

# Credit Risk Spreads in Local and Foreign Currencies

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## Abstract

**This Working Paper should not be reported as representing the views of the IMF.** The views expressed in this Working Paper are those of the authors and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

The paper shows how—in a Merton-type model with bankruptcy—the currency composition of debt changes the risk profile of a company raising a given amount of financing, and thus affects the cost of debt. Foreign currency borrowing is cheaper when the exchange rate is positively correlated with the return on the company's assets, even if the company is not an exporter. Prudential regulations should therefore differentiate among loans depending on the extent to which borrowers have "natural hedges" of their foreign currency exposures.

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I. Intr	oduction	3
II. Th	e Model	4
III. N	umerical Examples and Illustrations	7
IV. C	redit Spreads and Modigliani and Miller Propositions	9
V. Im	plications and Conclusions	.13
Refer	ences	.18
Table	S	
1.	The Euro-Denominated Debt Spread, Face Value, PD, and the Cost of Credit Risk a Function of Correlations	as 8
2.	Betas of Stocks and Foreign Currency Bonds for Various Correlations	.11
Figure	es	
1.	Spreads on Foreign-Currency Bonds and Correlations	9
2.	The Expected Return on Stock (yS) as a Function of the B/S Ratio	.12
3.	The Expected Return on Stock yS as a Function of the B/S Ratio and Correlation Coefficient o	.13
4.	FE as a Function of F	.15
5.	FE as a Function of F	.16
Apper	ndixes	
I.	Determination of the Face Value of Debt in the Foreign Currency	.15
II.	Firm Value, Exchange Rates, and Inflation	.17

Page

Contents

#### I. INTRODUCTION

The world today is characterized by growing globalization of enterprises, banks, and financial markets. Capital raising, once the province of local capital markets, is increasingly becoming a global activity. In the past it was rare to see firms access pools of financing beyond national boundaries. Today, companies operating internationally and dealing in many currencies have opportunities to finance capital investments and activities in diverse international markets. The ability to raise capital abroad compels companies to examine the economic implications of such opportunities.

Even when financing from abroad is not available, the practice of raising money locally, but denominated in foreign currency is widespread. Especially in countries that have suffered high inflation, dollarization can become pervasive (see for example Armas, Ize, and Levy Yeyati, 2006; Chan-Lau and Santos, 2006; and Havrylyshyn and Beddies, 2003).<sup>2</sup> Not only do people place their savings in foreign currency assets as a better store of value, many companies and households prefer to borrow in foreign currency. They are clearly willing to incur foreign exchange risk in exchange for a reduction in other elements of the cost of funding.

In this paper we study the micro-level factors that should be considered by a borrower when structuring debt denominated in various currencies. The macroeconomic implications of loan dollarization and foreign borrowing, the factors that contribute to the use of foreign currency as a medium of exchange, and the determinants of saving in foreign currency are not addressed.

The issue of currency composition of debt for both corporations and sovereign governments has attracted the attention of academics and policy-makers. Two dominant approaches to this issue can be found in financial literature. The first approach is to focus on "currency mismatch," i.e., discrepancies between the direct currency composition of assets and liabilities held by corporations and sovereigns.<sup>3</sup> Currency mismatch is often singled out as an important factor of financial crises, particularly in developing economies (Caballero and Krishnamurthy, 2005; Catao and Sutton, 2002; Duffie, Pederson, and Singleton, 2003; Gibson and Sundaresan, 2001; Gray, Merton, and Bodie, 2007; Longstaff, Pan, Pedersen, and Singleton, 2007; and Weigel and Gemmill, 2006. Armas and others (op. cit.) contains many papers on the causes and consequences of de facto dollarization in many South American

<sup>&</sup>lt;sup>2</sup> The phenomenon is not restricted to high-inflation countries. A significant portion of lending in Austria is denominated in foreign currency.

<sup>&</sup>lt;sup>3</sup> Google search of "currency mismatch" shows about one million results.

countries and elsewhere. Chan-Lau and Santos (2006) propose several structural models for measuring default risk for firms suffering from currency mismatches in their balance sheets.

The second approach to analyze the financial exposure stemming from the currency composition of debt is to focus on the hedging activities of firms and on the empirical relationship between the rate of return of a firm's shares and exchange rate fluctuations. In general, exposure to foreign currency risk can be mitigated by pricing policy, by "operational hedges," such as locating plants and suppliers in the foreign markets, and, by financial activities, including the determination of debt composition and use of financial derivatives. While some empirical studies estimate that the correlation between share prices and exchange rates is insignificant (e.g., Domingues and Tesar, 2006; Griffin and Stulz, 2001; and Jorion, 1990), theoretical models predict that foreign currency exposure for many corporations should be substantially larger than (see for example Bodnar, Dumas, and Maston, 2002).

The specific problem we address in this paper is whether the currency composition of debt matters. How should the choice of currency in bond offerings affect the credit spread for a given firm? Credit spread is defined as the yield to maturity of the bond, minus the default-free rate, in the relevant currency. One may have an intuition that, since the resulting financial structure when companies issue debt in local or foreign currency remains the same, the risk premium should be equal and the currency composition is irrelevant. However, we show that currency composition matters to the extent that it engenders or mitigates a firm's financial risk, and it should therefore be reflected in differential credit spreads for a given level of debt.

Hedging activity by multinational firms and the rationale behind it is discussed at length in the financial literature. Generally speaking, firms hedge currency exposure to mitigate risks inherent in mismatched revenues and expenses. The literature plays less attention to the other side of the coin, i.e., how financial decisions can be affected by the nature of corporate's "real" assets. Specifically, we incorporate the relationship between the statistical distribution of corporate assets' rate of return and the probability distribution of the currency of the debt. We show how the level of debt which minimizes the probability of default (POD) (and therefore the costs of borrowing) can be determined as a function of the uncertainty of a firm's investments and the correlation between this uncertainty and exchange rate uncertainty.

## II. THE MODEL

We analyze differential credit spreads arising from the choice of raising debt financing in local or foreign currency, by using the economic model proposed by Merton (1974). An American firm, with asset value V, floats pure discount debt with face value F, and market

value B, to be redeemed at time T, where the risk-free rate is a constant r. All the above parameters are in dollar terms.

It can be shown that, under the set of assumptions required for the Black-Scholes (1973) and Merton (1974) models to hold, that the yield to maturity for such a loan, y, is given by

$$y = -\frac{1}{T} \ln\left(\frac{B}{F}\right) = r - \frac{1}{T} \ln\left(\frac{V}{Fe^{-rT}} N(-d_1) + N(d_2)\right)$$
(1)

where

$$d_{1} = \frac{\ln\left(\frac{V}{Fe^{-rT}}\right) + \frac{\sigma^{2}}{2}T}{\sigma\sqrt{T}}, d_{2} = d_{1} - \sigma\sqrt{T},$$

 $\sigma$  is the standard deviation of the rate of return on the firm's assets, and  $N(\cdot)$  is the cumulative standard normal probability function. The spread between the yield to maturity of the local-currency bond and the safe local interest rate is

$$s \equiv y - r = -\frac{1}{T} \ln \left( \frac{V}{Fe^{-rT}} N(-d_1) + N(d_2) \right)$$
(2)

Following the standard Merton model, we can express the value of the bond as

$$B = Fe^{-rT} - Put(strike = F) = Fe^{-rT} + V \cdot N(-d_1) - Fe^{-rT} \cdot N(-d_2)$$
(3)

where  $Fe^{-rT}$  is the present value of a pure discount default-free bond maturing at *T* with face value of *F*, and Put (strike=*F*) is the present value of the credit risk of the pure discount corporate bond, promising to pay *F* at time *T*, with current assets valued at *V*. After simplification, we have

$$B = V \cdot N(-d_1) + Fe^{-rT} \cdot N(d_2)$$
(3')

Now assume that the same firm is considering issuing bonds representing the same present value of debt but denominated in euros, i.e.,  $x_0 \cdot B_E = B$  where subscript  $_E$  denotes the euro currency, and  $x_0$  is the current exchange rate (assumed for simplicity's sake to be 1:1). We further assume that the time to maturity, T, is unchanged, and the risk-free Euro-denominated

interest rate is  $r_E$ . Since we introduce another stochastic variable beyond V, namely, the exchange rate x between the dollar and the euro, we have to add an assumption that the exchange rate also follows a Wiener process with a constant  $\sigma_x$ , which represents the standard deviation of the rate of exchange rate fluctuation, and a constant correlation,  $\rho$ , with the rate of return on the firm's assets. Let  $\sigma_E$  denote the volatility of the firm's assets in euro terms, and  $V_E = V/x_0$ . By the definition of the correlation  $\rho$  one can easily show that

$$\sigma_E^2 = \sigma^2 + \sigma_X^2 - 2\sigma\sigma_X\rho. \tag{4}$$

We must simultaneously find the appropriate face value of the euro-denominated bond,  $F_E$ , and the yield to maturity of the bond,  $y_E$ , such that  $x_0 \cdot B_E = B$ . In order to find the face value of the foreign currency denominated debt,  $F_E$ , such that the current value of the debt in dollar terms is equal to B, we need to solve the following equation:

$$B = V \cdot N(-d_1) + Fe^{-rT} \cdot N(d_2) = x_0 \left( \cdot V_E N(-d_{1E}) + F_E e^{-r_E T} \cdot N(d_{2E}) \right) = x_0 B_E$$
(5)

The left hand side of (5) is from (3'), and the right hand side is the equivalent expression for a foreign-currency bond, and

$$d_{1E} = \frac{\ln\left(\frac{V_E}{F_E e^{-r_E T}}\right) + \frac{\sigma_E^2}{2}T}{\sigma_E \sqrt{T}}, \ d_{2E} = d_{1E} - \sigma_E \sqrt{T}.$$

 $F_E$  is the sole unknown in equation (5), and the right hand side of the equation is monotonic in this variable. Hence, a unique solution always exists.

The spread for a debt denominated in foreign currency is given by

$$s_{E} \equiv y_{E} - r_{E} = -\frac{1}{T} \ln \left( \frac{V_{E}}{F_{E} e^{-r_{E}T}} N(-d_{1E}) + N(d_{2E}) \right)$$
(6)

The question is whether credit risk spreads *s* and  $s_E$  are necessarily equal or is one larger than the other under certain conditions. This issue cannot be resolved analytically since deriving  $F_E$  from equation (3) can be accomplished only by a numerical solution; an analytic solution cannot be found because  $d_{1E}$  and  $d_{2E}$  are functions of  $F_E$ ).

#### **III.** NUMERICAL EXAMPLES AND ILLUSTRATIONS

Let us illustrate this approach with numerical examples. Assume that an American company with asset value of V=\$100, issuing a pure discount (zero coupon) bond in U.S. dollars with a current value of B=\$70, maturing in T=5 years. The standard deviation of the assets' rate of return (in dollars) is  $\sigma=20$  percent. We further assume that the risk less interest rate in the US is r=5 percent. Given these parameters, it can be shown using (3') that the face value of the bond must be F=\$98.27 in order to obtain a current value of \$70. From equation (2) for the bond spread, it can be shown that the yield to maturity on the bond is y=6.78 percent, or a spread of s=178bp. The risk-neutral probability of default (RNP) as measured by  $N(-d_2)$  is 35.4 percent. We can also calculate the present value of the credit risk of the bond (see Crouhy, Galai, and Mark, 1999), by the value of the 5-year put option on V with exercise price F:

$$P = V \cdot N(-d_1) + Fe^{-rT} N(-d_2) = N(-d_2) \left( Fe^{-rT} - V \frac{N(-d_1)}{N(-d_2)} \right)$$
(7)

The value in the bracket is the present value of the potential shortfall of the corporate bond from the promised value of F at time T (also called loss given default (LGD)). In our numerical example the present value of the credit risk is \$6.53, or 6.53 percent of the assets of the firm.

Now let us add that the current exchange rate  $x_0=1$ , the risk-free rate in euro is also 5 percent, and the standard deviation of the exchange rate is  $\sigma_X=6$  percent. In Table 1 we show the results for the bond, denominated in euros, with current value of \$70, for different correlation coefficients between the asset value, *V* and the exchange rate, *x*. (In Appendix I we show the relationship between the face value in foreign currency, *FE*, and in local currency, *F*, for various levels of exchange rate volatility and the correlation coefficient.) Thus, for a correlation of 20 percent, and a face value of debt of €97.9, the present value of the bond is \$70 (dollar). In this case the spread over the euro-denominated risk free rate is 171 bp (compared to 178 bp for the dollar-denominated bond), and the RNP of default is 34.6 percent (compared to 35.4 percent). The present cost of the credit risk is \$6.24 (compared to \$6.53). In other words, issuing euro-denominated bonds for the same present value in dollars can lead to lower credit risk and lower POD. The latter result is even more salient with a correlation parameter of 40 percent. In this case, the spread declines to 140 bp only, and the value of the credit risk is \$5.09.

Correlation	Face Value (in €)	Spread (basis points)	PD (risk neutral, percent)	Put Value
-0.4	102.6	265	43.0	9.92
-0.3	101.8	249	41.7	9.29
-0.2	101.0	233	40.4	8.66
-0.1	100.2	217	39.1	8.04
0.0	99.4	202	37.7	7.43
0.1	98.6	186	36.2	6.83
0.2	97.9	171	34.6	6.24
0.3	97.1	155	33.0	5.66
0.4	96.4	140	31.2	5.09

## Table 1. The Euro-Denominated Debt Spread, Face Value, PD, and the Cost ofCredit Risk as a Function of Correlations 1/

1/ given current value of debt (\$70); dollar and euro risk-free interest rates are 5 percent.

The results are reversed for low and negative correlations. For example, for  $\rho$ =-20 percent the face value of debt should be €101.0 and the credit spread should be 233 bp. The lower the correlation coefficient, the higher the credit spread and the RNP of default.

The intuition behind these findings is that, by issuing debt in a foreign currency (the euro), part of the asset risk is offset by currency risk, in cases in which the rate of return on assets, measured in local currency, and the exchange rate are significantly positively correlated. It serves as a "natural hedge" between the assets and the liabilities of the firm. When there is no or negative correlation, introducing foreign currency liabilities renders the firm riskier by adding a new risk factor. Due to the convexity of the bond payoff, additional risk must be compensated with higher return, even if the risk factors are uncorrelated and firms are risk neutral.

Figure 1 depicts the spread for foreign currency bonds as a function of the correlation between the firm's rate of return in local currency and the volatility of exchange rates, for various levels of volatility. The horizontal line at the 178bp spread (on the vertical axis) represents the spread for local currency-denominated debt, or equivalently, the case where the exchange rate remains constant,  $\sigma_X = 0$  percent, and all business risk stems entirely from the volatility of assets. The region below the flat line depicts spreads in euros falling below 178bp, i.e., where local currency spreads exceed those for foreign currency-denominated debt.

For sufficiently high correlation, the spread on foreign currency bonds falls below that on local currency bonds. The base case, as outlined in Table 1, is  $\sigma_X = 6$  percent. We also present the spreads for  $\sigma_X = 3$  percent and  $\sigma_X = 9$  percent. At first glance it looks odd that all three graphs intersect at positive correlation rates, although the spreads decline with  $\rho$ . To

understand the logic behind this intersection, recall from equation (4) that  $\sigma_E^2 = \sigma^2 + \sigma_X^2 - 2\sigma\sigma_X\rho$ . This function for firm volatility in euros demonstrates that  $\sigma_E^2$  is a declining function of  $\rho$ . It shows also that the product of  $\rho$  and  $\sigma_X$  (for a given  $\sigma$ ) affects the rate of this decline. The credit spread is an increasing function of  $\sigma_E^2$  (and  $\sigma^2$ ), such that when  $\rho$  and  $\rho\sigma_X$  increase, credit spreads decrease, but the sensitivity of spreads to  $\rho$  is smaller, the smaller is  $\sigma_X$ .





1/ Spreads in foreign currency for exchange rate volatility levels  $\sigma_X$ =3 percent, 6 percent, and 9 percent when *V*=100, *r*=5 percent,  $\sigma$ =20 percent, *T*=5, *r<sub>E</sub>*=5 percent.

#### IV. CREDIT SPREADS AND MODIGLIANI AND MILLER PROPOSITIONS

The question immediately raised is whether the above results contradict the famous Modigliani and Miller (M&M) Propositions (1958). M&M argued that, in perfect capital markets with no taxes, one should be indifferent to the capital structure of the firm. We have just demonstrated that the RNP is a function of the firm's investments and the currency used to finance these investments. For a given correlation coefficient, we can find the preferred currency mix for debt financing that reduces the probability of insolvency.

M&M Proposition I assumes that the investment decision is given and known (i.e., V is given and its distribution is known). They showed that, under the assumption of perfect capital markets, the value of the firm is unaffected by the method of financing. Under such conditions, capital structure decisions should not have an impact on the value of the firm. In our paper we assume that capital markets are perfect. We maintain a constant leverage ratio in market value terms, B/V, for both means of financing, but change the composition of the debt, from local currency to foreign currency. By holding the market value constant, we show that the probability of bankruptcy may change, as the notional value of debt changes with the chosen currency. Merton (1977) extends the Propositions to the case of debt with risk of default.

In our paper we maintain a constant leverage ratio in market value terms, B/V, for both means of financing, but change the composition of the debt, from local currency to foreign currency. By holding the market value constant, we show that the probability of bankruptcy may change, as the notional value of debt changes with the chosen currency. The argumentation starts from the proposition that V is unaffected by the financing decision, and it is shown how the foreign currency bond can be priced using non-arbitrage conditions and the requirement that the total value of debt B is constant. Thus, there is no contradiction with M&M Proposition I.

In the M&M analysis, shareholders are indifferent to the leverage ratio since they are compensated for assuming greater leverage, higher financial risk and higher POD by higher expected rates of return. M&M's Proposition II is a simple function: the expected rate of return on equity is a function of the leverage ratio B/V. It is a two dimensional function comprised solely of expected rate of return on equity and the leverage ratio.

In our model, we add another dimension,  $\rho$ , the correlation coefficient between the firm's assets in local currency and the exchange rate. Indifference to capital structure under the new assumptions is maintained. Shareholders and bondholders are fairly compensated for changes in credit risk and POD. At the same time, however, while capital structure does not matter, the structure of debt does. The POD can be reduced by selecting the appropriate (currency) composition of debt for any given capital structure. This leads to higher beta and hence higher expected rates of return on equity (while beta of debt and its expected rate of return decreases). The instantaneous beta for equity  $\beta_s$  and debt,  $\beta_B$ , in local currency are given, respectively, by

$$\beta_{S} = \frac{V}{S} \cdot N(d_{1}) \cdot \beta_{V},$$

and

$$\beta_B = \frac{V}{B} \cdot N(-d_1) \cdot \beta_V,$$

where  $\beta_V$  is the beta of the firm's assets.

In Table 2 we show the betas of equity and debt for different levels of the correlation coefficient for debt ratio of 70/100=0.7, the risk-free interest rate at 5 percent for both dollar and euro bonds, volatility of assets at 20 percent (in dollar terms), and volatility of the exchange rate at 6 percent.

Correlation	elation Beta of Equity Beta of Fore Currency De	Beta of Foreign	PD
		Currency Debt	(Risk neutral; in percent)
-0.4	2.5188	0.3491	43.0
-0.3	2.5397	0.3401	41.7
-0.2	2.5617	0.3307	40.4
-0.1	2.5847	0.3208	39.1
0	2.6090	0.3104	37.7
0.1	2.6346	0.2995	36.2
0.2	2.6616	0.2879	34.6
0.3	2.6903	0.2756	33.0
0.4	2.7207	0.2625	31.2

Table 2. Betas of Stocks and Foreign Currency Bonds for Various Correlations1/

1/V = 100 and the current value of the debt is \$70. Dollar and euro risk-free interest rates are 5 percent. In the case of domestic debt, the beta of equity is 2.6479, and beta of debt is 0.2938.

When the correlation coefficient is 0.20, the beta of equity is 2.66 and of debt 0.29 (given an asset beta of 1.0). For zero correlation, the beta of equity is lower and that of debt higher and the RNP of default increases. The equilibrating parameter in our model is the face value of the euro debt, FE, which is higher for zero correlation than for 0.2 correlation. One can argue that the debt capacity in the euro loan increases as the correlation coefficient rises.

Figure 2 is similar to the classical M&M Proposition II with expected yield on stock on the vertical axis and the debt/equity ratio (B/S) on the horizontal axis. This time we show the function for three levels of the correlation  $\rho$ : +0.5, 0, and -0.6. The graph shows that the greater the correlation, the steeper the function.



Figure 2. The Expected Return on Stock (y<sub>s</sub>) as a Function of the *B*/S Ratio 1/

1/  $\rho$ = 0.5, 0, and -0.6. *V*=100, r=5 percent,  $\sigma$ =20 percent, *T*=5, *r<sub>E</sub>*=5 percent,  $\sigma_X$ =6 percent, the expected return on the market portfolio is 10 percent, and  $\beta_V$ =1.

In Figure 3 we show the three dimensional M&M's Proposition II. This graph depicts the combined impact of B/S and  $\rho$  on the expected yield on equity,  $y_S$ .

The M&M Propositions, and the model presented here, abstract from certain real-world capital markets imperfections. In particular, bankruptcy does not in itself generate extra costs. Furthermore, no one is liquidity or credit constrained: any firm (or household) can get credit immediately in any currency it wishes, provided that its expected net worth is positive.

Borrowing at foreign currency may expose the firm to an additional source of uncertainty. All these factors will be fully priced in by the lender under our assumptions



Figure 3. The Expected Return on Stock  $y_s$  as a Function of the *B*/S Ratio and Correlation Coefficient  $\rho$  1/

1/ *V*=100, *r*=5 percent,  $\sigma$ =20 percent, *T*=5, *r<sub>E</sub>*=5 percent,  $\sigma_X$ =6 percent, expected return on the market portfolio=10 percent, and  $\beta_V$ =1.

## V. IMPLICATIONS AND CONCLUSIONS

Our analysis shows that credit spreads for companies with a given level of leverage in market value terms are not constant and are contingent on the currency composition of debt. The higher the correlation between the debt currency and the rate of return on the firm's assets, the lower the resulting credit spread. We show that in a multi-currency environment, a firm wishing to minimize the probability of insolvency (and thus the costs of financing) may select to finance activities with a currency that is highly correlated with the rate of return on the firm's assets.

We propose a structural model that can be used to estimate the currency mismatch and the availability of a "structural" or "natural hedge," which can guide the firm in its hedging policy, and can reduce a possible (over-) reliance on financial hedging. The model shows

also the negative consequences of over-using foreign currency denominated loans by corporations with a negative correlation with the exchange rate. All these results can be quantified under the set of assumptions we make. However, in the real world, the impact of non-zero bankruptcy costs and additional deviations from market perfection (such as credit constraints) can be significant, and would have to be taken into account in choosing the currency composition of financing.

What is also shown that the major factor is not whether a given company is an exporter or importer, and therefore exposed to exchange rate volatility. Rather, the major factor is the statistical correlation between the rate of return on the firm's assets and changes in the exchange rate. This correlation can stem from various sources and causes. In practice, one important factor that can cause positive correlation is the inflation rate, which affects simultaneously both the prices of products and hence, potentially, the nominal rate of return on assets, and the changes in the nominal exchange rate (Appendix II). It is therefore not surprising to find dollarization of loans in so many emerging markets with unstable inflation.

Our results have implications for prudential regulation of lending institutions. Regulators can assess the credit risk exposure of financial institutions who lend in foreign currency by evaluating the "natural hedges" of their borrowers. The model applies directly to a bank that lends in local currency but obtains financing in foreign currency either from abroad or from local depositors.

While the paper focused on currency mismatch and the possibility of raising debt capital in both local and foreign currencies, our approach can be applied to other forms of value linkage, such as the cost of living index. It can readily be shown, for example, that if the correlation between the rate of return on the firm's assets and the inflation rate is positive, firms can reduce the risk premium it is required to pay by issuing CPI-linked bonds. Indeed, indexed-linked bonds should be more attractive than exchange rate linked bonds to a firm without a natural hedge (Appendix III) because inflation should be better correlated with the value of the firm. This result helps explain why the introduction of indexed-bonds can contribute to de-dollarization (Holland and Mulder, 2006).

Finally, we show that, under the assumption of perfect capital markets with no bankruptcy costs, our results are not only consistent with M&M propositions, but even add to them. We extend M&M Proposition II from a two to three dimensional. The required rate of return on equity is shown to be a function of the leverage ratio (as in the original M&M paper) as well as of the correlation coefficient with exchange rate fluctuations.

#### Appendix I. Determination of the Face Value of Debt in the Foreign Currency

By using the equation (5) we can implicitly solve for the relationship between the face value of the debt in local and in the foreign currencies. Figure 4 shows the relationship between  $F_E$ and F for various levels of  $\sigma_X$  for a given total value of assets V=100. Each point on the graph corresponds to a different present value of debt. In this case, B increases with F (in contrast, in Table 1 we keep B constant). As can be expected,  $F_E$  increases with F and the rate of increase is a function of the of exchange rate volatility. The sensitivity of  $F_E(F)$ increases with  $\sigma_X$ .





1/ For V=100, *r*=5 percent,  $\sigma$ =20 percent, *T*=5*y*, *r*<sub>E</sub>=5 percent,  $\sigma_X$ =6 percent, 20 percent, 30 percent,  $\rho$ =10 percent.

Figure 5 depicts the relationship between  $F_E$  and F for various levels of the correlation  $\rho$  and fixed  $\sigma_X = 6$  percent.  $F_E$  is smaller relative to F, the high is the correlation  $\rho$  between the exchange rate and the value of the company's assets.



1/ For V=100, *r*=5 percent,  $\sigma$ =20 percent, *T*=5, *r<sub>E</sub>*=5 percent,  $\sigma_x$ =6 percent,  $\rho$ = -20 percent, 0, +20 percent.

Figure 5.  $F_E$  as a Function of F 1/

#### Appendix II. Firm Value, Exchange Rates, and Inflation

Suppose that the nominal rate of return of the firm  $R_V$  is the product of the price level  $\Pi$  and an underlying "real" value W, and, for simplicity, that the two components are uncorrelated. Then the variance of return on the assets of the firm, in obvious notation, is

$$\sigma^2 = \sigma_{\Pi}^2 + \sigma_W^2. \tag{8}$$

Suppose also that the exchange rate fluctuates randomly around its purchasing power parity (PPP) value. Thus

$$x = \upsilon \Pi / \Pi_E, \tag{9}$$

where v is the (random) deviation from PPP. Hence,

$$\sigma_X^2 = \sigma_v^2 + \sigma_\Pi^2 + \sigma_{\Pi_E}^2, \qquad (10)$$

where for simplicity we assume that the components are uncorrelated.

It follows that the correlation of the rate of return on V and x is

$$\rho = \frac{\sigma_{\Pi}^2}{(\sigma_{\Pi} + \sigma_{W})(\sigma_{v} + \sigma_{\Pi} + \sigma_{\Pi_{F}})}$$
(11)

It is easy to show that this expression is an increasing function of  $\sigma_{\Pi}$ , that is, the standard deviation of inflation. Hence, even when the firm's underlying business is unaffected by the exchange rate, higher inflation volatility can create a correlation that makes foreign currency borrowing worthwhile.

The correlation between the return on the firm's assets and domestic inflation is

$$corr(\dot{V}, \dot{\Pi}) = \frac{\sigma_{\Pi}}{(\sigma_{\Pi} + \sigma_{W})},$$
(12)

where again we have assumed zero correlation between risk factors and the superscript "." indicates a rate of change. It can been seen that, under these conditions, the correlation of asset returns with inflation is always greater than that with the exchange rate. Hence, indexed-linked debt is more attractive than foreign currency debt. However, this result may be reversed if the underlying real asset (captured by W) is strongly correlated with the exchange rate for an exporter or other firm with a natural hedge.

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