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Abstract

This paper develops a two-country monetary DSGE (dynamic stochastic general equilibrium) model in which households choose a portfolio of home and foreign equities, and a forward position in foreign exchange. Some goods prices are set without full information of the state. Home and foreign portfolios are not identical in equilibrium. In response to technology shocks, sticky prices generate a negative correlation between labor income and the profits of domestic firms, biasing portfolios in favor of home equities. In contrast, under flexible prices, labor income and the profits of the domestic firms are positively correlated.

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I. INTRODUCTION

In an open macroeconomy in which asset trade is possible, the portfolio choice of households may play an important role in understanding macro fluctuations. In contrast to a closed economy model—in which a representative agent simply holds the market portfolio—agents in each country may hold different portfolios depending on the country-specific risks and returns that they encounter. Portfolio choice might matter for a number of questions: Does the international transmission mechanism depend on who owns firms? Do changes in valuations of internationally traded assets play a role in the macroeconomic adjustment to shocks? Is there an interaction between the stock market and exchange rates?[2]

A successful model should be consistent with the empirical evidence that equity prices depend on the market's perception of monetary policy.[3] We model the channel through which monetary policy affects real returns as sticky nominal prices: some firms must set nominal prices without full information about the state.

We build a symmetric, two-country model in which agents have identical preferences in each country, firms use identical technologies, market structure is identical, and the stochastic processes of the driving variables (productivity and monetary) are identical. In equilibrium, we find, however, that home and foreign portfolios are not identical. This occurs because we assume that claims to human capital are not traded. In fact, the equilibrium portfolio may exhibit home bias in equities.

The "home bias" puzzle is one of the major puzzles in international finance. Empirical studies have found that foreign equities comprise a small proportion of investors' portfolios.[4] This finding is puzzling because it appears that investors are forgoing important opportunities for diversification of risk.[5] While there have been many suggested resolutions to the puzzle, none seem able to explain entirely the extent of home bias. Our model may contribute to an understanding of home bias. In a framework in which some nominal prices are sticky, it may be natural for households to bias their portfolios strongly toward home equities as a hedge against shocks to their labor income. We do not build the model specifically to explain the home bias puzzle, but we find that in the framework we examine, optimal portfolios may exhibit home bias.

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[2] On the first question, see for example Baxter and Crucini (1995), Kehoe and Perri (2002). Gourinchas and Rey (2005) and Tille (2004) have recently addressed the second question. Pavlova and Rigobon (2003), and Andersen, Bollerslev, Diebold, and Vega (2004) are recent papers that have tried to answer the third question.

[3] See for example Bernanke and Kuttner (2004), Andersen, Bollerslev, Diebold, and Vega (2004), Bordo and Wheelock (2004).

[4] French and Poterba (1991), Tesar and Werner (1995), and Warnock (2002), for example.

[5] Lewis (1999, 2000) surveys the literature on this puzzle and discusses the losses from non-diversification.

The intuition is straightforward: If all nominal prices are sticky, in the short run the level of output is demand determined. Productivity shocks have no effect on short-run output if the firm adjusts output only in response to changes in demand. For example, if home firms experience a positive productivity shock, their demand for labor will decline. Employment and wages will fall, but profits to the firm will increase. An effective hedge against employment and wage risk is ownership of the firm. If output is demand determined, the short-run returns to labor and firm owners are negatively correlated, in contrast to the usual presumption in neoclassical models.

The fact that productivity shocks create a negative correlation between returns to workers and those to firm owners is a key implication of the model. Gali (1999) builds a closed economy model under sticky prices and shows that it can generate a fall in labor hours in response to the positive technology shock, which rarely arises in a flexible price model.[6] His empirical work demonstrates that labor hours decline in response to positive technology shocks in most G-7 countries. The related empirical work by Bottazzi, Pesenti and van Wincoop (1996), and Julliard (2002) find that returns to human capital and equities are negatively correlated in most OECD countries.[7]

Our model is related to one thread of the literature that has attempted to explain home bias as a hedge against non-tradable risks.[8] Our nontradable risk is fluctuations in labor income. In neoclassical models, because labor income is correlated more with domestic firms' profits than with those of foreign firms, the optimal portfolio will be more foreign-weighted than the classical endowment model predicts, as shown in Baxter and Jermann (1997). So the introduction of nontradable risk generally has not been helpful in explaining home bias.[9]

Our chief aim is to provide a model of portfolio choice under sticky nominal prices in the open economy, not necessarily a model of home bias in equities. The literature has taken many different approaches to explain home bias. In addition to the papers cited above that

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[6] For example of flexible price models which generate a negative correlation, see Francis and Ramey (2003) and Dotsey (1999).

[7] The United States is one of the exceptions.

[8] For example, Eldor, Pines and Schwarz (1988), Stockman and Dellas (1989), Tesar (1993), Baxter, Jermann and King (1998), Serrat (2001) and Pesenti and van Wincoop (2002). A related analysis by Obstfeld and Rogoff (2001) argues that transactions costs to trade in international goods can help account for home bias in equities.

[9] See also Jermann (2002). However, Palacios-Huerta (2001) claims that a substantial fraction of home bias can be explained when the differential human capital of stockholders and nonstockholders is taken into account along with human capital frictions. Julliard (2002) claims that Baxter and Jermann did not properly account for return correlations across countries. Heathcote and Perri (2004) show that in a two-good model with investment that there may be home bias in an neoclassical setting.

consider diversification against nontradable risks, several other avenues have been explored. One group of studies has argued that the gains from international diversification are in fact small, so that small transactions costs of diversification will lead to heavily concentrated portfolios.[10] Others have claimed that acquisition of information about foreign firms is more costly than for information on home firms.[11] Another set of studies shows that home bias can be explained in the context of generalized preferences or prior beliefs.[12] Some claim that home bias is partly due to empirical mismeasurement.[13] All of these factors may help explain home bias.

In our model, both monetary shock and technology shocks lead to consumption risk, but monetary shocks can be hedged effectively with bond portfolios (or by taking a forward position in foreign exchange.) Unexpected changes in the relative supplies of money (at home and abroad) create nominal exchange rate changes that in turn alter the value of returns on home and foreign bonds. Monetary shocks lead to positively correlated changes in labor payments and profits, but that risk is not hedged with the equity portfolio.

Of course, nominal prices do not remain fixed forever when productivity or monetary shocks occur. Eventually an adjustment is made and neoclassical results obtain in the long run. Indeed, our model has real labor income positively correlated with productivity shocks in the long run. The degree of home bias depends on the persistence of price stickiness, the persistence of productivity shocks, and the weight that households assign to future consumption. We show that home bias is greater when prices adjust more slowly, when productivity shocks (in one country relative to the other) are less persistent, and when the future is discounted more heavily.

In the following sections, we present two kinds of models. The first is static and helps develop intuition. The second model is a more realistic dynamic one, in which we focus on persistent technology shocks and differential price stickiness.

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[10] For example, Cole and Obstfeld (1991), Tesar (1995), Butler and Joaquin (2002) and many others. However, van Wincoop (1994, 1999), for example, finds large unexploited gains from international risk sharing.

[11] For example, Kang and Stulz (1997) and Hasan and Simaan (2000). A related recent study is van Nieuwerburgh and Veldkamp (2005).

[12] For example, van Wincoop (1994), Aizenman (1999) as examples of the former and Pastor (2000) for the latter.

[13] For example, Rowland and Tesar (2004) find that multinationals may have provided diversification opportunities for some countries.

II. The Simple Static Model

We build a general-equilibrium, two-country model with sticky prices. We call the countries Home and Foreign. The world population is normalized to unity; half the population lives in Home and half in Foreign. Their preferences are identical. Households provide labor and own firms through equity. Firms use labor as the only input to produce a good monopolistically, and preset their prices in the consumers' currency. Markets are segmented so that only firms can export goods. All goods are tradable and perishable. In this section, the model is static.

We adopt local currency pricing here. We observe in the data, at least for developed countries, that consumer prices are sticky in the consumers' currencies rather than in the producers' currencies. However, the pricing assumption is not particularly important in determining the equity portfolio. In fact, we would have exactly the same equity portfolio when prices are preset in producers' currencies, even though the equilibrium number of forward contracts differs.[14]

In our model, we consider two kinds of shocks: monetary and technology shocks. The distribution of shocks is identical between Home and Foreign.

Finally, we assume that before the realization of shocks, only forward contracts in the foreign exchange and equities are traded.

A. Households

Households in both countries have identical preferences over the consumption basket, the real money of the domestic country, and leisure. There are two stages to the household decision problem. In the first stage, households choose a portfolio position: shares of Home equities (γ_h), shares of Foreign equities (γ_f), and a forward position in foreign exchange ($\tilde{\delta}$). These are chosen before the resolution of uncertainty. After shocks are realized, households choose consumption, labor supply and money balances to maximize

$$U\left(C_t, \frac{M_t}{P_t}, L_t\right) = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln\left(\frac{M_t}{P_t}\right) - \frac{\eta}{1+\psi} L_t^{1+\psi}, \tag{2.1}$$

$$\rho > 1,\ \chi > 0,\ \psi > 0, \text{ and } \eta > 0$$

subject to the constraint:

$$P_t C_t + M_t = \gamma_h \Pi_t + \gamma_f S_t \Pi_t^* + W_t L_t + \tilde{\delta}(S_t - F_t) + Tr_t. \tag{2.2}$$

[14] See Matsumoto (2004).

C_t denotes the consumption basket for Home; M_t denotes Home money; P_t, the price index; and L_t, the labor supply. C_t is a consumption basket of a representative Home household defined as

$$C_t \equiv \left(\tfrac{1}{2}\right)^{1/(\omega-1)} \left(C_{h,t}^{(\omega-1)/\omega} + C_{f,t}^{(\omega-1)/\omega}\right)^{\omega/(\omega-1)} \qquad (2.3)$$

where $\omega > 0$ is the elasticity of substitution between Home produced goods and Foreign produced goods. $C_{h,t}$ is the consumption basket of Home produced goods and $C_{f,t}$ is that of Foreign produced goods:

$$C_{h,t} \equiv \left[2^{1/\lambda} \int_0^{1/2} C_{h,t}(i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}, \quad C_{f,t} \equiv \left[2^{1/\lambda} \int_{1/2}^{1} C_{f,t}(i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}, \quad (2.4)$$

where λ denotes the elasticity of substitution among varieties, with $\lambda > 1$. Then we can write the CPI as follows:

$$P_t \equiv \left(\tfrac{1}{2}\right)^{1/(1-\omega)} \left(P_{h,t}^{1-\omega} + P_{f,t}^{1-\omega}\right)^{1/(1-\omega)}, \qquad (2.5)$$

where

$$P_{h,t} \equiv \left[2\int_0^{1/2} P_{h,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}, \quad P_{f,t} \equiv \left[2\int_{1/2}^{1} P_{f,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}, \qquad (2.6)$$

where $P_{h,t}(i)$ is the price of Home goods i sold in Home in terms of the Home currency, and $P_{f,t}(i)$ is the price of Foreign goods i sold in Home in terms of the Home currency. Home households receive the following: wages ($W_t L_t$, where W_t denotes the wage); dividends; transfers from the government (Tr_t) and the gains or losses from forward contracts. Equity dividends received by a Home household are given by

$$\gamma_h \Pi_t + \gamma_f S_t \Pi_t^*,$$

where Π_t is the profit (dividend) of Home firms and Π_t^* is that of Foreign firms in terms of the Foreign currency.[15] S_t is the Home currency price of Foreign currency. Home and Foreign households trade forward contracts in the foreign exchange. The forward rate, F_t, is known at the time the forward contract is entered into, prior to the realization of shocks. After the shocks are realized, the Home households receive $\tilde{\delta}(S_t - F_t)$ units of Home currency.

Foreign households have an analogous utility function for Foreign quantities and prices, which we will denote by superscript asterisks. Foreign prices are denominated in Foreign currency.

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[15] Theoretically, profits can be negative in the case of a loss, but we have to assume that the profits of both Home firms and Foreign firms are positive to take logarithms.

Prior to the realization of shocks, the households choose the portfolio position to maximize expected utility ($E_{t-1}U\left(C_t, \frac{M_t}{P_t}, L_t\right)$)[16] subject to the constraint:

$$\gamma_h + \gamma_f = 1. \quad (2.7)$$

Note that there is no constraint on the forward position, $\tilde{\delta}$. We assume that the ex ante distribution of shocks are identical between Home and Foreign. This assumption, together with the assumptions of identical size and identical preferences, gives us an equilibrium in which the equity prices of Home and Foreign firms are the same prior to the realization of shocks.[17] In our normalization, the representative household of each country is endowed with an ownership share of 1 of their own firms, but they may trade some of their shares with households in the other country, which implies constraint (2.7). Given the symmetry in the model, there is home bias when $\gamma_f < \frac{1}{2}$.

Given prices and the total consumption basket, C_t, the optimal consumption allocations are

$$C_{h,t} = \frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t, \qquad C_{f,t} = \frac{1}{2}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega} C_t, \qquad (2.8)$$

$$C_{h,t}(i) = 2\left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\lambda} C_{h,t}, \qquad C_{f,t}(i) = 2\left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\lambda} C_{f,t}. \quad (2.9)$$

The remaining first order conditions are

$$\frac{M_t}{P_t} = \chi C_t^{\rho}, \quad (2.10)$$

$$W_t = \frac{\eta}{\chi} M_t L_t^{\psi}, \quad (2.11)$$

$$E_{t-1}\left(S_t \frac{C_t^{-\rho}}{P_t}\right) = F_t E_{t-1}\left(\frac{C_t^{-\rho}}{P_t}\right), \qquad (2.12)$$

$$E_{t-1}\left(\Pi_t \frac{C_t^{-\rho}}{P_t}\right) = E_{t-1}\left(S_t \Pi_t^* \frac{C_t^{-\rho}}{P_t}\right). \quad (2.13)$$

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[16] We use the notation that expectations are taken at time t-1 in this section—even though the model is static—for notational convenience so that we can refer to some of the same equations that arise in the dynamic model.

[17] If prices are different, then one country is richer than the other *ex ante*, a situation that contradicts symmetry.

B. Firms

Firms engage in monopolistic competition as in Blanchard and Kiyotaki (1987). A firm in this economy monopolistically produces a specific good indexed by i using a linear technology:[18]

$$Y_t(i) = A_t L_t(i), \tag{2.14}$$

where $Y_t(i)$ is the production of firm i, A_t is the country-specific technology parameter and $L_t(i)$ is the labor input of firm i. Labor is assumed to be homogeneous and to be supplied elastically. Home and Foreign markets are segmented, and only the producer can distribute its product. Firms set prices one period in advance in the consumers' currencies for each country. Firms in each country set prices so as to maximize their expected profits, taking other firms' prices as given, which is equivalent to taking the price level as given since each firm has measure zero on interval [0,1].

Given the CES utility sub-function, the demand for Home good i from the Home market denoted by $Y_{h,t}(i)$ is

$$Y_{h,t}(i) = \left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\lambda} \left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t, \tag{2.15}$$

while the demand for Home good i from the Foreign market is

$$Y_{h,t}(i)^* = \left(\frac{P^*_{h,t}(i)}{P^*_{h,t}}\right)^{-\lambda} \left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t. \tag{2.16}$$

Firm i's profit maximization problem is

$$\max_{P_{h,t}(i),P^*_{h,t}(i)} E_{t-1}\left\{\tilde{D}_t(i)\left[P_{h,t}(i)Y_{h,t}(i) + S_t P^*_{h,t}(i)Y^*_{h,t}(i) - \frac{W_t}{A_t}(Y_{h,t}(i) + Y^*_{h,t}(i))\right]\right\},$$

where $\tilde{D}_t(i)$ is the stochastic discount factor for the firm i. For example, if firms are owned by Home residents, it will be $\frac{C_t^{-\rho}}{P_t}$. However, because firms are not always domestically owned, we use a more general notation.

The optimal price of Home goods for the Home market[19] is

$$P_{h,t} = \frac{\lambda}{\lambda - 1}\frac{E_{t-1}\left(\tilde{D}_t C_t \frac{W_t}{A_t}\right)}{E_{t-1}\left(\tilde{D}_t C_t\right)}. \tag{2.17}$$

[18] Using a Cobb-Douglas technology with other fixed inputs will not change the result if the returns on the other factors belong to the equity holders.

[19] We will omit index i since Home firms are identical.

Similarly, the optimal price of Home goods for the Foreign market is

$$P_{h,t}^{*}=\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(\tilde{D}_t C_t^{*}\frac{W_t}{A_t}\right)}{E_{t-1}\left(\tilde{D}_t C_t^{*} S_t\right)}. \quad (2.18)$$

Because firms are all alike, they will set the identical prices for each market.

The market clearing condition can be obtained by equating the output with the sum of the demands for Home goods:

$$A_t L_t=\frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t+\frac{1}{2}\left(\frac{P_{h,t}^{*}}{P_t^{*}}\right)^{-\omega} C_t^{*}. \quad (2.19)$$

Given these prices, we can calculate profits. Using the optimal consumption allocations, we can write the profits for the firms in each country in terms of the Home currency as

$$\Pi_t=\frac{1}{2} P_{h,t}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t+\frac{1}{2} S_t P_{h,t}^{*}\left(\frac{P_{h,t}^{*}}{P_t^{*}}\right)^{-\omega} C_t^{*}-W_t L_t, \quad (2.20)$$

$$S_t \Pi_t^{*}=\frac{1}{2} S_t P_{f,t}^{*}\left(\frac{P_{f,t}^{*}}{P_t^{*}}\right)^{-\omega} C_t^{*}+\frac{1}{2} P_{f,t}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega} C_t-S_t W_t^{*} L_t^{*}. \quad (2.21)$$

Firms will pay out all of their profits as dividends.

We assume that A_t and A_t^{*} are drawn from identical lognormal distributions with $\operatorname{var}_{t-1}\left(\ln\left(A_t\right)\right)=\operatorname{var}_{t-1}\left(\ln\left(A_t^{*}\right)\right)=\sigma_a^2$, and $\operatorname{cov}_{t-1}\left(\ln A_{t,} \ln A_t^{*}\right)=\sigma_{a,a^{*}}$ We also assume that M_t and M_t^{*} are drawn from identical lognormal distributions with $\operatorname{var}_{t-1}\left(\ln\left(M_t\right)\right)=\operatorname{var}_{t-1}\left(\ln\left(M_t^{*}\right)\right)=\sigma_m^2$, and $\operatorname{cov}_{t-1}\left(\ln M_{t,} \ln M_t^{*}\right)=\sigma_{m,m^{*}}$ We assume that the money shocks are independent of the technology shocks.

The labor market is competitive, and the wage moves freely to equate demand and supply of labor after the shocks. The output of each good is determined by demand. Firms adjust output after the shocks to satisfy demand, holding prices constant. The money market is assumed to equilibrate, so money demand equals money supply.

C. Solution of the Static Model

An equilibrium in the static model satisfies equations (2.2) and (2.5)-(2.21), and their foreign counterparts. These 39 equations (one is redundant by Walras' Law) solve for C_t, $C_{h,t}$, $C_{f,t}$, $C_{h,t}(i)$, $C_{f,t}(i)$, L_t, W_t, P_t, $P_{h,t}$, $P_{f,t}$, $P_{h,t}(i)$, $P_{h,t}^{*}(i)$, $Y_t(i)$, $Y_{h,t}(i)$, $Y_{h,t}^{*}(i)$, Π_t, γ_h, γ_f, and their foreign counterparts, and $\tilde{\delta}$, F_t, and S_t.[20]

[20] We have also implicitly assumed that there is a money market equilibrium condition, but we have not introduced separate notation for money demand and money supply and that there (continued…)

We will not in fact solve for this equilibrium, but will instead solve the equilibrium for a set of equations that approximate these 39. We take first-order approximations to the budget constraint (2.2), the definitions of the consumption and price indexes (2.3)-(2.6), the equilibrium condition (2.19), and the definition of profits (2.20)-(2.21). Under our assumption that the driving variables are lognormally distributed, and with the log-linearization of these equations, we can solve equations (2.7)-(2.18) exactly.

Our focus is on the equilibrium portfolio choice of equity shares and forward foreign exchange position. We proceed in this section to construct the equilibrium solutions for these variables in an intuitive manner. We will first derive the portfolio demands for households, taking prices as given. With these in hand, we will use equilibrium conditions in goods, labor, and asset markets to derive the equilibrium portfolio positions.

We rely on ex ante symmetry in the derivations below. Lower-case letters refer to logs of their upper case counterparts. We use "var" to denote variance, and "cov" covariance.[21] We use the notation $\bar{x} = E(x_t)$. In the linearized equations below, we suppress the intercept terms for convenience.

The household first-order condition (2.12) can be written as

$$-\rho \operatorname{cov}(c_t, s_t) + \frac{1}{2}\operatorname{var}(s_t) = 0, \quad (2.22)$$

where we have used ex ante symmetry to give us $f_t = 0$, and $E(s_t) = 0$.

We can use similar steps, and recognize that symmetry implies that $\bar{\pi} = \bar{\pi}^*$, $\operatorname{var}(\pi_t) = \operatorname{var}(\pi_t^*)$, and $\operatorname{cov}(s_t, \pi_t) = -\operatorname{cov}(s_t, \pi_t^*)$, to derive from equation (2.13):

$$\rho \operatorname{cov}(c_t, \pi_t - (s_t + \pi_t^*)) - \frac{1}{2}\operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*)) = 0. \quad (2.23)$$

We approximate the budget constraint (2.2), using condition (2.7) to arrive at

$$p_t + c_t = (1-\gamma)(1-\zeta)\pi_t + \gamma(1-\zeta)(s_t + \pi_t^*) + \zeta(w_t + l_t) + \delta s_t, \quad (2.24)$$

where $\zeta \equiv \dfrac{e^{\bar{w}+\bar{l}}}{e^{\bar{\pi}} + e^{\bar{w}+\bar{l}}}$, $\delta \equiv \dfrac{\tilde{\delta}}{e^{\bar{\pi}} + e^{\bar{w}+\bar{l}}}$, and $\gamma \equiv \gamma_f$. Here, we have approximated the budget constraint around a point where $x_t = \bar{x}$ for $x_t = s_t, c_t, \pi_t, \pi_t^*, w_t, l_t$.

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is a forward market clearing condition which can be guaranteed here by setting $\tilde{\delta}^* = F\tilde{\delta}$. As the Appendix demonstrates, equilibrium actually requires, but by symmetry in the static model, F equals one.

[21] We drop the $t-1$ subscript on expectations for the rest of this section.

We use equation (2.24) to substitute out for c_t in equations (2.22) and (2.23), and recognize that p_t is predetermined. Then we solve out for γ and δ:

$$\begin{aligned}
\gamma = & \frac{\operatorname{cov}(\pi_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} \\
& + \frac{\zeta}{1-\zeta}\frac{\operatorname{cov}(w_t + l_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} + \frac{1}{1-\zeta}(\delta - \frac{1}{2\rho})\frac{\operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} \\
\delta = & \frac{-(1-\zeta)\operatorname{cov}(\pi_t, s_t)}{\operatorname{var}(s_t)} - \frac{\zeta \operatorname{cov}(w_t + l_t, s_t)}{\operatorname{var}(s_t)} + \gamma\frac{(1-\zeta)\operatorname{cov}(\pi_t - (s_t + \pi_t^*), s_t)}{\operatorname{var}(s_t)} + \frac{1}{2\rho} \\
= & -(1-\zeta)\beta_{\pi,s} - \zeta\beta_{w+l,s} + \gamma(1-\zeta)\beta_{\pi-s-\pi^*,s} + \frac{1}{2\rho}
\end{aligned} \tag{2.25}$$

where we have used the notation $\beta_{x,s} \equiv \dfrac{\operatorname{cov}(x_t, s_t)}{\operatorname{var}(s_t)}$.

We can then use these two equations to solve out for γ, using the properties of orthogonal projections to get

$$\gamma = \frac{\operatorname{cov}(\pi_t - \beta_{\pi,s}s_t, \pi_t - \pi_t^*)}{\operatorname{var}(\pi_t - \pi_t^* - \beta_{\pi-\pi^*,s}s_t)} + \frac{\zeta}{1-\zeta}\frac{\operatorname{cov}(w_t + l_t - \beta_{w+l,s}s_t, \pi_t - \pi_t^*)}{\operatorname{var}(\pi_t - \pi_t^* - \beta_{\pi-\pi^*,s}s_t)}. \tag{2.26}$$

Consider expression (2.26). From the household's point of view, the equity position is determined by the covariances and variances of shocks to profits and labor income that are orthogonal to exchange rates. Any variance in the portfolio that is attributable to exchange rate changes is hedged through the forward position, so the equity position is determined only by those risks that are uncorrelated with exchange rate risk.

If the component of labor income that is orthogonal to exchange rates were uncorrelated with relative profits of Home and Foreign firms (or if labor's share were zero), the second term in equation (2.26) would drop out. Then the share γ of equities held in Foreign firms would increase as Home profits (orthogonal to the exchange rate) have a higher covariance with relative Home and Foreign profits. Under our symmetry assumption, this term will equal 1/2, so the portfolio would be balanced between Home and Foreign equities if only the first term mattered. It is the second term of equation (2.26) that will determine home bias.

That term tells us that the share of Foreign equities will be larger the greater the covariance between wage income and Home profits relative to Foreign profits. If this covariance is positive, there will be anti-home bias ($\gamma > \frac{1}{2}$), as in Baxter and Jermann (1997). In that case, returns to Home equities (compared to returns on Foreign equities) are positively correlated with labor income, so the variance of total income (returns to equities and human capital) is reduced by holding a relatively large share of Foreign equities. There is home bias when that covariance is negative. In that case, Home equities serve as a hedge against labor income shocks.

So far, to arrive at equation (2.26), we have only used the households' first-order conditions and budget constraints, along with the symmetry assumption and the assumption that nominal prices are fixed. Now we can bring in one more equation from the rest of the economy, the linearization of the profit equation for Home firms. We have from (2.20)

$$(1-\zeta)\pi_t + \zeta(w_t + l_t) = p_t + c_t^W + \frac{1}{2}s_t \,, \quad (2.27)$$

where $c_t^W = \frac{1}{2}(c_t + c_t^*)$. [22]

Taking covariances with $\pi_t - \pi_t^*$ on both sides of equation (2.27), we get

$$\text{cov}(\pi_t + \frac{\zeta}{1-\zeta}(w_t + l_t), \pi_t - \pi_t^*) = \frac{1}{2(1-\zeta)}\text{cov}(s_t, \pi_t - \pi_t^*)\,, \quad (2.28)$$

where we have used symmetry to infer that $\text{cov}(c_t^W, \pi_t - \pi_t^*) = 0$. Also,

$$\text{cov}(\pi_t + \frac{\zeta}{1-\zeta}(w_t + l_t), s_t) = \frac{1}{2(1-\zeta)}\text{var}(s_t)\,, \quad (2.29)$$

using symmetry to infer that $\text{cov}(c_t^W, s_t) = 0$. Dividing (2.29) through by $\text{var}(s_t)$, we can write

$$\beta_{\pi,s} + \frac{\zeta}{1-\zeta}\beta_{w+l,s} = \frac{1}{2(1-\zeta)}\,. \quad (2.30)$$

Substitute (2.28) and (2.30) into the right-hand side of (2.26) to derive $\gamma = 0$.

To get the equilibrium value of δ, substitute $\gamma = 0$ into equation (2.25), and use equation (2.29):

$$\delta = \frac{-(1-\zeta)\,\text{cov}(\pi, s)}{\text{var}(s)} - \frac{\zeta\,\text{cov}(w+l, s)}{\text{var}(s)} + \frac{1}{2\rho} = -\frac{1}{2} + \frac{1}{2\rho}\,. \quad (2.31)$$

We find complete home bias in equity holdings, $\gamma = 0$. Equation (2.26) indicates that the share of equities held in the foreign firm is determined by the covariance of the component of Home firm revenues $(1-\zeta)\pi_t + \zeta(w_t + l_t)$ that is orthogonal to the exchange rate with the relative profits of Home to Foreign firms. If that covariance is zero, then no foreign equities are held. In that case, returns to Home equities are a perfect hedge for labor income.

In fact, the residual from projecting $(1-\zeta)\pi_t + \zeta(w_t + l_t)$ on s_t is orthogonal to $\pi_t - \pi_t^*$. That is because equation (2.27) tells us that the revenue of the Home firm, $(1-\zeta)\pi_t + \zeta(w_t + l_t)$, is determined by world consumption and the exchange rate: $c_t^W + \frac{1}{2}s_t$. Output is demand determined. Demand depends on the overall level of consumption in both countries. Additionally, the Home-currency revenue of the Home firm increases when the currency

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[22] In deriving (2.27), we use symmetry to get $\bar{c} = \bar{c}^*$ and $p_t = p_t^*$. The Appendix shows that $p_{ht} - p_t = 0$, which we have used to derive (2.27).

depreciates, because the depreciation increases the Home-currency value of Foreign sales. The projection residual is simply world consumption, c_t^W , and that is uncorrelated with relative profits by symmetry.

Note that if we substitute the solutions for γ and δ back into the budget constraint (2.24), we obtain (using (2.27))

$$p_t + c_t = (1-\zeta)\pi_t + \zeta(w_t + l_t) + (\frac{1}{2\rho} - \frac{1}{2})s_t = p_t + c_t^W + \frac{1}{2\rho}s_t . \quad (2.32)$$

Using the definition of world consumption, this expression can be written as

$\rho c_t = s_t + \rho c_t^*$.(2.33)

This condition indicates that the linearized model replicates the equilibrium in which a full set of nominal contingent bonds is traded. As is well known, in this case (and assuming symmetry), the marginal utility of a unit of Home (or Foreign) currency is equalized between home and foreign residents:

$$\frac{C_t^{-\rho}}{P_t} = \frac{C_t^{*-\rho}}{S_t P_t^*} .$$

Equation (2.33) takes the log of this condition, using symmetry to infer $p_t = p_t^*$. The trading of Home and Foreign equities and forward contracts for foreign exchange are enough to deliver the same allocation as trading a full set of nominal contingent claims in the linearized economy.[23]

We have derived the complete home bias result using only the nominal price stickiness assumption, the definition of Home profits, the budget constraint of Home households, and the two first-order conditions (2.12 and 2.13) that pertain to asset choice. (The derivations in this subsection all arise from equations (2.22), (2.23), (2.24), and (2.27), which are the approximated versions of the two first-order conditions for asset choice, the household budget constraint, and the definition of firm profits. In performing the approximations, we have used the fact that prices are preset.)

We have not relied on other features of the model, so our home bias result is robust to alternative assumptions. For example, the result does not depend on money demand arising from real balances in the utility function. Other specifications that maintain equations (2.12) and (2.13) will deliver the same result. As long as symmetry is maintained, the result does not depend on the assumptions about monetary policy (that money supplies are determined exogenously with shocks that are independent of equity shocks.) The result also does not depend on our specification of the labor market as competitive with flexible wages. For

[23] Note the implication that the factor firms use in equations (2.17) and (2.18) to discount expected profits is identical (up to the linear approximation) for home and foreign households.

example, a sticky-wage model in which employment was demand determined would not alter the conditions that we used in the derivation of the home-bias result.

Further insights can be obtained from making use of some of the other equations of the model. Specifically, the first-order condition for holdings of money balances (and again using the fact that nominal prices are preset), is written as

$$m_t = \rho c_t . \qquad (2.34)$$

Using this equation along with its foreign counterpart, and equation (2.33), we derive

$$s_t = m_t - m_t^* . \qquad (2.35)$$

Exchange rates are determined by relative money supplies.

The fact that equity demand depends only on the covariances after projecting on the exchange rate means that the equity portfolio is used only to hedge productivity shocks. Productivity shocks do not influence the amount of product the firm sells, which is demand determined in a sticky-price model. Nor do productivity shocks affect the exchange rate, which influences firm revenue as well. So firm revenue depends only on monetary shocks. A positive productivity shock, for example, allows the firm to produce the quantity demanded with less labor. Both wages and employment fall in equilibrium. Profits increase by the exact amount of the drop in labor income. But the effect of those shocks on household income is fully hedged when Home households hold 100 percent of Home firms.

Monetary shocks have real consequences in this model. Indeed, equation (2.34) shows that in equilibrium, consumption is determined only by money supplies. As we have noted, productivity shocks only affect the distribution of revenues between labor income and profits, but in equilibrium, the effects of that redistribution is nullified by the complete home bias in equity holdings. The real effects of monetary shocks are hedged through the forward position in foreign exchange.

Suppose, for example, that there is a negative Home monetary shock. In equilibrium, income of Home households falls because both labor and profit income fall. But the drop in the Home money supply also causes a home currency appreciation (s_t declines.) The equilibrium value of δ is negative, given our assumption of $\rho > 1$. In this case, a decline in s_t leads to a positive payoff from the forward position. That is, when δ is negative, the Home resident is short in foreign currency and long in home currency. So an appreciation yields a positive payoff, which hedges the effects of monetary shocks on labor and profit income.

Notice that the forward position does not completely eliminate the effects of monetary shocks on income. From equation (2.27), we have that $(1-\zeta)\pi_t + \zeta(w_t + l_t)$ falls by $\frac{1}{2\rho} + \frac{1}{2}$ times the decrease in m_t (because c_t^W falls by $\frac{1}{2\rho}$ and $\frac{1}{2}s_t$ by $\frac{1}{2}$.) Including returns from the forward position solved from equation (2.31), $\delta = \frac{1}{2\rho} - \frac{1}{2}$, we find that income still falls by $\frac{1}{\rho}$ times the drop in m_t. Why? In this model, the Home and Foreign consumption markets are completely segmented. A change in the exchange rate causes a change in the relative prices paid by Home and Foreign households for identical goods, because nominal prices are set in advance in consumers' currencies and do not respond to shocks. So Home prices rise relative to Foreign prices (expressed in a common currency) when s_t falls. But households cannot trade goods to arbitrage the difference in goods prices. As is well known, when consumer products are not tradable, the efficient configuration of consumption (achievable when a full set of contingent nominal bonds is traded) has consumption levels lower in the Home country (relative to the Foreign country) in those states of the world in which its goods prices are higher than those in the Foreign country. That is why the equilibrium condition (2.33) does not achieve perfect consumption correlation. So with a negative Home monetary shock, *ceteris paribus*, Home income falls and Home consumption declines.

III. Dynamic Model

In this section, we build an infinite-horizon model, which allows us to examine the effects of persistent technology shocks and different degrees of price stickiness. Most of the assumptions are the same as in the static model.

The price-setting rule is modified as follows. A fraction τ of firms in each country set prices in advance, and the rest of the firms can adjust their prices in each period after the realization of shocks. This approach allows us to study the portfolio allocation with or without sticky prices, and we can learn how different degrees of price stickiness affect the portfolio. There are different types of firms in each country but we assume the equities of all firms in each country are bundled together.

An important question under the dynamic model is, how will persistent shocks affect the optimal portfolio? In a flexible price setting, the optimal portfolio is more foreign skewed than it is in the classic endowment economy case, as shown in Baxter and Jermann (1997). This effect decreases the degree of home bias in our model as well. In the dynamic model, when the elasticity of substitution between Home and Foreign goods is more than unity ($\omega > 1$), the optimal Home portfolio should be less home biased than it is in the static model because households must take into account the future after prices have been adjusted.

A. Household Problem

Home households maximize their expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, L_t \right)$$

subject to the following budget constraint:

$$P_tC_t + M_t + Q_t\gamma_{h,t+1} + S_tQ_t^*\gamma_{f,t+1}$$
$$= \gamma_{h,t}(Q_t + \Pi_t) + \gamma_{f,t}S_t(Q_t^* + \Pi_t^*) + (S_t - F_t)\tilde{\delta}_t + W_tL_t + M_{t-1} + Tr_t \quad (3.1)$$

where Q_t (Q_t^*) denotes the price of Home (Foreign) equities. The utility function and consumption baskets are the same as in the static model. Households enter time t with money M_{t-1}, equities ($\gamma_{h,t}, \gamma_{f,t}$), and forward contracts $\tilde{\delta}_t$. After the realization of shocks, households choose the consumption level, real money balances, and labor supply. The dividends from firms are paid at time t, and households get the payoff from the forward contract. They receive the transfer from the government as well. Finally, the households will choose forward contracts $\tilde{\delta}_{t+1}$ and equity holdings $\gamma_{h,t+1}$, $\gamma_{f,t+1}$, which will determine the dividends households receive at time $t+1$.

The first order conditions for the households are

$$\frac{\chi}{M_t} = \frac{C_t^{-\rho}}{P_t} - E_t\beta\frac{C_{t+1}^{-\rho}}{P_{t+1}}, \quad (3.2)$$

$$\eta L_t^{\psi} = \frac{C_t^{-\rho}}{P_t}W_t, (3.3)$$

$$E_{t-1}\left(\frac{C_t^{-\rho}}{P_t}S_t\right) = F_tE_{t-1}\left(\frac{C_t^{-\rho}}{P_t}\right), \quad (3.4)$$

$$\frac{C_{t-1}^{-\rho}}{P_{t-1}}Q_{t-1} = E_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t}(Q_t + \Pi_t)\right), \quad (3.5)$$

$$\frac{C_{t-1}^{-\rho}}{P_{t-1}}S_{t-1}Q_{t-1}^* = E_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t}S_t(Q_t^* + \Pi_t^*)\right). \quad (3.6)$$

First, let $D_{t,t+s} \equiv \left(\frac{C_{t+s}^{-\rho}}{P_{t+s}}\right) \Big/ \left(\frac{C_t^{-\rho}}{P_t}\right)$. The no-bubble solution for equity prices implies that

$$Q_t = \sum_{s=1}^{\infty}E_t\beta^sD_{t,t+s}\Pi_{t+s}, \quad S_tQ_t^* = \sum_{s=1}^{\infty}E_t\beta^sD_{t,t+s}S_{t+s}\Pi_{t+s}^* \quad (3.7)$$

Let

$$V_t \equiv \gamma_{h,t+1}Q_t + \gamma_{f,t+1}S_tQ_t^*, \quad (3.8)$$

$$H_t \equiv \sum_{s=1}^{\infty}\beta^sE_tD_{t,t+s}W_{t+s}L_{t+s}, \quad (3.9)$$

$$R_t \equiv \frac{\beta(Q_t + \Pi_t)}{Q_{t-1}}, \quad (3.10)$$

$$R_t^H \equiv \frac{\beta(H_t + W_tL_t)}{H_{t-1}}, \quad (3.11)$$

$$\gamma_{t+1} \equiv \frac{\gamma_{f,t+1}S_tQ_t^*}{V_t} = 1 - \frac{\gamma_{h,t+1}Q_t}{V_t}. (3.12)$$

These are, respectively, financial wealth, human capital, the rate of return on financial wealth and human capital (each multiplied by the utility discount factor for algebraic convenience) and the share of foreign equity in equity portfolio.

We can rewrite the budget constraint (3.1) for time t:

$$P_t C_t + V_t + H_t = V_{t-1}(1-\gamma_t)\beta^{-1}R_t + V_{t-1}\gamma_t\beta^{-1}\frac{S_t}{S_{t-1}}R_t^* + H_{t-1}\beta^{-1}R_t^H + \tilde{\delta}_t(S_t - F_t).$$

(3.13)

We will assume below a process for the money supply in which $E_t(M_{t+1}^{-1}) = M_t^{-1}$. We note this now, because under this assumption the first-order condition (3.2) can be simplified directly to get

$$\frac{C_t^{-\rho}}{P_t} = \chi M_t^{-1} + E_t\beta\frac{C_{t+1}^{-\rho}}{P_{t+1}} = \frac{\chi}{1-\beta}M_t^{-1}. \quad (3.14)$$

It follows from this that $D_{t,t+s} = \dfrac{M_t}{M_{t+s}}$. The first order conditions for equity holdings, (3.5) and (3.6), can be summarized as

$$E_{t-1}\left(\frac{M_{t-1}}{M_t}R_t\right) = E_{t-1}\left(\frac{M_{t-1}}{M_t}\frac{S_t}{S_{t-1}}R_t^*\right) = 1. \qquad (3.15)$$

B. Firms

Firms use the same linear technology as in the previous section. We have two types of firms in each country. A fraction τ of firms set the price in advance, and the rest set the price after the realization of shocks. The profit maximization problem of the Home firm with price flexibility is

$$\max P_{h,t}(i)Y_{h,t}(i) + S_t P_{h,t}^*(i)Y_{h,t}^*(i) - (\frac{W_t}{A_t})[Y_{h,t}(i) + Y_{h,t}^*(i)].$$

Because $Y_{h,t}(i)$ is not a function of $P_{h,t}^*(i)$, and $Y_{h,t}^*(i)$ is not a function of $P_{h,t}(i)$, the problem is easy to solve:

$$P_{h,t}(i) = \frac{\lambda}{\lambda-1}\frac{W_t}{A_t} \equiv P_{flex,h,t}, \qquad P_{h,t}^*(i) = \frac{\lambda}{\lambda-1}\frac{W_t}{A_t S_t} \equiv P_{flex,h,t}^*, \quad (3.16)$$

where $P_{flex,h,t}$ is the optimal price for the Home market of the Home goods produced by the firms that can adjust prices after they observe shocks. $P_{flex,h,t}^*$ is the optimal price for the Foreign market.

The other optimal prices are

$$P_{preset,h,t} \equiv \frac{\lambda}{\lambda-1} \frac{E_{t-1}\left[\tilde{D}_t \frac{W_t}{A_t}\left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t\right]}{E_{t-1}\left[\tilde{D}_t \left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t\right]}, \quad (3.17)$$

$$P^*_{preset,h,t} \equiv \frac{\lambda}{\lambda-1} \frac{E_{t-1}\left[\tilde{D}_t \frac{W_t}{A_t}\left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t\right]}{E_{t-1}\left[\tilde{D}_t \left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t\right]}, \quad (3.18)$$

where $\tilde{D}$ is the stochastic discount factor, and $P_{preset,h,t}$ is the optimal price for the Home market at time t of the goods produced by the firms that set prices in advance. Now we can rewrite the price indexes as follows:

$$P_{h,t} = [(1-\tau)P^{1-\lambda}_{flex,h,t} + \tau P^{1-\lambda}_{preset,h,t}]^{\frac{1}{1-\lambda}}, \quad (3.19)$$

$$P_{f,t} = [(1-\tau)P^{1-\lambda}_{flex,f,t} + \tau P^{1-\lambda}_{preset,f,t}]^{\frac{1}{1-\lambda}}. \quad (3.20)$$

Since we have CES sub-utility functions, the market clearing condition can be obtained by equating the output with the sum of the demands for Home goods:

$$A_t L_t = \frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t + \frac{1}{2}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t. \quad (3.21)$$

While flexible-price firms will have higher profit than preset-price firms in general, CES sub-utility makes the aggregate profit of each country the same as before:

$$\Pi_t = \frac{1}{2}P_{h,t}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t + \frac{1}{2}S_t P^*_{h,t}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t - W_t L_t, \quad (3.22)$$

$$S_t \Pi^*_t = \frac{1}{2}S_t P^*_{f,t}\left(\frac{P^*_{f,t}}{P^*_t}\right)^{-\omega} C^*_t + \frac{1}{2}P_{f,t}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega} C_t - S_t W^*_t L^*_t. \quad (3.23)$$

We assume that

$$m_{t+1} = m_t + v^m_{t+1}, \qquad m^*_{t+1} = m^*_t + v^{m^*}_{t+1}, \quad (3.24)$$

$$a^W_{t+1} = \vartheta_W a^W_t + v^W_{t+1}, \qquad a^R_{t+1} = \vartheta_R a^R_t + v^R_{t+1}, \quad (3.25)$$

where $\vartheta_W \in [0,1)$, $\vartheta_R \in [0,1)$ are degrees of persistence in world and relative technology levels and where v^x_t ($x = m, m^*, W, R$) are i.i.d. shocks. We denote $\ln(X_t)$ as x_t, the world variables as $x^W_t = \frac{1}{2}x_t + \frac{1}{2}x^*_t$, and the relative variables as $x^R_t = x_t - x^*_t$. We assume

$Ev_{t+1}^{m} = Ev_{t+1}^{m^*} = \frac{1}{2}\sigma_m^2$, so that $E_t(M_{t+1}^{-1}) = M_t^{-1}$ as mentioned above. We assume also $\mathrm{var}(v^m) = \mathrm{var}(v^{m*}) = \sigma_m^2$, and $\mathrm{cov}(v^m, v^{m*}) = \sigma_{m,m^*}$, $\mathrm{var}(v^W) = \sigma_W^2$, $\mathrm{var}(v^R) = \sigma_R^2$, and $\mathrm{cov}(v^W, v^R) = 0$. We assume initial symmetry between Home and Foreign: that is, $a_0^R = 0$, and $m_0^R = 0$. [24]

C. Solution of the Dynamic Model

To solve the model, we use approximations similar to those in the static model. The Appendix presents the solution to the model. There, the equilibrium is defined and solutions for all the endogenous variables are given. It shows that the equilibrium conditions are satisfied for those solutions. The derivation of the solution is extremely algebra intensive. Here we discuss the salient features of the solution.

An important feature of the solution is that we are able to replicate the allocation achieved when a full set of nominal bonds are traded in the linearly approximated model. We have two kinds of assets (equities and forward currency contracts) that span the space generated by a_t^R and m_t^R. In that case, we have

$$\rho(c_t - c_t^*) = s_t + p_t^* - p_t. \qquad (3.26)$$

This equation is the familiar condition that arises when there is a full set of contingent claims but in which consumer price levels are not equal (see, for example, Chari, Kehoe and Mcgrattan (2002).) Pushing the time subscripts one period forward and taking expectations at time t, we get

$$E_t(c_{t+1}) = E_t(c_{t+1}^*). \qquad (3.27)$$

This equation follows because prices are sticky for at most one period, so purchasing power parity holds in expectation.

Equation (3.27) demonstrates a key sort of stationarity that emerges from our dynamic solution. Even though consumption levels might differ between Home and Foreign households at any time, looking forward they are always expected to be equal. That follows because, as we show in the Appendix (where ζ is defined for the dynamic model),

$$\zeta v_t^R + (1-\zeta)h_t^R - s_t = 0. \qquad (3.28)$$

This equation means that relative total wealth, which is the sum of financial wealth and human capital, is equalized between Home and Foreign households. To be clear, V_t is defined as the value of equities that the Home household acquires at time t and carries into

[24] We assume that productivity shocks follow stationary processes. This is for convenience so that real variables have unconditional means—around which we log-linearize some equations. It would change nothing in our analysis, but be more notationally burdensome, if we allowed the world productivity shock to have a unit root. Then real variables defined in "effective" units would be stationary.

period $t+1$, and H_t is the expected value at time t of returns to work from $t+1$ onward. So equation (3.28) says that the wealth levels of Home and Foreign households at the end of period t are equal.

This equality of wealth occurs even though in equilibrium Home and Foreign households hold different equity portfolios. Since the conditionally expected return on equities depends on the realization of shocks, $v_t^R \neq 0$ in general. That is, the conditional expectations of discounted payoffs on the Home and Foreign equity portfolios differ. In addition, $h_t^R \neq 0$. The value of human capital for Home and Foreign households also depends on the realization of shocks, and so they are not in general equal.

Why then is relative total wealth equal? Suppose there is a positive relative technology shock, $a_t^R > 0$, but no change in world productivity so that Home productivity rises and Foreign productivity falls. Hold monetary shocks equal to zero. In this case, we can show that neither Home nor Foreign consumption levels will be changed by the a_t^R shock in equilibrium, which is convenient for this example.

Period t wage income of Home workers falls when prices are sufficiently sticky, and period t wage income of Foreign workers rises, as in the static model. The period t profits of Home firms rise and period t profits of Foreign firms fall. The current income of Home relative to Foreign might rise or fall. On the one hand, Home's relative labor income falls, but the profits Home households reap may be greater than that of Foreign households when there is home bias in equity holdings. Nonetheless, under the parameter configuration that delivers home bias, the overall income of Home falls relative to Foreign - the relative loss in wage income must outweigh any relative gain in profit income.

But, in this situation in which home bias arises, the relative decline in current income for Home is precisely offset by the gains Home gets in the value of its human wealth and the gain in the value of the equities that it carries into period t. The positive realization of a_t^R pushes up Q_t relative to Q_t^* and H_t relative to H_t^*. Home's total wealth - the sum of the income it receives in period t from labor and profits, plus the value (after the realization of a_t^R) of the equity position it carries into period t, plus the value of its human wealth - is unchanged relative to Foreign. Since consumption levels are not affected by a_t^R shocks, the relative wealth of Home and Foreign at the end of period t is unchanged.

As a result of this stationarity, we show that δ_t and γ_t are constant over time:

$$\delta \equiv \delta_t = \frac{1}{2}(\frac{1}{\rho}-1)\tau, \quad (3.29)$$

$$\gamma \equiv \gamma_t = \gamma_t^* = \frac{1}{2}\frac{\text{A}}{\zeta\text{B}+(1-\zeta)\text{A}}, \qquad (3.30)$$

where $\mathrm{A} \equiv (\omega - 1)[\frac{1-\tau}{1+\omega(1-\tau)\psi} + \frac{1}{\omega\psi + 1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}]$ and $\mathrm{B} \equiv \frac{\tau}{1+\omega(1-\tau)\psi}$.

The share of the equity portfolio held in foreign assets, γ, clearly is increasing in A, decreasing in B. Demand is also decreasing in ζ when $\mathrm{B} > \mathrm{A}$. In order to have home bias, or $\gamma < \frac{1}{2}$, we generally need $\mathrm{B} > \mathrm{A}$, which implies[25]

$$\frac{1-\omega(1-\tau)}{1+\omega(1-\tau)\psi} - \frac{\omega - 1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R} > 0. \quad (3.31)$$

Notice that the condition (3.31) does not depend on ρ or ζ, while ζ determines the level of home bias. There are intuitive explanations for how most of these parameters affect foreign equity demand.

As labor's share, ζ, rises, γ falls when there is home bias ($\mathrm{B} > \mathrm{A}$), and rises when there is anti-home-bias ($\mathrm{B} < \mathrm{A}$). The intuition is straightforward given our discussion above: When the short-run effects that lead to a negative covariance of Home profits and labor income are sufficiently large that there is home bias, the home bias is amplified the larger is labor's share. The benefits from hedging labor income risk are greater when labor's share is greater. But when the long-run effects dominate, and returns to human capital are hedged by having a foreign-equity bias, the effect is again amplified the larger is labor's share.

Next it is helpful to consider two special cases. $\beta\vartheta_R$ is, in a sense, a measure of the weight the future receives in the portfolio allocation decision. $\beta\vartheta_R$ is large when households place a high weight on the future, and when the relative productivity shocks have a very persistent influence. In the extreme case when all prices are sticky ($\tau = 1$) and the future does not matter ($\beta\vartheta_R = 0$), there is complete home bias ($\gamma = 0$.) This actually is just the static model we examined previously—that assumed full price stickiness and placed no weight on the future.

On the other hand, if all goods prices were flexible, $\tau = 0$, then the optimal equity portfolio is $\gamma = \frac{1}{2}\frac{1}{1-\zeta} > \frac{1}{2}$. This outcome is similar to the theoretical result obtained by Baxter and Jermann (1997)—"the international diversification puzzle is worse than you think."

Increasing price stickiness implies a larger value of τ—a greater fraction of firms set price in advance. A larger τ makes it more likely that the condition (3.31) for home bias is met. γ is decreasing in τ, when $\omega > 1$—which can be seen directly because an increase in τ raises

[25] We omit the case in which the denominator in equation (3.30) is non-positive: this case can happen only if the price is very flexible and $\omega \le 1$.

B and lowers A. We find, then that increasing price stickiness leads to greater home bias in equity holdings.

When $\omega > 1$, an increase in $\beta\vartheta_R$ leads to an increase in A, which implies a greater share of Foreign equities in the Home household's portfolio. In short, the more the future "matters", the larger the share of Foreign equities. In the limit, as $\beta\vartheta_R \to 1$, the portfolio approaches the flexible price value, $\gamma = \frac{1}{2}\frac{1}{1-\zeta}$. On the other hand, as $\beta\vartheta_R \to 0$, the portfolio approaches $\gamma = \frac{1}{2}\frac{(\omega-1)(1-\tau)}{\tau\zeta+(1-\zeta)(\omega-1)(1-\tau)}$. This latter value is precisely the level γ would take in the static model if a fraction τ of prices were preset.

When $\omega = 1$, the terms of trade adjustment insures against the effects on relative wealth from productivity shocks.[26] The share of Home or Foreign goods in consumption expenditure does not change because of the Cobb-Douglas sub-utility function. Hence, households care only about the distribution between labor and firms, as is the case in the static model. Therefore, we get 100 percent home bias: $\gamma = 0$. If $\omega = 1$, and all prices are flexible, then γ is indeterminate. This is similar to the models by Cole and Obstfeld (1991) and Obstfeld and Rogoff (2002), in which asset trade is not needed because of the Cobb-Douglas specification for the consumption index of Home and Foreign goods.

On the other hand, if β is close to one and $\tau = 1$, then ω, the elasticity of substitution between Home and Foreign goods, plays an important role. The technology shock will have a significant impact once prices adjust if Home and Foreign goods are substitutes for one another. When Home receives a negative technology shock, the demand for Home goods shifts to Foreign goods after prices are adjusted. This fall in demand for Home goods implies that Home firms will cut their labor inputs. In order to hedge against this employment risk, a Home household wants to have Foreign equities because Foreign firms will generate more profit than will Home firms suffering from the negative technology shock. Thus, sticky prices lead to home bias, as we have seen in static model, while flexible prices lead to foreign bias. If the effect from price stickiness is bigger, then home bias will be optimal. Under flexible prices, a positive technology shock enables firms to produce goods more cheaply and to sell them more cheaply so that nominal sales will increase if $\omega > 1$. Although the demand for labor will decrease from the direct effect of the technology shock, the demand for goods will increase and thus indirectly increase the demand for labor.

D. Properties of the Model

We can calibrate the amount of home bias implied by the model. Although the model is not realistic enough to capture some features of the macroeconomy, it still worthwhile to get a

[26] Heathcote and Perri assume $\omega = 1$, but assume that there is investment in capital and trade only in equities, and find that home bias can arise even with flexible goods prices.

sense of the magnitude of home bias implied by the solution in equation (3.31). The share of the Home household's equity portfolio held in foreign shares, γ, depends on the price stickiness parameter, τ ; labor's share, ζ ; the elasticity of substitution between Home and Foreign aggregates, ω; the discount factor, β; the persistence of relative productivity shocks, ϑ_R; and, the elasticity of labor supply, ψ.

We set $\tau = 1$ and then calibrate the length of a period by using estimates of the speed of price adjustment. With $\tau = 1$, the half-life of price adjustment is one-half of a period. In our model, the speed of price adjustment determines the rate of convergence toward purchasing power parity. Rogoff (1996) has noted that studies of purchasing power parity imply a half-life of the real exchange rate of 3-5 years. We will pick a much smaller half-life of 1 year, which is below the lower end of the range cited by Rogoff. This implies that one period is equal to two years.

Following Backus, Kehoe and Kydland (1992), we set $\zeta = 2/3$. The estimates of Backus, Kehoe and Kydland (1992) give us on quarterly data that the autocorrelation of relative productivity shocks is 0.855, so we set $\vartheta_R = (0.855)^8 \approx 0.286$. Likewise, the quarterly discount factor in Backus et al. is 0.99, so we take $\beta = (0.99)^8 \approx 0.923$. We follow Backus, Kehoe and Kydland (1994), and Chari, Kehoe and Mcgrattan (2002) and set $\omega = 1.5$. We follow Obstfeld and Rogoff (2002), and Bergin (2004) and set $\psi = 1$.

With this baseline set of parameters, we find $\gamma \approx 0.052$. That is, the model is capable of explaining a substantial amount of home bias. The model is perfectly symmetric between Home and Foreign countries, so an unbiased portfolio would be $\gamma = 0.5$.

In our model, negative conditional correlation between labor hours and productivity conditioning on productivity shock is the key driving force for home bias. However, because households can hedge demand shock through forward contracts, the unconditional correlation can be positive. It is important to distinguish between conditional and unconditional correlation in our model.

Gali (1999) has addressed precisely this issue. He has noted that real business cycle models tend to imply a positive correlation between hours and productivity. He shows in a simple closed-economy New Keynesian macroeconomic model that there is a negative correlation between hours and output per worker when there is a productivity shock. The reasoning is much the same as that in our model.

Gali goes on to derive empirical support for this implication of sticky-price models. He estimates a structural bivariate VAR on total labor hours and labor productivity using U.S. data.[27] The model was estimated on quarterly data from 1948:I to 1994:IV. There are two

[27] He also uses employment instead of labor hours, and finds the same result holds for all G-7 countries except Japan.

types of shocks in the model, which Gali classifies as technology shocks and non-technology shocks. The non-technology shocks can be associated with aggregate demand shocks. Under his identification scheme, only technology shocks can permanently increase labor productivity.

Gali finds that the conditional correlation between labor hours and productivity is negative for technology shocks, while the unconditional correlation is positive. Rotemberg (2003) finds similar results. If prices were flexible, in traditional real business cycle models, the correlation conditional on technology shocks would be positive—as it is in our model in the long run.

Gali's findings have not gone unchallenged.[28] Christiano, Eichenbum and Vigfusson (2003) substitute labor hours per capita for Gali's total labor hours and reverse Gali's finding on the conditional correlation. However, Francis and Ramey (2003) use the same measure, but quadratically detrended, and find the negative correlation between hours per capita and productivity conditional on technology shocks. Gali, Lopez-Salido and Valles (2003) find a similar result, using first-differences in hours per capita. Francis and Ramey (2004) create a new measure of hours per capita and confirm that a positive technology shock will reduce labor hours in the short run. While there is no consensus yet on the sign of the conditional correlation, there is some significant empirical support for the contention that it is negative.

Home bias does not require that the unconditional correlation of returns to human capital and returns to domestic equity be positive for two reasons: First, as we note above, productivity shocks may have a low variance relative to monetary shocks, but it is the covariance holding monetary shocks constant that matters for home bias. Second, it is the correlation of returns to human capital with the *relative* Home to Foreign equity returns that matters for productivity. If Home and Foreign productivity shocks are highly correlated, there may be home bias even when the conditional correlation of human capital returns and domestic equity returns is high. This is illustrated in the following table. Here, we use the parameter values above ($\zeta = 2/3$, $\vartheta_R = 0.286$, $\beta = 0.923$, $\omega = 1.5$, $\psi = 1$) which imply that the share of Foreign equities in the Home equity portfolio is equal to 0.052. In addition, we set the coefficient of relative risk aversion, ρ, equal to five, and set the autocorrelation of the world productivity shock $\vartheta_W = 0.75$. (Recall, a time period is equal to two years, so this implies an annual serial correlation of 0.866.)

[28] See Gali and Rabanal (2005) for details.

Correlation between the Return on Domestic Equities and Human Capital (under Different Assumptions on Technology Shocks)

	Standard Deviation of Home Productivity Shock Relative to Home Monetary Shock						
$cor(a_t,a_t^*)$	0.01	0.5	1	2	4	8	100
0.00	0.990	0.959	0.875	0.614	0.117	-0.314	-0.569
0.25	0.990	0.960	0.878	0.625	0.137	-0.296	-0.560
0.50	0.990	0.961	0.882	0.637	0.159	-0.287	-0.579
0.75	0.990	0.962	0.886	0.651	0.186	-0.290	-0.656
0.99	0.990	0.963	0.889	0.666	0.219	-0.317	-0.960

As the table indicates, these parameter values do not necessarily imply a negative correlation of returns to human capital and Home equities. Only when the standard deviation of productivity shocks is large relative to the standard deviation of monetary shocks do we find that the unconditional correlation must be negative.

The implication of our model for the Foreign equity share in the Home portfolio is sensitive, of course, to the assumption about the half-life of prices. Using the same values for the parameters of the utility function as above, we can calculate the foreign equity share for various values of the half-life of price adjustment:

Foreign Equity Share under Alternative Calculations of Speed of Price Adjustment

Half-life (quarters)	Foreign equity share	
(½-life when $\tau = 0.8$)	$\tau = 1$	$\tau = 0.8$
0.5 (0.375)	0.533	0.734
1 (0.75)	0.303	0.482
2 (1.5)	0.143	0.284
4 (3)	0.052	0.162
6 (4.5)	0.023	0.121
8 (6)	0.011	0.104

The second column shows that there is a substantial amount of home bias, even when the half-life of price adjustment is fairly rapid. If the half-life is 2 quarters, the foreign equity share is 14.3%, and it is only 30.3% when the half-life is 1 quarter. When the half-life shrinks to a half of a quarter, then we see the anti-home bias result of Baxter and Jermann (1997).

In these calculations, we have assumed that all prices are set in advance ($\tau = 1$), and adjust after one period. The half-life of price adjustment is then a half period, and we calibrate the model by setting the length of a period equal to twice the half-life of price adjustment. An alternative way to examine the effects of different degrees of price stickiness is to vary the fraction of firms that set prices in advance. In the third column of this table, we set $\tau = 0.8$. The implied aggregate half-life is reported in parentheses in the left-hand column.

IV. CONCLUSION

Our model provides a general equilibrium analysis of the factors that determine equilibrium portfolio choice in a dynamic setting. In both the static and dynamic models, the allocation in the linearly approximated model replicates the one achieved when a full set of state-contingent bonds is traded. This is in a sense a shortcoming of our model since this allocation leaves other puzzles unsolved—the high volatility of the observed exchange rate or the consumption-real exchange rate anomaly as described in Chari et al. (2002).

One possible way of extending our model may help to explain the anomalous behavior of real exchange rates and consumption, while maintaining our mechanism for home bias. Julliard (2004) argues, in a partial-equilibrium setting, that credit constraints (specifically, a constraint that prevents short selling of equities or bonds), may lead to substantial home bias when returns to human capital and relative equity returns are negatively correlated. His argument is that unconstrained households would prefer a portfolio weighted toward home equities for reasons similar to the ones discussed in this paper (though he takes as given the source of this negative correlation, rather than deriving it from a model.) Credit constrained households would like to go short in some assets. During the life-cycle of these households, they may move to a position in which they hold positive amounts of equities. Julliard demonstrates that these households that are just emerging from the credit constraint have a strong incentive to diversify their labor income risk, which they would do by acquiring a portfolio that is strongly biased toward domestic equities.

If such a model were embedded in a general equilibrium framework, the very tight link between the real exchange rate and relative consumption levels implied by our model would be broken. However, such a model would be much more difficult to solve (even numerically), and it is unlikely that one could obtain a closed-form solution for the foreign equity share such as our equation (3.30).

Although our model provides a theoretical foundation for home bias, we believe other factors, such as information costs, play important roles. The economic forces that lead to home bias in our model do not require the exclusion of other considerations that have been raised in the literature. We have not built a model that is intended to explain home bias, because it does not include any features that are designed explicitly to deliver home bias. Instead, we have found that home bias is a natural outcome in a symmetric model in which output is demand determined to some extent and claims to labor income are not traded. The model can be solved analytically in a straightforward way, and extensions of this framework may prove useful in examining other questions in international finance, such as the role of valuation effects in external adjustment; the effects of portfolio adjustment on macroeconomic and current account adjustment, the relationship between movements in stock prices and exchange rates, and so forth.

Table 1: List of Notations

β	Discount factor
ρ	Risk aversion parameter
χ	Real balance parameter
ψ	Labor supply parameter
ω	Elasticity of substitution between Home and Foreign goods
λ	Elasticity of substitution among Home goods and Foreign goods
ϑ_R, ϑ_W	Persistence of the technology shocks
ζ	Labor's share in national income
γ_h	Home equity share in the equity portfolio
$\gamma = \gamma_f$	Foreign equity share in the equity portfolio
$\tilde{\delta}$	Number of forward contracts
δ	Normalized number of forward contracts
τ	Ratio of firms setting price in advance in the dynamic model
Π_t	Nominal profit of Home firms = dividend
A_t	Productivity
C_t	Consumption basket
$C_{h,t}$	Consumption aggregate of Home-produced goods
$C_{f,t}$	Consumption aggregate of Foreign-produced goods
$C_{h,t}(i)$	Consumption of Home-produced good of variety *i*
$C_{f,t}(i)$	Consumption of Foreign-produced good of variety *i*
F_t	Forward rate for delivery at time t (set at $t-1$)
H_t	Value of human capital
L_t	Employment
M_t	Money balances
P_t	Price of consumption basket
$P_{h,t}$	Price of consumption aggregate of Home-produced goods
$P_{f,t}$	Price of consumption aggregate of Foreign-produced goods
$P_{h,t}(i)$	Price of consumption of Home-produced good of variety *i*
$P_{f,t}(i)$	Price of consumption of Foreign-produced good of variety *i*
Q_t	Price of home equity
R_t	Return on home equity
R_t^H	Return on human capital
S_t	Nominal exchange rate
Tr_t	Transfer from government
V_t	Value of equity portfolio
W_t	Nominal wage rate
Y_t	Output of Home goods

A. Solution of the Dynamic Model

An equilibrium satisfies the first order conditions, budget constraint and market clearing conditions. First we define an equilibrium formally. Then we will list the linearized first order conditions and redefine equilibrium in linearized form.

Definition A

An *equilibrium* is a set of sequences[29] $\{C_t, L_t, W_t, \tilde{\delta}_t, \gamma_t, C_{h.t}, C_{f.t}, C_{h.t}(i), C_{f.t}(i), P_{flex,h,t},$

$P_{flex,f,t}, P_{preset,h,t}, P_{preset,h,t}, P_t, P_{h,t}, P_{f,t}, Q_t, V_t, H_t, R_t, R_t^H, \Pi_t, \gamma_{h,t}, \gamma_{f,t}\}_{t=1}^{\infty}$ and their foreign counterparts and $\{S_t, F_t\}$, which solves the system of 50 equations[30] consisting of (2.5), (2.8), (2.9), (3.3), (3.4), (3.7)-(3.22), and their foreign counterparts plus 3 asset markets clearing conditions,[31] given stochastic sequences $\{A_t, A_t^*, M_t, M_t^*\}$ and initial conditions $A_0 = A_0^*$, $M_0 = M_0^*$, $\gamma_0 = 0$, and $\gamma_0^* = 0$.

Approximated System

In this section, we derive a log-linear version of the model, under the assumption that the stochastic driving variables (productivity and money) are lognormally distributed. Many of the equations of the model are linear in logs (without any approximation). But some of the equations in the model (the budget constraint for households, the definition of profits for the firms, and the market clearing conditions) are log-linearized around unconditional means. It is immediately apparent that our assumptions of stationary productivity processes and unit-root monetary processes imply that nominal variables have unit roots and real variables are stationary. So we log-linearize around the unconditional means of real variables.[32]

In some of the log-linearized equations below, the algebra is simplified considerably if we use the result that $\overline{p_h - p} = 0$. (In our notation, $\overline{x}$ represents the unconditional mean of x_t.) While we could proceed with the derivations without using this result, and then verify in the

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[29] There are $24 \times 2 + 2$ variables.

[30] The number of equations should be 51, but one is redundant by Walras' Law.

[31] $\gamma_{h,t} + \gamma_{h,t}^* = 1$, $\gamma_{f,t} + \gamma_{f,t}^* = 1$, and $F_t \tilde{\delta}_t = \tilde{\delta}_t^*$.

[32] We could easily accommodate unit-root processes in productivity. Then real variables expressed in "efficiency units" would be stationary. However, there is no real gain from this generalization, so we maintain stationary productivity shocks to simplify the algebra.

solutions that this result is true, it is easier to demonstrate this first and use it in some of the log-linearizations.

First, in the definition of profits for the home firm, divide both sides of equation (3.22) by P_t, then evaluate the equation at the point of expansion for the log-linearization:

$$\exp(\overline{\pi - p}) = \exp(\overline{c})[\tfrac{1}{2}\exp((1-\omega)(\overline{p_h - p})) + \tfrac{1}{2}\exp((1-\omega)(\overline{p_h^* - p^*}))] - \exp(\overline{w - p} + \overline{l}).$$

Here we have used symmetry to give us $\overline{c} = \overline{c^*}$ and $\overline{s + p^* - p} = 0$.

Divide the budget constraint (3.1) by P_t, then evaluate the equation at the point of expansion for the log-linearization:

$$\exp(\overline{\pi - p}) = \exp(\overline{c}) - \exp(\overline{w - p} + \overline{l}).$$

In deriving this expression, we have used symmetry to give us $\overline{s + q^* - q} = 0$, $\overline{s + \pi^* - \pi} = 0$, and $\overline{s - f} = 0$. We have also used $\gamma_{f,t} + \gamma_{h,t} = 1$ and $M_t = M_{t-1} + Tr_t$.

Now comparing the two equations we have derived, we must have

$$\tfrac{1}{2}\exp((1-\omega)(\overline{p_h - p})) + \tfrac{1}{2}\exp((1-\omega)(\overline{p_h^* - p^*})) = 1.$$

This can be written as

$$\tfrac{1}{2}\exp((1-\omega)(\overline{p_h - p})) + \tfrac{1}{2}\exp(-(1-\omega)(\overline{p_h - p})) = 1,$$

where we have used symmetry to give us that $\overline{p_h^* - p^*} = \overline{p_f - p}$, and linearized (2.5) to get $\overline{p_h - p} = -(\overline{p_f - p})$. It then follows that $\overline{p_h - p} = 0$, which is the result we will use below to simplify some of the log-linearizations.

A few more notational conventions: We denote $\hat{x}_t$ as the deviation from the conditional mean–that is, $\hat{x}_t \equiv x_t - E_{t-1}x_t$ and $\hat{E}x_{t+s} = E_t \ln x_{t+s} - E_{t-1} \ln x_{t+s}$. We will also denote the world variables as $x_t^W \equiv \frac{1}{2}x_t + \frac{1}{2}x_t^*$ and the relative variables as $x_t^R \equiv x_t - x_t^*$.

The first order conditions for households

Suppressing constant terms and taking logs, the first order condition for consumption (3.14) can be written as

$$c_t = \frac{1}{\rho}(m_t - p_t). \qquad \text{(A.1)}$$

Using equation (A.1), equation (3.3) can be expressed as

$$\psi l_t = -m_t + w_t. \qquad \text{(A.2)}$$

Some of the equations of the model are log-linear (such as (A.1) and (A.2)), and therefore, in the presence of lognormal distributions, offer exact solutions. But others (such as the budget constraint, the market clearing condition, and the expression for a firm's profits) require approximations. Because all shocks are lognormal, the solution of the approximated model will take on a lognormal distribution. We can use equation (3.14) to express (3.4) as

$$E_{t-1}(s_t)+\frac{1}{2}\text{var}_{t-1}(s_t)-\text{cov}_{t-1}(m_t,s_t)=f_t, \qquad \text{(A.3)}$$

$$E_{t-1}(r_t-(m_t-m_{t-1}))-\text{cov}_{t-1}(m_t,r_t)+\frac{1}{2}\text{var}_{t-1}(r_t)+\frac{1}{2}\text{var}_{t-1}(m_t)=0 \qquad \text{(A.4)}$$

$$\begin{aligned}&E_{t-1}(r_t^*+s_t-s_{t-1}-(m_t-m_{t-1}))+\frac{1}{2}\text{var}_{t-1}(r_t^*)+\frac{1}{2}\text{var}_{t-1}(m_t)+\frac{1}{2}\text{var}_{t-1}(s_t)\\ &\quad -\text{cov}_{t-1}(m_t,r_t^*)+\text{cov}_{t-1}(s_t,r_t^*)-\text{cov}_{t-1}(m_t,s_t)=0\end{aligned} \qquad \text{(A.5)}$$

The budget constraint

We log-linearize the budget constraint (3.13) to get

$$\begin{aligned}&p_t+c_t+\frac{\beta}{1-\beta}(1-\zeta)v_t+\frac{\beta}{1-\beta}\zeta h_t\\ &=\frac{1}{1-\beta}(1-\zeta)\{v_{t-1}+r_t-\gamma_t(r_t^R-\hat{s}_t)\}+\frac{1}{1-\beta}\zeta(h_{t-1}+r_t^H)+\delta_t(s_t-f_t)\end{aligned} \qquad \text{(A.6)}$$

Here, $\zeta\equiv\frac{\exp(\overline{w-p+l})}{\exp(\overline{c})}$, and $\delta_t\equiv\frac{\tilde{\delta}_tF_t}{M_{t-1}}\exp(\overline{m-p-c})$. In deriving this expression, we have used the fact that by symmetry, $\overline{v-p}=\overline{q-p}$, and then use equation (3.7) to derive $\exp(\overline{q-p})=\frac{\beta}{1-\beta}\exp(\overline{\pi-p})$. Similarly, from equation (3.9), we get $\exp(\overline{h-p})=\frac{\beta}{1-\beta}\exp(\overline{w-p+l})$. Then, evaluating the budget constraint at the point of expansion, we have $\exp(\overline{c})=\exp(\overline{w-p+l})+\exp(\overline{\pi})$.

The first order conditions for firms

Firms set their prices optimally. The first order conditions can be written as

$$p_{flex,h,t}=w_t-a_t, \qquad \text{(A.7)}$$

$$p_{flex,f,t}=(w_t^*-a_t^*+s_t), \qquad \text{(A.8)}$$

$$p_{preset,h,t}=E_{t-1}(w_t-a_t)+\frac{1}{2}Var_{t-1}(w_t-a_t)+Cov_{t-1}(w_t-a_t,\tilde{d}_t+(\lambda-\omega)p_{ht}+\omega p_t+c_t)$$

(A.9)

$$\begin{aligned}&p_{preset,h,t}=E_{t-1}(w_t-a_t-s_t)+\frac{1}{2}Var_{t-1}(w_t-a_t)-\frac{1}{2}Var_{t-1}(s_t)\\ &\qquad Cov_{t-1}(w_t-a_t-s_t,\tilde{d}_t+(\lambda-\omega)p_{ht}+\omega p_t+c_t^*)\end{aligned} \qquad \text{(A.10)}$$

Note that the conditional second moments in (A.9) and (A.10) are all constant over time, and will be treated as constant terms in subsequent linearizations.

Thus, the prices of each category of goods (3.19 and 3.20) can be expressed as following:

$$p_{h,t} = \tau p_{preset,h,t} + (1-\tau) p_{flex,h,t}\,, \qquad (A.11)$$

$$p_{f,t} = \tau p_{preset,f,t} + (1-\tau) p_{flex,f,t}\,. \qquad (A.12)$$

Combining these two and suppressing the constants, we get the expression for price index:

$$p_t = \frac{1}{2} p_{h,t} + \frac{1}{2} p_{f,t}\,. \qquad (A.13)$$

Goods market clearing

The goods market clearing condition, equation (3.21) can be linearized as

$$l_t = \frac{1}{2}\{-\omega(p_{h,t} - p_t) + c_t\} + \frac{1}{2}\{-\omega(p_{h,t}^* - p_t^*) + c_t^*\} - a_t\,. \qquad (A.14)$$

Other definitions

In rewriting the budget constraint (3.13), we introduced human capital. Linear zing (3.9) gives us

$$h_t = \frac{1-\beta}{\beta} \sum_{s=1}^{\infty} E_t \beta^s (w_{t+s} + l_{t+s})\,. \qquad (A.15)$$

Using the definition of R_t in equation (3.10), and the solution for Q_t in equation (3.7), we can write

$$r_t = (1-\beta)E_t\left(\sum_{s=0}^{\infty} \beta^s \pi_{t+s}\right) - (1-\beta)E_{t-1}\left(\sum_{s=0}^{\infty} \beta^s \pi_{t+s}\right) = (1-\beta)\sum_{s=0}^{\infty}(\beta^s \hat{E}_t \pi_{t+s})\,. \qquad (A.16)$$

The log of home firms' profits comes from linear zing (3.22):

$$\pi_t = \frac{1}{1-\zeta}\left[c_t^W + p_t^W + \frac{1}{2}s_t + \frac{1}{2}(1-\omega)(p_{h,t} - p_t) + \frac{1}{2}(1-\omega)(p_{h,t}^* - p_t^*) - \zeta(w_t + l_t)\right].$$

Similarly,

$$r_t^H = (1-\beta)\sum_{s=0}^{\infty}(\beta^s \hat{E}_t (w_{t+s} - l_{t+s}))\,. \qquad (A.17)$$

B. Definition of Approximated Equilibrium

Definition B

An approximated equilibrium is a set of sequences $\{c_t, l_t, w_t, r_t, r_t^H, \delta_t, \gamma_t, p_t, v_t, h_t\}$ and their foreign counterparts, and $\{s_t, f_t\}$ that solve the system of equation (A.1)-(A.6), (A.14)-(A.17), and their foreign counterparts, given sequences $\{m_t, m_t^*, a_t, a_t^*\}$ and initial conditions $a_0^R = 0$, $m_0^R = 0$, and $\gamma_0 = \gamma_0^* = 0$. An approximated equilibrium is a reduced form of *Definition A*. Most omitted part can be easily verified and should not be confusing. We present the solutions for x_t and x_t^* in the form of solutions for x_t^R and x_t^W to facilitate the demonstration that these satisfy the equilibrium conditions.

C. Equilibrium Allocation

We conjecture that the following allocation is an equilibrium.

$$\begin{aligned} l_t^R &= \frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi}\frac{\psi+1}{1+\omega\psi} E_{t-1} a_t^R \\ &= \frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi}\frac{\psi+1}{1+\omega\psi} \vartheta_R a_{t-1}^R \end{aligned} \quad \text{(A.18)}$$

$$\begin{aligned} l_t^W &= \frac{1}{\rho+(1-\tau)\psi}\left\{(1-\tau-\rho)a_t^W + \tau m_t^W + \tau E_{t-1}[\frac{\rho(\psi+1)}{\rho+\psi} a_t^W - m_t^W]\right\} \\ &= \frac{1}{\rho+(1-\tau)\psi}\left\{(1-\tau-\rho)a_t^W + \tau m_t^W + \tau[\frac{\rho(\psi+1)}{\rho+\psi} \vartheta_W a_{t-1}^W - m_{t-1}^W]\right\} \end{aligned} \quad \text{(A.19)}$$

$$w_t^R = \psi\left\{\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi}\frac{\psi+1}{1+\omega\psi}\vartheta_R a_{t-1}^R\right\} + m_t^R \quad \text{(A.20)}$$

$$w_t^W = \frac{\psi}{\rho+(1-\tau)\psi}\left\{(1-\tau-\rho)a_t^W + \tau\left[\frac{\rho(\psi+1)}{\rho+\psi}\vartheta_W a_{t-1}^W - m_{t-1}^W\right]\right\} + \frac{\rho+\psi}{\rho+(1-\tau)\psi} m_t^W \quad \text{(A.21)}$$

$$p_t^R = \tau m_{t-1}^R + (1-\tau)m_t^R \quad \text{(A.22)}$$

$$p_t^W = -\frac{\rho\tau}{\rho+(1-\tau)\psi}\left[\frac{\rho(\psi+1)}{\rho+\psi}\vartheta_W a_{t-1}^W - m_{t-1}^W\right] - (1-\tau)\frac{\rho+\psi}{\rho+(1-\tau)\psi}\left[\frac{\rho(\psi+1)}{\rho+\psi} a_t^W - m_t^W\right]$$
(A.23)

$$c_t^R = \frac{1}{\rho}\tau(m_t^R - m_{t-1}^R) \quad \text{(A.24)}$$

$$c_t^W = \frac{\tau}{\rho+(1-\tau)\psi}\left[\frac{\rho(\psi+1)}{\rho+\psi}\vartheta_W a_{t-1}^W + (m_t^W - m_{t-1}^W)\right] - (1-\tau)\frac{1+\psi}{\rho+(1-\tau)\psi} a_t^W \quad \text{(A.25)}$$

$$s_t = m_t^R. \quad \text{(A.26)}$$

$$f_t = m_{t-1}^R. \quad \text{(A.27)}$$

$$r_t^R = (1-\beta)(\psi+1)\left[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]\hat{a}_t^R + \hat{m}_t^R$$
(A.28)

$$r_t^W = (1-\beta)(\psi+1)\left[\frac{(1-\tau)(1-\rho)}{\rho+(1-\tau)\psi} + \frac{\zeta}{1-\zeta}\frac{\tau\rho}{\rho+(1-\tau)\psi} + \frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W}{1-\beta\vartheta_W}\right]\hat{a}_t^W \quad \text{(A.29)}$$

$$r_t^{H^R} = (1-\beta)(\psi+1)\left[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]\hat{a}_t^R + \hat{m}_t^R \quad \text{(A.30)}$$

$$r_t^{H^W} = (1-\beta)(\psi+1)\left[\frac{1-\tau-\rho}{\rho+(1-\tau)\psi} + \frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W}{1-\beta\vartheta_W}\right]\hat{a}_t^W + \left[\frac{(1-\beta)(\psi+1)\tau}{\rho+(1-\tau)\psi}+1\right]\hat{m}_t^W$$

(A.31)

$$\delta \equiv \delta_t = \frac{1}{2}(\frac{1}{\rho}-1)\tau \qquad \text{(A.32)}$$

$$\gamma \equiv \gamma_t = \gamma_t^* = \frac{1}{2}\frac{(\omega-1)\left[\frac{1-\tau}{1+\omega(1-\tau)\psi}+\frac{1}{\omega\psi+1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]}{\frac{\tau\zeta}{1+\omega(1-\tau)\psi}+(1-\zeta)(\omega-1)\left[\frac{1-\tau}{1+\omega(1-\tau)\psi}+\frac{1}{\omega\psi+1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]}$$

(A.33)

$$h_t^R = \frac{1-\beta}{\beta}\left[(\psi+1)\frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}a_t^R\right]+m_t^R \quad \text{(A.34)}$$

$$h_t^W = \frac{1-\beta}{\beta}\left[(\psi+1)\frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_w}{1-\beta\vartheta_w}a_t^W\right]+m_t^W \quad \text{(A.35)}$$

$$v_t^R = \frac{-\zeta}{1-\zeta}\frac{1-\beta}{\beta}\left[(\psi+1)\frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}a_t^R\right]+m_t^R \qquad \text{(A.36)}$$

$$v_t^W = \frac{1-\beta}{\beta}\left[(\psi+1)\frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W}{1-\beta\vartheta_W}a_t^W\right]+m_t^W \quad \text{(A.37)}$$

Notice that this allocation replicates the allocation when a full set of state-contingent bonds is traded:

$$\rho(c_t - c_t^*) = s_t + p_t^* - p_t\,. \qquad \text{(A.38)}$$

D. Proof

We will show this allocation satisfies the equilibrium conditions.

Fundamental Variables

We now prove that the first order conditions for fundamental variables and labor market clearing conditions are in fact satisfied.

It is immediate to confirm that equations (A.18)–(A.21) satisfy equation (A.2). Likewise it is straightforward to check that (A.22)–(A.25) satisfy (A.1).

We can also verify that (A.18), (A.20) and (A.26) satisfy the relative version of the labor market clearing condition (A.14):

$$l_t^R = -(1-\tau)\omega(w_t^R - a_t^R - s_t) - \tau\omega E_{t-1}(w_t^R - a_t^R - s_t) - a_t^R\,. \qquad \text{(A.39)}$$

It is tedious but straightforward to verify that (A.19) and (A.21) satisfy the world version of labor market clearing condition (A.14):

$$l_t^W = c_t^W - a_t^W . \quad \text{(A.40)}$$

Using equations (A.21) and (A.23), and using (A.20) and (A.26), we can show

$$p_t^W = \tau E_{t-1}(w_t^W - a_t^W) + (1-\tau)(w_t^W - a_t^W) \qquad \text{(A.41)}$$

$$p_t^R = \tau E_{t-1}s_t + (1-\tau)s_t \qquad \text{(A.42)}$$

are satisfied. Note that the variance and covariance terms in (A.9) and (A.10) are constant, from the solutions above. Substituting equations (A.7)–(A.12) into (A.13), and suppressing constant terms, we see that (A.37) and (A.38) are the solutions to the world and relative versions of (A.13).

So far, we have proved equations (A.1), (A.2), (A.13), and (A.14) are satisfied.

Returns on assets

In order to show that this allocation in fact satisfies the first order conditions for asset holdings, we want to calculate the rate of return on assets–human capital and equities. Since $w_{t+s} + l_{t+s} = (\psi+1)(l_{t+s}^W + \frac{1}{2}l_{t+s}^R) + m_{t+s}^W + \frac{1}{2}m_{t+s}^R$, the return on the human capital is

$$\begin{aligned} r_t^H &= (1-\beta)\sum_{s=0}^{\infty}\hat{E}_t\beta^s\left[(\psi+1)(l_{t+s}^W + \frac{1}{2}l_{t+s}^R) + m_{t+s}^W + \frac{1}{2}m_{t+s}^R\right] \\ &= (1-\beta)(\psi+1)\left\{\frac{1}{\rho+(1-\tau)\psi}\left[(1-\tau-\rho)\hat{a}_t^W + \tau\hat{m}_t^W\right]\right\} \\ &+(1-\beta)(\psi+1)\left\{\frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W}{1-\beta\vartheta_W}\hat{a}_t^W + \frac{1}{2}\left[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]\hat{a}_t^R\right\} \\ &+(\hat{m}_t^W + \frac{1}{2}\hat{m}_t^R). \end{aligned}$$

(A.43)

Subtracting the foreign counterpart, we get equation (A.30). Adding the foreign counterpart gives us the solution to $r_t^{H^W}$.

Following similar step as in the return on human capital, we get the return on equity:

$$\begin{aligned} r_t = (1-\beta)(\psi+1)&\left\{[\frac{(1-\tau)(1-\rho)}{\rho+(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau\rho\zeta}{\rho+(1-\tau)\psi} + \frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W}{1-\beta\vartheta_W}]\hat{a}_t^W\right. \\ &\left.+\frac{1}{2}[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}]\hat{a}_t^R\right\} \qquad \text{(A.44)} \\ &+\left\{1+\frac{1-\beta}{1-\zeta}\frac{1-\rho-\zeta(\psi+1)}{\rho+(1-\tau)\psi}\tau\right\}\hat{m}_t^W + \frac{1}{2}\hat{m}_t^R \end{aligned}$$

Subtracting the foreign counterpart, we get (A.28), and adding the foreign counterpart gives us that (A.29) is the solution for r_t^W. So, we have confirmed (A.16) and (A.17).

Asset Allocation

Since we replicate complete markets, these allocations should satisfy the first order conditions for the asset allocation as expressed in equations (3.4) and (3.15). We will prove that linearized version of them (A.3)–(A.4) are satisfied. From (A.26) and (A.27), we see $f_t = E_{t-1}s_t$. So, for equation (A.3) to be satisfied, we need

$$\mathrm{cov}_{t-1}(m_t^R, s_t) = \mathrm{var}_{t-1}(s_t), \qquad \text{(A.45)}$$

which follows since $s_t = m_t^R$.

Since from (A.28) and (A.29), r_t is i.i.d., we have $E_{t-1}(r_t - (m_t - m_{t-1}))$ is constant. Likewise, using (A.26), $E_{t-1}(r_t^* + s_t - s_{t-1} - (m_t - m_{t-1}))$ is constant. We can solve directly for these expectations from equations (A.4) and (A.5), using the covariances and variances implied by our solution in (A.18)–(A.33). But the following restriction links (A.4) and (A.5):

$$\mathrm{cov}_{t-1}(-m_t, r_t) + \frac{1}{2}\mathrm{var}_{t-1}(r_t) = \mathrm{cov}_{t-1}(-m_t, s_t + r_t^*) + \frac{1}{2}\mathrm{var}_{t-1}(s_t + r_t^*). \quad \text{(A.46)}$$

We verify this by using $r_t = r_t^W + \frac{1}{2}r_t^R$, and rewrite (A.44) as

$$\mathrm{cov}_{t-1}(m_t, r_t^R - m_t^R) + \frac{1}{2}\mathrm{var}_{t-1}(m_t^R + r_t^W - \frac{1}{2}r_t^R) - \frac{1}{2}\mathrm{var}_{t-1}(r_t^W + \frac{1}{2}r_t^R) = 0. \qquad \text{(A.47)}$$

It is easy to see that the first term is zero,

$$\mathrm{cov}_{t-1}(m_t, r_t^R - m_t^R) = \mathrm{cov}_{t-1}(m_t, \Upsilon a_t^R) = 0, \qquad \text{(A.48)}$$

where,

$$\Upsilon = (1-\beta)(\psi+1)[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}].$$

The intuition is the same as the static model. Because forward contracts provide a hedge against monetary shocks, the relative return on equity after adjusted monetary shocks is not correlated with home monetary shocks.

The second and third terms can be expressed as

$$\mathrm{var}_{t-1}(m_t^R + r_t^W - \frac{1}{2}r_t^R) - \mathrm{var}_{t-1}(r_t^W + \frac{1}{2}r_t^R) = \mathrm{var}_{t-1}(m_t^R) - 2\,\mathrm{cov}_{t-1}(m_t^R, \frac{1}{2}r_t^R)$$

(A.49)

Because $\mathrm{cov}_{t-1}(m_t^R, \frac{1}{2}r_t^R) = \frac{1}{2}\mathrm{var}_{t-1}(m_t^R)$, we confirm that this allocation in fact satisfies the first order conditions for asset allocations. So (A.3) – (A.5) are satisfied.

Human Wealth

To verify that (A.34) and (A.35) provide the solution for human wealth (A.15), we use (A.18)–(A.21) to write

$$\begin{aligned} h_t &= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty} E_t \beta^s (w_{t+s} + l_{t+s}) \\ &= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty} \beta^s \left\{(\psi+1)\left[l_{t+s}^W + \frac{1}{2}l_{t+s}^R\right] + m_{t+s}^W + \frac{1}{2}m_{t+s}^R\right\} \\ &= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty} \beta^s \left\{(\psi+1)\left[\frac{1-\rho}{\rho+\psi}\vartheta_W^s a_t^W + \frac{1}{2}\frac{\omega-1}{1+\omega\psi}\vartheta_R^s a_t^R\right] + m_t^W + \frac{1}{2}m_t^R\right\} \\ &= \frac{1-\beta}{\beta}\left\{(\psi+1)\left[\frac{1-\rho}{\rho+\psi}\frac{\beta\vartheta_W^s}{1-\beta\vartheta_W^s}a_t^W + \frac{1}{2}\frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R^s}{1-\beta\vartheta_R^s}a_t^R\right] + \frac{\beta}{1-\beta}\left[m_t^W + \frac{1}{2}m_t^R\right]\right\} \end{aligned} \quad \text{(A.50)}$$

Then subtracting the foreign counterpart of (A.50), we get (A.34), and adding the foreign counterpart gives us (A.35).

Budget Constraint

First, world budget constraint expressed in home currency is the following:

$$p_t^W + c_t^W + \frac{\beta}{1-\beta}\{(1-\zeta)v_t^W + \zeta h_t^W\} = \frac{1}{1-\beta}(1-\zeta)(r_t^W + v_{t-1}^W) + \frac{1}{1-\beta}\zeta(r_t^{H^W} + h_{t-1}^W)$$

(A.51)

where we have used $\gamma_t = \gamma_t^*$. We have also used $\delta_t(s_t - f_t) + \delta_t^*(-s_t + f_t) = 0$, which requires $\delta_t = \delta_t^*$. This requires some explanation. The home currency earnings, expressed in home currency, from the forward market are $\tilde{\delta}_t(S_t - F_t)$. That means that the foreign currency earnings for the foreign country are $\tilde{\delta}_t(\frac{F_t}{S_t} - 1)$, which can be written as $\tilde{\delta}_t F_t(\frac{1}{S_t} - \frac{1}{F_t})$. So, the foreign budget constraint, symmetrically to the home budget constraint, will contain the term $\tilde{\delta}_t^*(\frac{1}{S_t} - \frac{1}{F_t})$, where $\tilde{\delta}_t^* = \tilde{\delta}_t F_t$. Using this relationship, we can establish

$$\delta_t^* = \frac{\tilde{\delta}_t^*}{F_t M_{t-1}^*} e^{\overline{m-p}-\bar{c}} = \frac{\tilde{\delta}_t F_t}{M_{t-1}} e^{\overline{m-p}-\bar{c}} = \delta_t, \quad \text{(A.52)}$$

where we have used (A.27), and $\overline{m-p} = \overline{m^* - p^*}$ and $\bar{c} = \bar{c}^*$.

The world budget constraint holds with any realization of a_t^W and m_t^W since equation (A.51) simply indicates that total world wealth carried over into the next period is equal to the value of previous wealth, plus returns, less world consumption. More explicitly, because

$$v_t^W + h_t^W = \frac{1-\beta}{\beta}E_t\sum_{s=1}^{\infty}\beta^s(\pi_{t+s}^W + w_{t+s}^W + l_{t+s}^W) = \frac{1-\beta}{\beta}E_t\sum_{s=1}^{\infty}\beta^s(p_{t+s}^W + c_{t+s}^W), \quad \text{(A.53)}$$

both sides of the equation are the sum of future consumption.

Finally, we examine relative budget constraint:

$$p_t^R + c_t^R - s_t + \frac{\beta}{1-\beta}\{(1-\zeta)v_t^R + \zeta h_t^R - s_t\}$$
$$= \frac{1}{1-\beta}(1-\zeta)[r_t^R - s_t + v_{t-1}^R - (\gamma_t + \gamma_t^*)(r_t^R - \hat{s}_t)] + \frac{1}{1-\beta}\zeta(r_t^{H^R} - s_t + h_{t-1}^R) + 2\delta_t\hat{s}_t$$
(A.54)

Direct substitution from the solutions verifies this equation, but it is helpful to break this down into steps.

Using $\gamma = \gamma_t = \gamma_t^*$, and the solutions for c_t^R, p_t^R, and s_t, we can write

$$(\frac{1}{\rho}-1)\tau\hat{m}_t^R + \frac{\beta}{1-\beta}[(1-\zeta)v_t^R + \zeta h_t^R - m_t^R]$$
$$= \frac{1}{1-\beta}(1-\zeta)[r_t^R - \hat{m}_t^R - 2\gamma_t(r_t^R - \hat{m}_t^R)] + \frac{1}{1-\beta}\zeta(r_t^{H^R} - \hat{m}_t^R) + 2\delta_t\hat{m}_t^R \quad \text{(A.55)}$$
$$+\frac{1}{1-\beta}[(1-\zeta)v_{t-1}^R + \zeta h_{t-1}^R m_{t-1}^R]$$

Using relative returns (A.28) – (A.31), we get

$$\left[(\frac{1}{\rho}-1)\tau - 2\delta_t\right]\hat{m}_t^R + \frac{\beta}{1-\beta}[(1-\zeta)v_t^R + \zeta h_t^R - m_t^R]$$
$$= \left\{(1-2\gamma_t)(1-\zeta)(\psi+1)\left[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]\right\}\hat{a}_t^R$$
$$+\left\{\zeta(\psi+1)\left[\frac{(1-\tau)(\omega-1)-\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right]\right\}\hat{a}_t^R + \frac{1}{1-\beta}[(1-\zeta)v_{t-1}^R + \zeta h_{t-1}^R - m_{t-1}^R]$$
(A.56)

By substituting expressions for δ_t and γ_t from (A.32) and (A.33). into (A.56), we get

$$\beta[(1-\zeta)v_t^R + \zeta h_t^R - m_t^R] = (1-\zeta)v_{t-1}^R + \zeta h_{t-1}^R - m_{t-1}^R. \quad \text{(A.57)}$$

But (A.34) and (A.36) give us

$$(1-\zeta)v_t^R + \zeta h_t^R - m_t^R = 0, \qquad \text{(A.58)}$$

so (A.57) holds.

We have verified that equations (A.1)-(A.6) and (A.14)-(A.17) are satisfied.

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