WP/15/199

IMF Working Paper

Demand for Value Added and Value-Added Exchange Rates

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INTERNATIONAL MONETARY FUND

 WP/15/199

IMF Working Paper

Research Department

Demand for Value Added and Value-Added Exchange Rates

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Authorized for distribution by Thomas Helbling

September 2015

Abstract

We examine the role of cross-border input linkages in governing how international relative price changes influence demand for domestic value added. We define a novel value-added real effective exchange rate (REER), which aggregates bilateral value-added price changes, and link this REER to demand for value added. Input linkages enable countries to gain competitiveness following depreciations by supply chain partners, and hence counterbalance beggar-thy-neighbor effects. Cross-country differences in input linkages also imply that the elasticity of demand for value added is country specific. Using global input-output data, we demonstrate these conceptual insights are quantitatively important and compute historical value-added REERs.

JEL Classification Numbers: F1, F4
Keywords: Real effective exchange rate; global supply chains
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[1] This paper combines material from two papers previously circulated under the titles ``Value-Added Exchange Rates'' (NBER Working Paper No. 18498, October 2012) and ``International Prices and Demand for Value Added with Global Supply Chains'' (July 2014). We thank Olivier Blanchard, Menzie Chinn, Diego Comin, Robert Dekle, Julian Di Giovanni, Jordi Gali, Sarma Jayanthi, Rhys Mendes, Nikhil Patel, Steven Phillips, Jay Shambaugh, Martin Schmitz, Erol Taymaz, Kei-Mu Yi, and Jing Zhang for helpful comments, as well as seminar participants at the Banque de France, BICEPS (Riga), CREI (Pompeu Fabra), Dartmouth, the Geneva Graduate Institute, the IMF, the CEPR/SNB Conference on Exchange Rates and External Adjustment, the NBER ITM Summer Institute (2013), the Tsinghua International Conference on Global Value Chains and Structural Adjustments, the Mainz Workshop in Trade and Macroeconomics (2014), the NBER IFM Summer Institute (2014), the ECB/CBRT Conference on Assessing the Macroeconomic Implications of Financial and Production Networks, the BoE/CfM/CEPR Workshop on International Trade, Finance, and Macroeconomics, and the AEA Annual Meetings (2014). This work was partly carried out while Johnson was a Visiting Scholar in the IMF Research Department.

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I. INTRODUCTION

Global supply chains are important conduits for international trade [Feenstra (1998); Antràs (2014)]. Despite this, cross-border input linkages are largely absent in conventional macroeconomic analysis, which emphasizes demand-side expenditure switching as the key channel via which changes in international relative prices affect real economic activity and external balances [Johnson (1958); Corden (1960); Backus, Kehoe, and Kydland (1994); Obstfeld (2001)]. Global supply chains pose a challenge to this conventional view, because they link countries together on the supply side as well. These supply-side linkages alter how relative price changes influence international competitiveness – i.e., the ability to sell domestic goods, and ultimately domestic value added, on world markets.

Global supply chains introduce new, supply-side channels via which relative price changes affect demand for domestic goods. To fix ideas, consider how a yen depreciation affects Japan's Asian trading partners. The conventional logic is straightforward: Japanese goods become more competitive, so consumers switch expenditure toward them, which lowers demand for Asian-produced goods. When input trade is important, this conventional logic is incomplete. Because Japan supplies inputs to Asia, the yen depreciation also lowers production costs for downstream Asian producers, making their goods more competitive and stimulating demand for them. This counterbalances the demand-side expenditure switching channel, so which channel dominates is ultimately an empirical matter.

Global supply chains also alter the nature of international competition. In conventional macro-frameworks, each country's differentiated 'product' competes against 'products' from other countries on world markets [Armington (1969)]. The rise of global supply chains has made this product-centric view obsolete: countries increasingly specialize in adding value at particular stages of production, rather than in producing entire finished products [Hummels, Ishii, and Yi (2001); Yi (2003)]. This means that countries compete over supplying domestic value added to world markets, rather than products (final goods or gross exports) per se.

These observations highlight the potentially important role that global supply chains play in determining how international relative prices influence the competitiveness of, and hence demand for, domestic value added. In this paper, we put the role of supply chains under the microscope. We develop a framework to characterize demand for value added that includes trade in both final goods and intermediate inputs. Specifically, we elaborate on the supply side of workhorse international macro-models to distinguish gross output from value added in production and intermediate inputs from final goods in international trade.[1]

Using this framework, we derive an expression that links changes in demand for value added to value-added prices and final expenditure levels. This enables us to define a new value-added REER index, which aggregates bilateral value-added price changes into a composite multilateral price of domestic relative to foreign value added. We combine this value-added REER with two additional components – the price elasticity of demand for domestic value-added and an indicator for how open the economy is in value-added terms – to summarize the determinants of demand for value added. This analysis boils down the complex set of gross trade linkages across countries to describe how value-added price changes induce expenditure switching between home and foreign value added. Using global input-output data, we then characterize how input linkages shape these empirical building blocks of demand for value added and compute historical value-added REERs.

We focus our discussion around the the value-added REER index because REERs are a core piece of macroeconomic data, produced by most major statistical agencies and widely used in applications.[2] Existing REER indexes are based on product-centric theoretical foundations, which fail to account for cross-border input linkages. Our framework updates the theoretical foundations for constructing REERs to reflect the importance of global supply chain linkages in the modern global economy. In doing so, we develop an index that answers a well-defined economic question: *how much does demand for value added change following a change in relative value-added prices?* This value-added perspective is useful, because macro-policy objectives (employment, inflation, etc.) are conceptually linked to demand pressure in factor markets, to which demand for value added is directly linked.

Our value-added REER index differs conceptually from conventional REER indexes in three important ways. First, the weights attached to bilateral price changes in the value-added REER depend on both the global input-output structure *and* relative elasticities in production versus consumption. In contrast, conventional REER weights are based on gross trade flows and production alone. Incorporating input-output linkages and elasticities yields some initially surprising results.

[1] As in Armington (1969), each country produces a differentiated gross product. Departing from that framework, each country's product may be used as either an intermediate input or final good, and products themselves are produced by combining traded inputs with domestic factors. The resulting framework features three separate margins of substitution: among inputs from different source countries, between inputs and value added in production, and among final goods from alternative sources.

[2] REERs are produced by the Bank of International Settlements, the European Central Bank, the Federal Reserve, the International Monetary Fund, and the Organization for Economic Co-operation and Development (OECD), among others. For an overview of applications, see Chinn (2006).

For example, REER weights can actually be negative in our framework. The reason is that input linkages imply that a country *gains* competitiveness when prices in supply chain partners decline, which counteracts conventional beggar-thy-neighbor effects (as in the Japan-Asia example above).

Second, the mapping from the value-added REER to demand for value added depends on the country-specific elasticity of demand for value added. This value-added elasticity is a weighted average of primitive gross elasticities, with weights that depend on final and intermediate linkages across countries. For example, countries with larger shares of inputs in trade put larger weight on substitution elasticities among inputs. Because this elasticity is heterogeneous across countries, the value-added REER alone is an *incomplete* statistic for measuring the competitiveness of domestic value added. This contrasts with conventional REER indexes, where demand elasticities are assumed to be identical for all countries and hence normalized away in cross-country comparisons. Moreover, the value-added elasticity formula we provide is useful in its own right for calibrating macro-models. It describes how to aggregate gross elasticities of substitution, which can be estimated using conventional trade and production data, into composite value-added elasticities that are appropriate for value-added models.

Third, because our framework distinguishes between gross and value-added data concepts, it yields clear guidance about how to combine REER weights and prices in a theoretically consistent way to measure the price competitiveness of domestic value added. To summarize demand for value added in terms of its own price, we derive weights to attach to value-added price changes, measured using GDP deflators. In contrast, prominent conventional indexes mix gross trade weights with either consumer price changes, unit cost indexes, or GDP deflators in ways that cannot be rationalized in our framework.[3] Because our framework clarifies the link between theory and data, it yields an index that has a clear economic interpretation.

To quantify these conceptual contributions, we parameterize the framework using data on input-output linkages across countries and assign values for the substitution elasticities. We focus on two illustrative elasticity cases. The first is a case with equal elasticities in production and final demand. In this case, the framework behaves *as if* consumers have CES-Armington preferences directly defined over real value added purchased from alternative source countries. As a result, value-added REER weights can be computed using only value-added trade and production data. The second is a case with low elasticities of substitution in production (literally, Leontief pro-

[3]For example, the IMF REER and the US Federal Reserve's Broad Dollar index use consumer prices (CPIs), while the ECB's Harmonised Competitiveness Indicators include REERs based on unit cost indexes and GDP deflators. We describe the methods used by various statistical agencies in Section C.

duction). This second case reflects the commonly held view that global supply chains are inflexible in the short run. It also highlights the role that input linkages play in dampening beggar-thy-neighbor effects in the framework.

We first examine the individual building blocks underlying value-added competitiveness. We show that the weight attached to supply chain partners in the value-added REER index is typically smaller than in conventional indexes. Further, this effect is amplified when production elasticities are low. This reflects the role of input linkages in dampening the loss of competitiveness and insulating demand for value added following devaluations in supply chain partners. We also show that low production elasticities yield lower value-added elasticities for countries heavily integrated into global supply chains.

To complete the empirical analysis, we construct time series value-added REERs using historical data on input-output linkages across countries and observed price changes over the 1970-2009 period. We show that value-added indexes differ significantly from conventional REER indexes, both due to differences in weights and different measures of prices used in constructing each index. Moreover, value-added exchange rates capture competitiveness developments missed by conventional indexes in important episodes. For example, China's value-added REER appreciated by 20% during the 2000's, while its conventional REER was roughly unchanged. Value-added REERs also better capture pernicious changes in relative prices in the run-up to the Eurozone crisis – e.g., Germany experienced a substantially stronger depreciation (matched by stronger appreciations in Ireland, Spain, etc.) of its value-added REER than its conventional REER. Finally, we examine how value-added REERs and value-added elasticities combine to determine demand for value added and find that both components play a significant role.

Consistent with tradition and practice in the price index literature, our framework takes relative price changes as given. As a result, our analysis is partial equilibrium in nature, so we cannot examine the effects of particular identified shocks on relative prices or equilibrium output. Nonetheless, we are able to characterize historical shifts in competitiveness, using realized changes in relative prices observed in the data. Further, insights from our framework concerning the role of input linkages and elasticities in governing price spillovers and value-added expenditure switching can be carried over to the broader international macroeconomics literature.

In this way, our work contributes to an active literature on input linkages in international macroeconomics [Ambler, Cardia, and Zimmerman (2002); Huang and Liu (2007); Burstein, Kurz, and Tesar (2008); Di Giovanni and Levchenko (2010); Bussière et al. (2013); Bems (2014); Johnson

(2014)].[4] It also contributes to the analysis of beggar-thy-neighbor effects and competitive devaluations [Corsetti et al. (2000); Corsetti and Pesenti (2001); Tille (2001)]. Supplementing these monetary models, we identify a real channel (cross-border input linkages) that undermines standard beggar-thy-neighbor effects. More generally, our work also speaks to broad questions about the relationship between expenditure switching in gross versus value-added representations of the international macro-economy.

Our work is also naturally linked to the large macroeconomic literature on exchange rate indexes and demand-side measures of competitiveness, dating to Artus and Rhomberg (1973), Black (1976), and McGuirk (1987). Among recent work, our work is complementary in spirit to Thomas, Marquez, and Fahle (2008) and Lane and Shambaugh (2010), who develop new exchange rate indexes to capture important aspects of the modern global economy.[5] More directly, our focus on demand for value added echoes Neary (2006), who defines a competitiveness index as the change in the nominal exchange rate that would hold GDP constant, given price changes.[6] Further, Patel, Wang, and Wei (2014) build on the framework presented here to study how input linkages affect REER measurement in a multi-sector economy. This extension shifts attention toward cross-sector price adjustment, relative to our focus on international relative prices here.

The paper proceeds as follows. We present the demand for value added framework and define the real effective exchange rate in Section II. Section III discusses the economics underlying our framework, highlighting our conceptual contributions and contrasting our value-added REER to conventional REERs. We describe data and parameters in Section IV, present the empirical building blocks for measuring demand for value added in Section V, and discuss historical value-added REER indexes and competitiveness in Section VI. Section VII concludes.

II. Framework

This section presents a partial equilibrium framework that links changes in value-added prices to changes in demand for value added from each source country. Because we take value-added prices and real final expenditure as given in defining the value-added real effective exchange rate,

[4]See also Bems, Johnson, and Yi (2010) and Eaton et al. (2011), who analyze the great trade collapse in models with cross-border input linkages.

[5]Lane and Shambaugh (2010) examine how exchange rate indexes can be designed to capture the financial implications of currency movements. Thomas, Marquez, and Fahle (2008) develop an index of relative price levels that captures the competitiveness implications of rising trade with developing countries.

[6]Though related in spirit, Neary focuses on nominal GDP (and a GDP function representation of the economy), rather than real GDP, and does not consider cross-border input linkages.

we only need to specify three basic components of the economic environment: (1) preferences over final goods, (2) production functions for gross output, and (3) market clearing conditions for gross output.

A. Economic Environment

Suppose there are many countries indexed by $i, j, k \in \{1, \ldots, N\}$. Each country is endowed with a production function for an aggregate Armington differentiated good, which is used both as a final good and intermediate input. Gross output in country i, denoted Q_i, is produced by combining domestic real value added, denoted V_i, with a composite intermediate input, denoted X_i.[7] The composite input is a bundle of domestic and imported inputs, where inputs purchased by country i from country j are denoted X_{ji}.

We assume that the production structure takes the nested constant elasticity of substitution (CES) form:

$$Q_i = \left((\omega_i^v)^{1/\gamma} V_i^{(\gamma-1)/\gamma} + (\omega_i^x)^{1/\gamma} X_i^{(\gamma-1)/\gamma} \right)^{\gamma/(\gamma-1)} \tag{1}$$

$$\text{with} \quad X_i = \left(\sum_j \left(\frac{\omega_{ji}^x}{\omega_i^x} \right)^{1/\rho} X_{ji}^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}, \tag{2}$$

where the ω's are aggregation weights, γ is the elasticity of substitution between real value added and the composite input, and ρ is the elasticity of substitution among inputs.

We assume that agents in each country have CES preferences defined of over final goods.[8] Denoting final goods purchased by country i from country j as F_{ji}, preferences take the form:

$$F_i = \left(\sum_j (\omega_{ji}^f)^{1/\sigma} F_{ji}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{3}$$

where the ω's are preference weights and σ is the elasticity of substitution among final goods.

Gross output can be used as both a final good and intermediate input, so the market clearing condition for gross output is: $Q_j = \sum_{k=1}^N \left[F_{jk} + X_{jk} \right]$.

[7] Real value added can be thought of as a bundle of primary factor inputs (e.g., a Cobb-Douglas composite of capital and labor). Throughout the paper, we focus on demand for the bundle of domestic inputs.

[8] We define final goods as in the national accounts, including consumption, investment, and government spending. Therefore, we could alternatively describe Equation (3) as a final goods aggregator, which forms a composite final good used for consumption, investment, and by the government.

B. Linearization

The first order conditions for consumers and competitive firms are standard, as are the corresponding CES price indexes for gross output (p_i), the composite input (p_i^x), and the composite final good (P_F). To analyze these, we linearize and stack the first order conditions, price indexes, production functions, and market clearing conditions.

The final goods first order condition and final goods price index can be linearized as: $\hat{F}_{ji} = -\sigma(\hat{p}_j - \hat{P}_i) + \hat{F}_i$, with $\hat{P}_i = \sum_j \left(\frac{p_j F_{ji}}{P_i F_i}\right) \hat{p}_j$. We then define a vector $\mathbb{F}$ to be a N^2 dimensional vector that records final goods shipments: $\hat{\mathbb{F}} = [\hat{F}_{11}, \hat{F}_{12}, \ldots, \hat{F}_{1N}, \hat{F}_{21}, \hat{F}_{22}, \ldots]'$. This allows us to rewrite the first order conditions and price index as:

$$\hat{\mathbb{F}} = -\sigma M_1 \hat{p} + \sigma M_2 \hat{P} + M_2 \hat{F} \tag{4}$$

$$\text{with} \quad \hat{P} = W_f \hat{p}, \tag{5}$$

where $M_1 \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}$ and $M_2 \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}$. The weighting matrix W_f is an $N \times N$ matrix with ij elements $\frac{p_j F_{ji}}{P_i F_i}$ equal to country i's expenditure on final goods from country j as a share of total final goods expenditure in country i.

Turning to production, the first order conditions for intermediates linearize as: $\hat{X}_i = -\gamma(\hat{p}_i^x - \hat{p}_i) + \hat{Q}_i$ and $\hat{X}_{ji} = -\rho(\hat{p}_j - \hat{p}_i^x) + \hat{X}_i$. These can be stacked in a similar way:

$$\hat{X} = -\gamma \hat{p}^x + \gamma \hat{p} + \hat{Q} \tag{6}$$

$$\hat{\mathbb{X}} = -\rho M_1 \hat{p} + \rho M_2 \hat{p}^x + M_2 \hat{X} \tag{7}$$

$$\text{with} \quad \hat{p}^x = W_x \hat{p}, \tag{8}$$

where $\hat{\mathbb{X}} = [\hat{X}_{11}, \hat{X}_{12}, \ldots, \hat{X}_{1N}, \hat{X}_{21}, \hat{X}_{22}, \ldots]'$ is the N^2 dimensional vector of intermediate goods shipments.

The market clearing conditions can be linearized as:

$$\hat{Q} = S_F \hat{\mathbb{F}} + S_X \hat{\mathbb{X}}. \tag{9}$$

The S_F and S_X matrices collect shares of final and intermediate goods sold to each destination as a share of total gross output in the source country:

$$S_F \equiv \begin{pmatrix} s_1^f & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^f & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X \equiv \begin{pmatrix} s_1^x & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix}$$

$$\text{with} \quad s_i^f = [s_{i1}^f, \cdots, s_{iN}^f], \quad s_{ij}^f = \frac{p_i F_{ij}}{p_i Q_i}, \quad s_i^x = [s_{i1}^x, \cdots, s_{iN}^x], \quad \text{and} \quad s_{ij}^x = \frac{p_i X_{ij}}{p_i Q_i}.$$

Finally, we linearize components of the production function and the gross output price index as:

$$\hat{Q} = [diag(s_i^v)]\hat{V} + [diag(s_i^x)]\hat{X} \tag{10}$$

$$\hat{X} = W_X \hat{\mathbb{X}} \tag{11}$$

$$\hat{p} = [diag(s_i^v)]\hat{p}^v + [diag(s_i^x)]\hat{p}^x, \tag{12}$$

where $s_i^v \equiv \frac{p_i^v V_i}{p_i Q_i}$ and $s_i^x \equiv \frac{p_i^x X_i}{p_i Q_i}$ are the cost shares of real value added and the composite input in gross output. And $W_X = [diag(w_1^x), diag(w_2^x), \ldots]$ with $w_i^x = [w_{i1}^x, \cdots, w_{iN}^x]$ and $w_{ij}^x \equiv \frac{p_i X_{ij}}{p_j^x X_j}$ are shares of individual intermediates in the composite intermediate.

C. Demand for Gross Output

Demand for gross output depends on demand for both final and intermediate goods. To begin, we study how price changes influence production techniques for final goods. We then layer on endogenous changes in demand for final goods to arrive at demand for gross output.

1. Substitution in Input Use

Using Equations (6), (7), (8)), we substitute for bilateral input shipments ($\mathbb{X}$) in the gross output market clearing condition [Equation (9)] to yield:

$$\hat{Q} = S_F \hat{\mathbb{F}} + S_X \left[-\rho M_1 \hat{p} + \rho M_2 \hat{p}^x + M_2(-\gamma \hat{p}^x + \gamma \hat{p} + \hat{Q}) \right] \tag{13}$$

Note that there is an input-output loop in production here, as gross output appears on both sides of this expression. Pulling output to one side, and using $\hat{p}^x = W_x \hat{p}$ to eliminate the composite input price, we get:

$$\hat{Q} = [I - S_X M_2]^{-1} S_F \hat{\mathbb{F}} + [I - S_X M_2]^{-1} S_X \left[-\rho M_1 \hat{p} + \rho M_2 W_x \hat{p} - \gamma M_2 W_x \hat{p} + \gamma M_2 \hat{p}\right]. \tag{14}$$

The first term maps bilateral final goods shipments ($\hat{\mathbb{F}}$) through the initial input-output structure into changes in gross output. The second term captures how input choices, and hence the input-output structure, respond to gross output price changes. This input re-optimization, governed by γ and ρ, alters the mapping from final goods to gross output.

2. Substitution across Final Goods

Changes in demand for final goods depend on relative prices as well. Substituting for $\hat{\mathbb{F}}$ in Equation (14) using Equation (4), we get:

$$\begin{aligned}\hat{Q} = [I - S_X M_2]^{-1} S_F M_2 \hat{F} - \sigma [I - S_X M_2]^{-1} S_F (M_1 - M_2 W_f) \hat{p} \\ - \rho [I - S_X M_2]^{-1} S_X (M_1 - M_2 W_x) \hat{p} + \gamma [I - S_X M_2]^{-1} S_X M_2 (I - W_x) \hat{p}.\end{aligned} \tag{15}$$

The first term captures the role of changes in real final expenditure levels in altering demand for output. The second term picks up substitution in final goods purchases, hence the presence of the final goods elasticity σ there. As above, the third term picks up substitution within the input bundle, and the fourth term picks up substitution between real value added and inputs. In the end, how price changes feed through to demand for gross output depends on both supply side elasticities (γ, ρ) and the demand side elasticity (σ).

D. Demand for Value Added

To convert demand for gross output into demand for value added, we rearrange the production function (Equation (10)) and substitute for $\hat{X}$ using Equations (6) and (8):

$$\begin{aligned}\hat{V} &= [diag(s_i^v)]^{-1} \left[\hat{Q} - [diag(s_i^x)] \hat{X}\right] \\ &= \hat{Q} - \gamma [diag(s_i^x / s_i^v)] (I - W_x) \hat{p}.\end{aligned} \tag{16}$$

The first line corresponds to the definition of double-deflated real value added, which strips out changes in input use from changes in gross output to recover changes in real value added. The second line tells us that producers substitute towards produced inputs when country i's gross output price increases, and this lowers the share of real value added relative to gross output. Combining Equations (15) and (16), we arrive at:

$$\begin{aligned}\hat{V} = {} & [I - S_X M_2]^{-1} S_F M_2 \hat{F} \\ & - [I - S_X M_2]^{-1} \left[\sigma S_F (M_1 - M_2 W_f) + \rho S_X (M_1 - M_2 W_x) - \gamma S_X M_2 (I - W_x)\right] \hat{p} \\ & - \gamma [diag(s_i^x / s_i^v)](I - W_x)\hat{p}. \end{aligned} \tag{17}$$

This summarizes how demand for real value added depends on the level of real final expenditure in all countries ($\hat{F}$) and gross price changes ($\hat{p}$).

E. Linking Value-Added to Gross Output Prices

As a final step, we substitute for gross price changes to write demand for real value added in terms of value-added prices. To do this, we combine Equations (12) and (8) to write gross output price changes as function of value added price changes:

$$\hat{p} = [I - \Omega']^{-1} [diag(s_i^v)] \hat{p}^v, \tag{18}$$

where $\Omega' = diag(s_i^x) W_X$ is a global input-output matrix, with ij elements equal to the share of inputs from i purchased by j in total gross output of country j.

Two points about this formula are important to note. First, gross output price changes are a weighted average of value-added price changes in all countries ($\hat{p}^v$), where the weights reflect total cost shares.[9] Second, the mapping from value-added to gross output prices involves no elasticities, only production input shares.

F. Value-Added Real Effective Exchange Rates

Combining Equations (17) and (18), we have a complete description of demand for value added in terms of aggregate expenditure levels $\hat{F}$ and value-added prices $\hat{p}^v$. This linear system that

[9] The ij elements of $[I - \Omega']^{-1}$ describe the amount of gross output from country j used directly or indirectly in producing gross output in country i, and then the value-added to output ratios s_i^v rescale these to reflect how important value added from j is in producing gross output in j.

takes the stylized form:

$$\hat{V} = -\left[\sigma T_\sigma + \rho T_\rho + \gamma T_\gamma\right] \hat{p}^v + \hat{F}^w, \tag{19}$$

where the T-matrices and $\hat{F}^w$ are given by:

$$\begin{aligned}
T_\sigma &\equiv [I - S_X M_2]^{-1} S_F (M_1 - M_2 W_f)[I - \Omega']^{-1}[diag(s_i^v)],\\
T_\rho &\equiv [I - S_X M_2]^{-1} S_X (M_1 - M_2 W_x)[I - \Omega']^{-1}[diag(s_i^v)],\\
T_\gamma &= \left[[diag(s_i^x/s_i^v)] - [I - S_X M_2]^{-1} S_X M_2\right](I - W_x)[I - \Omega']^{-1}[diag(s_i^v)],\\
\hat{F}^w &\equiv [I - S_X M_2]^{-1} S_F M_2 \hat{F}.
\end{aligned}$$

To define the real effective exchange rate, we manipulate Equation (19), following standard practice [McGuirk (1987)]. First, we set changes in real final demand $\hat{F}$ to zero. This means that we focus on the influence of price changes on demand, holding levels of final demand constant. Second, we adopt a country-specific normalization so that weights on relative price changes sum to one. This normalization ensures that the real effective exchange rate depreciates by x% when all foreign prices increase by x% relative to the domestic price.

Focusing on country i, let T_x^{ij} be the ij element of matrix sub-scripted by x and define $T^{ii} \equiv \sigma T_\sigma^{ii} + \rho T_\rho^{ii} + \gamma T_\gamma^{ii}$. Then the change in demand for value added in country i is:

$$\begin{aligned}
\hat{V}_i &= -\sum_j \left[\sigma T_\sigma^{ij} + \rho T_\rho^{ij} + \gamma T_\gamma^{ij}\right] \hat{p}_j^v\\
&= -T^{ii} \sum_{j \neq i} \left[\frac{-(\sigma T_\sigma^{ij} + \rho T_\rho^{ij} + \gamma T_\gamma^{ij})}{T^{ii}}\right] \left(\hat{p}_i^v - \hat{p}_j^v\right),
\end{aligned} \tag{20}$$

where the second line uses $\sum_j \left[\sigma T_\sigma^{ij} + \rho T_\rho^{ij} + \gamma T_\gamma^{ij}\right] = 0$ (i.e., demand for value added is homogeneous of degree zero in value-added prices).

We then define the real effective exchange rate index as:

$$\widehat{REER}_i \equiv \sum_{j \neq i} \left[\frac{-(\sigma T_\sigma^{ij} + \rho T_\rho^{ij} + \gamma T_\gamma^{ij})}{T^{ii}}\right] \left(\hat{p}_i^v - \hat{p}_j^v\right). \tag{21}$$

Following convention, we refer to an increase in the REER index as an appreciation, and a decrease as a depreciation. The parameter T^{ii} translates changes in the REER index into changes in demand for value added: $\hat{V}_i = -T^{ii}\widehat{REER}_i$.

To sign the weights, we note that $\sigma T^{ii}_{\sigma} + \rho T^{ii}_{\rho} + \gamma T^{ii}_{\gamma}$ is always positive. Typically, though not always, $\sigma T^{ij}_{\sigma} + \rho T^{ij}_{\rho} + \gamma T^{ij}_{\gamma}$ for $j \neq i$ will be negative. This sign pattern is intuitive; own price increases lower demand, and foreign price increases raise demand for one's own value added. Together these signs would imply positive weights in the REER index. Note however, we said that $\sigma T^{ij}_{\sigma} + \rho T^{ij}_{\rho} + \gamma T^{ij}_{\gamma}$ for $j \neq i$ is *typically* negative. We discuss how under some elasticity parameter configurations $\sigma T^{ij}_{\sigma} + \rho T^{ij}_{\rho} + \gamma T^{ij}_{\gamma}$ can actually be positive, leading to the unconventional result that REER weights can be negative in our framework. We return to this important point in Section B.

This completes the formal definition of our proposed REER index. There are three key points to note, which serve as a launching point for our discussion of the index. The first is that the index treats value-added prices as primitives, and aggregates these into a composite multilateral index. The second is that the weights attached to individual bilateral prices depend on the interaction of both supply and demand side elasticities with the input-output structure. The third is that trade structure and elasticities, embodied in T^{ii}, also influence the mapping from the index into demand for value added. We now turn to explaining the economics underlying each of these observations.

III. The Mechanics of Demand for Value Added

In this section, we present economic interpretations of the value-added REER index, and the mapping from the index to demand for value added. We start in Section A with an instructive case with equal elasticities throughout the framework. With this restriction, demand for value added takes a familiar CES form, as if consumers have CES preferences defined directly over value added. We then describe how both the REER index and the mapping from the index to demand changes when elasticities of substitution are heterogeneous in Section B. To provide context, we discuss conventional product-based REER's in Section C.

A. Equal Elasticities

We open by analyzing demand for value added when substitution elasticities are identical throughout the model.

1. The Value-Added Armington-CES Model

Let $\varepsilon \equiv \gamma = \rho = \sigma$. Then, we can write demand for real value added from country i as:

$$\begin{aligned}
&\hat{V}_i = -\varepsilon \left(\hat{p}_i^v - \hat{P}_i^w\right) + \hat{F}_i^w \\
&\text{with} \quad \hat{P}_i^w = \sum_j \left(\frac{p_i^v V_{ij}}{p_i^v V_i}\right) \hat{P}_j \quad \text{where} \quad \hat{P}_j = \sum_k \left(\frac{p_k^v V_{kj}}{P_j F_j}\right) \hat{p}_k^v, \\
&\text{and} \quad \hat{F}_i^w = \sum_j \left(\frac{p_i^v V_{ij}}{p_i^v V_i}\right) \hat{F}_j.
\end{aligned} \tag{22}$$

Here V_{ij} denotes the amount of real value added produced by country i that is ultimately absorbed in country j, so $p_i^v V_{ij}$ is value-added exports from country i to country j [Johnson and Noguera (2012a)]. See Appendix A for derivation details.

Equation (22) tells us something familiar: each country faces a CES demand schedule for the value added it produces, as if each country sells value added to a single world market. The $\hat{P}_i^w$ and $\hat{F}_i^w$ are the aggregate price level and final demand level that each country faces in selling to the composite world market. Demand for value added from country i falls when the price of its own value added rises relative to P_i^w, with an elasticity of ε.

The perceived world price of value added itself is a weighted average of price changes for value added ($\hat{p}^v$) originating from all countries. The weighting scheme has two components. The first part is a value-added export weighted average of final goods price levels, and the second part links final goods price levels to value-added prices via value-added import weights. This formulation highlights that value-added trade patterns define which countries are important destination markets for a given source country, and which other countries provide competition in those destinations.[10]

This CES-demand interpretation suggests an alternative way to characterize demand for value added for this case. Rather than specifying the entire gross production and trade framework, we could instead assume that countries produce and trade value added directly.[11] Specifically, we could write preferences directly over value added from different countries, as in

[10]Note that the level of perceived demand ($\hat{F}^w$) is computed using value-added export shares. This weighting scheme is identical to the final demand weights in Bems, Johnson, and Yi (2010). In that paper, we assumed that technology and preferences were both Leontief ($\varepsilon = 0$). This is equivalent to assuming that price changes are zero (i.e., $\hat{p} = 0$).

[11]This bypasses the intermediate step via which value added is aggregated into commodities prior to being sold to consumers, and instead connects consumers to producers of value added directly.

$F_i = \left(\sum_j (\omega^v_{ji})^{1/\eta} V_{ji}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}$, where ω^v_{ji} is now a value-added preference weight. Together with market clearing conditions, these preferences generate a CES demand system that aggregates to yield Equation (22). Thus, one can re-interpret Equation (22) as if it were derived from an CES-Armington demand system for value added. This CES-Armington interpretation connects our gross framework to conventional value-added macro-models, which abstract from production and trade in intermediate inputs.

2. The VAREER

Setting $\hat{F}_j = 0$ for all j, we maniuplate (22) to write demand for value added as:

$$\begin{aligned} \hat{V}_i &= -\varepsilon T^{ii}_{VA} \widehat{REER}^{VA}_i, \\ \text{with} \quad \widehat{REER}^{VA}_i &\equiv \sum_{j \neq i} \left[\frac{1}{T^{ii}_{VA}} \sum_k \left(\frac{p^v_i V_{ik}}{p^v_i V_i} \right) \left(\frac{p^v_j V_{jk}}{P_k F_k} \right) \right] \left(\hat{p}^v_i - \hat{p}^v_j \right), \\ \text{and} \quad T^{ii}_{VA} &= 1 - \sum_k \left(\frac{p^v_i V_{ik}}{p^v_i V_i} \right) \left(\frac{p^v_i V_{ik}}{P_k F_k} \right). \end{aligned} \tag{23}$$

We refer to the $\widehat{REER}^{VA}_i$ index as the VAREER to emphasize that it is based on value-added data alone. The VAREER captures a normalized version of the relative price change $\hat{p}^v_i - \hat{P}^w_i$. The impact of changes in the VAREER on demand is governed by the elasticity of demand for value-added (ε) and the scaling parameter T^{ii}_{VA}. Roughly speaking, T^{ii}_{VA} captures the degree of openness measured in value added terms.[12]

B. Heterogeneous Elasticities

We now turn to evaluating the behavior of the REER index and demand for value added when elasticities are not equal.

[12] In practice, it is well approximated by $1 - \left(\frac{p^v_i V_{ii}}{p^v_i V_i}\right)\left(\frac{p^v_i V_{ii}}{P_i F_i}\right)$, since the $i \neq k$ terms in the summation tend to be small. As openness increases, both the share of value added sold by i to i and the share of final expenditure sourced by i from i fall, so T^{ii}_{VA} tends to increase. As a result, demand for value added becomes more sensitive to changes in the VAREER.

1. The IOREER

When elasticities are unequal, the value-added REER index weights differ in two ways from the previous VAREER case. First, we need to use the full global input-output framework to construct the index weights, not just value-added trade flows. Second, the index weights depend on the relative magnitudes of the elasticities in production versus demand. We refer to the general value-added REER index – defined in Equation (21) – as the IOREER (as in input-output REER) to emphasize these differences relative to the VAREER.

Elasticities serve as weights on different elements of the input-output structure, and therefore control the balance between different margins of substitution in the framework. As the elasticity between final goods rises, more weight is attached to substitution across final goods (T_{σ}^{ij}). In the opposite case, higher elasticities in production raise the weight attached to substitution between inputs from different sources (T_{ρ}^{ij}), and between inputs and value added in production (T_{γ}^{ij}). To provide intuition for how these elasticities govern the mapping from price changes influence competitiveness, we turn to a stylized three country example.

Example Consider a special case with three countries, depicted in Figure 1. Suppose that country 1 produces and exports all its output to 2, where it is used as an intermediate input to produce country 2's gross output. Country 2 consumes some of its own output, and exports the remainder to country 3. Exports from country 2 to country 3 are composed of final goods, which are consumed in country 3. Country 3 also consumes its own output, but does not export.

To illustrate the main issues, we focus on demand for value added from country 1. Market clearing for gross output from country 1 implies: $\hat{Q}_1 = \hat{X}_{12}$, with $\hat{X}_{12} = -\gamma(\hat{p}_1 - \hat{p}_2) + \hat{Q}_2$. In turn, $\hat{Q}_2 = s_{22}\hat{F}_{22} + s_{23}\hat{F}_{23}$, with $\hat{F}_{23} = -\sigma\left(\hat{p}_2 - \hat{P}_3\right) + \hat{F}_3$.[13] Putting these together yields: $\hat{Q}_1 = -\gamma(\hat{p}_1 - \hat{p}_2) - \sigma s_{23}\left(\hat{p}_2 - \hat{P}_3\right) + s_{22}\hat{F}_2 + s_{23}\hat{F}_3$. Further, our assumptions imply that $\hat{V}_1 = \hat{Q}_1$, $\hat{p}_1 = \hat{p}_1^v$, $\hat{p}_2 = (1-s^v)\hat{p}_1^v + s^v\hat{p}_2^v$, and $\hat{P}_3 = (1-w)\hat{p}_2 + w\hat{p}_3$ with $\hat{p}_3 = \hat{p}_3^v$.[14] Using these to substitute for prices, setting $\hat{F}_2 = \hat{F}_3 = 0$, and normalizing the weights on prices, we can write demand for value added in terms of the IOREER:

$$\hat{V}_1 = -T^{11}\underbrace{\left[\left(\frac{(\gamma - \sigma s_{23}w)\,s^v}{T^{11}}\right)(\hat{p}_1^v - \hat{p}_2^v) + \left(\frac{\sigma s_{23}w}{T^{11}}\right)(\hat{p}_1^v - \hat{p}_3^v)\right]}_{\widehat{REER}_1^{IO}}, \tag{24}$$

[13] Similar to the notation above, s_{ij} is the share of output shipped from i to j in country i's total output.

[14] For completeness, s^v is the share of own value added in gross output in country 2, and w is the expenditure share of country 3's own goods in its consumption.

with $T^{11} = \gamma s^v + \sigma s_{23} w(1 - s^v)$. Even in this stylized example, the IOREER weights are complicated functions of trade flows and elasticities. We highlight two features.

First, the IOREER depends (negatively) on prices in country 3. This might be surprising, since countries 1 and 3 do not compete head-to-head in *any* market. They do compete indirectly, however. Country 1 sells inputs to country 2, which are re-exported to country 3 embodied in country 2 goods. Therefore, a rise in prices in country 3 indirectly makes country 1 more competitive, and hence depreciates country 1's IOREER.[15]

Second, the sign of the IOREER weight attached to prices in country 2 is ambiguous. For example, if preferences are Leontief ($\sigma = 0$), then a fall in country 2's value-added price ($\hat{p}_2^v < 0$) is bad for country 1, appreciating its REER. This follows standard beggar-thy-neighbor intuition. Here, $\hat{p}_2^v < 0$ leads country 2 to switch expenditure away from country 1 inputs in production. Further, while country 2's final goods become more competitive in country 3, this leaves demand for country 2 goods unchanged since $\sigma = 0$. As a result, demand for inputs from country 1 unambiguously falls.

In contrast, if production is Leontief ($\gamma = 0$), then a fall in country 2's price causes the IOREER to depreciate, overturning beggar-thy-neighbor intuition. The reason is that as country 2's final goods become more competitive and it sells more to country 3, demand for country 1 inputs rises. As a result, country 1 inherits country 2's improvement in competitiveness. Further, country 1 experiences the full benefits of this because country 2 does not switch input expenditure in response to the price change.

In the general case, with both γ and σ greater than zero, both the beggar-thy-neighbor channel (input expenditure switching) and the input linkages channel are operative. The net response depends on how important input expenditure switching is relative to the competitiveness spillover via input linkages. This is fundamentally a quantitative question. When input elasticities are low, the input linkages channel is more important, and it is possible to obtain negative REER weights. In general cases, input linkages tend to lower REER weights for supply chain partners, as the positive competitiveness spillovers via input linkages counteract demand-side expenditure switching. In the empirical work below, we will focus on characterizing these weights and how they vary with relative elasticities given observed cross-border input and final goods linkages.

[15] To be clear, the VAREER index would pick this effect up as well, since value-added exports from 1 to 3 are positive, despite zero direct gross exports between them. To verify this, set $\gamma = \sigma$ here. In a conventional REER index, the absence of head-to-head competition – as in, country 1 and country 3 goods are never sold in the same market – would imply that country 3's prices would not matter to country 1.

2. Value-Added Elasticities

Relative elasticities are not only important for understanding how the IOREER behaves, but also in mapping the IOREER into demand for value added. As in Section F, demand for value added is given by: $\hat{V}_i = -T^{ii}\widehat{REER}_i^{IO}$, with $T^{ii} = \sigma T^{ii}_\sigma + \rho T^{ii}_\rho + \gamma T^{ii}_\gamma$. Elasticities matter in mapping the IOREER into demand because T^{ii} depends on elasticities.

To build intuition for how elasticities matter, it is helpful to compare how the VAREER versus the IOREER map into demand. When $\varepsilon \equiv \gamma = \rho = \sigma$, $\hat{V}_i = -\varepsilon T^{ii}_{VA}\widehat{REER}_i^{VA}$, with $T^{ii}_{VA} = T^{ii}_\sigma + T^{ii}_\rho + T^{ii}_\gamma$. So mapping the VAREER into demand requires knowing the value-added openness scaling term T^{ii}_{VA} and a value-added elasticity ε.

We can use this as a guide to interpreting IOREER changes. Specifically, let us re-write the mapping from the IOREER to demand as:

$$\begin{aligned}
&\hat{V}_i = -\tilde{\varepsilon}_i(\sigma,\rho,\gamma) T^{ii}_{VA}\widehat{REER}_i^{IO}, \\
&\text{with} \quad \tilde{\varepsilon}_i(\sigma,\rho,\gamma) \equiv \frac{T^{ii}}{T^{ii}_{VA}} = \left[\sigma\frac{T^{ii}_\sigma}{T^{ii}_{VA}} + \rho\frac{T^{ii}_\rho}{T^{ii}_{VA}} + \gamma\frac{T^{ii}_\gamma}{T^{ii}_{VA}}\right].
\end{aligned} \tag{25}$$

We will refer to $\tilde{\varepsilon}_i(\sigma,\rho,\gamma)$ as the effective "value-added elasticity." It summarizes the strength of aggregate value-added expenditure switching, telling us how sensitive demand for value added is to a change in the IOREER, controlling for value-added openness (encoded in T^{ii}_{VA}).

To interpret this elasticity further, suppose that there is a uniform 1% increase in home relative to foreign prices ($\hat{p}^v_i = 0.01$ and $\hat{p}^v_j = 0 \ \forall \ j \neq i$). Then, both the IOREER and VAREER would depreciate by 1%. In the heterogeneous elasticity framework, the change in demand for value added would be $\tilde{\varepsilon}_i(\sigma,\rho,\gamma)T^{ii}_{VA}$ percent. This is equal to the change in demand for value added for country i, following this multilateral appreciation, that one obtains in a pure value-added CES-Armington model with elasticity $\varepsilon = \tilde{\varepsilon}_i(\sigma,\rho,\gamma)$. In this sense, $\tilde{\varepsilon}_i(\sigma,\rho,\gamma)$ aggregates the heterogeneous fundamental elasticities into a composite value-added elasticity that is applicable to value-added models.

C. Conventional Real Effective Exchange Rates

To place the value-added REER in context, we pause to review how major statistical agencies currently compute REER indexes. Starting from the Armington (1969) demand system, with constant elasticity demand for products from each country, McGuirk (1987) derives the following Armington-REER formula:

$$\widehat{REER}_i^{Armington} = \sum_{j\neq i} \left[\frac{1}{\tilde{S}_i} \sum_k \left(\frac{Sales_{ik}}{p_i Q_i} \right) \left(\frac{Sales_{jk}}{\sum_l Sales_{lk}} \right) \right] (\hat{p}_i - \hat{p}_j)$$
$$\text{with} \quad \tilde{S}_i = 1 - \sum_k \left(\frac{Sales_{ik}}{p_i Q_i} \right) \left(\frac{Sales_{ik}}{\sum_l Sales_{lk}} \right), \tag{26}$$

where $Sales_{ij}$ is gross sales of products from country i to country j and $\hat{p}_i$ denotes changes in the price of products produced by country i (in a common currency).

This Armington-REER formula is the basis for all the major REER indexes, including those produced by the BIS, ECB, Federal Reserve, IMF, and OECD. It features "double export weights" for bilateral relative prices.[16] This scheme accounts for head-to-head competition between i and j in all destinations k – via $\left(\frac{Sales_{jk}}{\sum_l Sales_{lk}} \right)$ – and then weights each destination according to its importance in country i's total sales – via $\left(\frac{Sales_{ik}}{p_i Q_i} \right)$. All statistical agencies compute these weights using gross export and production data (i.e., $Sales_{ij} \equiv EX_{ij}$ for $i \neq j$ and $Sales_{ii} = p_i Q_i - \sum_{j\neq i} EX_{ij}$).

Our VAREER formula features a similar weighting scheme, with a major difference: the VAREER weights are double *value-added* export weights. Differences in weights between the VAREER and Armington-REER then reflect differences in value-added versus gross exports. More generally, the IOREER formula does not feature an explicit double weight scheme, and thus it represents a completely new approach to constructing REER weights.

A second difference between our indexes and conventional REERs is the measure of prices used. While our indexes use value-added price changes (measured by GDP deflators), the Armington-REER calls for using product prices, which correspond to gross output prices in our framework. Nonetheless, statistical agencies never use product prices in practice. The most common approach is to use consumer price indexes as a proxy for product prices, as in the IMF REER index or the Federal Reserve's Broad Dollar index.[17] We will therefore define the conventional

[16]Though these statistical agencies all use double export weights, they do not all implement the scheme in Equation (26) exactly. See Desruelle and Zanello (1997) and Bayoumi, Jayanthi, and Lee (2006) for the International Monetary Fund, Lorentan (2005) for the Federal Reserve, De Clercq et al. (2012) for the ECB, Durand, Simon, and Webb (1992) for the OECD, and Turner and Van't dack (1993) and Klau and Fung (2006) for the BIS.

[17]The OECD, ECB, and BIS also publish REER indexes based on consumer prices. Since the CPI includes both domestic and foreign goods, it is conceptually ill-suited to proxy for gross output prices. Some statistical agencies (e.g., the OECD and ECB) publish indexes based on unit labor costs or GDP deflators. While this is closer to our approach, these indexes aggregate price changes using gross trade weights, which mixes gross weights with value-added prices in a manner inconsistent with theory.

Armington-REER index to be the index in Equation (26) with $\left(\widehat{CPI}_i - \hat{E}_{i/j} - \widehat{CPI}_j\right)$ inserted in place of $\left(\hat{p}_i - \hat{p}_j\right)$, where $\widehat{CPI}_i$ and $\hat{E}_{i/j}$ are log changes in the CPI and nominal exchange rate. We will evaluate our VAREER and IOREER against this benchmark, which matches the widely-used IMF-REER index closely.

The final difference between our value-added indexes and conventional Armington-REER formulas concerns interpretation. Reflecting the product-based view of competition, Equation (26) should be interpreted as a measure of competitiveness for gross output, under the restriction that the demand for gross output takes the CES form. In contrast, our indexes focus on measuring price competitiveness for value added. We arrive at a different index for three basic reasons.[18] First, demand for gross output as a function of gross prices (Equation (15)) does not take the CES form in our framework, due both to input-output linkages across countries and heterogeneous elasticities in production and final demand. Second, we distinguish demand for gross output from demand for value added in our framework, due to the presence of imported intermediates. Third, we write demand for value added directly in terms of value-added prices, by linking gross prices to underlying value-added prices via the input-output framework.

IV. Data and Parameters

This section introduces the data and elasticities we use to parameterize the framework.

A. Global Input-Output and Price Data

We populate matrices $\{S_X, S_F, W_X, W_F, \Omega\}$ and production function shares $\{s_i^v, s_i^x\}$ using data on the value of gross output and value added by country, and the value of bilateral shipments of both final and intermediate goods.

We obtain these values from two data sets, depending on the time and country coverage needed in each application we examine.[19] The first is the World Input-Output Database (WIOD), which

[18] Our framework does not yield the REER formula in Equation (26) to describe demand for value added under *any* reasonable assumptions about input use. It is immediate that it does not emerge under the assumption that both domestic and foreign inputs are used in production. It also does not emerge under either the assumption that there are no inputs used in production, or the assumption that only domestic inputs are used in production. We discuss interpretation in these special cases in Appendix A.2.

[19] In one figure, where we drill in on Asian production chains, we switch to a third source – the Global Trade Analysis Project (GTAP) database, which covers 94 countries and 19 composite regions for one year (2004 in Ver-

covers 40 countries from 1995-2011 [Dietzenbacher et al. (2013)]. The second is the data set developed in Johnson and Noguera (2014), which covers 37 countries from 1970-2009. These data sets contain all the non-price information needed to parameterize the framework and build the REER weight matrices at an annual frequency.[20] We use both data sets in our time series analysis. While they provide similar answers during the period in which they overlap (1995-2009), the Johnson-Noguera data allows us to extend the analysis backward in time to 1970.

To construct historical REERs, we take price data – value-added (GDP) deflators, consumer price indexes, and nominal exchange rates – from the IMF's World Economic Outlook database. These data are annual (period averages) and available for all sample countries.[21]

B. Elasticity Parameters

Following Section III, we focus on two alternative elasticity parameterizations. First, we examine a homogeneous elasticity case, with $\sigma = \gamma = \rho = 1$. This sets the value-added elasticity to one, near typical values for Armington elasticities in international business cycle models. Second, we also examine a heterogeneous elasticity case, with limited input substitutability. We adopt an extreme parametrization and shut down substitution possibilities in production, as in Leontief production ($\rho = \gamma = 0$). To make this second case quantitatively comparable to the first, we choose σ so that the GDP-weighted mean value-added elasticity over the 1995-2009 period equal to one. This yields $\sigma = 3.00$ for the WIOD data, and $\sigma = 2.25$ for the Johnson-Noguera data.[22]

Our interest in this low production elasticity case is motivated by the commonly-held view that production chains are 'rigid' or 'inflexible', whereby producers find it difficult (if not impossible) to substitute across suppliers in the short run. This view has received attention in recent work on business cycle comovement [Burstein, Kurz, and Tesar (2008); Di Giovanni and Levchenko (2010)], and is supported by evidence on (the lack of) input substitution following the 2011 Japanese earthquake [Boehm, Flaaen, and Nayar (2015)]. Further, low elasticities between real value added

sion 7). This data set has wider country coverage in Asia than the other two data sets, and so is useful in this particular application.

[20] In each data set, we aggregate across sectors to define the values needed for our one sector framework.

[21] Each input-output database includes a rest-of-the-world region, in addition to individual countries. Because there is no price data for the rest-of-the-world region, we exclude this composite region from the REER computations. In doing so, we follow the standard practice in the construction of narrow indexes and re-normalize the weights for the remaining countries to add to 1.

[22] For the GTAP data, $\sigma = 2.90$. Differences in σ across data sets reflect differences in the share of input trade recorded in each data set, where WIOD reports a higher share of inputs in trade than do Johnson and Noguera. Differences in country samples also play a role.

and inputs are consistent with recent sector-level estimates [Atalay (2014)], as well as an older literature on the effects of raw materials and energy price shocks [Bruno (1984); Rotemberg and Woodford (1996)].

While we choose to set production-side elasticities to zero – a limiting case of the rigid supply chain view – the thrust of our analysis only requires inputs to be less substitutable than final goods. In particular, value-added REER weights depend on relative elasticities (ρ/σ) and (γ/σ).[23] Even if production elasticities are positive, the input linkage channels we emphasize will be important whenever production elasticities are low *relative to* the final goods elasticity. For given relative elasticities, the absolute level of the remaining third elasticity is then a free parameter, which can be set to yield whatever value-added elasticity one finds reasonable. This elasticity level matters for how determining sensitive demand for value added is to value-added REER changes, but not for the behavior of the REER itself.

There are two final points about elasticities worth emphasizing. First, from a quantitative perspective, σ and ρ are the key elasticities for pinning down REER weights and value-added elasticities. In contrast, the value of γ is less important, so our results are robust to alternative choices for this parameter. We discuss this issue carefully in Appendix B.1.

Second, for completeness, we discuss how our results differ if we impose Leontief demand, rather than Leontief production, in Appendix B.2. Based on the relative elasticities discussion above, it is not surprising that this case is essentially symmetric to the Leontief production case. Further, it is both less empirically plausible and less economically interesting than the low production elasticity case. A novel feature of our framework is that input linkages enable supply chain partners to gain competitiveness following depreciation by supply chain partners. This effect is shut down by the Leontief demand assumption, and so this eliminates the economically interesting role of input linkages in opposing standard beggar-thy-neighbor effects.

V. Building Blocks of Demand for Value Added

In this section, we examine the building blocks for measuring changes in demand for value added in cross-sectional data. The first building block is the REER index itself, and so we start by comparing the bilateral weights in the VAREER and IOREER indexes to conventional Armington-REER weights. The second is the value-added elasticity. We illustrate how aggregate value-

[23] Referring to Equation (21), the value-added REER weights can be written as: $\frac{-(T^{ij}_{\sigma}+(\rho/\sigma)T^{ij}_{\rho}+(\gamma/\sigma)T^{ij}_{\gamma})}{T^{ii}_{\sigma}+(\rho/\sigma)T^{ii}_{\rho}+(\gamma/\sigma)T^{ii}_{\gamma})}$.

added elasticities vary across countries when fundamental elasticities (σ, ρ, γ) are not equal. The third building block is value-added openness, which we show is substantially larger than gross openness measures.

A. Value-Added REER Weights

Each country's value-added REER index is a weighted average of bilateral relative price changes. To illustrate how supply chain linkages and relative elasticities influence these weights, we compare the weights attached to major trade partners (e.g., Germany, China, etc.) in VAREER, IOREER (with Leontief production), and Armington-REER indexes. Literally, the weight attached by country i to partner j tells us how much (in percent) its REER index appreciates when j's prices fall by 1% relative to i's own prices.

In Figure 2, we present the weights that various countries attach to Germany. The left panel includes the weights themselves, and the right panel reports differences between the value-added REER and Armington-REER weights. Consistent with standard intuition, countries that trade a lot with Germany attach large weights to Germany in their Armington-REER indexes.[24] For example, the core EU countries (Austria, Netherlands, Belgium, France, etc.) and the EU accession countries (the Czech Republic, Poland, etc.) put large weights on Germany in their Armington-REER indexes.

Relative to this benchmark, countries that are integrated into supply chains with Germany put less weight on Germany in both their VAREER and IOREER indexes. Looking first at the VAREER, the weight attached to Germany falls in all the EU accession countries, who import Germany inputs for assembly and export the resulting output. Going further, the IOREER accentuates this shift in REER weights away from German supply chain partners, leading REER weights attached to Germany to fall dramatically in the accession countries, and to rise substantially elsewhere. For example, moving from the Armington-REER to the IOREER roughly halves the weight that the Czech Republic attaches to Germany, and doubles the weight that Ireland attaches to Germany.

To summarize these observations, we plot these weight differences against measures of supply chain trade in Figure 3. The left panel plots the difference between the VAREER and the Armington-REER against the ratio of value-added to gross bilateral exports for each country

[24] This reflects the fact that – in the Armington view of the world – large bilateral gross trade flows signify intense head-to-head with German products, both in Germany and in their own market.

vis-à-vis Germany.[25] As is evident, VAREER weights are lower than Armington-REER weights mainly for countries where bilateral value-added exports are low relative to gross exports. For these countries, gross exports overstate the degree of bilateral head-to-head competition with Germany.

In the right panel of Figure 3, we demonstrate that bilateral trade composition – the mix of final versus intermediate goods – drives differences between IOREER and VAREER weights. We see that low production elasticities lower REER weights for countries that trade inputs with Germany, and raise weights for countries that trade final goods more intensively with Germany. This yields big changes in the ranking of which countries are hurt most by a real German devaluation. According to the Armington-REER index, the Czech Republic experiences a large decline in price competitiveness (ranking 2nd to Austria). However, in value-added terms, it is in fact relatively insulated (ranking 15th out of 20 countries in the figure) according to the IOREER. On the flip side, the relative importance of Germany for Irish price competitiveness is reversed.

The basic insights from the German example concerning supply chain linkages carry over to other cases. For illustration, we plot the REER index weights attached to China and South Korea in Figure 4. In both cases, the VAREER and IOREER weights fall the most for Asian countries that are linked to China or South Korea via supply chains in "Factory Asia." Further, these weight reassignments are larger than in the German example above. For example, a 20% fall in Chinese prices would induce a 4.6% (20×0.23) appreciation in Taiwan's Armington-REER, but translates into only a 0.08% (20×0.04) appreciation in the IOREER case. For Vietnam, a decline in Chinese prices actually *raises* Vietnamese competitiveness in the IOREER case, as captured by its negative IOREER weight. This illustrates that negative value-added REER weights – discussed in Section B – are not just a theoretical possibility, but do actually arise in empirical applications of our framework.

To summarize broader patterns beyond these examples, we report changes in weights by broad region in Table 1. For each country in a given source region, we compute weights attached to destination regions (Asia, EU, NAFTA, and Other) by summing across partners within those regions. Then, we compute the mean weight across countries within each source region. For both the VAREER and IOREER, the weight that a typical country attaches to regional trade partners in its value-added REER declines substantially relative to its Armington-REER, and correspondingly

[25] For country i, this ratio is defined as: $\frac{p_i^v V_{ij} + p_j^v V_{ji}}{EX_{ij} + EX_{ji}}$, with $j = $ Germany.

rises for extra-regional partners.[26] In the extreme, the total weight attached by a typical Asian country to its Asian partners is 15 percentage points lower in the IOREER index than in a conventional Armington-REER index.

In additional to regional boundaries, indicators of policy and non-policy barriers are also correlated with weight changes. First, regional trade agreements (RTAs) are associated with increased supply chain activity, lower value-added to export ratios, and hence declines in VAREER relative to Armington-REER weights. For example, the typical country (in 2005) attaches a VAREER weight that is about 4 percentage points lower for countries with which it had an RTA relative to countries with whom it has no RTA. Second, distance to trade partners is positively correlated with the difference between value-added REER weights and Armington-REER weights. This is mostly driven by large negative gaps between value-added and Armington REER weights among relatively close partners, with population-weighted distances of less than 5000km.

To sum up, global supply chains play two (related, but separate) roles in shaping REER weights. First, they distort bilateral value-added versus gross trade. This distortion gets picked up in comparisons between the VAREER and the conventional Armington-REER. Second, countries linked via supply chains trade inputs intensively. This influences weights heavily in the IOREER case, because trade composition interacts with elasticities. A subtle, but important, point to note is that differences between IOREER and Armington-REER weights can arise even if input trade does not distort the value added content of trade.[27] This implies that the IOREER case is relevant even when differences between value-added and gross trade are small. In turn, the IOREER weights will differ from conventional Armington-REER weights even in historical data, when value-added and gross trade were more similar than they are today.

B. Value-Added Elasticities and Openness

To link changes in the value-added REER to demand for value added, we need two additional building blocks. First, we need the value-added elasticity, which tells us how responsive demand

[26] For the VAREER, this reflects the observation that value-added to gross export ratios are lower for intra-regional trade than for trade across regions [Johnson and Noguera (2012b)]. For the IOREER, effects are driven by the fact that intra-regional trade is input intensive relative to extra-regional trade.

[27] To clarify, input trade is sometimes associated with double counting in trade, and hence differences between bilateral value-added and gross exports. For example, this is true when imports are used to produce exports, or when exports are used abroad to produce goods shipped to third countries. However, input trade does not necessarily generate gaps between value-added and gross exports. For example, if exported inputs are produced from entirely domestic factors, and then used abroad to produce goods that are consumed in the destination, this input trade represents trade in value added.

for value added is to relative price changes. Second, we need a measure of value-added openness, which determines how responsive total demand for value added is to value-added expenditure switching.

The value-added elasticity is given by: $\tilde{\varepsilon}_i(\sigma,\rho,\gamma) = \left[\sigma \frac{T^{ii}_{\sigma}}{T^{ii}_{VA}} + \rho \frac{T^{ii}_{\rho}}{T^{ii}_{VA}} + \gamma \frac{T^{ii}_{\gamma}}{T^{ii}_{VA}}\right]$, as in Equation (25). When $\sigma = \rho = \gamma$, then the value-added elasticity is equal across countries, invariant to trade and input-output linkages. In contrast, when elasticities are not equal, $\tilde{\varepsilon}_i(\sigma,\rho,\gamma)$ varies across countries, as the weights attached to different margins of substitution vary across countries.

In Figure 5, we examine value-added elasticities for the IOREER case with Leontief production. The left panel records $\tilde{\varepsilon}_i(3,0,0) - 1$, where $\tilde{\varepsilon}_i(3,0,0)$ is the value-added elasticity for the IOREER with Leontief production and 1 is the corresponding elasticity in VAREER case. There are significant downward adjustments in the value-added elasticity in the IOREER case for many countries. Glancing at country names, downward adjustments tend to occur in countries heavily engaged in global supply chains (e.g., Taiwan, Ireland, Indonesia, Hungary, etc.).[28] To confirm this, we plot $\tilde{\varepsilon}_i(3,0,0) - 1$ against the share of final goods in each country's trade in the right panel. As is evident, there is sizable variation in the final goods share of trade across countries, and a strong positive correlation between the final goods share of trade and $\tilde{\varepsilon}_i$. Countries that are more involved in global supply chains, and hence have larger shares of intermediate inputs in their trade, have lower effective value-added elasticities in the IOREER case. This will dampen the response of demand for value added to relative price changes in these countries.

Turning to value-added openness, we plot the values of T^{ii}_{VA} for various countries in Figure 6. Across countries, T^{ii}_{VA} varies a lot, from around 0.15 to 0.65 in the figure. As a result, a given change in the value-added REER has a much larger (up to 4 times larger) impact on demand for value-added for in the most "open" versus "closed" countries. In the figure, we also contrast T^{ii}_{VA} with the analog measure of gross openness (S_i) that appears in the Armington-REER framework [Equation (26)]. While T^{ii}_{VA} and S_i are highly correlated, there is an important difference between them: T^{ii}_{VA} is substantially larger – about 50% larger for the typical country – than S_i.[29] This implies that a uniform nominal devaluation, which would yield identical changes in value-added

[28]Note that there are upward adjustments in some countries, though these tend to be quantitatively smaller. These smaller upward adjustments counterbalance the downward adjustments in the aggregate, as the upward adjustments tend to occur in larger countries and we set the GDP-weighted average value-added elasticity to one.

[29]The reason that countries look more open in value-added than gross terms is that double-counting is more prevalent in domestic than cross-border transactions. As a result, total gross output is inflated by more relative to total value added (GDP) than gross exports are inflated relative to value-added exports. Put differently, the ratio of value-added exports to GDP tends to be larger than the ratio of gross exports to gross output. This is reflected in comparisons of S_i to T^{ii}_{VA}.

and conventional REERs, would yield larger changes in demand for value added than one would guess based on examining conventional gross measures of openness.

VI. Value-Added REERs and Expenditure Switching

Thus far, we have presented the framework in a cross-sectional context. We now turn to examining value-added REERs and competitiveness over time. We start by comparing our value-added REERs to conventional Armington-REERs in the time series. We then discuss how elasticities and openness influence how historical price changes influence demand for value added, on top of REER changes.

A. Value-Added REERs

Drawing on time-series input-output tables, GDP deflators, and exchange rates, we compute VA-REER and IOREER weights for each year, and bilateral value-added price changes between adjacent years. In year t, we then aggregate the price changes between $t-1$ and t using weights for year t, and chain these year-on-year REER changes together to generate a level series for each index.[30]

As a benchmark for comparison, we combine gross sales weights and CPI price changes to compute an Armington-REER index with time-varying weights (using the same chain index procedure). Because both weights and prices differ between the value-added and conventional REER indexes, we first discuss how these components compare over time, and then proceed to discuss the historical REER indexes.

REER Weights over Time In Section A, we compared VAREER and IOREER weights to Armingtion -REER weights for a single, representative year. While the cross-sectional section patterns we highlighted there carry over to other years, differences between value-added and conventional REER weights are larger now than in the past. To illustrate this, we compute the 'city-block distance' between VAREER and Armington-REER weights as:

$$d_{it}^{VAREER} = \sum_j \left| w_{ijt}^{VAREER} - w_{ijt}^{Armington} \right|, \tag{27}$$

[30] This follows the Federal Reserve Board's approach to time-varying weights, described in Lorentan (2005).

where w_{ijt} denotes the weight attached by country i to partner j in year t for each index. We also compute the distance between IOREER and VAREER weights:

$$d_{it}^{IOREER-VAREER} = \sum_j \left| w_{ijt}^{IOREER} - w_{ijt}^{VAREER} \right|. \tag{28}$$

Figure 7 plots these distance measures from 1970-2009 for Germany, Japan, and the United States, along with the cross-country median in each year. In the left panel, we see that the distance between VAREER and Armington-REER weights increased slowly during the 1970-1990 period and then rose more rapidly from 1990-2010.[31] In contrast to the left panel, there are no obvious trends in the distance between IOREER and VAREER weights in the right panel. That is, low elasticities in production generate important deviations between IOREER and VAREER weights throughout the sample period.

To understand why the trends differ between the left and right panel, it is helpful to know that the share of inputs in trade has not changed much over time [Chen, Kondratowicz, and Yi (2005)]. As discussed above, IOREER weights are dominated by the share of inputs in bilateral trade, and so inherit its stability.[32] In contrast, the extent of vertical specialization (i.e., the use of imports to produce exports) has risen over time. This rise in vertical specialization drives changes in value-added versus gross exports, and so leads to rising gaps between VAREER and Armington-REER weights over time.

Value-Added vs. Consumer Prices Turning from weights to prices, we now compare GDP deflators and consumer price indexes. In Figure 8, we plot the proportional difference between the GDP deflator and CPI for several representative countries.[33] The takeaway is that there are large and persistent differences in the alternative price measures, and that these differences will account for some of the gap between our value-added indexes and the CPI-based Armington-

[31] The turning point around 1990 is consistent with evidence that the value-added content of trade started falling rapidly around 1990, after not changing much in the earlier period [Johnson and Noguera (2014)].

[32] Changes the input share for particular countries, or changes in the bilateral pattern of input trade for a given country, can generate sizeable changes in IOREER versus VAREER weights. For example, the large movement in the distance between Japan's IOREER and VAREER weights in the right panel of Figure 7 is explained by changes in the share of inputs in Japanese trade.

[33] For each country, we normalize the relative price of value added to consumer prices to be one in 2000, so the axis should be read as the cumulative percentage change in value added relative to consumer prices from 2000 levels. Though country detail is not important for our general message, we note that in Japan and the United States the price of value added falls relative to consumer prices over the period, though relative prices level off for the United States after 2000. Spain and the United Kingdom see rising prices of value added relative to consumer prices. Finally, South Korea sees value added prices first rise then fall relative to consumer prices.

REER. To conserve space, we relegate more detailed discussion of these differences to Appendix B.3.

Historical REER Indexes Putting these pieces together, we plot the VAREER, IOREER, and Armington-REER indexes for several important, much-discussed exchange rates over the 1995-2011 period.[34] Figure 9 features selected Eurozone countries, and Figure 10 includes the United States and China.

Starting with Figure 9, there are large differences between value-added and conventional REERs in Eurozone countries. Starting with Germany, the VAREER and IOREER both depreciate more strongly than does the Armington-REER. In contrast, the VAREER and IOREER appreciate more strongly than the Armington-REER in the Greece, Ireland, Italy, Portugal, and Spain (GIIPS). As such, the value-added indexes indicate larger gains in German competitiveness, and correspondingly larger losses in competitiveness in the GIIPS group, than does the conventional index. For example, while Germany's value-added REER depreciated by 15 percentage points post-2000, the German Armington-REER was little changed. These value-added competitiveness changes are consistent with the conventional narrative underlying the build up of imbalances within the Eurozone, which set the stage for current policy conflicts.

Turning to Figure 10, we also see large differences between value-added and conventional REERs for China and the United States. While there is no obvious trend in China's Armington-REER, its VAREER and IOREER appreciate by over 20 percentage points during the 2000's. Again here, the value-added perspective paints a dramatically different picture of Chinese price competitiveness than the conventional approach, one in which China experienced a substantial real appreciation despite actively managing its nominal exchange rate. The United States is a mirror reflection of these relative price movements. Like China, the US value-added and conventional REERs diverge after 2000, but the United States sees a larger depreciation in the VAREER than is picked up by the conventional REER.

The divergence between value-added and conventional REERs in these important cases speaks to the "value added" by the value-added perspective on competitiveness; it contains useful information on price developments not captured by conventional REERs. The differences we highlight here are broadly representative of the larger sample. To illustrate this, consider the absolute gap

[34] REER series for all countries, based on both the WIOD and Johnson-Noguera input-output data are included in our online data set.

between year-on-year changes in the IOREER and Armington-REER:

$$\left|\left(\widehat{REER}_{it}^{IO} - \widehat{REER}_{it}^{Armington}\right) / \widehat{REER}_{it}^{Armington}\right|. \tag{29}$$

For the countries in Figures 9 and 10, the quartiles of this statistic are $\{0.13, 0.37, 1.06\}$. For all available countries, the quartiles are $\{0.16, 0.42, 1.25\}$. Thus, if anything, differences between the IOREER and Armington-REERs are slightly larger in the full data. Similar results hold for the VAREER versus Armington-REER.

Weights versus Prices Digging below the surface, we now examine the role that differences in weights versus differences in prices play in explaining deviations between value-added and conventional REERs. To do so, let us write changes in the value-added REERs as: $\widehat{REER}_{it}^{x} = \sum_{j\neq i} w_{ijt}^{x}\left(\hat{p}_{it}^{v} - \hat{E}_{i/j,t} - \hat{p}_{jt}^{v}\right)$, for $x = \{VAREER, IOREER\}$, where w_{ijt}^{x} denotes year-t weights and hats denote log changes from year $t-1$ to t.[35] Similarly, changes in the Armington-REER are: $\widehat{REER}_{it}^{Armington} = \sum_{j\neq i} w_{ijt}^{Armington}\left(\widehat{CPI}_{it} - \hat{E}_{i/j,t} - \widehat{CPI}_{jt}\right)$. Then we decompose the difference between the value-added and conventional indexes as:

$$\begin{aligned}\widehat{REER}_{it}^{x} - \widehat{REER}_{it}^{Armington} &= \underbrace{\sum_{j\neq i}\left(w_{ijt}^{x} - w_{ijt}^{Armington}\right)\left(\hat{p}_{it}^{v} - \hat{E}_{i/j,t} - \hat{p}_{jt}^{v}\right)}_{\text{weight differences}} \\ &+ \underbrace{\sum_{j\neq i} w_{ijt}^{Armington}\left[\left(\hat{p}_{it}^{v} - \widehat{CPI}_{it}\right) - \left(\hat{p}_{jt}^{v} - \widehat{CPI}_{jt}\right)\right]}_{\text{price differences}}.\end{aligned} \tag{30}$$

We compute this decomposition for each year from 1970 to 2009 for the VAREER and IOREER. In Table 2, we report the median share of the weight differences in explaining $\widehat{REER}_{it}^{x} - \widehat{REER}_{it}^{Armington}$. In the first column, we report medians at the one year horizon. In the second column, we aggregate the decomposition for overlapping 10 year horizons, cumulating the role of weight differences, and report corresponding medians across the thirty 10-year intervals in the sample.[36]

In Panel A, we see that weight differences play a small role in explaining differences between the VAREER and Armington-REER for the median country at both horizons. There are important

[35] In contrast to previous formulas, we make the currency conversion in bilateral value-added price comparisons explicit here, for comparability to the Armington-REER.

[36] To be clear, the cumulated 10-year change in the exchange rate is $\sum_{t}^{t+10}\left[\widehat{REER}_{it}^{x} - \widehat{REER}_{it}^{Armington}\right]$, and the cumulated role of weight differences is $\sum_{t}^{t+10}\sum_{j\neq i}\left(w_{ijt}^{x} - w_{ijt}^{Armington}\right)\left(\hat{p}_{it}^{v} - \hat{p}_{jt}^{v}\right)$. For each 10-year interval, we take the ratio between these two, and then take medians across all intervals in the sample.

exceptions, however. Weight differences play a significant role in explaining short run differences for several large countries, such as Germany, France, and the U.S. In general, however, deviations between the VAREER and Armington-REER are mostly driven by the shift from using consumer prices to value-added prices.[37]

In contrast, we see in Panel B that weight differences play a significant role in explaining differences between the IOREER and Armington-REER at both short and long horizons. At the one year horizon, weight differences account for 30% of the gap between IOREER and Armington-REER changes for the median country. The larger role for weight changes here reflects the fact that weight differences between the IOREER and the Armington-REER are themselves larger than for the VAREER versus the Armington-REER. Over the longer 10-year horizon, weight differences account for about 15% of the median gap.

While weight differences are important, price differences account for more than half of deviations between the value-added and conventional REERs. To explain why shifting weights do not play a larger role, we highlight three issues.

First, the reassignment of weights is a zero sum exercise (i.e., $\sum_{j\neq i}\left(w^x_{ij} - w^{Armington}_{ij}\right) = 0$). As a result, uniform devaluations induce identical changes in the value-added and Armington REERs. While exactly uniform depreciations are rare, it is common to see a country depreciate against many partners simultaneously, and this dampens differences between the VAREER, IOREER, and Armington-REER.

Second, even if bilateral price changes are heterogeneous across partners, there must be systematic variation between weight reassignments and changes in relative prices for the reassignment to matter. That is, $\left(w^x_{ij} - w^{Armington}_{ij}\right)$ must be correlated (either positively or negatively) with $\left(\hat{p}^v_i - \hat{E}_{i/j} - \hat{p}^v_j\right)$. In the historical data, this correlation turns out to be small for many countries.

Third, focusing on long horizons, trend differences in value-added versus consumer prices tend to assert themselves more strongly over the longer run. This explains the lower long run relative to short run role for weights in explaining the IOREER versus Armington-REER differential.

[37] One reason for this is that for most of the sample period, VAREER weights are similar to Armington-REER weights. As we pointed out in Figure 7, large differences between VAREER and conventional weights only emerge late in the sample period. This suggests that VAREER weight differences might matter more prospectively than they have historically.

B. Value-Added Expenditure Switching

While the real effective exchange rate measures multilateral price competitiveness, what matters in the end is how price competitiveness influences demand for value added. This depends on both how strongly demand responds to prices – the elasticity of demand for value added – and how open the economy is. In this section, we therefore discuss how changes in value-added REERs are linked to demand for value added.

For the IOREER case, demand for value added is given by $\hat{V}_i^{IO} = -\tilde{\varepsilon}_i(\sigma, \rho, \gamma) T_{VA}^{ii} \widehat{REER}_i^{IO}$. In the VAREER case, we have $\hat{V}_i^{VA} = -T_{VA}^{ii} \widehat{REER}_i^{VA}$, where we have imposed $\varepsilon = 1$ as above. Comparing these cases, we see that low production elasticities will influence demand for value added both because $\widehat{REER}_i^{IO}$ deviates from $\widehat{REER}_i^{VA}$, but also because $\tilde{\varepsilon}_i(\sigma, \rho, \gamma) \neq 1$. To illustrate the role of each factor separately, we construct a hybrid measure of demand for value added by combining the change in the IOREER with a value-added elasticity equal to one, as in the VAREER case: $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1) = -T_{VA}^{ii} \widehat{REER}_i^{IO}$. Comparing $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1)$ to $\hat{V}_i^{VA}$ demonstrates the role of shifting the REER measure, while comparing $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1)$ to $\hat{V}_i^{IO}$ illustrates the role of the value-added elasticity.

In Figure 11 we present $\hat{V}_i^{VA}$ and $\hat{V}_i^{IO}$ for actual price changes in 2005 in the left panel, with countries ordered by the size of deviations between the two measures. The first thing to note is that changes in demand for value added, as implied by our parameterized framework, can be sizable. For example, in case of Korea, demand for value added in 2005 decreased by 3.0-3.5 percentage points, depending on the demand measure used. At the same time, demand for value added in Germany increased by 1.6 percentage points.[38] Next, the gap between the two demand measures – $\widehat{V}_{it}^{IO}$ and $\widehat{V}_{it}^{VA}$ – can be sizable as well, on the order of -0.9 (Turkey) or 0.7 (Brazil) percentage points in the largest cases. Further, in Hungary and Taiwan $\hat{V}_i^{IO}$ and $\hat{V}_i^{VA}$ actually move in opposite directions.

In the right panel of Figure 11, we plot deviations $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1) - \hat{V}_i^{VA}$ and $\hat{V}_i^{IO} - \hat{V}_i^{IO}(\tilde{\varepsilon} = 1)$ to decompose the differences between $\hat{V}_i^{VA}$ and $\hat{V}_i^{IO}$. Differences in IOREER versus VAREER changes are captured by $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1) - \hat{V}_i^{VA}$, and these play an important role in explaining deviations between $\hat{V}_i^{VA}$ and $\hat{V}_i^{IO}$. For example, in case of China, both the IOREER and VAREER appreciated in 2005, but the IOREER appreciated by more, implying $\widehat{V}_{it}^{IO} - \widehat{V}_{it}^{VA} < 0$. The right panel of Figure 11 quantifies the negative demand impact of the larger IOREER appreciation for China at -0.35 percentage points. In addition to REER deviations, differences in value-added

[38] If GDP is entirely demand determined, then this would correspond to the change in equilibrium GDP. When supply matter, then actual GDP changes differ from the change in demand.

elasticities, captured by $\hat{V}_i^{IO} - \hat{V}_i^{IO}(\tilde{\varepsilon} = 1)$, also play an important role in some countries. For example, a value-added elasticity greater than one amplifies the decline in demand for value added in Turkey, while an elasticity of less than one attenuates the decline in demand for value added in Brazil.

In Appendix B.4 we pursue a more systematic examination of these elasticity effects and find that elasticity deviations across countries account for 1/2 of deviations between $\hat{V}_i^{IO}$ and $\hat{V}_i^{VA}$ for the median sample country over 1970-2009. These results highlight that measuring price competitiveness is necessary, but not sufficient, to evaluate how demand for value-added is changing. We also need information on the elasticity of demand for value added.

In making these comparisons, we have thus far ignored value-added openness. This is because T_{VA}^{ii} is embedded in both $\hat{V}_i^{VA}$ and $\hat{V}_i^{IO}$, so it plays no role in comparisons across elasticity parameterizations for a single country at a given point in time. However, changes in value-added openness do play a large role in determining how the value-added REER indexes map into demand at different points in time. In particular, value-added openness is rising over time in our data. To illustrate this, we plot the median value of T_{VA}^{ii} over time in Figure 12. We see it nearly doubles from 1970 to the present, rising from 0.26 to 0.42. What this means is that a given change in the VAREER or IOREER indexes is twice as influential today in terms of its ultimate effect on demand for value-added as it was in the past. This point is typically forgotten by users of REER indexes, but worth emphasizing given that the purpose of REER indexes is to measure changes in demand.

VII. Conclusion

This paper updates the conceptual foundations for assessing the impact of price changes on demand for domestic value added to allow for global supply chain linkages across countries. Input linkages open new channels that allow countries to benefit from improvements in the competitiveness of supply chain partners, which counterbalance standard beggar-thy-neighbor channels. Further, by distinguishing gross and value-added concepts, our framework emphasizes that what matters in the end is how changes in international relative prices induce expenditure switching over value added from different source countries.

Reflecting these insights, the framework yields new real effective exchange rate formulas, in which bilateral aggregation weights depend on both the global input-output structure and relative elasticities in production versus demand. The framework also delivers new results about how

REERs are linked to changes in demand for value added. Specifically, one needs to measure both value-added elasticities and openness to map REER changes into demand. Given observed input linkages, these conceptual insights are important quantitatively.

To conclude, we point out several avenues for future work. First, we have studied input linkages in partial equilibrium. More work is needed to incorporate input linkages into general equilibrium models (e.g., new open economy macro-models) in order to study input linkages as conduits for identified shocks. Second, we have relied heavily on Armington-CES foundations, following the bulk of the international macroeconomic literature. New concerns would arise in models that relax the Armington-CES assumptions to allow markups to be time varying and/or depart from the roundabout production structure. Third, from a quantitative perspective, there is considerable uncertainty regarding the true value of substitution elasticities, particularly in production. We have demonstrated that these elasticities matter for understanding price spillovers. Therefore, better elasticity estimates could contribute to improving macroeconomic policy.

References

Ambler, Steve, Emanuela Cardia, and Christian Zimmerman, 2002, "International transmission of the business cycle in a multi-sector model," *European Economic Review*, vol. 46, pp. 273–300.

Antràs, Pol, 2014, "Global Production: A Contracting Perspective," Unpublished Manuscript, Harvard University.

Armington, Paul, 1969, "A Theory of Demand for Products Distinguished by Place of Production," *IMF Staff Papers*, vol. 16, no. 1, pp. 159–178.

Artus, Jaques R., and Rudolf R. Rhomberg, 1973, "A Multilateral Exchange Rate Model," *IMF Staff Papers*, vol. 20, no. 3, pp. 591–611.

Atalay, Enghin, 2014, "How Important are Sectoral Shocks?" Unpublished Manuscript, University of Wisconsin.

Backus, David, Patrick Kehoe, and Finn Kydland, 1994, "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?" *The American Economic Review*, vol. 84, no. 1, pp. 84–103.

Bayoumi, Tamim, Sarma Jayanthi, and Jaewoo Lee, 2006, "New Rates from New Weights," *IMF Staff Papers*, vol. 53, no. 2, pp. 272–305.

Bems, Rudolfs, 2014, "Intermediate Inputs, External Rebalancing, and Relative Price Adjustment," *Journal of International Economics*, vol. 94, no. 2, pp. 248–262.

Bems, Rudolfs, and Robert C. Johnson, 2012, "Value-Added Exchange Rates," NBER Working Paper No. 18498.

Bems, Rudolfs, Robert C. Johnson, and Kei-Mu Yi, 2010, "Demand Spillovers and the Collapse of Trade in the Global Recession," *IMF Economic Review*, vol. 58, no. 2, pp. 295–326.

Black, Stanley W., 1976, "Multilateral and Bilateral Measures of Effective Exchange Rates in a World of Traded Goods," *Journal of Political Economy*, vol. 84, no. 3, pp. 615–622.

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai Nayar, 2015, "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the Tōhoku Earthquake," Unpublished Manuscript, University of Michigan.

Bruno, Michael, 1984, "Raw Materials, Profits, and the Productivity Slowdown," *The Quarterly Journal of Economics*, vol. 99, no. 1, pp. 1–30.

Burstein, Ariel, Christopher Kurz, and Linda Tesar, 2008, "Trade, production sharing, and the international transmission of business cycles," *Journal of Monetary Economics*, vol. 55, pp. 775–795.

Bussière, Matthieu, Giovanni Callegari, Fabio Ghironi, Guila Sestieri, and Norihiko Yamano, 2013, "Estimating Trade Elasticities: Demand Composition and the Trade Collapse of 2008-2009," *American Economic Journal: Macroeconomics*, vol. 5, no. 3, pp. 118–151.

Chen, Hogan, Matthew Kondratowicz, and Kei-Mu Yi, 2005, "Vertical Specialization and Three Facts about U.S. International Trade," *North American Journal of Economics and Finance*, vol. 16, pp. 35–39.

Chinn, Menzie D., 2006, "A Primer on Real Effective Exchange Rates: Determinants, Overvaluation, Trade Flows and Competitive Devaluation," *Open Economies Review*, vol. 115-143, pp. 115–143.

Corden, W. Max, 1960, "The Geometric Representation of Policies to Attain Internal and External Balance," *The Review of Economic Studies*, vol. 28, no. 1, pp. 1–22.

Corsetti, Giancarlo, and Paolo Pesenti, 2001, "Welfare and Macroeconomic Interdependence," *Quarterly Journal of Economics*, vol. 116, no. 2, pp. 421–445.

Corsetti, Giancarlo, Paolo Pesenti, Nouriel Roubini, and Cèdric Tille, 2000, "Competitive Devaluations: Toward a Welfare-Based Approach," *Journal of International Economics*, vol. 51, pp. 217–241.

De Clercq, Maarten, Michael Fidora, Bernadette Lauro, Christina Pinheiro, and Martin Schmitz, 2012, "Revisting the Effective Exchange Rates of the Euro," ECB Occasional Paper Series No. 134.

Desruelle, Dominique, and Alessandro Zanello, 1997, "A Primer on the IMF's Information Notice System," IMF Working Paper 97/71.

Di Giovanni, Julian, and Andrei Levchenko, 2010, "Putting the Parts Together: Trade, Vertical Linkages and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, vol. 2, no. 2, pp. 95–124.

Dietzenbacher, Erik, Bart Los, Robert Stehrer, Marcel Timmer, and Gaaitzen de Vries, 2013, "The Construction of World Input-Output Tables in the WIOD Project," *Economic Systems Research*, vol. 25, no. 1, pp. 71–98.

Durand, Martine, Jacques Simon, and Colin Webb, 1992, "OECD's Indicators of International Trade and Competitiveness," OECD Economics Department Working Papers No. 120.

Eaton, Jonathan, Samuel Kortum, Brent Neiman, and John Romalis, 2011, "Trade and the Global Recession," NBER Working Paper 16666.

Feenstra, Robert C., 1998, "Integration of Trade and Disintegration of Production in the Global Economy," *Journal of Economic Perspectives*, vol. 12, no. 4, pp. 31–50.

Huang, Kevin X. D., and Zheng Liu, 2007, "Business Cycles with Staggered Prices and International Trade in Intermediate Inputs," *Journal of Monetary Economics*, vol. 54, pp. 1271–1289.

Hummels, David, Jun Ishii, and Kei-Mu Yi, 2001, "The Nature and Growth of Vertical Specialization in World Trade," *Journal of International Economics*, vol. 54, pp. 75–96.

Johnson, Harry, 1958, *International Trade and Economic Growth* (London: Allen & Unwin).

Johnson, Robert C., 2014, "Trade in Intermediate Inputs and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, vol. 6, no. 4, pp. 39–83.

Johnson, Robert C., and Guillermo Noguera, 2012a, "Accounting for Intermediates: Production Sharing and Trade in Value Added," *Journal of International Economics*, vol. 86, no. 2, pp. 224–236.

———, 2012b, "Proximity and Production Fragmentation," *American Economic Review: Papers* *and Proceedings*, vol. 102, no. 3, pp. 407–411.

———, 2014, "A Portrait of Trade in Value Added over Four Decades," Unpublished Manuscript, Dartmouth College.

Klau, Marc, and San Sau Fung, 2006, "The New BIS Effective Exchange Rate Indices," *BIS Quarterly Review*, vol. March, pp. 51–65.

Lane, Philip R., and Jay C. Shambaugh, 2010, "Financial Exchange Rates and International Currency Exposures," *The American Economic Review*, vol. 100, no. 1, pp. 518–40.

Lorentan, Mico, 2005, "Indexes of the Foreign Exchange Value of the Dollar," *Federal Reserve Bulletin*, vol. 91, pp. 1–8.

McGuirk, Anne, 1987, "Measuring Price Competitiveness for Industrial Country Trade in Manufactures," IMF Working Paper No. 87/34.

Neary, Peter, 2006, "Measuring Competitveness," *The Economic and Social Review*, vol. 37, no. 2, pp. 197–213.

Obstfeld, Maurice, 2001, "International Macroeconomics: Beyond the Mundell-Fleming Model," *IMF Staff Papers*, vol. 47, no. Special Issue, pp. 1–39.

Patel, Nikhil, Zhi Wang, and Shang-Jin Wei, 2014, "Global Value Chains and Effective Exchange Rates at the Country-Sector Level," NBER Working Paper No. 20236.

Rotemberg, Julio J., and Michael Woodford, 1996, "Imperfect Competition and the Effects of Energy Price Increases on Economic Activity," *Journal of Money, Credit and Banking*, vol. 28, no. 4, pp. 549–577.

Thomas, Charles, Jaime Marquez, and Sean Fahle, 2008, "Measuring U.S. International Relative Prices: A WARP View of the World," Board of Governors of the Federal Reserve System, International Finance Discussion Paper Series, Number 917.

Tille, Cèdric, 2001, "The Role of Consumption Substitutability in the International Transmission of Monetary Shocks," *Journal of International Economics*, vol. 53, pp. 421–444.

Turner, Philip, and Jozef Van't dack, 1993, "Measuring International Price and Cost Competitiveness," BIS Economic Papers No. 39.

Yi, Kei-Mu, 2003, "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, vol. 111, pp. 52–102.

Table 1. Differences in REER Weights in 2005, by Region

Panel A: VAREER Weight minus Armington-REER Weight				
	Partner Region			
Source Region	Asia	EU	NAFTA	Other
Asia	-5.9	3.1	2.7	0.1
EU	1.5	-4.7	2.6	0.6
NAFTA	0.6	2.3	-3.6	0.7

Panel B: IOREER Weight minus Armington-REER Weight				
	Partner Region			
Source Region	Asia	EU	NAFTA	Other
Asia	-15.7	10.3	8.5	-3.1
EU	3.8	-5.7	2.2	-0.4
NAFTA	3.4	2.0	-5.3	-0.1

Note: Each entry records the total difference in trade weights (e.g., in Panel A VAREER weights - Armington-REER weights) for partners in each destination region, average across source countries within each region. Changes in weights are expressed in percentage points. Columns might not sum to zero due to rounding. Data from Johnson and Noguera (2014).

Table 2. Contribution of Weights to Differences Between Value-Added and Armington REERs.

	VAREER minus Armington-REER		IOREER minus Armington-REER	
Horizon	1-year	10-year	1-year	10-year
AUS	0.00	0.04	0.13	0.17
AUT	0.21	0.19	0.82	0.39
BEL	0.24	0.08	0.66	0.25
BRA	-0.01	-0.02	0.01	-0.04
CAN	-0.06	0.07	0.05	0.18
CHE	0.09	-0.03	0.27	0.05
CHN	-0.01	0.00	0.01	0.05
DEU	0.33	0.00	0.47	-0.02
ESP	0.03	0.13	0.28	0.24
FRA	0.27	0.05	0.60	0.30
GBR	0.08	0.06	0.20	0.20
IND	0.00	0.02	0.22	0.21
ITA	-0.01	0.00	0.31	0.03
JPN	0.03	0.09	0.45	0.19
KOR	0.01	0.01	0.29	0.11
MEX	-0.03	0.01	-0.02	-0.10
NLD	0.04	-0.01	0.47	0.13
SWE	0.10	0.13	0.55	0.44
TUR	-0.01	-0.03	0.01	-0.02
USA	0.08	0.04	0.37	0.15
Median	0.03	0.03	0.28	0.16

Note: Decomposition based on Equation (30). Each column contains the median contribution of REER weight differences, as a share of $\widehat{REER}_{it}^{x} - \widehat{REER}_{it}^{Armington}$ for $x \in \{VA, IO\}$ over 1970-2009. Values at the 10-year horizon are computed based on 30 overlapping 10-year intervals. Data from Johnson and Noguera (2014).

Figure 1. Three Country Example

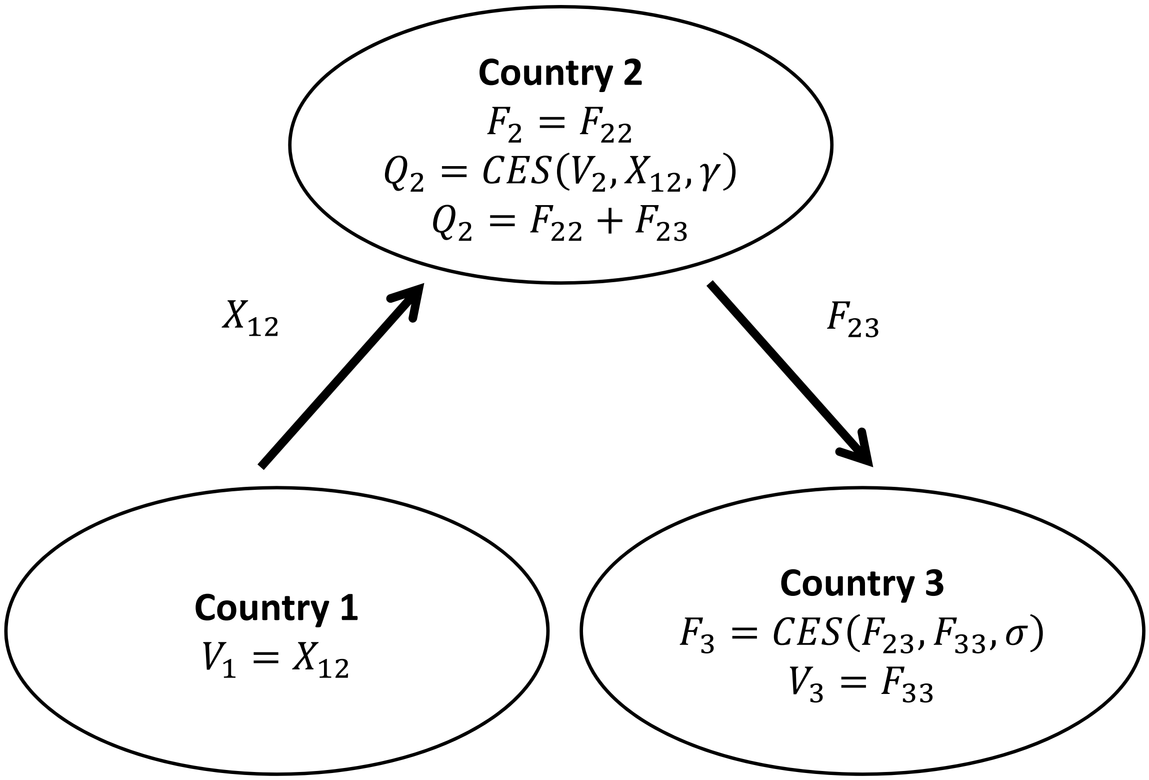
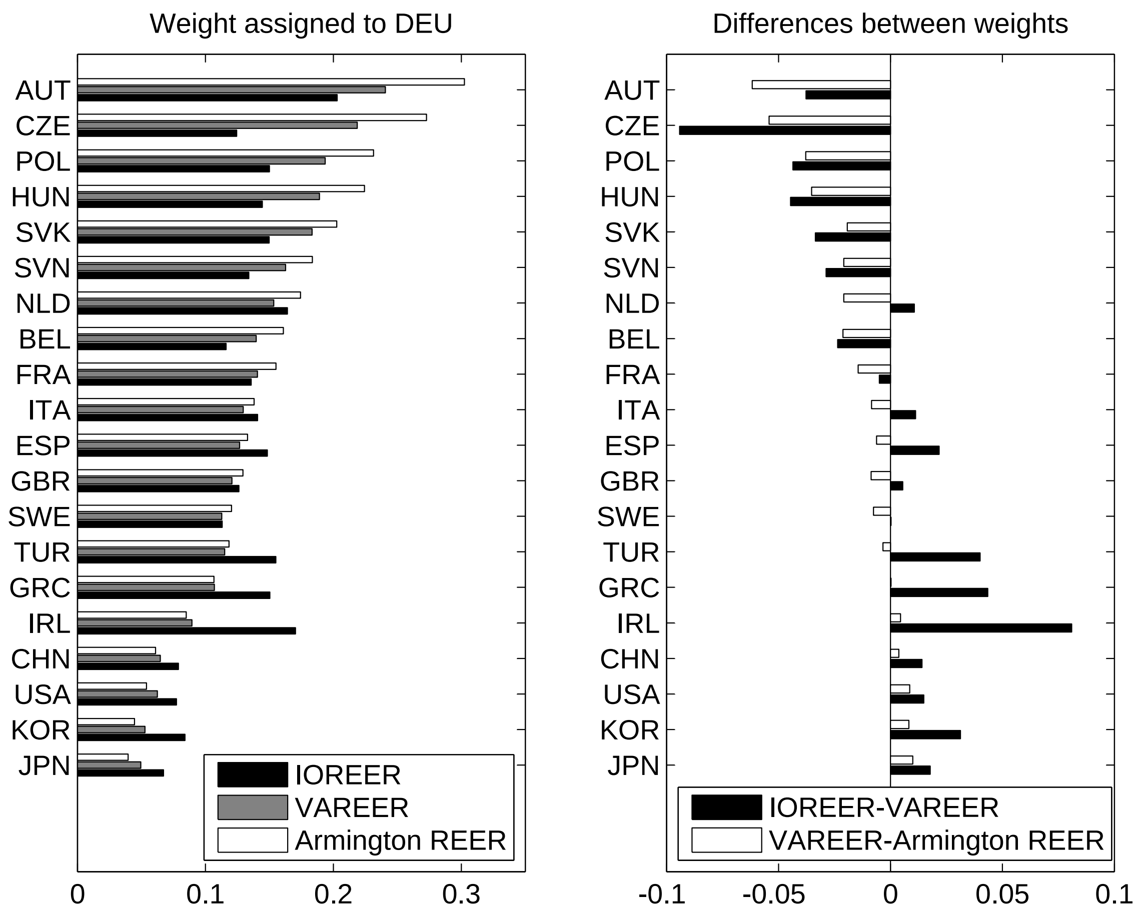
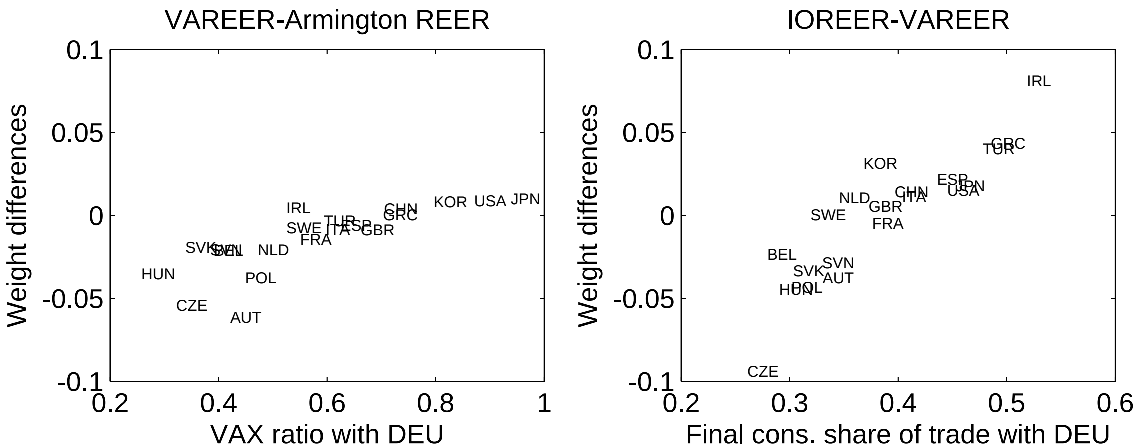


Figure 2. REER Weights Assigned to Germany, 2007



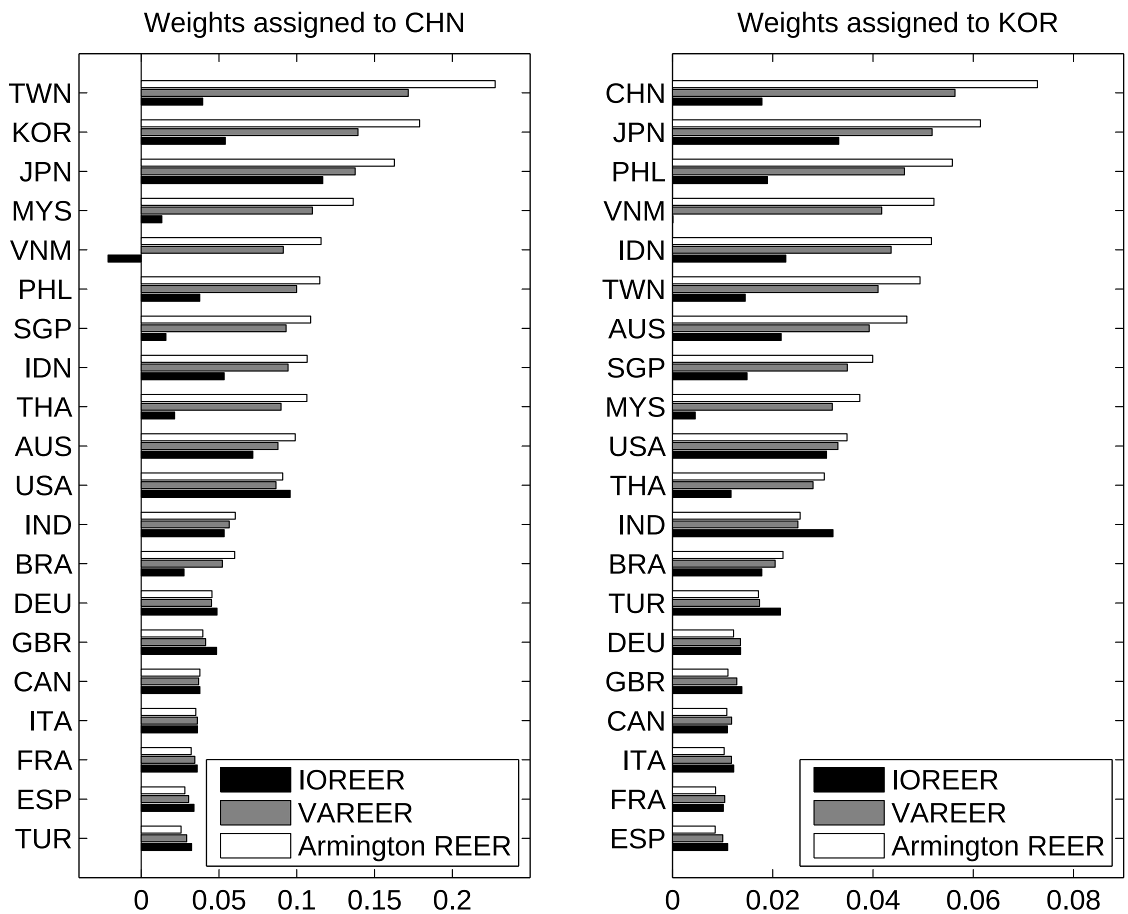
Note: VAREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{1,1,1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{3,0,0\}$. Data from WIOD.

Figure 3. Differences in REER Weights Assigned to Germany versus Bilateral Trade Composition with Germany, 2007



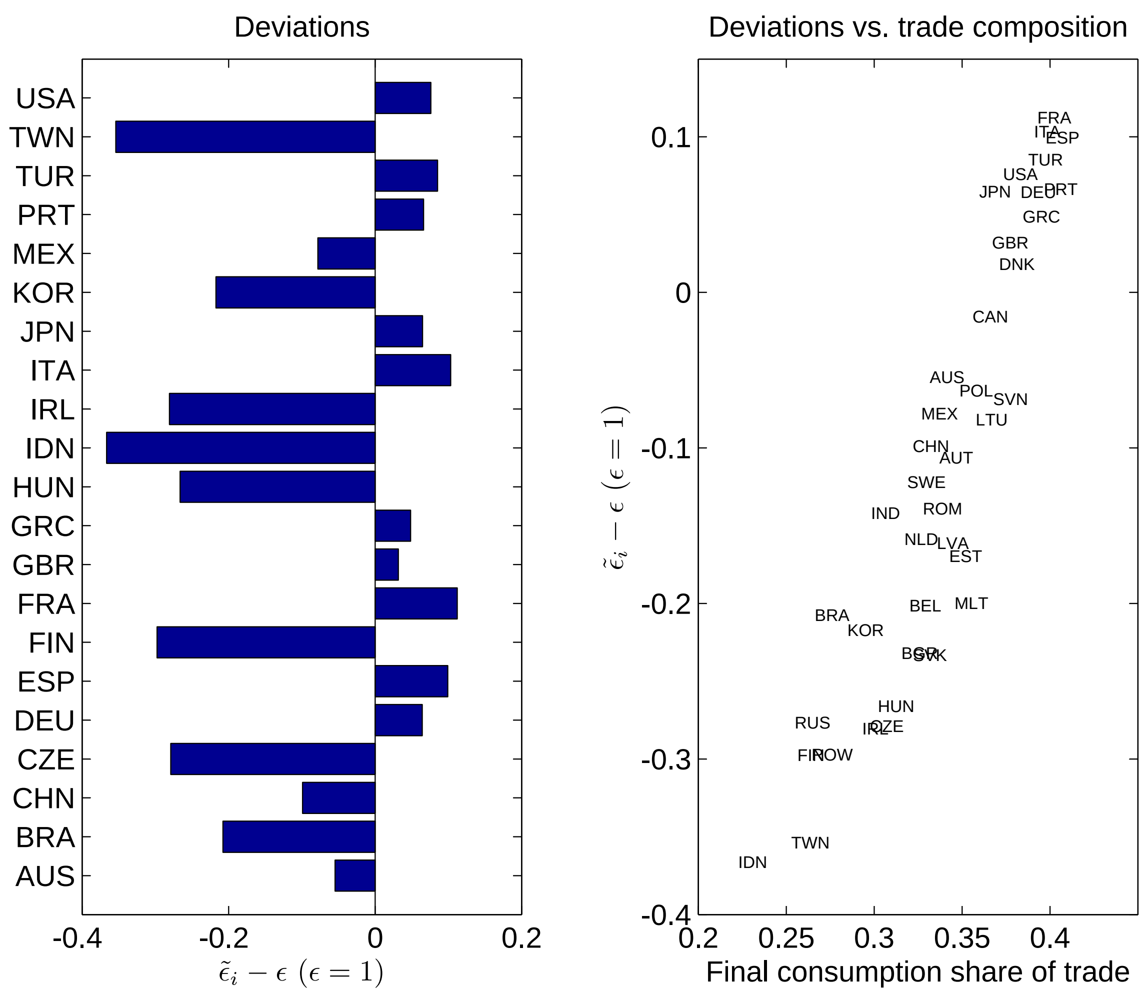
Note: VAREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{1, 1, 1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{3, 0, 0\}$. Bilateral final consumption share of trade with Germany includes both exports and imports. VAX ratio defined as bilateral trade in value added, relative to bilateral gross trade flows. Data from WIOD.

Figure 4. REER Weights Assigned to China and South Korea, 2004



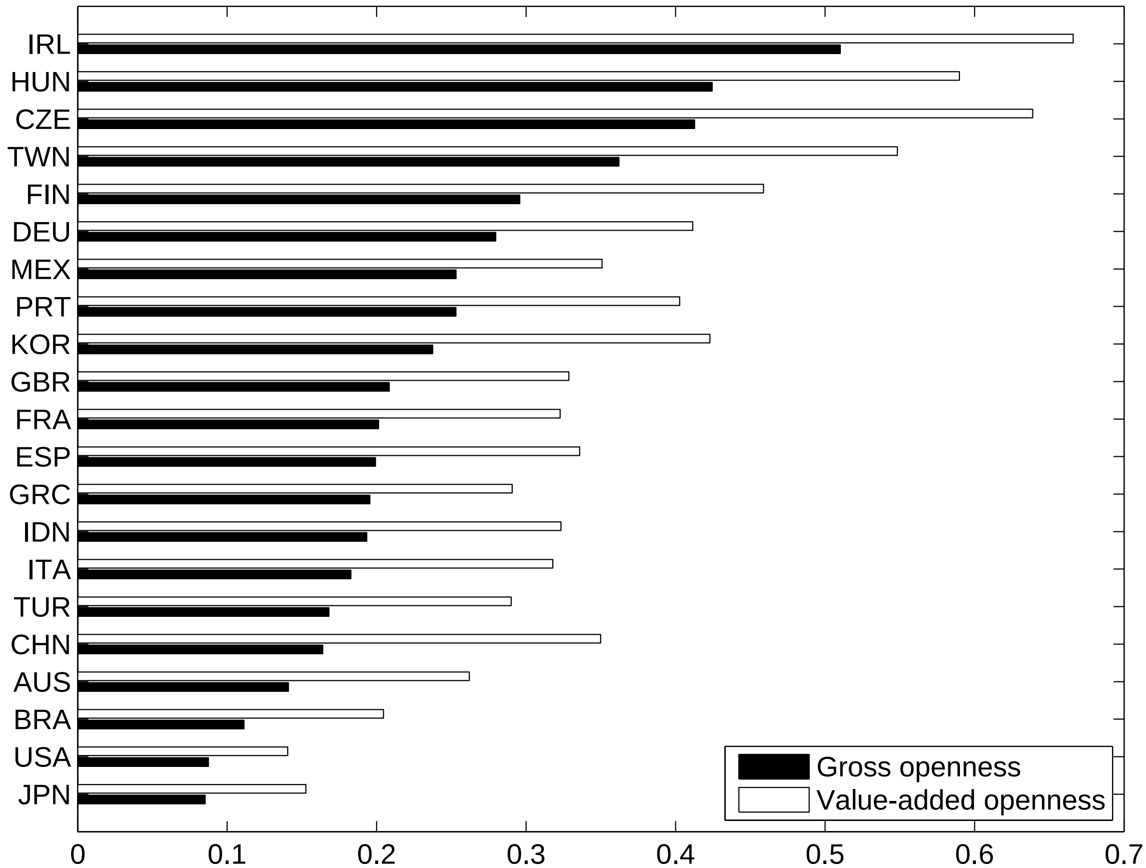
Note: VAREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{1, 1, 1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{2.9, 0, 0\}$. Data from GTAP.

Figure 5. Cross-Country Deviations in Effective Value-Added Elasticities with Inflexible Global Supply Chains, 2004



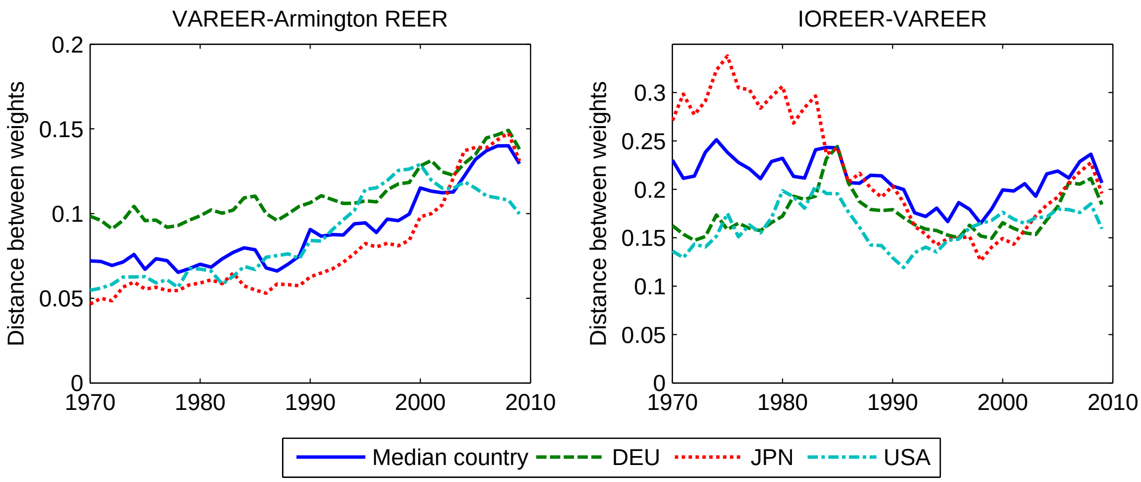
Note: Effective elasticities based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{3, 0, 0\}$. Final consumption share of trade includes both exports and imports. Data from WIOD.

Figure 6. Value-Added and Gross Measures of Openness, 2004



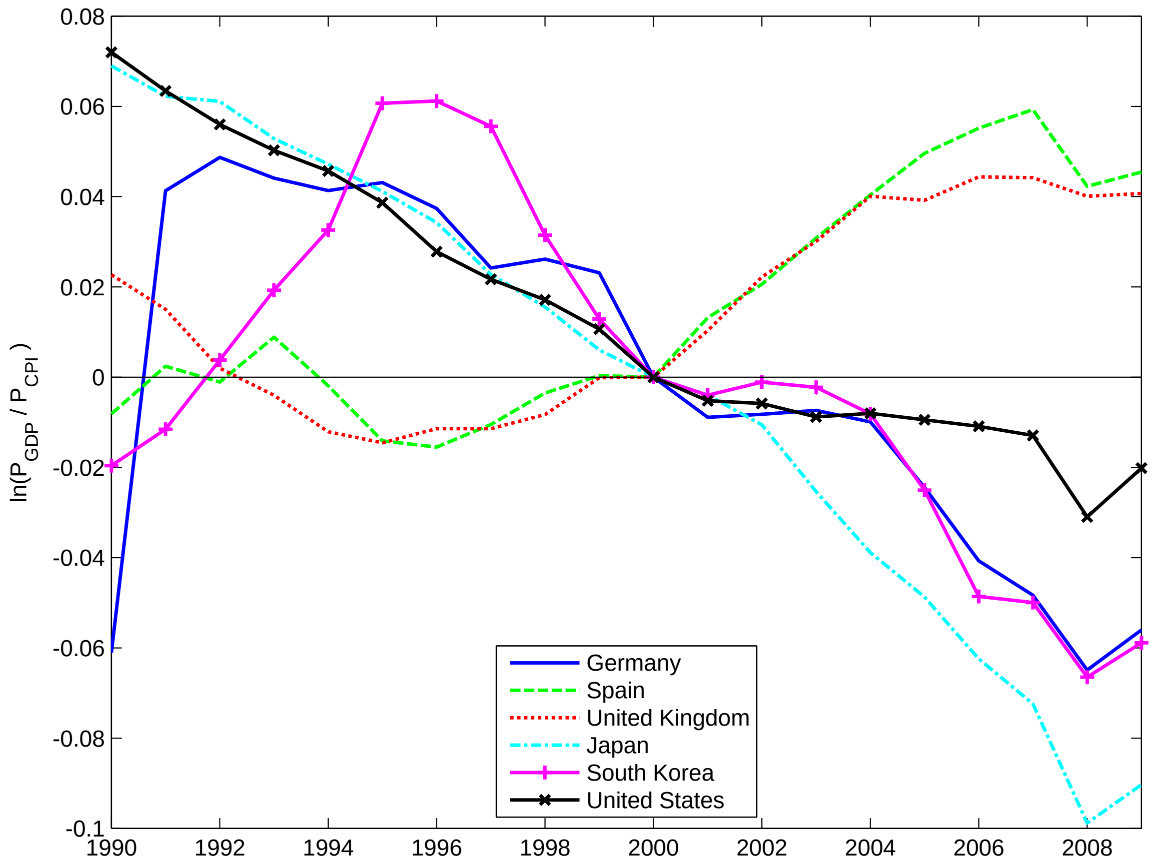
Note: Value-added openness defined as T_{VA}^{ii} [Equation (23)] and gross openness defined as S_i [Equation (26)]. Data from WIOD.

Figure 7. Reassignment of REER Weights over Time, 1970-2009



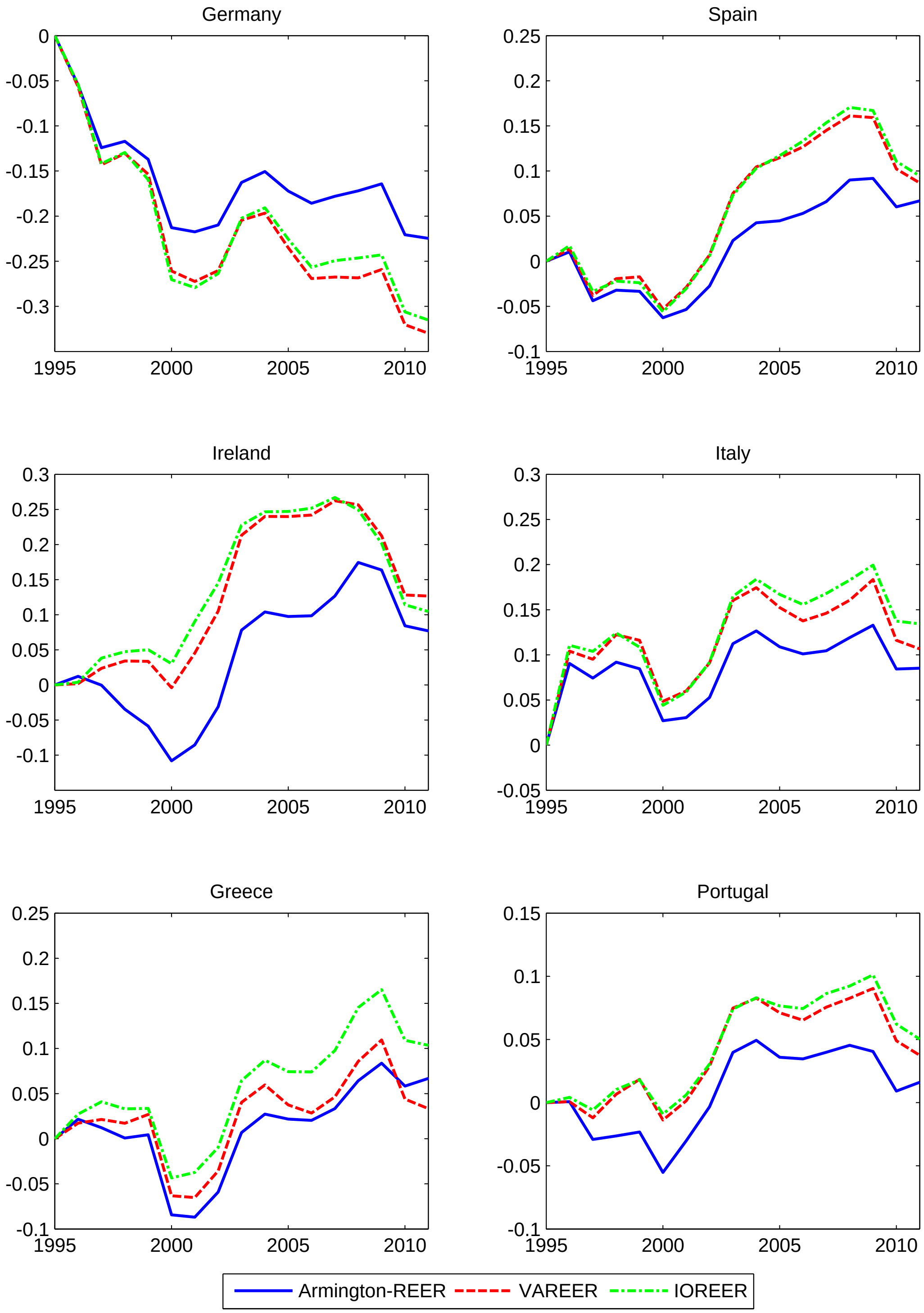
Note: VAREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{1,1,1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{2.25,0,0\}$. Distances between REER weights are measured using the city-block metric: $d_{it}^{VAREER} = \sum_j \left| w_{ijt}^{VAREER} - w_{ijt}^{Armington} \right|$ and $d_{it}^{IOREER-VAREER} = \sum_j \left| w_{ijt}^{IOREER} - w_{ijt}^{VAREER} \right|$, where w_{ijt} denotes the weight attached by country i to partner j in year t for each index. Data from Johnson and Noguera (2014).

Figure 8. Difference between GDP and CPI Price Deflators, 1990-2009



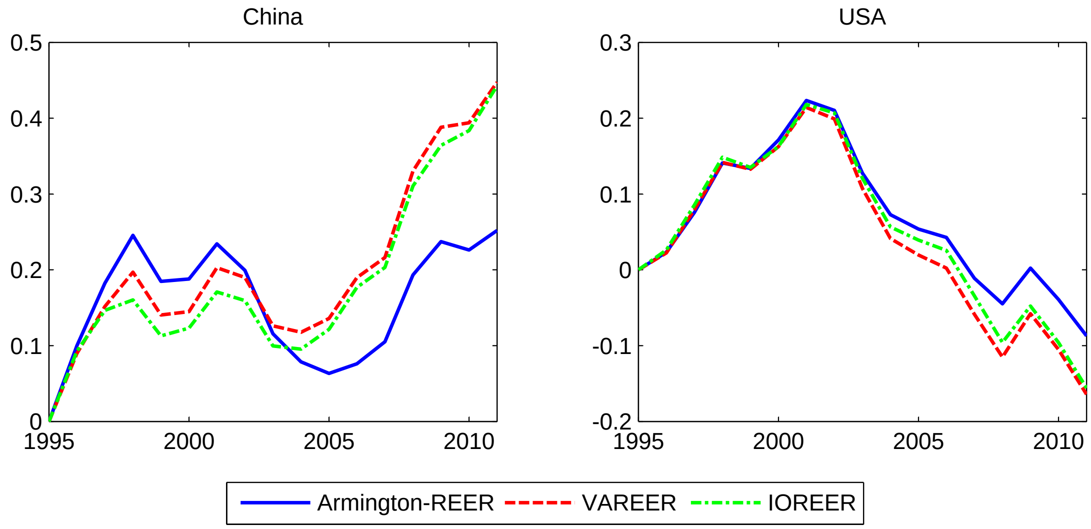
Note: Log relative price of GDP to CPI is normalized to zero in 2000. Data from IMF's World Economic Outlook database.

Figure 9. Real Effective Exchange Rates for Select EMU Countries, 1995-2011



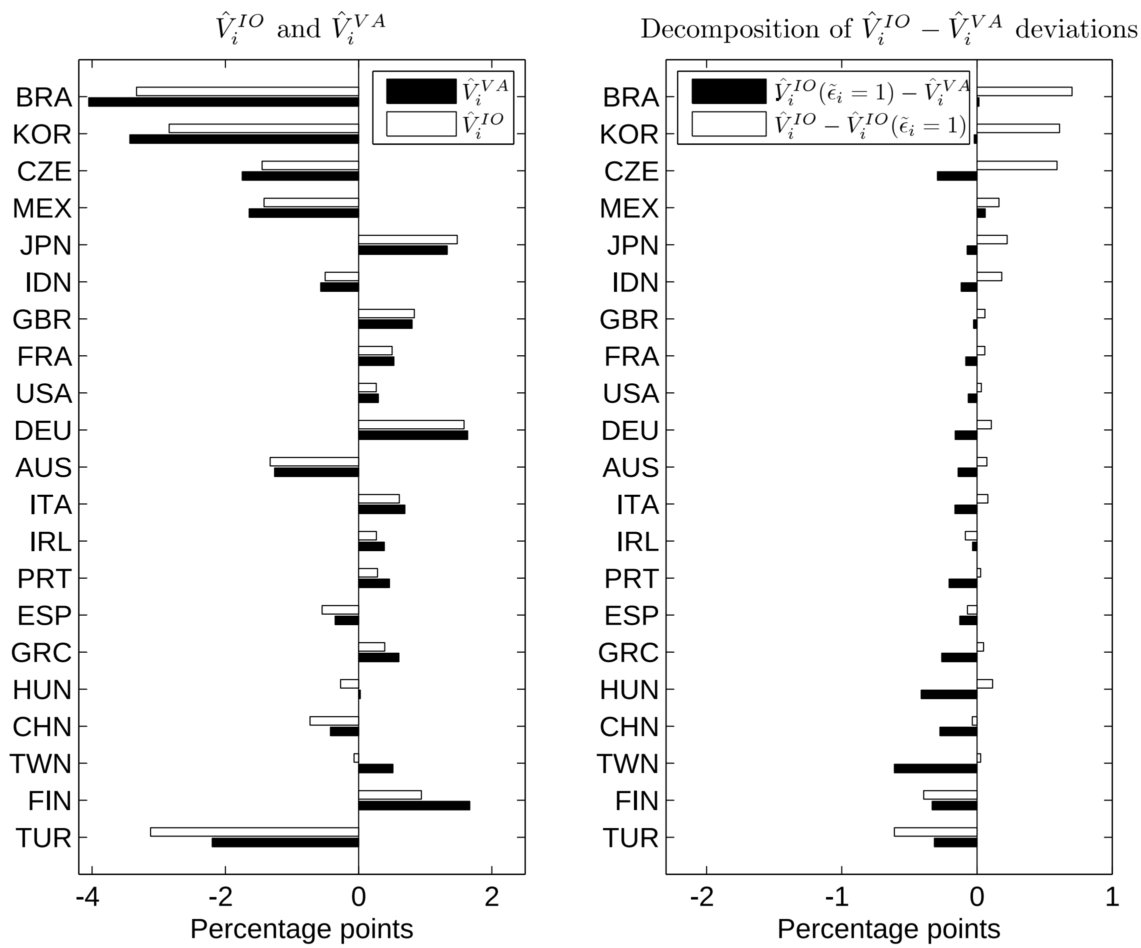
Note: VAREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{1, 1, 1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{3, 0, 0\}$. The level of the log REERs is normalized to zero in 1995. Data from WIOD.

Figure 10. Real Effective Exchange Rates for China and United States, 1995-2011



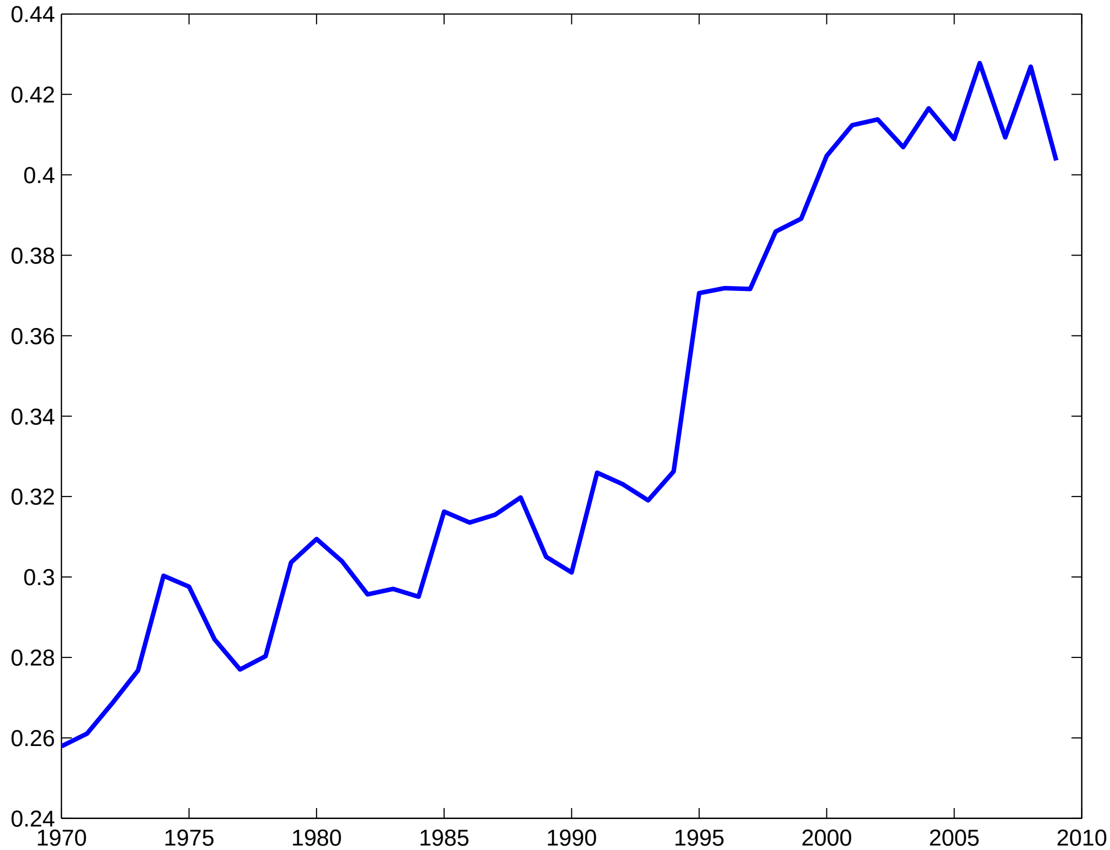
Note: VAREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{1, 1, 1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{3, 0, 0\}$. The level of the log REERs is normalized to zero in 1995. Data from WIOD.

Figure 11. Changes in Demand for Value Added and Contributing Factors to Deviations, 2005



Note: $\hat{V}_i^{VA}$ denotes changes in demand for value added when final demand and production elasticities are set to $\{\sigma,\gamma,\rho\} = \{1,1,1\}$. $\hat{V}_i^{IO}$ denotes changes in demand for value added when final demand and production elasticities are set to $\{\sigma,\gamma,\rho\} = \{3,0,0\}$. Decomposition components, $\hat{V}_i^{IO}(\tilde{\varepsilon} = 1) - \hat{V}_i^{VA}$ and $\hat{V}_i^{IO} - \hat{V}_i^{IO}(\tilde{\varepsilon} = 1)$, capture correspondingly contributions from deviations in IOREER-VAREER and contributions from deviations in effective value-added elasticities. Data from WIOD.

Figure 12. Value-Added Openness over Time, 1970-2009



Note: Value-added openness defined as T_{VA}^{ii} [Equation (23)]. The line records the cross-country median for each year. Data from Johnson and Noguera (2014).

Appendix A. Framework Appendix

In this appendix, we provide supplemental results related to Sections II and III. The first part provides algebraic details underlying special case with equal elasticities, leading to Equation (22). The second part discusses demand for value added in three special cases of the framework with restricted input trade.

A.1. Demand for Value Added with Equal Elasticities

This section sketches the derivation of the VAREER, as a special case of the general framework in Section II.

When $\varepsilon \equiv \gamma = \rho = \sigma$, then the price level of inputs falls out of Equation (13), so it reduces to:

$$\hat{Q} = S_F \hat{\mathbb{F}} + S_X \left[-\varepsilon M_1 \hat{p} + \varepsilon M_2 \hat{p} + M_2 \hat{Q}) \right]. \tag{31}$$

Using Equation (4) to replace $\hat{\mathbb{F}}$, and noting that $(S_F + S_X) M_1 = I$, then:

$$\hat{Q} = -\varepsilon \hat{p} + \varepsilon (S_F M_2 W_f + S_X M_2) \hat{p} + S_X M_2 \hat{Q} + S_F M_2 \hat{F}, \tag{32}$$

where $(S_F M_2 W_f + S_X M_2)\hat{p}$ summarizes the effective aggregate gross output price that each country facts on the world market, and $S_X M_2 \hat{Q} + S_F M_2 \hat{F}$ summarizes the level of demand for gross output. Going one step further, we can re-write demand for gross output as:

$$\hat{Q} = -\varepsilon \hat{p} + \varepsilon [I - S_X M_2]^{-1} S_F M_2 W_f \hat{p} + \hat{F}^w, \tag{33}$$

with $\hat{F}^w$ defined as in the main text.

To convert demand for gross output into demand for real value added, we use the production side of the framework. Combining Equation (16) with (33) yields:

$$\begin{aligned} \hat{V} &= -\varepsilon \hat{p} + \varepsilon [I - S_X M_2]^{-1} S_F M_2 W_f \hat{p} - [diag(s_i^x / s_i^v)](I - W_x)\hat{p} + \hat{F}^w \\ &= -\varepsilon [I - \Omega'][diag(s_i^v)]^{-1} \hat{p} + \varepsilon [I - S_X M_2]^{-1} S_F M_2 W_f \hat{p} + \hat{F}^w, \end{aligned} \tag{34}$$

where the second line follows because $I + [diag(s_i^x / s_i^v)] = [diag(s_i^v)]^{-1}$ and $diag(s_i^x) W_x = \Omega'$.

Finally, we substitute out for gross prices, using Equation (18). Canceling terms in the resulting expression yields:

$$\begin{aligned} &\hat{V} = -\varepsilon \left(\hat{p}^v - \hat{P}^w \right) + \hat{F}^w \\ &\text{with} \quad \hat{P}^w \equiv [I - S_X M_2]^{-1} S_F M_2 W_f [I - \Omega']^{-1} [diag(s_i^v)] \hat{p}^v \\ &\text{and} \quad \hat{F}^w \equiv [I - S_X M_2]^{-1} S_F M_2 \hat{F}. \end{aligned} \tag{35}$$

The perceived world prices of value added ($\hat{P}^w$) are weighted averages of price changes for value added ($\hat{p}^v$) originating from all countries. The first component is $W_f[I-\Omega']^{-1}[diag(s_i^v)]$. This combines Equations (5) and (18) to compute changes in the final goods price level in each destination ($\hat{P}_F$). The second component, $[I-S_X M_2]^{-1} S_F M_2$, aggregates $\hat{P}_F$ into the perceived world price of value added ($\hat{P}^w$). Specifically, each ij element of $[I-S_X M_2]^{-1} S_F M_2$ records the share of gross output from each source country i used directly or indirectly to produce final goods absorbed in destination j. These weights are equal to the share of value added from source i embodied in final goods in destination j: $\frac{p_i^v V_{ij}}{p_i^v V_i}$. That is, they are value-added export shares. The same weighting scheme applies in computing $\hat{F}^w$. Recognizing this, we arrive at Equation (22) in the main text, the Armington-CES model that is the basis for the VAREER.

A.2. Demand for Value Added with Restrictions on Input Trade

To aid in understanding the value-added REER formulas, we discuss three special cases. The first case has no intermediate inputs in production. The second case assumes that domestic inputs are used in production, but there is no input trade. The third case allows for input trade, but assumes imports are used to produce exports for only one bilateral pair and that elasticities are equal throughout the model.

A.2.1. Case I: no intermediate inputs

Suppose that we modify the framework to remove intermediate inputs entirely, so that $\{S_X, \Omega\}$ and s_i^X are zeros. Then, Equation (19) can be written as: $\hat{V} = -\eta \left[I - S_F M_2 W_f\right] \hat{p}^v + S_F M_2 \hat{F}$, where we recognize that $s_i^V = 1$ and $S_F M_1 = I$. Setting $\hat{F}$ to zero and re-writing this in summation notation using results from Section A, we arrive at:

$$\hat{V}_i = -\sigma \hat{p}_i^v + \eta \sum_j \left(\frac{p_i^v F_{ij}}{p_i^v V_i} \right) \hat{P}_j \quad \text{with} \quad \hat{P}_j = \sum_k \left(\frac{p_k^v F_{kj}}{P_j F_j} \right) \hat{p}_k^v. \tag{36}$$

Note that exports consist entirely of final goods in this case, so we can replace $p_i^v F_{ij}$ with gross exports EX_{ij}. This leads to a straightforward interpretation. This formula is a double export weighted index of bilateral relative value-added prices. The weights are based on exports as a share of GDP and/or final expenditure, because exports are comparable to GDP when not inputs are used in their production.

An alternative interpretation is as follows. Note that value added equals gross output in this special case, so $Q_i = V_i$ and $p_i = p_i^v$. Together with the fact that $p_i^v F_{ij} = EX_{ij}$, then we can re-write Equation (36) as:

$$\hat{V}_i = -\sigma \hat{p}_i + \eta \sum_j \left(\frac{EX_{ij}}{p_i Q_i} \right) \hat{P}_j \quad \text{with} \quad \hat{P}_j = \sum_k \left(\frac{EX_{kj}}{P_j F_j} \right) \hat{p}_k. \tag{37}$$

That is, we can trivially re-write Equation (36) to look like it involves gross prices, exports, and output, because there is no distinction between final goods, gross output, and value added in this case.

It follows that one could define a value-added REER as in Equation (26), by defining $EX_{ij} = Sales_{ij}$ and $P_j F_j = \sum_k Sales_{kj}$. This provides a possible rationale for interpreting conventional REERs as describing demand for value added. However, we believe this interpretation is problematic – and therefore do not advance it – for three reasons. First, it rests on an obviously counterfactual assumption – that no inputs are used in production. Second, this special case provides no guidance about how to construct REERs in practice. Given this view, one would go out to data and observe differences between value added and gross variables in the real world, and have no way of deciding which to use in building the REER. In contrast, our framework makes the mapping from model to data crystal clear. Third, this interpretation is not consistent with the Armington product-based approach on which conventional REERs are based.

A.2.2. Case II: domestic inputs only

Let us now assume that domestic inputs are used in production, but there is no input trade. In this case, Ω is a diagonal matrix with elements ω_{ii} equal to the share of domestic intermediates in gross output in each country (i.e., $\omega_{ii} = s_i^X$). Then, Equation (19) can be written as just as in the previous case: $\hat{V} = -\sigma \left[I - S_F M_2 W_f\right] \hat{p}^v + S_F M_2 \hat{F}$. Thus, we arrive at Equation (36) in this case as well, despite the introduction of intermediates.

One way to understand this is that Equation (36) can be re-written as:

$$\hat{V}_i = -\eta \hat{p}^v + \eta \sum_j \left(\frac{(1-\omega_{ii})^{-1} p_i F_{ij}}{p_i Q_i} \right) \hat{P}_j \quad \text{with} \quad \hat{P}_j = \sum_k \left(\frac{p_k F_{kj}}{P_j F_j} \right) \hat{p}_k^v. \tag{38}$$

Gross output equals final goods plus domestic intermediates, which implies that $\sum_j \frac{p_i F_{ij}}{p_i Q_i} < 1$. The $(1-\omega_{ii})^{-1}$ adjustment in the formula above takes final goods and converts them into the amount of gross output needed to produce those final goods. Then the weights on individual destination markets records the amount of gross output needed to produce final goods shipped to a given destination $(1-\omega_{ii})^{-1} p_i F_{ij}$ as a share of gross output $p_i Q_i$. Noting that the ratio of value added to gross output is $1-\omega_{ii}$, then these shares are equivalent to the share of value-added exports in total value added.

A.2.3. Case III: restricted input trade and homogeneous elasticities

We now turn to a case in which there are no domestic intermediates, but there is restricted trade in inputs. We assume that country 1 exports inputs to country 2, and no other country exports or imports inputs. Put differently, $\Omega_{12} > 0$ is the only non-zero element of Ω. Further, let us also assume that elasticities are equal throughout the framework, so we are in the Armington value-added case. This means that Equation (22) describes demand for value added, so we are interpreting that formula here in a special case.

Starting with destination price indexes ($\hat{P}_j$), computing $W_f[I-\Omega']^{-1}[diag(s_i^v)]$ yields the weights to attach to value added prices. These can be written in the form:

$$\hat{P}_j = \left(\frac{p_1F_{1j} + p_2F_{2j}\Omega_{12}}{P_jF_j}\right)\hat{p}_1^v + \left(\frac{p_2F_{2j}(1-\Omega_{12})}{P_jF_j}\right)\hat{p}_2^v + \sum_{k\neq 1,2}\left(\frac{p_kF_{kj}}{P_jF_j}\right)\hat{p}_k^v. \tag{39}$$

Here the weight on $\hat{p}_1^v$ is adjusted upwards and the weight on $\hat{p}_2$ is adjusted downward relative to the share of final goods imported from each country by j. This reflects the fact that country 1 ships inputs to country 2 that are embodied in final goods shipments F_{2j}. Therefore, the fraction Ω_{12} of F_{2j} is value added originating in country 1.

These price indexes get weighted by $[diag(p_iQ_i)]^{-1}[I-\Omega]^{-1}[diag(p_iQ_i)]S_FM_2$ in constructing the hypothetical world price index. For country 1, demand for real value added can be written as:

$$\hat{V}_1 = -\eta\hat{p}_1^v + \eta\sum_j\left(\frac{p_1F_{1j} + \Omega_{12}p_2F_{2j}}{p_1Q_1}\right)\hat{P}_j, \tag{40}$$

where $\hat{P}_j$ is given by Equation (39).

How do we interpret the destination weights? Note that $p_1Q_1 = p_1^vV_1$ and $p_1F_{1j} + \Omega_{12}p_2F_{2j} = p_1^vV_{1j}$ for country 1, so these destination weights are simply equal to the share of value added from country 1 consumed in country j (i.e., $\frac{p_1^vV_{1j}}{p_1^vV_1}$). Some of the value added from country 1 ($p_1^vV_{1j}$) is consumed directly in final goods shipped from country 1 (p_1F_1j), and some of it is consumed indirectly embodied in final goods shipped from country 2 ($\Omega_{12}p_2F_{2j}$).

Turning to country 2, demand for real value added can be written as:

$$\begin{aligned}\hat{V}_2 &= -\eta\hat{p}_2^v + \eta\sum_j\left(\frac{p_2F_{2j}}{p_2Q_2}\right)\hat{P}_j,\\ &= -\eta\hat{p}_2^v + \eta\sum_j\left(\frac{(1-\Omega_{12})p_2F_{2j}}{(1-\Omega_{12})p_2Q_2}\right)\hat{P}_j.\end{aligned} \tag{41}$$

From the first to the second line, we simply multiply and divide the destination weight by $1-\Omega_{12}$ to convert the gross output share $\frac{p_2F_{2j}}{p_2Q_2}$ into a value added share $\frac{p_2^vV_{2j}}{p_2^vV_2}$. So these weights also equal the share of value added from country 2 consumed in j.

In both cases, destinations are weighted by value-added trade shares, which means that these shares tell us how important destination j is as a source of demand for country i. Further, the share of value added from i in final spending in j captures how important price changes in i are in determining the price level in j. The takeaway from this example is trade measured in value added terms captures how production linkages influence evaluations of competitiveness. When inputs are traded, neither final goods shipments nor gross exports suffice to evaluate competitiveness.

APPENDIX B. EMPIRICAL APPENDIX

This appendix provides supplemental empirical results. Section B.1 explores how sensitive value-added elasticities and REER weights are to different elasticity parameters. Section B.2 compares results from our framework with Leontief final demand to results from the main text with Leontief production. Section B.3 examines the sources of deviations between value-added deflators and consumer price indexes (CPIs). Finally, Section B.4 quantifies the role that effective value-added elasticities play in explaining deviations in demand for value added between the heterogeneous and homogeneous elasticity cases.

B.1. How Sensitive are Value-Added Elasticity and REER Weights to σ, γ and ρ?

This section examines the relative importance of each elasticity – σ, γ and ρ – in the key building blocks of our framework: the value-added elasticity and bilateral REER weights.

Referring to Equation (25), the value-added elasticity is a country-specific weighted average of the primitive gross elasticities:

$$\tilde{\varepsilon}_i(\sigma,\gamma,\rho) = \frac{T^{ii}_{\sigma}}{T^{ii}_{VA}}\sigma + \frac{T^{ii}_{\rho}}{T^{ii}_{VA}}\rho + \frac{T^{ii}_{\gamma}}{T^{ii}_{VA}}\gamma. \tag{42}$$

Bilateral REER weights also depend on the elasticities. For country i, the bilateral REER weight assigned to country j is $\left(\sigma T^{\sigma}_{ij} + \rho T^{\rho}_{ij} + \gamma T^{\gamma}_{ij}\right)/T_{ii}(\sigma,\gamma,\rho)$. Therefore, by construction

$$\sum_{j\neq i} \frac{-(\sigma T^{\sigma}_{ij} + \rho T^{\rho}_{ij} + \gamma T^{\gamma}_{ij})}{T_{ii}(\sigma,\gamma,\rho)} = 1.$$

We can regroup these $N-1$ bilateral weights according to contributions attached to each elasticity as follows:

$$\sigma\frac{\sum_{j\neq i} -T^{\sigma}_{ij}}{T_{ii}(\sigma,\gamma,\rho)} + \gamma\frac{\sum_{j\neq i} -T^{\gamma}_{ij}}{T_{ii}(\sigma,\gamma,\rho)} + \rho\frac{\sum_{j\neq i} -T^{\rho}_{ij}}{T_{ii}(\sigma,\gamma,\rho)} = 1,$$

where terms multiplying each elasticity summarize the overall sensitivity of REER weights in country i to each elasticity. Finally, we note that $\sum_j T^{x}_{ij} = 0$ for $x = \{\sigma,\gamma,\rho\}$. This is because final demand, the demand for the composite input, and input demand are each homogeneous of degree zero in prices. With this insight, we can write:

$$\sigma\frac{T^{ii}_{\sigma}}{T^{ii}(\sigma,\gamma,\rho)}+\gamma\frac{T^{ii}_{\rho}}{T^{ii}(\sigma,\gamma,\rho)}+\rho\frac{T^{ii}_{\gamma}}{T^{ii}(\sigma,\gamma,\rho)}=1. \tag{43}$$

Examining Equations (42) and (43) reveals that the same three diagonal elements of T-matrices – T^{ii}_{σ}, T^{ii}_{ρ} and T^{ii}_{γ} – determine how sensitive the value-added elasticity and REER weights are to the three elasticity parameters.[39] In Figure 13, we plot the relative size of T^{ii}_{σ}, T^{ii}_{ρ}, and T^{ii}_{γ} for selected countries in 2005. The weight attached to ρ is the largest, with a median value of 0.52 in the full sample. The weight attached to σ has a country median of 0.29, while the weight attached to γ accounts for the remainder with median value of 0.17. This means that the value-added elasticity and REER weights are most sensitive to the choice of ρ – the elasticity of substitution among inputs. At the same time, γ – the elasticity of substitution between domestic value added and inputs – is least important.

What explains the relative size of T^{ii}_{σ}, T^{ii}_{ρ} and T^{ii}_{γ}? The economic forces that determine the size of T^{ii}_{σ} and T^{ii}_{ρ} are essentially identical. Both terms capture substitution between domestic and imported goods in correspondingly final demand and among inputs in production. T^{ii}_{ρ} has twice the impact on demand for value added simply because trade in inputs is twice the trade in final consumption goods.

The contribution of T^{ii}_{γ} is considerably smaller because there are two opposing effects at work. First, there is substitution effect between value added and inputs in production: if the price of domestic value added increases, it is substituted for inputs. As a result, demand for value added falls. Second, domestic value added is itself a major component of inputs used in production, so that substituting towards inputs increases demand for value added.

More formally, one can identify these opposing effects by examining Equation (17), where the direct substitution effect is captured with $-\gamma[diag(s^x_i/s^v_i)](I-W_x)$ (which has a negative sign), while the off-setting effect is captured by $\gamma[I-S_XM_2]^{-1}S_XM_2(I-W_x)$ (which has a positive sign). We can then decompose T^{ii}_{γ} into $T^{ii}_{\gamma}=T^{ii,V}_{\gamma}+T^{ii,X}_{\gamma}$, where $T^{ii,V}_{\gamma}\equiv[diag(s^x_i/s^v_i)](I-W_x)$ and $T^{ii,X}_{\gamma}\equiv[I-S_XM_2]^{-1}S_XM_2(I-W_x)$. Looking at these two terms empirically, we find that values for the median sample country are $T^{ii,V}_{\gamma}/T^{ii}_{VA}=0.57$ and $T^{ii,X}_{\gamma}/T^{ii}_{VA}=-0.40$. Thus, the role of the substitution effect between value added and inputs, $T^{ii,V}_{\gamma}$, is comparable to the role of the substitution effect among inputs, T^{ii}_{ρ}. This is to be expected, as both value added and inputs are important components of production. However, the off-setting effect, $T^{ii,X}_{\gamma}$, is also sizable, because domestic inputs are a major component of overall inputs. Putting these together, the overall sensitivity of REER weights and effective value-added elasticity to γ is muted, consistent with Figure 13.

[39] Note that differences in the common denominator – $T_{ii}(\sigma,\gamma,\rho)$ versus T^{ii}_{VA} – do not affect relative size of elasticity weights.

B.2. Leontief Final Demand: $\sigma = 0$ and $\gamma = \rho > 0$

This appendix examines how low final demand elasticities, as in Leontief final demand ($\sigma = 0$), alter the building blocks of the framework and historical value-added REERs. Given this restriction we set $\gamma = \rho = 1.50$ to make the global, GDP-weighted value-added elasticity equal to 1 over the 1995-2009 period in the WIOD data (similar to what we did in the Leontief case in Section IV). Rather than replicating all our results for this case, we focus on a few key results with this alternative parametrization.

In Figure 14, we present bilateral REER weights attached to Germany when $\sigma = 0$, and compare them to the VAREER and Armington-REER weights. The main finding is that IOREER weights with Leontief demand tend to undo the adjustment we see in VAREER relative to Armington-REER weights. That is, Leontief demand moves us back toward conventional REER weights. The economics underlying this case are symmetric to the case of low production elasticity. When final demand elasticity is low, bilateral partners with larger final goods share in trade are assigned increased weights, while partners with larger input trade share are assigned smaller weights. Consequently, the correlation between input trade intensity and IOREER-VAREER weight differences is negative, as reported in the lower right panel of Figure 14. In the Leontief production case, this correlation was positive. Figure 15 confirms this result for bilateral weights attached to China and Korea as well. For most countries, the IOREER weights under Leontief demand are very close to the Armington-REER weights.

Figure 16 examines value-added elasticities in the Leontief demand case. Again, the results and economic logic are symmetric to the Leontief production case. Recall that Leontief production reduced value-added elasticities for countries with large input shares in their trade. Leontief demand generates the opposite result: value-added elasticities are lower for countries that have higher final goods shares in trade, as depicted in Figure 16. Further, the overall magnitudes for deviations of the value-added elasticity from 1 are about half as large with Leontief final demand as they were with Leontief production. This reflects the fact that input trade shares are twice as large as final goods trade shares in the data.

Finally, we compare deviations between IOREER and VAREER indexes for the Leontief production versus Leontief final demand cases in Figure 17. The main takeaway again is the symmetry in the results. For example, consider the figure for China. Figure 10 showed that IOREER with Leontief production generated a smaller appreciation than VAREER, and this is reflected in the dashed line in the top-left panel in Figure 17. In contrast, the IOREER with Leontief demand generates a smaller appreciation than the VAREER, the mirror image of the Leontief production case. This basic results extends throughout the country sample. This finding should come as no surprise, given the results regarding REER weight differences and value-added elasticities above.

B.3. Deviations Between Prices of Value Added and CPI

In Section A, we observed that there are large and persistent differences between GDP deflators and consumer price indexes. To interpret these differences, it is instructive to decompose them into (a) differences between value added versus gross output prices ($\hat{p}^v - \hat{p}$), and (b) differences between gross output and consumer prices ($\hat{p} - \widehat{p}^{CPI}$):

$$\hat{p}^v - \hat{p}^{cpi} = \underbrace{\hat{p}^v - \hat{p}}_{\text{VA terms of trade}} + \underbrace{\hat{p} - \widehat{p}^{cpi}}_{\text{approximation}}. \tag{44}$$

The first component ($\hat{p}^v - \hat{p}$) captures differences between gross output and value-added prices. Because gross output prices are a cost-share-weighted average of value-added prices, then $\hat{p}^v - \hat{p}$ captures changes in the value added terms of trade. The second component $\hat{p} - \widehat{p}^{cpi}$ captures differences between each country's gross output price and its consumer prices. Because conventional REER measures use consumer prices rather than gross output prices for pragmatic reasons (e.g., data availability), we think of this price gap as simply reflecting approximation error – i.e., consumer price changes are typically a bad proxy for gross output price changes.[40] Together, these two components lead relative consumer prices to deviate from value-added prices, and this will account for part of the gap between our value-added indexes and currently published (CPI-based) REER indexes.

To illustrate how the gap between value added and consumer prices breaks down in practice, we plot the components of Equation (44) in Figure 18, focusing on the same six countries depicted in Figure 8.[41] Both components are important in explaining differences between value added and consumer prices, though the relative importance of each component differs across countries. For example, gross output and value added prices track each other closely in Germany, but growth in consumer prices persistently outstrips growth in either prices for value added or output over this period. Other countries like Spain see the exact opposite pattern, where gross output and CPI prices track each other, and the gap between value added and gross output prices is large.

Overall, this evidence points to both the distinction between gross output and value added, as well as the approximation of output prices with consumer prices, as important in understanding gaps between value added and consumer prices. Explaining differences in price measures in detail for individual countries lies outside the scope of this paper.

[40] There are several reasons why we might expect consumer price changes to be a bad proxy for gross output price changes. First, the terms of trade factor in here as well. Consumer prices are weighted averages of gross output prices from all countries (i.e., $\hat{P} = W_f \hat{p}$), so changes in the gross output terms of trade drive a wedge between consumer prices and a country's own gross output price. Second, the CPI measures consumer prices rather than supply-side prices. So, further deviations can be attributed to differences in weights that the CPI assigns to components of total demand. For example, CPI assigns zero weight to expenditures on nonresidential investment.

[41] We take gross output price indexes from the EU KLEMS database (http://www.euklems.net/).

B.4. Quantifying the Role of Value-Added Elasticities

In Section B, we described how differences in country-specific value-added elasticities in the IOREER case influence demand for value-added, relative to the VAREER case with a homogeneous elasticity. To study this more systematically, we can decompose $\hat{V}_{it}^{IO} - \hat{V}_{it}^{VA}$ as follows:

$$\widehat{V}_{it}^{IO} - \widehat{V}_{it}^{VA} = \underbrace{T_{iit}^{VA}\left(\widehat{REER}_{it}^{IO} - \widehat{REER}_{it}^{VA}\right)}_{\text{REER effect}} + \underbrace{T_{iit}^{VA}\widehat{REER}_{it}^{IO}\left(\tilde{\varepsilon}_{it}(\sigma,\rho,\gamma)-1\right)}_{\text{Elasticity effect}}. \tag{45}$$

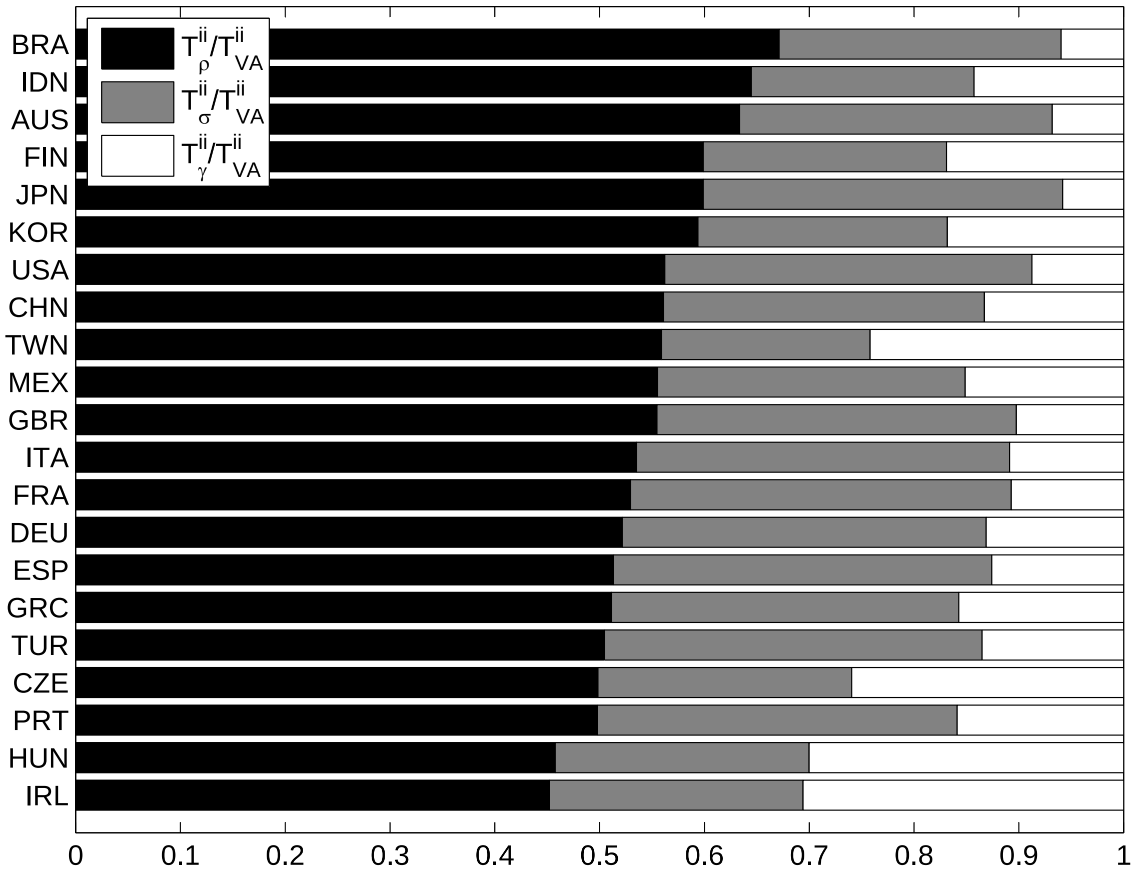
The first term captures differences between IOREER and VAREER changes, which we label the *REER effect*. The elasticity that governs how the IOREER influences demand is fixed here at $\tilde{\varepsilon}_{it} = 1$. The second term accounts for the effect of the country-specific value-added elasticity on demand for value added. The contribution of this *Elasticity effect* to deviations in demand for value added depends on interaction of deviations in $\tilde{\varepsilon}_{it}$ across countries and the change in the IOREER. To translate aggregated price changes into changes in demand for value added, both terms are adjusted for country-specific openness.[42]

Figure 19 provides a more systematic evidence about the Elasticity effect, using the decomposition reported in (45). On x-axis in the left panel we plot the absolute size of the median Elasticity effect for each sample country over 1970-2009. The value of 0.1 implies that the Elasticity effect in a given year contributes 0.1 percentage points to demand for value added. The resulting statistic shows that the Elasticity effect is large in economic terms. For example, in Japan the median one-year contribution of the elasticity effect to change in demand for value added is 0.13 percentage points. For some countries the impact is considerably higher. We plot these contributions against median values of effective elasticities in each country (i.e., $\widetilde{\varepsilon}_{it}(\sigma,\rho,\gamma)-1$) to show that the Elasticity effect is larger in countries in which there are more sizable deviations in the effective elasticity.

The right panel of Figure 19 looks at the relative importance of Elasticity and REER effects. The panel reports median contribution of the Elasticity effect to deviations in demand for value added for each sample country over 1970-2009. We find, in line with results in Figure 11, that the contribution of the Elasticity effect varies considerably across countries and for the median country is close to 0.5.

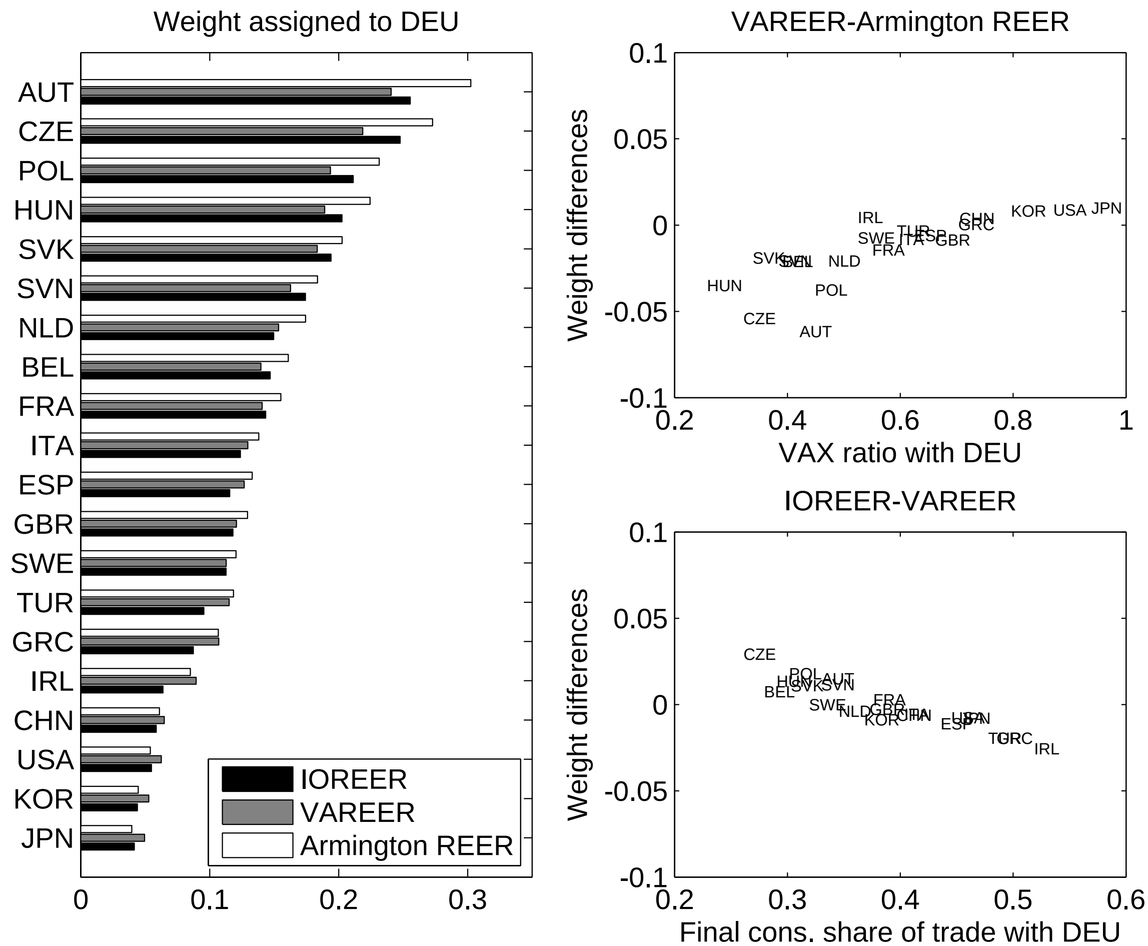
[42]Note that the two decomposition terms are identical to the two contributing factors reported in the right panel of Figure 11, but are expressed so as to highlight deviations in value-added REERs and value-added elasticities.

Figure 13. Sensitivity of REER Weights and Effective Value-Added Elasticity to Elasticities in Production and Final Demand, 2005



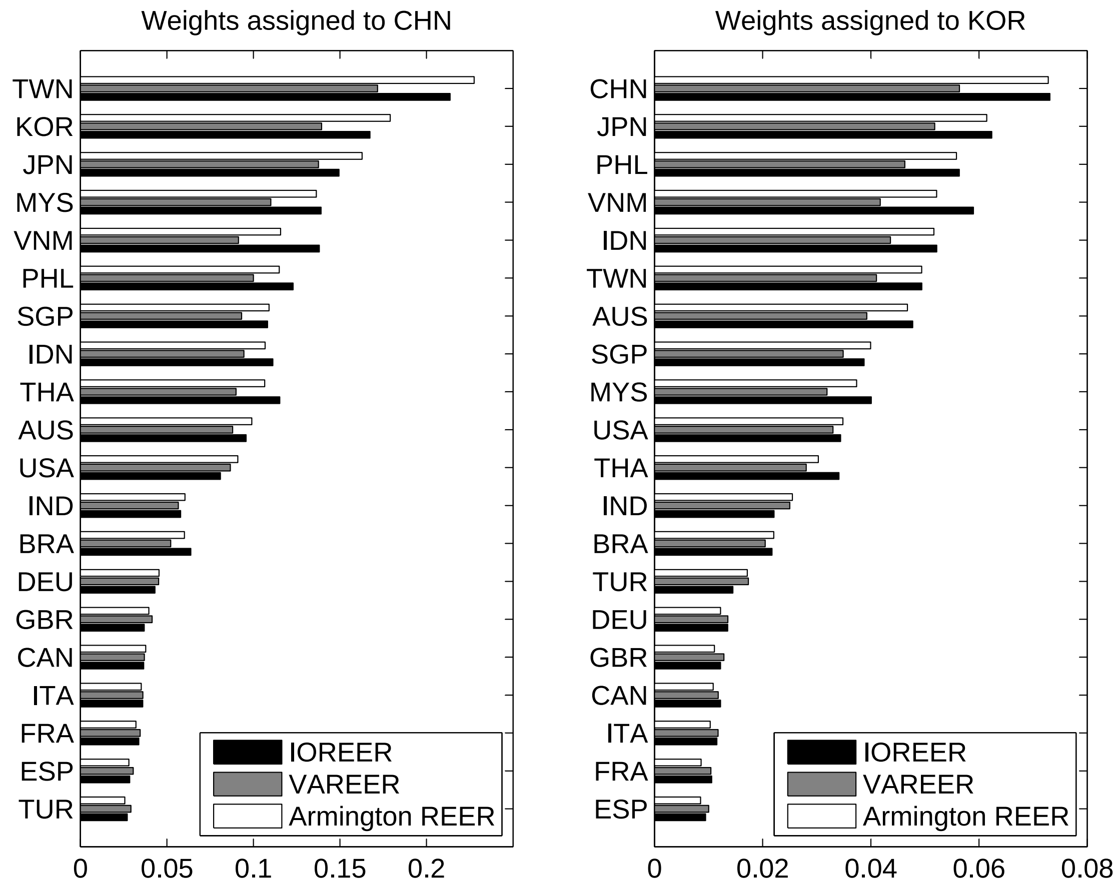
Notes: $T^{ii}_{\rho}/T^{ii}_{VA}$, $T^{ii}_{\sigma}/T^{ii}_{VA}$ and $T^{ii}_{\gamma}/T^{ii}_{VA}$, where $T^{ii}_{VA} = T^{ii}_{\rho} + T^{ii}_{\sigma} + T^{ii}_{\gamma}$, represent weights attached to the framework's three margins of substitution: (i) among inputs from different source countries, ρ, (ii) among final goods from alternative sources, σ, and (iii) between domestic value added and inputs in production, γ. Data from WIOD.

Figure 14. REER Weights Assigned to Germany and Differences in REER Weights versus Bilateral Trade Composition with Germany, 2007



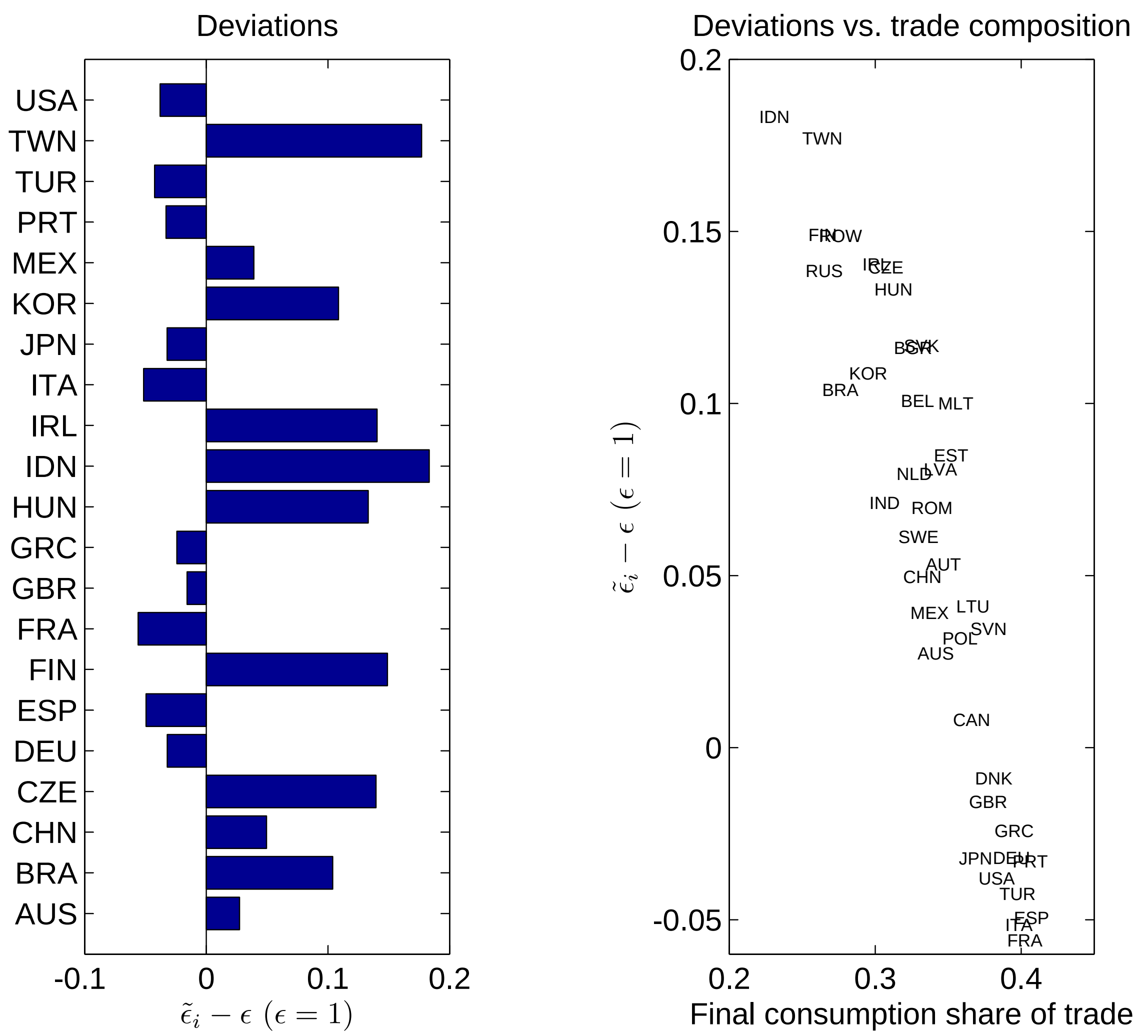
Note: VAREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{1,1,1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma,\gamma,\rho\} = \{0,1.5,1,5\}$. Bilateral final consumption share of trade with Germany includes both exports and imports. VAX ratio defined as bilateral trade in value added, relative to bilateral gross trade flows. Data from WIOD.

Figure 15. REER Weights Assigned to China and South Korea, 2004



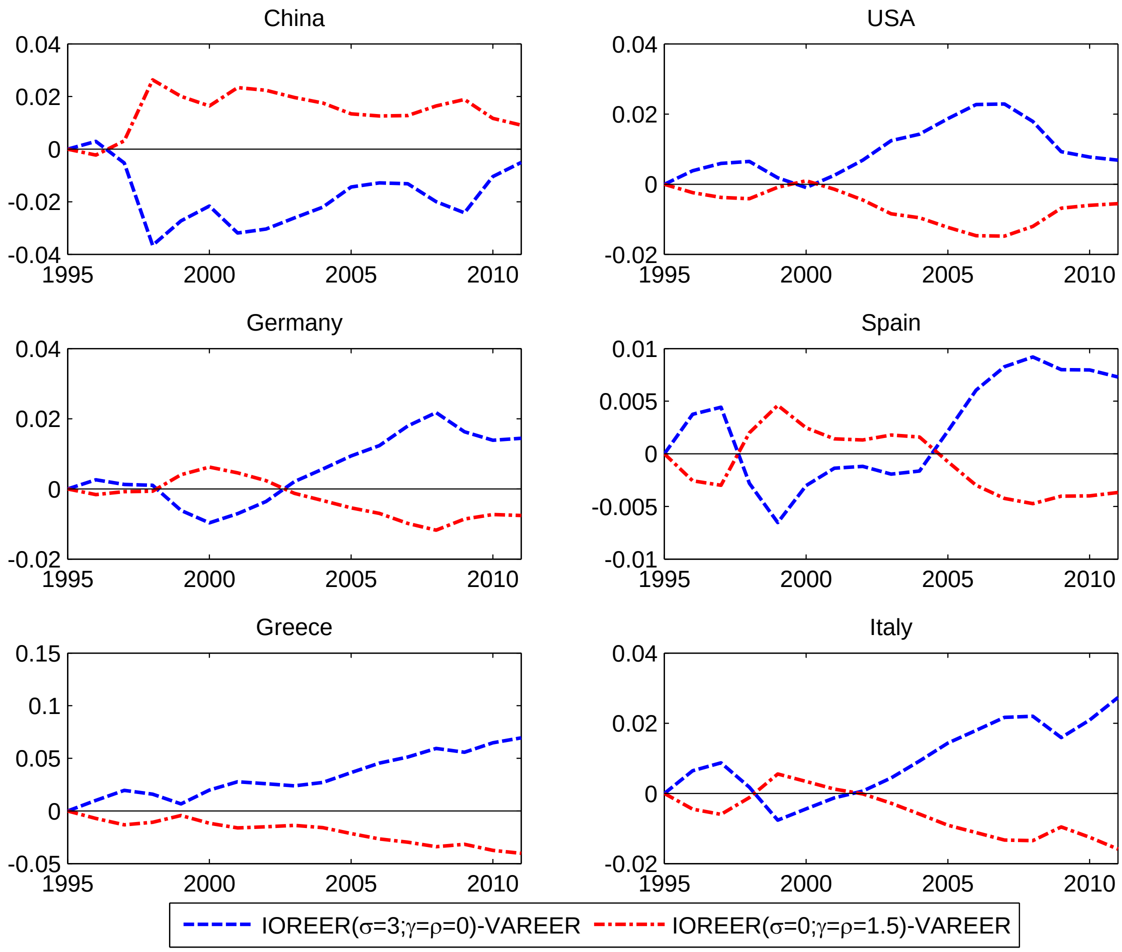
Note: VAREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{1, 1, 1\}$. IOREER weights are based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{0, 1.53, 1.53\}$. Data from GTAP.

Figure 16. Cross-Country Deviations in Effective Value-Added Elasticities with Inflexible Global Supply Chains, 2004



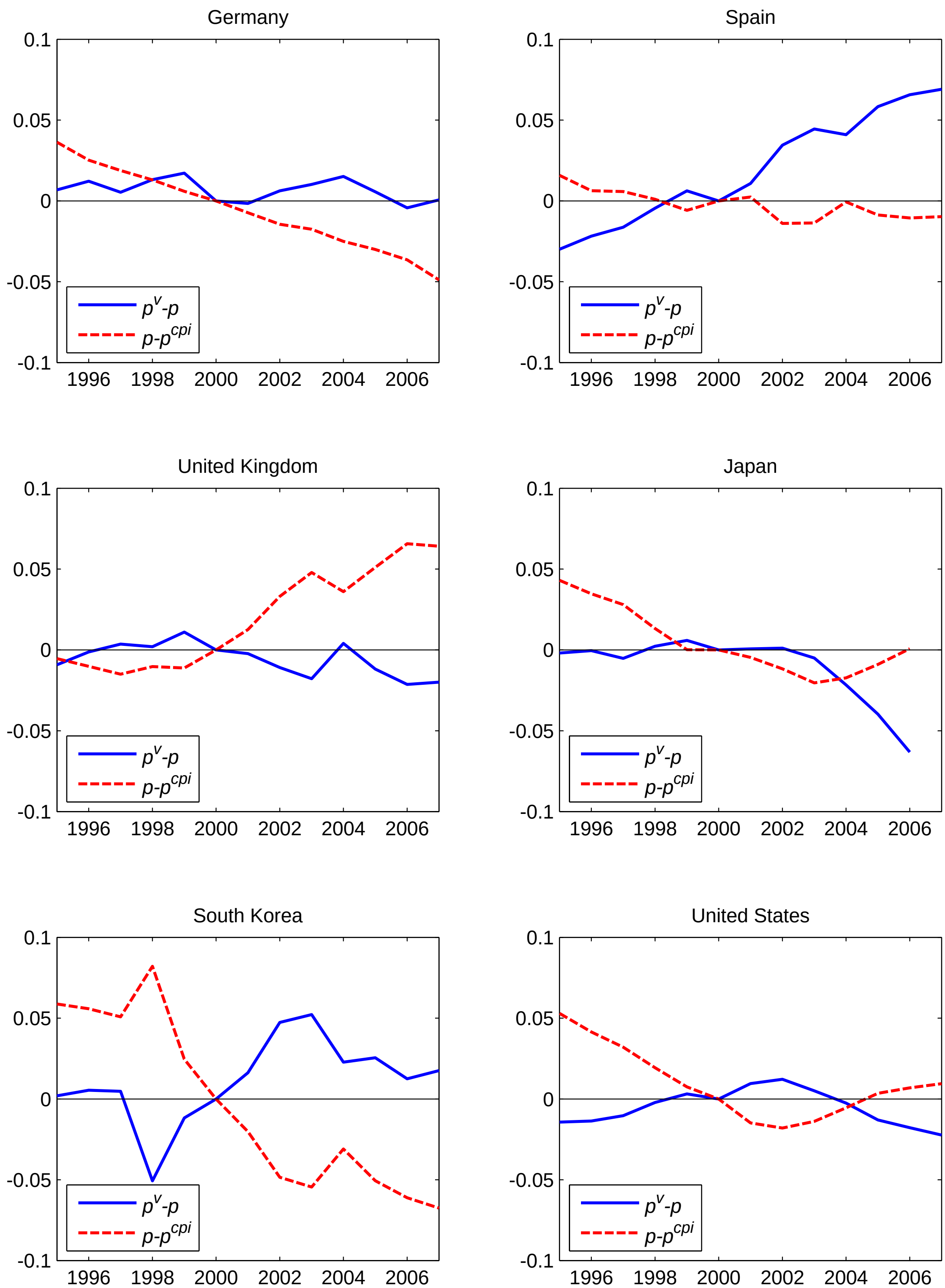
Note: Effective elasticities based on final demand and production elasticities $\{\sigma, \gamma, \rho\} = \{0, 1.5, 1.5\}$. Final consumption share of trade includes both exports and imports. Data from WIOD.

Figure 17. Cumulative Deviations between IOREER and VAREER Indexes, 1995-2011



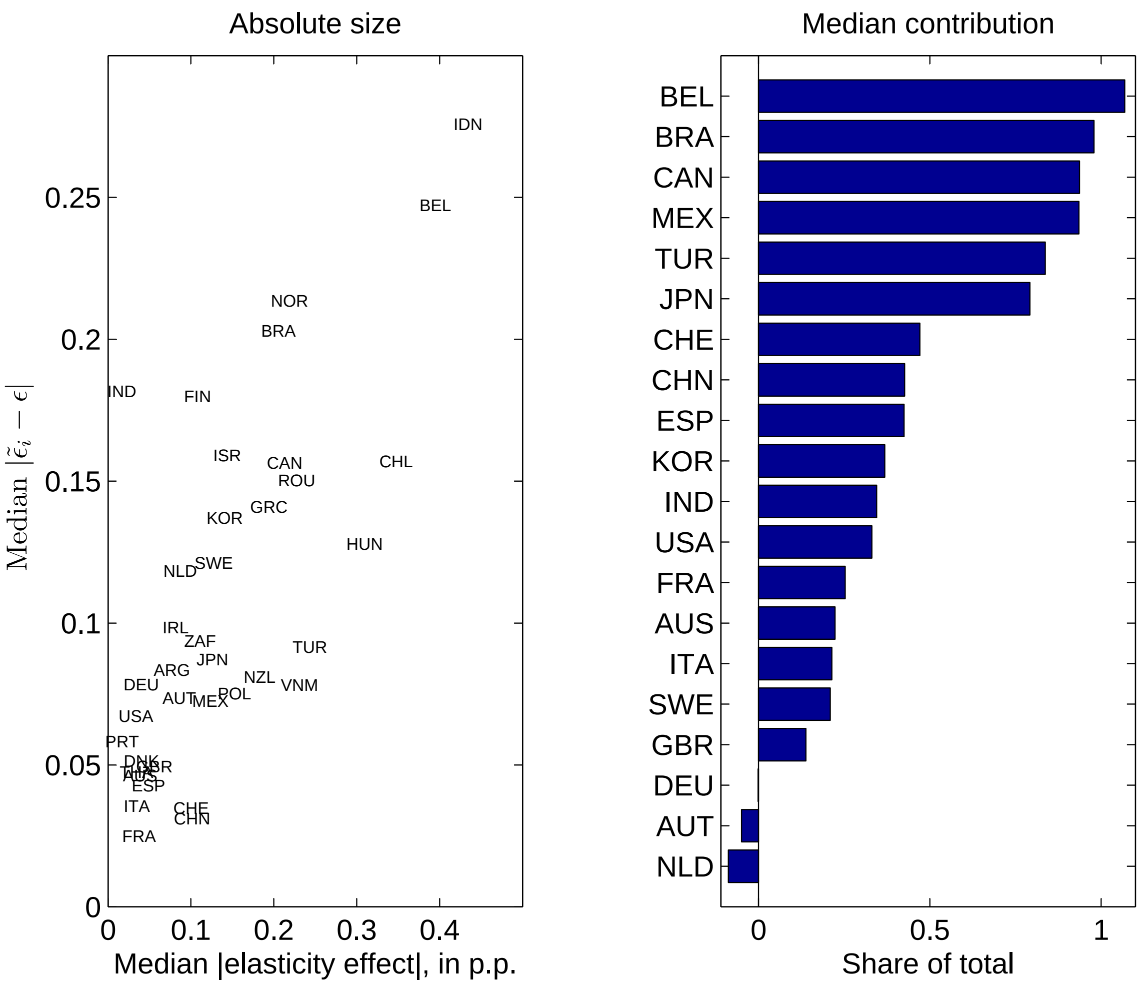
Note: All log REER indexes normalized to zero in 1995. Data from WIOD.

Figure 18. Decomposition of Differences Between GDP Deflator and CPI, 1995-2007



Notes: Log relative price is normalized to zero in 2000. Data from IMF World Economic Outlook and EU KLEMS databases.

Figure 19. Absolute Size of the "Elasticity Effect" and Its Contribution to Deviations in Demand for Value Added, 1970-2009



Note: Size of the median elasticity effect for each country computed as median absolute value of elasticity effects, in percentage points, at 1-year horizon over 1970-2009. Median contribution of the elasticity effect computed from a decomposition that attributes deviations in demand for value added to deviations in effective elasticities and deviations in REERs (i.e., IOREER-VAREER).Data from Johnson and Noguera (2014).