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Search in the Labor Market under Imperfectly Insurable Income Risk

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Search in the Labor Market under Imperfectly Insurable Income Risk

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Abstract

This paper develops a general equilibrium model with unemployment and noncooperative wage determination to analyze the importance of incomplete markets when risk-averse agents are subject to idiosyncratic employment shocks. A version of the model calibrated to the U.S. shows that market incompleteness affects individual behavior and aggregate conditions: it reduces wages and unemployment but increases vacancies. Additionally, the model explains the average level of unemployment insurance observed in the U.S. A key mechanism is the joint influence of imperfect insurance and risk aversion in the wage bargaining. The paper also proposes a novel solution to solve this heterogeneous-agent model.

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I. INTRODUCTION

The search and matching model of the labor market has been widely used in macroeconomics to explain the determination of unemployment. The key assumption of the model is that agents need to spend resources to form a successful employment match and engage in a productive activity. This realistic conjecture about the labor market has also been incorporated in dynamic stochastic general equilibrium models to analyze the effects of macroeconomic shocks on labor market flows. However, due to technical difficulties, most studies lack a treatment of the risks associated with income variation due to the presence of unemployment. These general equilibrium models usually assume that agents are either risk neutral or that they are perfectly insured against income fluctuations. Paradoxically, a model devised to explain the existence of unemployment leaves aside what is arguably the most important concern derived from it. This omission is especially critical if we wish to use such models to analyze the consequences of unemployment insurance.

Indeed, several empirical studies show that households are not perfectly insured and that this could be of particular importance in the presence of unemployment. For instance, Dynarski, Gruber, Moffitt, and Burtless (1997) find that the ability of a household to smooth consumption across employment states is related to its wealth level; the decline in earnings for those households in the lower end of the wealth distribution has a stronger negative effect on consumption expenditure. Gruber (2001) also shows that accumulated wealth and unemployment benefits are used as consumption-smoothing devices.

This paper analyzes a model of equilibrium unemployment with incomplete markets. Risk-averse agents search for jobs in the labor market and face idiosyncratic employment shocks. It studies the characteristics of the stationary equilibrium and the optimal level of unemployment insurance under this assumption. To achieve this, the paper develops a novel approach for solving for the aggregate dynamics in the presence of heterogeneity resulting from different labor market outcomes.

We characterize optimal agent behavior using a general equilibrium model with frictions in the labor market. The model consists of many risk-averse utility-maximizing consumers, and risk-neutral profit-maximizing firms that trade in the markets for goods and factors of production. The presence of frictions in the labor market explains the existence of equilibrium unemployment. Consumers randomly become unemployed during certain periods of their life. Since they are risk averse, they would like to insure against these risks. In the tradition of Bewley (1977, 1980), Hugget (1993) and Aiyagari (1994), we assume that they do not have sufficient financial instruments to successfully diversify the risks associated with random spells of unemployment. Consequently, ex-ante homogeneous consumers accumulate different levels of wealth ex-post due to their dissimilar employment histories.

Since it is costly to hire a worker, there is a rent associated with any established productive job. We follow Mortensen and Pissarides (1994), and most related literature, in assuming that the wage is determined by a bargain between firms and workers over this surplus. What is novel about our analysis is that we consider the effect of idiosyncratic risks on the bargaining process. In the

wage negotiations, agents compare what is offered with their outside options. In the presence of incomplete markets, the amount of wealth that they have accumulated to be able to smooth consumption over time and across states of nature becomes relevant. The outside options are different for each agent. This contrasts with the bargaining under perfect income insurance where everyone enjoys the same outside option.

Solving a model with an endogenous wealth distribution is a difficult task. We develop and implement a novel solution based on perturbation methods. The logic of these methods is to first solve the model for a particular case—which has a known, maybe analytical, solution—and then use the information obtained from that particular solution to find an approximation to the model's general solution. Translated to our specific framework, the objective is to find a particular solution to a case with a degenerate wealth distribution, in which all agents have the same accumulated wealth, and then use this solution to approximate the general solution to the model with a nondegenerate wealth distribution.

The existence of unemployment generates two obstacles to applying this kind of methodology. First, unemployment is a discrete state: in any period of time, agents can be either employed or unemployed. Since perturbation methods rely on differentiability of the model equilibrium conditions, the existence of a discrete variable makes their application difficult. Second, in our model a degenerate distribution can only be achieved in equilibrium if all agents make the same consumption and saving decisions and hold the same amount of wealth. In other words, for such a distribution to exist in equilibrium all agents must be not only ex-ante, but also ex-post, homogeneous. But this situation will not obtain in the presence of unemployment, which creates differences in income that unevenly affect agents decisions, and ultimately over time, their accumulated wealth.

We tackle these problems simultaneously. By allowing the turnover rate between employment statuses to vary with the length of the considered time period, we achieve a degenerate wealth distribution in the limit as the intervals are made infinitely small. In our model, as we shrink the length of the period, the speed at which agents change employment status increases, and the mean duration of the time spent in each state decreases. In this fast-turnover limit, agents change their status infinitely fast, and consequently over a given unit of time they all share the same income from labor. Since all agents have the same budget constraint, they become ex-post homogeneous. Consequently, this particular limit can be solved as if it where a representative agent model. To approximate the general solution to the heterogeneous agent model, we perturb the equilibrium conditions for the case of a small time interval in which agents spend a finite amount of time in each employment status. Under these conditions, their budget constraints are no longer identical, and consequently they accumulate different amounts of wealth, thereby introducing ex-post heterogeneity.[2]

A couple of remarks regarding our methodology are in order here. First, the equilibrium conditions

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[2] Because unemployment spells are typically short in an economy like that of the U.S., the short-interval approximation appears to be a useful construct.

characterizing our fast-turnover limit are comparable to those obtained under the assumption that agents share the same budget constraint because there exists some mechanism through which they pool their income. The difference is that when we depart from that limit, we are able to characterize individual decisions and aggregate conditions in the absence of insurance markets. This permits isolating the effects of idiosyncratic risk. Second, in approaching the fast-turnover limit, we do not approach the continuous-time Mortensen–Pissarides model. In their model, the probabilities of changing employment status are given by a Poisson process, so that there is a fixed probability of a transition over any finite time interval. Consequently, the heterogeneity does not disappear in their continuous-time limit. Our assumption about transition probabilities instead delivers identical incomes from labor for all agents in the limit.

The results show that the lack of complete insurance has noticeable effects on agents' optimal decisions and on aggregate conditions. Compared to the perfect-insurance benchmark, the introduction of uninsurable income risk reduces consumption upon becoming unemployed. It is worth noting that we obtain these results in a model lacking a consumption–leisure trade-off or home production,[3] so that all consumption changes can be attributed to insufficient insurance. It also follows that differences in consumption are immediately translated into differences in welfare. We observe that welfare depends positively on individual assets but, ceteris paribus, welfare of unemployed agents is lower than that of the employed.

An important mechanism in our framework that has not been analyzed in previous work is the effect of incomplete insurance on the wage bargain. Agents holding different levels of accumulated wealth will have different outside options. Consequently, there is a direct effect on prices derived from heterogeneity: aggregate wages decline as a result of idiosyncratic risk. For comparison, consider a perfectly competitive market for labor. In this case, even if agents are heterogeneous and the wealth distribution may affect their optimal decisions, all agents maximize subject to the same prices; in consequence, the effects of heterogeneity are diminished.

Unemployment insurance, in the form of subsidies to the unemployed, operates through two opposing mechanisms in this framework. To the extent that it serves to complete markets, it reduces precautionary savings and increases both employed and unemployed agents' consumption. This insurance effect of policy is associated with a positive effect on welfare. However, in increasing the outside option of an employed worker, it raises wages and this has detrimental effects on aggregate welfare. A higher unemployment subsidy increases average wages that in turn reduces the number of offered vacancies, and consequently, employment. The capital–labor ratio is moderately affected and aggregate capital decreases along with employment. This efficiency effect of policy has a negative effect on welfare. Note that the insurance effect of policy arises due to the presence of idiosyncratic risk, but the efficiency effect is independent of that risk. Hence, in an economy with idiosyncratic risk, the optimal level of unemployment insurance is higher than in an economy with perfect income insurance. In this regard, a model constructed under the assumption of complete private insurance would underestimate the positive effects of this type of

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[3] The reduction in consumption could in principle be explained in a model with any of these features under the assumption of complete markets and the appropriate preference specification.

policy.

This paper is related to an extensive literature that analyzes the macroeconomic implications of the existence of trading frictions in the labor market.[4] We follow Merz (1995), Andolfatto (1996), and den Haan, Ramey, and Watson (2000) in embedding a search and matching model of the labor market, as presented in Mortensen and Pissarides (1994), in a general equilibrium framework. Even though the central purposes of these models is the analysis of unemployment, they share the shortcoming of not taking into account the effects of income risks on agents' optimal decisions, and through their aggregation, on macroeconomic conditions. To ensure tractability these models are forced to make the strong assumption that all agents are risk neutral, or that risk averse agents belong to extended households within which they obtain perfect insurance against income fluctuations, or alternatively, that there exist a complete set of financial instruments that allows them to perfectly insure against unemployment risk. In departing from these assumptions, our work relates to the literature on heterogeneity among agents resulting from the presence of uninsurable risk originated by Bewley (1977, 1980).

A smaller literature has sought to analyze the effects of unemployment insurance in models with risk aversion and incomplete markets. However, most of these papers have done so using different modeling of the labor market that lack the noncooperative wage setup that our model incorporates. Acemoglu and Shimer (1999) use a model of directed search and wage posting to analyze the efficiency of unemployment insurance; Alvarez and Veracierto (2001) use a model of search with rigid labor contracts to study the effects of severance payments; Gomes, Greenwood, and Rebelo (2001) use a search model in which agents choose whether to work or not at the prevailing wage. Finally, Lentz (2009) uses a partial equilibrium model of search with savings and exogenous wages to empirically estimate the optimal level of unemployment insurance using Danish data.

To the best or our knowledge, Valdivia (1996) and Costain and Reiter (2004) are the only studies to use a model of search and matching, with non-cooperative wage determination and savings. However, a crucial difference with our work is that they do not take into account the effects of idiosyncratic risk in the wage bargaining process.[5] Our findings indicate that this is an important mechanism that cannot be ignored in the presence of incomplete markets because heterogeneity affects not only consumption smoothing, but also price determination.

Our methodology based on perturbation methods allows us to analyze the effects of incomplete markets in wages. These methods go back to Kydland and Prescott (1982) for first order approximation, and they have been since developed by Judd (1998), Kim, Kim, Schaumburg, and Sims (2004), and Schmitt-Grohe and Uribe (2004) for representative agents models. Preston and Roca (2007) show how to apply this methodology to models with aggregate and idiosyncratic

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[4] See Mortensen and Pissarides (1999) and Yashiv (2006) for a survey of this literature. Rogerson, Shimer, and Wright (2005) presents a survey of labor search models.

[5] In analyzing the case with incomplete markets Valdivia (1996) uses the same wage rule derived under the complete markets assumption. Costain and Reiter (2004) deliberately assume that bargaining occurs at the sectoral level to avoid the dependence of wages on individual asset holdings.

shocks. In the present paper, the latter approach is adapted to the particular characteristics of a model of matching in the labor market. A particular challenge to using perturbation methods, which rely on differentiability, is the treatment of unemployment because it introduces a discrete state space. In this paper, we show how to deal with this issue in developing and implementing a methodology that can also be applied to other types of models involving discrete state variables.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the solution methodology. Section 4 performs a quantitative analysis to evaluate the effects of incomplete markets on agents' optimal decision rules and aggregate conditions. It also determines optimal unemployment insurance in this framework. Section 5 concludes. Appendices explain the main derivations.

II. THE MODEL

We use a general equilibrium model with flexible prices and perfect competition in the market for the homogeneous consumption good. This good is produced by the many firms in the economy through the transformation of capital and labor supplied by consumers. These agents optimally allocate part of their income to buy the consumption good and save the rest in the form of physical capital — the only available means for transferring wealth across periods. While the rental market for capital is Walrasian, we assume that there are frictions in the labor market that makes it costly for firms and workers to meet and form a successful match. Time is discrete, with period length given by Δ.

A. Labor market

There is a constant labor force, normalized to one, and a fraction u_t of its members are unemployed and looking for a job at period t. There is also a number v_t of vacant jobs offered by firms wanting to increase their productive positions. During every time interval, some unemployed workers are randomly matched with some vacancies. The total number of matches is determined by the aggregate relation

$$m_t = m(v_t, u_t). \tag{1}$$

This matching function is assumed to be increasing in each of its arguments, concave and homogeneous of degree one. Using these properties, the fraction of vacancies that are filled in each interval, the "job-filling rate," is given by

$$q_t = \frac{m(v_t, u_t)}{v_t}. \tag{2}$$

Similarly, the fraction of unemployed workers that are matched with a vacancy in each interval, the "job-finding rate," is given by

$$p_t = \frac{m(v_t, u_t)}{u_t}. \tag{3}$$

By the properties of the matching function, q is decreasing, and p increasing, in the vacancy–

unemployment ratio. This implies that a market with relatively higher vacancies makes it more difficult for firms to fill a position, and consequently, easier for an unemployed worker to be hired. Similarly, a market with relatively more unemployment makes it more difficult for a worker to form a match with a firm. In other words, unemployed agents face higher negative search externalities as the number of agents looking for a job increases, but they face lower externalities as the number of posted vacancies rises. The opposite is true for firms seeking to fill vacancies.

Every period, some agents become unemployed because a fraction λ of productive jobs are severed for exogenous reasons. Together with the matching function, this flow of agents into the unemployment pool explains the existence of equilibrium unemployment. Consequently, in our framework variations in the unemployment rate are explained by firm's job creation decisions rather than by any variations in the rate of job destruction.

Matches and separations occur at the end of each time interval. The net change in jobs then determine the number of workers available for production at the beginning of the following interval. Under these assumptions, the law of motion of the unemployment rate is characterized by

$$u_{t+\Delta} = \lambda(1 - u_t) - p_t u_t. \tag{4}$$

B.   Consumers

There is a continuum of infinitely lived consumer-workers indexed by $i \in I = [0, 1]$, that maximize the present value of the expected sum of intertemporal utilities, discounted at rate ρ. Risk averse agents derive utility from consumption of the homogenous good; this is represented by an increasing, strictly concave felicity function $U(.)$. To smooth consumption over time, agents accumulate assets in the form of physical capital, that is rented to firms at an endogenous rate r_t in a perfectly competitive market, and depreciates at an exogenous rate δ. These individual holdings of capital stock are represented by a_t^i. In addition, agents' optimal saving decisions are subject to the constraint that individual asset holdings can not fall below an exogenous borrowing limit $\tilde{a} \geq 0$.[6]

Employment status affects the income that agents receive in each period. A subset of agents, $i \in E \subset I$, are employed in any given period. They receive a compensation for their labor of w_t^i determined through individual bargaining, and face probability λ of becoming unemployed in the following period. The model of wage bargain developed below implies that $w_t^i = w(a_t^i)$, that is, wages are consumer specific through their dependence in the consumer's wealth a_t^i.

The remainder of agents, $i \in I - E$, are unemployed. They receive unemployment insurance in the form or monetary compensation b_t, and face probability p of becoming employed. Additionally, all consumers receive an equal share π_t of firms' profits, and pay lump-sum taxes τ_t.

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[6]   We assume that consumers can only save through holdings of physical capital; hence, they cannot hold negative quantities of it.

We can express the maximization problem of each type of agent in terms of a Bellman equation. Denoting by $W^e(a_t^i, t)$ the value function of an employed worker with assets a_t^i at beginning of period t, we have

$$W^e(a_t^i, t) = \max_{c_t^e, \dot{a}_t^e} \left\{ \begin{array}{c} U(c_t^e)\Delta + e^{-\rho\Delta} \left[\lambda W^u(a_t^i + \Delta\dot{a}_t^e, t + \Delta) + (1 - \lambda)W^e(a_t^i + \Delta\dot{a}_t^e, t + \Delta) \right] \\ -\varepsilon B \left(a_t^i + \Delta\dot{a}_t^e\right)\Delta \end{array} \right\},$$

(5)

where

$$\dot{a}_t^e = (r_t - \delta)a_t^i + w(a^i) + \pi_t - \tau_t - c_t^e$$

(6)

is the rate of asset accumulation per unit of time, and c_t^e is the consumption rate per unit of time. Equation (5) shows that employed agents optimally choose consumption to maximize intertemporal per-period utility $U(.)\Delta$, given their current accumulated wealth a_t^i. The expected next period continuation value is determined by the probability λ of changing employment status, and by next period accumulated wealth. This is the result of adding per-period savings $\Delta\dot{a}_t^e$ to current wealth.

The last term imposes the borrowing limit $\tilde{a}$ through the interior function

$$B(a) = \frac{(a - \tilde{a})^{-\nu}}{\nu},$$

(7)

where $\nu > 0$.[7] This function has the property that as individual asset holdings approach the borrowing constraint $\tilde{a}$ the interior function leads the value function $W^e(a_t^i, t)$ to take large negative values. The composite function therefore penalizes consumption-savings decisions that lead to an asset position near the borrowing limit.[8]

Similarly, by letting $W^u(a_t^i, t)$ be the value function of an unemployed agent that is searching for a job and has accumulated level of assets a_t^i, we have

$$W^u(a_t^i, t) = \max_{c_t^u, \dot{a}_t^u} \left\{ \begin{array}{c} U(c_t^u)\Delta + e^{-\rho\Delta} \left[(1 - p_t)W^u(a_t^i + \Delta\dot{a}_t^u, t + \Delta) + p_tW^e(a_t^i + \Delta\dot{a}_t^u, t + \Delta) \right] \\ -\varepsilon B \left(a_t^i + \Delta\dot{a}_t^u\right)\Delta \end{array} \right\},$$

(8)

where

$$\dot{a}_t^u = (r_t - \delta)a_t^i + b_t + \pi_t - \tau_t - c_t^u$$

(9)

---

[7] In introducing this interior function we follow a vast literature in the linear programming and non-linear optimization fields of applied mathematics on interior methods for optimization problems subject to inequality constraints — see Forsgren, Gill, and Wright (2002) for a review and detailed references therein. The idea is to replace maximizations problems subject to inequality constraints with unconstrained maximization problems. This is achieved by defining a composite function that reflects the properties of the unconstrained objective functions and the constraints.

[8] Forsgren, Gill, and Wright (2002) provide theorems under which the maximand of the composite function converge to the maximand of the original problem as $\varepsilon \to 0$. Moreover, bounds can be determined on the magnitude of the error in the maximand obtained from the modified problem with small ε. Note that interior methods are very close in spirit to penalty functions used to solve the original Bellman equation when using numerical methods and value function iteration. Such penalty functions appear in various literatures. In related work Kim, Kim, and Kollmann (2005) introduce a penalty term in the utility function to enforce the same kind of borrowing constraint.

is the rate of asset accumulation, and c_t^u represents the consumption rate per unit of time.

For later use, note that the first order condition for optimality for employed agents is given by

$$
\begin{aligned}
U_c(c^e(a_t^i)) \;=\; & \frac{1}{1+(r-\delta)\Delta} e^{-\rho\Delta} \left[\lambda U_c(c^u(a_t^i + \Delta\dot{a}^e(a_t^i))) + (1-\lambda)U_c(c^e(a_t^i + \Delta\dot{a}^e(a_t^i)))\right] \\
& + e^{-\rho\Delta}(1-\lambda)U_c(c^e(a_t^i + \Delta\dot{a}^e(a_t^i)))w_a(a_t^i + \Delta\dot{a}^e(a_t^i))\Delta \\
& + \varepsilon B_a(a_t^i + \Delta\dot{a}^e(a_t^i))\Delta,
\end{aligned}
\tag{10}
$$

where subscripts indicate a derivative with respect to that variable.

Similarly, the first order condition for unemployed agents is given by

$$
\begin{aligned}
U_c(c^u(a_t^i)) \;=\; & \frac{1}{1+(r-\delta)\Delta} e^{-\rho\Delta} \left[(1-p_t)U_c(c^u(a_t^i + \Delta\dot{a}^u(a_t^i))) + p_t U_c(c^e(a_t^i + \Delta\dot{a}^u(a_t^i)))\right] \\
& + e^{-\rho\Delta}p_t U_c(c^e(a_t^i + \Delta\dot{a}^u(a_t^i)))w_a(a_t^i + \Delta\dot{a}^u(a_t^i))\Delta \\
& + \varepsilon B_a(a_t^i + \Delta\dot{a}^u(a_t^i))\Delta.
\end{aligned}
\tag{11}
$$

These conditions state that agents attempt to smooth consumption across intervals gauging the marginal utility of current consumption against its opportunity cost. They have the interpretation of standard stochastic Euler equations. The first term on the right hand side represents the discounted expected marginal utility of future consumption. The second term emerges because of the assumed form of wage bargaining. It states that assets not only provide a return in the rental market, but since they provide self-insurance in the event of job loss, they can also potentially influence the outcome of the wage bargaining, and consequently have an effect on future labor earnings.[9]

C. Firms

There is a continuum of large firms indexed by $j \in J = [0,1]$, that seek to maximize an expected discounted sum of current and future profits. Firms sell to consumers a homogenous good produced using labor, n^j, and capital, k^j. The production function $F(.)$ is increasing in each input, concave, homogeneous of degree one, and satisfies the Inada conditions.

To hire employees, the firm posts v^j vacancies at a unit cost ϕ. These vacancies are matched with an unemployed agent and become a productive job during the following interval with probability q. Additionally, during each interval, some jobs will be destroyed with probability λ. At the beginning of each interval, after a match has been formed, but before production starts, the firm negotiates with each worker a salary that will be specific to that position. The average salary paid by the firm is w^j.

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[9] Note that this effect occurs only because we assume re-negotiation of the wage each period during an employment relation. While this assumption is common in Mortensen-Pissarides style models, because of the presence here of incomplete insurance, this assumption is not innocuous.

Denoting by $\Pi(.)$ the value function of the firm, its maximization problem can be represented as

$$\Pi(n_t^j, t) = \max_{k^j, v^j} \left\{ \left[F(k_t^j, n_t^j) - r_t k_t^j - w_t^j n_t^j - \phi v_t^j \right] \Delta + E_t Q_{t,t+\Delta} \Pi(n_{t+\Delta}^j, t + \Delta) \right\}, \quad (12)$$

subject to

$$n_{t+\Delta}^j = (1 - \lambda) n_t^j + q_t v_t^j$$

where $Q_{t,t+\Delta}$ is the stochastic discount factor used by the firm to price future profits. Since we assume, for simplicity, that consumers own an equal part of each firm, and receive a common share of profits,[10]

$$\pi_t = \int_J \left[F(k_t^j, n_t^j) - r_t k_t^j - w_t^j n_t^j - \phi v_t^j \right] dj, \quad (13)$$

we assume that the firm uses average marginal utilities to value future profits

$$Q_{t,t+\Delta} = e^{-\rho\Delta} \left[\frac{\int_{E'} U_c(c^e(a_{t+\Delta}^i)) di + \int_{I-E'} U_c(c^u(a_{t+\Delta}^i)) di}{\int_E U_c(c^e(a_t^i)) di + \int_{I-E} U_c(c^u(a_t^i)) di} \right]. \quad (14)$$

Note that this assumption is of little relevance in a stationary equilibrium like the one we are solving for. In this case, the discount factor is just determined by the discount rate ρ and the length of the time interval Δ because the bracketed expression always equals one.[11]

The first order conditions for optimality of firm j are given by

$$r_t = F_k(k_t^j, n_t^j), \quad (15)$$

and

$$\frac{\phi}{q_t} \Delta = E_t Q_{t,t+\Delta} \Pi_n((1 - \lambda) n_t^j + q_t v_t^j). \quad (16)$$

The first condition is the usual optimality condition under perfect competition and states that firms hire capital until they equate its marginal product with the economy wide rental rate. The second condition states that firms post vacancies to equate their average cost per interval — given by the fixed flow cost ϕ multiplied by the expected duration of a vacancy — with the expected benefit of hiring additional workers. This is the usual condition that firms post vacancies until their contribution to profits is driven to zero.

Finally, since firms are homogeneous, to simplify the notation we drop the superscript j from these optimality conditions and from now on use the conditions for a representative firm.

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[10]   This assumption rules out the existence of a market for equities. This is not restrictive because if such a market did exist, by an arbitrage condition the return on firm's equity would equate the return on physical capital in equilibrium. As a consequence, consumers would be indifferent between portfolios composed of different proportions of the two assets. In our case, firm ownership is fixed and agents adjust the proportion of the assets they hold in their portfolios by varying their holdings of physical capital.

[11]   We include this general treatment because in the presence of aggregate shocks or if we wish to analyze the transition dynamics resulting from a policy change, that simplification would not be possible. We present the general case to allow a complete statement of the model equations before defining the stationary equilibrium.

Wage determination

Due to the existence of search costs, any job has an associated rent that is divided between worker and firm through a negotiated wage. Since each agent would like to maximize his share of that surplus, they bargain over the wage. The negotiation takes place at the beginning of the following period after a match has been formed, but before production starts. The wage is renegotiated every period while the worker is employed at the firm.[12] Following Binmore, Rubinstein, and Wolinsky (1986), the bargaining is described by an strategic model with exogenous risk of breakdown. According to this bargaining protocol each agent alternates in making an offer that could be accepted or rejected by the other agent. In the latter case, there exists an exogenous probability that the bargain falls through, and the match is dissolved. This risk of breakdown acts as an incentive for agents to reach an agreement.[13] Let α_w be the probability that the bargain fails if the worker's offer is rejected, and α_f be the corresponding probability in the case that the firm's offer is rejected.

From the envelope condition to problem (12) the firm's valuation of an additional worker who receives a wage w_t^i is given by

$$\Pi_n(w_t^i) = \left(F_n(k_t, n_t) - w_t^i + (1 - \lambda)\frac{\phi}{q_t} \right) \Delta. \qquad (17)$$

This condition states that the firm values the worker for her current and future marginal contributions to output. The last term indicates that with probability $1 - \lambda$ the position will remain occupied the following period and the firm will obtain its discounted expected value, equal to the average cost of posting vacancies, by condition (16).

The valuation of the job by the worker is determined by the difference in the value functions across different employment states

$$W^{e-u}(a_t^i, w_t^i, t) = W^e(a_t^i, w_t^i, t) - W^u(a_t^i, t). \qquad (18)$$

This valuation of the job depends on the negotiated wage and the unemployment compensation.[14]

In appendix A.1. we show that the solution to the bargaining is given by

$$\frac{W^{e-u}(a_t^i, w_t^i, t)}{U_c(c^e(a_t^i))} = \frac{\alpha_w}{\alpha_f}\Pi_n(w_t^i). \qquad (19)$$

---

[12] Since individual and aggregate conditions changes every period, this assumption prevents the introduction of commitment issues that could potentially be interesting in this setting but go beyond the scope of this paper.

[13] Note that there is no time involved in the bargain process and consequently there is no possibility of delays. In a bargaining that extends over time, the impatience of the agents for the desired outcomes acts as an incentive to reach an agreement. This is important because in this model the outside options of the agents in the bargain change over time and consequently the solution could be affected if time was considered. Coles and Wright (1998) analyze dynamic bargaining in a non-stationary environment.

[14] We make explicit the dependence of the value function on the negotiated wage since we are solving the wage function for any level of individual assets.

Since the right hand side is monotonically increasing in wages but the left hand side is monotonically decreasing, there exists a unique solution for the individual wage $w^i = w(a^i)$ for a given exogenous bargaining power and aggregate conditions. The solution states that the parties divide the total surplus generated by the occupied position according to the relative bargaining power $\frac{\alpha_w}{\alpha_f}$.[15]

There are two central aspects of this solution. First, the valuation of the job by the worker depends not only on the negotiated wage but also on her accumulated wealth since her ability to smooth consumption depends on these assets under incomplete insurance. The higher the amount of wealth, the lower would be the difference in consumption between the two employment states and the lower would the valuation of the job by the worker. Second, this valuation is expressed in units of current consumption by normalizing it by her marginal utility of consumption. This determines the effect in the bargain of risk aversion. Note that under linear preferences that divisor would just equal one.

Finally, the average wage paid by the representative firm is given by

$$w_t = \int_E w(a_t^i)di. \tag{20}$$

This completes the description of the labor market.

D. Government

The government follows a balanced budget policy in each time interval collecting lump-sum taxes from consumers and providing unemployment insurance. We consider an unemployment insurance scheme in which unemployed agents receive a constant replacement ratio χ of the average wage in the economy. Hence, we have the additional constraints

$$\tau_t = u_t b_t, \tag{21}$$
$$b_t = \chi w_t. \tag{22}$$

This completes the description of the model.

E. Stationary Equilibrium

To define a stationary equilibrium we consider invariant measures over individual asset levels μ^e and μ^u for the fraction of the population that is respectively employed and unemployed. To characterize these measures we introduce the inverse mappings $\Phi^e(a)$ and $\Phi^u(a)$, which determine today's asset holdings as a function of tomorrow's, and defined using the asset accumulation

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[15] As can be expected, the solution is consistent with the Nash axioms for a solution to a non-cooperative bargaining; we make explicit the bargain protocol because valuations are not transferable under risk aversion and to explain the determination of the relative bargaining power.

equations as

$$\Phi^e(a) + \Delta\dot{a}^e(\Phi^e(a)) = a, \tag{23}$$
$$\Phi^u(a) + \Delta\dot{a}^u(\Phi^u(a)) = a. \tag{24}$$

The condition for μ^e and μ^u to be invariant measures for any interval (a_1, a_2) are

$$(1) \; \mu^e((a_1, a_2)) = (1-\lambda)\mu^e((\Phi^e(a_1), \Phi^e(a_2))) + p\mu^u((\Phi^u(a_1), \Phi^u(a_2))), \tag{25}$$
$$(2) \; \mu^u((a_1, a_2)) = \lambda\mu^e((\Phi^e(a_1), \Phi^e(a_2))) + (1-p)\mu^u((\Phi^u(a_1), \Phi^u(a_2))). \tag{26}$$

where $\mu^e((a_1, a_2))$ and $\mu^u((a_1, a_2))$ are the fractions of the population consisting respectively of employed and unemployed persons with asset holdings between a_1 and a_2. The first condition states that the measure of workers that keep their jobs plus the measure of unemployed agents that are matched must be equal to the measure of employed in the following interval; the second, states a similar relation for unemployed agents.

Definition

A stationary equilibrium consists of:

- constant values for r, w, π, τ, b, k, n, u, v, q, p;

- measures μ^e and μ^u, defined on the Borel sets of some compact interval of asset values A; and

- policy functions $c^e(a)$, $c^u(a)$, $\dot{a}^e(a)$, $\dot{a}^u(a)$, value functions $W^e(a)$, $W^u(a)$, and a wage function $w(a)$ defined for all $a \in A$; such that

1. the policy functions $\dot{a}^e$ and $\dot{a}^u$ imply that for any $a \in A$, $a + \Delta\dot{a}^e(a)$, $a + \Delta\dot{a}^u(a) \in A$, as well;

2. given the constant values r, π, τ and the time invariant wage function $w(a)$, the policy functions $c^e(a)$ and $\dot{a}^e(a)$ are time-invariant solutions to problem (5), and $W^e(a)$ is the associated value function;

3. given the constant values r, π, τ, b the policy functions $c^u(a)$ and $\dot{a}^u(a)$ are time-invariant solutions to problem (8), and $W^u(a)$ is the associated value function;

4. given the policy functions $\dot{a}^e$ and $\dot{a}^u$ the inverse mappings $\Phi^e(a)$ and $\Phi^u(a)$ satisfy (23) and (24) for each $a \in A$;

5. given the constant values q, r, w and discount factor $Q = e^{-\rho\Delta}$ each period, and given an initial labor force n, the solution to problem (12) involves constant values k, v, n each period;

6. for each $a \in A$, $w(a)$ solves (19);

7. the values of v, u, p, q satisfy (2) and (3);

8. the values of n, u, p satisfy

$$u = 1 - n = \frac{\lambda}{\lambda + p} \tag{27}$$

which is the stationary version of (4);

9. the values of w, τ, b, u, satisfy (21) and (22);

10.the values of π, r, w, k, n, v, satisfy

$$\pi = F(k,n) - rk - wn - \phi v; \tag{28}$$

11.given the wage function $w(a)$ and the invariant measure μ^e, the average wage w satisfies (20);

12.given the invariant measures μ^e and μ^u the constant value of k satisfies

$$k = (1-u)\int a\mu^e(da) + u\int a\mu^u(da); \tag{29}$$

and

13.the invariant measures μ^e and μ^u satisfy (25) and (26).

III. SOLUTION METHOD

To analyze the behavior of this economy, we must obtain a solution for the equilibrium conditions explained above. This is a difficult task because of the wealth distribution that emerges due to agents being subject to different employment histories in the absence of complete insurance. Preston and Roca (2007) show how to apply perturbation methods to characterize state dependent agents' optimal decision rules in the presence of continuously distributed idiosyncratic and aggregate shocks that engender a dynamic wealth distribution. The key conceptual issue is that the aggregate constraint (29) imposes restrictions on the computation of the elasticities of individual decisions functions. In other words, the aggregation of the individual decision rules must be consistent with the aggregate conditions of the economy.

The main idea behind perturbation methods is to first solve the model for a particular case that has a simple, known solution. Given the information provided by that solution we then find an approximation to the more complex general solution to the problem of interest. For instance, when this methodology is applied to problems with aggregate uncertainty, the solution consists in a perturbation around the non-stochastic steady state. In the present model, we obtain an approximation around the fast-turnover limit obtained when the length of the time interval is made infinitively small. As we explain in detail below, heterogeneity vanishes in that limit and we are able to compute a solution using the usual methods for representative agent models. Then we perturb the equilibrium conditions for a positive time interval to approximate the solution to the model with heterogeneous agents.

The utilization of this approach has several advantages. First, from the determination of analytical optimal decision rules we will be able to identify how intrinsic characteristics of heterogeneous agents — such as employment status or accumulated wealth — diversely affect agents' behavior, and through aggregation, the macroeconomy. Second, in contrast to other purely numerical procedures that must resort to simulations to analyze the effects of different policies, we will be able to obtain a measure of individual welfare that explicitly takes into consideration both the

aggregate and distributional effects of policy.

We adapt the methodology presented in Preston and Roca (2007) to our specific problem of interest. That paper models idiosyncratic risk as a shock that follows a continuous stochastic process. That treatment is suitable for the use of perturbation methods because they rely on differentiability of the equilibrium conditions with respect to the state variables. In the present paper we face the complication that the idiosyncratic employment risk is a discrete variable, agents can be either employed or unemployed, creating a complication for the use of those methods. Finally, the restrictions that aggregate conditions impose on the coefficients of the individual decision functions need to be made explicit. The following sections explain in detail how we deal with these issues.

A. Fast-turnover limit

To characterize the equilibrium around which we approximate the solution to the economy with uninsurable idiosyncratic risk, we proceed in two steps. The first step, described in this section, considers the limit when all agents are changing employment states infinitely fast and the wealth distribution becomes degenerate at the aggregate level of capital. In the next section, we describe the steady state as a special case of the former in which individual and aggregate wealth remain constant.

Consider an interval of size Δ. During this interval, an endogenous fraction p of unemployed agents form a match with a vacancy posted by the representative firm, and an exogenous fraction λ of workers losses their work. We can define the turnover rates between employment status as $\frac{p}{\Delta}$ and $\frac{\lambda}{\Delta}$ respectively. These rates indicate the speed at which agents are changing employment status. As we shrink the size of the time interval Δ, the turnover rates increase, producing a decline in the mean duration that agents spend in each state. In the limit when $\Delta \rightarrow 0$, agents change status infinitely fast.

In this fast-turnover limit, agents receive the same average income from labor and accumulate equal amounts of wealth per unit of time. Thus, their consumption and saving decisions are alike. It is worth noting that this case is consistent with the commonly used assumption, after Merz (1995), that individuals belong to extended households in which they pool their income. An important difference is that in our model agents become homogeneous only in the limit, and further, we are able to analyze their behavior outside the limit when heterogeneity becomes important. It is this device that permits one to treat employment and unemployment as discrete states yet still apply perturbation methods that rely on differentiability.

An important aspect of our modeling is that unemployment and vacancies are endogenously determined in a general equilibrium model according to agents' optimizing decisions. As such, they are influenced not only by other endogenous variables but also by the length of the time interval under consideration. Consequently, the endogenous rates q and p are also dependent on the length of the time interval, but only through to the operation of the general equilibrium

model.[16]

In this respect, our model differs from the standard Mortensen and Pissarides model. In their model, the transition probabilities follow a Poisson process that implies—using similar notation for ease of comparison—that the number of matches in any given interval of size Δ is given by $m\Delta$, and that the job-filling, job-finding and job destruction rates are respectively given by $q\Delta$, $p\Delta$, and $\lambda\Delta$. All of these rates approach zero per period as the period length Δ is made increasingly small, though there remains a non-zero (and well-defined) rate per unit of time. In our model, instead, the rates approach a positive limit as Δ approaches zero.

In appendix A.2. we show that taking the limit $\Delta \to 0$ to the first order conditions of (10) and (11), the consumers optimality conditions can be expressed as

$$(\rho - r_t + \delta)U_c(c(a_t^i)) = U_{cc}(c(a_t^i))c_a(a_t^i)\dot{a}(a_t^i) + \varepsilon B_a(a_t^i), \tag{30}$$

where

$$\dot{a}(a_t^i) = (r_t - \delta)a_t^i + \frac{p_t}{\lambda + p_t}w(a_t^i) + \frac{\lambda}{\lambda + p_t}b_t + \pi_t - \tau_t - c(a_t^i) \tag{31}$$

is the common rate of asset accumulation. Note that these are similar to the standard conditions that would be obtained in a representative agent model subject to the same borrowing constraint. This is the expected result since agents in this state are homogeneous and do not face any income risk. In this limit, it is therefore a deterministic model.

To solve for the steady state wage we use the bargaining solution (19). In appendix A.2.2. we show that taking the limit $\Delta \to 0$ to the difference in the value functions (5), and (8), and to the valuation of the job by the firm (17) we obtain the following wage function

$$w_t = \frac{\alpha_f}{\alpha_f + \alpha_w}b_t + \frac{\alpha_w}{\alpha_f + \alpha_w}\left(F_n(k_t, n_t) + (1 - \lambda)\frac{\phi}{q_t}\right). \tag{32}$$

This relation is similar to the one obtained under the assumption of complete insurance against labor income risk or linear preferences, and consequently is independent of individual wealth. It states that the wage is a linear combination between the two extremes of the bargaining range with a weight given by the exogenous bargaining power of the two agents. When the worker has no relative power ($\frac{\alpha_w}{\alpha_f} \to 0$) the firm appropriates the whole surplus of the job by paying the minimum wage possible, the one that equates the unemployment subsidy. On the other extreme, when the relative bargaining power of the firm is nil ($\frac{\alpha_w}{\alpha_f} \to \infty$), the worker gets all the surplus by negotiating a wage that equates the marginal product of labor plus the savings in posting vacancies.

From the analysis of the conditions characterizing the fast-turnover limit we can observe the

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[16]   All endogenous variables in our model depend on the size of the time interval. In the case of the transition probabilities we should write $p(\Delta)$ and $q(\Delta)$ but for clarity in the exposition we do not show explicitly that dependence.

limited influence of unemployment under the prevailing assumption of complete insurance. In that case, welfare of employed and unemployed agents is the same, and correspond to the predictions of the model assuming a representative household. The only departure from a model with competitive labor markets is that labor income is different from the marginal product of labor due to the presence of search frictions.

B. Approximation

Our objective is to obtain a linear approximation to the decision functions around the described steady state. A departing point is the value of those functions at that steady state computed in the previous section. Then we determine the behavior of these functions around that expansion point by computing the derivatives of the equilibrium conditions with respect to the state variable a and the length of the time interval Δ. These derivatives form a system of equations. The solution to this system gives the desired coefficients determining the behavior of the functions around the steady state. Hence, this is just the usual first order approximation solution to a system of differential equations. What is innovative is that we take the approximation with respect to the size of the time interval; from the methodological point of view this is treated as another state variable.

Consider for instance the function describing the consumption of the employed agent. We want to obtain a first order approximation to that function around the steady state $(a, \Delta) = (\bar{a}, 0)$ of the following form

$$c^e(a, \Delta) \approx \bar{c} + c_a(a - \bar{a}) + c_\Delta^e \Delta \tag{33}$$

That is, the approximate value that the function $c^e(a, \Delta)$ takes around its steady state value $\bar{c} = c^e(\bar{a}, 0)$ depends on the deviation of individual assets from their steady state level and the exposure to uninsurable risk, measured by Δ. The coefficients c_a and c_Δ^e quantify these responses. An interesting feature of our methodology is that we can identify the effects of idiosyncratic risk with the last term in equation (33); without considering this term the approximation would correspond to the one obtained under the complete insurance assumption.

Steady state

The steady state is a particular case of the fast-turnover limit defined with the additional condition $\dot{a}(\bar{a}) = 0$. In this case, conditions (30) and (31) become

$$(\rho - r + \delta)U_c(\bar{c}) = \varepsilon B_a(\bar{a}), \tag{34}$$

and

$$\bar{c} = (r - \delta)\bar{a} + \frac{p}{\lambda + p}w + \frac{\lambda}{\lambda + p}b + \pi - \tau. \tag{35}$$

Furthermore, individual and aggregate capital are equal and constant,

$$k = \bar{a}. \tag{36}$$

From these conditions together with (2), (3), (15), (16), (21), (22), (27), (28), and (32) we solve for the steady state values $\bar{c}$, $\bar{a}$, k, q, p, r, v, τ, b, u, n, π, and w.

Approximation around steady state

To obtain the coefficients c_a and a_a determining the response of consumption and savings to individual assets we obtain a local approximation to the saddle-path determining the dynamics of the individual consumption-savings decision in a standard way. For completeness, details of the calculations and conditions for determinacy are provided in appendix A.3.1.

To obtain the coefficients determining the response of all endogenous variables to the length of the time interval Δ we take the derivatives of the equilibrium conditions with respect to this variable and then we solve the resulting system of linear equations for those coefficients. One important innovation of our methodology is the treatment of the aggregate condition, and consequently this is explained in detail in the next subsection. The rest of the calculations are standard and explained in appendix A.3.2.

Aggregate conditions and stationary distribution

When dealing with heterogeneous agent models it is important to explicitly consider the restrictions given by aggregate conditions in order to ensure consistency with the individual decision functions. Since our objective is to obtain a linear approximation, we only need to impose that consistency between the linearized aggregate conditions and the linear approximation to the individual decision rules. In this case, the analysis of these relations will also shed light about the dependence of the distribution on the length of the time interval Δ.

Consider the linear approximation to the functions characterizing the rate of asset accumulation of employed and unemployed agents

$$\dot{a}^e(a) \approx a_a(a - \bar{a}^e), \tag{37}$$
$$\dot{a}^u(a) \approx a_a(a - \bar{a}^u), \tag{38}$$

where $\bar{a}^e$ and $\bar{a}^u$ are defined by

$$\dot{a}^e(\bar{a}^e) = 0, \text{ and } \dot{a}^u(\bar{a}^u) = 0,$$

or using (6) and (9)

$$\bar{a}^e = \frac{w + \pi - \tau - c^e}{r - \delta}, \text{ and } \bar{a}^u = \frac{b + \pi - \tau - c^u}{r - \delta}. \tag{39}$$

Conditions (25), and (26) can be expressed in terms of the corresponding (normalized) density

functions f^e and f^u

$$\int a f^e(a) da = (1-\lambda) \int f^e(\Phi^e(a)) \Phi^{e'}(a) da + \frac{u}{1-u} p \int f^u(\Phi^u(a)) \Phi^{u'}(a) da,$$

$$\int a f^u(a) da = \frac{1-u}{u} \lambda \int f^e(\Phi^e(a)) \Phi^{e'}(a) da + (1-p) \int f^u(\Phi^u(a)) \Phi^{u'}(a) da.$$

Noting from (23) and (24) that

$$\Phi^{e'}(a) = \frac{1}{1 + \Delta a_a^e(\Phi^e(a))}, \text{ and } \Phi^{u'}(a) = \frac{1}{1 + \Delta a_a^u(\Phi^u(a))},$$

the previous conditions can be expressed as

$$\int a f^e(a) da = (1-\lambda) \int (a + \Delta \dot{a}^e(a)) f^e(a) da + \frac{u}{1-u} p \int (a + \Delta \dot{a}^u(a)) f^u(a) da,$$

$$\int a f^u(a) da = \frac{1-u}{u} \lambda \int (a + \Delta \dot{a}^e(a)) f^e(a) da + (1-p) \int (a + \Delta \dot{a}^u(a)) f^u(a) da.$$

Next, by replacing the rate of asset accumulation by their approximations given by (37) and (38), and appropriately reordering we get

$$\lambda k^e - (1-\lambda) \Delta a_a (k^e - \bar{a}^e) = \lambda k^u + \lambda \Delta a_a (k^u - \bar{a}^u), \tag{40}$$

$$p k^u - (1-p) \Delta a_a (k^u - \bar{a}^u) = p k^e + p \Delta a_a (k^e - \bar{a}^e), \tag{41}$$

where k^e and k^u are the average asset holdings of employed and unemployed agents respectively. These equations can alternatively be expressed as the set of linear restrictions

$$\begin{bmatrix} k^e \\ k^u \end{bmatrix} = \left[I_2 - \left(\frac{1}{\Delta a_a} - 1 \right) \begin{bmatrix} \lambda & -\lambda \\ -p & p \end{bmatrix} \right]^{-1} \begin{bmatrix} 1-\lambda & \lambda \\ p & 1-p \end{bmatrix} \begin{bmatrix} \bar{a}^e \\ \bar{a}^u \end{bmatrix}. \tag{42}$$

Finally, the aggregate level of capital is given by

$$k = \frac{1}{\lambda + p} (p k^e + \lambda k^u). \tag{43}$$

These conditions characterize the wealth distribution in a first order approximation. We replace equilibrium condition (29) with these linear restrictions in order to obtain our approximate solution.

It is worth noting how the distribution depends on the length of the time interval, the mechanism that permits the application of our methodology. From (40), and (41) we observe that when $\Delta \to 0$, the mean asset level held by the employed agents k^e approximates that of the unemployed k^u, and consequently both values approximates the aggregate asset level. In other words, the distribution becomes degenerate. For positive values of Δ, these mean asset levels held by each type of agent diverge and the distribution becomes nondegenerate. Moreover, for a given Δ, it is also stationary.

IV. QUANTITATIVE ANALYSIS

While this type of approximation could be performed analytically, we proceed to work with a calibrated version of the model to obtain easily interpretable coefficients.

A.   Calibration

We consider a time period of one month. Preferences are of the CRRA type

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{44}$$

with coefficient of relative risk aversion σ set to 2. The consumers' discount rate ρ is set to 0.3356% per month, implying a quarterly discount factor of 0.99.

The production function is Cobb-Douglas

$$F(k, n) = zk^{\alpha}n^{1-\alpha}, \tag{45}$$

with share of capital α set equal to 0.36. The constant z is used to normalize wages to one in steady state. We consider an annual depreciation rate of capital δ of 10% or equivalently 0.83% per month.

The parameter ν of the barrier function (7) is set to 2, while ε is 0.1, and the borrowing limit $\tilde{a}$ is set to 0 implying that agents cannot borrow.

Regarding the parameters characterizing the labor market, we calibrate the job destruction rate λ to 0.034 per period. This value implies that when $\Delta = 1$ jobs last on average thirty months, consistent with the empirical evidence for the U.S. presented on Shimer (2005). The matching function $m(u, v)$ is implicitly defined by the equation

$$m = A(u - m)^{\gamma}(v - m)^{1-\gamma}. \tag{46}$$

Note that this specification reduces to the commonly used Cobb-Douglas as A is made infinitely small. As such, it retains its main properties, like linear homogeneity. However, it has the advantage that the job-finding and the job-filling rates originated by this function are always contained in the unit interval. This must always hold since the number of matches cannot be larger than the number of unemployed agents or the number of vacancies. In a discrete time setting like ours, the Cobb-Douglas specification does not assure that these rates remain in the unit interval. For a Cobb-Douglas specification, Petrongolo and Pissarides (2001) reports that a suitable value for γ lies between 0.5 and 0.7. We set γ equal to 0.5.

We initially follow Shimer (2005) and set the replacement ratio χ to 40%, that is considered to be an upper bound for the U.S.; O.E.C.D. (2006) reports that the average replacement rate for the U.S. is 36%. Later, we investigate the effects of varying levels of this policy instrument and determine its optimal level in this economy.

Since a priori, there is not indication about what could be an appropriate level of the relative bargaining power of the firm and workers, we give the same bargaining strength to each party, setting $\frac{\alpha_w}{\alpha_f}$ equal to one.

Finally we calibrate the flow cost of posting vacancies ϕ and the matching function constant A to target the job-finding rate of 0.45 per month estimated by Shimer (2005). Since in this model, we are free to set the value of the vacancy-unemployment ratio,[17] we target a value of 1. Together with the desired job-finding rate, this pins down the constant of the matching function to 0.8181, and implies that the job-filling rate equals the job-finding rate in steady state. Then, using these values in the optimizing condition of the firm (16), we pin down the flow cost of posting vacancies ϕ to be equal to 0.2699. This value represents approximately one fourth of steady state monthly wages.

B.   The effects of idiosyncratic risk

In this section we analyze the effects of introducing idiosyncratic risk on aggregate conditions and individual optimal decisions. In Table 1 we present the effects on some variables directly related to the labor market. We compare the values of those variables in the fast-turnover limit, that is for the limit in which Δ approaches 0, and in a first-order approximation to an equilibrium with a small positive value of Δ. The positive-Δ case that we consider is one in which Δ equals one month. Giver our other parameters, this case corresponds to roughly the rate of flows in and out of unemployment in the U.S.

Table 1. Effects of Idiosyncratic Risk on the Labor Market

		Fast-Turnover	Idiosyncratic Risk	Variation
		$\Delta \to 0$	$\Delta = 1$	(%)
Wage (w)*		1.0000	0.9999	-0.01
Vacancies (v)		0.0702	0.0706	0.57
Unemployment (u)		0.0702	0.0699	-0.43
Job-finding Rate (p)	per period	0.4500	0.4523	0.51
	per month	∞	0.4523	
Job-filling Rate (q)	per period	0.4500	0.4477	-0.51
	per month	∞	0.4477	
* Average over employed agents				

The last rows of Table 1 are useful to illustrate our method. In there we present the different values of the job-finding and job-filling rates under these two assumptions In the fast-turnover limit, while the per period rates have a finite positive value equal to 0.45, the rates per month are infinite and consequently the mean duration of each state goes to zero. When we introduce positive delay, the rates per unit of time become finite, and consequently the mean duration becomes positive. When $\Delta = 1$ the rate per period equals the rate per month at an endogenously determined value.

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[17]   Choosing another vacancy-unemployment ratio would only produce a proportionate adjustment in the level of vacancies. For appropriately chosen parameter values, the job finding rate and the unemployment rate would not be affected.

Note that the resulting job-finding rate is still close to our target value of 0.45.

The introduction of incomplete markets primarily affects the outcome of the wage bargaining because it changes the outside option that workers would experience in case they do not arrive to an agreement with the firm. This is reflected in a change in the worker's valuation of the job that will be specific to each agent because her asset holdings determine her capacity of smoothing consumption between employment states. The average valuation increases in going from the fast-turnover limit to the new equilibrium with idiosyncratic risk producing an increment in the total surplus of the match. This additional surplus is divided according to the exogenous relative bargaining power through a reduction in wages. In Table 1 we observe that average wages decline by 0.01%.

The higher surplus obtained by firms gives them incentives to post more vacancies, that in turn increases the number of employed agents. As a consequence, the trading externalities for unemployed agents decrease, and the trading externalities for firms increase. In Table 1 we observe that the job-finding rate increases by 0.51% and the job-filling rate decreases by the same percentage.

We can analyze the effects of incomplete markets on agent behavior from the approximation to the individual decision functions. Figure 1 presents consumption profiles with different exposure to uninsurable risk and different employment status. We observe that in the fast-turnover limit, in which there is no effect of idiosyncratic risk, all agents share the same consumption function that is positively correlated with variations in individual wealth. As can be expected in this case, the aggregate consumption function coincides with the individual ones. The introduction of idiosyncratic risk generates a decline in the consumption profile of unemployed agents but an increase in that of employed agents. This reflects the fact that in the presence of incomplete markets agents are unable to perfectly smooth their consumption across the different employment states. It is also qualitatively consistent with the empirical evidence documented by Gruber (1997), who shows that consumption declines upon becoming unemployed. Moreover, taking the unconditional expectation of those consumption functions we obtain a consumption profile that lies below that of the fast-turnover limit. In other words, on average, consumption is reduced at any wealth level because of the presence of uninsurable risk.

In Table 2 we present the general equilibrium effects generated by the introduction of incomplete insurance. We observe that the average consumption of employed agents increases by 0.02% in comparison with the fast-turnover limit benchmark, but the average consumption of the unemployed population decreases by 1.13%. The latter pulls down the aggregate consumption of the whole population that decreases by 0.06%.

The effect of incomplete markets is particularly noticeable in the saving decision. We observe that the average wealth of unemployed agents is reduced by 4.36%. In contrast, the average wealth of employed agents increases by 0.37%. The aggregation of these effects produce an increase in total wealth of 0.04%. This is due to two different mechanisms. One is that each consumer expends more time in each employment state and consequently this is translated to differences in

Figure 1. Approximation to Consumption Functions
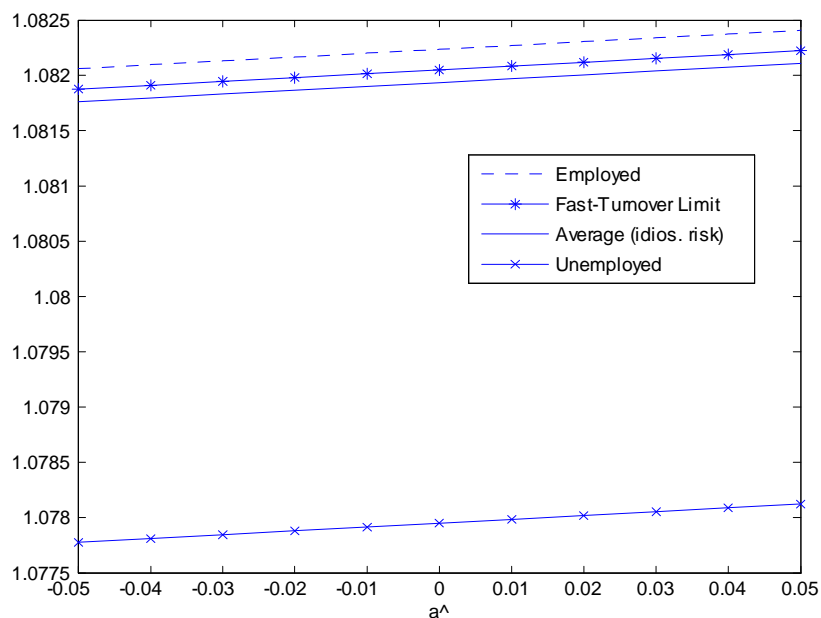
the accumulated level of assets. The second is the usual precautionary motive to save. Employed agents increase their savings so when they become unemployed they can adjust the level of assets in order to smooth consumption between employment states.

C. Optimal replacement rate

In this section we analyze the effects of unemployment insurance on welfare in order to determine the optimal level of the replacement rate χ. Our objective is to consider in particular how that optimal level is affected by the presence of incomplete markets.[18]

To evaluate policy changes we use the following social welfare criterion

$$W = (1 - u) \int W^e(a)\mu^e(da) + u \int W^u(a)\mu^u(da), \tag{47}$$

where $W^e(a)$, and $W^u(a)$ are the value functions (5), and (8).[19]

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[18]   We restrict our analysis to the policy described in (22). Whenever we refer to optimal policy we mean the optimal level of the replacement rate. Consideration of more general policies is left for future work.

[19]   This measure of welfare has the useful alternative interpretation of the ex-ante welfare of an individual before employment status and asset level have been realized. In that sense, it is egalitarian because it gives the same weight to every individual. However, it has the drawback that it does not evaluate the transition dynamics of changes in policy. We use this criterion here in order to compare stationary equilibriums. Future research will consider a more general criterion that evaluates those transition dynamics.

Table 2. Effects of Idiosyncratic Risk on Consumption and Capital

|		Fast-Turnover $\Delta \to 0$	Idiosyncratic Risk $\Delta = 1$	Variation (%)
Consumption (c)*	Aggregate	1.0821	1.0815	-0.06
	Employed	1.0821	1.0823	0.02
	Unemployed	1.0821	1.0699	-1.13
Capital (a)*	Aggregate	45.5493	45.5678	0.04
	Employed	45.5493	45.7185	0.37
	Unemployed	45.5493	43.5629	-4.36
* Average over respective population				

We solve the model for different values of the replacement rate and compare the value attained by (47). The optimal policy rate is determined by the value that maximizes this criterion. Additionally, to have a sense of the magnitude of the welfare changes associated with variations in policy we express the welfare gains in terms of percentage changes in consumption using as a benchmark the value obtained under our baseline calibration with $\chi = 0.40$. Figure 2 presents these welfare gains for the fast-turnover limit and for the economy with idiosyncratic risk. In the case of the fast-turnover limit the welfare gains are maximized for a replacement rate of 0%, that is, any positive level of unemployment insurance is detrimental to welfare. In contrast, with idiosyncratic risk the optimal replacement rate is positive and it equals 30%.

To understand the differences in optimal policy we need to analyze deeper the mechanisms through which policy operates.

In this framework, unemployment insurance has a direct effect on wages. In altering the income that an agent receives while unemployed, it affects the outside option for a current employee and consequently it influences the outcome of the wage bargaining. Figure 3 presents the general equilibrium effects on different endogenous variables of varying levels of the replacement rate in the fast-turnover limit. We observe that a higher replacement rate induces higher wages since it increases the worker's threat point in the bargain. This reduces the firms' valuation of a filled position, and consequently they reduce the number of posted vacancies. This has negative consequences on employment producing an increment in the unemployment rate. The reduction in labor pressures for a decline in the return on capital, and ultimately reduces the stock of aggregate capital. This can be explained by the low response of the interest rate to changes of policy. We observe that the wedge between the discount rate and the net interest rate $\rho - r + \delta = 0.03\%$, remains practically unaltered. This implies that the capital-labor ratio also suffers little modification and consequently aggregate capital must adjust to changes in labor. Overall, the reduction in the inputs of production generates a decline in output, and in consumption.

Hence, we observe that the lowest level possible of unemployment insurance is desired in the fast-turnover limit because any positive level is inefficient due to the influence that it exerts in the wage bargaining. It is important to remark that the analysis of the general equilibrium effects is crucial in this analysis, in a partial equilibrium model that takes as given the marginal product of

Figure 2. Variations in Welfare
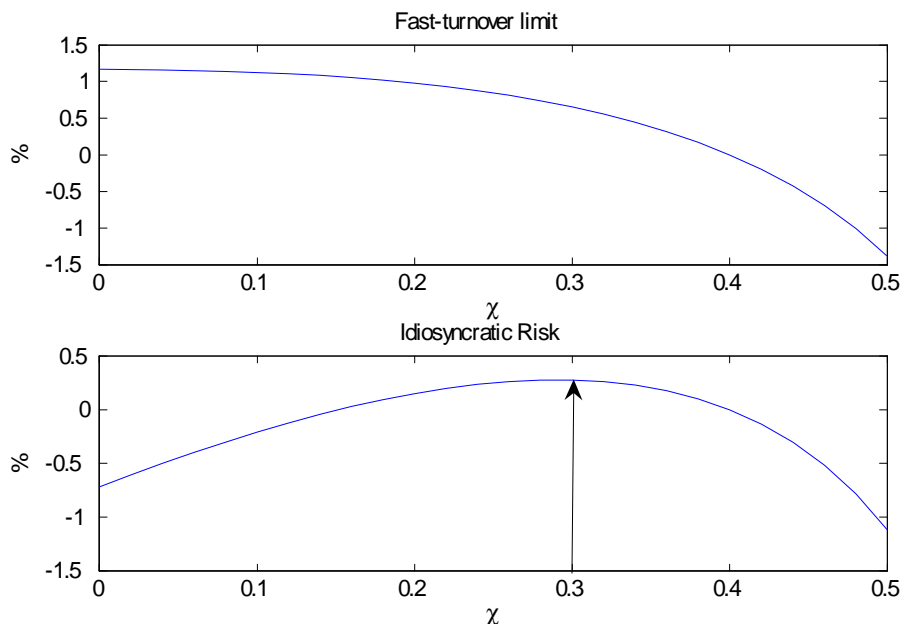
labor the effects of the wage bargain on aggregate capital would be missed and consequently such an analysis could lead to erroneous evaluation of the insurance policy.

The introduction of idiosyncratic risk into the analysis produces the same qualitative effects on endogenous variables as those presented in tables 1 and 2. The different levels of the replacement rate only modify their relative magnitude. Figure 4 shows the effects of idiosyncratic risk on individual decision functions and, throw their aggregation, on aggregate conditions for different levels of the replacement rate. All effects are measure as percentage of the corresponding steady state values. In panel A we observe that the consumption profile of unemployed agents is reduced by the presence of the uninsurable risk, but also that such effect is inversely related to the magnitude of the replacement rate. Panel B shows that the average consumption of the unemployed population is affected in the same way. This implies that the insurance policy effectively helps to complete the market even when the general equilibrium effects are taken into account. As a consequence agents have less incentive to save because of precautionary motives and the reduction in consumption relative to that of the fast turnover limit is diminished.

It is important to note the usefulness of our methodology in this analysis, as it permits us to isolate the variation in consumption due to different levels of idiosyncratic risk from the variation in consumption due to general equilibrium effects discussed above. With other numerical methods, disentangling these two effects would not be immediate.

This positive effect of policy in consumption is also translated to a positive effect on welfare. The optimal level of the replacement rate under incomplete markets balances the trade-off between

Figure 3. Effects of Unemployment Insurance
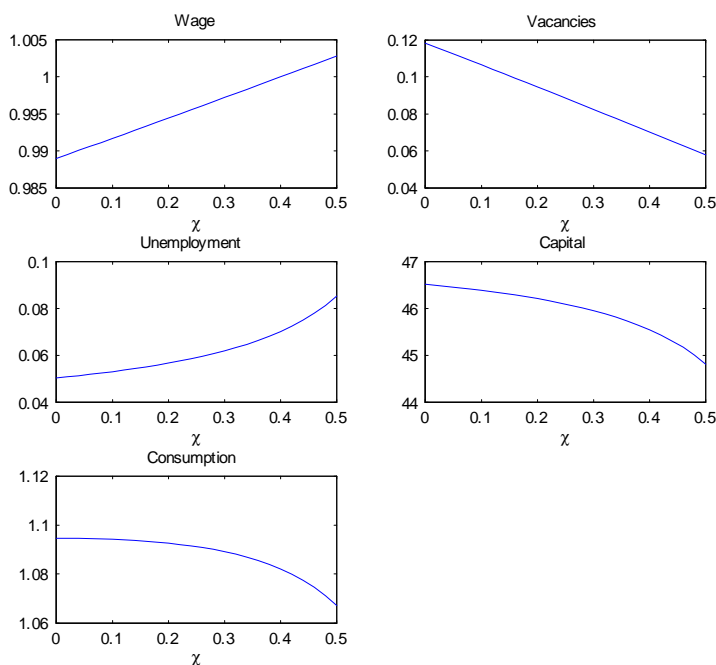
the (negative) efficiency effects and the (positive) insurance effects of policy. Since the latter effect is not present under the complete insurance assumption the optimal level of the replacement rate under incomplete markets will always be as least as high as the obtained under the first assumption, and will generally be higher. In other words, the assumption of complete markets underestimates the positive effects of unemployment insurance and hence leads to misleading policy recommendations for an economy with incomplete markets.

V. CONCLUSIONS

This paper has analyzed the importance of incomplete insurance in a general equilibrium model with search frictions in the labor market and ex-post bargaining over the rents created by a match. We show that policy recommendations are markedly different when the effects of idiosyncratic income risk are taken into account. For a calibration of the model to the U.S. economy, we find that the optimal level of the replacement ratio is about 30%. In contrast, under the assumption of complete income insurance, the model would not justify a positive replacement ratio.

Our model can also explain the existence of a decline in consumption upon becoming unemployed that has been documented in the empirical literature. Employed agents in this model save due to their precautionary motives. We find that employed agents continuously increase their accumulated wealth and as a consequence they have a better threat point in the bargaining over

Figure 4. Effects of Idiosyncratic Risk

A. Effect on Unemployed Agents` Consumption Function

B. Effect on Average Consumption of Unemployed Agents
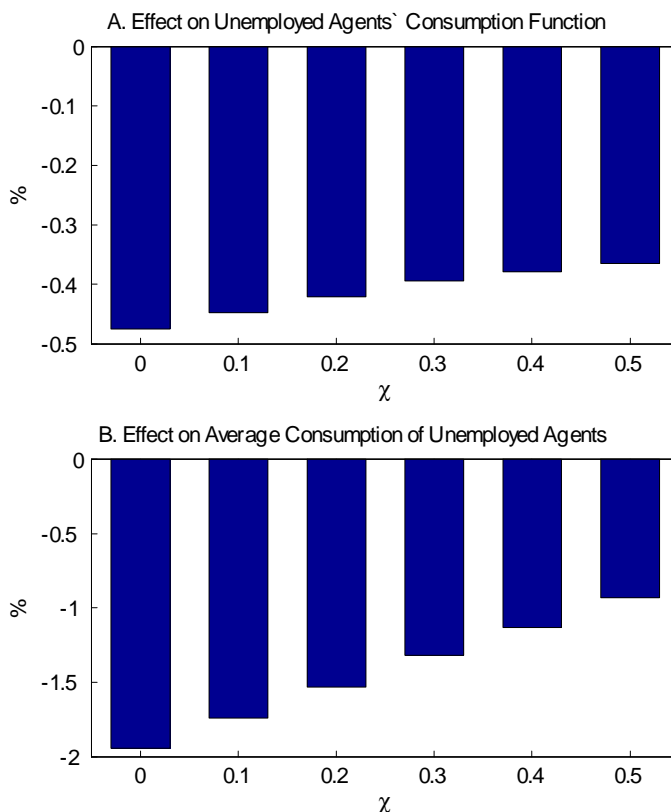
time. In contrast, unemployed agents run down their savings to buffer consumption from the income shock. As other theoretical works analyzing the importance of incomplete markets,[20] our qualitative results are consistent with empirical findings but the magnitude of those results seems to be relatively small. This is originated in the fact that agents in our model can get considerable self-insurance against income fluctuations by moderately increasing their savings. This could indicate that the magnitude and/or the persistence of the idiosyncratic risk in our model fall short of those observed in reality. Moreover, the assumption of ex-ante homogeneity in conjunction with an infinite horizon reduces the effect of that risk. If agents were ex-ante heterogeneous and they faced different risks due to this heterogeneity, both the persistence and magnitude of those risks could be increased. If they faced a finite horizon, their decisions would be more pronounced in order to spread a given income shock in a shorter horizon. In summary, the fact that our quantitative results are relatively small but our qualitative results are consistent with observable facts indicates that the modeled economy is not risky enough, or that markets are still too close to completion, and not that a model constructed under the assumptions of complete

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[20]   See for instance Aiyagari (1994).

markets and/or linear utility could be a good approximation. As we have demonstrated, the policy recommendation derived from such a model could be markedly different.

Another contribution of this paper is the proposed methodology to solve heterogeneous-agent models with a discrete state variable such as unemployment through the consideration of a fast-turnover limit. This methodology has several advantages. First, it allows us to analytically characterize the responses of individual decision functions and aggregate conditions to changes in the states of the model. Second, our approach permits the isolation of the effects of idiosyncratic risk and consequently allows us to easily evaluate the effects of incomplete markets in this framework. Third, we have focused on the analysis of idiosyncratic risk in a stationary environment, but the model and the methodology that we have used can easily incorporate aggregate shocks to analyze the business cycle. In such a model, it would natural to also analyze the response of wages with respect to the economy-wide shock under the additional frictions generated by wealth heterogeneity in the wage bargaining. Additionally, one could easily analyze the dynamics of the aggregate capital stock. This also allows the consideration of the transition dynamics originated from changes in policy in the welfare analysis of alternative unemployment insurance policies.

We have restricted our analysis to a constant unemployment insurance policy. Under this policy, unemployed workers receive the same subsidy regardless of their wealth or the duration of an unemployment spell. In future research, it would be worth investigating the more general problem of optimal insurance without restriction to this particular class of policies.

The object of this paper has been to analyze the effects of relaxing the strong assumption of complete insurance in a model that shares the main features of the commonly used Mortensen-Pissarides search and matching model with noncooperative wage determination. We have not attempted to analyze other possible labor contracts that might be preferable in this framework. Since our model includes incomplete markets and risk-averse workers, an employment contract between the firm and the worker could well be mutually beneficial and superior in terms of efficiency. The aim of the present paper is simply to investigate the effects of idiosyncratic risk on the most commonly used model of wage bargaining and determine how these would influence the aggregate equilibrium. The analysis of optimal labor contracts in such a setting is undoubtedly a topic for future research.

For the purposes of this paper it has also sufficed to assume the existence of incomplete markets without inquiring into the causes that could lead to them. Future research will investigate how incomplete insurance can arise endogenously. Some possible causes for incompleteness could be the existence of limited enforcement or private information.

APPENDICES

I. DERIVATION OF THE SOLUTION TO THE WAGE BARGAINING

The valuation of the job by each party in the bargain is given by equations (17) and (18), that we repeat here for ease of exposition,

$$\Pi_n(w_t^i) = \left(F_n(k_t, n_t) - w_t^i + (1-\lambda)\frac{\phi}{q_t} \right) \Delta,$$

$$W^{e-u}(a_t^i, w_t^i, t) = W^e(a_t^i, w_t^i, t) - W^u(a_t^i, t).$$

A higher negotiated wage increases the valuation of the job by the worker, but it has a negative effect on the valuation of the position by the firm. Hence, during the bargaining process the worker will offer the maximum wage possible such that the firm gets at least a valuation for the position that equals its expected value; for a higher wage the firm will prefer to reject the offer. Similarly, the firm will offer the minimum wage possible such that the worker gets at least the expected value of the job. Denote by $w^{i,\max}$ the maximum wage that will be offered by the worker, and by $w^{i,\min}$ the minimum wage that will be offered by the firm. Hence, the offers of the worker and of the firm must satisfy

$$\Pi_n(w_t^{i,\max}) = (1 - \alpha_w)\,\Pi_n(w_t^{i,\min}), \tag{A-1}$$

and

$$W^{e-u}(a_t^i, w_t^{i,\min}, t) = (1 - \alpha_f)W^{e-u}(a_t^i, w_t^{i,\max}, t). \tag{A-2}$$

The difference in the wage offers decrease along with the probabilities α_w and α_f. For sufficiently small values of these probabilities, (A-1) and (A-2) can be expressed as

$$dw_t^i = \alpha_w \Pi_n(w_t^i),$$

and

$$\frac{\partial W^{e-u}(a_t^i, w_t^i, t)}{\partial w_t^i} dw_t^i = \alpha_f W^{e-u}(a_t^i, w_t^i, t).$$

Combining these conditions we obtain the bargaining solution

$$\frac{W^{e-u}(a_t^i, w_t^i, t)}{U(c^e(a_t^i))} = \frac{\alpha_w}{\alpha_f}\Pi_n\left(w_t^i\right).$$

where we have replaced $\frac{\partial W^{e-u}(a_t^i, w_t^i, t)}{\partial w_t^i} = \frac{\partial W^e(a_t^i, w_t^i, t)}{\partial w_t^i} = U(c^e(a_t^i))$ by application of the envelope theorem to (5). This is equation (19) in the text.

II. FAST-TURNOVER LIMIT

Derivation of the Euler condition

Taking the limit $\Delta \to 0$ to the first order condition for optimality for employed agents (10) we get

$$(\rho - r_t + \delta - (1 - \lambda)w_a(a_t^i))U_c(c^e(a_t^i)) = U_{cc}(c^e(a_t^i))c_a^e(a_t^i)\dot{a}^e(a_t^i) + \lambda\psi(a_t^i) + \varepsilon B_a(a_t^i), \quad \text{(A-1)}$$

where $\psi(.)$ is a term of order Δ defined as

$$\Delta\psi(a_t^i) = U_c(c^u(a_t^i)) - U_c(c^e(a_t^i)).$$

Similarly, in the limit $\Delta \to 0$ the first order condition (11) becomes

$$(\rho - r_t + \delta - p_t w_a(a_t^i))U_c(c^u(a_t^i)) = U_{cc}(c^u(a_t^i))c_a^u(a_t^i)\dot{a}^u(a_t^i) - p_t\psi(a_t^i) + \varepsilon B_a(a_t^i). \quad \text{(A-2)}$$

Finally, by application of the Law of large numbers and using that in the limit $c^u(a_t^i) = c^e(a_t^i) = c(a_t^i)$ the unconditional expectation of (A-1) and (A-2) is given by

$$(\rho - r_t + \delta - p_t w_a(a_t^i))U_c(c(a_t^i)) = U_{cc}(c(a_t^i))c_a(a_t^i)\dot{a}(a_t^i) + \varepsilon B_a(a_t^i), \quad \text{(A-3)}$$

where the rate of asset accumulation is given by

$$\begin{aligned}
\dot{a}(a_t^i) &= \frac{p_t}{\lambda + p_t}\dot{a}^e(a_t^i) + \frac{\lambda}{\lambda + p_t}\dot{a}^u(a_t^i) \\
&= (r_t - \delta)a_t^i + \frac{p_t}{\lambda + p_t}w(a_t^i) + \frac{\lambda}{\lambda + p_t}b_t + \pi_t - \tau_t - c(a_t^i).
\end{aligned} \quad \text{(A-4)}$$

Derivation of the wage equation

From the difference between (5) and (8) we get that the valuation of a job by a worker is given by

$$\begin{aligned}
W^{e-u}(a_t^i) &= [U(c^e(a_t^i)) - U(c^u(a_t^i))]\Delta + e^{-\rho\Delta}\{(1 - \lambda - p)(W^e(a_t^i) - W^u(a_t^i)) \\
&\quad + [(1 - \lambda)W_a^e(a_t^i) + \lambda W_a^u(a_t^i)]\dot{a}^e(a_t^i)\Delta \\
&\quad - [pW_a^e(a_t^i) + (1 - p)W_a^u(a_t^i)]\dot{a}^u(a_t^i)\Delta\} + \vartheta(\Delta^2)
\end{aligned} \quad \text{(A-5)}$$

Considering only terms of order Δ, and using the envelope conditions

$$\begin{aligned}
W_a^e(a_t^i) &= U_c(c^e(a_t^i))(1 + (r_t - \delta + w_a(a_t^i))\Delta) \\
W_a^u(a_t^i) &= U_c(c^u(a_t^i))(1 + (r_t - \delta)\Delta),
\end{aligned}$$

the valuation of the job by a worker can be expressed as

$$(\lambda + p_t)W^{e-u}(a_t^i) = U_c(c(a_t^i))(w(a_t^i) - b_t)\Delta + \vartheta(\Delta^2).$$

Finally, let $V(.)$ be defined by

$$W^{e-u}(a_t^i) = \Delta V(a_t^i),$$

such that in the limit $\Delta \to 0$ we obtain

$$V(a_t^i) = \frac{U_c(c(a_t^i))}{\lambda + p_t}[w(a_t^i) - b_t].$$

Replacing this expression together with the valuation of the job by the firm (17) in the wage bargaining solution (19) we get

$$w_t = \frac{1}{1 + \frac{\alpha_w}{\alpha_f}}b_t + \frac{\frac{\alpha_w}{\alpha_f}}{1 + \frac{\alpha_w}{\alpha_f}}\left(F_n(k_t, n_t) + (1 - \lambda)\frac{\phi}{q_t}\right);$$

this is equation (32) in the text.

From this condition we observe that the wage solution does not depend on individual assets in the fast-turnover limit. This is consistent with the individuals not being exposed to idiosyncratic risk in this limit. Hence, we have that $w_a(a_t^i) = 0$, and conditions (A-3) and (A-4) can respectively be expressed as

$$(\rho - r_t + \delta)U_c(c(a_t^i)) = U_{cc}(c(a_t^i))c_a(a_t^i)\dot{a}(a_t^i) + \varepsilon B_a(a_t^i),$$

and

$$\dot{a}(a_t^i) = (r_t - \delta)a_t^i + \frac{p_t}{\lambda + p_t}w_t + \frac{\lambda}{\lambda + p_t}b_t + \pi_t - \tau_t - c(a_t^i);$$

these are equations (30) and (31) in the text.

III. APPROXIMATION AROUND THE STEADY STATE

Response to individual asset holdings

The local approximation to the saddle-path of the individual consumption-savings problem around the steady state conditions (34) and (35) can be represented as

$$
\begin{bmatrix} \frac{da}{dt} \\ \frac{dc}{dt} \end{bmatrix} = \begin{bmatrix} r - \delta & -1 \\ -\frac{\varepsilon B_{aa}(\bar{a})}{U_{cc}(\bar{c})} & \rho - r + \delta \end{bmatrix} \begin{bmatrix} a - \bar{a} \\ c - \bar{c} \end{bmatrix}
$$

with transversality condition $\lim\limits_{t \to \infty} \frac{da}{dt} = \bar{a}$.

Under the condition for determinacy

$$
\frac{B_{aa}(\bar{a})}{B_a(\bar{a})} > (r - \delta)\frac{U_{cc}(\bar{c})}{U_c(\bar{c})},
$$

the solution is given by

$$
c(a) = \bar{c} + c_a(a - \bar{a}),
$$

where c_a is the appropriate element of the eigenvector associated with the negative eigenvalue a_a:

$$
\begin{bmatrix} r - \delta & -1 \\ -\frac{\varepsilon B_{aa}(\bar{a})}{U_{cc}(\bar{c})} & \rho - r + \delta \end{bmatrix} \begin{bmatrix} 1 \\ c_a \end{bmatrix} = a_a \begin{bmatrix} 1 \\ c_a \end{bmatrix}.
$$

Response to the length of the time interval Δ

To obtain the response of all endogenous variables with respect to the length of the time interval we solve the system of equations resulting from taking the first derivative to the equilibrium conditions with respect to Δ. We repeat those conditions below to explain these calculations in greater detail.

The first order condition for optimality of employed agents is given by equation (10)

$$
\begin{aligned}
U_c(c^e(a)) &= \frac{1}{1 + (r - \delta)\Delta} e^{-\rho\Delta} \left[\lambda U_c(c^u(a + \Delta\dot{a}^e(a))) + (1 - \lambda)U_c(c^e(a + \Delta\dot{a}^e(a))) \right] \\
&\quad + e^{-\rho\Delta}(1 - \lambda)U_c(c^e(a + \Delta\dot{a}^e(a))w_a(a + \Delta\dot{a}^e(a))\Delta \\
&\quad + \varepsilon B_a(a + \Delta\dot{a}^e(a))\Delta,
\end{aligned}
$$

where savings per unit of time $\dot{a}^e(a)$ is given by equation (6)

$$
\dot{a}^e = (r - \delta)a + w(a) + \pi - \tau - c^e(a).
$$

Similarly, the first order condition for optimalilty of unemployed agents is given by equation (11)

$$U_c(c^u(a)) = \frac{1}{1 + (r - \delta)\Delta}e^{-\rho\Delta}\left[(1 - p)U_c(c^u(a + \Delta\dot{a}^u(a)) + pU_c(c^e(a + \Delta\dot{a}^u(a))\right]$$
$$+ e^{-\rho\Delta}pU_c(c^e(a + \Delta\dot{a}^u(a))w_a(a + \Delta\dot{a}^u(a))\Delta$$
$$+ \varepsilon B_a(a + \Delta\dot{a}^u(a))\Delta,$$

where savings per unit of time $\dot{a}^u(a)$ is given by equation (9)

$$\dot{a}^u = (r - \delta)a + b + \pi - \tau - c^u(a).$$

The first order conditions for optimalilty of the representative firms are given by equation (15)

$$r = F_k(k, n),$$

and the stationary version of equation (16)

$$\frac{\phi}{q} = \frac{1}{\lambda + e^{\rho\Delta} - 1}(F_n(k, n) - w),$$

while profits are determined by equation (28)

$$\pi = F(k, n) - rk - wn - \phi v.$$

Individual wages are determined by the bargaining solution (19)

$$\frac{W^{e-u}(a, w(a))}{U_c(c^e(a))} = \frac{\alpha_w}{\alpha_f}\left(F_n(k, n) - w(a) + (1 - \lambda)\frac{\phi}{q}\right)\Delta,$$

where $W^{e-u}(a, w(a))$ is determined by (A-5).

The labor market is characterized by equations (2), (3), and (27)

$$q = \frac{m(v, u)}{v},$$

$$p = \frac{m(v, u)}{u},$$

$$u = 1 - n = \frac{\lambda}{\lambda + p}.$$

Policy is determined by equations (21)

$$\tau = ub,$$

and (22)

$$b = \chi w.$$

Finally we need to take into account the aggregate conditions. Since all individual wages are equally affected by Δ, the aggregate effect is the same as the individual effect, we impose that condition in the resulting system of linear equations. The aggregate condition (29) characterizing the asset distribution is replaced with equations determined the average capital of each type of population (42)

$$
\begin{bmatrix} k^e \\ k^u \end{bmatrix} = \left[I_2 - \left(\frac{1}{\Delta a_a} - 1 \right) \begin{bmatrix} \lambda & -\lambda \\ -p & p \end{bmatrix} \right]^{-1} \begin{bmatrix} 1-\lambda & \lambda \\ p & 1-p \end{bmatrix} \begin{bmatrix} \frac{w+\pi-\tau-c^e(a)}{r-\delta} \\ \frac{b+\pi-\tau-c^u(a)}{r-\delta} \end{bmatrix},
$$

and the aggregate level of capital (43)

$$
k = \frac{1}{\lambda+p}(pk^e + \lambda k^u).
$$

We solve for the coefficients c_Δ^e, c_Δ^u, r_Δ, v_Δ, π_Δ, w_Δ, q_Δ, p_Δ, u_Δ, n_Δ, τ_Δ, b_Δ, k_Δ^e, k_Δ^u, and k_Δ by solving the system of equations that results of taking the derivative with respect to Δ to the above conditions. In this system, all variables are evaluated at their steady state levels.

As a last remark, it is important to notice a practical complication: the first derivatives of the euler conditions (10) and (6) with respect to Δ are linearly dependent. To avoid that dependency we replace the resulting equations by linear combinations of derivatives of the mentioned euler conditions. These linear combinations still provide the same information about the coefficients of interest and they have the additional property of being linearly independent. The first linear combination it is just the difference between the first order derivatives of the euler conditions with respect to Δ. The second it is the sum of p times the second order derivative of (10) and λ times the second order derivative of (6). In that combination all the second order terms cancel out and only the first order terms remain.

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