

# Monetary Policy Analysis and Forecasting in the World Economy: A Panel Unobserved Components Approach

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# Monetary Policy Analysis and Forecasting in the World Economy: A Panel Unobserved Components Approach

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#### **Abstract**

This paper develops a panel unobserved components model of the monetary transmission mechanism in the world economy, disaggregated into its fifteen largest national economies. This structural macroeconometric model features extensive linkages between the real and financial sectors, both within and across economies. A variety of monetary policy analysis and forecasting applications of the estimated model are demonstrated, based on a novel Bayesian framework for conditioning on judgment.

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## I. Introduction

The practice of monetary policy analysis and forecasting is becoming increasingly informed by structural macroeconometric models. Indeed, leveraging recent advances in theoretical and empirical macroeconomics, the leading central banks have all developed sophisticated models of the monetary transmission mechanisms in their respective economies which serve as inputs into the conduct of monetary policy, as discussed in survey papers by Sims and Tovar (2008). Effective surveillance over the conduct of monetary policy in these economies by the International Monetary Fund demands that it develop comparable quantitative tools.

Recent research and development work at the leading central banks has focused on addressing theoretical and empirical deficiencies common among existing structural macroeconometric models. As observed by Tovar (2008), these include limited theoretical linkages between the real and financial sectors within economies, which are becoming increasingly relevant for bilateral surveillance. Another area of theoretical deficiency is limited trade and financial linkages across economies, which are becoming increasingly pertinent to multilateral surveillance. Finally, as emphasized by Sims (2008), a unified empirical framework for conditioning on judgment in estimation and forecasting having good statistical properties is desired.

This paper develops a panel unobserved components model of the monetary transmission mechanism in the world economy, disaggregated into its fifteen largest national economies. This structural macroeconometric model is designed to inform bilateral and multilateral surveillance over the conduct of monetary policy in these economies, and accordingly features extensive linkages between the real and financial sectors, both within and across economies. The major advanced and emerging market economies under consideration are Australia, Brazil, Canada, China, France, Germany, India, Italy, Japan, Korea, Mexico, Russia, Spain, the United Kingdom, and the United States. A variety of monetary policy analysis and forecasting applications of the estimated model are demonstrated. These include the measurement of monetary conditions, the analysis of the monetary transmission mechanism, and the generation of conditional forecasts of inflation and output growth. They are based on a novel Bayesian framework for incorporating judgment concerning the paths of unobserved variables into estimation and forecasting.

The organization of this paper is as follows. The next section describes a panel unobserved components model of the world economy. Estimation of this model is the subject of section three. Monetary policy analysis within the framework of the estimated model is conducted in section four, while forecasting is undertaken in section five. Finally, section six offers conclusions and recommendations for further research.

## II. THE PANEL UNOBSERVED COMPONENTS MODEL

Our panel unobserved components model of the world economy consists of multiple economies connected by trade and financial linkages. Within each economy, cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism derived from postulated behavioral relationships. These behavioral relationships approximately nest those associated with a variety of alternative structural macroeconomic models derived from microeconomic foundations, conferring robustness to model misspecification. In the interest of parsimony, cross economy equality restrictions are imposed on the structural parameters of these behavioral relationships, the response coefficients of which vary across economies with their degree of trade openness. Trend components are modeled as independent random walks, conferring robustness to intermittent structural breaks.

The monetary transmission mechanism in each economy operates via interest rate and exchange rate channels, both of which link the short term nominal interest rate, which serves as the instrument of monetary policy, to consumption price inflation and the output gap, which are target variables. Under the interest rate channel, monetary policy affects the output gap and by implication inflation by inducing intertemporal substitution in domestic demand in response to changes in the long term real interest rate. Under the exchange rate channel, monetary policy both directly affects inflation, and indirectly affects the output gap and by implication inflation via intratemporal substitution between domestic and foreign demand, by inducing changes in the real effective exchange rate. A financial accelerator mechanism linked to the real value of an internationally diversified equity portfolio amplifies and propagates both of these channels.

In what follows,  $\hat{x}_{i,t}$  denotes the cyclical component of variable  $x_{i,t}$ , while  $\overline{x}_{i,t}$  denotes the trend component of variable  $x_{i,t}$ . Cyclical and trend components are additively separable, that is  $x_{i,t} = \hat{x}_{i,t} + \overline{x}_{i,t}$ . Furthermore,  $E_t x_{i,t+s}$  denotes the rational expectation of variable  $x_{i,t+s}$  associated with economy i, conditional on information available at time t. Finally,  $x_{i,t}^f$  denotes the trade weighted average of variable  $x_{i,t}$  across the trading partners of economy i,  $x_{i,t}^p$  denotes the portfolio weighted average of domestic currency denominated variable  $x_{i,t}$  across the investment destinations of economy i, and  $x_t^w$  denotes the output weighted average of variable  $x_{i,t}$  across all economies.

## A. Cyclical Components

The cyclical component of output price inflation  $\hat{\pi}_{i,t}^{Y}$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical component of output according to domestic supply relationship,

$$\hat{\pi}_{i,t}^{Y} = \phi_{1,1}\hat{\pi}_{i,t-1}^{Y} + \phi_{1,2} E_{t} \hat{\pi}_{i,t+1}^{Y} + \theta_{1,1} \ln \hat{Y}_{i,t} + V_{i,t}^{\hat{p}^{Y}}, \tag{1}$$

where domestic supply shock  $v_{i,t}^{\hat{p}^{y}} = \rho_{\hat{p}^{y}} v_{i,t-1}^{\hat{p}^{y}} + \varepsilon_{i,t}^{\hat{p}^{y}}$  with  $\varepsilon_{i,t}^{\hat{p}^{y}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{p}^{y}_{i}}^{2})$ .

The cyclical component of consumption price inflation  $\hat{\pi}_{i,t}^{C}$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical component of output according to supply relationship,

$$\hat{\pi}_{i,t}^{C} = \phi_{1,1} \hat{\pi}_{i,t-1}^{C} + \phi_{1,2} \, \mathbf{E}_{t} \, \hat{\pi}_{i,t+1}^{C} + \theta_{1,1} \ln \hat{Y}_{i,t} \\
+ \theta_{2,1} \, \frac{M_{i}}{Y_{i}} \phi_{1}(L) \Delta \ln \hat{Q}_{i,t} + \theta_{2,2} \, \frac{M_{i}}{Y_{i}} \phi_{1}(L) \Delta \ln (\hat{S}_{i,t}^{USA} \hat{P}_{t}^{COM}) + v_{i,t}^{\hat{p}^{Y}} + \phi_{1}(L) v_{i,t}^{\hat{p}^{M}},$$
(2)

where foreign supply shock  $v_{i,t}^{\hat{P}^M} = \rho_{\hat{P}^M} v_{i,t-1}^{\hat{P}^M} + \varepsilon_{i,t}^{\hat{P}^M}$  with  $\varepsilon_{i,t}^{\hat{P}^M} \sim$  iid  $\mathcal{N}(0,\sigma_{\hat{P}^M,i}^2)$ . The cyclical component of consumption price inflation also depends on contemporaneous, past, and expected future changes in the cyclical components of the real effective exchange rate and the domestic currency denominated price of commodities, where polynomial in the lag operator  $\phi_1(L) = 1 - \phi_{1,1} L - \phi_{1,2} \to E_t L^{-1}$ . The response coefficients of this relationship vary across economies with their degree of trade openness, measured by the ratio of imports to output  $\frac{M_i}{Y_i}$ .

The cyclical component of output  $\ln \hat{Y}_{i,t}$  follows a stationary first order autoregressive process driven by a monetary conditions index according to demand relationship,

$$\ln \hat{Y}_{i,t} = \phi_{3,1} \ln \hat{Y}_{i,t-1} + \theta_{3,1} \left( 1 - \frac{M_i}{Y_i} \right) \left( \hat{r}_{i,t}^L + \theta_{3,2} \ln \frac{\hat{P}_{i,t}^{STK,p}}{\hat{P}_{i,t}^C} \right) + \theta_{4,1} \frac{X_i}{Y_i} \phi_3(L) \ln \hat{D}_{i,t}^f + \theta_{4,2} \frac{X_i + M_i}{Y_i} \phi_3(L) \ln \hat{Q}_{i,t} + \left( 1 - \frac{M_i}{Y_i} \right) v_{i,t}^{\hat{D}} + \frac{X_i}{Y_i} \phi_3(L) v_{i,t}^{\hat{X}},$$
(3)

where foreign demand shock  $v_{i,t}^{\hat{X}} = \rho_{\hat{X}} v_{i,t-1}^{\hat{X}} + \varepsilon_{i,t}^{\hat{X}}$  with  $\varepsilon_{i,t}^{\hat{X}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{X},i}^2)$ . Reflecting the existence of international trade and financial linkages, this monetary conditions index is defined as a linear combination of a financial conditions index and the contemporaneous and past cyclical components of the real effective exchange rate. The cyclical component of output also depends on the contemporaneous and past cyclical components of foreign demand, where polynomial in the lag operator  $\phi_3(L) = 1 - \phi_{3,1}L$ . The response coefficients of this relationship vary across economies with their degree of trade openness, measured by the ratio of exports to output  $\frac{X_i}{Y_i}$  or imports to output  $\frac{M_i}{Y_i}$ .

This monetary conditions index  $\hat{I}_{i,t}^{MCI}$  is defined as  $\hat{I}_{i,t}^{MCI} = \hat{I}_{i,t}^{FCI} + \frac{\theta_{4,2}}{\theta_{3,1}} \frac{X_i + M_i}{Y_i} \left(1 - \frac{M_i}{Y_i}\right)^{-1} \phi_3(L) \ln \hat{Q}_{i,t}$ , where financial conditions index  $\hat{I}_{i,t}^{FCI}$  satisfies  $\hat{I}_{i,t}^{FCI} = \hat{r}_{i,t}^L + \theta_{3,2} \ln \frac{\hat{P}_{i,t}^{FCI}}{\hat{P}_{i,t}^C}$ .

The cyclical component of domestic demand  $\ln \hat{D}_{i,t}$  follows a stationary first order autoregressive process driven by a financial conditions index according to domestic demand relationship,

$$\ln \hat{D}_{i,t} = \phi_{3,1} \ln \hat{D}_{i,t-1} + \theta_{3,1} \left( \hat{r}_{i,t}^L + \theta_{3,2} \ln \frac{\hat{P}_{i,t}^{STK,p}}{\hat{P}_{i,t}^C} \right) + v_{i,t}^{\hat{D}}, \tag{4}$$

where domestic demand shock  $v_{i,t}^{\hat{D}} = \rho_{\hat{D}} v_{i,t-1}^{\hat{D}} + \varepsilon_{i,t}^{\hat{D}}$  with  $\varepsilon_{i,t}^{\hat{D}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{D},i}^2)$ . This financial conditions index is defined as a linear combination of the contemporaneous cyclical components of the long term real interest rate and the real value of an internationally diversified equity portfolio.

The cyclical component of the short term nominal interest rate  $\hat{i}_{i,t}^{S}$  depends on a weighted average of its past and desired cyclical components according to monetary policy rule,

$$\hat{i}_{i,t}^{S} = \phi_{5,1} \hat{i}_{i,t-1}^{S} + (1 - \phi_{5,1})(\theta_{5,1,t} \hat{\pi}_{i,t}^{C} + \theta_{5,2,t} \ln \hat{Y}_{i,t} + \theta_{5,3,t} \ln \hat{S}_{i,t}^{USA}) + \varepsilon_{i,t}^{\hat{i}^{S}},$$
(5)

where monetary policy shock  $\varepsilon_{i,t}^{i^S} \sim \operatorname{iid} \mathcal{N}(0,\sigma_{i^S,j}^2)$ . The desired cyclical component of the short term nominal interest rate depends on the contemporaneous cyclical components of consumption price inflation, output, and the nominal bilateral exchange rate. The response coefficients of this rule vary across economies, nesting flexible inflation targeting and fixed exchange rate regimes as special cases of the conduct of monetary policy. In cases where multiple economies are members of a currency union, the target variables entering into their common monetary policy rule are expressed as output weighted averages across union members. The cyclical component of the short term real interest rate  $\hat{r}_{i,t}^S$  satisfies  $\hat{r}_{i,t}^S = \hat{i}_{i,t}^S - \operatorname{E}_t \hat{\pi}_{i,t+1}^C$ .

The cyclical component of the long term real interest rate  $\hat{r}_{i,t}^L$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical component of the short term real interest rate according to term structure relationship,

$$\hat{r}_{i,t}^{L} = \phi_{6,1} \hat{r}_{i,t-1}^{L} + \phi_{6,2} \, \mathcal{E}_{t} \, \hat{r}_{i,t+1}^{L} + \theta_{6,1} \hat{r}_{i,t}^{S} + \varepsilon_{i,t}^{\hat{i}^{L}}, \tag{6}$$

where liquidity risk premium shock  $\varepsilon_{i,t}^{\hat{i}^L} \sim \mathrm{iid} \ \mathcal{N}(0,\sigma_{\hat{i}^L,i}^2)$ . The cyclical component of the long term nominal interest rate  $\hat{i}_{i,t}^L$  satisfies  $\hat{r}_{i,t}^L = \hat{i}_{i,t}^L - \mathrm{E}_t \ \hat{\pi}_{i,t+1}^{\hat{C}^L,i}$ .

The cyclical component of the price of equity  $\ln \hat{P}_{i,t}^{STK}$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical components of output and the short term nominal interest rate,

$$\ln \hat{P}_{i,t}^{STK} = \phi_{7,1} \ln \hat{P}_{i,t-1}^{STK} + \phi_{7,2} E_t \ln \hat{P}_{i,t+1}^{STK} + \theta_{7,1} \ln \hat{Y}_{i,t} + \theta_{7,2} \hat{i}_{i,t}^S + \varepsilon_{i,t}^{\hat{p}^{STK}}, \tag{7}$$

where equity risk premium shock  $\varepsilon_{i,t}^{\hat{p}^{STK}} \sim \mathrm{iid} \ \mathcal{N}(0,\sigma_{\hat{p}^{STK}_{i}}^{2})$ .

The cyclical component of the real bilateral exchange rate  $\ln \hat{Q}_{i,t}^{USA}$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical component of the real bilateral interest rate differential,

$$\ln \hat{Q}_{i,t}^{USA} = \phi_{8,1} \ln \hat{Q}_{i,t-1}^{USA} + \phi_{8,2} E_t \ln \hat{Q}_{i,t+1}^{USA} + \theta_{8,1} (\hat{r}_{i,t}^S - \hat{r}_{USA,t}^S) + \varepsilon_{i,t}^{\hat{S}},$$
(8)

where exchange rate risk premium shock  $\varepsilon_{i,t}^{\hat{S}} \sim \operatorname{iid} \mathcal{N}(0,\sigma_{\hat{S},i}^2)$ . The cyclical component of the nominal bilateral exchange rate  $\ln \hat{S}_{i,t}^{USA}$  satisfies  $\ln \hat{Q}_{i,t}^{USA} = \ln \hat{S}_{i,t}^{USA} + \ln \hat{P}_{USA,t}^C - \ln \hat{P}_{i,t}^C$ .

The cyclical component of the change in the price of commodities  $\ln \hat{P}_t^{COM}$  depends on a linear combination of its past and expected future cyclical components driven by the contemporaneous cyclical component of world output,

$$\Delta \ln \hat{P}_{t}^{COM} = \phi_{9,1} \Delta \ln \hat{P}_{t-1}^{COM} + \phi_{9,2} E_{t} \Delta \ln \hat{P}_{t+1}^{COM} + \theta_{9,1} \ln \hat{Y}_{t}^{w} + \varepsilon_{t}^{\hat{P}^{COM}}, \tag{9}$$

where commodity price shock  $\varepsilon_t^{\hat{p}^{com}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{p}^{com}}^2)$ . As an identifying restriction, all innovations are assumed to be independent, which combined with our distributional assumptions implies multivariate normality.

### **B.** Trend Components

The growth rates of the trend components of the price of output  $\ln \overline{P}_{i,t}^Y$ , the price of consumption  $\ln \overline{P}_{i,t}^C$ , output  $\ln \overline{Y}_{i,t}$ , domestic demand  $\ln \overline{D}_{i,t}$ , the price of equity  $\ln \overline{P}_{i,t}^{STK}$ , and the price of commodities  $\ln \overline{P}_{i}^{COM}$  follow random walks:

$$\Delta \ln \overline{P}_{i,t}^{Y} = \Delta \ln \overline{P}_{i,t-1}^{Y} + \varepsilon_{i,t}^{\overline{P}^{Y}}, \ \varepsilon_{i,t}^{\overline{P}^{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^{Y}, i}^{2}),$$

$$\tag{10}$$

$$\Delta \ln \overline{P}_{i,t}^C = \Delta \ln \overline{P}_{i,t-1}^C + \varepsilon_{i,t}^{\overline{P}^C}, \ \varepsilon_{i,t}^{\overline{P}^C} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^C, i}^2), \tag{11}$$

$$\Delta \ln \overline{Y}_{i,t} = \Delta \ln \overline{Y}_{i,t-1} + \varepsilon_{i,t}^{\overline{Y}}, \ \varepsilon_{i,t}^{\overline{Y}} \sim \text{iid } \mathcal{N}(0,\sigma_{\overline{Y},i}^2),$$
(12)

 $<sup>^3</sup>$  It can be shown that the cyclical component of the nominal effective exchange rate  $\ln \hat{S}_{i,t}$  satisfies  $\ln \hat{S}_{i,t} = \ln \hat{S}_{i,t}^{USA} - \sum_{j=1}^{N} w_{i,j} \ln \hat{S}_{j,t}^{USA} \text{ , while the cyclical component of the real effective exchange rate } \ln \hat{Q}_{i,t}$  satisfies  $\ln Q_{i,t} = \ln \hat{Q}_{i,t}^{USA} - \sum_{j=1}^{N} w_{i,j} \ln \hat{Q}_{j,t}^{USA} \text{ , where } w_{i,j} \text{ denotes the bilateral trade weight for economy } i \text{ with respect to economy } j \text{ , and } N \text{ denotes the number of economies. Note that } \ln \hat{Q}_{i,t} = \ln \hat{S}_{i,t} + \ln \hat{P}_{i,t}^{C,f} - \ln \hat{P}_{i,t}^{C} \text{ .}$ 

$$\Delta \ln \overline{D}_{i,t} = \Delta \ln \overline{D}_{i,t-1} + \varepsilon_{i,t}^{\overline{D}}, \ \varepsilon_{i,t}^{\overline{D}} \sim \text{iid} \ \mathcal{N}(0, \sigma_{\overline{D}_i}^2), \tag{13}$$

$$\Delta \ln \overline{P}_{i,t}^{STK} = \Delta \ln \overline{P}_{i,t-1}^{STK} + \varepsilon_{i,t}^{\overline{P}^{STK}}, \ \varepsilon_{i,t}^{\overline{P}^{STK}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^{STK}, i}^2), \tag{14}$$

$$\Delta \ln \overline{P}_{t}^{COM} = \Delta \ln \overline{P}_{t-1}^{COM} + \varepsilon_{t}^{\overline{P}^{COM}}, \ \varepsilon_{t}^{\overline{P}^{COM}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^{COM}}^{2}). \tag{15}$$

The trend components of the short term nominal interest rate  $\overline{i}_{i,t}^S$ , long term nominal interest rate  $\overline{i}_{i,t}^L$ , and nominal bilateral exchange rate  $\overline{S}_{i,t}^{USA}$  also follow random walks:

$$\overline{i}_{i,t}^S = \overline{i}_{i,t-1}^S + \varepsilon_{i,t}^{\overline{i}^S}, \ \varepsilon_{i,t}^{\overline{i}^S} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{i}^S}^2),$$

$$(16)$$

$$\overline{i}_{i,t}^{L} = \overline{i}_{i,t-1}^{L} + \varepsilon_{i,t}^{\overline{i}^{L}}, \ \varepsilon_{i,t}^{\overline{i}^{L}} \sim \text{iid} \ \mathcal{N}(0, \sigma_{\overline{i}^{L},i}^{2}), \tag{17}$$

$$\ln \overline{S}_{i,t}^{USA} = \ln \overline{S}_{i,t-1}^{USA} + \varepsilon_{i,t}^{\overline{S}}, \quad \varepsilon_{i,t}^{\overline{S}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{S}_i}^2).$$

$$\tag{18}$$

The trend component of the short term real interest rate  $\overline{r}_{i,t}^S$  satisfies  $\overline{r}_{i,t}^S = \overline{i}_{i,t}^S - E_t \overline{\pi}_{i,t+1}^C$ , the trend component of the long term real interest rate  $\overline{r}_{i,t}^L$  satisfies  $\overline{r}_{i,t}^L = \overline{i}_{i,t}^L - E_t \overline{\pi}_{i,t+1}^C$ , and the trend component of the real bilateral exchange rate  $\ln \overline{Q}_{i,t}^{USA}$  satisfies  $\ln \overline{Q}_{i,t}^{USA} = \ln \overline{S}_{i,t}^{USA} + \ln \overline{P}_{USA,t}^C - \ln \overline{P}_{i,t}^C$ . As an identifying restriction, all innovations are assumed to be independent.

## III. ESTIMATION

The traditional econometric interpretation of this panel unobserved components model of the world economy regards it as a representation of the joint probability distribution of the data. We employ a Bayesian estimation procedure which respects this traditional econometric interpretation while conditioning on prior information concerning the values of structural parameters, and judgment concerning the paths of trend components. In addition to mitigating potential model misspecification and identification problems, exploiting this additional information may be expected to yield efficiency gains in estimation.

## A. Estimation Procedure

Let  $x_i$  denote a vector stochastic process consisting of the levels of  $N_x$  nonpredetermined endogenous variables, of which  $N_y$  are observed. The cyclical components of this vector stochastic process satisfy second order stochastic linear difference equation

$$A_0 \hat{\mathbf{x}}_t = A_1 \hat{\mathbf{x}}_{t-1} + A_2 \mathbf{E}_t \hat{\mathbf{x}}_{t+1} + A_3 \hat{\mathbf{v}}_t, \tag{19}$$

where vector stochastic process  $\hat{\mathbf{v}}_t$  consists of the cyclical components of  $N_v$  exogenous variables. This vector stochastic process satisfies stationary first order stochastic linear difference equation

$$\hat{\boldsymbol{v}}_{t} = \boldsymbol{B}_{1} \hat{\boldsymbol{v}}_{t-1} + \boldsymbol{\varepsilon}_{1t}, \tag{20}$$

where  $\varepsilon_{1,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \Sigma_1)$ . If there exists a unique stationary solution to this multivariate linear rational expectations model, then it may be expressed as:

$$\hat{\boldsymbol{x}}_{t} = \boldsymbol{C}_{1} \hat{\boldsymbol{x}}_{t-1} + \boldsymbol{C}_{2} \hat{\boldsymbol{v}}_{t}. \tag{21}$$

This unique stationary solution is calculated with the procedure due to Klein (2000).

The trend components of vector stochastic process  $x_t$  satisfy first order stochastic linear difference equation

$$\boldsymbol{D}_0 \overline{\boldsymbol{x}}_t = \boldsymbol{D}_1 \boldsymbol{u}_t + \boldsymbol{D}_2 \overline{\boldsymbol{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \tag{22}$$

where  $\varepsilon_{2,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \Sigma_2)$ . Vector stochastic process  $\mathbf{u}_t$  consists of the levels of  $N_u$  common stochastic trends, and satisfies nonstationary first order stochastic linear difference equation

$$\boldsymbol{u}_{t} = \boldsymbol{u}_{t-1} + \boldsymbol{\varepsilon}_{3t}, \tag{23}$$

where  $\varepsilon_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \Sigma_3)$ . Cyclical and trend components are additively separable, that is  $\mathbf{x}_t = \hat{\mathbf{x}}_t + \overline{\mathbf{x}}_t$ .

Let  $y_t$  denote a vector stochastic process consisting of the levels of  $N_y$  observed nonpredetermined endogenous variables. Also, let  $z_t$  denote a vector stochastic process consisting of the levels of  $N_x - N_y$  unobserved nonpredetermined endogenous variables, the cyclical components of  $N_x$  nonpredetermined endogenous variables, the trend components of  $N_x$  nonpredetermined endogenous variables, the cyclical components of  $N_y$  exogenous variables, and the levels of  $N_y$  common stochastic trends. Given unique stationary solution (21), these vector stochastic processes have linear state space representation

$$\mathbf{y}_{t} = \mathbf{F}_{1}\mathbf{z}_{t},\tag{24}$$

$$\boldsymbol{z}_{t} = \boldsymbol{G}_{1}\boldsymbol{z}_{t-1} + \boldsymbol{G}_{2}\boldsymbol{\varepsilon}_{4,t}, \tag{25}$$

where  $\varepsilon_{4,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \Sigma_4)$  and  $\mathbf{z}_0 \sim \mathcal{N}(\mathbf{z}_{0|0}, \mathbf{P}_{0|0})$ . Let  $\mathbf{w}_t$  denote a vector stochastic process consisting of alternative estimates or forecasts of  $N_w$  linearly independent linear combinations of unobserved state variables. In an extension of Vitek (2008a, 2008b) to allow

for a time varying innovation covariance matrix, suppose that this vector stochastic process satisfies

$$\mathbf{w}_{t} = \mathbf{H}_{1}\mathbf{z}_{t} + \mathbf{\varepsilon}_{5,t},\tag{26}$$

where  $\varepsilon_{5,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \Sigma_{5,t})$ . Conditional on known parameter values, this signal equation imposes judgment on linear combinations of unobserved state variables in the form of a set of stochastic restrictions of time dependent tightness. The signal and state innovation vectors are assumed to be independent, while the initial state vector is assumed to be independent from the signal and state innovation vectors, which combined with our distributional assumptions implies multivariate normality.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector  $\mathbf{z}_t$  and its mean squared error matrix  $\mathbf{P}_t$  may be calculated with the filter due to Kalman (1960). Given initial conditions  $\mathbf{z}_{0|0}$  and  $\mathbf{P}_{0|0}$ , estimates conditional on information available at time t-1 satisfy prediction equations:

$$\mathbf{z}_{t|t-1} = \mathbf{G}_1 \mathbf{z}_{t-1|t-1}, \tag{27}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{G}_1 \boldsymbol{P}_{t-1|t-1} \boldsymbol{G}_1^{\mathsf{T}} + \boldsymbol{G}_2 \boldsymbol{\Sigma}_4 \boldsymbol{G}_2^{\mathsf{T}}, \tag{28}$$

$$y_{t|t-1} = F_1 z_{t|t-1}, (29)$$

$$\mathbf{Q}_{t|t-1} = \mathbf{F}_1 \mathbf{P}_{t|t-1} \mathbf{F}_1^{\mathsf{T}}, \tag{30}$$

$$\mathbf{w}_{t|t-1} = \mathbf{H}_1 \mathbf{z}_{t|t-1}, \tag{31}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{H}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{H}_1^{\mathsf{T}} + \boldsymbol{\Sigma}_{5,t}. \tag{32}$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time t, and judgment concerning the paths of linear combinations of state variables in sample, satisfy Bayesian updating equations

$$\mathbf{z}_{t|t} = \mathbf{z}_{t|t-1} + \mathbf{K}_{\mathbf{y}_t} (\mathbf{y}_t - \mathbf{y}_{t|t-1}) + \mathbf{K}_{\mathbf{w}_t} (\mathbf{w}_t - \mathbf{w}_{t|t-1}), \tag{33}$$

$$P_{t|t} = P_{t|t-1} - K_{v_t} F_1 P_{t|t-1} - K_{w_t} H_1 P_{t|t-1},$$
(34)

where  $K_{y_t} = P_{t|t-1}F_1^{\mathsf{T}}Q_{t|t-1}^{-1}$  and  $K_{w_t} = P_{t|t-1}H_1^{\mathsf{T}}R_{t|t-1}^{-1}$ . Given terminal conditions  $z_{T|T}$  and  $P_{T|T}$  obtained from the final evaluation of these prediction and updating equations, estimates conditional on information available at time T, and judgment concerning the paths of linear combinations of state variables in sample, satisfy Bayesian smoothing equations

$$\mathbf{z}_{t|T} = \mathbf{z}_{t|t} + \mathbf{J}_{t}(\mathbf{z}_{t+1|T} - \mathbf{z}_{t+1|t}), \tag{35}$$

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_{t} (\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t}) \boldsymbol{J}_{t}^{\mathsf{T}}, \tag{36}$$

where  $J_t = P_{t|t}G_2^T P_{t+1|t}^{-1}$ . Under our distributional assumptions, recursive forward evaluation of equations (27) through (34), followed by recursive backward evaluation of equations (35) and (36), yields mean squared error optimal conditional estimates of the unobserved state vector.

Let  $\theta \in \Theta \subset \mathbb{R}^K$  denote a K dimensional vector containing the parameters associated with the signal and state equations of this linear state space model. The Bayesian estimator of this parameter vector has posterior density function

$$f(\theta \mid \mathcal{I}_T) \propto f(\mathcal{I}_T \mid \theta) f(\theta),$$
 (37)

where  $\mathcal{I}_t = \{\{y_s\}_{s=1}^t, \{w_s\}_{s=1}^t\}$ . Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, conditional density function  $f(\mathcal{I}_T | \theta)$  satisfies:

$$f(\mathcal{I}_T \mid \boldsymbol{\theta}) = \prod_{t=1}^T f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) \cdot \prod_{t=1}^T f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}).$$
(38)

Under our distributional assumptions, conditional density functions  $f(y_t | \mathcal{I}_{t-1}, \theta)$  and  $f(w_t | \mathcal{I}_{t-1}, \theta)$  satisfy:

$$f(\mathbf{y}_{t} \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{N_{y}}{2}} | \mathbf{Q}_{t|t-1} |^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\mathbf{y}_{t} - \mathbf{y}_{t|t-1})^{\mathsf{T}} \mathbf{Q}_{t|t-1}^{-1} (\mathbf{y}_{t} - \mathbf{y}_{t|t-1}) \right\},$$
(39)

$$f(\mathbf{w}_{t} \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{N_{\mathbf{w}}}{2}} \mid \mathbf{R}_{t|t-1} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{w}_{t} - \mathbf{w}_{t|t-1})^{\mathsf{T}} \mathbf{R}_{t|t-1}^{-1}(\mathbf{w}_{t} - \mathbf{w}_{t|t-1})\right\}. \tag{40}$$

Prior information concerning parameter vector  $\theta$  is summarized by a multivariate normal prior distribution having mean vector  $\theta_1$  and covariance matrix  $\Omega$ :

$$f(\boldsymbol{\theta}) = (2\pi)^{-\frac{K}{2}} |\boldsymbol{\Omega}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}_1)^{\mathsf{T}} \boldsymbol{\Omega}^{-1} (\boldsymbol{\theta} - \boldsymbol{\theta}_1)\right\}. \tag{41}$$

Independent priors are represented by a diagonal covariance matrix, under which diffuse priors are represented by infinite variances.

Inference on the parameters is based on an asymptotic normal approximation to the posterior distribution around its mode. Under regularity conditions stated in Geweke (2005), posterior mode  $\hat{\theta}_T$  satisfies

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \stackrel{d}{\to} \mathcal{N}(\mathbf{0}, -\boldsymbol{\mathcal{H}}_0^{-1}), \tag{42}$$

where  $\theta_0 \in \Theta$  denotes the pseudotrue parameter vector. Following Engle and Watson (1981), Hessian  $\mathcal{H}_0$  is estimated by:

$$\hat{\mathcal{H}}_{T} = -\frac{1}{T} \sum_{t=1}^{T} \left[ \nabla_{\theta} \mathbf{y}_{t|t-1}^{\mathsf{T}} \mathbf{Q}_{t|t-1}^{-1} \nabla_{\theta} \mathbf{y}_{t|t-1} + \frac{1}{2} \nabla_{\theta} \mathbf{Q}_{t|t-1}^{\mathsf{T}} (\mathbf{Q}_{t|t-1}^{-1} \otimes \mathbf{Q}_{t|t-1}^{-1}) \nabla_{\theta} \mathbf{Q}_{t|t-1} \right] 
- \frac{1}{T} \sum_{t=1}^{T} \left[ \nabla_{\theta} \mathbf{w}_{t|t-1}^{\mathsf{T}} \mathbf{R}_{t|t-1}^{-1} \nabla_{\theta} \mathbf{w}_{t|t-1} + \frac{1}{2} \nabla_{\theta} \mathbf{R}_{t|t-1}^{\mathsf{T}} (\mathbf{R}_{t|t-1}^{-1} \otimes \mathbf{R}_{t|t-1}^{-1}) \nabla_{\theta} \mathbf{R}_{t|t-1} \right] - \frac{1}{T} \mathbf{\Omega}^{-1}.$$
(43)

This estimator of the Hessian depends only on first derivatives and is negative semidefinite.

#### **B.** Estimation Results

Joint estimation of the parameters and unobserved components of our panel unobserved components model of the world economy is based on the levels of a total of one hundred fourteen endogenous variables observed for fifteen economies over the period 1999Q1 through 2009Q2. The economies under consideration are Australia, Brazil, Canada, China, France, Germany, India, Italy, Japan, Korea, Mexico, Russia, Spain, the United Kingdom, and the United States. The observed endogenous variables under consideration are the price of output, the price of consumption, the quantity of output, the quantity of domestic demand, the short term nominal interest rate, the long term nominal interest rate, the price of equity, the nominal bilateral exchange rate, and the price of commodities. For a detailed description of the data set, please refer to the Appendix.

## **Parameters**

The set of parameters associated with our panel unobserved components model is partitioned into two subsets. Those parameters associated with the conditional mean function are estimated conditional on informative independent priors, while those parameters associated exclusively with the conditional variance function are estimated conditional on diffuse priors.

The marginal prior distributions of those parameters associated with the conditional mean function are centered within the range of estimates reported in the existing empirical literature, where available. Tight priors are imposed on a subset of these structural

parameters to ensure the existence of a unique stationary rational expectations equilibrium. The marginal prior distributions of the monetary policy rule response coefficients represent a flexible inflation targeting regime for all economies except for China, for which they represent a fixed exchange rate regime. Great ratios and bilateral trade and equity portfolio weights entering into the conditional mean function are calibrated to match their observed values in 2005. All world output shares and bilateral trade and equity portfolio weights are normalized to sum to one.

Judgment concerning the paths of trend components is generated by passing the levels of all observed endogenous variables through the filter described in Hodrick and Prescott (1997). Stochastic restrictions on the trend components of all observed endogenous variables are derived from these preliminary estimates, with a time varying innovation covariance matrix set proportional to that obtained from unrestricted estimation. Reflecting the considerable uncertainty surrounding these preliminary trend component estimates, the factor of proportionality is set equal to 3<sup>2</sup>. Initial conditions for the cyclical components of exogenous variables are given by their unconditional means and variances, while the initial values of all other state variables are treated as parameters, and are calibrated to match functions of initial realizations of the levels of observed endogenous variables, or preliminary estimates of their trend components calculated with the filter due to Hodrick and Prescott (1997).

The posterior mode is calculated by numerically maximizing the logarithm of the posterior density kernel with a modified steepest ascent algorithm. Parameter estimation results pertaining to the period 1999Q3 through 2009Q2 are reported in Table 1 of the Appendix. The sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Klein (2000) is satisfied in a neighborhood around the posterior mode, while our estimator of the Hessian is not nearly singular at the posterior mode, suggesting that the linear state space representation of our panel unobserved components model is locally identified.

The posterior modes of structural parameters are generally close to their prior means. For some structural parameters, this result reflects the imposition of tight priors, while for others it reflects weak identification. In particular, the monetary policy rule response coefficients are weakly identified, presumably because they are not estimated subject to cross economy equality restrictions. The estimated variances of shocks driving variation in cyclical components are all well within the range of estimates reported in the existing empirical literature, after accounting for data rescaling. The estimated variances of shocks driving variation in trend components vary considerably across economies and observed endogenous variables

# **Unobserved Components**

The output gap and the monetary conditions gap are both theoretically motivated candidate indicators of inflationary pressure. The monetary conditions gap is also a theoretically motivated candidate indicator of business cycle fluctuations. Smoothed estimates of the

output gap are plotted in Figure 1, while smoothed estimates of the monetary conditions gap are plotted in Figure 2. These estimates are conditional on past, present and future information.

The output gap is a measure of the position of the business cycle. Within the framework of our panel unobserved components model, the estimated output gap may be decomposed into contributions from domestic demand and net exports. Our output gap estimates reveal that business cycle fluctuations have become increasingly synchronized across economies during the estimation sample under consideration, highlighting the rising relevance of global considerations to monetary policy analysis and forecasting. Indeed, a strong global synchronized expansion followed by a precipitous global synchronized contraction, associated with the ongoing global economic crisis, marks the end of this sample period. In most economies, this contraction in the level of economic activity was primarily attributable to a collapse in domestic demand. Notable exceptions are China and Germany, where robust domestic demand was dominated by a collapse in net exports. Our terminal output gap estimates are uniformly negative and generally large. Small estimates are associated only with Brazil, China and India.

The monetary conditions gap is a composite of those determinants of the output gap, and by implication output price inflation, over which monetary policy exerts relatively direct control. Within the framework of our panel unobserved components model, the estimated monetary conditions gap may be decomposed into contributions from the estimated financial conditions gap and the real effective exchange rate. The estimated financial conditions gap may in turn be decomposed into contributions from the long term real interest rate and the real value of an internationally diversified equity portfolio. Loose monetary and financial conditions prevailed in the Euro Area and the United States during the sustained build up to the global economic crisis, attributable originally to low long term real interest rates, and subsequently to high real equity prices. During the crisis which ensued, collapses in real equity prices and real effective currency appreciations contributed to an abrupt tightening of monetary conditions in these economies.

## IV. MONETARY POLICY ANALYSIS

We analyze the interaction between inflation, the business cycle, and monetary policy in the world economy within the framework of our estimated panel unobserved components model with reference to the empirical properties of key target and indicator variables. In particular, we quantify dynamic interrelationships among consumption price inflation, the output gap, and the monetary conditions gap with vector autocorrelations and impulse response functions. We also identify the structural determinants of these target and indicator variables with forecast error variance decompositions and historical decompositions.

## A. Vector Autocorrelations

Business cycle fluctuations are manifestations of the cumulative effects of a variety of nominal and real shocks originating domestically and abroad. Vector autocorrelations measure the strength of comovement between endogenous variables at particular lags, on average over the business cycle. Estimated vector autocorrelations between inflation, the output gap, and the monetary conditions gap are plotted in Figure 3.

Estimated vector autocorrelations reveal that the output gap and the monetary conditions gap are both leading indicators of inflation. However, the usefulness of these indicators for predicting deviations of inflation from its implicit target rate varies across economies and horizons. These deviations are relatively unpredictable in Australia, Canada and the United Kingdom, which are small open economies with well established flexible inflation targeting regimes. In contrast, they are relatively predictable within the Euro Area, the member economies of which lack autonomous monetary policies. Reflecting lags in the monetary transmission mechanism, the output gap is generally more useful for predicting inflation at short horizons, while the monetary conditions gap tends to be more useful at longer horizons.

Estimated vector autocorrelations also indicate that the monetary conditions gap is a leading indicator of the position of the business cycle. The monetary conditions gap is most useful for predicting the output gap in China, as a consequence of its fixed exchange rate regime. This indicator is also relatively useful for predicting the output gap within the Euro Area.

# **B.** Impulse Response Functions

Impulse response functions measure the dynamic responses of endogenous variables to isolated structural shocks. Within the framework of our panel unobserved components model, the effects of nominal and real shocks are transmitted throughout the world economy via trade and financial linkages, necessitating monetary policy responses to spillovers. The estimated impulse responses of inflation, the output gap, the short term nominal interest rate gap, and the real effective exchange rate gap to domestic and foreign supply, demand, monetary policy, and commodity price shocks are plotted in Figure 4 through Figure 10.

In response to a domestic supply shock, inflation rises and the output gap falls in all economies, confronting flexible inflation targeting central banks with a monetary policy tradeoff. To control inflation at the cost of exacerbating the decline in the level of economic activity, monetary conditions are tightened by raising the short term nominal interest rate, inducing an appreciation of the currency in real effective terms. Following a foreign supply shock which raises inflation and reduces the output gap in the United States, inflation falls in all other economies except for China, as a consequence of its fixed exchange rate regime. Meanwhile, the output gap falls in only Canada and Mexico, reflecting their high degrees of trade integration with the United States.

In response to a domestic demand shock, inflation and the output gap both rise in all economies, and flexible inflation targeting central banks do not face a monetary policy tradeoff. Monetary conditions are tightened to control inflation and moderate the level of economic activity by raising the short term nominal interest rate, which induces an appreciation of the currency in real effective terms. Following a foreign demand shock which raises inflation and the output gap in the United States, inflation and the output gap both rise in all other economies. The strength of transmission of this foreign demand shock is increasing in the degree of trade integration with the United States, with the largest output response exhibited by Canada, followed by Mexico.

In response to a domestic monetary policy shock, the short term nominal interest rate rises in all economies, inducing an appreciation of the currency in real effective terms. This tightening of monetary conditions causes inflation and the output gap to fall. For those economies in which the conduct of monetary policy is represented by a flexible inflation targeting regime, peak inflation control effects are realized after six to eight quarters, while peak output stabilization effects occur with a delay of four to six quarters. Following a foreign monetary policy shock which reduces inflation and the output gap in the United States, inflation rises in all other economies except for China, reflecting its fixed exchange rate regime, while the output gap falls in all other economies. The strength of transmission of this foreign monetary policy shock is increasing in the degree of trade integration with the United States, and is decreasing in the degree of nominal bilateral exchange rate flexibility, with the largest output response exhibited by China, followed by Canada.

In response to a world commodity price shock, inflation rises and the output gap falls in all economies. Monetary conditions are tightened to control inflation at the cost of exacerbating the decline in the level of economic activity by raising the short term nominal interest rate. This induces real effective currency appreciations in relatively open economies, and depreciations in relatively closed ones.

## C. Forecast Error Variance Decompositions

Forecast error variance decompositions measure the proportion of unpredictable variation in endogenous variables at particular horizons attributable to different structural shocks. Estimated forecast error variance decompositions of inflation, the output gap, and the monetary conditions gap are plotted in Figure 11 through Figure 13.

Estimated forecast error variance decompositions reveal that world risk premium shocks account for the majority of unpredictable variation in inflation at short horizons in all economies except for the United States, reflecting its relatively low degree of trade openness, and the invoicing of commodity prices in its currency. These world risk premium shocks encompass liquidity risk premium shocks, equity risk premium shocks, and exchange rate risk premium shocks. At longer horizons, domestic supply shocks also contribute substantially to inflation forecast errors, as do domestic monetary policy shocks.

With regards to the unpredictable component of business cycle fluctuations, estimated forecast error variance decompositions indicate that domestic demand shocks account for the majority of unpredictable variation in the output gap at short horizons in all economies. These contributions of domestic demand shocks are closely followed by those of foreign demand shocks in selected economies having a high degree of trade openness, namely Canada, Germany and Korea. At longer horizons, domestic supply shocks also contribute substantially to output gap forecast errors, as do domestic monetary policy shocks and world risk premium shocks.

In parallel with our forecast error variance decompositions of inflation, world risk premium shocks are estimated to account for the majority of unpredictable variation in the monetary conditions gap at short horizons in all economies. At intermediate horizons, domestic monetary policy shocks also contribute substantially to monetary conditions gap forecast errors, while domestic supply shocks also contribute substantially at long horizons. The only economy in which foreign monetary policy shocks contribute substantially at any horizon is China, reflecting its fixed exchange rate regime.

# **D.** Historical Decompositions

Historical decompositions measure the contributions of different structural shocks to realizations of endogenous variables. Estimated historical decompositions of inflation, the output gap, and the monetary conditions gap are plotted in Figure 14 through Figure 16.

Estimated historical decompositions reveal diverse economy specific sources of deviations of inflation from its implicit target rate. The relative contributions of domestic versus foreign factors varies systematically across economies with their degree of trade and financial openness, with inflation in relatively open economies such as Germany and the United Kingdom driven to a greater extent by shocks originating abroad. Estimated implicit inflation targets have recently stabilized at low levels in most economies, particularly those with well established flexible inflation targeting regimes. The end of the estimation sample period under consideration is marked by a synchronized global rise in inflation which overlaps with the end of the sustained build up to the ongoing global economic crisis, followed by a synchronized global fall in inflation coincident with the onset and intensification of the crisis. While the sources of these global inflation dynamics were predominantly economy specific, they were strongly amplified by world commodity price shocks.

With regards to the sources of business cycle fluctuations, estimated historical decompositions of the output gap attribute them primarily to economy specific combinations of domestic and foreign demand shocks. In parallel with our historical decompositions of inflation, business cycle dynamics in relatively open economies such as China and Germany have been driven to a greater extent by foreign demand shocks. During the build up to the global economic crisis, positive domestic demand shocks contributed to the accumulation of excess demand pressure in all economies, often amplified by foreign demand shocks and world risk premium shocks. During the precipitous global synchronized contraction which

ensued, economy specific combinations of negative domestic and foreign demand shocks rapidly eliminated this excess demand pressure, supplanting it with excess supply pressure to varying degrees. Notable exceptions are China and Germany, where small positive domestic demand shocks were dominated by large negative foreign demand shocks.

Estimated historical decompositions of the monetary conditions gap expose factors which contributed to the global economic crisis. Exceptionally loose monetary policies in the Euro Area and the United States at the beginning of the build up to the crisis were followed by reinforcing decreases in world risk premia. The contribution of declines in world risk premia to a loosening of monetary conditions was particularly large in the United States, which experienced large falls in domestic liquidity and equity risk premia. At the onset of the crisis, abrupt world risk premia reversals contributed to a tightening of monetary conditions in these economies, in spite of large unsystematic monetary policy interventions in the United States. These increases in world risk premia subsequently intensified, and only recently started to unwind.

#### V. FORECASTING

The world economy is complex, and any structural macroeconometric model of it is necessarily misspecified to some extent, while any forecasts generated by such a model are necessarily based on an incomplete information set. In order to mitigate these problems while respecting monetary policy relevant constraints, we employ a Bayesian forecasting procedure which combines restricted forecasts generated by our estimated panel unobserved components model with judgment.

## **A.** Forecasting Procedure

Consider the linear state space model consisting of signal equations (24) and (26), and state equation (25). Given initial conditions  $z_{T|T}$  and  $P_{T|T}$ , dynamic out of sample forecasts at horizon h conditional on information available at time T, and judgment concerning the paths of linear combinations of state variables in sample, satisfy prediction equations:

$$\mathbf{z}_{T+h|T+h-1} = \mathbf{G}_1 \mathbf{z}_{T+h-1|T+h-1},\tag{44}$$

$$\boldsymbol{P}_{T+h|T+h-1} = \boldsymbol{G}_1 \boldsymbol{P}_{T+h-1|T+h-1} \boldsymbol{G}_1^{\mathsf{T}} + \boldsymbol{G}_2 \boldsymbol{\Sigma}_4 \boldsymbol{G}_2^{\mathsf{T}}, \tag{45}$$

$$y_{T+h|T+h-1} = F_1 z_{T+h|T+h-1}, (46)$$

$$\boldsymbol{Q}_{T+h|T+h-1} = \boldsymbol{F}_1 \boldsymbol{P}_{T+h|T+h-1} \boldsymbol{F}_1^{\mathsf{T}}, \tag{47}$$

$$\mathbf{w}_{T+h|T+h-1} = \mathbf{H}_1 \mathbf{z}_{T+h|T+h-1},\tag{48}$$

$$\mathbf{R}_{T+h|T+h-1} = \mathbf{H}_{1} \mathbf{P}_{T+h|T+h-1} \mathbf{H}_{1}^{\mathsf{T}} + \boldsymbol{\Sigma}_{5,T+h}. \tag{49}$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, dynamic out of sample forecasts at horizon h conditional on information available at time T, and judgment concerning the paths of linear combinations of state variables both in and out of sample, satisfy Bayesian updating equations

$$\mathbf{z}_{T+h|T+h} = \mathbf{z}_{T+h|T+h-1} + \mathbf{K}_{\mathbf{w}_{T+h}} (\mathbf{w}_{T+h} - \mathbf{w}_{T+h|T+h-1}), \tag{50}$$

$$P_{T+h|T+h} = P_{T+h|T+h-1} - K_{w_{T+h}} H_1 P_{T+h|T+h-1},$$
(51)

where  $\mathbf{K}_{w_{T+h}} = \mathbf{P}_{T+h|T+h-1} \mathbf{H}_{1}^{\mathsf{T}} \mathbf{R}_{T+h|T+h-1}^{-1}$ . Under our distributional assumptions, recursive forward evaluation of equations (44) through (51) yields mean squared error optimal conditional forecasts.

## **B.** Forecasting Results

We generate dynamic out of sample forecasts of inflation and output growth conditional on monetary policy relevant constraints and judgment concerning the paths of these variables. The results of this forecasting exercise are plotted in Figure 17 and Figure 18.

The restricted forecasts of inflation and output growth are generated by an augmented linear state space representation of our panel unobserved components model subject to the constraint that the levels of all real effective exchange rates remain constant out of sample. Imposing these deterministic restrictions occasionally results in substantial revisions to the unrestricted forecasts of inflation and output growth. The size of these revisions is generally increasing across economies with the size of the estimated real effective exchange rate gap at the forecast origin, and tends to be larger for inflation than for output growth.

The judgmental forecasts of inflation and output growth were produced by the International Monetary Fund. To ensure compatible forecast origins, they were obtained from the October 2009 vintage of the World Economic Outlook database. These alternative forecasts were also generated subject to the constraint of constant real effective exchange rates. Stochastic restrictions on the paths of inflation and output growth out of sample are derived from these judgmental forecasts, with a time varying forecast error covariance matrix set proportional to that associated with our restricted forecasts. Reflecting the uncertainty surrounding these alternative forecasts, the factor of proportionality is set equal to 1<sup>2</sup>.

The combined forecasts of inflation and output growth are composites of the restricted and judgmental forecasts. To elaborate, these combined forecasts are recursive weighted averages of the restricted forecasts and the judgmental forecasts. The weight assigned to the restricted forecasts is decreasing in the uncertainty surrounding them, and is increasing in the uncertainty associated with the judgmental forecasts. Reflecting this relative uncertainty, the

combined forecasts of inflation and output growth generally lie further from the restricted forecasts than the judgmental forecasts, which tend to be smoother. Nevertheless, the restricted and judgmental forecasts of inflation and output growth generally have similar profiles, and unanimously point towards a synchronized global economic recovery. Although the restricted forecasts indicate that this recovery will be stronger than is envisaged in the judgmental forecasts, they also suggest that it will be less durable.

## VI. CONCLUSION

This paper develops a panel unobserved components model of the monetary transmission mechanism in the world economy, disaggregated into its fifteen largest national economies. A variety of monetary policy analysis and forecasting applications of the estimated model are demonstrated. These include the measurement of monetary conditions, the analysis of the monetary transmission mechanism, and the generation of conditional forecasts of inflation and output growth. They are based on a novel Bayesian framework for incorporating judgment concerning the paths of unobserved variables into estimation and forecasting.

This panel unobserved components model of the world economy implicitly treats fiscal policy as a source of business cycle fluctuations to which monetary policy must respond. Yet the ongoing global economic crisis has accentuated the countercyclical stabilization role of fiscal policy. Augmenting this model of the monetary transmission mechanism with a fiscal transmission mechanism remains an objective for future research.

## **Appendix. Description of the Data Set**

Estimation is based on quarterly data on several macroeconomic and financial market variables for fifteen economies over the period 1999Q1 through 2009Q2. The economies under consideration are Australia, Brazil, Canada, China, France, Germany, India, Italy, Japan, Korea, Mexico, Russia, Spain, the United Kingdom, and the United States. This data was obtained from the GDS database maintained by the International Monetary Fund where available, and from the CEIC database maintained by Internet Securities Incorporated otherwise.

The macroeconomic variables under consideration are the price of output, the price of consumption, the quantity of output, the quantity of domestic demand, and the price of commodities. The price of output is measured by the seasonally adjusted gross domestic product price deflator, while the price of consumption is proxied by the seasonally adjusted total consumer price index. The quantity of output is measured by seasonally adjusted real gross domestic product, while the quantity of domestic demand is measured by seasonally adjusted real domestic demand. The price of commodities is proxied by a broad commodity price index denominated in United States dollars.

The financial market variables under consideration are the short term nominal interest rate, the long term nominal interest rate, the price of equity, and the nominal bilateral exchange rate. The short term nominal interest rate is measured by the policy rate where available, and the treasury bill yield otherwise, expressed as a period average. The long term nominal interest rate is measured by the long term government bond yield where available, and the long term lending or deposit rate otherwise, expressed as a period average. The price of equity is proxied by a broad stock price index denominated in local currency units. The nominal bilateral exchange rate is measured by the domestic currency price of one United States dollar expressed as a period average.

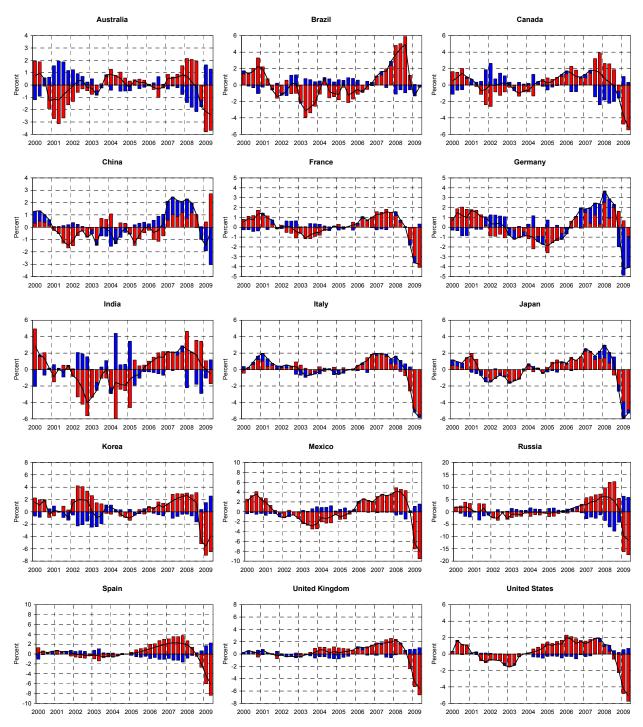
Calibration is based on annual data extracted from databases maintained by the International Monetary Fund where available, and from the World Bank Group otherwise. Great ratios are derived from the WEO database, bilateral trade weights are derived from the DOTS database, and bilateral equity portfolio weights are derived from the CPIS and WDI databases.

Table 1. Parameter Estimation Results

	Prior	r														Poste	erior															
			World	Australia	Br	azil	Cana	ada	Chi	na	Fran	ce	Gem	nany	Ind	lia	Ita	ıly	Jap	an	Ko	rea	Mex	kico	Rus	sia	Spa	in	United K	Lingdom	United	States
	Mean	SE	Mode SE	Mode SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE
$\phi_{1,1}$	0.495 5	5.0e-5	0.503 4.9e-5	5																												
$\phi_{1,2}$			0.504 4.9e-5																													
$\theta_{\scriptscriptstyle 1,1}$	0.010 1	.0e-2	0.008 3.0e-4	l																												
$\theta_{\scriptscriptstyle 2,1}$	0.100 1	.0e-1	0.350 2.5e-3	3																												
$\theta_{\scriptscriptstyle 2,2}$			0.165 9.3e-4																													
$\phi_{3,1}$			0.495 4.9e-5																													
$\theta_{\scriptscriptstyle 3,1}$			-0.399 1.5e-2																													
$\theta_{\scriptscriptstyle 3,2}$			-0.009 9.4e-4																													
$\theta_{\scriptscriptstyle 4,1}$			0.992 2.8e-2																													
$\theta_{\scriptscriptstyle 4,2}$			0.049 1.5e-3																													
$\phi_{5,1}$			0.749 7.5e-5																													
$\theta_{\scriptscriptstyle 5,1}$	1.500 1			1.497 3.3e-										3.8e-2		2.8e-2				4.2e-2		2.6e-2				2.4e-2				3.3e-2		
$\theta_{\scriptscriptstyle 5,2}$	0.125 1			0.125 2.5e-											0.125						0.125		,								0.125	2.4e-2
$\theta_{\scriptscriptstyle 5,3}$	0.000 1			0.001 9.2e	-4 0.001	8.4e-4	0.001	1.0e-3	0.249	1.9e-2			0.001	8.8e-4	0.001	3.3e-3			0.001	2.3e-3	0.001	1.1e-3	0.001	2.5e-3	0.001	1.0e-3			0.001	1.2e-3		
$\phi_{6,1}$			0.493 4.9e-5																													
$\phi_{6,2}$			0.493 4.9e-5																													
$\theta_{\scriptscriptstyle 6,1}$			0.071 6.7e-4																													
$\phi_{7,1}$			0.494 4.9e-5																													
$\phi_{7,2}$			0.494 4.9e-5																													
$\theta_{7,1}$			0.936 7.2e-2																													
$\theta_{7,2}$			-0.980 7.4e-2																													
$\phi_{8,1}$			0.495 4.9e-5																													
$\phi_{8,2}$			0.495 4.9e-5																													
$\theta_{_{8,1}}$			-0.099 9.9e-3																													
$\phi_{9,1}$			0.495 4.9e-5																													
$\phi_{9,2}$			0.495 4.9e-5																													
$\theta_{9,1}$			0.100 1.8e-2																													
$ ho_{\hat{P}^{\scriptscriptstyle{y}}}$			0.250 2.5e-5																													
$ ho_{\hat{P}^{\scriptscriptstyle M}}$			0.250 2.5e-5																													
$ ho_{\hat{X}}$			0.500 5.0e-5																													
$ ho_{\hat{\scriptscriptstyle D}}$	0.500 5	5.0e-5	0.500 5.0e-5																													
$\sigma_{\hat{\ell}^y}^{\scriptscriptstyle 2}$		00		2.5e-4 4.9e-																												
$\sigma_{\hat{P}_2^M}^{\scriptscriptstyle 2}$		00		1.8e-4 2.3e-																												
$\sigma_{\hat{X}}^{_{2}}$		00		4.4e-2 2.7e-																												
$\sigma_{\hat{\mathcal{Q}}}^{\scriptscriptstyle 2}$		00		2.4e-2 5.1e-																												
$\sigma_{\hat{i}_{2}^{s}}^{z}$		00		8.8e-3 2.0e-											9.1e-3				9.0e-3										8.6e-3			
$\sigma_{i^L}^2$		00		1.0e-4 2.1e-																												
$\sigma_{\hat{P}_{2}^{STK}}^{z}$		00		1.6e+1 3.7e-																												
$\sigma_{\hat{s}}^{z}$		00		4.2e-1 8.6e-	-2 4.3e-1	1 3.3e-2	4.2e-1	9.1e-2	4.1e-1	9.2e-2				9.4e-2	4.2e-1	9.2e-2			4.2e-1	9.2e-2						6.2e-2			4.2e-1	9.0e-2		
$\sigma_{\hat{p}com}^2$		00	5.7e-2 1.1e-2																													
$\sigma_{\overline{\ell}^{r}}^{2}$		00		1.1e-5 1.8e-																												
$\sigma_{\overline{P}_2^C}^2$		00		6.5e-6 1.2e-																												
$\sigma_{\overline{\chi}}^{2}$		00		2.4e-6 1.1e-																												
$\sigma_{ar{\mathcal{D}}}^{\scriptscriptstyle 2}$		00		3.1e-5 3.4e-																												
$\sigma_{\overline{\zeta}^s}^z$		00		8.9e-6 2.4e-																												
$\sigma_{\bar{i}^L}^2$		00		2.8e-6 5.2e-																												
$\sigma_{\overline{P}^{STK}}^2$		00		2.3e-3 2.7e-																												
$\sigma_{\overline{s}}^2$		00		6.2e-2 4.9e-																5.3e-3	7.3e-2	4.7e-3	2.9e-3	1.2e-4	1.4e-1	4.2e-3			5.4e-2	6.7e-3		
$\sigma_{\overline{p}_{COM}}^{2}$		00	1.8e-3 7.0e-5	5																												

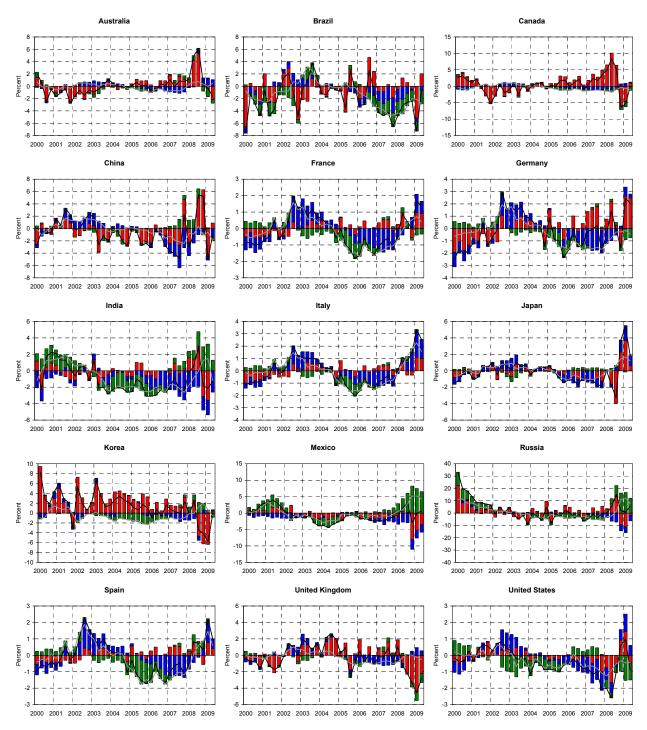
Note: All priors are normally distributed, while all posteriors are asymptotically normally distributed. The marginal prior distributions of the response coefficients of the monetary policy rule for China are  $\theta_{5,1,CHN} \sim \mathcal{N}(0,1)$ ,  $\theta_{5,2,CHN} \sim \mathcal{N}(0,1)$  and  $\theta_{5,3,CHN} \sim \mathcal{N}(1/S,1/S^2)$ , where S denotes the seasonal frequency. All observed endogenous variables are rescaled by a factor of 100.

Figure 1. Output Gap Estimates



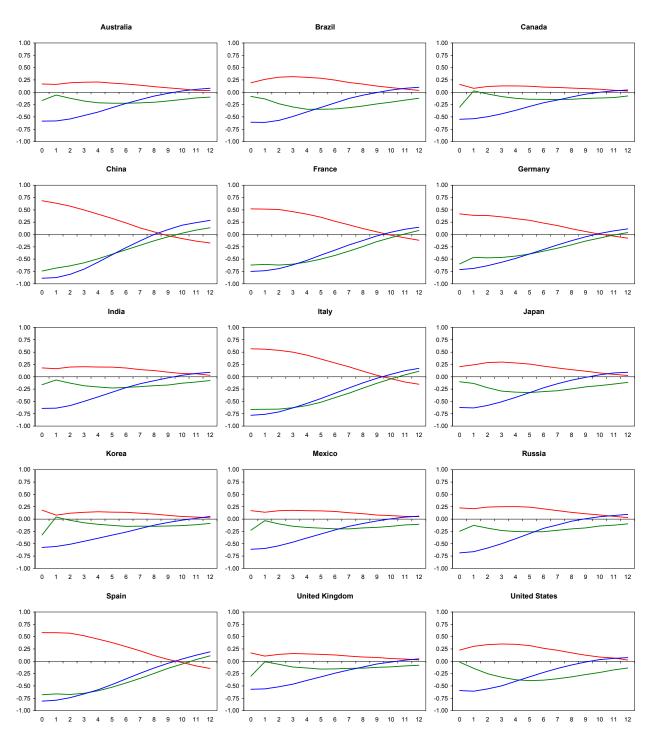
Note: Decomposes smoothed estimates of the output gap ■ into contributions from domestic demand ■ and net exports ■.

Figure 2. Monetary Conditions Gap Estimates



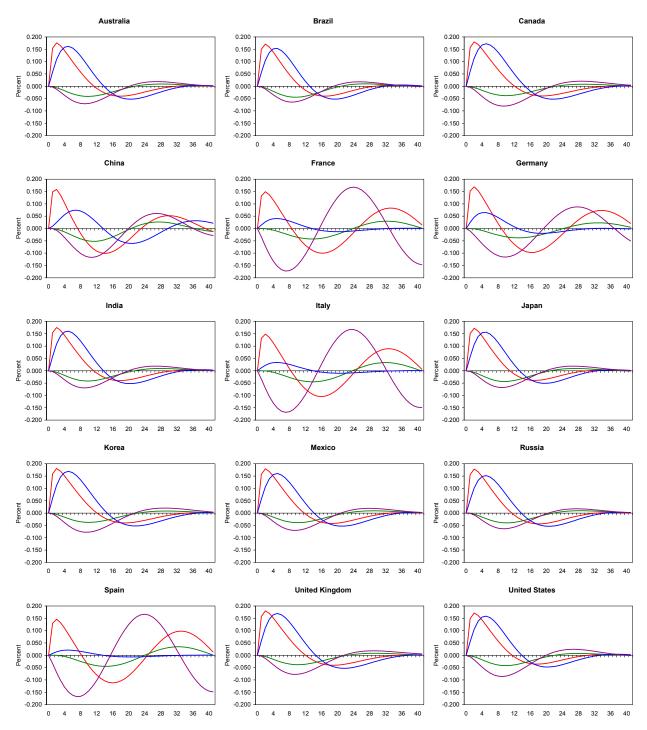
Note: Decomposes smoothed estimates of the monetary conditions gap  $\blacksquare$  into contributions from the financial conditions gap  $\blacksquare$  and the real effective exchange rate  $\blacksquare$ . Smoothed estimates of the financial conditions gap  $\blacksquare$  are decomposed into contributions from the long term real interest rate  $\blacksquare$  and the real value of an internationally diversified equity portfolio  $\blacksquare$ .

Figure 3. Vector Autocorrelations



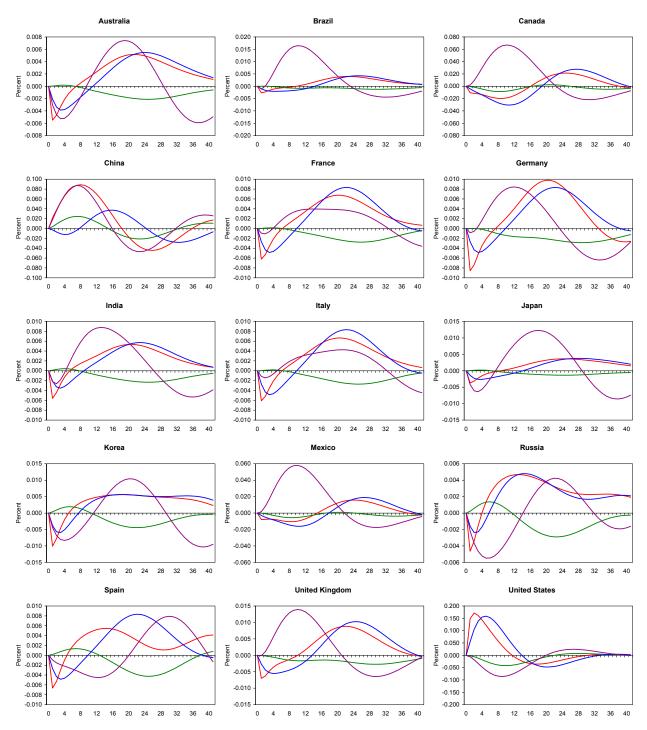
Note: Depicts the correlations of consumption price inflation with the lagged output gap  $\blacksquare$ , consumption price inflation with the lagged monetary conditions gap  $\blacksquare$ , and the output gap with the lagged monetary conditions gap  $\blacksquare$ . These correlations are calculated with a Monte Carlo simulation with 999 replications for 2T periods, discarding the first T simulated observations to eliminate dependence on initial conditions, where T denotes the observed sample size.

Figure 4. Impulse Responses to a Domestic Supply Shock



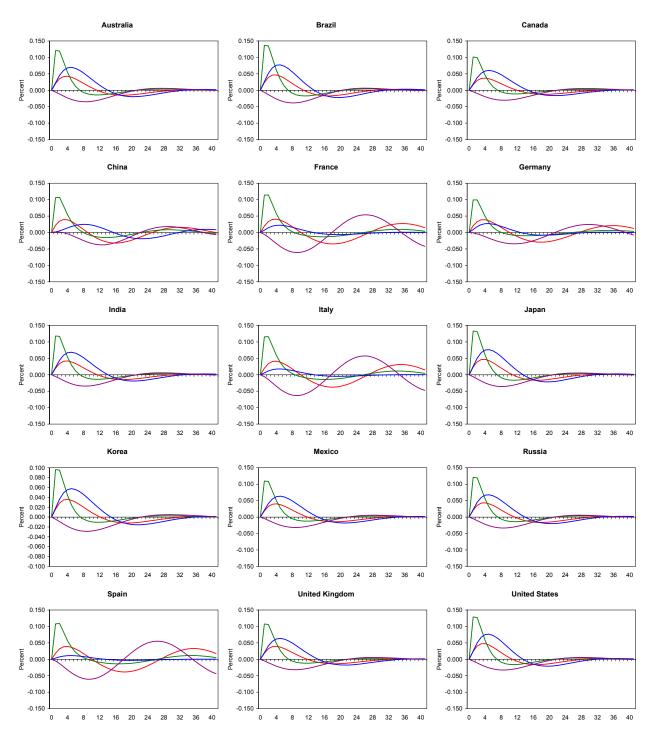
Note: Depicts the impulse responses of consumption price inflation ■, the output gap ■, the short term nominal interest rate gap ■, and the real effective exchange rate gap ■ to domestic supply shocks of size normalized to one standard deviation in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 5. Impulse Responses to a Foreign Supply Shock



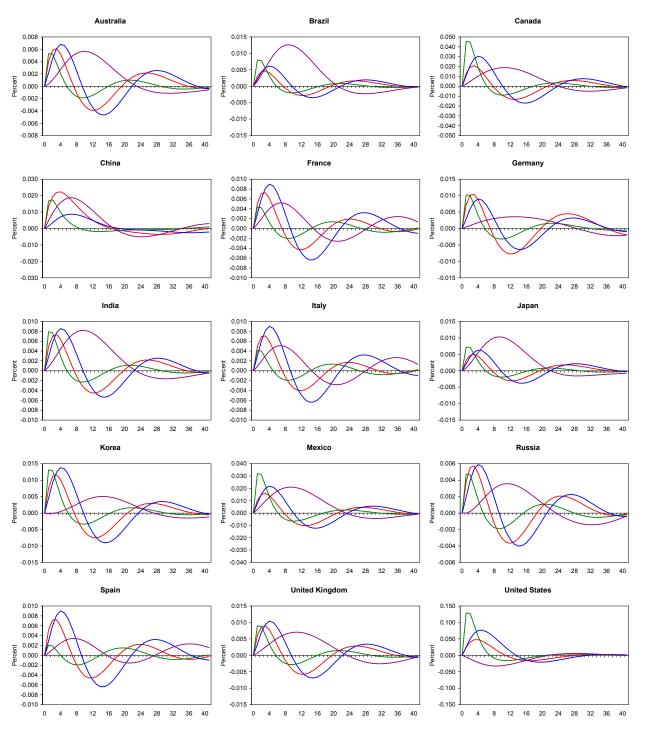
Note: Depicts the impulse responses of consumption price inflation ■, the output gap ■, the short term nominal interest rate gap ■, and the real effective exchange rate gap ■ to a one standard deviation supply shock in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 6. Impulse Responses to a Domestic Demand Shock



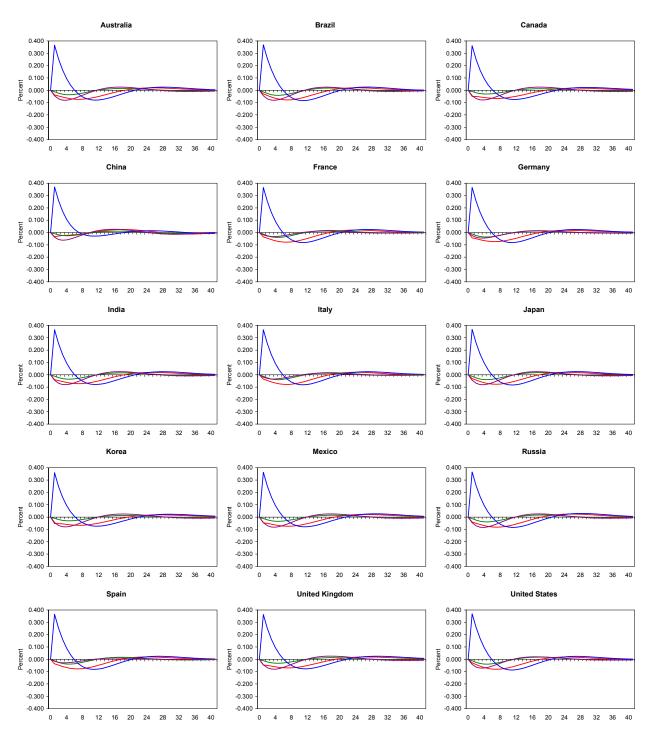
Note: Depicts the impulse responses of consumption price inflation ■, the output gap ■, the short term nominal interest rate gap ■, and the real effective exchange rate gap ■ to domestic demand shocks of size normalized to one standard deviation in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 7. Impulse Responses to a Foreign Demand Shock



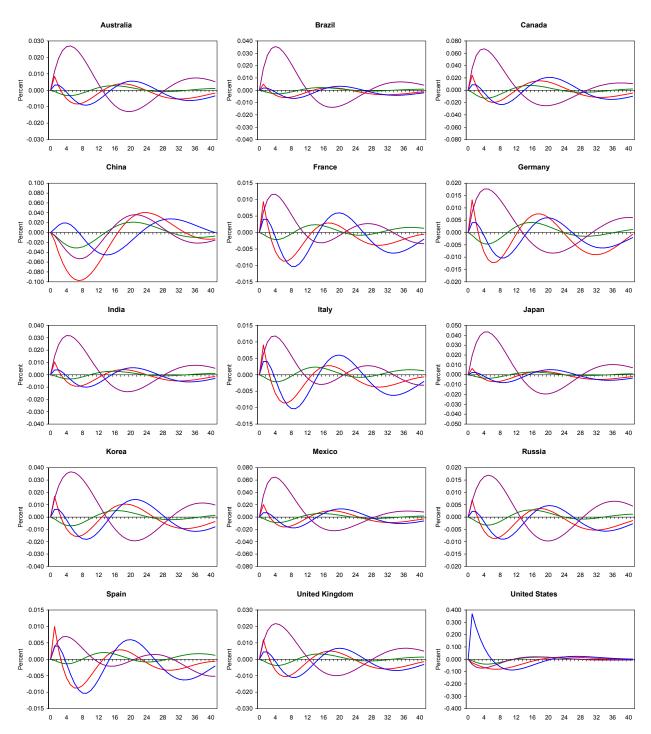
Note: Depicts the impulse responses of consumption price inflation ■, the output gap ■, the short term nominal interest rate gap ■, and the real effective exchange rate gap ■ to a one standard deviation demand shock in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 8. Impulse Responses to a Domestic Monetary Policy Shock



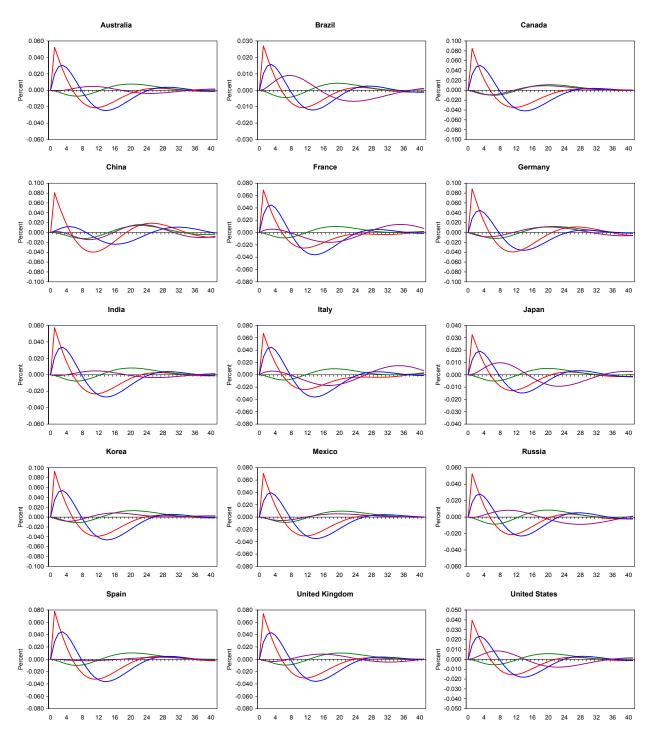
Note: Depicts the impulse responses of consumption price inflation **u**, the output gap **u**, the short term nominal interest rate gap **u**, and the real effective exchange rate gap **u** to domestic monetary policy shocks of size normalized to one standard deviation in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 9. Impulse Responses to a Foreign Monetary Policy Shock



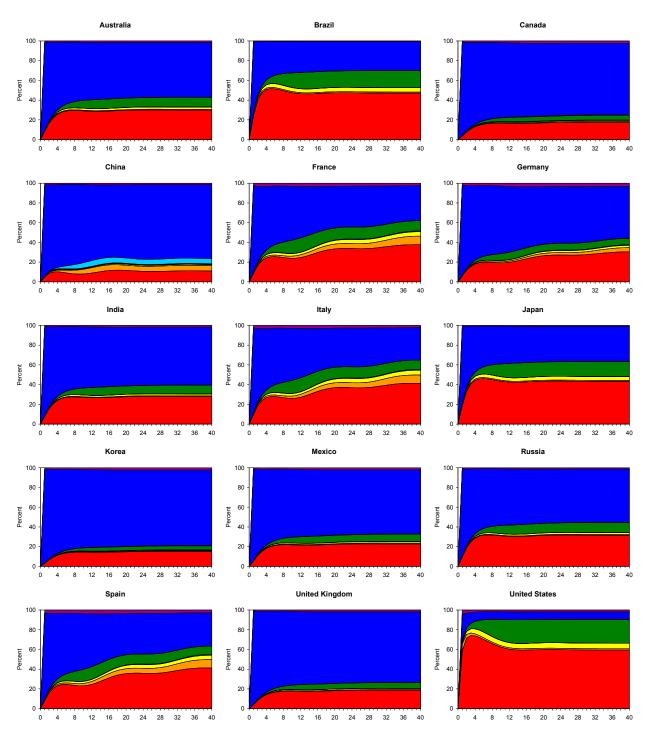
Note: Depicts the impulse responses of consumption price inflation **•**, the output gap **•**, the short term nominal interest rate gap **•**, and the real effective exchange rate gap **•** to a one standard deviation monetary policy shock in the United States. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 10. Impulse Responses to a World Commodity Price Shock



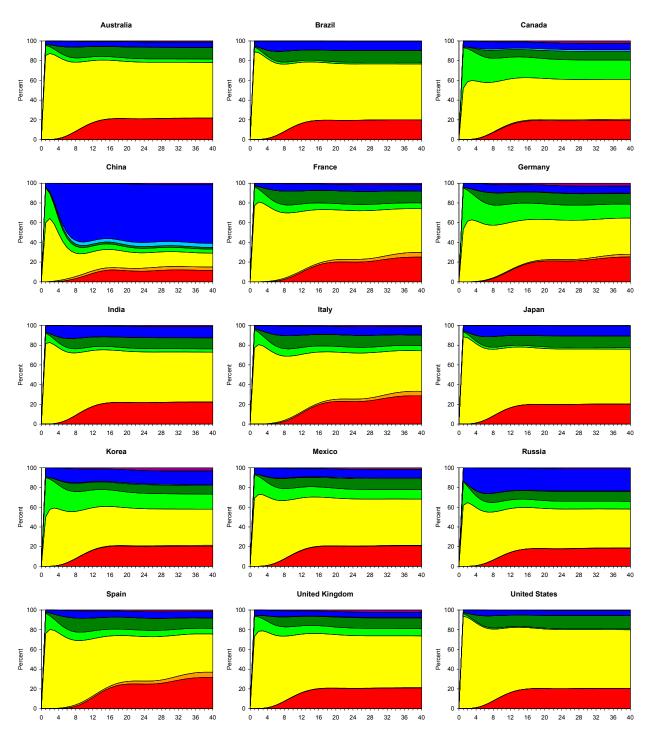
Note: Depicts the impulse responses of consumption price inflation ■, the output gap ■, the short term nominal interest rate gap ■, and the real effective exchange rate gap ■ to a one standard deviation world commodity price shock. Inflation and the short term nominal interest rate are expressed as annual percentage rates.

Figure 11. Forecast Error Variance Decompositions of Inflation



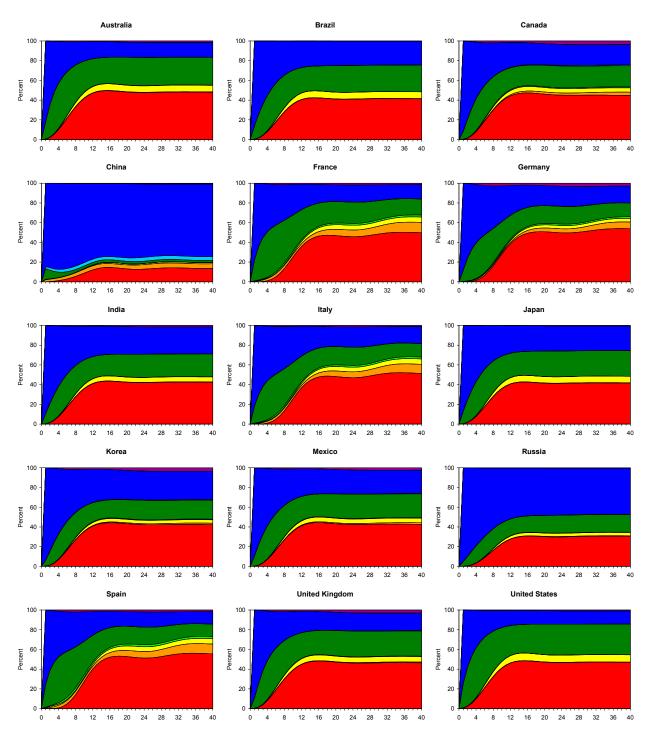
Note: Decomposes the horizon dependent forecast error variance of consumption price inflation into contributions from domestic supply  $\blacksquare$ , foreign supply  $\blacksquare$ , domestic demand  $\blacksquare$ , foreign demand  $\blacksquare$ , domestic monetary policy  $\blacksquare$ , foreign monetary policy  $\blacksquare$ , world risk premium  $\blacksquare$ , and world commodity price  $\blacksquare$  shocks.

Figure 12. Forecast Error Variance Decompositions of the Output Gap



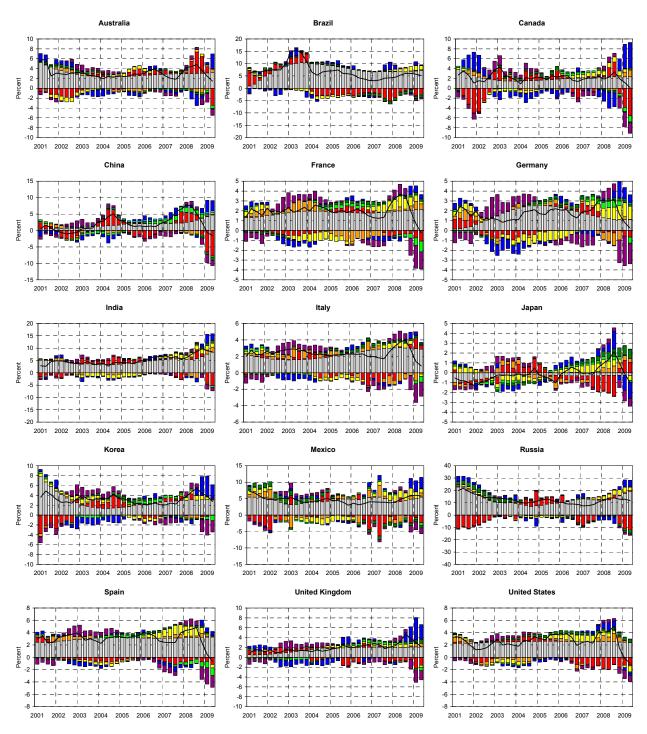
Note: Decomposes the horizon dependent forecast error variance of the output gap into contributions from domestic supply , foreign supply , domestic demand , foreign demand , domestic monetary policy , foreign monetary policy , world risk premium , and world commodity price shocks.

Figure 13. Forecast Error Variance Decompositions of the Monetary Conditions Gap



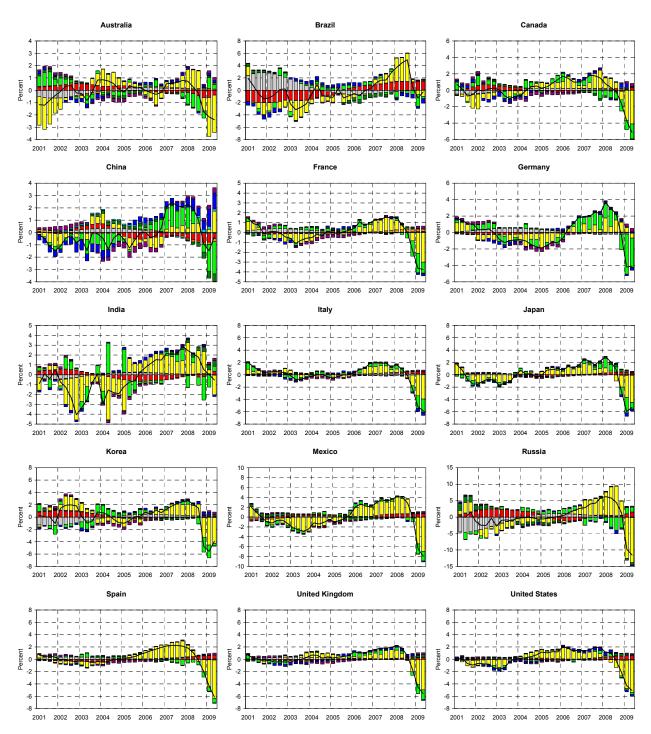
Note: Decomposes the horizon dependent forecast error variance of the monetary conditions gap into contributions from domestic supply **a**, foreign supply **a**, domestic demand **b**, foreign demand **c**, domestic monetary policy **c**, world risk premium **c**, and world commodity price **c** shocks.

Figure 14. Historical Decompositions of Inflation



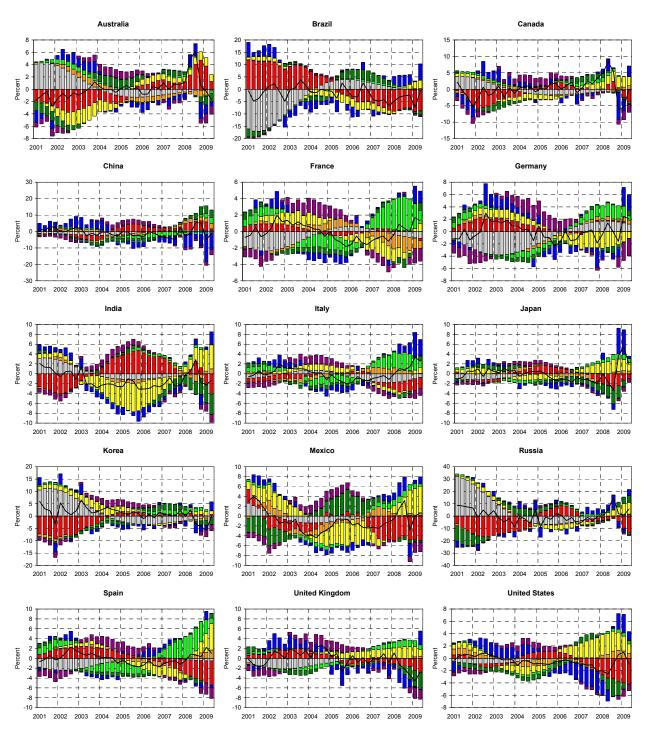
Note: Decomposes observed consumption price inflation  $\blacksquare$  as measured by the seasonal logarithmic difference of the consumption price level into the sum of a trend component  $\blacksquare$  and contributions from domestic supply  $\blacksquare$ , foreign supply  $\blacksquare$ , domestic demand  $\blacksquare$ , foreign demand  $\blacksquare$ , domestic monetary policy  $\blacksquare$ , foreign monetary policy  $\blacksquare$ , world risk premium  $\blacksquare$ , and world commodity price  $\blacksquare$  shocks.

Figure 15. Historical Decompositions of the Output Gap



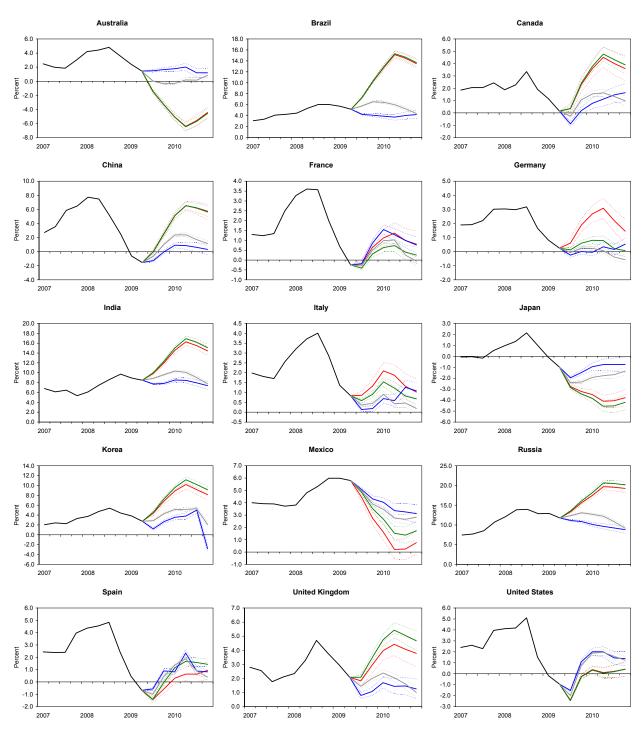
Note: Decomposes smoothed estimates of the output gap ■ into the sum of a deterministic component ■ and contributions from domestic supply ■, foreign supply ■, domestic demand ■, foreign demand ■, domestic monetary policy ■, foreign monetary policy ■, world risk premium ■, and world commodity price ■ shocks.

Figure 16. Historical Decompositions of the Monetary Conditions Gap



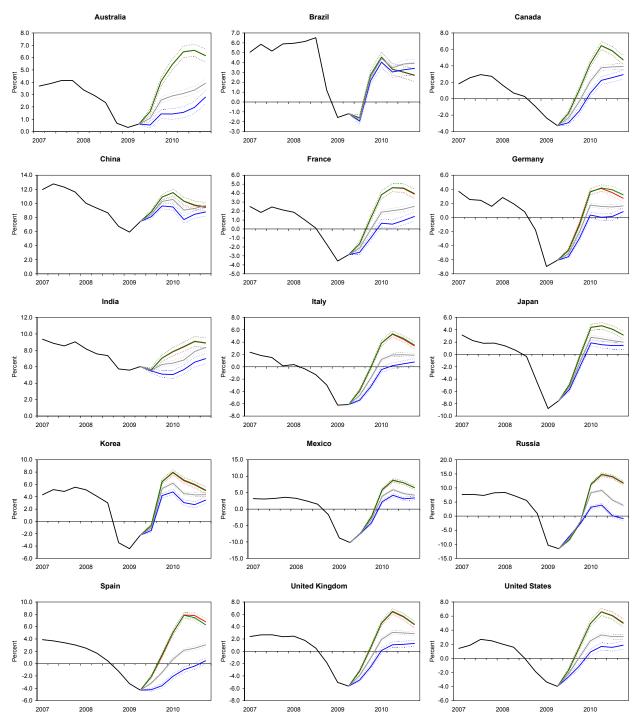
Note: Decomposes smoothed estimates of the monetary conditions gap  $\blacksquare$  into the sum of a deterministic component  $\blacksquare$  and contributions from domestic supply  $\blacksquare$ , foreign supply  $\blacksquare$ , domestic demand  $\blacksquare$ , foreign demand  $\blacksquare$ , domestic monetary policy  $\blacksquare$ , foreign monetary policy  $\blacksquare$ , world risk premium  $\blacksquare$ , and world commodity price  $\blacksquare$  shocks.

Figure 17. Conditional Forecasts of Inflation



Note: Depicts observed consumption price inflation ■ as measured by the seasonal logarithmic difference of the consumption price level together with unrestricted forecasts ■, restricted forecasts ■, judgmental forecasts ■, and combined forecasts ■. Symmetric 95% confidence intervals represented by dashed lines assume normally distributed innovations and known parameters.

Figure 18. Conditional Forecasts of Output Growth



Note: Depicts observed output growth ■ as measured by the seasonal logarithmic difference of the level of output together with unrestricted forecasts ■, restricted forecasts ■, judgmental forecasts ■, and combined forecasts ■. Symmetric 95% confidence intervals represented by dashed lines assume normally distributed innovations and known parameters.

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