Can Debt Crises Be Self-Fulfilling?
Marcos Chamon

# IMF Working Paper 

Research Department

# Can Debt Crises Be Self-Fulfilling? 

Prepared by Marcos Chamon ${ }^{1}$<br>Authorized for distribution by Eduardo Borensztein

June 2004


#### Abstract

This Working Paper should not be reported as representing the views of the IMF. The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

Several papers argue that debt crises can be the result of self-fulfilling expectations that no one will lend to a country. I show this type of coordination failure can be eliminated by a combination of state-contingent securities and a mechanism that allows investors to promise to lend only if enough other investors do so as well. This suggests that runs on the debt of a single borrower (such as the government) can be eliminated, and that self-fulfilling features are more plausible when articulated in a context in which externalities among many decentralized borrowers allow for economy-wide debt runs to occur.


JEL Classification Numbers: E60, F30, F34, G13
Keywords: Debt crises, multiple equilibria, liquidity, financial innovation
Author's E-Mail Address: mchamon@imf.org

[^0]
## Contents Page

I. Introduction................................................................................................................................ 3
II. The Basic Environment.............................................................................................................. 4
III. Solving the Coordination Problems.......................................................................................... 9
IV. Discussion............................................................................................................................... 16
V. Conclusion................................................................................................................................. 17

References...................................................................................................................................... 19

## I. Introduction

The concept of a "solvent but illiquid" debtor has frequently been used to explain debt crises. In these models, a borrowing country is willing to repay its debts provided it can spread repayments over time by issuing new debt in order to roll over part of the debt coming due. However, if the country is unable to issue new debt, it is assumed to either be forced to default for not being able to repay the debt out of its current output or to find it optimal to default, because repaying the debt would leave very little for current consumption. Such seemingly arbitrary loss of access to credit can be the result of a self-fulfilling creditor panic even if all agents are rational. This view has become one of the dominant frameworks used to discuss alternative policies and crisis prevention measures. This paper shows that this type of self-fulfilling feature often articulated in the debt crisis literature can in theory be eliminated by simple market mechanisms of voluntary participation.

The argument that self-fulfilling expectations can trigger runs on the short-term debt of a country that are analogous to a Diamond and Dybvig (1983) bank run was first formally articulated by Sachs (1984). Cole and Kehoe (1996, 2000) present a much richer model, where the coordination failure is not limited to existing creditors and involves new potential lenders as well, and the coordination failure can occur not only in the current period but also in future periods. They show that if every investor expects other investors not to lend to the country, and if such credit exclusion would trigger default, then no individual investor would be willing to lend new funds. As a result, the pessimistic expectations become self-fulfilling. I show that if this is the only reason investors are unwilling to lend, the coordination failure can be easily overcome by offering the new debt through a mechanism where investors are allowed to condition their participation on a large enough amount being successfully issued. The borrowing country can then choose any outcome for that issue provided it satisfies the participation constraints laid out by the investors in their contingent bids. Since every investor is willing to lend provided the others do so as well, the country is always able to select the "good equilibrium." Similar procedures are observed in practice for different types of financial intermediation. For example, when underwriting a bond issue, an investment bank guarantees its size, holding whatever amount it cannot find a buyer for. A very similar mechanism to the one proposed is used by some start-up firms to raise funds. Investors often commit to providing a share of the amount that needs to be raised by the start-up firm, provided the remaining part can be raised from other investors at similar terms.

The repayment of today's investors may also depend on whether or not future investors will lend to the country. Alesina, Prati and Tabellini (1990) argue that if investors today expect their future counterparts not to lend and if this prospect implies a default on the debt offered today, then today's investors are not willing to hold the country's debt. This triggers a default, and, following it, future investors will not lend to the country, making the initial expectations self-fulfilling. The present paper shows that this type of coordination failure is eliminated if the setting is modified to one where the country is forced to either pay down a large enough share of its debt by a given deadline or default. As a result, investors at that future date would be willing to lend to a fundamentally sound country even if they expected their future counterparts not to.

That is enough to ensure through a simple backward-induction argument that the country is able to borrow today. The country can self-impose the constraint that it must pay down its debt before the deadline or be forced to default. This is done by issuing state-contingent securities that pay a large amount (enough to force a default) if the country fails to meet the targeted debt level, and pay zero otherwise.

The findings of this paper cast doubt on debt crisis explanations that emphasize a self-fulfilling run on the public debt, since such run could be prevented by the mechanisms described. ${ }^{2}$ However, it is not clear whether these coordination mechanisms would be of help in a context where the lending is made to several decentralized borrowers. Externalities among those borrowers may allow for economy-wide debt runs that are more robust to the introduction of the coordination mechanisms. That is the case in some "twin crises" models, where a debt/financial crisis is coupled with a currency crisis. For example, in the "third generation" currency crisis models of Aghion, Bacchetta and Banerjee (2001a, 2001b) and Krugman (1999), credit constraints interact with balance sheet mismatches in such a way that foreign investors are only willing to lend to a given borrower if the other investors lend to the other borrowers in that economy. Since no individual borrower internalizes the effects of his or her actions on aggregate lending, the coordination mechanisms described in this paper may not be applicable.

By challenging explanations based on self-fulfilling runs at the level of an individual borrower, this paper suggests that "investor panic" arguments are more likely to be relevant when articulated in the context of broader types of economy-wide coordination failures. This has a number of interesting implications. For example, the maturity of the public debt should not have first order effects on a country's vulnerability to this type of investor panic. If a run were to occur only on the public debt, then it could be stopped by the coordination mechanisms proposed. But if such run is the result of a broader coordination failure in that economy that renders the public debt unsustainable, then its maturity only matters insofar as a longer maturity allows for the possibility that positive shocks prevent a default before the debt comes due. But short-term debt remains undesirable for reasons that are unrelated to liquidity considerations.

The remainder of the paper is organized as follows: Section II lays out a basic environment to illustrate the standard arguments for self-fulfilling liquidity crises. Section III presents the mechanisms that can eliminate such crises. Section IV briefly discusses the implications of this paper's findings. Section V concludes.

## II. The Basic Environment

This section presents a simple model of self-fulfilling liquidity crises to provide a context for the mechanism proposed in the next section. ${ }^{3}$

[^1]Consider a small open economy with a continuum of measure one of identical, infinitely lived residents. Their preferences are represented by a utility function that is additively separable across time. The instantaneous utility function $u()$ is continuous and strictly concave, and the future is discounted at rate $\beta$. This economy has a stock of foreign debt $B_{t}$ at time $t$. Its output depends on a productivity shock, on investment, and on whether or not the country has ever defaulted in the past. All borrowing, repayment and investment decisions are made by a benevolent social planner. If the country has never defaulted before time $t$, its output in that period is given by:

$$
Y_{t}=A_{t} f\left(I_{t-1}\right)
$$

where $A_{t}$ is the realization of an i.i.d. productivity shock, $I_{t-1}$ is the number of units of the consumption good invested in the previous period, which are consumed in the production process, and $f^{\prime}>0, f^{\prime \prime}<0$. If the country has ever defaulted on its debt before $t$, the output of this technology is given by $\alpha A_{t} f\left(I_{t-1}\right)$ instead, where $\alpha<1$. The parameter $\alpha$, the function $f()$, and the distribution that generates the realizations of the productivity shock are common knowledge to all agents.

This paper takes the decline in output following a default as given. One possible way of motivating this assumption is to allow for the general reputation considerations presented in Cole and Kehoe (1998) where, following a default, the government is viewed as untrustworthy in the other relationships in which it is involved. This can lead to several types of distortions in an economy. For example, if the model is extended to one where each individual decides how much to invest on his or her own production which is then taxed by the government, they can become reluctant to invest because they expect the untrustworthy government to ex post tax them more heavily than promised. The lower tax base will require higher tax rates, confirming those fears. But for the sake of simplicity, this output cost of default is just taken as given. ${ }^{4}$

Once the country incurs the reputational cost of defaulting it has no incentives to repay its debt and no creditor is willing to provide new loans. ${ }^{5}$ In principle, the country may still be able to smooth its consumption through a storage technology or through cash-in-advance contracts, such as the ones described in Bulow and Rogoff (1989b). But for simplicity, I assume that following a default the country's consumption is always equal to its output minus the investment in that period. This additional assumption does not change the qualitative results of the model. It can be

[^2]justified if the creditors the country defaulted on can attach any foreign payments to the country, which discourages the use of cash-in-advance insurance. ${ }^{6}$

The value of the objective function following a default at time $t$ is given by:

$$
V^{D}\left(Y_{t}\right)=\max _{I_{t}}\left\{u\left(Y_{t}-I_{t}\right)+\beta E_{t}\left[V^{D}\left(\alpha A_{t+1} f\left(I_{t}\right)\right)\right]\right\} .
$$

The country can only borrow by issuing bonds with a one period maturity. Jeanne (2000) presents a model where a short debt maturity is a constrained optimum due to its disciplining effect. ${ }^{7}$ While the short-term maturity can be endogenized by extending the model to incorporate disciplining features, this paper just takes that assumption as given. The timing of events is the following: At the beginning of each period $t$, the productivity shock $A_{t}$ is realized. The country then offers $B_{t+1}$ bonds that pay one unit of the consumption good at the end of period $t+1$, and investors decide whether or not to buy the bonds. Output becomes available, and at the end of the period debt repayment, consumption and investment decisions take place. The bonds are offered to risk neutral atomistic foreign investors, whose total wealth is much larger than the country's financing needs. These investors also discount the future at the rate $\beta$, so the world risk-free interest rate is $1 / \beta$. The country announces the amount $B_{t+1}$ that it is willing to issue and investors submit bids stating how much they are willing pay for the bonds and how much of it they are willing to hold. I initially assume that the country is always able to sell new debt at a price $p_{t}$ that corresponds to its actuarially fair price. ${ }^{8}$ This assumption is later relaxed with investors becoming strategic players in the model. The country can use the proceeds $p_{t} B_{t+1}$ to repay the debt $B_{t}$ that is due at the end of period $t$. I assume the proceeds from the new bond offer can be directly transferred to the existing creditors. While $p_{t}$ depends on the amount $I_{t}$ that the country invests in the production of the following period's output, I assume that amount cannot be contracted. Once the country sells its debt, it will choose the optimal $I_{t}$ taking $B_{t+1}$ and $p_{t}$ as given. But the foreign investors anticipate that choice, and take it into account when deciding the price $p_{t}$ at which they are willing to hold the bonds. The value of the objective function given a decision to repay is:

[^3]\[

$$
\begin{align*}
& V^{R}\left(Y_{t}, B_{t}\right)= \max _{B_{t+1}}\left\{u\left(Y_{t}+p_{t} B_{t+1}-B_{t}-I_{t}\right)\right.  \tag{1}\\
&\left.+\beta E_{t}\left[\max \left[V^{R}\left(A_{t+1} f\left(I_{t}\right), B_{t+1}\right), V^{D}\left(A_{t+1} f\left(I_{t}\right)\right)\right]\right]\right\}, \\
& \text { s.t. } I_{t}= \arg \max _{I_{t}}\left\{u\left(Y_{t}+p_{t} B_{t+1}-B_{t}-I_{t}\right)\right.  \tag{2}\\
&+\left.\beta E_{t}\left[\max \left[V^{R}\left(A_{t+1} f\left(I_{t}\right), B_{t+1}\right), V^{D}\left(A_{t+1} f\left(I_{t}\right)\right)\right]\right]\right\}, \\
& p_{t}= \beta \operatorname{Pr}_{t}\left(V^{R}\left(A_{t+1} f\left(I_{t}\right), B_{t+1}\right) \geq V^{D}\left(A_{t+1} f\left(I_{t}\right)\right)\right),  \tag{3}\\
&  \tag{4}\\
& \lim _{s \rightarrow \infty} \beta^{s-t} E_{t}\left[B_{s}\right]=0,
\end{align*}
$$
\]

where (2) is the incentive compatibility constraint for $I_{t}$, (3) is the participation constraint for the foreign investors and (4) is a transversality condition that rules out Ponzi-games. I assume throughout the paper that, when indifferent between repaying its debt or defaulting on it, a borrower chooses to repay it. If the country has fully repaid its debt, then it is allowed to save at the world interest rate.

Suppose that for whatever reason, investors are unwilling to hold the new bonds no matter the premium offered. Let $V_{L C}^{R}\left(Y_{t}, B_{t}\right)$ denote the value of the objective function given a decision to repay under the liquidity constraint that the country cannot borrow at $t$, but assuming normal access to credit from $t+1$ onwards:

$$
V_{L C}^{R}\left(Y_{t}, B_{t}\right)=\max _{I_{t}}\left\{u\left(Y_{t}-B_{t}-I_{t}\right)+\beta E_{t} V^{R}\left(A_{t+1} f\left(I_{t}\right), 0\right)\right\}
$$

If $Y_{t}<B_{t}$ then it is impossible for the country to repay the debt that matures at $t$ without new financing (and $u\left(Y_{t}-B_{t}\right.$ ) is not defined). But even if it is possible to repay the debt without new financing, doing so may imply such a low consumption in period $t$ that the country prefers to default. The region of interest of this problem occurs when:

$$
\begin{equation*}
V^{R}\left(Y_{t}, B_{t}\right) \geq V^{D}\left(Y_{t}\right)>V_{L C}^{R}\left(Y_{t}, B_{t}\right) \tag{5}
\end{equation*}
$$

which immediately implies the following result:

Lemma 1 If (5) holds, $B_{t}$ is low enough so that the country chooses to repay its debt provided it is allowed to roll over a large enough part of it. But $B_{t}$ is large enough so that the country will default if called upon to repay the entire stock of debt without new financing.

Once strategic behavior is introduced on the investors' choice of whether or not to hold the debt, this sudden loss of access to credit can be the result of a coordination failure somewhat analogous to a Diamond and Dybvig (1983) bank run. That is the case if an investor makes a loss when she lends to the country when others do not. I incorporate this feature in the model by assuming that if the country defaults after a bond offer is held, the proceeds from that offer are divided among all the creditors the country defaulted on (including both the old and the new bond holders). ${ }^{9}$

An equilibrium for this economy is defined as a situation where the government's decision to sell debt $B_{t+1}$ at a price $p_{t}$ and its decision to repay or to default are optimal given its rational expectations for future outcomes and each investor's decision of whether or not to hold the debt offered is optimal given $B_{t+1}, p_{t}$ and her rational expectations for future outcomes.

Proposition 1 If (5) holds and every investor expects the other investors not to buy the new bonds, then these expectations become self-fulfilling and the country defaults.

Proof. Lemma 1 implies that if every investor expects the other investors not to buy the new bonds, then every investor expects the country to default with certainty. As a result, no investor is willing to buy the new bonds, self-fulfilling the initial expectations and triggering a default.

This argument that debt crises can be the result of self-fulfilling expectations was first formally articulated in Sachs (1984), where existing creditors do not roll over their debt if they expect the other creditors not to. Cole and Kehoe $(1996,2000)$ present a much richer model, where among other things the coordination failure is not limited to existing creditors and involves new potential lenders as well, and the coordination failure can occur not only in the current period but also in future periods.

Even if the investors lend at time $t$, whether or not they are repaid at $t+1$ depends on the country's access to credit in that period. If the country is expected to lose access to credit at time $t+1$, it will only find investors willing to hold its new debt for an issue $B_{t+1}$ that satisfies the following conditions:

$$
\begin{align*}
u\left(Y_{t}+p_{t} B_{t+1}-\right. & \left.B_{t}-I_{t}\right) \\
& +\beta E_{t}\left[\max \left(V_{L C}^{R}\left(A_{t+1} f\left(I_{t+1}\right), B_{t+1}\right), V^{D}\left(A_{t+1} f\left(I_{t+1}\right)\right)\right)\right] \geq V^{D}\left(Y_{t}\right), \tag{6}
\end{align*}
$$

[^4]\[

$$
\begin{gather*}
I_{t}=\arg \max _{I_{t}}\left\{u\left(Y_{t}+p_{t} B_{t+1}-B_{t}-I_{t}\right)\right. \\
 \tag{7}\\
\left.\quad+\beta E_{t}\left[\max \left(V_{L C}^{R}\left(A_{t+1} f\left(I_{t+1}\right), B_{t+1}\right), V^{D}\left(A_{t+1} f\left(I_{t+1}\right)\right)\right)\right]\right\},  \tag{8}\\
p_{t}=\beta \operatorname{Pr}_{t}\left(V_{L C}^{R}\left(A_{t+1} f\left(I_{t+1}\right), B_{t+1}\right) \geq V^{D}\left(A_{t+1} f\left(I_{t+1}\right)\right)\right) .
\end{gather*}
$$
\]

Proposition 2 If there is no $B_{t+1}$ that can simultaneously satisfy (6), (7) and (8) and investors at time $t$ expect their $t+1$ counterparts not to lend to the country, then investors at time $t$ are not willing to hold the new debt, leading to a default and self-fulfilling their expectations.

Proof. If the country has no access to credit at $t+1$ and (6), (7) and (8) cannot be simultaneously satisfied for any $B_{t+1}$, then there is no new bond issue that when priced at its actuarially fair value (8) can satisfy the country's incentive compatibility constraints for debt repayment (6) and investment (7). As a result, investors at time $t$ do not lend, and the country defaults. Subsequently, no future investors will lend to that country, self-fulfilling the initial expectations.

This coordination failure among investors at different dates was proposed by Alesina, Prati and Tabellini (1990).

The higher the amount of debt $B_{t}$ due at time $t$, the higher the repayment the country would need to make in order not to default, and the more likely it is that (6), (7) and (8) cannot be simultaneously satisfied. The range of values of $B_{t}$ for which that occurs is a subset of the range for which (5) holds, since the second type of credit exclusion is less severe than the first one described. In the second case, the country can at least have some access to credit in this period, while in the first case the loss of access to credit is complete.

The analysis in this section has focused on whether or not these coordination failures can occur in equilibrium. If that is the case, then investors must be compensated at every point in time for the coordination risk they face. The international finance literature usually assumes that investors coordinate on the realization of a sunspot variable, and are paid a premium that compensates them for the probability of a realization implying a loss of their investment (for example, Cole and Kehoe 1996, 2000). Since the next section argues that these coordination failures can be eliminated, I do not elaborate on the properties of crisis prone equilibria.

## III. Solving the Coordination Problems

This section proposes market mechanisms that solve the coordination problems discussed above.
The coordination failure where the only reason why investors do not lend to the country is because they expect the other investors in the same period not to lend seems to rely on strong assumptions
for a number of reasons. First, the benefits for a large enough mass of investors coordinating among themselves to lend to the country are very large. If that coalition were to be pivotal in preventing a default, it could charge the country an amount equal to the equivalent variation of not defaulting: $V^{R}\left(Y_{t}, B_{t+1}\right)-V^{D}\left(Y_{t}\right)$. Second, creating a market for bond issues is precisely what investment banks are supposed to do. If instead of auctioning its debt, a country issues it through an investment bank underwriter, that bank guarantees the size of the issue. Once the underwriter sets the bond price and the size of the issue, it pledges to hold the bonds it cannot find buyers for. If an investment bank underwrites a large enough debt issue then the investors know for sure the country will have access to credit this period. One possible criticism to this argument is that the size of the issue necessary to stem that liquidity crisis may be too large and that no investment bank is willing to underwrite such a large amount. ${ }^{10}$

But it is possible to design a fairly simple auction that addresses the coordination problem without requiring any institution to commit to holding those bonds. In the traditional set-up, each atomistic investor submits how much he is willing to buy and at what price (which she takes as given). But suppose that when offering the bonds, the country allows the investors to submit bids that are also contingent on the total amount of debt issued. Then the country chooses which bids to accept and how much to issue subject to the investor's participation constraints laid out in their bids.

Suppose we are in the region where the country would like to repay but defaults if a credit crunch occurs. In order to focus on the intra-period coordination problem, suppose everyone at time $t$ expects the country to have normal access to credit in the future periods. The minimum amount of new debt the country needs to issue in order not to choose to default when borrowing at the actuarially fair rate is $\underline{B_{t+1}}$, given by the smallest solution to the equations below: ${ }^{11}$

$$
\begin{gathered}
u\left(Y_{t}+p_{t} \underline{B_{t+1}}-B_{t}-I_{t}\right)+\beta E_{t}\left[\max \left[V^{R}\left(A_{t+1} f\left(I_{t}\right), \underline{B_{t+1}}\right), V^{D}\left(A_{t+1} f\left(I_{t}\right)\right)\right]\right]=V^{D}\left(Y_{t}\right) \\
\begin{array}{r}
I_{t}=\arg \max _{I_{t}} u\left(Y_{t}+p_{t} \underline{B_{t+1}}-B_{t}-I_{t}\right) \\
\\
+\beta E_{t}\left[\max \left[V^{R}\left(A_{t+1} f\left(I_{t}\right), \underline{B_{t+1}}\right), V^{D}\left(A_{t+1} f\left(I_{t}\right)\right)\right]\right], \\
p_{t}=\beta \operatorname{Prg}_{t}\left(V^{R}\left(Y_{t+1}, \underline{B_{t+1}}\right) \geq V^{D}\left(Y_{t+1}\right)\right)
\end{array}
\end{gathered}
$$

[^5]Each investor $j$ with wealth $\omega_{j, t}$ submits a bid function $p_{j, t}\left(b_{j, t+1}, B_{t+1}\right)$ stating the price $p_{j, t}$ that she is willing to pay for $b_{j, t+1}$ bonds when the size of the total issue is $B_{t+1}$. Since $B_{t+1}<\underline{B_{t+1}}$ implies a default with certainty, there is no $p_{j, t}>0$ for which an investor is willing to hold the bonds under these circumstances. ${ }^{12}$ I assume the country always rejects bids where $p_{j, t}=0$. If on the other hand $B_{t+1} \geq \underline{B_{t+1}}$, then investor $j$ knows for sure that a default will not take place at time $t$.

Investors are competitive and all bids in equilibrium have zero expected profits. Since investors would then be indifferent between participating or not, I assume the borrowing country provides an infinitesimal transfer to the bidders in order to ensure their participation. Investor $j$ will bid: ${ }^{13}$

$$
\begin{gathered}
p_{j, t}\left(b_{j, t+1}, B_{t+1}\right)=\beta \underset{t}{\operatorname{Pr}\left(V^{R}\left(Y_{t+1}, B_{t+1}\right) \geq V^{D}\left(Y_{t+1}\right)\right),} \\
\quad b_{j, t+1}=\omega_{j, t} / p_{j, t} \text { if } B_{t+1} \geq \underline{B_{t+1}}, \\
p_{j, t}\left(b_{j, t+1}, B_{t+1}\right)=0 \text { for all } b_{j, t+1} \text { if } B_{t+1}<\underline{B_{t+1}} .
\end{gathered}
$$

Given this bidding strategy, the borrowing country is always able to issue the optimal amount of debt $B_{t+1} \geq \underline{B_{t+1}}$, which is preferable to defaulting, and to satisfy the participation constraints laid out in the investors' bids, implying: ${ }^{14}$

Lemma 2 If the country allows investors at time to bid on the new bonds contingent on the resulting issue $B_{t+1}$, then it cannot be rational for an investor not to lend because she expects the other investors at t not to.

It is possible that under some circumstances the country prefers not to allow investors to make their bids fully contingent on the amount issued. Although that does not make any difference in this particular model, it could matter if there were informational asymmetries. The result above still holds in this model if instead of $p_{j, t}$ being allowed to vary continuously on $B_{t+1}$, it is only allowed to be a step-function, provided the size of the offer is bounded by a value that is close

[^6]enough to its efficient level. The country would set that bound at the optimal level of $B_{t+1}$, and investors would offer $p_{j, t}=0$ for a final issue lower than $B_{t+1}$, and a positive value corresponding to the actuarial fair price when the optimal level of debt is issued for issues where $B_{t+1} \geq \underline{B_{t+1}}$. Thus, the result above can be restated in more general terms:

Lemma 3 If the country offers its optimal level of debt and allows investors at time to bid on the new bonds contingent on the resulting issue $B_{t+1}$ being higher or lower than a threshold of their choice, then it cannot be rational for an investor not to lend because she expects the other investors at t not to.

As mentioned previously, many start-up firms use a similar mechanism to raise funds where investors commit to providing part of the capital contingent on the entrepreneurs being able to raise the remaining part from other investors at similar terms. Given that this mechanism can be used for start-up firms, it seems unlikely that any fixed costs involved would be a problem in the context of sovereign debt.

Finally, the country could just organize the bond offer in a way that automatically cancels all bids and fully refunds the buyers if the total amount of debt issued falls below $B_{t+1}$. This can be done by having a reputable third party organize an auction following this specific rule. Again, in the context of this model this would not make a difference, but in other settings this generality may come at some cost.

Lemma 4 If the country offers its optimal level of debt and allows investors at time to bid on the new bonds through an auction that cancels and fully refunds all bids if the resulting issue $B_{t+1}$ were to fall below $\underline{B_{t+1}}$, then it cannot be rational for an investor not to lend because she expects the other investors at t not to.

A country's vulnerability to a intra-period investor coordination failure in the absence of this coordination mechanism depends only on whether its debt level lies within the range prone to that type of crisis today, given by (5). Whether or not the country's debt level will exit that crisis prone region in the future does not eliminate the vulnerability today. But this is not the case for the inter-period coordination failure once investors can coordinate with the other investors in their own period. Given intra-period coordination, it is easy to show that the inter-period coordination failure is eliminated if, in the absence of a crisis, the country's debt level is expected to leave the crisis prone region with certainty at some point in the future. That corresponds to lowering the debt to a level low enough such that (6), (7) and (8) can be satisfied even for the worst possible realization of $A_{t}$. I initially assume that the country is forced to either lower its debt level below that range by a future date or to default. Given that constraint, investors would indeed expect with certainty that unless a default occurs by that future date, the country's debt level would have exited the crisis prone range. I later present a market mechanism through which the country can impose that constraint on itself. Note that even after removing the scope for coordination risk, a
crisis can still occur if the country's fundamentals ${ }^{15}$ deteriorate to the point where its debt cannot be sustained at an actuarially fair rate (which reflects the risk of a fundamentals-driven crisis occurring).

Lemma 5 Suppose the country is forced to either lower its debt level below the range prone to an inter-period investor coordination failure by time $T>t$ or to default. Then it cannot be rational for an investor at time $t$ not to lend because she expects the investors at $t+1$ not to.

Proof. Investors can always coordinate on lending with the other investors in their own period through the contingent biding mechanism described above. The investors at time $t$ expect that if the country is able to continue borrowing up to $T$, then the investors at time $T$ will lend at the actuarially fair rate even if they expect their counterparts at $T+1$ not to. But then the investors at $t$ expect the investors at time $T-1$ to expect that if they lend, their counterparts at $T$ will lend at the actuarially fair rate. It follows immediately from backward induction that the investors at time $t$ expect that if they lend, their counterparts at $t+1$ will lend at the actuarially fair rate. The actuarially fair rate reflects only the risk that the country's fundamentals deteriorate to the point where it prefers to default since there is no longer a coordination risk.

It is easy to show that the cost to the country of facing an additional constraint that its debt must exit that crisis prone region in a large number of periods becomes arbitrarily small as the horizon becomes arbitrarily large. An even stricter condition than requiring the debt level to exit that region by time $T$ is that the debt be fully repaid by date $T$ (i.e., the country repays its debt at $T$ without issuing new bonds due at $T+1$ ). Suppose the country follows such a plan but is allowed to borrow at the actuarially fair rate throughout that time. Let $V_{L C, T}^{R}\left(Y_{t}, B_{t}, T\right)$ denote the corresponding value of the country's objective function under such constraint. That value is given by the solution of the maximand (1) subject to constraints (2), (3), (4), as well as to the added constraint:

$$
\begin{equation*}
B_{T+1}=0 . \tag{9}
\end{equation*}
$$

The only difference between this maximization problem and the one from Section II is the introduction of constraint (9). Since it was assumed that a country that has fully repaid its debt can accumulate foreign assets at the world interest rate (i.e. have negative values for $B_{t}$ ), the cost of imposing constraint (9) can be no larger than that of increasing the country's debt at time $t$ by the bound of its discounted time $T+1$ debt. ${ }^{16}$ The transversality condition (4) implies that for any $\varepsilon$, there exists $S$ large enough such that $\beta^{s-t} B_{s} \leq \varepsilon$ for all $s \geq S$. Thus, the discounted value at time $t$ of the time $T+1$ debt becomes arbitrarily small as $T$ becomes arbitrarily large.

[^7]Lemma 6 If the country strictly prefers to repay its debt to default when it can spread repayments across time, then the country strictly prefers to follow a plan that lowers its time $T$ debt level below the threshold for which an inter-period coordination failure can occur than to default, provided $T$ is large enough.

Proof. If $V^{R}\left(Y_{t}, B_{t}\right)>V^{D}\left(Y_{t}\right)$, then $V_{L C, T}^{R}\left(Y_{t}, B_{t}, T\right) \geq V^{R}\left(Y_{t}, B_{t}+\varepsilon\right)>V^{D}\left(Y_{t}\right)$ for large enough $T$.

If the country can commit itself to either exiting that crisis prone region by time $T$ or incurring the cost of a default, then it would not be rational for the time $t$ investors to expect their $t+1$ counterparts not to lend, as shown by Lemma 5. While the country cannot commit its future actions, there are ways through which it can distort its future incentives so that it would indeed find it optimal to follow such a debt plan.

Lemma 7 The country can impose the constraint $B_{T+1}=0$ on itself by issuing state-contingent securities that pay at time $T$, yielding zero if $B_{T+1}=0$ and a very large amount if $B_{T+1}>0$

Proof. As long as the payment of this security in the states of the world where $B_{T+1}>0$ is large enough to force the country to default, no investor will be willing to hold the country's debt at time $T$ since any such claims would be defaulted on with certainty. That amounts to imposing the constraint $B_{T+1}=0$

While the country cannot coordinate future investors on lending, it can coordinate them on not lending, forcing itself to either pay the debt by that future date or default. By issuing the state-contingent securities described above, the country changes the original game to a new one where the inter-period investor coordination failure cannot occur. The pay-off at time $t$ from repaying the debt becomes $V_{L C, T}^{R}\left(Y_{t}, B_{t}, T\right)$. Since $V_{L C, T}^{R}\left(Y_{t}, B_{t}, T\right)>V^{D}\left(Y_{t}\right)$, if the country were to face a situation where investors would expect their future counterparts not to lend if such beliefs can be rationalized, the country would issue the state contingent security in order to change the game so that those beliefs are not rationalizable, preventing a crisis from occurring. The mechanisms proposed to address the intra-period coordination failure still work in this modified game.

One may argue that the reputational loss incurred from defaulting on such state-contingent securities could be smaller than that of defaulting on the regular bondholders. But that would not matter as long as it is not possible for the country to repay its regular bondholders while defaulting on the state-contingent securities. That can be achieved by including a pari passu clause on the contract of the state-contingent security stating that no future lender may be senior to the holders of that security. The courts in the creditor countries would not allow a group of creditors to be repaid while another group with equal priority is not. ${ }^{17}$

17 While the courts in the creditor countries cannot enforce any contract on the sovereign

It is worth noting that any disciplining role that short-term debt might play (such as in Jeanne 2000) is not compromised by using neither of the mechanisms described above, since the debt is still short-term. It seems reasonable to assume that bond offers are scheduled such that there is enough time following a failed offer to organize a new one before the old bonds become due. Under that assumption, all actions discussed need only occur off the equilibrium path in order to rule out liquidity crises in equilibrium. Having made that last assumption, the following result can be stated:

Proposition 3 If the country strictly prefers to repay its debt when it can spread repayments across time, self-fulfilling liquidity crises caused by a coordination failure among investors cannot occur in equilibrium.

Proof. Lemma 2 implies that the borrowing country can eliminate the scope for coordination failures among investors within a same period by allowing them to bid on the new bonds contingent on the resulting size of the issue. Given that result, Lemma 5 implies that the coordination failure among today's investors and their future counterparts can be eliminated if the constraint $B_{T+1}=0$ is imposed. By Lemma 7, that constraint can be self-imposed by issuing state-contingent securities, and doing so is preferable to a default for large enough $T$ as shown by Lemma 6. Thus, an investor at time $t$ will always be willing to lend to the country since even if the other investors do not, the country will take the measures described above to ensure the success of a second bond offer at time $t$ and in the subsequent periods.

If the model is extended to a richer setting with informational asymmetries, then a failed bond offer may lead some investors to update their beliefs regarding the country's fundamentals. If there is a deterioration in the perceived fundamentals, investors will demand a larger premium when lending to the country. But in order for credit to completely dry out (as in the case of a liquidity crisis) this deterioration in the perceived fundamentals has to be large to the point where no lending can be sustained at the actuarially fair rate. But presumably investors know the reason why they did not buy the bonds in the first place (i.e. whether they did not buy because they expected others not to, or whether they were concerned about the country's fundamentals). That reduces the scope for such deterioration, especially considering that a starting assumption was that investors do not question the country's fundamentals and their only source of concerns are coordination problems with other investors. But even if a significant deterioration in perceived fundamentals is possible following a failed offer, liquidity crises will still be ruled out provided the country's fundamentals are strong enough to begin with.

Finally, beginning with the work of Morris and Shin (1999), there is a trend in the macroeconomics literature to model self-fulfilling events as the result of higher order beliefs (players' beliefs about other players beliefs, players' beliefs about other players' beliefs about other players' beliefs, and so on). For example, a creditor may be unwilling to lend not because she believes

[^8]the country's fundamentals are unsound, but because she believes other investors believe they are (or even because she believes other investors believe other investors believe the fundamentals are unsound, and so on). Morris and Shin (2001) apply this framework to a problem where a group of creditors finances a firm through short-term debt that needs to be rolled-over at a later date. The firm's ability to meet short-term claims from creditors that do not roll over their debt is given by a fundamental variable. In the traditional set-up, there is common knowledge about that fundamental variable and multiplicity of equilibria arises (analogously to the problem presented in Section II). But they show that if creditors have a private signal on the value of that fundamental in addition to their prior, the introduction of an epsilon amount of noise in that signal can lead to a unique equilibrium in switching strategies with the outcome of the game depending on the fundamental variable. That uniqueness is the result of a complex interplay between fundamental uncertainty (which concerns the payoff relevant state of nature) and strategic uncertainty (which concerns the actions of the other creditors). The coordination mechanism proposed in this section to address the intra-period coordination failure removes strategic uncertainty by allowing investors to condition their decisions on an outcome which maps into the strategies of the other investors. As a result, it is robust to the introduction of higher order belief considerations. However, the solution to the inter-period coordination failure can be compromised if there is no common knowledge that all investors are forward-looking.

## IV. DISCUSSION

This paper has focused only on runs on the debt of an aggregate borrower (such as the government). But matters can become more complicated if the debt is contracted by several decentralized private borrowers and there are strategic complementarities between the decisions of creditors lending to different borrowers in this economy. There are a number of channels through which such strategic complementarities can arise. One such channel that is becoming increasingly popular in the international finance literature is through balance sheet effects. Krugman (1999) presents a model with a home and a foreign good where the decline in investment following a credit crunch leads to a decline in the relative price of the home good. Entrepreneurs produce the home good while their liabilities are denominated in terms of the foreign one. They are also credit constrained in the sense that their ability to borrow depends on their current net-worth. ${ }^{18}$ As a result, the relative price adjustment lowers their ability to borrow, rationalizing the credit crunch. Aghion, Bacchetta and Banerjee (2001a, 2001b) articulate a similar self-fulfilling argument in an environment that includes the monetary side of the economy. Their earlier work (Aghion, Bacchetta and Banerjee 1999) had shown how the interplay of credit constraints and balance sheet effects can lead to endogenous volatility, with stable limit cycles occurring in economies at an intermediate level of financial development. Many other papers have also explored these balance sheet effects. ${ }^{19}$ Another potential source for strategic complementarities is the presence of contract enforcement externalities. It seems reasonable to assume that an investor's prospect of recouping her loan may decrease if there is widespread default in the economy. A previous version of this

[^9]paper presented a decentralized economy model of private debt, where contract enforcement externalities generated a "rush to the exits" effect, allowing economy-wide debt runs to occur.

In the presence of the externalities mentioned above, we may have a situation where an investor does not find it optimal to lend to a borrower if the other investors do not lend to the other borrowers in the economy. The coordination mechanisms presented in this paper may not be able to address these economy-wide coordination failures. In order to eliminate the coordination failure among investors within the same period, investors must be allowed to condition their lending on the aggregate lending to the home economy. If they move simultaneously, such coordination will be difficult, and require additional mechanisms. Problems are even more severe in the case of the inter-period coordination failure. Since a small private investor does not internalize the effect of his or her actions on the aggregate liabilities of the home economy, the strategy proposed in the previous section to commit to lower that debt over a long but finite horizon cannot be implemented. Moreover, even if such barriers can be overcome, a free-rider problem will arise if there are private costs to participating in such mechanisms. Since a small borrower's vulnerability to this type of crisis does not depend on whether or not she joins the mechanism, only on whether enough other borrowers do so, even a small private cost can compromise a coordination effort.

The one agent in the economy that would internalize those externalities is the government. In principle, the government could consolidate all the private liabilities and then use the mechanisms from Section III to coordinate the economy in the good equilibrium. In theory, the government could issue new debt in order to take over those liabilities and eventually collect the repayments from the private borrowers it rescued. These two transactions would offset each other in the government's inter-temporal budget constraint. But even if such action does not affect the government's ability to repay the additional debt, it could affect its willingness to do so, since the government can keep the repayments from the borrowers it rescued while defaulting on the public debt. Thus, what matters is not only the net public liabilities but also the gross ones, and the government may not be able to borrow against future claims if doing so would bring its liabilities beyond the threshold for which a strategic sovereign default occurs. If that is the case, government intervention cannot remove the investor coordination failure in a decentralized economy. ${ }^{20}$ These issues are discussed in detail in a previous version of this paper and in Chamon (2003).

## V. CONCLUSION

This paper has argued that self-fulfilling debt runs at the level of an individual borrower (in this case an aggregate borrower) can be prevented by market mechanisms that modify the game being played to one where creditor coordination failures are eliminated. That suggests investor panic arguments are not likely to be relevant when articulated in the context of a run only on the government's debt. Self-fulfilling features can still play a major role when articulated in the context of economy-wide coordination failures, which may be difficult to overcome through the mechanisms proposed in this paper. This implies, among other things, that having many

[^10]small illiquid borrowers can make an economy more vulnerable to self-fulfilling investor panics than having a single aggregate illiquid borrower would. This may seem counterintuitive since liquidity problems are usually associated with large borrowers, who are more likely to borrow from many creditors, increasing the a priori scope for coordination failures. This paper also suggests that, at an individual borrower level, short-term debt per se is not as risky as much of the existing literature suggests since the mechanisms presented can eliminate the "roll-over risk" associated with it. However, short-term debt remains undesirable for several reasons, including the vulnerability to changes in the terms at which it is rolled-over.

Finally, some of the insights from this paper may be applicable to other problems. There are many nonexistent financial instruments that could substantially improve risk sharing if well-functioning markets for them were developed. Market thinness considerations are often evoked as an explanation for the nonexistence of some of those instruments. When deciding to enter a market, investors may demand a higher premium if that market is illiquid. That can generate multiplicity of equilibria in market participation as shown in Pagano (1989) and Allen and Gale (1994). If a new instrument is introduced using the contingent bidding mechanism described in Section III, investors would price their bids according to the expected level of market thickness associated with the different issue sizes. It is possible that would allow the efficient market thickness to be achieved, and the mechanisms proposed can be a useful tool for encouraging financial innovation.

## References

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee, 1999, "Capital Markets and the Instability of Open Economies," CEPR Discussion Paper No. 2083 (London: Centre for Economic Policy Research).
—— (2001a). "A Simple Model of Monetary Policy and Currency Crises," European Economic Review, Vol. 44, 728-738.
—_ (2001b). "A Balance-Sheet Approach to Currency Crises," Working Paper No. 01.05 (Switzerland: Studienzentrum Gerzensee).

Alesina, Alberto, Alessandro Prati, and Guido Tabellini, 1990, "Public Confidence and Debt Management: A Model and a Case Study of Italy," in Public Debt Management: Theory and History, Rudiger Dornbusch and Mario Draghi eds., pp. 94-118 (Cambridge: Cambridge University Press).

Allen, Franklin, and Douglas Gale, 1994, "Liquidity Preference, Market Participation and Asset Price Volatility" American Economic Review, Vol. 84, pp. 933-955.

Bagnoli, Mark, and Barton Lipman, 1988, "Successful Takeovers without Exclusion," Review of Financial Studies, Vol. 1, pp. 89-110.

Bernanke, Ben and Mark Gertler, 1989, "Agency Costs, Net Worth, and Business Fluctuations," American Economic Review, Vol. 79, pp. 14-31.

Bulow, Jeremy, and Kenneth Rogoff, 1989a, "A Constant Recontracting Model of Sovereign Debt," Journal of Political Economy, Vol. 97, pp. 155-178.
——, 1989b, "Sovereign Debt: Is to Forgive to Forget?" American Economic Review, Vol. 79, pp 43-50.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, 2001, "Prospective Deficits and the Asian Currency Crises," Journal of Political Economy, Vol. 109, pp. 1155-97.

Calvo, Guillermo, 1988, "Servicing the Public Debt: The Role of Expectations," American Economic Review, Vol. 78, pp. 647-661.

Chamon, Marcos, 2003, "On the limitations of government financial intermediation," in Essays on International Debt (Ph.D. dissertation; Cambridge, Massachusetts: Harvard University).

Chang, Roberto and Andres Velasco, 2000, "Financial Fragility and the Exchange Rate Regime," Journal of Economic Theory, Vol. 92, pp. 1-34.

Cole, Harold and Patrick Kehoe, 1998, "Models of Sovereign Debt: Practical vs. General Reputations," International Economic Review, Vol. 39, pp. 55-70.

Cole, Harold, and Timothy Kehoe, 1996, "A Self-fulfilling Model of Mexico's 1994-1995 Debt Crisis," Journal of International Economics, Vol. 41, pp. 309-330.
—_, 2000, "Self-fulfilling Debt Crises," The Review of Economic Studies, Vol. 67, pp. 91-116.

Corsetti, Giancarlo, Paolo Presenti, and Nouriel Roubini, 1999, "What Caused the Asian Currency and Financial Crisis?," Japan and the World Economy, Vol. 11, pp. 305-73.

Detragiache, Enrica, 1996, "Rational Liquidity Crises in the Sovereign Debt Market: In Search of a Theory," Staff Papers, International Monetary Fund, Vol. 43, pp. 545-570.

Diamond, Douglas, and Philip Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy, Vol. 91, pp. 401-419.

Eaton, John, and Raquel Fernandez, 1995, "Sovereign Debt," in Handbook of International Economics, Vol. 3, Gene Grossman and Kenneth Rogoff eds., pp. 2031-77 (Amsterdam; New York and Oxford: Elsevier, North-Holland).

Jeanne, Olivier, 2000, "Debt Maturity and the Global Financial Architecture," CEPR Discussion Paper No. 2520 (London: Centre for Economic Policy Research).
——_, and Jeromin Zettelmeyer, 2002, "Original Sin, Balance Sheet Crises and the Roles of International Lending," Working Paper WP/02/234 (Washington: International Monetary Fund).

Krugman, Paul, 1985, "International Debt Strategies in an Uncertain World," in International Debt and the Developing Countries, Gordon W. Smith and John T. Cuddington eds., pp. 79-100 (Washington: World Bank).
——, 1999, "Balance Sheets, The Transfer Problem, and Financial Crises" in International Finance and Financial Crises, Essays in Honor of Robert P. Flood, Peter Isard, Assaf Razin, and AndrewRose eds., pp. 31-44 (Dordrecht: Kluwer).

Morris, Stephen, and Hyun Shin, 1999, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," American Economic Review, Vol. 88, pp. 587-597.
——, 2001, "Coordination Risk and the Price of Debt," European Economic Review, forthcoming.

Obstfeld, Maurice, 1986, "Rational and Self-Fulfilling Balance-of-Payment Crises," American Economic Review, Vol. 76, pp. 72-81.
_- 1996, "Models of currency crises with self-fulfilling features," European Economic Review, Vol. 40, pp. 1037-1048.
_-_, and Kenneth Rogoff, 1996, Foundations of International Macroeconomics, (Cambridge: MIT Press).

Pagano, Marco, 1989, "Endogenous Market Thinness and Stock Price Volatility." Review of Economic Studies, April 1989, Vol. 56, pp. 269-88.

Rodrik, Dani, and Andres Velasco, 1999, "Short-term Capital Flows," NBER Working Paper No. 7364, (Cambridge: National Bureau of Economic Research).

Schneider, Martin, and Aaaron Tornell, 2004, "Balance Sheet Effects, Bailout Guarantees and Financial Crises," Review of Economic Studies, forthcoming.

Sachs, Jeffrey, 1984, "Theoretical Issues in International Borrowing," Princeton Studies in International Finance, Vol. 54, International Finance Section, Department of Economics, Princeton University.
——, Aaron Tornell, and Andres Velasco, 1996, "The Mexican Peso Crisis: Sudden Death or Death Foretold?," Journal of International Economics, Vol. 41, pp. 265-283.

Segal, Ilya, 1999, "Contracting With Externalities," Quarterly Journal of Economics, Vol. 114, pp.337-388.

Shiller, Robert, 1993, Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks (Oxford and New York: Oxford University Press).

Tirole, Jean, 2002, "Inefficient Foreign Borrowing: A Dual-and Common-Agency Perspective," American Economic Review, Vol. 93, pp. 1678-1702.


[^0]:    ${ }^{1}$ This paper was written while I was a graduate student at Harvard University. I am especially grateful to my thesis advisors: Philippe Aghion, Michael Kremer and Kenneth Rogoff for their guidance and support. I am also very grateful to Gustavo Adler, Abhijit Banerjee, Richard Cooper, Ricardo Hausmann, Olivier Jeanne, Markus Mobius, Adam Szeidl, Andres Velasco, Jeromin Zettelmeyer and participants in various seminars for useful comments. Any remaining errors are solely mine. Financial support from the Social Science Research Council Program in Applied Economics and from the MacArthur Network on the Effects of Inequality on Economic Performance is gratefully acknowledged.

[^1]:    2 While the mechanisms proposed are a way of achieving coordination in the extreme case of atomistic investors, in practice, meetings and conference calls among a few key players may well be just as effective.

    3 For a very detailed model of this type of crises, please refer to Cole and Kehoe (1996, 2000).

[^2]:    4 Another commonly used argument to motivate output losses following a default is based on trade disruptions. Suppose there are two varieties of the final consumption good. The home country produces a variety that sells in the world market at a price higher than that of the other variety which is a perfect substitute for home consumers. If trade is disrupted following a default, the value of the country's output to its consumers declines. But this allows for ex post renegotiations (where the country offers a partial repayment to its creditors in exchange for not being punished), which can substantially complicate the analysis. For the model of trade-based cost of default with debt renegotiations please refer to Bulow and Rogoff (1989a).

    5 In practice most defaults are partial, and even countries that default on most of their debt are able to eventually regain access to credit markets. Allowing for a cost of default that is both temporary and proportional to the share of the debt that is not repaid would make the model's assumptions more realistic but would not change its qualitative results. For simplicity, I assume the cost of defaulting is permanent and fixed, an assumption commonly made in the sovereign debt literature.

[^3]:    6 This assumption in itself is enough to sustain some level of lending in equilibrium, although it is too strong an assumption to rely on for that purpose. I chose to use an output loss to sustain lending and repayment in equilibrium, and given that assumption, the additional assumption of financial autarky is only simplifying the model (without changing any of its qualitative features).
    7 In Jeanne (2000) lending to a country can only take place at an actuarially fair rate if the country pursues a certain policy that would not be pursued under a long-term debt contract. But if the debt is short-term the country is forced to pursue the policy since it would not be able to roll-over its debt otherwise. As a result, short-term debt is a constrained optimum despite its risks.

    8 The implied interest rate is $\left(1 / p_{t}\right)-1$. Note that by announcing $B_{t+1}$ as opposed to the amount $p_{t} B_{t+1}$ it wants to raise at time $t$, the country eliminates the possibility of multiple equilibria in the interest rate à la Calvo (1988).

[^4]:    9 A more natural way to incorporate this feature might be to allow the country's decision of whether or not to default to take place after it collects the proceeds from the bond offer. But that would violate the assumption already made that these proceeds can be directly transfered to its creditors.

[^5]:    10 This criticism does not apply in the case of very small countries.
    11 There must be at least one solution to these equations. Otherwise, that would imply that there is no debt level for which the country does not default. But then, there is only one equilibrium (where a default occurs), contradicting the assumption that we are in the region of interest where there are two equilibria.

[^6]:    12 While at first sight it may seem that an investor should be willing to bid an infinitesimal amount, that is actually not the case. Even though the investor will be able to obtain some return on the bond once the proceeds from the failed offer are divided among the country's creditors, such return comes precisely at the expense of those who bought the bond in the failed offer. Buying the bond can only provide a positive return if $p_{j, t}$ is sufficiently lower than $p_{-j, t}$. In equilibrium every investor bids $p_{j, t}=0$.
    $13 \quad Y_{t+1}$ is given by $A_{t+1} f\left(I_{t}\right)$ where $I_{t}$ is the solution to (2).
    14 One may argue that there could still be an equilibrium where no investor submits bids because they expect the other investors not to. But since an investor loses nothing by submiting a bid even when others do not, and receives a small pay-off for lending, submiting a bid is a weakly dominant strategy. Therefore, as long as investors believe others will bid with any non-zero probability they will submit bids. Moreover, the country can always provide a small transfer to investors submiting bids in order to ensure their participation.

[^7]:    15 The country's fundamentals consist of its output, debt level and the distribution that generates the realizations of the productivity shock. A sequence of bad productivity shocks can make the debt unsustainable.

    16 Suppose the country were to choose the same levels of consumption and investment as in its original problem (with the additional increase in its debt level). If the additional debt is set in an escrow account and compounded at the world risk-free interest rate, then it will be enough to fully repay the country's debt by date $T$, satisfying constraint (9).

[^8]:    borrowing country, they can attach payments made to a group of creditors within their jurisdiction in order to share that amount with the other creditors the country defaulted on.

[^9]:    18 As in Bernanke and Gertler (1989).
    19 Jeanne and Zettelmeyer (2002) provide an excellent survey of that literature.

[^10]:    20 Whether or not that is the case will depend on the costs of a sovereign default, and on how large the public liabilities are to begin with.

