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A Stochastic Framework for Public Debt Sustainability Analysis

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A Stochastic Framework for Public Debt Sustainability Analysis

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Abstract

This paper proposes a framework for public debt sustainability analysis (DSA) that is complementary to that generally used by IFIs. The DSA in this paper has three components: (i) an integrated and consistent accounting framework for the Consolidated Public Sector (CPS); (ii) the estimation of an appropriate, and country-specific debt threshold, following the approach proposed by Reinhart, Rogoff and Savastano (2003); and (iii) a method for the calculation of the CPS primary balance to achieve the desired debt targets, without resorting to ad-hoc assumptions for the values of the macroeconomic variables during the planning horizon, in the spirit of Garcia and Rigobon (2004) and Celasun, Debrun and Ostry (2006). The paper uses this approach to analyze the sustainability of the Dominican Republic's Public Debt.

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Contents Page

I. INTRODUCTION

The purpose of this paper is to present a framework for debt sustainability analysis (DSA) that is complementary to that generally used by IFIs. The DSA in this paper has three components: (i) an integrated and consistent accounting framework; (ii) the estimation of an appropriate, and country-specific debt threshold; and (iii) a method for the calculation of the CPS primary balance to achieve the desired debt targets, without resorting to ad-hoc assumptions for the values of the macroeconomic variables during the planning horizon. The paper will then use this approach to analyze the sustainability of the Dominican Republic's public debt.

Regarding the first component, standard DSAs do not explicitly consider the effects on public finances of quasi-fiscal operations of the Central Bank. In other words, the costs associated with the implementation of monetary policy, usually linked to the sterilization of excess liquidity, are not explicitly incorporated into the analysis.[2] In the case of the Dominican Republic, the relatively large level of Central Bank debt suggests that the DSA framework would need to be adapted to incorporate the Central Bank accounts in a manner as detailed as possible. This is done in Section II.

On the second component, standard DSAs do not explicitly take into account whether the initial level of public debt exceeds (or is close to exceeding) what history suggests is that country's tolerable debt burden (Reinhart, Rogoff and Savastano, 2003). In this regard, standard DSAs focus more on debt trends and not on the level of debt compared with what the market is prepared to absorb.[3] In the case of the Dominican Republic, this calls for calculating a country-specific debt threshold, one that would take into consideration the particularities of the country's historical experience. This is done in Section III.

Finally, regarding the third component, standard DSAs usually associate a primary surplus of a given size with a targeted debt-to-GDP ratio; however, the likelihood of the baseline scenario coming to fruition is not assessed. Moreover, standard DSAs include a set of stress tests that assume shocks to one or more variables, but without considering any covariance among the variables. The framework in this paper links the size of the primary surplus with the (cumulative) probability of achieving a targeted debt-to-GDP ratio. This in turn, enables policymakers to assess the risk in terms of the increased vulnerability that comes with a larger debt ratio behind any decision in terms of the size of the primary surplus target, as stressed by Celasun, Debrun and Ostry (2006). Moreover, it allows for stress tests that

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[2] Excess liquidity could be the consequence, for instance, of large capital inflows and/or the result of large monetary expansions of quasi-fiscal (or fiscal) origin (such as those resulting from bailouts of the private sector during an economic crisis).

[3] The templates for public DSA used at the International Monetary Fund (IMF) and the World Bank (WB) differ depending on whether the analysis is being carried for a low-Income country (LIC) or not. In this connection, the LIC-External DSA defines debt thresholds according to the CPIA index which rates countries against a set of criteria grouped in four clusters: economic management; structural policies; policies for social inclusion and equity; and public sector management. See IMF and IDA (2005).

consider the covariance among macroeconomic variables, an essential feature of the economy in times of stress (as suggested by Garcia and Rigobon, 2004). Section IV deals with these issues.

Section V summarizes and concludes.

II. The Accounting Framework

This section develops an integrated framework for analyzing debt sustainability of the Consolidated Public Sector (CPS), where most flows/stock variables of both the non-financial public sector (NFPS) and the central bank (CB) balance sheets are considered explicitly (including all flows/stock variables linked, if needed, to the recapitalization of the CB).

A. The Budget Constraint for the Consolidated Public Sector

Non-Financial Public Sector

$$\begin{aligned} &T_t + V_t^{CB} - S_t^{G,CB} - S_t^{G,H} + i_{t-1}^{Q}\cdot Q_{1t-1}^{d,G} - i_{t-1}^{F}\cdot F_{t-1}^{s,G} - i_{t-1}^{H}\cdot H_{t-1}^{s,G} - i_{t-1}^{U}\cdot U_{t-1}^{s,G} - i_{t-1}^{E}\cdot E_{t-1}^{s,G} - p_t\cdot Z_t^{d,G} - w_t\cdot N_t^{d,G} = \\ &= \Delta Q_{1t}^{d,G} + \Delta Q_{2t}^{d,G} - \Delta H_t^{s,G} - \Delta F_t^{s,G} - \Delta E_t^{s,G} - \Delta U_t^{s,G} \end{aligned} \tag{1}$$

Equation (1) shows the flow budget constraint for the NFPS (or G as denoted in all expressions).[4] "Above the line" flow variables (the left hand side–LHS-of (1)) include: Taxes (T_t), dividends from the CB(V_t^{CB}), interest from deposits (domestic currency-denominated) in the domestic banking system ($i_{t-1}^{Q}\cdot Q_{1t-1}^{d,G}$), wages ($w_t\cdot N_t^{d,G}$), goods and services ($p_t\cdot Z_t^{d,G}$), transfers to the CB ($S_t^{G,CB}$), transfers to the households ($S_t^{G,H}$), interest payments on domestic currency-denominated bonds ($i_{t-1}^{F}\cdot F_{t-1}^{s,G}$), interest payments on domestic currency-denominated CB recapitalization bonds ($i_{t-1}^{U}\cdot U_{t-1}^{s,G}$), interest payments on domestic currency-denominated CB domestic credit ($i_{t-1}^{H}\cdot H_{t-1}^{s,G}$), and interest payments on foreign currency-denominated external debt ($i_{t-1}^{E}\cdot E_{t-1}^{s,G}$).[5,6] "Below the line" flows (the right

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[4] For a given stock variable ($X_j^{h,i}$) and its respective flow ($\Delta X_j^{h,i}$), $h=\{s,d\}$ indicates whether the variable refers to a "demand" or a supply, $i=\{G,CB,CPS\}$ indicates the economic agent that is demanding/supplying, while j denotes the time period to which the associated stock/flow corresponds; when necessary, in order to separate between subcomponents of a given variable, a subscript 1 is used with the time period to denote that the corresponding variable is originally denominated in domestic currency, while a subscript 2 is used to denote that the corresponding variable is originally denominated in foreign currency. All variables are expressed in nominal terms, in domestic currency units. Finally, an asterisk ($*$) denotes that the associated variable is measured in foreign currency. It is assumed throughout the analysis that there is only one foreign currency.

[5] Note that the NFPS holds non-remunerated foreign currency-denominated deposits in the CB ($Q_{2t-1}^{d,G}$).

hand side–RHS-of (1)), includes changes in NFPS' financial assets including domestic currency-denominated deposits in the domestic banking sector ($Q_{1t}^{d,G}$) and foreign currency denominated deposits in the CB ($Q_{2t}^{d,G}$); it also includes the changes in the NFPS' liabilities including domestic credit from the CB ($H_t^{s,G}$), domestic currency-denominated bonds ($F_t^{s,G}$), domestic currency-denominated CB recapitalization bonds ($U_t^{s,G}$), and foreign currency-denominated external debt ($E_t^{s,G}$).

Central Bank

$$\begin{aligned} &S_t^{G,CB} - V_t^{CB} + i_{t-1}^{R} \cdot R_{t-1}^{d,CB} + i_{t-1}^{F} \cdot F_{t-1}^{d,CB} + i_{t-1}^{H} \cdot H_{t-1}^{d,CB} + i_{t-1}^{U} \cdot U_{t-1}^{d,CB} - i_{t-1}^{E} \cdot E_{t-1}^{s,CB} - i_{t-1}^{D} \cdot D_{t-1}^{s,CB} - w_t \cdot N_t^{d,CB} = \\ &= \Delta R_t^{d,CB} + \Delta H_t^{d,CB} + \Delta F_t^{d,CB} + \Delta U_t^{d,CB} - \Delta B_{1t}^{s} - \Delta B_{2t}^{s} - \Delta D_t^{s,CB} - \Delta E_t^{s,CB} - \Delta Q_{2t}^{s,CB} \end{aligned} \tag{2}$$

Equation (2) shows the flow budget constraint for the CB. Many of the terms in this equation are merely the counterparts of the respective terms in (1) namely, V_t^{CB}, $S_t^{G,CB}$, $i_{t-1}^{U} \cdot U_{t-1}^{d,CB}$, $Q_{2t}^{s,CB}$, and $U_t^{d,CB}$. There are other terms in (1) that have a "partial" counterpart in (2): $i_{t-1}^{F} \cdot F_{t-1}^{d,CB}$, $F_t^{d,CB}$ (as the domestic currency-denominated bonds supplied by the NFPS are held both by the CB and the private sector), and $i_{t-1}^{H} \cdot H_{t-1}^{d,CB}$, $H_t^{d,CB}$ (as the CB lends both to the NFPS and the banking sector). In addition, there are other variables that do not have a counterpart in (1): Foreign currency- denominated gross international reserves ($R_t^{d,CB}$), interest income derived from such international reserves ($i_{t-1}^{R} \cdot R_{t-1}^{d,CB}$), foreign currency-denominated external debt ($E_t^{s,CB}$), and interest paid on such debt ($i_{t-1}^{E} \cdot E_{t-1}^{s,CB}$), domestic currency-denominated CB debt ($D_t^{s,CB}$), interest paid on such debt ($i_{t-1}^{D} \cdot D_{t-1}^{s,CB}$), domestic currency-denominated monetary base ($B_{1t}^{s,CB}$), foreign currency-denominated monetary base ($B_{2t}^{s,CB}$), and wages paid to CB employees ($w_t \cdot N_t^{d,CB}$).

Consolidated Public Sector

Solving for V_t^{CB} in (2), replacing in (1), aggregating/consolidating CB and NFPS flows when appropriate, and taking into consideration that both (1) and (2) are *ex-post* expressions, yields the consolidated public sector (CPS) budget constraint (3):

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[6] In (1)-(3), w_j is the wage rate per labor unit (which, to simplify, is assumed equal for all economic agents) in period j, $N_j^{h,i}$ denotes de amount of labor units demanded/supplied by sector i in period j, p_j is the price for the only good in the economy in period j, $Z_j^{h,i}$ denotes de amount of units of goods demanded/supplied by sector i in period j, while i_j^{x} denotes the nominal interest rate for financial instrument x in period j.

$$T_t + i_{t-1}^R \cdot R_{t-1}^{s,CB} + i_{t-1}^Q \cdot Q_{1t}^{d,G} + i_{t-1}^H \cdot H_{t-1}^{d,CPS} - i_{t-1}^F \cdot F_{t-1}^{s,CPS} - i_{t-1}^E \cdot E_{t-1}^{s,CPS} - i_{t-1}^D \cdot D_{t-1}^{s,CB} - p_t \cdot Z_t^{d,G} - w_t \cdot N_t^{d,CPS} = \\ = \Delta R_t^{d,CB} + \Delta H_t^{d,CPS} + \Delta Q_{1t}^{d,G} - \Delta B_{1t}^s - \Delta B_{2t}^s - \Delta D_t^{d,CB} - \Delta F_t^{s,CPS} - \Delta E_t^{s,CPS} \qquad (3)$$

Expression (3) indicates a number of points: (i) the overall CPS balance (LHS) should be equal to the increase in the CPS (net) assets (RHS); (ii) increases in the overall CPS debt may be explained by a CPS overall deficit, but also, by increases in CPS financial assets, most notably, by increases in international reserves; (iii) an overall CPS deficit may be financed by increases in debt, and/or by seignorage, and/or by decreases in financial assets; (iv) increases in international reserves can be financed by monetary base expansions, and/or increases in CPS debt, and/or an overall CPS surplus; (v) increases of international reserves in excess of increases in monetary base may be sterilized by increases in CB debt, and/or NFPS debt, and/or an overall CPS surplus, underscoring the desirability for the coordination of monetary and fiscal policies; (vi) sterilization is costly provided it is done by a means different than an overall CPS surplus and the effective interest rate paid on CPS debt (in peso terms) is larger than the effective interest rate (in peso terms) obtained from the placement of international reserves; and (vii) the recapitalization of the CB does not have any impact for the CPS as a whole, as it only implies flows between the NFPS and the CB that cancel out after consolidation.[7]

B. Target Public Debt Ratios and the Size of the Primary Balance

Algebraic manipulation of (3) reveals the factors contributing to CPS debt accumulation. In this regard, expression (4) aggregates, on the LHS, all CPS liabilities (in stock terms, Φ_t), while the RHS includes all variables explaining CPS debt levels.

$$\Phi_t = \Phi_{t-1} \cdot \left(1 + \widetilde{i_{t-1}^{\Phi}}\right) + R_{t-1} \cdot \left(1 + \widetilde{\beta_t}\right) + H_{t-1} \cdot \left(1 + \widetilde{h_t}\right) - B_{t-1} \cdot \left(1 + \widetilde{y_t}\right) - \overline{PB_t^{CB}} - \overline{PB_t^G} \qquad (4)$$

In (4), $\widetilde{\beta_t}$ is a function of the differential (in domestic currency terms) between the rate of growth of international reserves and the interest rate on international reserves (the risk free rate). Analogously, $\widetilde{h_t}$ is a function of the rate differential between the rate of growth of CB domestic credit and interest charged on such credit. In turn, $\widetilde{y_t}$ is a function of the nominal GDP growth while $\overline{PB_t^G}$, and $\overline{PB_t^{CB}}$ are the primary balances of the NFPS and CB respectively, after consolidating the crossed terms.

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[7] In (3), consolidation implies that: $F_t^{s,CPS} = F_t^{s,G} - F_t^{d,CB}$, $H_t^{d,CPS} = H_t^{d,CB} - H_t^{s,G}$ and $i_{t-1}^U \cdot U_{t-1}^{d,CB} = i_{t-1}^U \cdot U_{t-1}^{s,G}$; in turn, aggregation implies that, $E_t^{s,CPS} = E_t^{s,G} + E_t^{s,CB}$ and $N_t^{d,CPS} = N_t^{d,G} + N_t^{d,CB}$; finally consolidation plus the ex-post condition imply that $Q_{2t}^{s,CB} = Q_{2t}^{d,G}$, and $U_t^{d,CB} = U_t^{s,G}$.

Equation (4) can be expressed in terms of GDP as follows:

$$\phi_t = \phi_{t-1} \cdot \left(1 + \overline{i_{t-1}^{\Phi}}\right) + r_{t-1} \cdot \left(1 + \overline{\beta_t}\right) + h_{t-1} \cdot \left(1 + \overline{h_t}\right) - b_{t-1} \cdot \left(1 + \overline{y_t}\right) - \overline{pb_t^{CB}} - \overline{pb_t^{G}} \qquad (5)$$

In (5), lowercase letters denote corresponding variables expressed in terms of GDP, while $\overline{i_{t-1}^{\Phi}}$, $\overline{\beta_t}$, $\overline{h_t}$, and $\overline{y_t}$ are functions of the rate differential between the corresponding variables and (nominal) GDP growth.

Recursive substitution in (5) yields an expression relating the NFPS primary balance to a target debt ratio to be reached in J periods, $\phi_{t+J} = \overline{\overline{\phi}}$:

$$\overline{pb^G} = \frac{\overline{i^{\Phi}}}{\left(1+\overline{i^{\Phi}}\right)^J - 1} \cdot \left[\phi_t \cdot \left(1+\overline{i^{\Phi}}\right)^J + r_t \cdot \left(1+\overline{\beta}\right) \cdot \frac{\left[\left(1+\overline{i^{\Phi}}\right)^J - \left(1+\overline{\beta^*}\right)^J\right]}{\left(\overline{i^{\Phi}} - \overline{\beta^*}\right)} + h_t \cdot \left(1+\overline{h}\right) \cdot \frac{\left[\left(1+\overline{i^{\Phi}}\right)^J - \left(1+\hat{\overline{h}}\right)^J\right]}{\left(\overline{i^{\Phi}} - \hat{\overline{h}}\right)} - \overline{\overline{\phi}} \right] - \qquad (6)$$
$$- b_t \cdot \left(1+\overline{y}\right) - \overline{pb^{CB}}$$

In this regard, (6) assumes that (i) $\overline{pb_t^{CB}}$ is exogenous and constant, and (ii) that all relevant macroeconomic variables remain constant through time. The CPS primary balance needed to reach $\overline{\overline{\phi}}$ would simply result from adding (the assumed constant) $\overline{pb_t^{CB}}$ plus the NFPS primary balance resulting from (6).[8,9]

In contrast, if sequences of macroeconomic variables are allowed to change through time, expression (6) turns into:

$$\overline{pb^G} = \left[\Theta(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \overline{\overline{\phi}}\right] \qquad (7)$$

In turn, if $\overline{pb_t^G}$ is allowed to change with the cycle, $\overline{pb_t^G} = \overline{\overline{pb^G}} \cdot \left[1 + \eta \cdot \left(\gamma_t - \overline{\gamma}\right)\right]$, where $\overline{\overline{pb^G}}$ is the (constant) structural primary balance, $\overline{\eta}$ is the elasticity of the primary balance with respect to the output gap $\left(\gamma_t - \overline{\gamma}\right)$, and $\overline{\gamma}$ is the long-term GDP growth rate, then (7) turns into:

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[8] Note that in (6) $\widehat{h_t}$ and $\overline{\beta_t^*}$ are also functions of the differentials between the corresponding variables and the nominal GDP growth.

[9] Burnside (2005) (Chapter 3) derives a similar, though less general, formula.

$$\overline{\overline{pb^{G}}} = \left[\overline{\overline{\Theta}}(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \overline{\overline{\phi}}\right] \tag{8}$$

The intuition behind (6), (7) and (8) is similar, in the sense that the CPS primary balance needed to reach a given debt ratio target will be larger: (i) the larger the initial debt ratio; (ii) the larger the size of any realized contingent liability shocks during the J periods; (iii) the larger the expected increase in CPS gross assets (most notably international reserves); (iv) the lower the expected seignorage (including that coming from legal reserve requirements on bank deposits); (v) the lower the target for the debt ratio; (vi) the faster the speed at which such debt ratio is to be reached (i.e., the lower is J); (vii) the larger the differential between the (real) interest rate on CPS debt and the real GDP growth (which would be larger the larger the RER depreciation expected); (vii) the larger the difference between the expected rate of increase in international reserves and the risk-free interest rate; and (viii) the larger the difference between the expected rate of increase in CB domestic assets and their rate of return. Finally, note that expressions (7) and (8) only differ in their denominator.[10]

III. Establishing a Target Debt Ratio

Reinhart, Rogoff and Savastano (RRS, 2003), argue that a country's track record at meeting its debt obligations (measured by the country's default history) and managing its macro economy (measured by the country's historical inflation rate) have an influence on their ability to access voluntary debt markets. In this regard, RRS introduce the concept of "debt intolerance" to describe the problems that some economies experience when reaching debt levels that would seem manageable by the standards of more advanced economies. Even though their analysis of debt intolerance is mainly focused on external debt, they argue that overall debt intolerance (i.e., including domestic and external debt) may be viewed as linked to a common set of factors.

This section draws on (and at times adapts) the concepts, sources of data and terminology used by RRS, but extends their analysis to consider domestic debt for a sample of 38 countries for the period 1989-2005. In turn, this analysis will be used to establish a "safe threshold" for the Dominican Republic's CPS debt.[11]

A. Clubs and Regions of Debt Intolerance

Following RRS, this paper uses the country ratings (IIR) published bi-annually by the "Institutional Investors" magazine to organize countries in "Clubs". The IIR ranges from 0 to

[10] See Appendix I for a fully detailed explanation as to how expressions (4)-(8) were derived, as well as for definitions of each of their terms, including the differences between expressions (7) and (8); the Appendix also include a derivation of the factors contributing to the change in the liabilities of the CPS, along the lines of IMF (2005), but including those derived from the implementation of monetary policy.

[11] IMF (2003) for a nice summary of available methods to derive a "sustainable" public debt ratio.

100, with 100 being given to countries with the least chance of default. The IIR for a given country is then used as a proxy to measure that country's creditworthiness, or conversely, (100-IRR) would proxy its sovereign risk. To define such Clubs, the IRR mean (51.7) and standard deviation (23.0) were calculated for 38 industrial and developing countries included in a sample over the period 1989–2005. As shown in Table 1, Club A includes those countries whose average IIR is larger than the mean plus one standard deviation; Club C includes those countries whose average IIR is lower than the mean less one standard deviation; Club B includes all countries in the intermediate range, which in turn is divided in two sub-ranges, Club B(I) includes countries whose average IIR is larger than the mean but lower than the mean plus one standard deviation, while Club B(II) includes those countries whose average IIR is lower than the mean but higher than the mean less one standard deviation.

Table 1. "Institutional Investors' Country Credit Survey"
(Annual ratings average, 1989-2005)

A (IIR >=74.8)	**B**		**C** (IIR <28.7)
	I (51.7=< IIR < 74.8)	**II** (28.7=< IIR < 51.7)	
Canada (85.0)	Chile (56.4)	Argentina (30.4)	Bolivia (23.2)
Denmark (82.1)	Czech Republic (57.7)	Brazil (35.5)	**Dominican Republic (25.4)**
Finland (80.2)	Greece (58.4)	Colombia (41.8)	Kenya (25.7)
Ireland (77.4)	Hungary (52.5)	Indonesia (40.4)	Nigeria (18.4)
Italy (79.0)	Korea (66.5)	India (45.8)	Pakistan (26.0)
Japan (89.1)	Malaysia (61.5)	Sri Lanka (30.4)	Tanzania (17.5)
Norway (84.9)	Thailand (57.2)	Mexico (47.2)	Zimbabwe (23.5)
Singapur (82.5)		Philippines (37.3)	Ghana (28.4)
USA (91.0)		Poland (44.4)	Peru (28.2)
		South Africa (45.5)	
		Turkey (40.3)	

Source: Fund Staff calculations using data from *Institutional Investors magazine*

RRS argue that members of Club A have continuous access to voluntary debt markets, while members of Club B only enjoy intermittent access to voluntary debt markets. In contrast, members of Club C would be able to access voluntary markets only rarely, mainly resorting to bilateral and multilateral financing. Thus, members of Club A are the least debt intolerant, while countries in Club C are the most debt intolerant. RRS argue that even though graduation to higher clubs is possible, it is not easy, as it would require many years of uninterrupted debt repayment, good macroeconomic management as measured by continuously low inflation rates, and relatively low public debt levels.

The Dominican Republic, with an average IRR of 25.4, is well within Club C. In addition, during 1989–2005, it alternated between Club C (1989–1998 and again in 2005 in the aftermath of the banking crisis of 2003) and Club B(II) (1999–2004). It should be noted that in 2006-2007, as the Dominican Republic recovered from the banking crisis, its IRR increased and is consistent with that of Club B(II).

B. Deriving a Country-Specific Debt Threshold

This section establishes a link between a country's sovereign risk (as proxied by the IIR) and its history of default and inflation. Table 2 shows three different specifications, whose results are similar to those in RRS: higher default rates and higher inflation rates both result in lower IIRs, or in other words, a higher country risk. The public debt ratio enters with a negative and significant coefficient for all countries in Clubs B and C, while the coefficient is positive for the countries in Club A; the Dominican Republic dummy enters specifications 2 and 3 with a negative (and significant) coefficient. The rationale for including a Dominican Republic dummy was to "catch" the additional country-risk premium that the IIR seems to include in the case of the Dominican Republic. RRS argue that to identify countries that may be plausible candidates to graduate from a lower to a higher Club, one should look at those countries in which actual IIRs are consistently higher than those predicted by models as such included in Table 2 (RRS use the examples of Greece, Portugal, Thailand, Malaysia and Chile). In the case of the Dominican Republic, all specifications that exclude a Dominican Republic dummy (i.e., specification 1 in Table 2 and others not presented in this paper) result in IIR predictions that are consistently larger than the actual IIRs. This would suggest that despite the progress in stabilizing the economy observed during the last years, including the recovery from the banking crisis, the Dominican Republic would still be relatively far from consistent graduation from Club C to Club B(II).

The estimated coefficients from the third specification in Table 2, together with the actual values of the regressors, are used to predict the IIR for the Dominican Republic for varying ratios of CPS public debt. This exercise, shown in Table 3, suggests that given the Dominican Republic's historical performance with inflation and default, as well as the additional risk premium that investors seem to have placed on the country during the period under consideration, a CPS debt level of just under 30 percent of GDP marks the boundary between the country belonging to Club B(II) or Club C.[12] While about 30 percent of GDP marks the boundary between Club C and Club B, prudence would suggest a lower debt ratio. In this regard, 25 percent of GDP seems like a good "focal point" for the following reasons: (i) it would allow the accommodation of short-term shocks without compromising the "country's membership" to Club B; and (ii) it was the public debt ratio prevalent before the banking

[12] Debt ratios for the Dominican Republic are calculated using the GDP, base 1970, series, in order to ensure consistency with figures in Stand By Agreement documents (see also footnote 10). However, the same analysis was performed with debt ratios that were calculated using the new GDP series (base 1991). As the new GDP levels that are about 13 percent larger than those in the 1970 series, the country-specific debt threshold for the Dominican Republic lowers to about 27 percent of GDP (base 1991).

Table 2. The Role of History and Clubs: Cross Section Results

Following RRS the regression is:

$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \beta_6 X_{6i} + \beta_7 X_{7i} + u_i$

Y = IIR, 1989-2005 average
X_1 = DR Dummy
X_2 = Percent of 12-month periods of inflation at or above 40 percent since 1989
X_3 = Percent of years in a state of default or restructuring since 1946
X_4 = Public Debt / GDP (1989-2005 average) x Club A Dummy
X_5 = Public Debt / GDP (1989-2005 average) x Club B Dummy
X_6 = Public Debt / GDP (1989-2005 average) x Club C Dummy
X_7 = Public Debt / GDP (1989-2005 average) x Club Not A Dummy

38 observations

Regression Number	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Adjusted R^2
				Least Squares Estimates, Robust errors				
1		-0.28	-0.21	0.21	-0.23	-0.44		0.78
		(-2.21)	(-1.27)	(2.32)	(-2.99)	(-6.07)		
2	-22.43	-0.27	-0.17	0.17	-0.27	-0.46		0.80
	(-2.05)	(-2.40)	(-1.18)	(1.95)	(-3.83)	(-6.12)		
3	-25.86	-0.25	-0.31	0.14			-0.33	0.75
	(-2.14)	(-1.97)	(-1.97)	(1.48)			(-4.77)	

Notes: t-statistics in parenthesis. Clubs are defined above and broadly follow the criteria established by RRS.

Sources: Reinhart, Rogoff and Savastano (2003), *Institutional Investor* , IMF's International Financial Statistics

Table 3: Predicted IIR for the Dominican Republic

Public Debt / GDP	Predicted IIR	Club
0	37.8	B II
5	36.3	B II
10	34.8	B II
15	33.2	B II
20	31.7	B II
25	30.2	B II
30	**28.6**	**C**
35	27.1	C
40	25.6	C
50	22.5	C

Note: Fund staff calculations based on the results of specification 3 in Table 2

crisis. Thus, if the Dominican Republic wants to graduate from Club C to Club B, it should maintain (in the short to medium term) a relatively low debt ratio while keeping the current good performance, both in terms in macroeconomic management and debt service. Such a profile would result in progressively larger IIRs, as well as (when extending the sample period) lower values for the regressors associated with default and inflation rates. Graduation from Club C would allow the Dominican Republic to have a larger public debt threshold, reflecting a larger "appetite" of investors for exposure on Dominican Republic's paper.

IV. Assessing the Appropriate CPS Primary Balance

Having defined the conceptual framework that will be used for the DSA calculations (Section II), as well as a country-specific debt threshold (Section III), what remains is to determine the size of the CPS primary balance that would result in achieving such a debt target.

Values for the macroeconomic variables in the expressions included in Section II were obtained using a data generating process that reflects the observed historical behavior of the economy. As noted earlier, this captures the covariance among the macroeconomic variables, which is an essential feature of the economy in times of stress. Recommendations arising from this framework link the size of the primary surplus with the (cumulative) probability of achieving a targeted debt-to-GDP ratio. This allows policymakers to assess the risk, in terms of the increased vulnerability that comes with a larger debt ratio, behind any decision in terms of the size of the primary surplus target.

In addition, the analysis assumes: (i) a decrease of 5 percentage points in the legal reserve requirements on commercial bank deposits (one percentage point per year beginning in 2010) as recommended by successive Fund TA missions to the Dominican Republic during 2006 and 2007; (ii) that the economy is subject (on average) to a shock costing the government the equivalent of 5 percent of GDP once every ten years, reflecting the experience of the Dominican economy during the last 30 years; and (iii) for 2008, it is assumed that the CPS primary balance will reach 1.2 percent of GDP, consistent with the authorities' budget.[13]

A. The Data Generating Process

The macroeconomic data used to calculate the value of expressions in Section II were calculated using a three-stage process. *First* a VAR with 2 lags was computed with (yearly) data for the period 1961-2007; the variables (all in log first differences) included in the VAR were international reserves (measured in foreign currency, $\widehat{\beta_t^*}$), the nominal exchange rate ($\widehat{\varepsilon_t}$), the price level (π_t), and the real GDP (γ_t), in that order. After computing the (reduced form) VAR, the coefficients of the corresponding structural VAR were calculated. This allows the recovery of its structural innovations that, in turn, allow the recovery of the variance-covariance matrix of the structural innovations (Table 4).[14]

[13] This way to analyze the risks to public debt sustainability seems to go in the direction proposed by Gapen, Gray, Lim and Xiao (2005).

[14] Although the results in Section III and IV have been calculated using the 1970 GDP series, (in order to facilitate comparability and continuity with previous SBA documents), all calculations in these sections were also performed using the 1991 GDP series. In this regard, a DSA seminar presenting the results of this paper was held in the context of Article IV consultation last November; in such seminar, all the results were presented using the 1991 GDP series. It is worth pointing out that the main consequence of using the 1991 GDP series is one of scale, as cumulative GDP growth rates since 1991 do not differ significantly between the two series.

Table 4: Variance-Covariance Matrix VAR(2)

	International Reserves	Exchange Rate	Inflation	Real GDP Growth
International Reserves	0.2156	-0.0324	-0.0073	0.0043
Exchange Rate		0.0183	0.0026	-0.0011
Inflation			0.0022	0.0000
GDP Growth				0.0017

Source: Fund Staff calculations

The signs of the covariances are as expected: increases in international reserves (capital inflows) result in exchange rate appreciations, decreases in the inflation rate and increases in GDP growth. Shocks resulting in exchange rate depreciations result in increases in the inflation rate and decreases in the rate of GDP growth.[15]

Second, two separate regressions were estimated linking foreign capital flows (proxied by the log first difference of international reserves), with the sovereign risk premium, and the exchange rate risk with the level of the sovereign risk premium. As expected, capital outflows result in increases in sovereign risk, which in turn, result in increases in exchange rate risk

Third, using the estimated variance-covariance matrix for the VAR, 1,000 sequences were generated for each of its structural innovations covering the period 2008-2015; each of the sequences generated were fed into the VAR, resulting in 1,000 different sequences for the log first differences of the macroeconomic variables included in the VAR (i.e., international reserves, the nominal exchange rate, the price level and the real GDP), for the period 2008-2015. In turn, using the sequences for the (log first differences) of international reserves, and the regression coefficients of the sovereign and exchange rate risk premia, 1,000 sequences were generated for such premia. Finally, using an uncovered interest rate parity model (like the one in Furman and Stiglitz, 1998), 1,000 sequences for interest rates on NFPS foreign currency debt, BCRD peso-denominated debt and NFPS peso-denominated debt covering the period 2008–2015 debt were generated.[16]

Figure 1 shows the (kernel density) distribution of the macroeconomic variables included in the VAR (yearly averages for the period 2008-2015 resulting from the 1,000 sequences generated), plus the resulting distribution of RER changes as well as for $\overline{i_t^{\Phi}}$ (the differential

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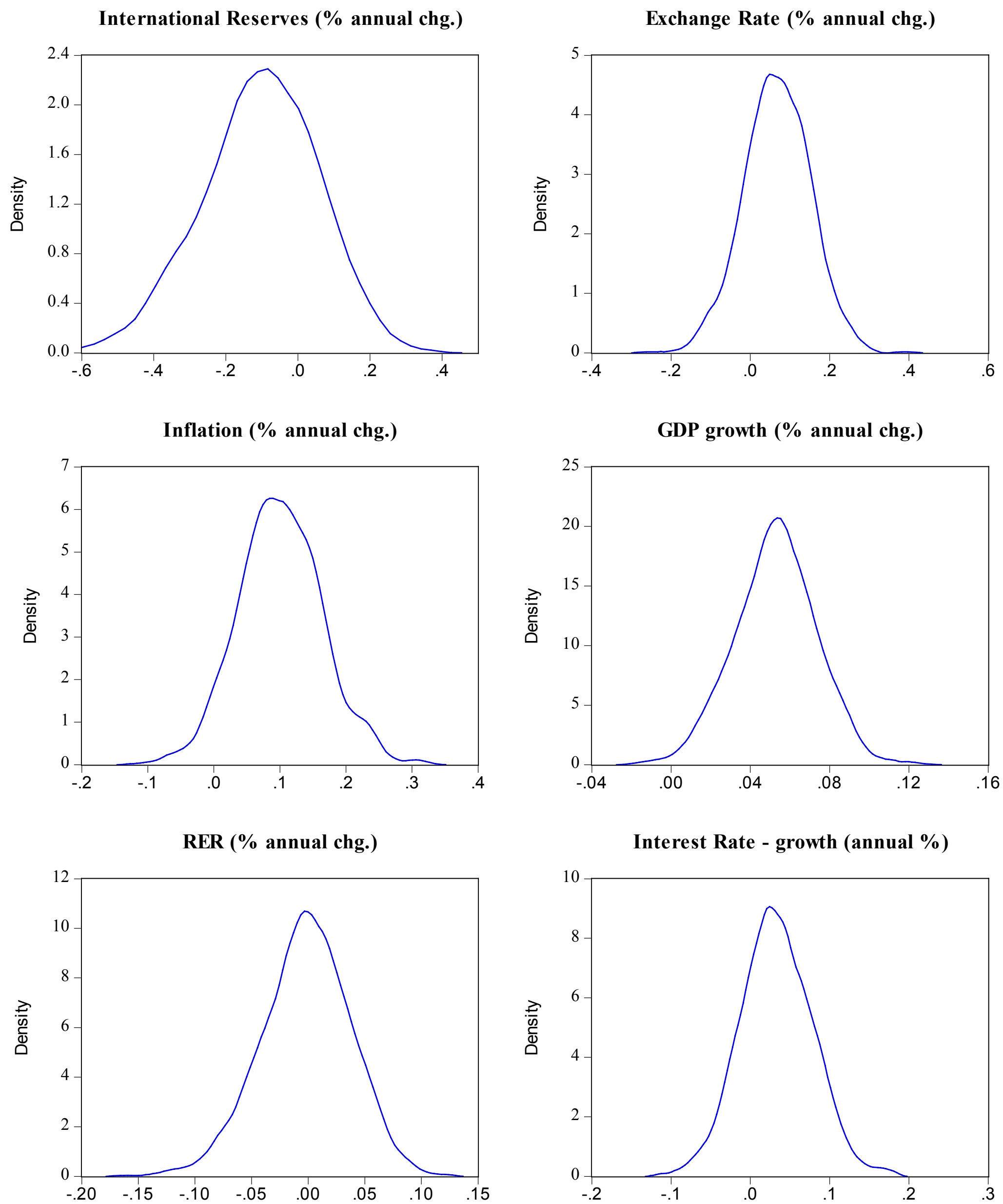
[15] An interesting extension would be to use data generated out of a model for RER determination (which usually include all variables that are relevant for DSA). In such a case, the model pointing to a RER overvaluation would translate in a larger fiscal effort, in anticipation of a RER depreciation (with a needed effort that would be larger, the larger the share of foreign-denominated debt in total debt, and the RER overvaluation). For the rationale between fiscal sustainability and the RER see, e.g., Burnside (2005), Chapter 8.

[16] Ideally, interest rates should have formed part of the VAR, which would have eliminated one step of the data generation process. The problem with incorporating interest rates into the VAR was one of data availability: relevant interest rate data is only available since the mid 1990s, while data on other macroeconomic variables is available since early 1960s. See Appendix.

between the weighted average interest rate and GDP growth, see Section II and Mathematical Appendix).

Figure 1. Dominican Republic: (Generated) Macroeconomic Data through 2012

(Kernel Densites)



Source: Fund Staff calculations

The relative large volatility of the Dominican economy during the last three decades is reflected in distributions for the generated macroeconomic variables that have relatively large variances; this is particularly true in the case of international reserves, where the (kernel) distribution seems to be the flatter of all the macroeconomic variables considered.

B. The Results

This section uses the sequences of generated data referred to in the previous section to calculate the CPS primary balance needed to achieve a debt ratio like that suggested in Section II (i.e. 25 percent of GDP), which, as noted, is also the debt ratio that prevailed before the banking crisis (all calculations were also performed for the boundary debt ratio, i.e., 30 percent of GDP). In the calculations it was assumed that the targets are to be achieved in 5 years (i.e., by end-2012) or 8 years (i.e., by end-2015). As the primary balance for 2008 is already set in the budget, the calculations were made assuming that the primary balances suggested by the framework starting in 2009. As the sequences of macroeconomic variables described in the previous section are not constant through time, the CPS primary balance was calculated using expression (7) in Section II.[17]

Table 5: CPS Primary balance needed to decrease the Debt/GDP ratio to:

Probality (cummulative)	Debt / GDP Target			
	25%		30%	
	end- 2012	end- 2015	end- 2012	end- 2015
50	3.1%	1.7%	1.7%	1.2%
60	3.5%	2.1%	2.3%	1.6%
70	4.0%	2.6%	2.9%	2.1%
75	4.3%	2.8%	3.3%	2.3%

Source: Fund staff calculations

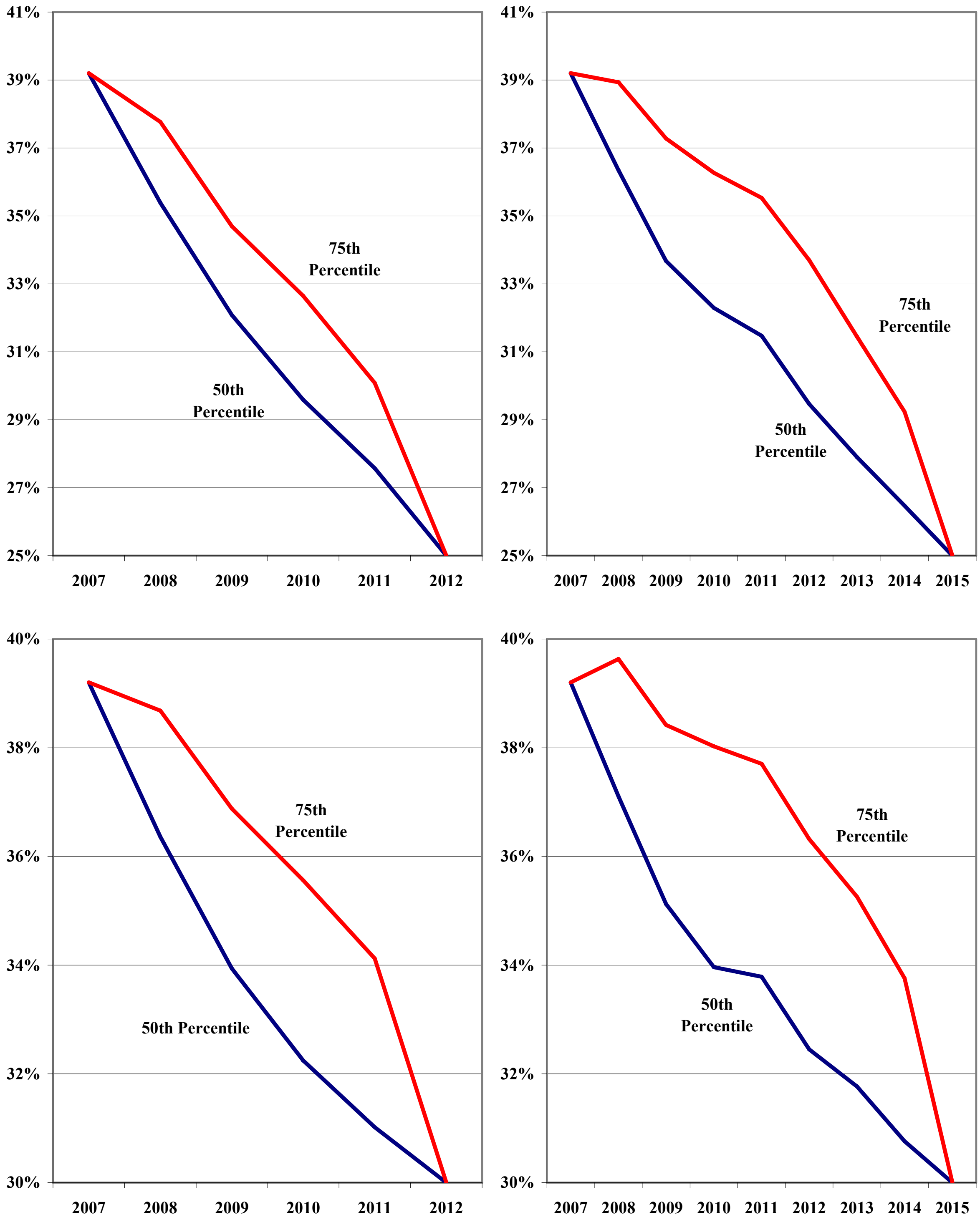
Table 5 summarizes the CPS primary balance needed to achieve the debt targets referred to above, with the delays specified. For example, achieving a debt ratio of 25 percent of GDP by 2015 with a 70 percent probability, would require a CPS primary balance of around 2.5 percent of GDP. In the case that a CPS primary surplus of this size was chosen for 2009, the analysis suggests that this would result in a larger than expected decrease in debt ratios in 70 percent of the cases, i.e., in all those cases in which the effective realization of the macroeconomic variables is better than that that was planned. Thus, if authorities choose such a level for the CPS primary balance, this would likely result, beginning in 2010, in a lower CPS primary balance needed to achieve the same debt targets.[18]

---

[17] As the BCRD primary balance (its operating expenses) is assumed constant and exogenous, what is calculated in reality are 1,000 values for the NFPS primary balance, each one mapped to one of the generated sequences for the macroeconomic variables.

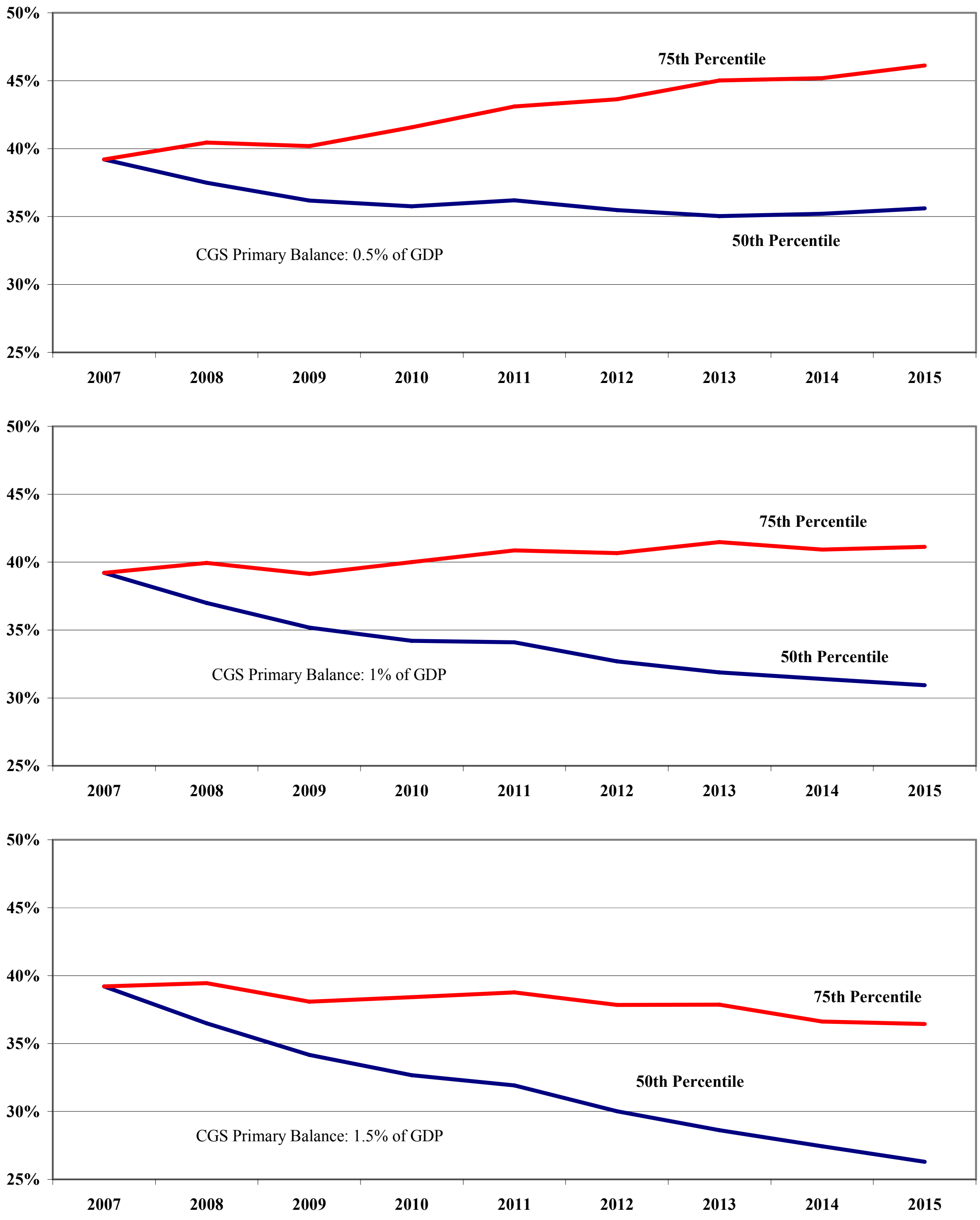
[18] These calculations were shared with the Dominican authorities and are available upon request.

Figure 2. Dominican Republic: Evolution of the CGS Debt Ratio
(for different debt ratio targets and time horizons)



Source: Fund Staff calculations

Figure 3. Dominican Republic: Evolution of the CGS Debt Ratio
(for different CGS primary balances)



Source: Fund Staff Calculations

In turn, Figure 2 shows the paths for the CPS debt ratios for each combination of target debt ratios and periods to achieve them. Note that each of the charts in the panel include the 50th percentile and 75th percentile paths for the debt ratios. Evidently, the 75th percentile reflects a situation of more macroeconomic stress than that implicit in the path at the 50th percentile.

Instead of calculating the exact CPS primary balance to achieve a given debt target in a number of periods, Figure 3 shows the path for the debt ratios for a number of different CPS primary balances (0.5 percent of GDP, 1.0 percent of GDP and 1.5 percent of GDP). This was done by calculating expression (6) in Section II using each of 1000 generated sequences for the macroeconomic variables. Note that even though in the median (or 50th percentile) scenario, CPS primary balances of these sizes result in a decrease of debt ratios, they do not result in significant decreases of debt ratios in stress situations (reflected by the 75th percentile), and indeed, they result in increases in debt ratios for CPS primary surpluses of 0.5 percent of GDP and 1 percent of GDP.[19]

Data for the period 1980-2007 suggests that when GDP growth exceeds long term growth, fiscal revenues (in GDP terms) would increase by a factor of about 0.2 times the output gap;[20] in case primary spending remained constant (in GDP terms), this would result in an improvement in the primary balance (in GDP terms) in times of positive output gaps. This is consistent with evidence for other emerging markets, as pointed out in IMF (2003).

Table 6: CPS Primary balance that decreases the debt ratio, allowing for automatic stabilization

Probality (cummulative)	Debt / GDP target: 25% by end- 2015				
	Output Gap				
	-2	-1	0	1	2
50	0.6%	0.8%	1.6%	1.2%	1.4%
60	1.0%	1.2%	2.0%	1.6%	1.8%
70	1.6%	1.8%	2.5%	2.2%	2.4%
75	1.9%	2.1%	2.7%	2.5%	2.7%

Source: Fund staff calculations

With this background, it is possible to calculate the CPS *structural* primary balance ($\overline{\overline{pb^G}}$) needed to achieve the target debt ratios mentioned above in the specified delays. This can be done by calculating expression (8) in Section II for each of the 1,000 sequences of macroeconomic variables. The results are shown in Table 6. The actual CPS primary balance would then be equal to the CPS structural primary balance plus a term that will be positive

[19] The charts in Figure 3 could be formatted as “fan charts” as in Celasun, Debrun and Ostry (2006).

[20] Results available upon request.

when the output gap is expected to be positive and negative when the output gap is expected to be negative. The application of such a rule provides space for macroeconomic stabilization, without ignoring the objective of CPS debt reduction.[21]

V. Summary and Conclusions

This paper develops and alternative (and complementary) framework for debt sustainability analysis and applies it to the Dominican Republic. The explicit incorporation of Central Bank accounts and operations into debt sustainability analysis results in policy recommendations that account for the expected cost of conducting monetary policy. DSA recommendations regarding the size of the CPS primary surplus can be also adapted to take into consideration the need for macroeconomic stabilization. Both factors are especially relevant in the case of the Dominican Republic, given the relatively large quasi-fiscal losses, as well as the relatively large variability in aggregate demand observed during the last decade.

The arguments and extension of the analysis in Reinhart, Rogoff and Savastano (2003), implies that a public debt threshold of around 25 percent of GDP (and not higher than 30 percent of GDP) would be appropriate for the Dominican Republic. Should the Dominican Republic maintain its good macroeconomic management and timely servicing of its public debt, the country's debt threshold could increase in the future.

The size of the CPS primary balance needed to achieve a 25 percent of GDP debt threshold by 2015, would be about 2.5 percent of GDP (beginning in 2009), with about 70 percent probability. In turn, if automatic stabilization is allowed, the CPS structural primary balance needed to achieve such target would also be about 2.5 percent of GDP that would increase/decrease by 0.2 percentage points of GDP for each positive/negative percentage point of output gap.

Consolidated Public Sector primary balances of lower magnitudes would still decrease debt ratios in the median scenario, but would result in increases in this ratio if the macro situation were subject to sustained stress. The magnitude of the additional fiscal effort required to increase the probability that the debt decreases to a given target ratio reflects the relatively large historical variability of the macroeconomic variables. As the economy stabilizes, and thus, such variability decreases, the "cost" in terms of the additional fiscal effort to increase the probability of achieving any targeted debt ratio will decrease.

This analysis underscores the importance of choosing an appropriate baseline scenario. If the economy remains vulnerable to policy ad/or market shocks, choosing a relatively optimistic baseline may result in a CPS primary surplus that is not large enough to accommodate such shocks. In these circumstances, it is more prudent to pick a baseline scenario associated with a primary surplus of a size that is large enough to achieve the target debt ratio with a larger probability (say 70 percent), rather than choosing the "median" baseline scenario (i.e., that linked with a primary surplus that results in achieving the debt target 50 percent of the time).

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[21] See Hostland and Karam (2005) for an analysis on how to incorporate an endogenous fiscal rule in a stochastic environment.

AUXILIARY CALCULATIONS TO SECTION II

Algebraic manipulation of (3) allows to establish the factors contributing to CPS debt accumulation. In this regard, expression (9) separates, on the LHS, all CPS liabilities (in stock terms), noting whether such liabilities belong to the NFPS or to the BCRD, while the RHS includes all variables explaining CPS debt levels.

$$\begin{aligned} E_t^G + E_t^{CB} + F_t^G + D_t^{CB} &= E_{t-1}^G \cdot \left(1+\widetilde{i_{t-1}^E}\right) + E_{t-1}^{CB} \cdot \left(1+\widetilde{i_{t-1}^E}\right) + F_{t-1}^G \cdot \left(1+i_{t-1}^F\right) + D_{t-1}^{CB} \cdot \left(1+i_{t-1}^D\right) + R_{t-1} \cdot \left(1+\widetilde{\beta_t}\right) + \\ &+ H_{t-1} \cdot \left(1+\widetilde{h_t}\right) - B_{t-1} \cdot \left(1+\widetilde{y_t}\right) - \overline{PB_t^{CB}} - \overline{PB_t^G} \end{aligned} \tag{9}$$

In (9), note that, $B_t = B_{t-1} \cdot \left(1+\widehat{y_t}\right)$, $R_t^* = R_{t-1}^* \cdot \left(1+\widehat{\beta_t^*}\right)$ and $H_t = H_{t-1} \cdot \left(1+\widehat{h_t}\right)$; thus, the monetary base is assumed to remain constant in GDP terms (i.e. it grows at a rate equal to the nominal GDP, $\widehat{y_t} = \pi_t + \gamma_t + \pi_t \cdot \gamma_t$, where π_t is the inflation rate and γ_t is the real GDP growth rate).[22] Also, $\widehat{\beta_t^*}$ is the rate growth of international reserves (measured in foreign currency) and $\widehat{h_t}$ is the (policy determined) rate of growth of BCRD's domestic credit. In addition, $\widetilde{i_{t-1}^E} = i_{t-1}^E + \widehat{\varepsilon_t}$, where $i_{t-1}^E = \left(1+\widehat{\varepsilon_t}\right) \cdot i_{t-1}^{E*}$, $\widehat{\varepsilon_t}$ is the depreciation of the exchange rate; and, $\widetilde{\beta_t} = \left(1+\widehat{\varepsilon_t}\right) \cdot \left(\widehat{\beta_t^*} - i_{t-1}^{R*}\right) - 1$, i.e., the rate differential (in domestic currency terms) between the rate of growth of international reserves and the interest rate on international reserves (the risk free rate). Analogously, $\widetilde{h_t} = \widehat{h_t} - i_{t-1}^H - 1$ is the rate differential between the rate of growth of BCRD domestic credit and interest charged on such credit. Finally, $\widetilde{y_t} = \widehat{y_t} - 1$, while $\overline{PB_t^G} = T_t - S_t^{G,H} - p_t \cdot Z_t^{d,G} - w_t \cdot N_t^{d,G}$, and $\overline{PB_t^{CB}} = -w_t \cdot N_t^{d,CB}$, i.e., the primary balances of the NFPS and BCRD respectively, after consolidating the crossed terms.

Further simplifying (9) results in expression (10) (which is equivalent to (4) in Section II), where $E_t = E_t^G + E_t^{CB}$, $\Phi_t = E_t + D_t^{CB} + F_t^G$ and $\widetilde{i_{t-1}^\Phi} = \theta_{t-1}^E \cdot \widetilde{i_{t-1}^E} + \theta_{t-1}^D \cdot i_{t-1}^D + \theta_{t-1}^F \cdot i_{t-1}^F$, i.e., a weighted average for CPS debt, where $\theta_{t-1}^z = \frac{z_{t-1}}{\Phi_{t-1}}$ and $z = \{E, D, F\}$:

$$\Phi_t = \Phi_{t-1} \cdot \left(1+\widetilde{i_{t-1}^\Phi}\right) + R_{t-1} \cdot \left(1+\widetilde{\beta_t}\right) + H_{t-1} \cdot \left(1+\widetilde{h_t}\right) - B_{t-1} \cdot \left(1+\widetilde{y_t}\right) - \overline{PB_t^{CB}} - \overline{PB_t^G} \tag{10}$$

Equation (10) can be expressed in terms of GDP as follows:

[22] Note that for simplicity the monetary base is assumed to be denominated in domestic currency only, and that the interest rate on external debt is assumed equal for the NFPS and the BCRD.

$$\phi_t = \phi_{t-1} \cdot \left(1 + \overline{i_{t-1}^{\Phi}}\right) + r_{t-1} \cdot \left(1 + \overline{\beta_t}\right) + h_{t-1} \cdot \left(1 + \overline{h_t}\right) - b_{t-1} \cdot \left(1 + \overline{y_t}\right) - \overline{pb_t^{CB}} - \overline{pb_t^{G}} \tag{11}$$

In (11) (which is equivalent to (5) in Section II), lowercase letters denote corresponding variables expressed in terms of GDP. In addition, note that, $\frac{1+\widetilde{i_{t-1}^{\Phi}}}{1+\widehat{y_t}} = \frac{1}{1+\gamma_t} \cdot \frac{1+\widetilde{i_{t-1}^{\Phi}}}{1+\pi_t}$, so if $\widetilde{\widetilde{i_{t-1}^{\Phi}}} = \frac{\widetilde{i_{t-1}^{\Phi}} - \pi_t}{1+\pi_t}$, then $\frac{1+\widetilde{i_{t-1}^{\Phi}}}{1+\widehat{y_t}} = \frac{1+\widetilde{\widetilde{i_{t-1}^{\Phi}}}}{1+\gamma_t}$, or, $\overline{i_{t-1}^{\Phi}} = \frac{\widetilde{i_{t-1}^{\Phi}} - \widehat{y_t}}{1+\widehat{y_t}} = \frac{\widetilde{\widetilde{i_{t-1}^{\Phi}}} - \gamma_t}{1+\gamma_t}$; moreover, $\overline{\beta_t} = \frac{\widetilde{\beta_t} - \widehat{y_t}}{1+\widehat{y_t}}$, $\overline{h_t} = \frac{\widetilde{h_t} - \widehat{y_t}}{1+\widehat{y_t}}$, and $\overline{y_t} = \frac{\widetilde{y_t} - \widehat{y_t}}{1+\widehat{y_t}}$.

Now, if the authorities want to reach a target debt ratio in J periods, $\phi_{t+J} = \overline{\overline{\phi}}$, and assuming that (i) $\overline{pb_t^{CB}}$ is exogenous, and (ii) that all relevant macroeconomic variables remain constant through time, recursive substitution allows to transform (11) into (12):

$$\begin{aligned}\phi_{t+J} = {} & \phi_t \cdot \left(1+\overline{i^{\Phi}}\right)^J + r_t \cdot \left(1+\overline{\beta}\right) \cdot \sum_{j=0}^{J-1} \left(\frac{1+\widehat{\beta^*}}{1+\widetilde{i^{\Phi}}}\right)^j + h_{t-1} \cdot \left(1+\overline{h}\right) \cdot \sum_{j=0}^{J-1} \left(\frac{1+\hat{h}}{1+\widetilde{i^{\Phi}}}\right)^j - \\ & - b_{t-1} \cdot \left(1+\overline{y}\right) \cdot \sum_{j=0}^{J-1} \left(\frac{1+\hat{y}}{1+\widetilde{i^{\Phi}}}\right)^j - \overline{pb^{CB}} \cdot \sum_{j=0}^{J-1} \left(1+\overline{i^{\Phi}}\right)^j - \overline{pb^{G}} \cdot \sum_{j=0}^{J-1} \left(1+\overline{i^{\Phi}}\right)^j\end{aligned} \tag{12}$$

Then the (constant) NFPS primary balance, in terms of GDP, needed to achieve such debt target is given by (13), which is the same as (6) in Section II.

$$\begin{aligned}\overline{pb^G} = {} & \frac{\overline{i^{\Phi}}}{\left(1+\overline{i^{\Phi}}\right)^J - 1} \cdot \left[\phi_t \cdot \left(1+\overline{i^{\Phi}}\right)^J + r_t \cdot \left(1+\overline{\beta}\right) \cdot \frac{\left[\left(1+\overline{i^{\Phi}}\right)^J - \left(1+\overline{\beta^*}\right)^J\right]}{\left(\overline{i^{\Phi}} - \overline{\beta^*}\right)} + h_t \cdot \left(1+\overline{h}\right) \cdot \frac{\left[\left(1+\overline{i^{\Phi}}\right)^J - \left(1+\hat{\overline{h}}\right)^J\right]}{\left(\overline{i^{\Phi}} - \hat{\overline{h}}\right)} - \overline{\overline{\phi}}\right] - \\ & - b_t \cdot \left(1+\overline{y}\right) - \overline{pb^{CB}}\end{aligned} \tag{13}$$

Note that in (13) $\hat{\overline{h_t}} = \frac{\widehat{h_t} - \widehat{y_t}}{1+\widehat{y_t}}$ and $\overline{\beta_t^*} = \frac{\widehat{\beta_t^*} - \widehat{y_t}}{1+\widehat{y_t}}$.

In turn, if sequences of macroeconomic variables are not constant through time, expression (12) turns into (14):

$$
\begin{aligned}
&\phi_{t+J} = \phi_t \cdot \prod_{j=0}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\\
&+r_t \cdot \left[\left(1+\overline{\beta_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{\beta_{t+j+1}}\right)\cdot \prod_{m=1}^{j}\left(1+\overline{\beta^{*}_{t+m}}\right)\cdot \prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{\beta_{t+J}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{\beta^{*}_{t+j}}\right)\right]+\\
&+h_t \cdot \left[\left(1+\overline{h_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{h_{t+j+1}}\right)\cdot \prod_{m=1}^{j}\left(1+\widehat{h_{t+m}}\right)\cdot \prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{h_{t+J}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\widehat{h_{t+j}}\right)\right]-\\
&-b_t \cdot \left[\left(1+\overline{y_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{y_{t+j+1}}\right)\cdot \prod_{m=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left(1+\overline{y_{t+J}}\right)\right]-\\
&-\sum_{j=1}^{J-1}\overline{pb^{CB}_{t+j}}\cdot \prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)-\sum_{j=1}^{J-1}\overline{pb^{G}_{t+j}}\cdot \prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)-\overline{pb^{CB}_{t+J}}-\overline{pb^{G}_{t+J}}
\end{aligned}
\tag{14}
$$

Expression (14) can be simplified and presented as in (15), which is equivalent to (7) in Section II:

$$
\overline{pb^{G}} = \left[\Theta(J)\right]^{-1}\cdot\left[\phi(J)+r(J)+h(J)-b(J)-\overline{pb^{CB}}(J)-\overline{\overline{\phi}}\right] \tag{15}
$$

In (15):

$$
\begin{aligned}
&\phi(J) = \phi_t \cdot \prod_{j=0}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right),\\
&r(J) = r_t \cdot \left[\left(1+\overline{\beta_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{\beta_{t+j+1}}\right)\cdot \prod_{m=1}^{j}\left(1+\overline{\beta^{*}_{t+m}}\right)\cdot \prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{\beta_{t+J}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{\beta^{*}_{t+j}}\right)\right],\\
&h(J) = h_t \cdot \left[\left(1+\overline{h_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{h_{t+j+1}}\right)\cdot \prod_{m=1}^{j}\left(1+\widehat{h_{t+m}}\right)\cdot \prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{h_{t+J}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\widehat{h_{t+j}}\right)\right],\\
&b(J) = b_t \cdot \left[\left(1+\overline{y_{t+1}}\right)\cdot \prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{y_{t+j+1}}\right)\cdot \prod_{m=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left(1+\overline{y_{t+J}}\right)\right],\\
&\overline{pb^{CB}}(J) = \sum_{j=1}^{J-1}\overline{pb^{CB}_{t+j}}\cdot \prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\overline{pb^{CB}_{t+J}},
\end{aligned}
$$

and,

$$
\Theta(J) = \left[1+\sum_{j=1}^{J-1}\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)\right] \tag{16}
$$

In turn, if $\overline{pb^{G}_t}$ is allowed to change with the cycle, $\overline{pb^{G}_t} = \overline{\overline{pb^{G}}}\cdot\left[1+\eta\cdot\left(\gamma_t-\overline{\gamma}\right)\right]$, where $\overline{\overline{pb^{G}}}$ is the (constant) structural primary balance, $\overline{\eta} = \overline{\gamma}\cdot\eta$ is the elasticity of the primary balance with respect to the output gap $\left(\gamma_t - \bar{\gamma}\right)$, and $\bar{\gamma}$ is the long-term GDP growth rate, then (15) turns into (17), which is equivalent to (8) in Section II.

$$\overline{\overline{pb^G}} = \left[\bar{\bar{\Theta}}(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \bar{\bar{\phi}}\right] \tag{17}$$

Note that expressions (15) and (17) only differ in their denominator: If automatic stabilization is to be allowed, the NFPS primary balance would be given by $\overline{pb^G}(J) = \overline{\overline{pb^G}} \cdot \left\{\sum_{j=1}^{J-1}\left[1+\eta\cdot\left(\gamma_t - \bar{\gamma}\right)\right]\cdot\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left[1+\eta\cdot\left(\gamma_{t+J}-\bar{\gamma}\right)\right]\right\}$ and thus, expression (16) needs to be replaced by (18) in expression (15), giving rise to (17).

$$\bar{\bar{\Theta}}(J) = \left\{\sum_{j=1}^{J-1}\left[1+\eta\cdot\left(\gamma_t - \bar{\gamma}\right)\right]\cdot\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left[1+\eta\cdot\left(\gamma_{t+J}-\bar{\gamma}\right)\right]\right\} \tag{18}$$

Factors contributing to the Evolution of the CPS Debt Ratio

Expression (3) can be re-written as in (19), with lowercase letters expressing corresponding variables in GDP terms:

$$\begin{aligned}
&\left[\left(e_t^{*s,CPS} + b_{2t}^{*s}\right)\cdot\varepsilon_t + d_t^{s,CB} + b_{1t}^{s} + f_t^{s,CGS}\right] = h_t^{d,CPS} + q_{1t}^{d,G} + r_t^{*d,CB}\cdot\varepsilon_t + e_{t-1}^{*s,CPS}\cdot\varepsilon_{t-1}\cdot\frac{\left(1+i_{t-1}^{E}\right)\cdot\left(1+\hat{\varepsilon}_t\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)}+\\
&+b_{2t-1}^{*s}\cdot\varepsilon_{t-1}\cdot\frac{\left(1+\hat{\varepsilon}_t\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)} - r_{t-1}^{*d,CB}\cdot\varepsilon_{t-1}\cdot\frac{\left(1+i_{t-1}^{R}\right)\cdot\left(1+\hat{\varepsilon}_t\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)} + d_{t-1}^{s,CB}\cdot\frac{\left(1+i_{t-1}^{D}\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)}+\\
&+b_{1t-1}^{s}\cdot\frac{1}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)} + f_{t-1}^{s,CPS}\cdot\frac{\left(1+i_{t-1}^{F}\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)} - h_{t-1}^{d,CPS}\cdot\frac{\left(1+i_{t-1}^{H}\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)} - q_{1t-1}^{d,G}\cdot\frac{\left(1+i_{t-1}^{Q}\right)}{\left(1+\pi_t\right)\cdot\left(1+\gamma_t\right)}-\\
&-pb_t
\end{aligned} \tag{19}$$

In turn, (19) can be re-expressed in absolute changes (in GDP terms), as in (20):

$$\begin{aligned}
&\left[\left(\Delta e_t^{*s,CGS} + \Delta b_{2t}^{*s}\right)\cdot\varepsilon_t + \Delta d_t^{s,CB} + \Delta b_{1t}^{s} + \Delta f_t^{s,CGS}\right] = \Delta h_t^{d,CGS} + \Delta q_{1t}^{d,G} + \Delta r_t^{*d,CB}\cdot\varepsilon_t+\\
&+e_{t-1}^{*s,CGS}\cdot\varepsilon_{t-1}\cdot\frac{\left(i_{t-1}^{E}+\hat{\varepsilon}_t+\hat{\varepsilon}_t\cdot i_{t-1}^{E}-y_t\right)}{\left(1+y_t\right)} + b_{2t-1}^{*s,CGS}\cdot\varepsilon_{t-1}\cdot\frac{\left(\hat{\varepsilon}_t-y_t\right)}{\left(1+y_t\right)} - r_{t-1}^{*d,CB}\cdot\varepsilon_{t-1}\cdot\frac{\left(i_{t-1}^{R}+\hat{\varepsilon}_t+\hat{\varepsilon}_t\cdot i_{t-1}^{R}-y_t\right)}{\left(1+y_t\right)}+\\
&+d_{t-1}^{s,CB}\cdot\frac{\left(i_{t-1}^{D}-y_t\right)}{\left(1+y_t\right)} + b_{1t-1}^{s}\cdot\frac{-y_t}{\left(1+y_t\right)} + f_{t-1}^{s,CGS}\cdot\frac{\left(i_{t-1}^{F}-y_t\right)}{\left(1+y_t\right)} - h_{t-1}^{d,CGS}\cdot\frac{\left(i_{t-1}^{H}-y_t\right)}{\left(1+y_t\right)} - q_{1t-1}^{d,G}\cdot\frac{\left(i_{t-1}^{Q}-y_t\right)}{\left(1+y_t\right)} - pb_t
\end{aligned} \tag{20}$$

Thus, the change in the CPS gross liabilities in any given period can be decomposed in a number of contributing factors, namely, the exchange rate contribution $\frac{\hat{\varepsilon}_t}{(1+y_t)}\cdot\varepsilon_{t-1}\cdot\left(e_{t-1}^{*s,CGS}+b_{2t-1}^{*s}-r_{t-1}^{*d,CB}\right)$, the GDP growth contribution $\frac{-y_t}{(1+y_t)}\left[\left(e_{t-1}^{*s,CGS}+b_{2t-1}^{*s,CGS}-r_{t-1}^{*d,CB}\right)\cdot\varepsilon_{t-1}+d_{t-1}^{s,CB}+b_{1t-1}^{s}+f_{t-1}^{s,CGS}-h_{t-1}^{d,CGS}-q_{1t-1}^{d,G}\right]$, the primary balance contribution, $-pb_t$, the interest rate contribution $\frac{1}{(1+y_t)}\left[e_{t-1}^{*s,CGS}\cdot\varepsilon_t\cdot i_{t-1}^{E}-r_{t-1}^{*d,CB}\cdot\varepsilon_t\cdot i_{t-1}^{R}+d_{t-1}^{s,CB}\cdot i_{t-1}^{D}+f_{t-1}^{s,CGS}\cdot i_{t-1}^{F}-h_{t-1}^{d,CGS}i_{t-1}^{H}-q_{1t-1}^{d,G}\cdot i_{t-1}^{Q}\right]$, and the asset change contribution, $\Delta h_t^{d,CGS}+\Delta q_{1t}^{d,G}+\Delta r_t^{*d,CB}\cdot\varepsilon_t$. Note that the cost of sterilized intervention would be $\Delta r_t^{*d,CB}\cdot\varepsilon_t\cdot\left(i_t^R-i_t^D\right)$.

Calculation of the Interest Rate Sequences

Assuming that investors are risk averse, the foreign-currency interest rate paid by the sovereign, i_t^{E*} could be expressed as $\left(1-\delta^*\right)\cdot\left(1+i_t^{E*}\right)-v=\left(1+i_t^{R*}\right)$, where v is the sovereign risk premium, δ^* is the default probability on external public debt, and i_t^{R*} is the risk-free interest rate. Analogously, following Furman and Stiglitz (1998), one can express the relationship between the interest rates paid on domestic and external public debt as $\frac{E\left[\varepsilon_{t+1}\right]}{\varepsilon_t}\cdot\left(1+i_t^{E*}\right)+x=\left(1+i_t^{D}\right)$, where i_t^D is the interest rate paid on domestic debt, x is the exchange rate risk, and $E\left[\varepsilon_{t+1}\right]$ is the expected value of the exchange rate next period. These expressions can be simplified and expressed as $i_t^{E*}=i_t^{R*}+\omega(\cdot)$ and $i_t^{D}=i_t^{E*}+\rho(\cdot)$, where $\omega(\cdot)$ and $\rho(\cdot)$ are functions of economic variables. Assuming that investors look at reserve coverage as a good indicator for a country's capacity to service its external debt, one should observe that $\omega(\cdot)$ is a function of the changes in the level of international reserves, with increases in international reserves associated with decreases in sovereign risk. Analogously, if investors associate increases in sovereign risk with a higher probability of a disruption in exchange rate markets, $\rho(\cdot)$ and $\omega(\cdot)$ should be linked, with increases in sovereign risk associated with increases in exchange rate risk. Table 7 shows that indeed, such associations appear to exist when using interest rate data for the Dominican Republic during the last 15 years.

Then, to calculate the sequences for external and domestic public debt, the risk-free interest rate was assumed constant, $\omega(\cdot)$, was calculated using the sequences for the changes in international reserves generated by the VAR, and in turn $\rho(\cdot)$ was calculated from the sequences obtained for $\omega(\cdot)$.

Table 7: Sovereign and Exchange Rate Risk Regressions

The regression (1) for the sovereign risk is:

$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$

Y = sovereign risk premium (DR US$ debt yield - LIBOR)
X_1 = First Log difference of international reserves (lagged one period)
X_2 = First Log difference of international reserves (lagged two periods)

The regression (2) for the exchange rate risk is:

$Z_i = \alpha + \beta 3 X_{3i} + v_i$

Z = exchange rate risk (DR peso debt yield- DR US$ debt yield)
X_3 = sovereign risk

12 observations after adjustments

Regression Number	X_1	X_2	X_3	Adjusted R^2
		Least Squares Estimates, Robust errors		
1	-0.03 (-4.32)	-0.03 (-4.97)		0.78
2			0.80 (2.56)	0.33

Notes: t-statistics in parenthesis.

Sources: Fund staff calculations on BCRD data.

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