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Market Volatility as a Financial Soundness Indicator: An Application to Israel

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Abstract

Financial decisions of economic agents are based on volatility considerations. However, no aggregate indicators have been used by policymakers and regulators to assess the market risk environment. This paper applies a market volatility indicator to analyze the Israeli's transition toward inflation targeting. Unlike conventional measures of volatility, it shows a substantial decline once volatility is measured against the minimum variance for the same returns on assets. Using a conventional Multivariate GARCH model, we find that interest rates sensitivity to changes in the risk environment may be important for a correct identification of volatility patterns of individual assets.

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I. INTRODUCTION

Central banks and governments are paying increasing attention to monitoring the health and efficiency of financial institutions and markets.[2] Market risk is an important factor affecting the vulnerability of the financial system and its main users, comprising exposure and volatility components. In the analysis of systemic risks, market-risk analysis has been based on overall exposure indicators, partly because related variables are easily observable and difficult to reverse within a short time frame. By contrast, volatility analysis has been used only to a limited extent in financial risk assessments at the country level. The reasons for this are the difficulty to predict the extent and periodicity of volatility shocks, the high speed at which volatility correlations change in periods of turmoil, the delay in volatility indicators to showing clear patterns (normally these do not change before instability has already erupted), and doubts about the additional information content of volatility indicators relative to more conventional indicators.

This preference for exposure indicators over volatility indicators to assess market risk at the aggregate level has resulted in a gap between the aggregate measurement and analysis of market risk and the rapid development of methodologies to measure and model financial asset returns' volatility and correlations. Recent advances in the latter have moved from the academic community to industry, and then to the regulatory framework, at the level of individual bank regulation.[3] While more financial decisions of economic agents are made based on volatility considerations, until now no aggregate indicators have been used by policymakers and regulators to assess the market-risk environment.

This paper aims at filling this void by devising a reasonable working methodology to address aggregate market volatility using a standard framework for the analysis of volatility patterns, such as Multivariate GARCH models, applied to the case of Israel. In addition, the paper presents an additional aggregate micro indicator of market risk whose impact on the evolution of individual volatility patterns is assessed using the Multivariate GARCH framework. The information content of this indicator would be valuable, since its fluctuations would provide information about changes in the risk environment, even when exposure remains unaltered.

The plan of this paper is as follows: Section II discusses the relevance of a market volatility indicator in the context of the IMF's effort to develop financial soundness indicators. Then, it discusses and addresses the concept of portfolio volatility based on a representative country portfolio, including a comparison with alternative indicators. Section III contains an introduction to the risk conditions of Israeli financial markets during the period 1992–2000

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[2]Sundararajan and others (2002).

[3]As in JP Morgan's Risk Metrics, and the Basel Capital Accord recommendation to use VaR measures to determine capital requirements.

for three representative assets in the context of Israel's transition to an inflation-targeting framework and analyzes the patterns of the proposed market volatility indicator for Israel in this period. Section IV presents a Multivariate GARCH model for Israel's representative assets and for the interactions between the Israel stock market and other stock markets from which the Israel stock market is expected to receive spillover effects, namely those in the United States and Argentina. This framework allows for the incorporation of the proposed market volatility indicator as an illustration of the influence of the risk environment on specific asset-volatility patterns. Section V comprises possibilities of future work and concludes.

II. Market Volatility as a Financial Soundness Indicator

A. Relevant Analytical Aspects Related to Financial Soundness Indicators

A recent IMF paper on Financial Soundness Indicators (FSI)[4] identifies two sets of indicators for the purpose of periodic monitoring and for compilation and dissemination efforts by national authorities: A *core set* of banking sector indicators that would have priority in compilation and monitoring and an *encouraged set* including additional banking indicators and data on other institutions and markets that are relevant in assessing financial stability. Market volatility indicators belong to the second group, as they relate to the behavior of other institutions and markets that are relevant in assessing financial stability, for example the stock and the foreign exchange markets. These are however not unrelated to banking activity, especially if interest rate volatility is incorporated. The choice of specific market volatility indicators also conditions the appropriate measurement of bank sensitivity to market risk.[5] The following considerations are relevant:

- **Justification of the use of an MVI:** The construction of a market volatility indicator (MVI) is justified by the need to have a parsimonious indicator that reflects market risk environment conditions. By focusing on relevant markets,[6] and using widely available information, a market volatility indicator could even fulfill some of the conditions of the core set indicators. More extensive application would test its usefulness and analytical significance.

- **Information content of MVI:** An MVI would be based on the volatility and correlations of prices and yields to convey market perceptions and risk expectations.

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[4]Sundararajan and others (2002).

[5]It should be mentioned that duration, included in the core set to measure sensitivity to market risk for deposit-taking institutions, has proven not so easy to compile.

[6]In the case of the application to Israel, we use price and yield information from the stock, foreign exchange and bond markets.

Ideally, information from the MVI should be complementary to that of early warning indicators to monitor vulnerabilities and prevent crises. This is a challenge, because volatility normally increases markedly only once financial disruption prevails. Another challenge is to devise an indicator that could provide information useful as an early warning and, at the same time, easily applicable for cross-country comparisons.

- **Measurement limitations:** Volatility indicators could be affected by regulations that limit flexibility of prices, which makes the selection of relevant prices an important consideration. The most obvious case is the exchange rate, which could be fixed or predetermined making the analysis of nominal exchange rate volatility irrelevant. In some cases, volatility of indicators other than prices could be used as a proxy of the unobservable price volatility.[7] However, an advantage of price volatility indicators is that prices are less affected by accounting norms and other related regulations than exposure indicators.

- **Gross versus "net" risk:** Absolute risk levels "may not by themselves fully indicate financial institutions' or a system's vulnerabilities." [8] While qualitative analysis compensates for that, it is desirable to construct indicators that are already at least partially corrected for differences in the risk environment (or "net risk," to use the FSI terminology).

B. Proposed Market Volatility Indicator

Volatility analysis requires controlling for exposure, that is, a neutral exposure is required as a reference.[9] While bank, corporate and household exposure to market risk will show different degrees of risk aversion, an indicator of aggregate volatility would allow approaching the "volatility environment." We construct an indicator of volatility for a neutral exposure to be used as a benchmark, using the weights of each asset relative to total market capitalization, namely foreign exchange deposits, stock holdings and interest rate-bearing bank deposits (time and savings).

A straight country portfolio volatility indicator may still not be suitable to reflect differences in risk environment across countries. For example, a given value of a volatility indicator in two kinds of environment, one unstable and another one stable, implies differences in risk-

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[7]For example, volatility of international reserves in a managed exchange rate regime could proxy hypothetical volatility of the exchange rate if it were allowed to move.

[8]Sundararajan and others (2002).

[9]Likewise, analysis of exposure is normally done at different levels of aggregation, taking volatility as given.

taking. The same volatility indicator in an unstable environment would imply much less risk being undertaken relative to a calmer environment. In the same fashion, the same portfolio volatility in unstable and stable periods would indicate a lower risk aversion in the second period. Thus, an indicator of relative volatility appears necessary to assess market risk relative to the risk environment.

As an initial step, we calculate the actual portfolio volatility as the product of the corresponding weights of different assets in the portfolio and the variance covariance matrix. In matrix notation:

$$\text{AV (aggregate volatility)} = \boldsymbol{w}'\mathbf{V}\ \boldsymbol{w}$$

Where w_1, w_2 and w_3 are the actual weights of the corresponding assets and **V** is the corresponding variance and covariance matrix.

To construct a relative volatility indicator, actual volatility should be compared with an alternative volatility indicator that would serve as a benchmark. A minimum-variance portfolio is constructed based on the following procedure:

$$\text{MV (minimum variance)} = \underset{w}{Min.}\ \ w'\mathbf{V}w,$$

$$\text{s.t.}\quad \begin{aligned} & w_1 \geq 0 \\ & w_2 \geq 0 \\ & w_3 \geq 0 \\ & w_1 + w_2 + w_3 = 1 \end{aligned}$$

For a return equal to $y = w_1 \times y_1 + w_2 \times y_2 + w_3 \times y_3$

where $w' = w_1, w_2, w_3$ are the unknown weights of three corresponding assets in the portfolio that would result from minimizing the portfolio variance for the same **V**, in order to obtain the same corresponding historical returns y_1, y_2 and y_3. In other words, rather than the absolute minimum, this indicator shows the minimum variance to obtain the same return. This portfolio would be the one that would have given the same return as the actual portfolio by undertaking minimum risk. Relative volatility would be the result of comparing both indicators. The corresponding market volatility indicator would be the following:

$$\text{MVI} = \frac{\text{AV - MV}}{\text{AV}}$$

This indicator expresses the following: Higher relative volatility would result if a larger share of total volatility corresponds to deviations with respect to the minimum variance. This would mean that, on average, agents are willing to undertake higher risk. Likewise, if most volatility is almost equivalent to the minimum variance for the same returns, it would indicate maximum risk aversion.

Several advantages result from the use of this indicator:

- It is a *parsimonious* indicator. It is constructed with high-frequency data, but for easily observed variables (mostly price volatility indicators).
- It is constructed based on *conventional statistical concepts* such as the variance-covariance matrix.
- It is a *forward-looking* indicator, to the extent that it relies on values relative to a benchmark rather than on absolute values.
- It provides *additional information content* relative to other "level" variables such as exposure indicators or mere rates of change.
- It is at the same time able to be standardized and easily adaptable to specific circumstances, for example by selecting the relevant variables and the relevant weights.

C. Comparison with Alternative Indicators

Some operational problems common to other alternative indicators affect the proposed MVI, namely how to determine the optimal time period and how to account for regime shift effects. However there are some important differences to be considered, especially with duration and risk appetite indicators.

Duration indicators

Duration is widely used to measure bank sensitivity to market risk. Duration is the weighted average life of an asset or liability. It adjusts maturity to account for "risks to financial system stability that can derive from developments in nonbank financial intermediaries, the corporate sector, households, and real estate markets." (Sundararajan and others). Duration rises with maturity, falls with the frequency of coupon payments, and falls as the yield rises.

Duration is a useful tool to assess interest rate risk. It provides an estimate of the change in the market value of the portfolio due to changes in interest rates. However, there are some advantages to use instead an aggregate indicator of volatility such as the one we propose:

- In the event of interest rate shocks, duration does not account for the impact of portfolio shifts.
- More generally, lower volatility in some asset prices may be more than compensated by higher volatility in other asset prices, which is not captured by independent

duration indicators. The proposed MVI provide an aggregate concept of volatility that implicitly incorporates this kind of effect.

- Moreover, duration models are generally complex for countries with less sophisticated statistical systems, paradoxically for which they would be more useful (sophisticated financial institutions have more complex models to assess market risk).

- Incorporating duration data to VaR models to account for correlations and disaggregations is useful for individual institutions and for short-term risk assessment. However, unlike our proposed MVI, VaR techniques are not suitable for aggregate risk and the identification of medium-term trends.

Risk appetite indicators

A variety of indicators have been used to assess investor risk appetite.[10] Commonly used measures of risk appetite comprise the yield spread between high-low rated bonds, and options implied vs. historical volatility. However, despite being relative measures of risk, these proxies still combine absolute risk with risk appetite. The benchmarks are economic units less affected by risk fluctuations, or time-varying component of risk. MVI has several advantages over these risk appetite indicators (regardless of how effectively they measure risk appetite):

- MVI could be applied to broader asset categories (bonds with different rating or option prices are of limited availability in most developing economies).

- The corresponding reference benchmark is obtained based on the same information on which the actual volatility is measured, which reduces the absolute risk component.

However, we prefer not to jump to label MVI as a risk-appetite indicator, which would require an analysis of other statistical properties. A proper risk appetite index is constructed for currency markets by Kumar and Persaud using a Spearman's rank correlation of current returns and past risks based on spot and forward exchange rates. They calculate the average volatility of excess returns over a period of a year, and then average excess returns against the major currencies. This indicator proves significant in explaining systemic crises. The MVI is a much more parsimonious index based on more conventional measures that facilitates the consolidation of different asset prices into one measure. To differentiate it from risk appetite indicators, we refer to it as a risk exposure indicator.

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[10]See Kumar and Persaud (2001).

III. Market Volatility in Israel, 1992–2000.

A. Exchange Rate, Interest Rates, and Stock Prices: Evolution and Volatility Patterns

In this section we analyze three representative asset daily returns for 1992–2000: The exchange rate expressed in Israeli Shekels per U.S. dollar; the interest rate on three-month treasury bills; and the Tel-Aviv 25 stock index (TA-25: weighted average index stock price for the 25 main Israeli corporations). The economic policy framework in Israel was characterized by the transition from a nominal exchange rate anchor to increasing exchange rate flexibility toward the adoption of inflation-targeting.

Inflation and interest rates rapidly converged toward that of industrial countries in that period. In the subperiod 1992–96, a crawling exchange rate band was implemented and an explicit inflation target was adopted. The inflation rate declined to about 10 percent per year (from 16–20 percent a year in the previous five-year period). In 1997–2000, continuous single-digit inflation rates consolidated the disinflation process, despite a temporary volatility shock coming from unrest in the world financial markets at the end of 1998. Currently, inflation targeting coexists with a commitment to keep the exchange rate within a crawling exchange rate band.

Exchange rate volatility has been contained by a deliberate policy of maintaining the exchange rate close to the central parity rate between 1992 and 1996, with the central bank operating an inner intervention band until February 1996. This led to sizable purchases of foreign exchange by the central bank in the face of progress in the Middle East peace process. The domestic-foreign interest rate differential attracted additional capital inflows that reinforced the tendency to exchange rate appreciation, which was offset by central bank intervention in the foreign exchange market.[11] A moderate reversal of capital flows in late 1998 caused a sizable depreciation of the shekel, reflecting the impact of the Russian crisis in the context of the relaxation of most foreign exchange controls that year.

Interest rate volatility (as measured by the volatility of 3-month treasury bill interest rates) has been on average much more limited than that of the exchange rate (see Table 1), suggesting that the authorities focus on interest rates as the intermediate target of monetary policy. Lower interest rate volatility reflects the reduction in inflation, the weakening of the pass-through from the exchange rate to prices, and the cautious reduction of central bank interest rates along with the progress toward price stability. In contrast with what occurred between 1988 and 1991, smaller interest rate changes proved sufficient to counteract speculative attacks on the exchange rate. The interest rate differential between Israeli and

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[11]The exchange rate band was widened from 5 percent to 7 percent in 1995, 14 percent in 1996 and 28 percent in June 1997, with further gradual widening bringing the band above 35 percent in the second half of 2000.

foreign interest rates has declined from a peak of 12 percentage points in mid-1996 to below 4 percentage points since the beginning of 2000.

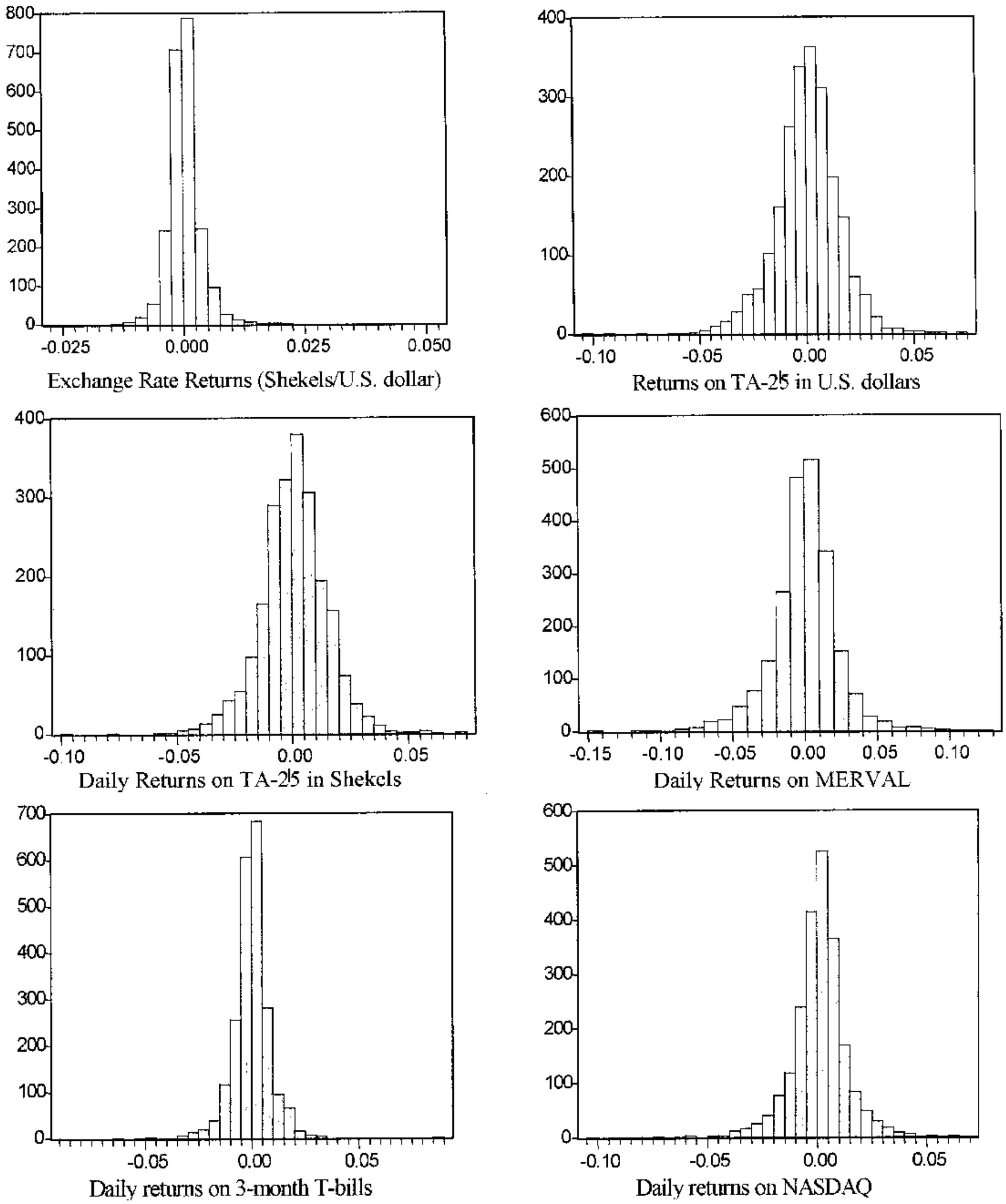
Table 1. Israel: Returns on Alternative Assets

	Exchange Rate Returns	Interest Rate Return on T-Bills	
Mean	0.000	0.000	0.001
Median	0.000	0.000	0.001
Maximum	0.051	0.023	0.071
Minimum	-0.027	-0.021	-0.099
Std. Dev.	0.004	0.003	0.015
Skewness	1.851	-0.440	-0.196
Kurtosis	27.379	11.105	5.751
Jarqie=Bera	56,371.9	6,161.3	715.9
Probability	0.0	0.0	0.0
Observations	2,225	2,225	2,225
Average one-month volatility	5.031	3.911	21.874
(Standard deviation)	2.843	1.833	8.405
Average three-month volatility	5.275	4.018	22.544
(Standard deviation)	2.402	1.507	6.448
Average six-month volatility	5.427	4.095	22.734
(Standard deviation)	2.106	1.337	5.452

Sources: Central Bank of Israel, Israel Stock Exchange; and IMF staff calculations.

Returns on the TA-25 index show the widest dispersion relative to other assets, as is normally observed in other stock markets. At the same time the distribution of returns on the TA-25 index shows the lowest skewness and kurtosis, but the distribution is still far from normal (as reflected in the Jarque-Berra coefficient). Average daily returns on foreign exchange, treasury bills and stocks are close to zero as expected. (Table 1 and Figure 1). This is also true for other countries, as illustrated by the returns on the Argentine MERVAL and the US NASDAQ, which also show thick tails relative to the normal distribution. Interestingly, the unconditional volatility of the TA-25 in the period under consideration is only slightly higher than for the NASDAQ (considered on the high side in the United States). By contrast, the unconditional volatility of the Argentine MERVAL is almost twice that of the NASDAQ's.

Figure 1. Daily Returns on Israel's Alternative Assets, the Argentina's MERVAL, and the U.S. NASDAQ



Sources: Central Bank of Israel, Argentina, Israel and U.S. Stock Exchanges, and staff calculations.

Table 2. Unconditional Distribution Statistics: Stock Price Returns
(Sample period: January 3, 1992–July 12, 2000) 1/

	DLNNASDAQ	DLNSEARG	DLNSEISUSD
Mean	0.000874	-0.00018	0.000503
Median	0.00138	0.000606	0.000879
Maximum	0.069418	0.120719	0.074227
Minimum	-0.101684	-0.147649	-0.101947
Standard Deviation	0.013155	0.023522	0.015096
Skewness	-0.692714	-0.301655	-0.311813
Kurtosis	9.585421	7.15781	6.023661
Jarque-Berra	4,196.611	1,635.693	883.2462

Sources: National Stock Exchanges; and IMF staff calculations.

1/ DLNNASDAQ, DLNSEARG, and DLNSEISUSD stand for daily returns on the U.S. NASDAQ, the Argentinean MERVAL and the Israeli TA-25 respectively, expressed in U.S. dollars.

Figure 2 summarizes the main patterns of the evolution of the level and volatility of the exchange rate, the three-month interest rate on treasury bills and the TA-25 stock price index:

- In terms of levels, the exchange rate shows a rather smooth path up to the end of 1998, consistent with central bank policy. After that, the pace of nominal exchange rate depreciation decelerates, consistent with the consolidation of disinflation.

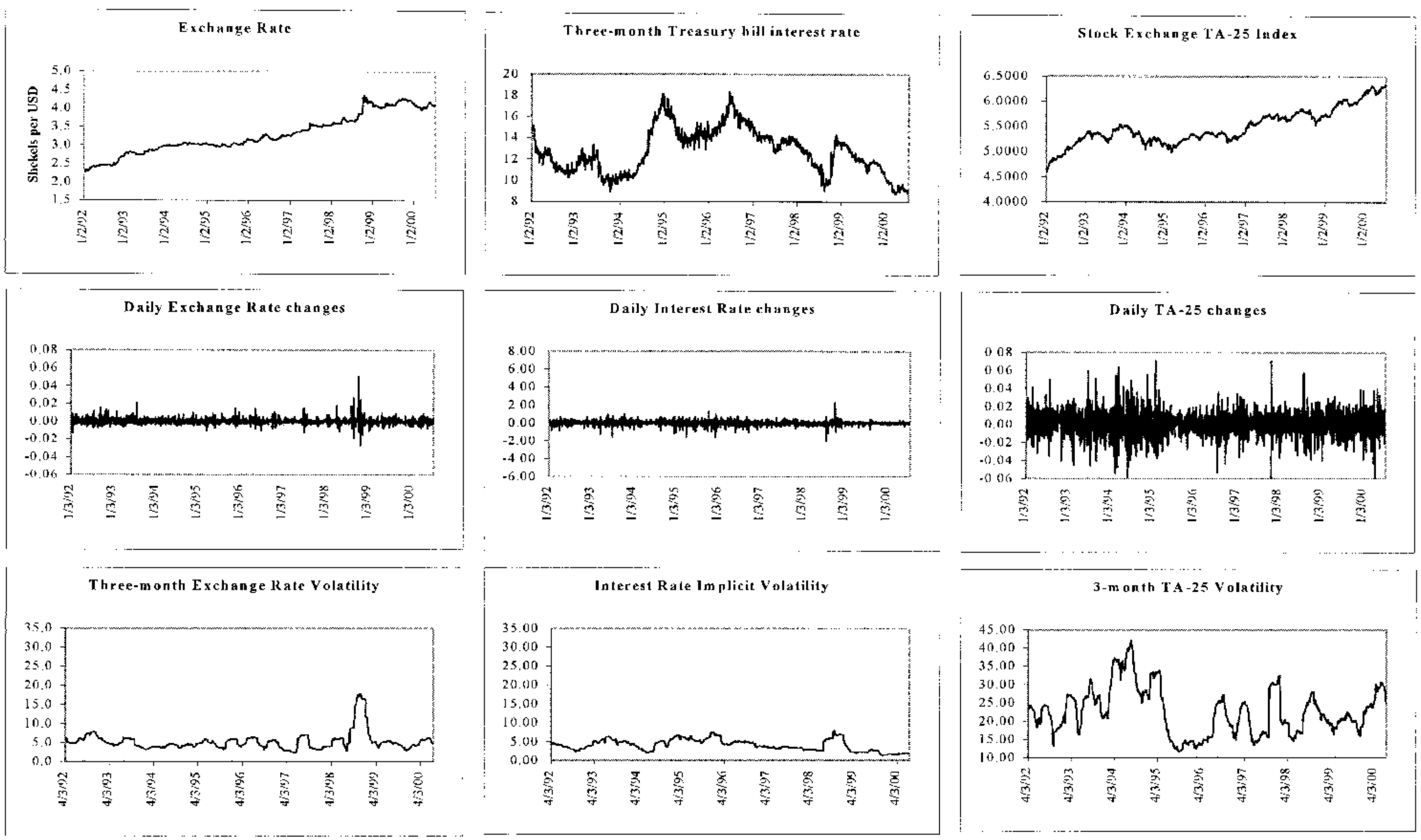
- Interest rates remain relatively high between 1995–1996, a period in which budget jitters were followed by uncertainties surrounding general elections, to show a declining trend thereafter to reach one-digit interest rates in 2000 (process only briefly interrupted by the October 1998 turmoil following the Russian crisis).

- Stock prices show an upward trend since 1997, helped by economic stability and a liberal approach toward foreign investment, attracted by the extensive knowledge base and developed high-tech sector. The average increase in foreign direct investment reached 35 percent per year from 1995 to 1999.

- Peaks in three-month exchange rate volatility are rarely above 5 percent with the exception of the 1998 economic unrest following the Russian crisis.[12] The same is true for interest rates, which experienced a more limited impact after the1998 crisis. By contrast, stock prices show large volatility swings, with three-month volatility always above 10 percent (it reached 40 percent in 1994) with a somewhat higher average volatility between 1992 and 1995.

[12] All volatility calculations are based on 260 working days per year.

Figure 2. Israel: Evolution and Volatility Patterns of the Exchange Rate, Interest Rates, and Stock Prices, 1992–2000



Sources: Central Bank of Israel; Israel Stock Exchange; and IMF staff calculations.

B. A Market Volatility Indicator for Israel

Table 3 shows the variance-covariance matrix and corresponding correlation matrices used to construct an MVI for Israel. While the variance increases for the exchange rate and decreases for the interest rate and stock prices, the covariances show drastic swings. Correlations even change signs (for both exchange rate and the interest rate against stock prices) and show abrupt changes (for the exchange rate and interest rates), despite corresponding to overlapping periods. Relative weights decline for foreign currency and stock holdings in the period, in favor of interest-bearing assets, which is consistent with a positive response of expectations to policy changes in the period.

Annualized four-year moving-average returns in combination with the corresponding weights of each asset in the total portfolio are used to construct average portfolio returns. As can be observed in Figure 3, portfolio returns show an upward trend, having fluctuated between 1995 and 2000 in the range of 8-16 percent in nominal terms. Using the variance-covariance matrix for each period, we obtain the corresponding moving portfolio variance shown in Figure 3. Portfolio variance declines slightly since 1998, from almost 4 percent per month in the preceding years to slightly more than 2 percent.

The minimum variance for the same average returns shows a process of convergence toward the minimum variance portfolio in the second half of the decade resulted, consistent with lower risk exposure, in part as a result of a reduction of foreign currency holdings at an aggregate level.[13] As can be observed in the bottom figure in Figure 3, while the fluctuations of actual portfolio volatility are small, the distance with respect to the minimum volatility for the same rate of return narrows after 1998. This coincided with a period of overall increasing Sharpe ratio (risk-adjusted returns). Israel shows then an atypical benign situation: Higher returns were achievable with less risk exposure. An implication of this is that the necessary adjustment to jump toward the minimum-variance portfolio in case of a shock declined, as measured by the square difference of the corresponding weights for the actual and the minimum variance portfolio (Table 4).

Let's call "risk exposure" the ratio of actual over minimum variance (which would be a transformation of the MVI, as this is the difference of actual minus minimum variance divided by actual variance). The risk exposure ratio in Figure 3, as based on moving averages, gives an idea of medium-term trends concerning risk-taking for Israel's economy.[14]

[13]It should be noted that the calculation of the minimum variance often implies corner solutions (i.e., disposing of all assets of the most risky class). In the sample period, this would imply the (unrealistic) frequent disposal of foreign exchange holdings.

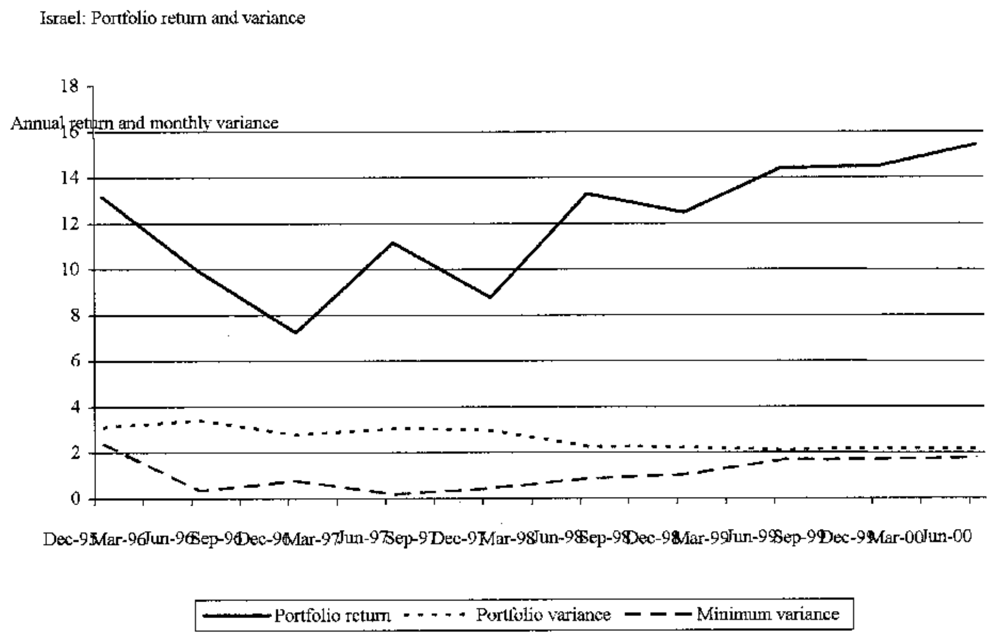
[14]We consider that shorter periods would not allow for adjustments in the asset portfolio as a response to changes in returns.

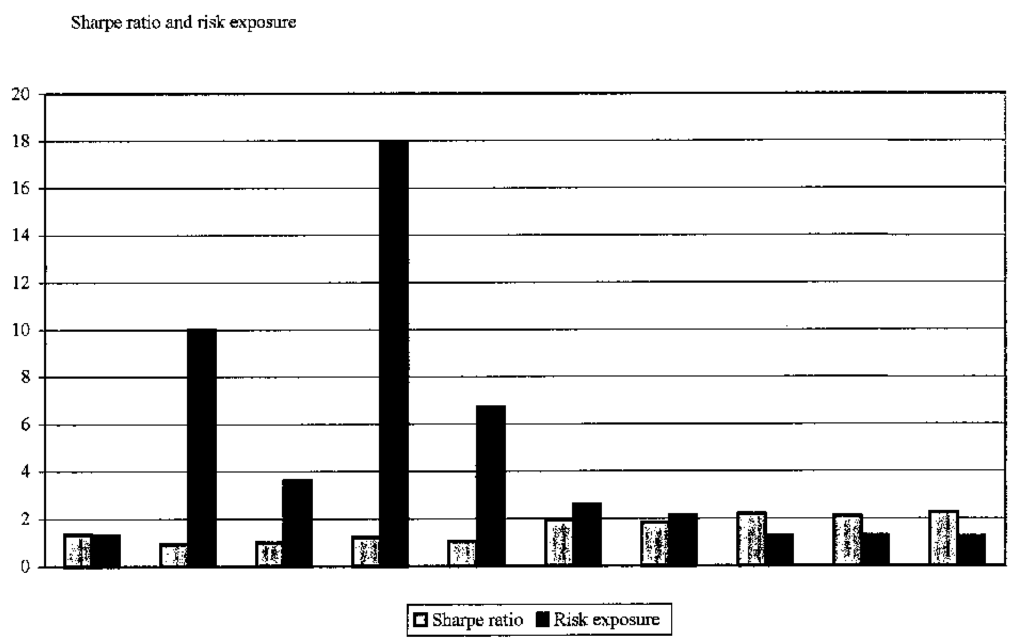
Table 3. Israel: Correlation of Asset Returns and Shares of Assets

	Foreign Currency	Stocks	Interest-Bearing Assets	Foreign Currency	Stocks	Interest-Bearing Assets	Foreign Currency	Stocks	Interest-Bearing Assets
	Jan 1992-Dec 1995			Jan 1994-Dec 1997			Jan 1996-Dec 1999		
				Variance-Covariance Matrix					
Foreign currency	0.000229	6.01E-05	-6.59E-06	0.000216	1.99E-04	-1.92E-06	0.000485	-7.75E-05	-7.55E-06
Stock	6.01E-05	0.005381	-1.63E-05	1.99E-04	0.0052	4.28E-06	-7.75E-05	0.00349	1.22E-06
Interest-bearing assets	-6.59E-06	-1.63E-05	2.40E-06	-1.92E-06	4.28E-06	1.58E-06	-7.55E-06	1.22E-06	1.60E-06
Annual std. dev.	5.24%	25.41%	0.54%	5.09%	24.98%	0.44%	7.63%	20.46%	0.44%
				Correlation Matrix					
Foreign currency	1.00000	0.05414	-0.28110	1.00000	0.18777	-0.10393	1.00000	-0.05957	-0.27103
Stock	0.05414	1.00000	-0.14343	0.18777	1.00000	0.04722	-0.05957	1.00000	0.01633
Interest-bearing assets	-0.28110	-0.14343	1.00000	-0.10393	0.04722	1.00000	-0.27103	0.01633	1.00000
				Share of Assets in Portfolio					
Weights	27	42	31	24	40	37	23	36	41

Source: IMF staff calculations.

Figure 3. Israel: Country Portfolio Return and Volatility, 1992–2000





Source: IMF staff calculations.

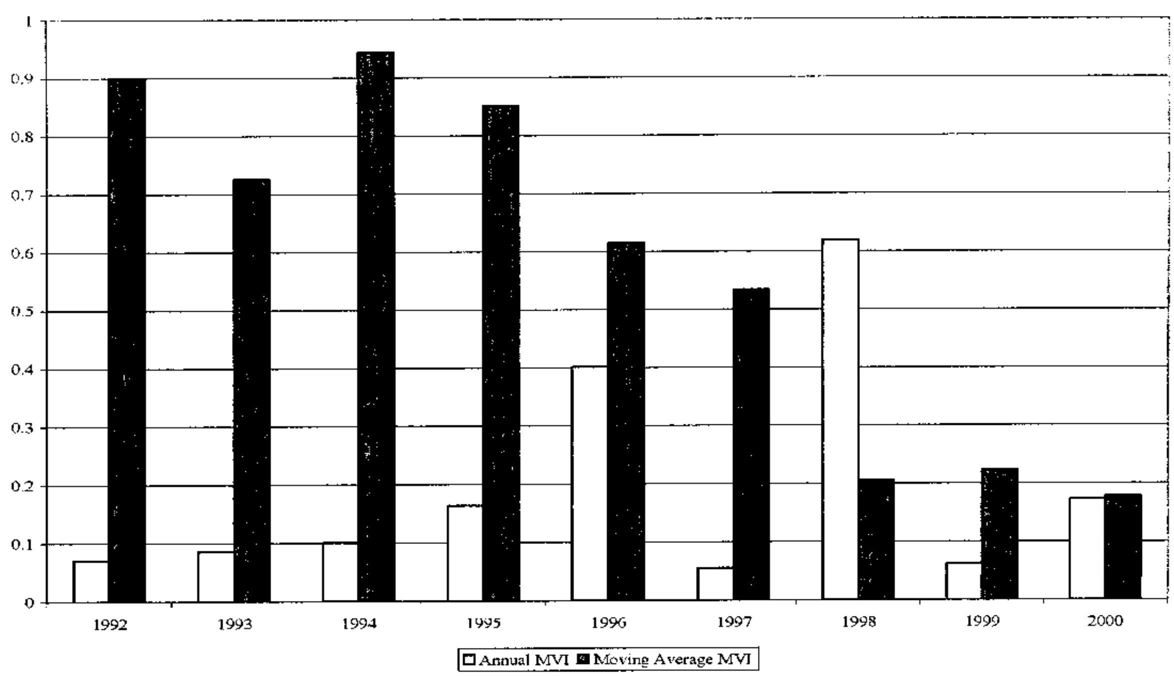
Table 4. Israel's Actual and Minimum-Variance Portfolio Composition

	Actual			Minimum Variance			Squared Portfolio Adjustment
	Foreign currency	Stocks	Time and savings	Foreign currency	Stocks	Time and savings	
Dec 91–Dec 95	26.8	41.9	31.3	0.0	32.6	67.4	2,113.1
Jun 1992–Jun 96	27.1	47.0	25.9	25.4	0.0	74.6	4,581.7
Dec 1992–Dec 1996	23.1	38.2	38.7	56.8	1.1	42.1	2,516.9
Jun 1993–Jun 1997	24.3	40.9	34.8	9.5	0.0	90.5	4,994.6
Dec 1993–Dec 1997	23.7	39.8	36.5	30.3	0.0	69.7	2.729.0
Jun 1994–Jun 1998	24.1	36.3	39.6	0.0	14.4	85.6	3,169.4
Dec 1994–Dec 1998	24.3	35.4	40.4	0.0	17.1	82.9	2,726.6
Jun 1995–Jun 1999	23.9	34.7	41.4	0.0	28.0	72.0	1,549.6
Dec 1995–Dec 1999	23.2	36.1	40.7	0.0	28.4	71.6	1,552.3
Jun 1996–Jun 2000	22.9	35.8	41.4	0.0	29.8	70.2	1,393.0

Source: IMF staff calculations.

Figure 4 shows the corresponding transformation of the risk exposure ratio in Figure 3 to an MVI. In addition, another MVI is calculated using annual data. Clearly, the annual MVI shows significant fluctuations in recent periods with sharp increases followed by sharp decreases, even below minimum levels historically observed. This reflects portfolio adjustments taking place within a one-year lag toward the minimum-variance portfolio, to get rid of the "undesired" or "excess volatility." One interpretation may be that the four-year MVI reflects medium-term trends in risk exposure, while the one-year MVI shows the pattern of volatility adjustment. Both indicators provide valuable information. However, for the purposes of assessing the risk environment, a moving average seems more appropriate.

Figure 4. Israel: Four-Year and Annual Market Volatility Indicators, 1992–2000



IV. A Multivariate-GARCH Model for Israel

Auto-Regressive Conditional Heteroskedasticity models (ARCH) are useful to infer volatility patterns and volatility properties of high-frequency data (See Appendix). As an econometric tool, it also helps identify additional variables explaining volatility patterns. ARCH models allow for the modeling of volatility persistence or asymmetries related to the direction of volatility changes, based on some stylized facts usually observed in high-frequency time series of asset returns, among them the presence of thick tails, time-varying correlations, and volatility clustering. Multivariate GARCH models have the additional advantage of formalizing the pattern of covariances and correlations. The reason to include a GARCH model in this paper is twofold: To identify other features of volatility related to statistical features of the data, and to use it as a framework for an illustration about the impact on volatility patterns related to overall risk exposure (as reflected in changes in MVI).

A. Multivariate GARCH Modeling of Market Volatility for Israel

Two Multivariate GARCH models are estimated for Israel: one to assess the volatility patterns and correlations for the exchange rate, interest rates and stock prices (cross-asset model) and one to assess the volatility patterns and correlations relative to other markets (cross-country model). In this way, we approximate the different volatility patterns and correlations that are relevant for the dedicated and the crossover investor respectively. Specifically, two trivariate GARCH models are used with one lag for both ARCH and GARCH terms, that is, GARCH (1,1) models. In order to ensure positive definiteness and relative generality, we adopt the parameterization and associated methodology of the multivariate GARCH model proposed by Engle and Kroner (1995).[15]

Data

The cross-asset model is applied to daily returns for the exchange rate, treasury bills, and the TA-25 stock price index at the close of each market expressed in domestic currency. The cross-country model is applied to daily stock price returns expressed in US dollars for the Israeli TA-25, the Argentine MERVAL and the U.S. NASDAQ. Following, the features of the selected specifications for each model are summarized.

[15]Engle and Kroner (1995) prove that their parameterization is not only able to guarantee the positive definiteness but also relative general and inclusive by studying equivalence relations between two widely adopted parameterizations: vech representation and BEKK representation.

Cross-asset model

For the cross-asset model, a **trivariate TGARCH (1,1)** formulation was used to allow for asymmetries related to the sign of returns (positive or negative returns lead to different degrees of volatility persistence).[16] The representation of the model is as follows:

(1) $\mathbf{r}_t = \mu + \gamma \mathbf{xm}_t + \varepsilon_t$,

where $\varepsilon_t \mid \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$

(2) $\mathbf{H}_t = \psi'\psi + B'\mathbf{H}_{t-1} B + A'\mathbf{e}_{t-1}\, \mathbf{e}_{t-1}'\, A + \theta\varepsilon_t^2 \mathbf{d}_{t-1} + \Xi'\, \mathbf{x}_t\, \mathbf{x}_t'\, \Xi$

where daily returns r_{1t}, r_{2t}, and r_{3t} represent exchange rate, stocks and treasury bill returns, respectively. The vector of returns is denoted by $\mathbf{r}_t' = [r_{1t}, r_{2t}, r_{3t}]$, and the vector of residuals by $\varepsilon_t' = [\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}]$, with its corresponding conditional covariance matrix $\{\mathbf{H}_t\}_{3x3} = h_{ij,t}$. ε_t is assumed as a column vector of forecast errors of the best linear predictor of $\mathbf{r}_t$ conditional on past information, denoted by Ω_{t-1}.

The main features of this specification are the following:

- Returns are represented by random walk processes with constant mean, except for the three-month treasury bill that is affected by a policy rate (the central bank auction rate).

- Exogenous variables affecting exchange rate volatility include dummies for periods after the exchange rate band was widened within the sample period (BAND1 since June 1995 and BAND2 since June 1997) and the real interest rate (interest rate on indexed 3-month treasury bills), which affects the stock price volatility, as a proxy for liquidity in financial markets.

- The customary weekend effect [17] is made more complicated by the fact that weekends in Israel comprise Friday and Saturday, unlike other financial markets. Two dummy variables were tested to represent that kind of effect: SUNDAY (the first day of the week in the Israeli calendar) and MONDAY (the fist day of the week in international markets).

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[16]Based on the statistical evidence, no threshold effects are applied to stock returns when measured in domestic currency. Interestingly, they appear to be present when measured in U.S. dollars.

[17]Volatility observations for dates falling at the beginning of the week reflect responses to information corresponding to a three-day period. See French and Roll (1986), French, Schwert, and Stambaugh (1987), Nelson (1989, 1990c), Connolly (1989).

The corresponding parameter vectors and matrices of the mean returns equation are defined as $\mu' = [\mu_1, \mu_2, \mu_3]$ for the constant and $\gamma \mathbf{xm}_t' = [0, 0, \gamma_1 xm_{3t}]$ for the exogenous variable vector. Here xm_{3t} represents the one-day lagged central bank auctions interest rate.

The parameter matrices for the variance equation (2) are defined as $\{\psi\}_{3x3} = \omega_{ij}$ for the constant, which is restricted to be lower triangular; $\{B\}_{3x3} = \beta_{ij}$ and $\{A\}_{3x3} = \alpha_{ij}$ for the GARCH term and the ARCH term, which are restricted to be diagonal; $\{\theta\varepsilon_{t-1}{}^2 d_{t-1}\}_{3x3} = \{\theta^2\varepsilon_{t-1}{}^2 d_{t-1}\}_{ij}$ for the TARCH term, which is assumed to be zeros except $\{\theta^2\varepsilon_{t-1}{}^2 d_{t-1}\}_{11} = \theta_1{}^2\varepsilon_{1,t-1}{}^2 d1_{t-1}$ and $\{\theta^2\varepsilon_{t-1}{}^2 d_{t-1}\}_{33} = \theta_2{}^2\varepsilon_{3,t-1}{}^2 d3_{t-1}$; and $\{\Xi\}_{3x4} = \xi_{ij}$ for exogenous variable vector $\mathbf{x}_t' = [x_{1t}, x_{2t}, x_{3t}, x_{4t}]$. Variances depend solely on past own squared residuals and covariances depend solely on past own cross-products of residuals, consistent with the diagonal matrices $\{B\}_{3x3}$ and $\{A\}_{3x3}$. The notation for the four exogenous variables are (x_{1t}) for MONDAY, (x_{2t}) for the dummy variable for the exchange rate band effective since June 1995, (x_{3t}) for the dummy variable for the exchange rate band effective since June 1997, and (x_{4t}) for the one-day lagged real interest rate differential.

Taking the vech of each matrix, this system translates into the following equations:

The mean return equations

$$r_{1t} = \mu_1 + \varepsilon_{1t}$$
$$r_{2t} = \mu_2 + \varepsilon_{2t}$$
$$r_{3t} = \mu_3 + \gamma_1 xm_{3t} + \varepsilon_{3t}$$

and the variance equations

$$h_{11t} = \omega_1{}^2 + \beta_1{}^2 h_{11,t-1} + \alpha_1{}^2\varepsilon_{1t-1}{}^2 + \theta_1{}^2\varepsilon_{1t-1}{}^2 d1_{t-1} + (\xi_1 x_{1t} + \xi_2 x_{2t} + \xi_3 x_{3t})^2$$
$$h_{22t} = \omega_2{}^2 + \omega_3{}^2 + \beta_2{}^2 h_{22,t-1} + \alpha_2{}^2\varepsilon_{2t-1}{}^2 + (\xi_4 x_{4t})^2$$
$$h_{33t} = \omega_4{}^2 + \omega_5{}^2 + \omega_6{}^2 + \beta_3{}^2 h_{33,t-1} + \alpha_3{}^2\varepsilon_{3t-1}{}^2 + \theta_2{}^2\varepsilon_{3t-1}{}^2 d2_{t-1}$$

$$h_{12t} = \omega_1\omega_2 + \beta_1\beta_2\, h_{12,t-1} + \alpha_1\alpha_2\,\varepsilon_{12,t-1} + (\xi_1 x_{1t} + \xi_2 x_{2t} + \xi_3 x_{3t})\,(\xi_4 x_{4t})$$
$$h_{13t} = \omega_1\omega_4 + \beta_1\beta_3\, h_{13,t-1} + \alpha_1\alpha_3\,\varepsilon_{13,t-1}$$
$$h_{23t} = \omega_2\omega_4 + \omega_3\omega_5 + \beta_2\beta_3\, h_{23,t-1} + \alpha_2\alpha_3\,\varepsilon_{23,t-1}$$

Cross-country model

For the cross-country model, a **trivariate GARCH-M (1,1)** formulation was used to allow for a feedback of forecasted volatility on corresponding returns (larger volatility should explain higher returns). The representation of the model is as follows:

(3) $$\mathbf{r}_t = \mu + \lambda\mathbf{h}_t + \varepsilon_t,$$

where $\varepsilon_t \mid \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$

(4) $$\mathbf{H}_t = \psi'\psi + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} + \mathbf{A}'\mathbf{e}_{t-1}\mathbf{e}_{t-1}'\mathbf{A} + \Xi'\mathbf{x}_t\mathbf{x}_t'\Xi$$

The GARCH-in-Mean term in equation (3) is represented by the endogenous term $\lambda \mathbf{h}_t$. Features of this model that are different from the formulation for the TGARCH model are the following:[18]

- Returns were aligned in such a way as to account for different working weeks and time zone differences. As end-of-the-day stock price quotes were used, observations corresponding to a given date in Argentine and US markets were matched with observations for the following date in the Israeli stock market.

- Exogenous variables only affect the volatility of the Israeli TA-25: the SUNDAY dummy variable and the real interest rate of indexed 3-month treasury bills.

The parameter vector $\lambda \mathbf{h}_t' = [\lambda_1 h_{11t}, \lambda_2 h_{22t}, \lambda_3 h_{33t}]$ reflects the impact of volatility (or risk) on expected returns. In the variance equation (4) the exogenous variable vector is $\mathbf{x}_t' = [x_{1t}, x_{2t}]$, (x_{1t}) for SUNDAY and (x_{2t}) for the one-day lagged change in real interest rate.

Taking the vech of each matrix, the above-discussed GARCH-M system translates into the following equations:

The mean return equations

$$r_{1t} = \mu_1 + \lambda_1 h_{11t} + \varepsilon_{1t}$$
$$r_{2t} = \mu_2 + \lambda_2 h_{22t} + \varepsilon_{2t}$$
$$r_{3t} = \mu_3 + \lambda_3 h_{33t} + \varepsilon_{3t}$$

and the variance equations

$$h_{11t} = \omega_1^2 + \beta_1^2 h_{11,t-1} + \alpha_1^2 \varepsilon_{1t-1}^2 + \theta_1^2 \varepsilon_{1t-1}^2 d1_{t-1} + (\xi_1 x_{1t} + \xi_2 x_{2t})^2$$
$$h_{22t} = \omega_2^2 + \omega_3^2 + \beta_2^2 h_{22,t-1} + \alpha_2^2 \varepsilon_{2t-1}^2$$
$$h_{33t} = \omega_4^2 + \omega_5^2 + \omega_6^2 + \beta_3^2 h_{33,t-1} + \alpha_3^2 \varepsilon_{3t-1}^2$$

$$h_{12t} = \omega_1 \omega_2 + \beta_1 \beta_2 h_{12,t-1} + \alpha_1 \alpha_2 \varepsilon_{12,t-1}$$
$$h_{13t} = \omega_1 \omega_4 + \beta_1 \beta_3 h_{13,t-1} + \alpha_1 \alpha_3 \varepsilon_{13,t-1}$$
$$h_{23t} = \omega_2 \omega_4 + \omega_3 \omega_5 + \beta_2 \beta_3 h_{23,t-1} + \alpha_2 \alpha_3 \varepsilon_{23,t-1}$$

[18] No threshold effects are incorporated to keep the model parsimonious while allowing for GARCH-M components.

Tests

Unit root tests shown in Table 5[19] indicate that all the first differences of the log price series are stationary. The sample autocorrelation functions for the daily raw returns series and their respective squared-returns series up to two lags with Ljung-Box (LB) statistics up to 6 and 12 lags are shown in Table 6. Consistent with other studies, the autocorrelations in the asset returns are statistically different from 0 for the first and possibly higher lags. The LB statistics for the raw and squared returns series easily reject the null hypothesis of white noise. The autocorrelations for the squared daily returns may be evidence of nonlinear dependence in the returns series possibly due to changing conditional volatility over time. Finally, the lead and lag correlations for the raw and squared returns series for three assets in the two portfolios demonstrate significant interdependence among these series.

Table 5. Augmented Dickey-Fuller Unit-Root Test
(Sample period: January 3, 1992–July 12, 2000)

Variable	No Trend with Intercept	With Trend and Intercept	No Trend and no Intercept
Prices (Levels)			
LNUSD	-1.74	-2.554	2.95
LNSEIS	-0.969	-2.111	2.24
ISTEB3M	-1.855	-1.878	-0.966
LNSEISUSD	-1.275	-2.086	1.371
LNSEARG	-2.908	-2.965	-0.464
LNNASDAQ	0.877	-2.239	3.002
Returns (Differences)			
DLNUSD	-20.879	-20.914	-20.589
DLNSEIS	-21.794	-21.789	-21.638
DISTEB3M	-25.726	-25.722	-25.719
DLNSEISUSD	-21.769	-21.767	-21.712
DLNSEARG	-20.439	-20.446	-20.44
DLNNASDAQ	-21.873	-21.933	-21.625
Critical Values			
1% Critical value	-3.436	-3.967	-2.566
5% Critical value	-2.863	-3.414	-1.939
10% Critical value	-2.567	-3.129	-1.615

Source: IMF staff calculations.

[19]LNUSD, LNSEIS AND ISTEB3B stand for the log of the exchange rate expressed in Shekels per U.S. dollars, the log of the TA-25 stock price index and the 3-month treasury bill rate, respectively, accompanied by their corresponding differences.

Table 6. Correlations in Raw and Squared Returns Series

	DISTEB3M	DLNNASDAQ	DLNSEARG	DLNSEIS	DLNUSD	DLNSEISUSD
Raw returns correlations						
Rho (lag=1)	-0.117	0.068*	0.120*	0.056*	0.067*	0.055*
Rho (lag=2)	-0.108	-0.003	-0.084	-0.0015	0.025	0.016
LB(6)	58.051*	15.704	44.824*	19.256*	21.876*	23.727*
LB(12)	67.176*	37.541*	70.548*	41.084	58.023*	46.924*
Squared returns correlations						
Rho (lag=1)	0.1214*	0.207*	0.173*	0.195*	0.220*	0.207*
Rho (lag=2)	0.042	0.352*	0.187*	0.114*	0.028	0.101*
LB(6)	75.403*	1161.7*	510.92*	273.55*	190.94*	268.89*
LB(12)	88.527*	1971.0*	851.63*	342.20*	348.09*	319.33*
	USD*SEIS	USD*TEB3M	SEIS*TEMB3M	SEISUSD*SEARG	SEISUSD*NASDAQ	NASDAQ*SEARG
Raw returns cross-correlations						
Rho (lag=-2)	-0.0209	-0.0424	-0.0062	-0.0209	0.0048	-0.0159
Rho (lag=-1)	-0.0317	-0.0258	-0.0157	0.1173	-0.0134	-0.0419
Rho (lag=0)	-0.0361	0.1193	-0.1134	0.1112	0.202	0.0303
Rho (lag=1)	0.0291	0.08	-0.048	0.0211	0.1752	0.3084
Rho (lag=2)	-0.0503	0.0451	-0.0223	0.0544	0.0529	0.019
Squared returns cross-correlations						
Rho (lag=-2)	0.0058	0.0144	0.0035	0.0965	0.0745	0.0459
Rho (lag=-1)	0.0278	0.0362	0.0092	0.2042	0.1359	0.1811
Rho (lag=0)	0.0666	0.4395	0.0873	0.107	0.1609	0.0452
Rho (lag=1)	0.0015	0.0587	0.016	0.0815	0.1929	0.0324
Rho (lag=2)	0.0032	0.0053	-0.0054	0.0996	0.1063	0.057

Results

As is customary for Multivariate GARCH models, the dependent variables were standardized to facilitate convergence in the iteration process. Table 7 shows the econometric results, with the subscripts 1,2, and 3 denoting the equations for the exchange rate, stock prices and treasury bills interest rates respectively for the cross-asset model. The same subscripts stand for the TA-25, MERVAL and NASDAQ indices for the cross-country model. The coefficients in Table 7 represent the following variables:

MU	:	Mean returns.
LAMBDA	:	GARCH-in-mean term (values only for the cross-asset model).
ALPHA	:	Coefficients of ARCH terms.
BETA	:	Coefficients of GARCH terms.
THETA	:	Threshold effects.
GAMMA	:	Coefficient of exogenous variables in mean equations.
OMEGA	:	Components of the unconditional variance.
ZETA	:	Components of coefficients of exogenous variables in variance equations.

Coefficients are generally significant with comfortable margins, confirming that the model is well suited to represent the dynamics of the main Israeli risk factors and also the pattern of global spillovers from other stock markets into the Israeli stock market. Low significance of some mean returns is consistent with the hypothesis that they are not significantly different from zero, and low significance of some unconditional variances is consistent with standardization performed on modeled variables. Table 8 shows the results of the transformations in line with the equations described in the former section.

The main findings for each model are the following:

Cross-asset model:

- Asset volatility shows persistent changes over time and a generally significant response to news in each market. Persistence is stronger for the interest rate, and is also apparent in the covariance parameters. Volatility behaves asymmetrically: The threshold effects in the exchange rate and treasury bill equations indicate that volatility is higher at times of exchange rate depreciation relative to appreciation and interest rate increases relative to decreases. This is consistent with findings in other emerging markets. The threshold effect on the exchange rate appears to be stronger.

- In the mean equations, the equation for the treasury bill interest rate mean shows a positive impact of the policy interest rate as expected, while the hypotheses that mean returns are equal to zero can not be rejected for any asset return equation at 95 percent confidence level.

Table 7. Multivariate GARCH Coefficients

Coefficients	Cross-Asset Model	Cross-Country Model
MU(1)	-0.0256	-0.0061 **
MU(2)	0.0321 *	0.0251
MU(3)	-0.0225	-0.0309
LAMBDA(1)		0.0359 *
LAMBDA(2)		0.0368
LAMBDA(3)		0.0517
ALPHA(1)	0.2126 **	0.2748 **
ALPHA(2)	0.3548 **	0.3421 **
ALPHA(3)	0.1694 **	0.1969 **
BETA(1)	0.8538 **	0.9292 **
BETA(2)	0.8999 **	0.9251 **
BETA(3)	0.9595 **	0.9792 **
GAMMA(3)	0.0719**	
OMEGA(1)	0.1903 **	0.1478 **
OMEGA(2)	0.0235	0.0246 *
OMEGA(3)	0.2453 **	0.1725 **
OMEGA(4)	0.0199 **	0.0483 **
OMEGA(5)	-0.0322 **	0.0015
OMEGA(6)	0.0075 **	0.0352 *
THETA(1)	0.4139 **	
THETA(2)	0.3311 **	
ZETA(1)	0.6337 **	0.399 **
ZETA(2)	-0.1259 **	0.0942 **
ZETA(3)	-0.1034 *	
ZETA(4)	-0.0717 **	
Log likelihood	-8,702.75	-8,463.12
Average log likelihood	-3.9166	-3.8089
Number of coefficients	22	20
Akaike info criterion	7.8531	7.6356
Schwarz criterion	7.9096	7.6356
Hannan-Quinn Criterion	7.8737	7.6543

- Among exogenous variables affecting variance equations, the widening of the exchange rate band proves significant, with a decreasing impact as expected. Increases in the real interest rate appear to be related to increases in stock price volatility, reflecting short-term liquidity restrictions. The variable MONDAY captures in this model the weekend effect on the foreign exchange market.

- Modeled covariance is stronger between the exchange rate and interest rates relative to any of these variables and stock prices. While the covariance of interest rates and the exchange rate is positive, it is positive for the exchange rate and stock prices and negative for interest rates and stock prices, reflecting the substitutability between stocks and bonds.

Table 8. Multivariate Garch Equations for Cross-Asset and Cross-Country Models

Equation Set 1

	Constant	DRIBEFEBI
DLNUSD	-0.026	
DLNSEIS	0.032	
DISTEB3M	-0.023	0.072

	Constant	GARCH(1)	ARCH(1)	RES<0 (TARCH)	MONDAY	BAND1	BAND2	DREALI	MONDAY* BAND1	MONDAY* BAND2	BAND1* BAND2	MONDAY* DREALI	BAND1* DREALI	BAND2* DREALI
VAR(DLNUSD)	0.036	0.727	0.044	0.170	0.400	0.015	0.010		-0.158	-0.130	0.028			
VAR(DLNSEIS)	0.060	0.808	0.125					0.005						
VAR(DISTEB3M)	0.007	0.919	0.028	0.109										
COV(Y1,Y2)	0.004	0.766	0.075									-0.044	0.008	0.007
COV(Y1,Y3)	0.003	0.818	0.035											
COV(Y2,Y3)	-0.006	0.826	0.059											

Equation Set 2

	Constant	GARCH-in-Mean
DLNSEIS	-0.006	0.035
DLNSEARG	0.025	0.036
DLNNASDAQ	-0.030	0.051

	Constant	GARCH(1)	ARCH(1)	SUNDAY	DREALI	SUNDAY* DREALI
VAR(DLNSEISUSD)	0.022	0.863	0.075	0.158	0.008	0.074
VAR(DLNSEARG)	0.029	0.855	0.116			
VAR(DLNNASDAQ)	0.003	0.958	0.038			
COV(Y1, Y2)	0.003	0.859	0.093			
COV(Y1, Y3)	0.007	0.909	0.053			
COV(Y2, Y3)	0.001	0.905	0.067			

- Forecasted volatility of returns is somewhat significant as an explanatory variable of mean stock price returns in the three stock markets (GARCH-M effects), showing that higher risk is generally associated with higher returns. It is significant for Israel at 90 percent confidence, and the coefficients show the appropriate sign. Presumably related to market efficiency, the magnitude of the impact of risk on mean stock price returns in the U.S. market is larger.[20]

- As in the cross-asset model, conditional volatility dominates the unconditional terms in all countries. Volatility in the three markets clearly changes over time, and the TA-25 shows a significant response to news and a strong persistence of innovations over time. Persistence is also apparent in the covariance parameters.

- As for exogenous variables affecting the variance equations, the real interest rate has a relatively stronger impact on the variance of the TA-25 expressed in U.S. dollar terms. SUNDAY captures weekend effects, reflecting the compounded time-zone effects added to weekend effects.

- As expected, there are significant and persistent spillover effects from the NASDAQ and the MERVAL into the Israeli stock market. Based on the unconditional variance, changes in the MERVAL index of 1 percent are expected to result in changes of 0.3 per cent in the TA-25. Changes in the NASDAQ index of 1 percent would result in a stronger impact, causing a 0.7 percent change in the TA-25. At least for this sample period, the TA-25 shows a higher behavioral correlation with NASDAQ relative to the MERVAL.

Out-of-sample data

Looking at out-of-sample data, the most prominent event was the decline of the TA-25 index of 22 percent between June 2000 and June 2001. However, in the preceding annual period, this same index had increased by 37 percent. This means that actual stock price volatility remained basically stable in both periods. Concerning the portfolio variance, it increased from 14 to 16 percent in annual terms in both periods.

Figure 5 summarizes the evolution of the variance-covariance matrix in graphical form, to help analyze the above-mentioned behavior: The continuous straight line is the actual variance or covariance for the two periods (in- and out-of-sample), the dim horizontal line is the unconditional variance according to the model and the line that

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[20]It is worth noting that GARCH-M effects were tested unsuccessfully for this variable in the cross-asset model, while it shows acceptable significance in the cross-country model, probably reflecting that these effects are driven by valuations of stock prices in foreign currency.

fluctuates more noticeably is the sum of conditional plus unconditional variance from the model. The following observations are worth noticing:

- According to the model, almost all volatility action comes from the conditional variance (the unconditional variance is very close to the horizontal line). Again, volatility remained basically at the same level as in the in-sample subperiod. This shows that average volatility shows persistence even beyond the immediate short term.

- The largest difference between the two periods is the drop in the covariance between the exchange rate and interest rates, which declines closer to the modeled unconditional variance. In this period, the deposit interest rate declined from 8.9 to 6.3 percent without a major impact on exchange rate depreciation (1.4 percent in nominal terms). This shows that covariance levels are much less persistent, especially if volatility conditions remain relatively stable for prolonged periods.

- By contrast, both the covariance of the exchange rate and interest rates relative to stock prices decline only sluggishly. This may reflect that covariances are more persistent relative to asset prices that show larger fluctuations in absolute terms (in this case, stock prices).

B. Incorporation of MVI into the Multivariate GARCH Model

In order to further explore what additional information content could be provided by the alternative MVI, this variable was incorporated into the Multivariate GARCH model taking into account the following considerations:[21]

- The medium-term measurement of risk exposure is more appropriate as a measurement tool of the risk environment. As was seen in the preceding chapter, annual measurements of this indicator shows swings related to the dynamics of portfolio adjustment.

- Although including aggregate volatility in a GARCH framework is in some sense a way to regress "volatility against volatility," the medium-term nature of the indicator, and the standardization against minimum variance, makes it suitable to work as a proxy variable for the risk environment.

- Because overlapping periods result from the use of moving averages, a bootstrapping mechanism may be required to assign values for each particular

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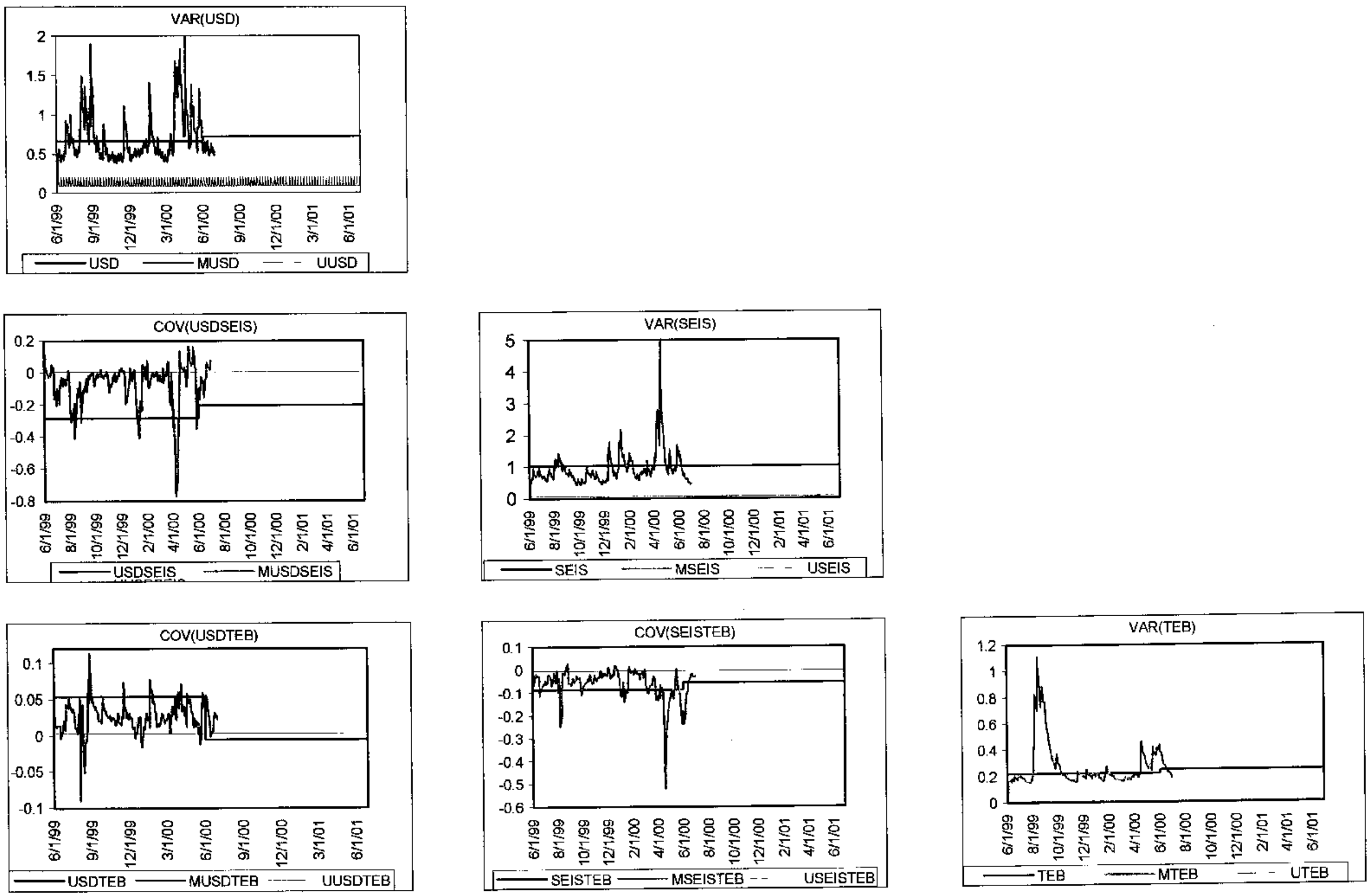
[21]The risk exposure variant of the indicator was used, as it allowed for convergence in the mutivariate GARCH model (unlike the MVI itself).

Figure 5. Israel: Graphical Variance-Covariance Matrix



year. Moreover, since risk exposure is not expected to be a highly fluctuating variable (fluctuations would be incorporated in the dynamics of the model as such), the way bootstrapping was proxied is by filtering the risk exposure indicator using Hodrick-Prescott. As the exercise is illustrative rather than an exhaustive in identifying the role of risk exposure in Multivariate GARCH models, this loosely defined indicator serves the purpose of the paper of showing possible additional information content resulting from the use of the indicator.

Table 9 shows the result of incorporating MVI to the model. A general observation is that the significance of the original variables is basically not affected. MVI shows significance as an explanatory variable of the variance of the interest rate, at 99 percent confidence (ZETA7). While some significance is also found relative to the exchange rate (significance of ZETA4 at just 80 percent confidence), MVI is not significant to explain volatility patterns of stock prices (ZETA6). A preliminary conclusion is that the risk environment is more closely reflected in changes in the interest rate than in changes in other variables (as expected), and that stock prices (subject to contagion and affected by real sector variables) evolve more unrelated to the overall risk environment.

Table 10 allows a comparison between the original formulation of the cross-asset model and the one incorporating MVI once all matrix transformations are performed. The following observations are worth noting:

- The mean equations are generally not altered, with the exception of the mean interest rate equation, which shows a lower magnitude of both the constant term and the coefficient for the central bank interest rate.

- The impact of MVI is noticeable in the case of the interest rate equation, marginal for the exchange rate equation and basically zero for the stock price equation. The risk environment seems to have a stronger effect on nominal interest rates.

- The unconditional variance of interest rates is much larger when MVI is included. It was by far the lowest in the original formulation. It is still lower than for the exchange rate and stock prices in the alternative formulation, but closer to the range observed originally for these two variables. This may indicate that the risk environment may be an important "missing variable" when formulating a Multivariate GARCH model for Israel.

- Persistence coefficients declined for the GARCH term in the alternative formulation. They are somewhat compensated by increases in the coefficient of the ARCH term, except again for interest rates, which on aggregates show lower volatility persistence in the original formulation. This could mean that apparent persistence in interest rate volatility was related to changes in the risk environment.

- There is a much stronger threshold effect for interest rates in the alternative formulation, very close to the one observed for the exchange rate. This reinforces the "missing variable" hypothesis.

- The main impact on the exchange rate equation is on the coefficients of dummy variables. The weekend effect is weaker (although still significant as shown in Table 9). And the impact of the latest widening of the exchange rate band may not be as high, once changes in the risk environment are incorporated.

Table 9. Multivariate GARCH Coefficients After Incorporating MVI

	Coefficient
MU(1)	-0.0270
MU(2)	0.0308 *
MU(3)	-0.0175
GAMMA(1)	0.0600 **
OMEGA(1)	0.2756 **
BETA(1)	0.8292 **
ALPHA(1)	0.2393 **
THETA(1)	0.4460 **
ZETA(1)	0.5560 **
ZETA(2)	-0.0434
ZETA(3)	-0.0731 *
ZETA(4)	-0.0053
OMEGA(3)	0.2555 **
OMEGA(2)	0.0204
BETA(2)	0.8951 **
ALPHA(2)	0.3606 **
ZETA(5)	-0.0680 **
ZETA(6)	0.0011
OMEGA(4)	-0.0035
OMEGA(5)	-0.0608 **
OMEGA(6)	0.1431 **
BETA(3)	0.8860 **
ALPHA(3)	0.1813 **
THETA(2)	0.4452 **
ZETA(7)	0.0480 **
Log likelihood	-8678.9924
Average log likelihood	-3.9059
Number of Coefficients	25

Notes: ** indicates significant level of 95 percent and * indicates 90 percent.

Table 10. Comparative Cross-Asset Models [1/]

	Exchange rate		Stock Prices		Interest Rates	
	Original	w/MVI	Original	w/MVI	Original	w/MVI
			Mean Equations			
Constant	-0.026	-0.027	0.032	0.0308	-0.023	-0.0175
Central bank interest rate (change)					0.072	0.06
			Variance Equations			
Constant	0.036	0.036	0.06	0.066	0.007	0.0242
GARCH(1)	0.727	0.6875	0.808	0.8012	0.919	0.7849
ARCH(1)	0.044	0.0573	0.125	0.13	0.028	0.0329
RES<0 (TARCH)	0.17	0.1989			0.109	0.1982
MONDAY	0.4	0.3901				
BAND1	0.015	0.019				
BAND2	0.01	0.0053				
Real interest rate (change			0.005	0.0046		
MVI		0.00003		0.000		0.0023

1/ Cross-product coefficients are excluded for simplicity.

V. Conclusions and Areas for Further Work

We propose a parsimonious forward-looking market volatility indicator to fill the gap between market-risk monitoring, normally based on different measures of exposure, and market-risk decisions by economic agents, normally based on volatility assessments. The indicator is easily adaptable to focus on the relevant markets of different countries and seems to provide additional information content as a measurement of "net" risk (relative to the risk environment). All these characteristics make this indicator useful in the context of an early-warning system.

This paper applies a market volatility indicator to Israel in the transition toward inflation targeting. The decline in volatility following conventional measures of volatility does not seem to reflect the more substantial decline once volatility is measured against the minimum variance for the same returns on assets. This would seem to indicate that this process may have facilitated a reduction of risk exposure that was not so evident when looking at conventional indicators.

As an illustration, the proposed MVI is used in a conventional Multivariate GARCH model to further assess its information content. The Multivariate GARCH model proves generally appropriate to characterize volatility patterns in Israel. Adding MVI as a variable shows a noticeable impact on the explanatory power of interest rate equations, presumably because this variable captures more immediately changes in the risk environment.

On the other hand, some findings of this paper are consistent with other work using a multivariate GARCH framework. Correlation between stocks prices and interest rates is found to be weak also for Korea (Kim and Chang, 1996). On the other hand, no "transfer of volatility" to the interest rate at times of limiting exchange rate fluctuations was found for European Monetary System (EMS) countries, which is presumably explained by net 'credibility gains' (Sarno, 1997; Artis and Taylor, 1994). There is widespread evidence of relatively weak but consistent spillover effects within a GARCH-like framework (Theodossiou, Kahya, and Christofi, 1997) from the United States and Japan to the United Kingdom. Weak spillovers are also found in Scandinavian stock markets within an E-GARCH model (Booth, Martikainen, and Tse, 1997); and between the United States and Canada (Karolyi, 1995).

Further areas for work are multiple, chiefly the use of MVI for cross-country analysis, which would also help to assess the usefulness of this indicator, especially as an early-warning indicator. Moreover, MVI could help identify bank vulnerabilities, for example by experimenting with alternative weights related to the banks' portfolios. In terms of the methodology, formal bootstrapping methods may be devised to allow for the incorporation of this indicator in formal models.

Appendix I: GARCH Models: Volatility Clusters, Persistence, and Asymmetry

ARCH models allow for the modeling of volatility persistence or asymmetries related to the direction of volatility changes, based on some stylized facts usually observed in high-frequency time series of asset returns, among them the presence of thick tails, time-varying correlations, and volatility clustering (i.e., volatility jumps to a different plateau for consecutive periods).[22] Time-varying volatility is a function of news about volatility from the previous period reflected in the lag of the squared residual from a given mean equation. The generalized ARCH model (GARCH) adds as an explanatory variable the last period forecast variance, which allows for a parsimonious parameterization to estimate the time-varying variance (conditional variance) because it usually requires shorter lag lengths of variances and squared errors (and consequently less parameters to be estimated than in an ARCH). Total forecasted variance would therefore be equal to the sum of a non-varying variance (unconditional variance) and the conditional variance, which would contain ARCH and GARCH terms. These models allow for the testing of different properties of volatility:

- Volatility persistence is reflected in the magnitude and significance of the coefficients for the ARCH and GARCH terms (more persistence when the sum of significant coefficients closer to one).
- The inclusion of risk as an explanatory variable of returns is allowed by ARCH-in-Mean (ARCH-M) models introduced by Engle, Lilien, and Robins (1987). In such

[22]This family of models was developed by Engle (1982) and generalized by Bollerslev (1986). For a comprehensive review on ARCH modeling in finance, see Bollerslev, Chou, and Kroner (1992).

models, mean returns are an explicit function of the conditional variance of the process.

- The asymmetric impact (or leverage effect) of good news against bad news is allowed by Threshold ARCH (TARCH) models, developed independently by Zakoian (1994) and Glosten, Jaganathan, and Runkle (1993).[23]

One additional feature of volatility of great importance for portfolio analysis is the correlation of variances of different assets, among them and across markets for each individual asset. The widely used multivariate form of ARCH models [24] allows for time-varying volatility correlations in a framework of complex second-order dynamics of short-run movements of financial variables. The identification of volatility patterns in this framework is more important for economies showing increasing portfolio diversification and increased participation of crossover investors and international institutions in domestic financial markets, as they are susceptible to more acute portfolio swings.

[23]Nelson (1989, 1990c) developed the exponential GARCH (EGARCH) model to explain the leverage effect represented as an exponential rather than a linear function. This was first noted by Black (1976) and further investigated by Christic (1982). The term "leverage effect" relates to the differential effects of changes in one given direction of financial variables on the debt-to-equity ratio.

[24]See Bollerslev, Engle, and Wooddrige (1988), Schwert and Seguin (1990), Lin et al.(in press), Chan, Chan, and Karolyi (1991), Chan, Karoyi, and Stulz (1992), and Engle and Susmel (1993).

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