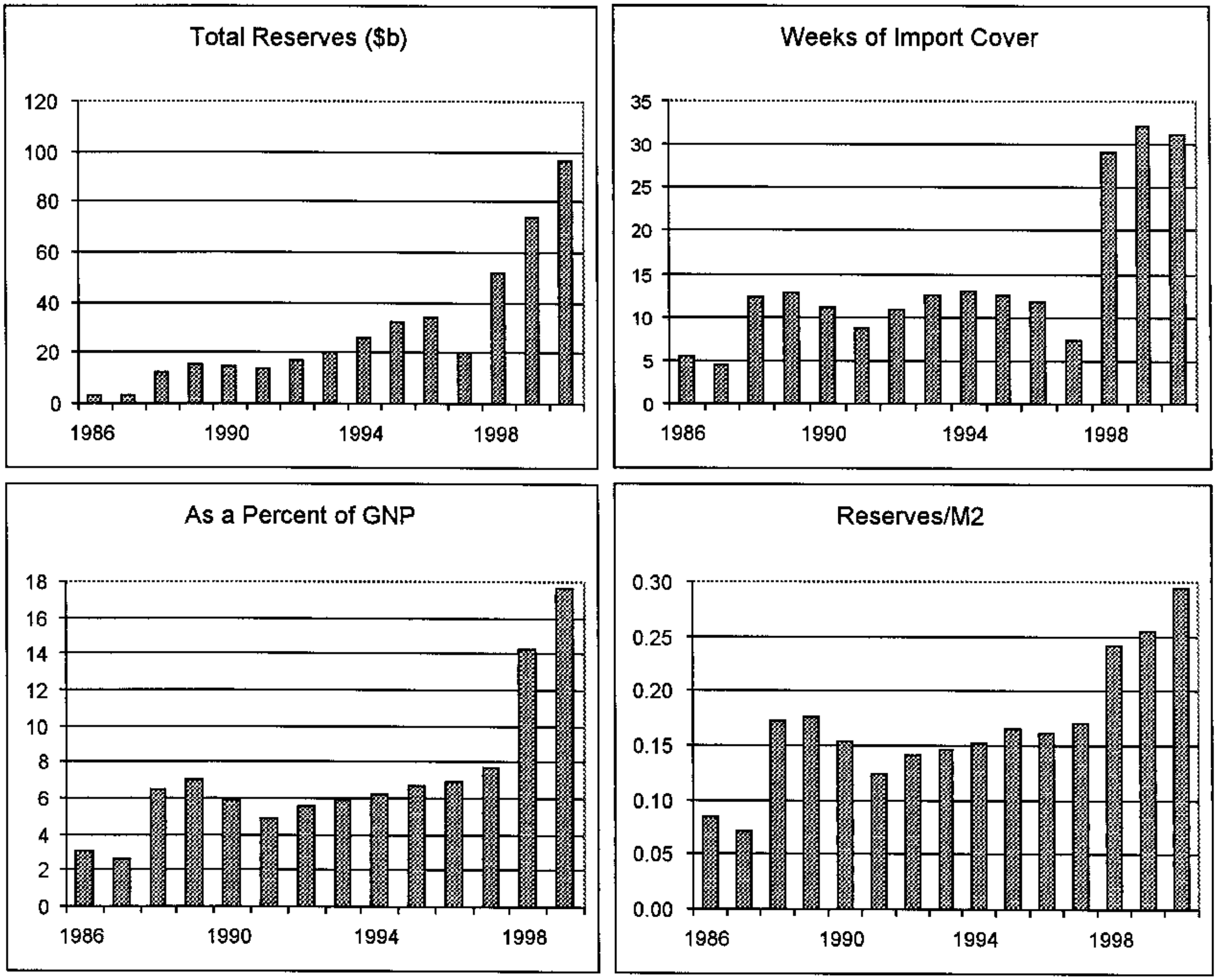
Case Study: Korea

South Korea's international reserve holdings plummeted during the 1997-98 Asian financial crisis but have since grown dramatically. From a low of $6 billion in usable international reserves at end-1997, reserves rose 766 percent over the next year, to $52 billion. Usable reserves rose a phenomenal 1500 percent over the 1998-2000 period, to $96.2 billion by the end of 2000. The leading European credit rating agency (Fitch IBCA) called this remarkable turnaround in international reserves "unprecedented in modern rating history." At the end of 2000, Korea held five times more total reserves than it did at the beginning of the 1990s.

The increase in Korea's reserves started in earnest in 1988, with liberalization of capital transactions, capital inflows and intervention in the foreign-exchange market by the monetary authorities to prevent serious appreciation of the won. Between 1987 and 1988, reserves almost quadrupled. They also increased sharply between 1993 and 1995. When Thailand abandoned its fixed exchange rate in the summer of 1997, Korea's reserve position seemed strong. Its end-of-June, 1997 reserve holdings were put at $33 billion. However, the central bank had placed foreign-currency deposits with foreign branches of domestic Korean banks. While this practice had actually begun in the late 1980s, the amounts had stayed small (less than 10 percent of total reserves) until 1997. During 1997, the share of reserves placed with these foreign branches increased.

The swing in Korea's current-account balance from deficit to surplus in 1998 amounted to 15 percent of GDP, far outweighing the Mexican experience in 1994-95. Usable international reserves rebounded, reflecting the move to current-account surplus, and, as investor confidence returned, inflows from foreign investors. Korea's stock market rebounded in mid-1998 and closed that year as the best performing emerging market. Although the won is still below its pre-crisis level, concerns about the effects of its appreciation on export competitiveness mounted in early 1999. The central bank has intervened to limit the appreciation, increasing its reserve holdings. The central bank also attributes the increase in its reserve holdings in 2000 to repayments by domestic financial institutions of their foreign-exchange loans borrowed from the central bank. (*Korea Economic Weekly*, various issues.)

Figure 8. Korea's Reserve Holdings



Source: IMF

Case Study: China

In the second half of the1990s, China's current-account surpluses and relatively strong foreign direct investment (FDI) inflows allowed it to increase its international reserve holdings. During the 1980s, reserves averaged about 4.5 percent of GNP. In the second half of the 1990s, reserves averaged a much larger 14 percent of GNP.

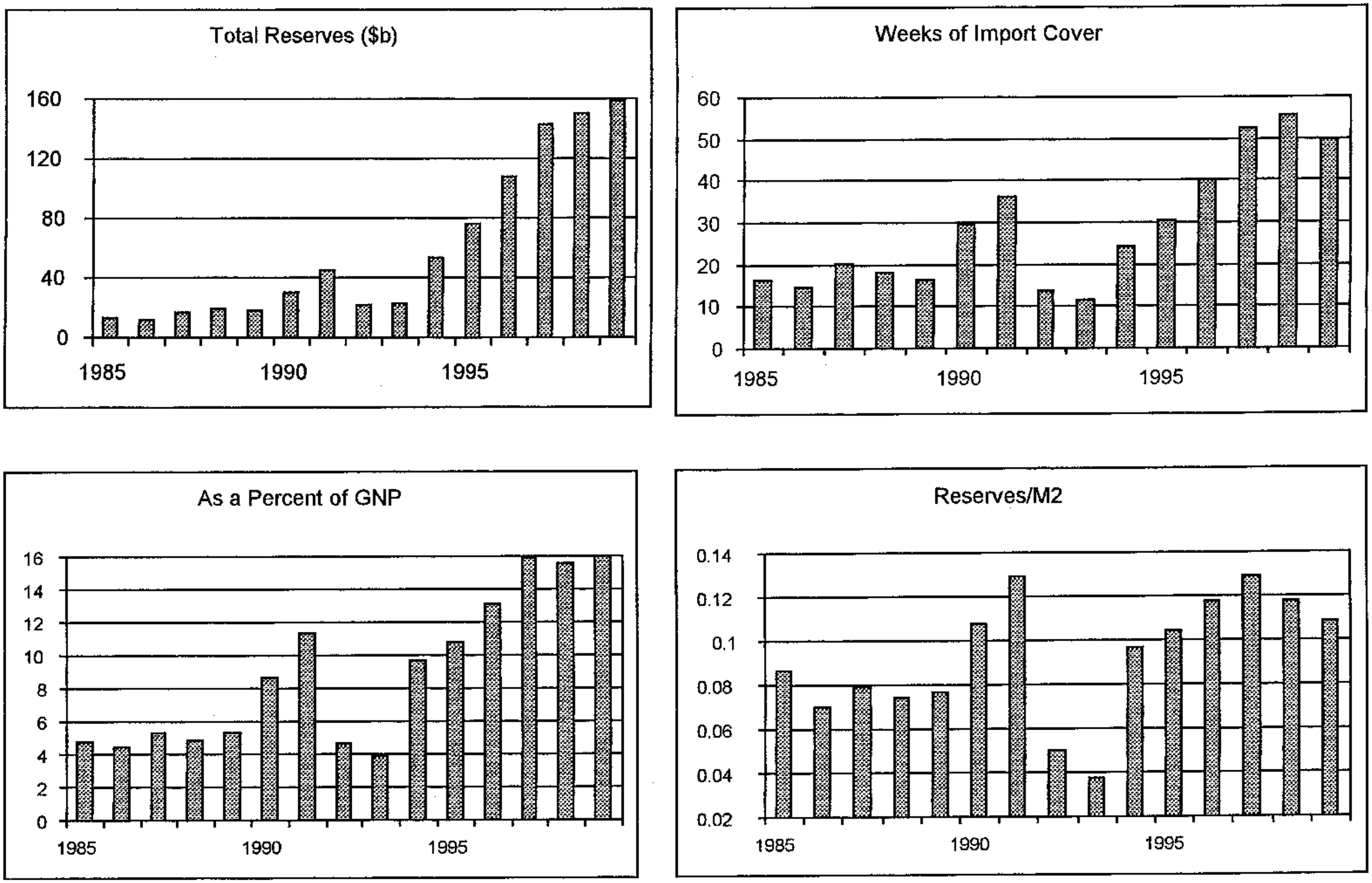
At the end of 1999, China had $158 billion in reserves, which represented 16 percent of its GNP, 50 weeks of import cover, 10 percent of M2 and about four times its short-term foreign-currency debt. By the end of January, 2001, reserves had exceeded $170 billion.

The domestic currency is convertible only on current account; strict controls apply to all capital transactions. In 1995 and 1996, China's current-account surplus was 0.2 percent of GNP and 0.9 percent of GNP, respectively. In 1997, the surplus rose to 3.2 percent; it was 2.5 percent in 1998 and 1.5 percent in 1999. In response to these surpluses, as well as to the FDI inflows, the authorities have tried to keep the currency stable, allowing only a very modest appreciation since 1994. China chose not to devalue during the Asian crisis, and it has continued to keep its currency fixed since early 1995 at 8.3 renminbi to the U.S. dollar.

China did not face financial turmoil during the 1997-98 Asian financial crisis because it had capital controls. Witnessing the damage inflicted on the financial and corporate sectors of its neighbors, it took measures to tighten its already strict foreign debt management. At the end of 1997, its foreign debt was US$134 billion, about 80 percent of it long term and about half of it borrowed from international organizations and foreign governments. Its capacity to service foreign debt, as measured by the growth of its international reserves, has kept pace with the growth of foreign liabilities. Consequently, the key debt ratios have been kept well within the generally accepted safety limits. (Barclay's Bank *Country Report*, August, 1998). In particular, total foreign debt is well under 20 percent of GDP, the debt service ratio is under 11 percent and international reserves are four times larger than short-term debt.

China favors having sizeable reserve holdings. It believes that international reserves help it maintain the foreign confidence needed for attracting foreign direct investment and securing foreign loans at good terms. (Ford and Huang (1994)).

Figure 9. China's Reserve Holdings



Source: IMF

Data Appendix

[All series are obtained from the IMF's *International Financial Statistics* Database]

- ***Reserves Excluding Gold.*** Series '.1LDZF', quoted in $. Global reserves excluding gold is global reserves including gold, Series ' 001.1..DZF' minus global gold holdings, Series '001.1AD.ZF' (gold in ounces), multiplied by 35 (gold in SDRs), then valued in $ using the $/SDR exchange rate, Series '111..AA.ZF'.
- ***Gold.*** Series '.1AD.ZF' (Gold in ounces), multiplied by 35 (=gold in SDR), and then valued in $ using the end-of-period $/SDR exchange rate (Series '111..AA.ZF').
- ***Reserves Including Gold.*** Sum of *Reserves Excluding Gold* and *Gold.* Global reserves including gold is series ' 001.1..DZF'.
- ***'Real' Reserves.*** Reserves are deflated with U.S. GNP deflator (series '111..99BIR', extended backwards using older publications, 1975=100).
- ***Period Average Exchange Rate.*** Series ',,RF.ZF', quoted in national currency per $.
- ***Nominal Effective Exchange Rate.*** Monthly data for trade-weighted nominal exchange rate based on *period average exchange rate*, weights obtained from IMF. The rate is expressed as an index (1957:1=100), with an increase indicating an appreciation. This series was used to construct the annual measures of exchange-rate volatility.
- ***Interest Rates.*** For the FJ replication, the government bond yield in series '61...ZF' was used for most countries. The discount rate in series '60...ZF' was used for Austria, Sweden, Japan, Finland, Greece and Iceland. For the later regressions, the choice of interest rate used was based on maximum availability. The Money Market Rate in series '60B..ZF' was used for Belgium, Brazil, Denmark, India, Korea, Malaysia, Netherlands, South Africa, Spain, Thailand, Japan and Pakistan. The deposit rate in series '60L..ZF' was used for Argentina, Chile, Colombia, Costa Rica, Indonesia, Jamaica, Venezuela. The Treasury Bill Rate in series '60C..ZF' was used for Mexico, Philippines, and Sweden. The Lending Rate in series '60P..ZF' was used for Finland and Israel. The Government Bond Yield in series '61...ZF' was used for Australia, Austria, Canada, Germany, France, Ireland, Italy, New Zealand, Norway, Portugal, Switzerland and United Kingdom. The interest rate differential was constructed as $\ln((1+i)/(1+is))$, where *(is)* is the United States interest rate corresponding to the definition used for the national interest rate.
- ***GNP.*** Series '99A.ZF' or '99AC.ZF' (Gross National Income) for most countries. For Japan, Switzerland, Hong Kong SAR, Egypt, Nicaragua, Peru, United Kingdom, Portugal, Tunisia, Mexico and South Africa, series '99B..ZF' or '99BC.ZF' (GDP) was used. For Turkey, the GDP series '186..99B..ZF' was extended backwards using the real GDP volume index '186..99BVPZF' (1995=100), inflated with the CPI '186..64.XZF', linking the two series in 1995. All measures are quoted in national currency, and converted into $ using the *period average exchange rate*. Global GDP is from IMF's *World Economic Outlook*, Series 'W001NGDPD'.

- ***Imports.*** Series '71.DZF' (Imports, c.i.f., quoted in \$). For Belgium, series '12498C..ZF' (Imports of Goods and Services) was used, converted into \$ using the *period average exchange rate*. Global import weeks is Series ' 0171.DZF' divided by 52.
- ***Exports.*** Series '70.DZF' (Exports, quoted in \$).
- ***Monetary Base.*** Series '14...ZF', quoted in National Currency. For Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain the sum of series '14A..ZF' (Currency in Circulation) and '14C..ZF' (Liabilities to Banking Institutions in the Country) was used. For the United Kingdom, series '11259MC.ZF' (M0) was used.
- ***M1***. Series '34...ZF', quoted in national currency. For Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands and Portugal the sum of '34A..NZF' (Currency in Circulation) and '34B..NZF' (Demand Deposits) was used. For Spain the series '18459MA.ZF' (M1) was used. For Sweden the series '144..39M.ZF' (Broad Money) was used.
- ***Quasi-Money.*** Series '35...ZF', quoted in National Currency. For Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain the series '35..NZF' was used.
- ***M2***. The sum of *M1* and *Quasi-Money*. For Sweden and UK the series '35L..ZF' (Money plus Quasi-Money) was used. For France and Germany the series '39MBCZF' (M2, Seasonally Adjusted) was used. For the Netherlands the series '39.CZF' (M2, Seasonally Adjusted) was used. The *M2* series are quoted in national currency and were converted into \$ using the *period average exchange rate*.
- ***Shadow-Rate Fundamental F.*** First Domestic Credit *D* was constructed by subtracting *Reserves Including Gold* (expressed in national currency using the *period average exchange rate*) from the *Monetary Base*. This difference was then multiplied by (100/US CPI), where the US CPI is represented in the series '111..64..ZF'. The monthly series is used to construct annual measures for the *Volatility of the Shadow-Rate Fundamental*.
- ***Capital Inflows***. Sum of absolute values of series '78BEDZF', '78BGDZF' and '78BIDZF' (quoted in \$).
- ***Capital Outflows.*** Sum of absolute value of series '78BDDZF', '78BFDZF', '78BHDZF' and '78CADZF' (quoted in \$).
- ***Real Openness***. Sum of (absolute value of) *imports* and (absolute value of) *exports.*
- ***Financial Openness***. Sum of *capital inflows* and *capital outflows*.

Technical Appendix

Bias from Skewness

The potential upward bias in the estimated coefficient on volatility arises from the interaction between reserve volatility and measurement error in reserves, the dependent variable. To see the nature of the possible bias, consider an example. Suppose that reserves follow a random walk with drift:

$$R_t = \mu + R_{t-1} + v_t \tag{6}$$

where μ is the constant drift and v a zero-mean disturbance with non-constant variance. Between discrete adjustments, reserves follow the Wiener process described earlier. But when R_{t-1} is at its lower limit, the value of v is special. It is large and it is positive. It is the inventory restocking under the FJ null. We therefore discard the assumption that v is distributed normally and instead let it be skewed positively.

To compute reserve volatility for our example, we use a one-year rolling variance of reserves rather than the15-year rolling standard deviation used by FJ. Thus our volatility measure is simply the square of the once-lagged reserves disturbance, $\sigma_t^2 = v_{t-1}^2$.

Our FJ-style reserve estimating equation is[33]:

$$R_t = \beta_0 + \beta_1 \sigma_t^2 + \varepsilon_t \tag{7}$$

In equation (7), the β_i are regression coefficients and ε_t is a regression disturbance assumed to be uncorrelated with the volatility measure.

If FJ are correct in their assumption that all reserve observations are very close to the optimal value, R_0, then we know from the definition of a least squares estimator that $\hat{\beta}_1 = \text{cov}(R_0, \sigma_t^2)/\text{var}(\sigma_t^2)$. We can then use our estimated value for $\hat{\beta}_1$ to test the theory against the alternative that $\hat{\beta}_1 = 0$.

But suppose, realistically, there is measurement error in our reserves variable. Observed reserves are off a bit from their optimal value. Perhaps two periods ago they were at their optimal level, $R_{t-2} = R_0$, but given the random walk process in (A1), last period they were at $R_{t-1} = R_0 + \mu + v_{t-1}$ and this period they are at $R_t = R_0 + 2\mu + v_{t-1} + v_t$. Now our

[33] For clarity in the example, we have suppressed the opportunity cost effect and made the estimating equation linear in levels using a variance measure rather than a standard deviation.

estimated coefficient will be $\hat{\beta}_1 = \text{cov}(R_t, \sigma_t^2)/\text{var}(\sigma_t^2)$ where $\text{cov}(R_t, \sigma_t^2) = \text{cov}(R_0, \sigma_t^2) + E(v^3)$. Since v is skewed positively by large reserve adjustments, $Ev^3 > 0$ simply as a statistical artifact of restocking. An investigator, therefore, would find $\beta_1 > 0$ regardless of optimal inventory behavior in the presence of uncertainty.[34].

A Model of the Shadow Exchange Rate and International Reserves

A warning is appropriate here. Our reinterpretation of the reserve inventory model and its empirical implementation requires us to rely on a macroeconomic model of exchange-rate determination—hardly the most robust building block in the economist's tool-kit.[35] This is the price we pay for choosing to work with volatility measures that involve endogenous variables. We must rely on some sort of "first stage model" that then generates the final volatility measure.

The macro model we use in the text and in our empirical work to specify shadow exchange-rate behavior is the monetary model of the shadow rate introduced by Flood and Garber (1984) and estimated by Blanco and Garber (1986). The model consists of the following equations:

$$\frac{M}{P} = \beta - \alpha i + \varepsilon \tag{8}$$

$$P = P^* S \delta \tag{9}$$

$$i = i^* + \frac{\dot{S}}{S} \tag{10}$$

In equation (8), M is the high-powered money supply; P is the domestic price level; i is the domestic-currency interest rate, and ε is a money-demand error term that captures all factors influencing real money demand other than the interest rate. In equation (9), P^* is the foreign price level; S is the exchange rate quoted as the domestic-currency price of foreign exchange and δ is the real exchange rate. Equation (10) imposes uncovered interest parity. The domestic interest rate equals the foreign rate i^* plus the expected (actual) rate of change of the nominal exchange rate. This model is well known and we use it here as a starting point for that reason.

[34] Regression biases induced by skewness are confronted frequently in the finance literature since researchers often attempt to explain asset returns, in part, by the variability of those returns. See e.g., Shin and Stulz (2000).

[35] See e.g., Flood and Rose (1995)

The model brings in international reserves through the asset backing of high power money, $M \equiv R + D$, where R is the domestic-currency book value of international reserves.and D is domestic credit. Rearranging equation (8), we see that while the exchange rate is controlled, reserves follow:

$$R_t = P_t^* \bar{S} \delta_t (\beta - \alpha i_t + \varepsilon_t) - D_t \ . \tag{11}$$

The shadow exchange rate equilibrates the money market in equation (8) under the condition that reserves have been driven to their constant lower boundary (assumed here to be zero) and the exchange rate is allowed to float freely in the sense that the reserve authority does not intervene in the foreign-exchange market.

We now substitute equation (9) and equation (10) into equation (8), substitute D for the money base since reserves have been driven to their zero lower bound, and solve for the shadow exchange rate, $\tilde{S}$. We find that the shadow exchange rate at time t is equal to:

$$\tilde{S}_t = F_t + \lambda \dot{\tilde{S}}_t \tag{12}$$

where the fundamentals driving the shadow exchange rate are $F_t = (D_t / P_t^*)/\gamma_t$, with $\gamma_t \equiv \delta\,[\beta - \alpha i^* + \varepsilon_t]$, $\lambda = \delta\alpha/\bar{\gamma}$, and $\bar{\gamma}$ representing the long-term average value of γ_t.[36]

A complete solution for the shadow exchange rate described by equation (12) requires information about the time-series process of fundamentals. We assume that fundamentals follow a random walk with drift,

$$F_t = F_{t-1} + \mu + v_t \tag{13}$$

---

[36] Our shadow rate fundamental is $D/(P^*\gamma)$ in theory. In our empirical implementation, γ is held constant at $\bar{\gamma}$. This is a potentially important simplification that is exactly consistent with Flood and Garber (1984), but removes all real volatility from our new volatility measure. Expunging real volatility contrasts with early studies that captured primarily real-side volatility. In the 1960s and 1970s, extensive capital controls meant that volatility in external payments represented primarily volatility in trade flows. Indeed, some early empirical studies focused solely on volatility in export earnings.

where u is the drift parameter and v is a white noise disturbance.[37] When fundamentals follow equation (13), the shadow exchange rate, in turn, must follow:

$$\tilde{S}_t = F_t + \lambda\mu \,. \tag{14}$$

Its rate of change is equal to the growth rate of fundamentals.

If our replacement for the variance of reserve increments is to match FJ exactly, then increments to the right-hand-side of equation (11) must have the same variance as increments to the shadow-rate fundamental defined in equation (13). This is true exactly (for small increments and a very short-term interest rate) in a log linear shadow rate model (as in Flood and Marion (2000)) but can hold here only as an approximation since we are working with a level-linear exchange rate model.

Our ability to substitute fundamentals volatility for reserve volatility is assured once we recognize that the expected value of reserves conditional on their not having hit the lower boundary since they were reset at R_0 is just the conditioning information we need to exclude both upward restocking reserve jumps and downward successful speculative attacks. Consequently, the expected value of reserves depends only on its initial optimal value and the distribution of the fundamentals that drive the shadow rate until it hits its upper bound. From equation (13), the increments to the fundamentals are distributed normally, just as FJ had hoped their reserve increments would be.[38]

Coskewness

When we estimated the reserve-holding equation using the a reserves volatility measure, we tested for skewness. When the fundamentals volatility measure is used instead, the appropriate analog is coskewness. If v is the reserve-increment and ε is the fundamentals increment, then skewness is Ev^3 and coskewness is $Ev\varepsilon^2$. We were unable to find a simple coskewness statistic analogous to Pearsons SK statistic, so we made up our own break-point measure based on $\rho = Cov(v,\varepsilon^2)/(Var(v) * Var(\varepsilon^2))$ for v, ε as above.

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[37] We assume that v is distributed normally, so $v_t \sim (0, \sigma_v^2)$. There is no skewness problem. If there are big jumps in fundamentals, or if these jumps are correlated with reserve jumps, as might be the case if the reserve authority follows a policy of sterilized intervention in the foreign-exchange market, then skewness problems will arise and again contaminate the estimation of reserve holdings. We address this issue in Section VI of the text.

[38] In our empirical implementation, we try two different replacements for reserve volatility. When $\bar{S}$ is *piecewise stable* (e.g., moving with time-to-time devaluations.), we use the volatility of shadow-rate fundamentals. This is the option we report in the text. Our method is not appropriate, however, when $\bar{S}$ is moving stochastically but nearly continuously (e.g., when $\bar{S}$ is stabilized partially or smoothed.). In the latter situation, the relevant volatility measure is the volatility of fundamentals that influence $\tilde{S} - \bar{S}$.

References

Baumol, William J., (1952), "The Transactions Demand for Cash: an Inventory Theoretic Approach," *Quarterly Journal of Economics*, Vol. 66 No. 4 (November), pp. 545-56.

Bahmani-Oskooee, Mohsen, (1985), "Demand for International Reserves: Survey of Recent Empirical Studies," *Applied Economics*, No. 17, pp. 359-75.

Ben-Bassat, Avraham and Daniel Gottlieb, (1992), "Optimal International Reserves and Sovereign Risk," *Journal of International Economics*, No. 33, pp. 345-62.

Blanco, Herminio and Peter Garber, (1986), "Recurrent Devaluation and Speculative Attacks on the Mexican Peso," *Journal of Political Economy*, No. 94, pp. 148-66.

Bussiere, Matthieu and Christian Mulder, (1999), "External Vulnerability in Emerging Market Economies: How High Liquidity Can Offset Weak Fundamentals and the Effects of Contagion," IMF Working Paper No. 99/99, (July), (Washington: International Monetary Fund).

Calvo, Guillermo, and Carmen Reinhart, (2000), "Fear of Floating," NBER Working Paper No. 7993, (November), (Cambridge, Massachusetts: MIT Press).

——, (1996), "Capital Flows and Macroeconomic Management: Tequilla Lessons," *International Journal of Finance and Economics*, (July).

Carroll, Chris, (1992), "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence," Brookings Papers on Economic Activity, No. 2, pp. 61-156.

Claassen, Emil, (1975), "Demand for International Reserves and the Optimum Mix and Speed of Adjustment Policies," *American Economic Review*, No. 65, pp. 446-53.

Detragiache, Enrica, (1996), "Fiscal Adjustment and Official Reserves in Sovereign Debt Negotiations," *Economica*, No. 63, (February), pp. 81-95.

Disyatat, Piti and Donald Mathieson, (2001), "Currency Crises and the Demand for Foreign Reserves," mimeo, (April), (Washington: International Monetary Fund).

Elbadawi, Ibrahim, "The Sudan Demand for International Reserves: A Case of a Labour-exporting Country," *Economica*, No. 57, (February), pp. 73-89.

Economist, (2000), "Intervention: Divine or Comic?" (September), Nos. 23-29, pp. 89-90.

Edwards, Sebastian, (1983), "The Demand for International Reserves and Exchange Rate Adjustments: The Case of LDCs, 1964-72," *Economica,* No. 50 (August), pp. 269-80.

——, (1985), "On the Interest-Rate Elasticity of the Demand for International Reserves: Some Evidence from Developing Countries," *Journal of International Money and Finance,* No. 4, pp. 287-95.

Eichengreen, Barry, (1992), "Golden Fetters: The Gold Standard and the Great Depression, 1919-39," (New York: Oxford University Press).

Flood, Robert and Peter Garber, (1984), "Collapsing Exchange-Rate Regimes: Some Linear Examples," *Journal of International Economics,* No. 17, pp. 1-13.

——, and Nancy Marion, (1999), "Perspectives on the Recent Currency Crisis Literature," *International Journal of Finance and Economics,* Vol. 4 No. 1, pp. 1-26, reprinted in Guillermo Calvo, Rudi Dornbusch and Maurice Obstfeld, Money, Capital Mobility, and Trade: Essays in Honor of Robert A. Mundell, (Cambridge: MIT Press 2001), pp. 207-50.

——, (2000), "A Model of the Joint Distribution of Banking and Exchange-Rate Crises," IMF Working Paper No. 01/213, (Washington: International Monetary Fund).

——, and Andrew Rose, (1995), "Fixing Exchange Rates," *Journal of Monetary Economics.*

——, and Olivier Jeanne, (2000), "An Interest-Rate Defense of a Fixed Exchange Rate?" CEPR Discussion Paper No. 2057 (July) and IMF Working Paper No. 00/159, (Washington: International Monetary Fund).

Fischer, Stanley, (2001), "Distinguished Lecture on Economics in Government—Exchange Rate Regimes: Is the Bipolar View Correct?" *Journal of Economic Perspectives,* (Spring), Volume 15 No. 2, pp. 3-24.

Ford, J. L. and Guobo Huang, (1994), "The Demand for International Reserves in China: An ECM Model with Domestic Monetary Disequilibrium," *Economica* No. 67 (August), pp. 379-97.

Frenkel, Jacob, and Boyan Jovanovic, (1981), "Optimal International Reserves: A Stochastic Framework," *Economic Journal* No. 91 (June), pp. 507-14.

——, (1983), "International Liquidity and Monetary Control," in George M. von Furstenberg (ed.) International Money and Credit: The Policy Roles, (Washington: International Monetary Fund), pp. 65-109.

Frenkel, Jacob, (1980), "The Demand for International Reserves under Pegged and Flexible Exchange Rate Regimes and Aspects of the Economics of Managed Float," in David Bigman and Teizo Taya (eds.) *The Functioning of Floating Exchange Rates: Theory, Evidence, and Policy Implications,* Ballinger Publishing Company.

——, (1974), "The Demand for International Reserves by Developed and Less-Developed Countries," *Economica,* No. 41, (February), pp. 14-24.

Grimes, Arthur, (1993), "International Reserves under Floating Exchange Rates: Two Paradoxes Explained," *The Economic Record,* Vol. 69 No. 207 (December), pp. 411-15.

Grubel, Herbert (1971). "The Demand for International Reserves: A Critical Review of the Literature," *Journal of Economic Literature* Vol. IX No. 4, (December), pp. 1148-66.

Hamada, Koichi and Kazuo Ueda, (1977), "Random Walks and the Theory of Optimal International Reserves," *Economic Journal* No. 87, (December), pp. 722-42.

Hausmann, Ricardo, Ugo Panizza and Ernesto Stein, (2000), "Why Do Countries Float the Way They Float?" Working Paper, Research Department, Inter-American Development Bank, (March).

Heller, Robert, (1966), "Optimal International Reserves," *Economic Journal,* No. 76, pp. 296-311.

——, and Mohsin Khan, (1978), "The Demand for International Reserves Under Fixed and Floating Exchange Rates," *IMF Staff Papers*, pp. 623-49.

Huang, Tai-Hsin and Chung-Hua Shen, (1999), "Applying the Seasonal Error Correction Model to the Demand for International Reserves in Taiwan Province of China," *Journal of International Money and Finance,* No. 18, pp. 107-31.

Jung, Chulho, (1995), "Optimal Management of International Reserves," *Journal of Macroeconomics*, (Fall), Vol. 17 No. 4, pp. 601-21.

Kenen, Peter and Elinor Yudin, (1965), "The Demand for International Reserves," *Review of Economics and Statistics,* No. 47, (August), pp. 242-50.

Krugman, Paul, (1979), "A Model of Balance-of-Payments Crises," *Journal of Money, Credit, and Banking,* No. 11, pp. 311-25.

Kuo, Shirley W.Y. and Christina Y. Liu, (1998), "Taiwan," in Ross H. McLeod and Ross Garnaut (eds.), East Asia in Crisis: From Being a Miracle to Needing One?, (New York: Routledge Press), pp. 179-88.

Lizondo, Jose and Don Mathieson, (1987), "The Stability of the Demand for International Reserves," *Journal of International Money and Finance,* No. 6, pp. 251-82.

Levy, Victor, (1983), "Demand for International Reserves and Exchange-Rate Intervention Policy in an Adjustable-Peg Economy," *Journal of Monetary Economics,* No. 11, pp. 89-101.

Miller, Merton and Daniel Orr, (1966), "A Model of the Demand for Money By Firms," *Quarterly Journal of Economics,* No. 80, pp. 413-35.

Radelet, Steven and Jeff Sachs, (1998), "The Onset of the East Asian Financial Crisis," in National Bureau of Economic Research volume Asian Crisis and International Solutions, Ch. 20.

Saidi, Nasser, (1981), "The Square-Root Law, Uncertainty and International Reserves Under Alternative Regimes: The Canadian Experience, 1950-76," *Journal of Monetary Economics,* No. 7, pp. 271-90.

Salant, Steve and Dale Henderson, (1978), "Market Anticipation of Government Policy and the Price of Gold," *Journal of Political Economy,* No. 86, pp. 627-48.

Shin, Hyun-Han and Rene M. Stulz, (2000), "Shareholder Wealth and Firm Risk," mimeo, (December), (Ohio: Ohio State University).

Tobin, James, (1956), "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics,* Vol. 38 No. 3, (August), 241-70.