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How Delaying Fiscal Consolidation Affects the Present Value of GDP

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Abstract

We develop a simple model to examine the conditions under which delaying fiscal consolidation can affect the present value of GDP via the fiscal stance's effects on the output gap and hysteresis. We find that the absolute size of the fiscal multiplier—the focus of much empirical investigation and policy debate—is likely inconsequential in this regard. Rather, what matters is the degree to which the multiplier during the initial period of fiscal stimulus differs from the multiplier when the stimulus is withdrawn. If the multiplier is constant over time, delaying consolidation is unlikely to significantly boost the present value of GDP via effects on the output gap and hysteresis. The potential success of such efforts relies instead on exploiting time-variation in multipliers.

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Contents

List of Tables

List of Figures

I. Introduction

The aftermath of the Great Recession has left many advanced economies facing the challenge of closing wide output gaps while at the same time reducing large structural fiscal deficits. The tension between these two objectives has heightened the debate over the optimal pace of fiscal consolidation. This paper aims to contribute to this discussion by using a simple model and numeric simulations to examine the conditions under which delaying consolidation is likely to increase the present value (PV) of GDP.

In doing so, this paper builds on a recent influential and important paper by DeLong and Summers (2012). In their paper, DeLong and Summers posit that fiscal multipliers and hysteresis effects are currently substantial in most advanced economies.[2] Consequently, temporary fiscal stimulus will boost output not only in the short run, but also in the long run due to lessened hysteresis effects. DeLong and Summers' model also shows that, under their preferred parameters, these positive effects on GDP more than offset the negative effect on GDP from the higher distortionary taxes that are required to stabilize the debt-to-GDP ratio at a higher level following the stimulus. A temporary fiscal stimulus thus increases the PV of GDP in their model.

This paper adds to this discussion by extending this work in at least two ways:

- *First, we examine the case in which an initial stimulus is followed by fiscal consolidation in order to return the debt-to-GDP ratio to its baseline path*. One potential issue with DeLong and Summers' approach is that they assume that the government raises new taxes to pay only the growth-adjusted interest rate (r-g) on the debt issued to finance the stimulus. Consequently, the debt-to-GDP ratio permanently rises to a higher level as result of the stimulus.

  This assumption could be problematic if higher debt ratios have costs other than the distortions associated with the extra taxes required to pay the growth-adjusted interest. For example, higher debt may increase borrowing costs, raise risks of a sovereign crisis, or reduce the fiscal space to adopt future stimulus. Because of these costs, the scenario of a stimulus that permanently raises the debt-to-GDP ratio may not be fully comparable to a baseline scenario with a lower debt-to-GDP ratio.

  We attempt to address this issue by developing a simple model where an initial fiscal stimulus is followed by a delayed consolidation that brings the debt-to-GDP ratio back to the level that the country would have had in the absence of the initial stimulus. By incorporating the consolidation phase that is required to return the debt-to-GDP ratio to its baseline path, the model highlights that the positive effects of the initial stimulus through the fiscal multiplier and hysteresis effects operate in reverse during the subsequent consolidation.

[2] Hysteresis occurs when the temporary underutilization of productive resources leads to a permanent reduction in potential output. This may occur, for example, as a result of the scrappage of idle capital or a deterioration in labor skills or labor force participation due to extended periods of unemployment.

- *Second, we are explicit about our assumptions regarding (i) how multipliers vary over time and (ii) the persistence of the output gap*. In DeLong and Summers' model, the economy implicitly reverts to its pre-stimulus cyclical state when the temporary stimulus is removed. This reversion could be interpreted as an implicit assumption that either (i) the multiplier is the same during both the expansion phase when stimulus is applied and the contraction phase when stimulus is withdrawn or (ii) the multiplier is lower during the contraction phase, but the output gap closes at least partially on its own, though DeLong and Summers are not explicit about the assumptions in this regard.

  To better examine how these assumptions affect the results, we explicitly model the degree to which multipliers vary over time and the speed at which the output gap closes naturally.

In our model and numeric simulations and under some basic assumptions (e.g., that the interest rate equals the time-preference discount rate), we find that a temporary fiscal stimulus followed by a fiscal consolidation calibrated to return the debt-to-GDP ratio to its baseline path has the following effects on the PV of GDP:

- If the multiplier is constant over time and there is no hysteresis, then delaying consolidation has no effect on the PV of GDP, even if the multiplier is substantial. This is because, with a constant multiplier, the GDP gains from the initial stimulus are exactly offset by the losses from the subsequent consolidation. Under these conditions, the absolute size of the multiplier is thus irrelevant to assessing the merit of stimulus in terms of boosting the PV of GDP.

- Under a constant multiplier and hysteresis, delaying consolidation increases the PV of GDP. However, the magnitude of these gains is quite small—likely smaller than various costs and risks to stimulus not captured by our model (more on these below).

- If the multiplier is larger today than it will be during the future consolidation phase, then the gains from delaying consolidation can become considerable, especially in the presence of hysteresis and if the output gap closes slowly on its own.

- Scope for exploiting time-variation in multipliers depends in part on the nature of this variation. For example, simulations suggest that time-variation in multipliers may be easier to exploit if multipliers vary with a relatively slow-adjusting variable (e.g., the output gap) than if they vary with a faster-adjusting variable (e.g., the growth rate).

These results suggest that the decision about delaying fiscal consolidation should not be shaped mainly by views on the average size of the fiscal multiplier, but rather by the degree to which multipliers vary over the cycle and the degree to which this time-variation can be exploited by discretionary fiscal policy.

Unfortunately, the empirical literature provides only limited evidence on how multipliers vary over the cycle. As discussed in Parker (2011), the estimation of fiscal multipliers has been generally conducted with VARs that have essentially constrained the multiplier to being independent of economic conditions.[3] Therefore, we can interpret much of the empirical literature as having estimated the average multiplier over the cycle, which is largely inconsequential for the decision to implement a fiscal stimulus that needs to be subsequently reversed by consolidation. Similarly, the DSGE literature has extensively relied on linear solution techniques that remove any possible dependence of the multiplier from the state of the economy.

That said, relevant progress has been made over the last few years along two dimensions. First, Christiano, Eichenbaum, and Rebelo (2011) have shown that the multiplier can become very high in a calibrated model with sticky prices if monetary policy is constrained by the zero lower bound on nominal rates.[4] Second, Auerbach and Gorodnichenko (2011, 2012) have developed a VAR approach that allows for variations in fiscal multipliers over the cycle and found that multipliers are significantly higher in recessions, and possibly in the presence of a negative output gap.[5] It is fair to say, though, that the understanding of precisely how multipliers vary over time is still preliminary. We hope that our paper will provide further motivation for research in this area.

Finally, it is critical to emphasize that our model aims to capture—in a parsimonious way—the potential effects of delaying consolidation on the PV of GDP via the fiscal stance's effects on the output gap and hysteresis. In so doing, we have not incorporated all of the factors that policymakers would need to consider in deciding upon the optimal fiscal stance in any specific situation. Such considerations might include, for example, (i) effects on external balance; (ii) a view that certain public investments have a high return, relative to the costs of financing such investment, in terms of boosting potential GDP (e.g., see IMF, 2014); or (iii) various risks to delaying consolidation. The latter could include risks of higher sovereign yields arising from temporarily higher debt; political-economy risks that stimulus may not be reversed once the economy recovers; and the stochastic nature of the economy, with the risk that negative shocks may unexpectedly push the economy back into recession and increase multipliers during the phase of delayed consolidation when fiscal space for stimulus has been reduced.

Given these considerations and other factors that may need to be taken into account in assessing any specific policy action, this paper should not be construed as advocating for or against any specific fiscal path in any specific country. Rather, the objective is to better clarify the key criteria for assessing how a given path of the fiscal stance might affect the PV of GDP.

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[3] Seminal papers include Blanchard and Perotti (2002) and Ramey and Shapiro (1998).

[4] Other contributions on the role of fiscal policy at the zero lower bound include Krugman (1998), Eggertsson and Woodford (2003), Eggertsson (2008), and Woodford (2011).

[5] Other recent studies finding some evidence of time-varying multipliers include IMF (2012), Batini et al. (2012), Baum et al. (2012), and Giavazzi and McMahon (2012). The latter find, using household-level data, that fiscal multipliers are larger when the unemployment rate is higher.

The paper is structured as follows. Section II presents the model. Section III uses the model to assess the merit of delaying consolidation under constant fiscal multipliers. Section IV considers the case of time-varying multipliers using simulations based on data for the aggregate of G7 economies. Section V discusses risks from delaying consolidation that are not modeled in our simulations. Section VI concludes.

II. A Model of Delayed Fiscal Consolidation

In this section, we present a simple discrete-time model to solve for the conditions under which delaying fiscal consolidation may increase the PV of GDP. The model's key features include the following:

- We assume that the government implements fiscal stimulus at time 1 and delays consolidation to time 2. The size of the consolidation is such to bring the debt-to-GDP ratio back to the level that the country would have had in the absence of the stimulus. As discussed in the introduction, this is to ensure an even-handed comparison between alternative fiscal paths.

- The impact of fiscal policy on GDP is determined by the size of the fiscal multiplier.

- The model allows for hysteresis, whereby the output gap has a permanent effect on potential output. Hysteresis could result from various effects, such as the scrappage of idle capital or a deterioration in the skills of the workforce due to unemployment.

- For simplicity, we first present the model under the assumption that the output gap fully closes "naturally" from one period to the next. Natural closing is defined as closing of the output gap not driven by discretionary fiscal policy. This closing could occur, for example, as a result of active monetary policy or the gradual adjustment of prices.

- This assumption of full natural closing is then relaxed in subsequent sections.

We first describe the model's dynamics in the absence of discretionary fiscal policies, which we refer to as the "baseline" scenario. We denote actual and potential GDP at time 1 as y_1 and p_1, respectively. The time-1 output gap, γ_1, is thus simply given by

$$\gamma_1 = y_1 - p_1 \tag{1}$$

The model results are robust to the sign of the output gap, but we focus on the case in which the initial output gap is negative, consistent with current conditions in most advanced economies.

Hysteresis implies that the output gap affects potential GDP. We formalize this idea by assuming that time-2 potential GDP is given by

$$p_2 = G\,p_1 + h\,\gamma_1 \tag{2}$$

where $G = 1 + g$, with g being the underlying growth rate of potential GDP (i.e., potential GDP growth in the absence of hysteresis effects).

The parameter h captures the strength of hysteresis by specifying a relationship between potential GDP and the size of the output gap in the previous period. For example, if h = 0.1, this implies that an output gap of -5 percent for one period permanently reduces potential GDP by 0.5 percent due to hysteresis effects.

Under the assumption of full closing of the output gap, the time-2 output gap is zero regardless of the time-1 output gap, and GDP is equal to potential output:

$$\gamma_2 = 0 \text{ and } y_2 = p_2 \tag{3}$$

Ruling out possible shocks, actual GDP remains equal to potential output from time 2 onwards, with the latter growing at the rate g:

$$y_t = p_t\ ,\ \gamma_t = 0\ \text{, and }\ p_{t+1} = G\ p_t \text{ for all } t \geq 2 \tag{4}$$

Regarding the transition dynamics for the stock of public debt, d_t, we simply assume that

$$d_t = R\ d_{t-1} - \tau\ y_t \tag{5}$$

where d_{t-1} is debt at the beginning of period t, τ is the tax rate, and R is equal to $1 + r$, with r being the real yield on sovereign debt.

Government spending could be added to equation (5) as an additional term that also adds to debt and grows at an exogenous rate. However, this addition would not affect the merits of delaying consolidation, as adding a stream of exogenous government spending to both the baseline and alternative scenarios would not affect the choice between them. We thus abstract from government spending for simplicity.

We now consider how the dynamics change under an alternative scenario in which the government implements a fiscal stimulus, S, at time 1 to reduce the output gap, followed by a consolidation, C, at time 2 to bring the debt-to-GDP ratio back to the baseline level.

Using the upper-case letters Y_1 and Γ_1 to denote GDP and the output gap, respectively, after fiscal intervention, we have

$$Y_1 = y_1 + S\mu_1 \tag{6}$$
$$\Gamma_1 = \gamma_1 + S\mu_1 \tag{7}$$

where μ_1 is the fiscal multiplier. This multiplier incorporates the whole impact of the fiscal stimulus on output, including the effects of any offsetting monetary policy actions.

Fiscal stimulus also increases the stock of public debt, which becomes equal to the following:

$$D_1 = R\ d_0 - \tau\ Y_1 + S = d_1 + S(1 - \tau\mu_1) \tag{8}$$

Note that the fiscal cost of stimulus is partly immediately offset as higher GDP leads to higher tax revenue in time 1.

Fiscal stimulus has no immediate impact on potential GDP, so $P_1 = p_1$. However, stimulus increases potential output in time 2 by reducing hysteresis effects in time 1. Specifically, time-2 potential output equals

$$P_2 = G\,P_1 + h\,\Gamma_1 = p_2 + h\,S\mu_1 \tag{9}$$

At time 2, fiscal stimulus is withdrawn and, unlike DeLong and Summers (2012), we assume that the government implements an additional fiscal consolidation C in order to bring the debt-to-GDP ratio back to the level that the country would have had in the absence of stimulus. The withdrawal of the stimulus and the additional fiscal consolidation lead to a policy-induced contraction in the fiscal balance equal to $S+C$, so that time-2 GDP is equal to

$$Y_2 = P_2 - \mu_2(S + C) = y_2 + S(h\,\mu_1 - \mu_2) - C\,\mu_2 \tag{10}$$

where μ_2 is the time-2 fiscal multiplier. The consolidation reduces the debt stock at the end of time 2 to

$$D_2 = R\,D_1 - \tau\,Y_2 - C = d_2 + S\big(R(1 - \tau\mu_1) - \tau\,(h\,\mu_1 - \mu_2)\big) - C(1 - \tau\mu_2) \tag{11}$$

Note that some of the fiscal gains from the consolidation dissipate in time 2 as a result of lower tax revenue due to the contraction of GDP.

Importantly, the contractionary effects from the withdrawal of the stimulus and the fiscal consolidation widen again the output gap:

$$\Gamma_2 = Y_2 - P_2 = -\mu_2(S + C) \tag{12}$$

This wider output gap feeds, via hysteresis, into lower potential output at time 3:

$$P_3 = G\,P_2 + h\,\Gamma_2 = p_3 + S\,h\,(G\,\mu_1 - \mu_2) - C\,h\,\mu_2 \tag{13}$$

The withdrawal of fiscal consolidation at time 3 generates a final stimulative effect on the economy so that GDP in time 3 is given by

$$Y_3 = P_3 + \mu_3 C = y_3 + S\,h\,(G\,\mu_1 - \mu_2) - C\,(h\,\mu_2 - \mu_3) \tag{14}$$

and the debt stock becomes

$$\begin{aligned} D_3 &= R\,D_2 - \tau\,Y_3 \\ &= d_3 + S\big(R^2(1 - \tau\mu_1) - R\,\tau\,(h\,\mu_1 - \mu_2) - \tau\,h\,(G\,\mu_1 - \mu_2)\big) \\ &\qquad - C\big(R(1 - \tau\mu_2) - \tau\,(h\,\mu_2 - \mu_3)\big) \end{aligned} \tag{15}$$

The stimulative effect from the scaling down of the consolidation leads to a positive output gap

$$\Gamma_3 = Y_3 - P_3 = \mu_3 C \tag{16}$$

which increases time-4 potential output through the hysteresis effect:

$$P_4 = G\, P_3 + h\, \Gamma_3 = p_4 + S\, G\, h\, (G\, \mu_1 - \mu_2) - C\, h\, (G\, \mu_2 - \mu_3) \tag{17}$$

The absence of further fiscal impulses from time 4 onwards implies that GDP remains equal to potential output, which, as in the baseline version of the model, keeps growing at rate g:

$$Y_t = P_t\,,\ \Gamma_t = 0\,, \text{and}\ P_{t+1} = G\, P_t \text{ for all } t \geq 4 \tag{18}$$

Therefore, denoting with β the intertemporal discount factor and imposing that $G\beta < 1$, the present value of GDP (PV) equals

$$\mathrm{PV} = Y_1 + \beta\, Y_2 + \beta^2 Y_3 + \frac{\beta^3}{1-G\beta} Y_4 \tag{19}$$

We can now solve for the parameter restrictions under which the initial stimulus and the subsequent consolidation increase the PV of GDP. The impact of the fiscal stimulus is given by

$$\frac{\partial PV}{\partial S} = \left(1 + \frac{h\,\beta}{1-G\beta}\right)\left((\mu_1 - \beta\, \mu_2) - \beta \frac{\partial C}{\partial S}(\mu_2 - \beta\, \mu_3)\right) \tag{20}$$

which is positive if and only if

$$(\mu_1 - \beta\, \mu_2) > \frac{\partial C}{\partial S} \beta\, (\mu_2 - \beta\, \mu_3) \tag{21}$$

where the left and right sides of the inequality capture, respectively, the positive impact from the initial stimulus and the contractionary effect from the delayed consolidation.

Regarding the partial derivative of the consolidation with respect to the stimulus, we require the consolidation to be sufficiently large to bring the debt-to-GDP ratio back to the level that the country would have had in the absence of the initial stimulus. This is necessary to fairly compare the baseline scenario with the outcomes under alternative fiscal paths.[6] More specifically, we solve for the consolidation size that ensures that both the baseline and alternative scenarios yield the same ratio between debt at the start of period 4 (i.e., debt at the end of period 3) and period 4 GDP

$$\frac{D_3}{Y_4} = \frac{d_3}{y_4} \tag{22}$$

[6] Note that, while we target the same final debt-to-GDP ratio as in the baseline scenario, the final stock of debt can be higher as long as it is matched by a proportionally higher GDP level.

which also implies equality for any subsequent period.[7] This can be seen by considering that

$$\frac{D_4}{Y_5}-\frac{d_4}{y_5}=\frac{RD_3-\tau Y_4}{GY_4}-\frac{Rd_3-\tau y_4}{Gy_4}=\frac{R}{G}\left(\frac{D_3}{Y_4}-\frac{d_3}{y_4}\right) \tag{23}$$

The required consolidation size is given by

$$C=S\frac{R^2(1-\tau\mu_1)-R\,\tau\,(h\,\mu_1-\mu_2)-\tau\,h\,(G\,\mu_1-\mu_2)-G\,h\,(G\,\mu_1-\mu_2)\frac{d_3}{y_4}}{R(1-\tau\mu_2)-\tau\,(h\,\mu_2-\mu_3)-\,h\,(G\,\mu_2-\mu_3)\frac{d_3}{y_4}} \tag{24}$$

Taking the partial derivative of *C* with respect to *S* and inserting the result into equation (21), we find that an initial stimulus offset by a delayed consolidation increases the PV of GDP as long as

$$\begin{aligned}&(\mu_1-\beta\mu_2)\\&>\frac{R^2(1-\tau\mu_1)-R\tau(h\mu_1-\mu_2)-\tau h(G\mu_1-\mu_2)-Gh(G\mu_1-\mu_2)\frac{d_3}{y_4}}{R(1-\tau\mu_2)-\tau(h\mu_2-\mu_3)-\,h(G\mu_2-\mu_3)\frac{d_3}{y_4}}\;\beta\,(\mu_2-\beta\mu_3)\end{aligned} \tag{25}$$

In the next two sections, we analyze the tightness of this condition under alternative parameter values. In particular, we first consider the simple case of constant multipliers, both with and without hysteresis effects, in Section III. We then examine the more complicated case of time-varying multipliers using simulations based on data for advanced economies in Section IV.

III. Constant Fiscal Multipliers

***Case 1*: No hysteresis and full natural closing of the output gap**

To assess whether delaying consolidation can increase the PV of GDP, we consider first the simple case of constant multipliers and no hysteresis effects by imposing that $\mu_1=\mu_2=\mu_3=\mu$ and $h=0$. Under these parameters, delaying consolidation increases the PV of GDP if and only if the discount rate is higher than the interest rate:

$$R<1/\beta \tag{26}$$

This is a very intuitive condition. Stimulus involves increasing GDP today at the expense of reducing GDP in the future when the stimulus must be withdrawn and paid back with interest in order to return the debt ratio to its baseline level (assuming constant multipliers and no hysteresis). A higher interest rate thus requires more future consolidation and therefore a bigger reduction in future GDP, reducing the attractiveness of stimulus. In contrast, a higher discount rate reduces the value placed on future GDP, increasing the attractiveness of stimulus. The benefit of stimulus will thus be larger than the cost of consolidation if and only if the discount rate is higher than the interest rate.

[7] This condition is also equivalent to equalizing debt at the end of time 4 as a ratio to time-4 GDP.

It is important to note that under these parameters (constant multipliers and no hysteresis), the size of the fiscal multiplier is inconsequential to assessing the merit of delaying fiscal consolidation. This is because a larger multiplier increases the stimulative effect of fiscal expansion, but also strengthens the contractionary effect from the subsequent consolidation, and these effects exactly offset each other.

***Case 2*: Hysteresis and full natural closing of the output gap**

Can the presence of hysteresis relax the above condition? Allowing for hysteresis, delaying consolidation increases the PV of GDP if and only if

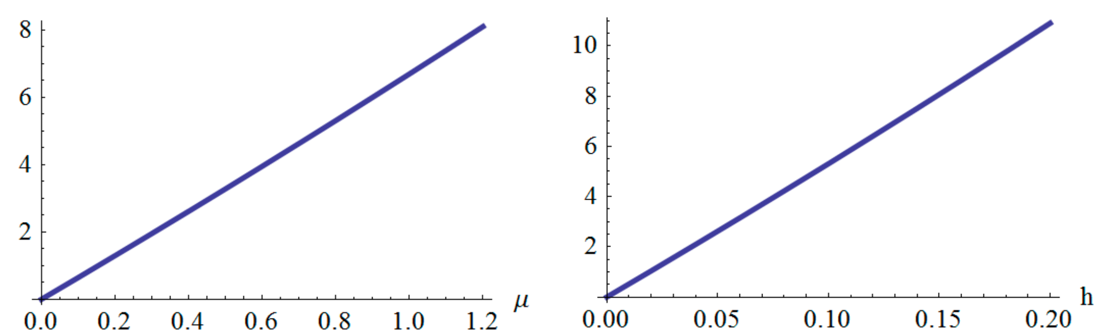
$$R < \frac{1}{\beta} + \frac{h\,\mu\,g}{R(1-\tau\,\mu)+\mu\,\tau\,(1-h)-h\,\mu\,g\,d_3/y_4}\left(\tau - \frac{d_3}{y_4}(r-g)\right) \tag{27}$$

Note that, under reasonable parameter values, the new additive term on the right-hand side of the inequality is positive. This implies that hysteresis does relax the interest rate threshold under which delaying consolidation may be beneficial. The key intuition behind this result is that the initial fiscal stimulus generates higher tax revenue in the following period by increasing potential output through hysteresis. Therefore, this positive fiscal gain allows for a smaller consolidation and possibly leads to positive net gains from delaying consolidation even if the interest rate exceeds the discount rate.

However, the effect of hysteresis on the interest rate threshold in equation (27) is quantitatively negligible under reasonable parameter values. For example, setting the fiscal multiplier to 0.8, the tax rate to 1/3, the growth rate to 2 percent, the baseline debt-to-GDP to 0.8, and the hysteresis parameter to 10 percent, the interest rate threshold exceeds the discount rate $1/\beta$ by only 5 basis points.

As shown in Figure 1, increasing the fiscal multiplier to 1.2 or the hysteresis parameter to 20 percent raises the interest rate threshold over the discount rate by only 10 basis points. Therefore, under constant multipliers and full natural closing of the output gap, the size of hysteresis does not appear to have an important bearing on the decision of whether or not to delay consolidation.

Figure 1: Interest rate threshold with constant multipliers and full natural closing of the output gap (basis points above $1/\beta$)



Note: The *x*-axis indicates different assumptions for the multiplier (μ) and hysteresis (h) in the left-hand and right-hand charts, respectively. The corresponding interest rate threshold for each assumption is on the *y*-axis. Other parameters are as assumed in the main text above.

***Case 3*: Slow natural closing of the output gap, with and without hysteresis**

One important caveat regarding the results so far is about the dynamics of the output gap. For the sake of simplicity, we developed our analysis assuming that the output gap entirely closes from one period to the next. We now relax this assumption by considering the opposite case of no natural closing of the output gap until time 4 when there are no further fiscal impulses.

Formally, we assume that for $t \leq 3$ the output gap fully carries over into the definition of next period actual GDP:

$$\begin{aligned} y_{t+1} &= p_{t+1} + \gamma_t \\ Y_{t+1} &= P_{t+1} + \Gamma_t \pm \text{fiscal impulses} \end{aligned} \tag{28}$$

We then allow for the closing of the output gap at time 4, setting $\gamma_4 = \Gamma_4 = 0$, so that we can derive a closed-form expression for the PV of GDP, which grows from then onwards at the rate g.[8]

The main difference relative to the previous analysis is that the initial fiscal stimulus has a persistent impact not only on potential output through hysteresis, but also on actual GDP. This can be seen from the new definition of time-2 GDP

$$Y_2 = P_2 + \overbrace{\gamma_t + S\,\mu_1}^{\Gamma_1} - (S + C)\mu_2 = y_2 + S\,h\,\mu_1 + S\,\mu_1 - (S + C)\,\mu_2 \tag{29}$$

which includes the additional term $S\mu_1$. With constant multipliers, this persistent positive effect exactly offsets the contractionary impulse from the withdrawal of stimulus at time 2. The same considerations apply to the subsequent consolidation phase, with the persistent contractionary effects from the consolidation in time 2 offsetting the boost to GDP from the removal of consolidation in time 3.

The assumption of no closing of the output gap modifies the condition under which delaying consolidation increases the PV of GDP to the following:

$$R < \frac{1}{\beta} + \frac{h\,\mu\,G}{R(1-\tau\,\mu) - \mu\,h\,(\tau + G\,d_3/y_4)}\left(\tau - \frac{d_3}{y_4}(r - g)\right) \tag{30}$$

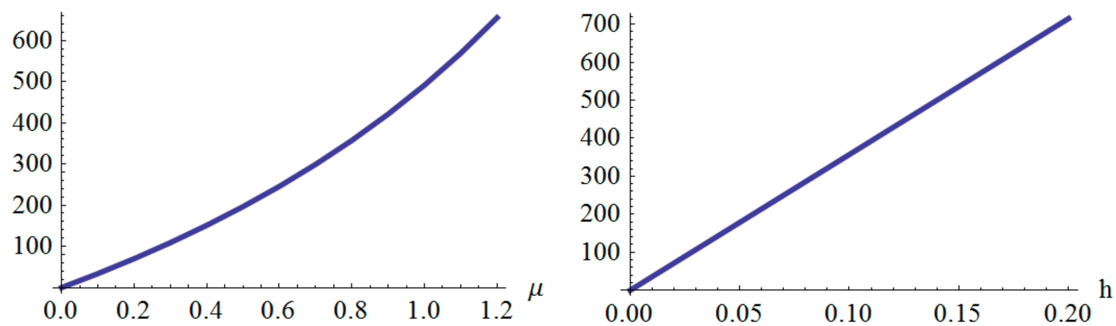
A first observation is that inequality (30) is identical to inequality (26) if there is no hysteresis ($h = 0$). Hence, allowing for no natural closing of the output gap does not change the finding that the size of the multiplier is inconsequential to the decision to delay consolidation if multipliers are constant and there is no hysteresis.

[8] Closing the output gap at time 4 is also required to ensure equality in the debt-to-GDP ratio between the baseline and delayed consolidation model from time 4 onwards. We will avoid imposing the full closing of the output gap in the simulations presented in Section IV.

A second observation is that, with hysteresis and with plausible values for other parameters, this modified inequality can imply a significantly higher interest rate threshold than in the previously considered case of immediate natural closing of the output gap.

Figure 2 shows indeed that delaying fiscal consolidation may increase the PV of GDP even if the interest rate exceeds the intertemporal discount rate by several percentage points. Therefore, the speed of closing of the output gap is an important element to be considered when assessing the benefits of different fiscal paths. This implication will also be confirmed in the analysis of time-varying multipliers in the next section. But before moving to this discussion, there are two other important considerations.

Figure 2: Interest rate threshold with constant multipliers and no natural closing of the output gap until Time 4
(basis points above $1/\beta$)

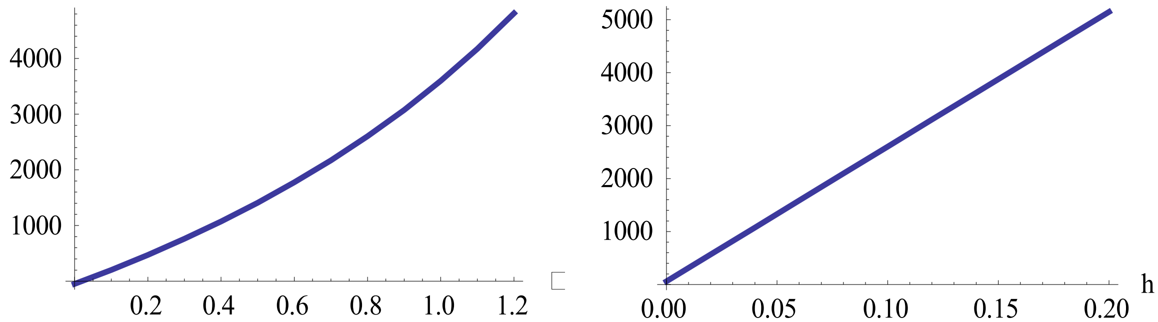


Note: The x-axis indicates different assumptions for the multiplier (μ) and hysteresis (h) in the left-hand and right-hand charts, respectively. The corresponding interest rate threshold for each assumption is on the y-axis. Other parameters are as assumed in the main text above.

First, it is important to note that the interest rate thresholds in Figure 1 and Figure 2 are much lower than those derived in a model à la DeLong and Summers (2012), which we show in Figure 3.[9] Instead of implementing a delayed consolidation that brings the debt-to-GDP ratio back to the baseline level, in DeLong and Summers' model the government only pays the growth-adjusted interest rate (i.e., the interest rate minus the growth rate) on the debt issued to finance the initial stimulus. This implies that after the stimulus the economy moves to a permanently higher debt-to-GDP ratio and thus does not experience the negative multiplier and hysteresis effects from the delayed consolidation. Neglecting these effects may be appropriate if the only cost of higher debt is the distortionary cost of taxation required to pay the growth-adjusted interest on the higher debt, as in DeLong and Summers' model. However, if there are other costs to permanently higher debt, it may be useful to instead compare alternative fiscal paths that yield a common ultimate debt-to-GDP ratio, as we have done in our analysis so far. As the comparison between Figure 2 and Figure 3 indicates, requiring such a common ultimate debt-to-GDP ratio significantly reduces the interest rate threshold from the very high levels found by DeLong and Summers. Nonetheless, the interest rate threshold can still be substantial, especially under moderately strong hysteresis and multiplier effects, as shown in Figure 2.

[9] Appendix A presents the version of the model by DeLong and Summers that we used to derive the interest rate thresholds in Figure 3.

Figure 3: Interest rate threshold in a model à la DeLong and Summers
(basis points above $1/\beta$)



Note: The *x*-axis indicates different assumptions for the multiplier (μ) and hysteresis (h) in the left-hand and right-hand charts, respectively. The corresponding interest rate threshold for each assumption is on the *y*-axis. Other parameters are as assumed in the main text above.

Second, it is important to note that even if interest rates are currently below the thresholds predicted by our model, delaying fiscal consolidation may still not be optimal. Indeed, the decision to postpone consolidation should consider not only whether the model predicts positive gains, but also the expected size of the gains. This is because, as discussed in the concluding section of our paper, delaying consolidation might expose the country to several risks not captured in our model. Therefore, delaying consolidation is worthwhile only if the expected gains are sufficiently large to compensate for the risks.

To examine this issue, we measure the gains from delaying consolidation in terms of the percentage increase in the PV of GDP predicted by the model from delaying consolidation relative to the baseline. We focus in particular on the percentage increase in the PV of GDP over the first three periods, $\alpha_{1,2,3}$, when fiscal policy directly affects GDP

$$\alpha_{1,2,3} = \frac{Y_1+\beta Y_2+\beta^2 Y_3}{y_1+\beta y_2+\beta^2 y_3} - 1 \tag{31}$$

and on the percentage increase at time 4, α_4

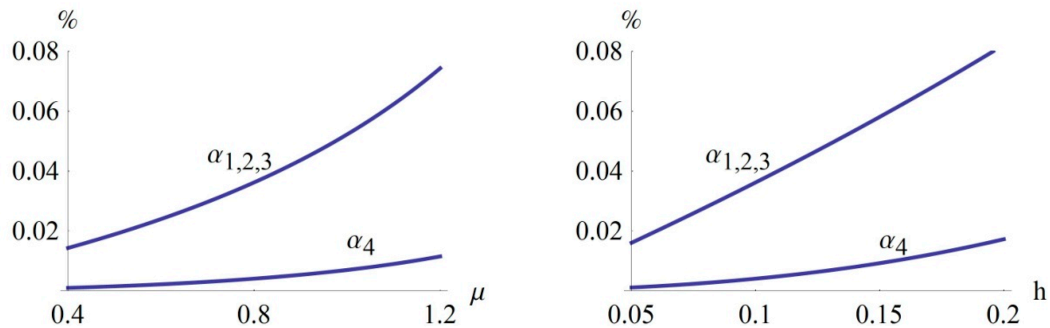
$$\alpha_4 = \frac{Y_4}{y_4} - 1 \tag{32}$$

Note that any difference in time-4 GDP between the two scenarios is due only to differences in potential output, as we assume that the output gap closes at time 4, ending any further cyclical effects. Consequently, the percentage difference in time-4 GDP is permanent for all future periods. With the vast majority of the PV of GDP occurring after time 3 (unless the discount rate is very high), α_4 will thus be very close to the percentage difference in the PV of GDP across all periods (i.e., from time 1 to infinity). Nonetheless, we show the effects on the PV of GDP in the first three periods separately, given the interest in these periods when fiscal policy is active and natural closing of the output gap may be slow.

Figure 4 shows the GDP gains generated by the model with no natural closing of the output gap (until time 4) from a fiscal stimulus equal to 1 percent of baseline GDP, y_1, as a function of the multiplier and hysteresis. We assume an initial negative output gap of 4 percent under the baseline and use the same parameter values as in Figure 1, setting $\mu = 0.8$, $\tau = 1/3$, $g = 2$ percent, $d_3/y_4 = 0.8$, and $\theta = r = 1$ percent, where $\theta = 1/\beta - 1$.[10]

Under such parameters and the continued assumption of constant multipliers, delaying consolidation produces only small increases in GDP, despite the assumption of a highly persistent output gap. Even with fiscal multipliers equal to 1.2 or hysteresis up to 20 percent, a 1 percent of GDP fiscal stimulus raises the PV of GDP in the first three periods by only about 0.08 percent. The impact on time-4 GDP is even smaller—less than 0.02 percent.

Figure 4: Increase in GDP from a fiscal stimulus of 1 percent of GDP



Summing up, the model finds that, if fiscal multipliers are constant, the potential gains in terms of the PV of GDP from delaying consolidation appear to be modest, even under substantial fiscal multipliers and hysteresis effects and with slow natural closing of the output gap. The simple intuition behind these results is that the positive effects from the initial fiscal stimulus are offset by the contractionary impact of the subsequent consolidation that is required to bring the debt-to-GDP ratio back down to its baseline level.

In the next section, we consider how the results differ if we allow multipliers to not be constant, but instead to vary over time.

IV. Time-Varying Multipliers: Simulations for the G7

The analysis so far suggests that delaying consolidation is likely to have small effects on the PV of GDP if fiscal multipliers are constant. The case for delaying consolidation would intuitively strengthen if the multiplier associated with the initial stimulus is higher than the multiplier during the subsequent consolidation. For example, the fiscal multiplier might be higher when the economy is in a severe recession or when the monetary authorities are unwilling or unable to offset fiscal policies.

[10] The gains in the left-hand chart are computed fixing the hysteresis parameter to 10 percent, while those in the right-hand chart are based on a fiscal multiplier equal to 0.8.

To quantitatively analyze the role of time variation in fiscal multipliers, we further extend the model in three ways to enhance its realism.

- First, we allow multipliers to depend on the state of the economy.

- Second, we calibrate the model using more realistic paths of delayed consolidation that span a number of years. This is because a crucial issue is the extent to which reasonable paths of delayed consolidation can truly exploit time variation in multipliers.

- Third, we allow a fraction of the output gap to close naturally each period, as such gradual and continuous closing is more realistic than the two extreme assumptions (full closing after one period and no closing until the end of the simulation) modeled in the previous section.

These enhancements to the model make it more realistic. However, they also preclude closed-form solutions. We thus analyze the model via numeric simulations.[11]

A. Simulation Assumptions

The simulations analyze the effects of delaying consolidation by comparing two fiscal scenarios for the next 25 years:

- *Baseline*. For the baseline, we use projections for the aggregate of G7 economies through 2017, as published in the October 2012 edition of the *World Economic Outlook* (IMF, 2012).[12] This baseline entails significant fiscal consolidation over the medium term, with the cyclically-adjusted primary balance (CAPB) improving from -4¼ percent of potential GDP in 2011 to –½ percent of potential GDP by 2015. After 2017, we assume that the CAPB converges to zero and remains there until the end of the simulation period (2027).

- *Alternative scenario of delayed consolidation*. An alternative scenario of delayed consolidation examines how the path would change if instead a stimulus had been implemented in 2012, such that the ratio of the CAPB to potential GDP had fallen by 1 percent of GDP (relative to its value in 2011) and stayed at this lower level in 2013. This alternative scenario provides a cumulative stimulus of 4¼ percent of potential

[11] The simulations in this section build on those initially presented in Fletcher and Sandri (2012) and share some similarities with those in Abbas and others (2013) and Bi, Qu, and Roaf (2013). A key difference with the latter is that we constrain our simulations to focus on comparing alternative fiscal routes to the same terminal debt-to-GDP and CAPB ratios. In contrast, Bi, Qu, and Roaf (2013) show results for alternative fiscal paths that differ in the terminal values of one or both of these ratios.

[12] We use the October 2012 projections because this is a point in time that is both relatively recent and a point at which advanced economies still planned significant fiscal consolidation going forward. Although the numbers do not represent actual outcomes for 2012 onward, this is immaterial, as the objective of the simulations is simply to investigate how the PV of GDP varies under alternative realistic fiscal paths and parameter assumptions.

> GDP over 2 years relative to the baseline scenario in which fiscal policy is tightened (i.e., the CAPB is increasing) during these two years. The alternative scenario then assumes gradual consolidation starting in 2014 in order to bring the debt-to-GDP and primary balance-to-GDP ratios back to their baseline levels by 2027.

The basic mechanics of the simulated alternative scenario follow the previous section's model. Specifically: (i) the output gap deviates from the baseline scenario in response to differences in the fiscal impulse, in line with multiplier assumptions; (ii) potential GDP deviates from the baseline scenario in response to differences in the output gap, in line with hysteresis assumptions; and (iii) fiscal balances deviate from the baseline scenario in response to differences in the fiscal impulse and GDP growth (automatic stabilizers).

The parameter values underlying the simulations are presented in Table 1:

- Potential output is assumed to grow as projected by the WEO for the years 2012-2017. As WEO projections in the October 2012 edition are not available past 2017, for subsequent years we assume that potential output grows at 2 percent per year before hysteresis effects.

- Hysteresis effects are assumed to equal 10 percent of the output gap each period.

- The relationship between output and the primary balance (PB) is refined to make it more realistic. In the previous sections, the model followed DeLong and Summers (2012) in assuming that higher GDP raises government revenue and the PB by the coefficient τ. To enhance realism, the simulations in this section now differentiate between (i) the coefficient τ_1, which relates the PB to increases in potential output above the baseline level and which we keep equal to 1/3 (roughly the average tax rate), and (ii) the coefficient τ_2, which relates the PB to the output gap and is set to 0.7. The higher value for the latter coefficient captures the pro-cyclicality of revenue as a percent of GDP and the counter-cyclicality of some expenditure items.

- 40 percent of the output gap is assumed to close naturally from one year to the next.[13]

- The real interest rate on sovereign debt is set to the rates implicit in the 2012 WEO projections until 2017 and to 1 percent from 2018 onwards.

- The intertemporal discount rate is set to 2 percent.

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[13] This implies that, for example, if potential growth is 2 percent and the output gap is -2 percent (a sizeable gap by historical standards), then actual growth will be 2.8 percent in the absence of exogenous shocks. To check the plausibility of this pace of natural closing, we model the OECD output gap estimates for the US and UK for 1980-2011 as an AR(1) and ARMA(1,1) process, with the coefficient on the AR term interpreted as the "natural closing" and the MA term interpreted as capturing persistent exogenous shocks to the output gap. We find AR coefficients in the range of 0.3 to 0.7. Given that the output gap may close more slowly than normal under the present constrained conditions for monetary policy, we use a rate of output gap persistence (0.6) at the higher end of these estimates. The sensitivity of the results to this assumption is presented later in the paper.

Table 1: Calibration of Simulation Parameters

Growth of potential GDP before hysteresis effects	g	2012-2017: WEO assumptions 2018-2027: 2 percent
Hysteresis effect	h	0.10
Derivative of the PB to potential output	τ_1	1/3
Derivative of the PB to the output gap	τ_2	0.7
Persistence of the output gap	ρ	0.6
Real interest rate	r	2012-2017: 1.1 percent on average 2018-2027: 1 percent
Intertemporal discount factor	β	1/1.02

The simulations examine three alternative assumptions regarding the relationship between the fiscal multiplier and the state of the economy. Figure 5 illustrates these alternatives:

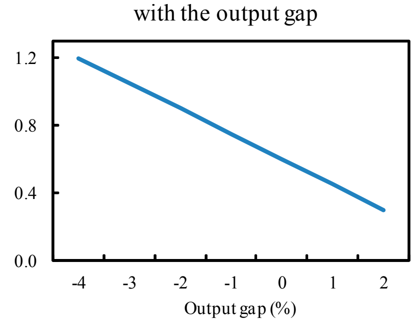
- *Alternative 1*: the multiplier is constant at 0.8.
- *Alternative 2*: the multiplier varies inversely and linearly with the output gap.
- *Alternative 3*: the multiplier varies inversely and linearly with real GDP growth, as suggested by Auerbach and Gorodnichenko (2012).

The calibration of the multiplier under Alternatives 2 and 3 is such that the average multiplier during the phase of delayed consolidation is equal to 0.8, as under Alternative 1. In this way, the three alternatives isolate the effect of changing the relationship of the multiplier to the business cycle rather than changing the average size of the multiplier.

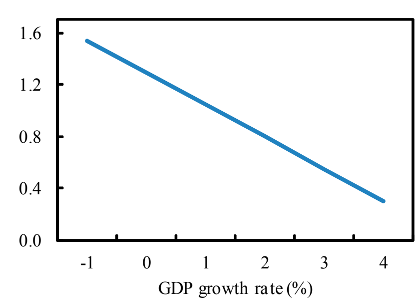
Figure 5: Alternative Assumptions about Fiscal Multipliers

Alternative 1: the multiplier is constant at 0.8

Alternative 2: the multiplier varies with the output gap



Alternative 3: the multiplier varies with the GDP real growth rate



B. Simulation Results

Figure 6 shows the evolution of the CAPB and government debt under the baseline and alternative scenarios. By construction, the alternative scenarios of delayed consolidation entail first a substantial loosening of fiscal policy relative to the baseline and then a longer phase of consolidation to return the debt-to-GDP ratio to the baseline level by 2027. The slightly different dynamics of the CAPB across Alternatives 1, 2, and 3 reflect the different assumptions regarding the effects of the business cycle on the fiscal multiplier, which in turn affect how much consolidation is required to return debt to the baseline path.

Figure 6: Alternative Fiscal Paths

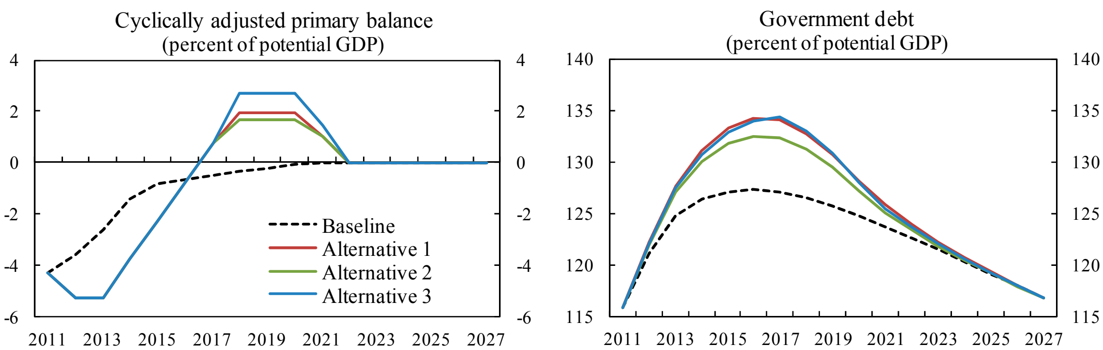


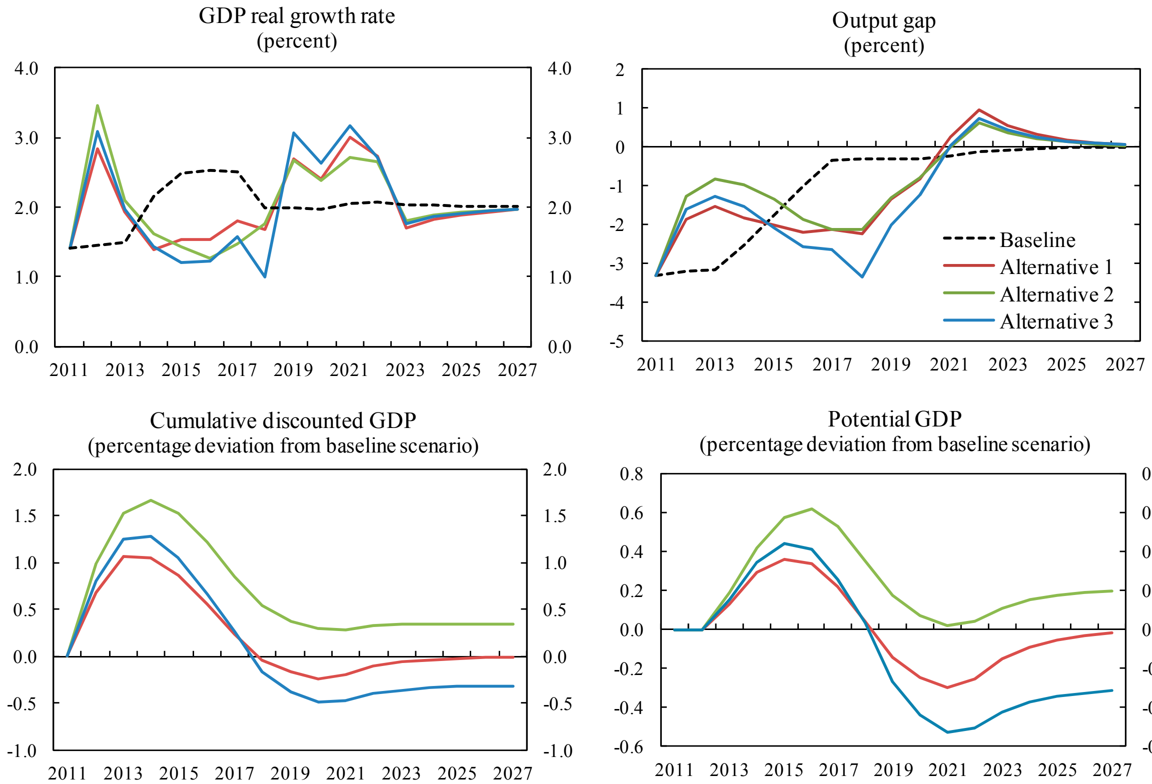
Figure 7 shows the effects of delayed consolidation on GDP and the output gap. As with the model in the previous sections, delayed consolidation shapes GDP dynamics over three main phases:

- First, the initial stimulus considerably increases GDP growth and allows for a faster reduction in the output gap than under the baseline. This is especially the case under Alternatives 2 and 3, as fiscal multipliers are large given the initial wide negative output gap and modest GDP growth rates.

- Second, a subsequent phase of deeper and more prolonged fiscal consolidation (relative to the baseline) is necessary to offset the initial stimulus. This results in an extended period of weak growth and renewed widening of the output gap.

- Finally, a third phase begins with the reduction of the fiscal surplus once the debt-to-GDP ratio approaches the baseline path. This generates a stimulative impulse that raises growth and rapidly closes the output gap. Once the CAPB levels off at the baseline level, fiscal impulses end, and the output gap converges to zero at the $1-\rho$ rate.

Figure 7: Implications of Alternative Fiscal Paths for Actual and Potential GDP



Effects on the PV of GDP

What are the overall effects of these alternative fiscal paths on the PV of GDP through 2027? The bottom-left chart of Figure 7 shows that delayed consolidation has virtually no effect on the PV of GDP through 2027 if fiscal multipliers are constant (Alternative 1), as the output losses from the consolidation phase offset the gains from the stimulus phase. Similarly, delayed consolidation has virtually no effect on potential GDP at the end of the simulations (bottom-right chart). The more elaborate simulations in this section thus confirm a key result from the previous sections: even under reasonably large multipliers and hysteresis effects, delaying consolidation does not yield meaningful gains in cumulative output if multipliers are constant over time.

In contrast, delaying consolidation does yield cumulative output gains in the scenario in which fiscal multipliers vary with the output gap (Alternative 2). Specifically, the PV of GDP is 0.3 percent higher than in the baseline. Delaying consolidation also permanently increases potential output by 0.2 percent. Therefore, if multipliers depend on the output gap, delaying consolidation in countries that face a large negative output gap has the potential to generate non-negligible GDP gains under the assumptions in this scenario.

However, the simulation results also reveal that the effects of delaying consolidation can be quite different if multipliers vary with the growth rate of GDP (Alternative 3). In this case, both the PV of GDP and potential GDP in 2027 are lower under delayed consolidation than in the baseline.

The intuition behind this result is the following: the initial stimulus makes the subsequent consolidation, which is required to return debt to baseline levels, larger and longer than in the baseline. During this consolidation phase, the fiscal tightening reduces growth below that in the baseline. Because multipliers vary negatively with growth in Alternative 3, this lower growth raises the fiscal multiplier, which further depresses growth. The result is a downward spiral of lower growth and higher multipliers. This adverse dynamic has a negative effect on GDP during the consolidation phase that more than offsets the benefits during the initial stimulus.

More generally, if the multiplier varies with growth, fiscal stimulus will exhibit diminishing marginal benefits (since the multiplier shrinks as stimulus boosts growth) while consolidation will exhibit increasing marginal costs (since the multiplier increases as consolidation dampens growth).

Such decreasing marginal benefits to stimulus and increasing marginal costs of consolidation also occur if the multiplier depends on the output gap (Alternative 2). However, these "second-derivative effects" are less dramatic in this case because the output gap is a more sluggish variable than the growth rate.

For example, during the consolidation phase of the alternative scenario, growth immediately falls below the baseline rate as soon as consolidation starts. In contrast, it takes several periods for this below-baseline growth to make the output gap, which starts the consolidation narrower than the baseline level due to the positive effects of stimulus, more negative than in the baseline. Hence, in Alternative 2 the "second-derivative effects" are not strong enough to offset the benefits from starting the initial stimulus at a time when the output gap is negative and multipliers are high. In contrast, the "second-derivative effects" kick-in more rapidly when multipliers vary with growth, and hence more than offset the benefits of starting the initial stimulus when growth is weak.

We should emphasize that our finding that the PV of GDP worsens under delayed consolidation if multipliers vary with growth is a result specific to the scenario investigated in this paper. If instead the starting position were to be not a period of modestly below-average growth, but instead a period of deep recession, then the benefits of the initial stimulus may indeed be large enough to offset the "second-derivative" effects.[14]

Whether multipliers actually depend on the output gap, growth, or other economic conditions and, if so, the exact nature of these relationships remain open questions. Auberbach and Gorodnichenko (2012) provide some empirical evidence in favor of fiscal multipliers varying more closely with growth than the output gap. More specifically, they find that "the response of output [to fiscal policies] seems to be larger when the economy starts to contract than when it reaches a bottom." However, these results appear far from conclusive.

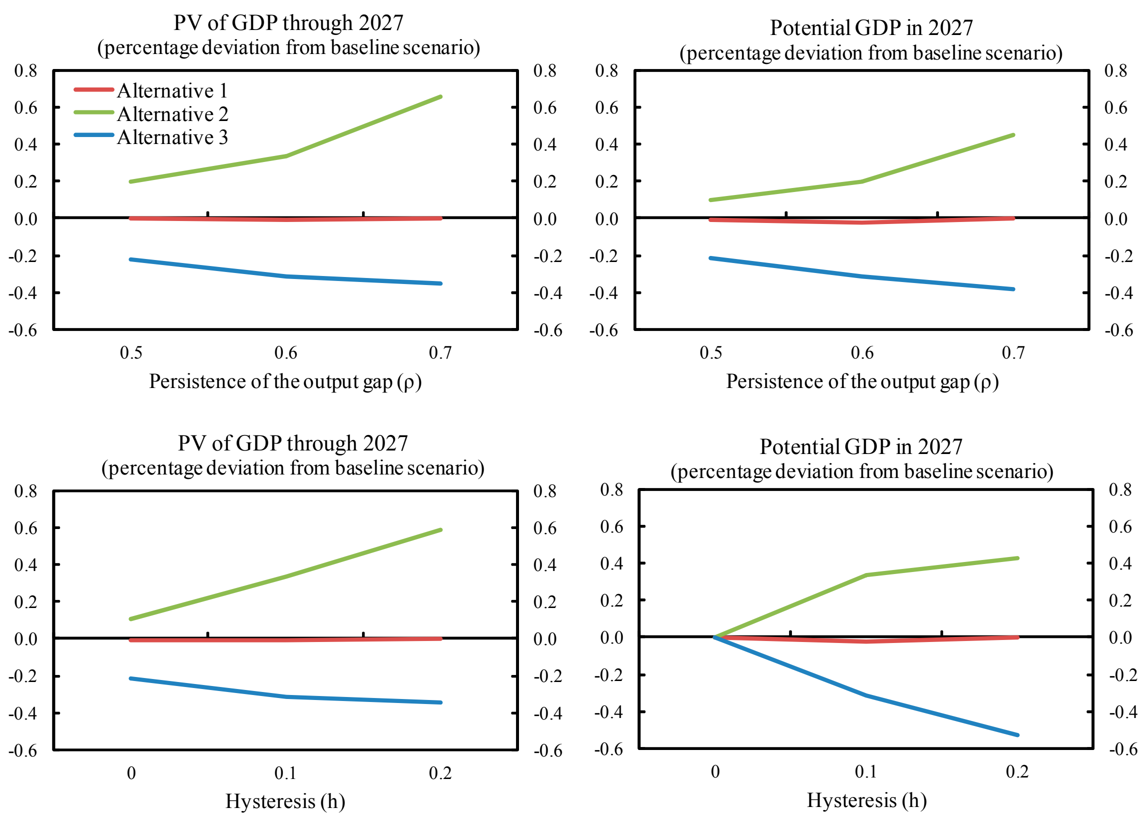
[14] Time-variations in fiscal multipliers raises interesting issues about how to design an optimal path of fiscal consolidation. We leave this topic for future research, but our analysis suggests that a continuous, but gradual, path of consolidation may have advantages if "second-derivative effects" are particularly strong.

Alternative assumptions for output gap persistence and hysteresis effects

Figure 8 shows how the simulation results vary in response to alternative assumptions regarding the persistence of the output gap and the strength of hysteresis. The top two plots show the sensitivity of the PV of GDP through 2027 and of potential GDP in 2027 to variations in the persistence of the output gap.

We first observe that if fiscal multipliers are constant (Alternative 1), delaying fiscal consolidation has negligible effects irrespective of the speed at which output gaps close naturally. The persistence of the output gap plays instead an important role if multipliers vary over the business cycle. In particular, higher persistence increases both the gains when multipliers vary with the output gap (Alternative 2) and the losses when multipliers depend on the growth rate (Alternative 3). Varying the hysteresis parameter produces similar effects, as shown in the bottom two plots of Figure 8: stronger hysteresis effects magnify gains and losses from asymmetric multipliers, while they have little bearing on the case of constant multipliers.

Figure 8: Sensitivity of the Simulation Results to Output Gap Persistence and Hysteresis



Note also that we have modeled hysteresis effects as a linear function of the output gap. If instead hysteresis effects are time-varying—with large adverse effects when the output gap is negative, but no positive effects when the output gap is positive—this could magnify gains from delaying consolidation to periods of positive output gaps. However, as the simulations above demonstrate, delaying consolidation until a period of positive output gaps while still achieving debt sustainability within a reasonable period of time may not be straightforward, especially under initial conditions of a large negative output gap and large structural fiscal deficits. It is also unclear why the adverse effects of downturns on potential GDP (e.g., as workers lose skills while being unemployed) should not also work in reverse during booms. Indeed, Clark and Summers (1982) find evidence of positive hysteresis during World War II, as more employment of women led to sustained increases in their labor force participation rates. Similarly, Kannan (2012) finds little evidence of nonlinear hysteresis effects.

Declining marginal utility

Another extension to our analysis is to consider how declining marginal utility of consumption might affect the results. Declining marginal utility might add to the benefits of delaying consolidation given that output and consumption are lower at the start of the simulation (in part due to the wide output gap) and hence the marginal utility of boosting output and consumption may be higher than during the subsequent consolidation phase.

To examine this effect, we calculate the PV of utility rather than the PV of GDP in each scenario, using a standard constant relative risk aversion utility function with a coefficient of two and assuming consumption is a constant share of output. We find that the results do not change materially, with the PV of utility differing from the PV of GDP by less than a tenth of one percent in any given scenario. This is perhaps not surprising given that output gaps are typically only a few percentage points of output and hence do not have large effects on the marginal utility of consumption in most standard utility functions.

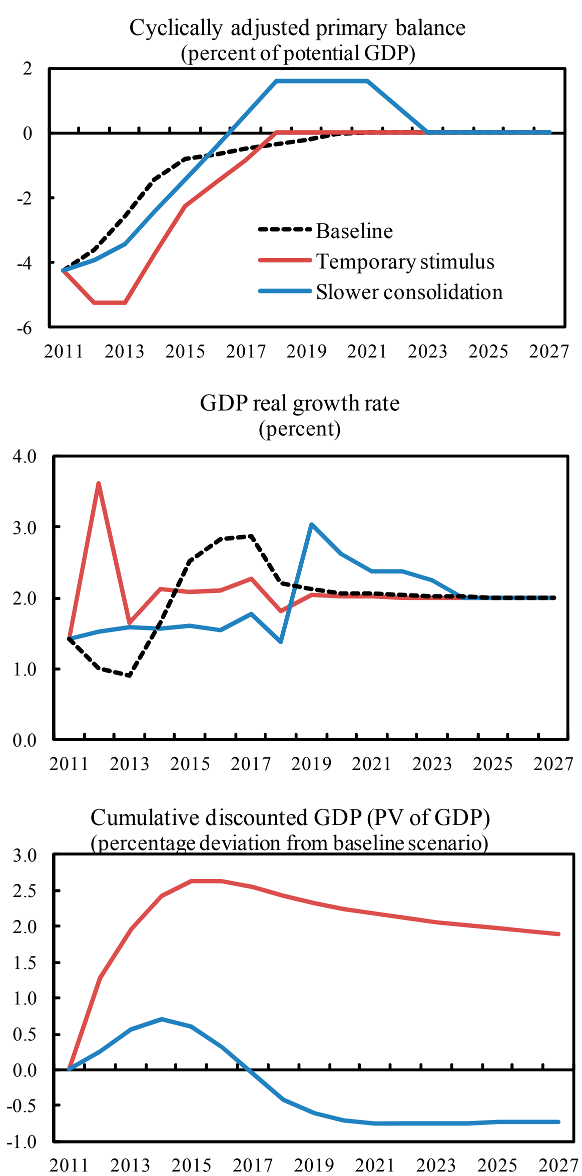
Liquidity traps and discontinuous multipliers

Some economists have argued that many advanced economies may be caught in a liquidity trap, a situation in which the zero lower bound on nominal interest rates renders monetary policy unable to provide further stimulus. Such a situation may introduce discontinuities into fiscal multipliers, with multipliers being small above the zero lower bound (since monetary policy can reduce interest rates to offset fiscal tightening) and multipliers becoming large once the zero lower bound is hit (since monetary policy may have a more difficult time offsetting fiscal tightening at the zero lower bound).

Fully modeling the effects of a liquidity trap would require a more detailed macroeconomic model that captures the determinants of interest-rate dynamics and hence is beyond the scope of this paper. However, we can examine the more narrow issue of how discontinuities in multipliers might affect our results. Specifically, we examine how the results change if we assume that the fiscal multiplier is normally a low 0.2, but rises to a high 1.5 when the economy enters a deep slump, which we define as an output gap more negative than -2 percent. All other parameters are as assumed in the baseline simulations above.

Under these assumptions, a temporary fiscal stimulus of 1 percent of GDP, as simulated in the previous stimulus scenarios, is able to push the economy out of its initial slump and substantially narrows the output gap (Figure 9). As a result, the multiplier shrinks to 0.2, following the assumptions outlined above. The subsequent consolidation phase to return the CAPB and debt ratio to their baseline levels occurs at these lower multipliers. Consequently, the drag on growth from consolidation does not fully reverse the boost to growth from the initial stimulus. Not surprisingly, the effects on the PV of GDP are large in this scenario, with the stimulus boosting the PV of GDP by about 2 percent of GDP.

Figure 9: Effects of Different Fiscal Paths under Discontinuous Multipliers



A second scenario ("Slower consolidation") examines what happens under these parameter assumptions with a pace of initial consolidation that is slower than the baseline, but faster than under the previous "Temporary stimulus" scenario. In this case, the smaller stimulus is insufficient to push the economy out of the deep slump. Consequently, the multiplier remains high during the subsequent consolidation phase. The consolidation phase thus exacts a significant toll on growth, and the economy stays in the wide output gap/high multiplier state for even longer than in the baseline. As a result, the PV of GDP is significantly lower in this scenario than in the baseline.

Interestingly, these simulations show that, under certain assumptions, a slow-paced consolidation could exact higher costs than either a rapid consolidation or an initial stimulus followed by consolidation. Undoubtedly, this outcome is quite specific to the assumptions and scenarios investigated. However, this result further underscores that the effect of delaying consolidation on the PV of GDP depends crucially on the exact manner in which multipliers vary over time and the results may be counterintuitive in some cases.
We note two important caveats to these scenarios:

- First, there is much debate about whether the zero lower bound is truly binding: if unconventional monetary policies can keep monetary policy effective, then the assumptions underlying the scenarios in this section may not hold.

- Second, we have examined the case in which output gap persistence and multipliers vary discontinuously with the output gap. However, a liquidity trap is more strictly a case in which these parameters vary discontinuously with the full-employment interest rate. As noted above, a more detailed macroeconomic model would be needed to capture such interest-rate dynamics and could produce different results. In particular, if fiscal stimulus is the only force pushing the full-employment interest rate upward and thus the only force pulling the economy out of a liquidity trap, then the reversal of such stimulus to return the structural fiscal balance to its initial level may return the full-employment interest rate back below the zero lower bound, such that the economy re-enters the liquidity trap. In this case, fiscal policy may not be able to produce a fiscally sustainable exit from the liquidity trap.

For these reasons, the results in this section should be viewed mainly as an illustration of the sensitivity of the results to the specific assumptions regarding the time-varying nature of the multiplier.

Summary of simulation results

In sum, the simulation results reiterate the absence of substantive gains from delaying consolidation if multipliers are constant, even in the presence of hysteresis effects. If multipliers instead vary over the business cycle and are higher during the stimulus phase than during the subsequent consolidation phase, delaying consolidation can potentially lead to meaningful gains. However, the sign and magnitude of these gains hinge crucially on the nature of the variability of the multipliers and on the specific macroeconomic and fiscal conditions.

V. Other Considerations

Risks from delaying consolidation

It is important to highlight that the analysis has so far focused on identifying possible output gains from delaying consolidation. To keep the analysis tractable, it has neglected several risks associated with delaying consolidation that are difficult to quantify but nonetheless relevant for real-world fiscal policy decisions. In assessing the merit of delaying consolidation, any output gains indicated by the model would thus have to be weighed against these risks that the model neglects.

For example, one such risk is that a temporarily higher debt-to-GDP ratio under delayed consolidation may lead to crowding-out effects by absorbing financial resources that would otherwise have been used to support private investment. The extent of such crowding out is likely to be limited in the presence of a negative output gap, but may rise rapidly as the economy recovers. That said, possible costs from this effect may be small in our scenario of delayed consolidation in which the debt ratio is only temporarily elevated above baseline levels.

A second risk is that delaying consolidation may lead to higher sovereign risk premia and external borrowing costs (e.g., if such a decision is misinterpreted as a signal of weak fiscal discipline, thereby raising expectations of future fiscal difficulties), with adverse effects both on public debt dynamics and the real economy. Based purely on fundamentals, a relatively moderate stimulus like the one in the above simulations should not elicit a strong response in risk premia, since the increase in the debt-to-GDP ratio is fairly small (roughly 5 percent of GDP) and temporary. Indeed, in the presence of large, asymmetric multipliers and hysteresis effects, delaying consolidation could even reduce risk premia by increasing the PV of GDP. Nonetheless, such risks would need to be considered in any real-world policymaking, especially in countries already under market pressure.

A third risk is one of political economy. Though fiscal consolidation is sometimes popular in the abstract, it is usually unpopular once actual spending cuts and tax increases are identified and implemented. In addition, fiscal consolidation may paradoxically become even more difficult once the economy is experiencing healthy growth and the cyclical improvement in the fiscal balance masks the real extent of the structural fiscal problem. Such political biases against fiscal rectitude may argue for undertaking more consolidation while there is political support for it (e.g., early in a consolidation before reform fatigue sets in (Blanchard, 2012)), even if a more delayed path of consolidation could in theory yield a better economic outcome. This is because political economy considerations imply a risk that a delayed consolidation may never actually occur, resulting in higher long-run deficit and debt levels.

A fourth source of concern regards the possibility of future negative shocks. The potential gains from delaying consolidation identified by the model are derived under a perfect foresight setting that involves a steady recovery. The delayed phase of consolidation is thus implemented during a period in which the baseline entails higher growth rates and a smaller output gap than in the initial periods when stimulus is adopted. But in reality the baseline economic path is very uncertain, and we cannot rule out the possibility that the economy may experience future negative shocks that could push it back into recession. If we consider that many economies are currently facing elevated debt ratios and assume that they have fiscal space to postpone consolidation only once, then delaying consolidation today implies losing the option value of using fiscal stimulus against possible future shocks.[15] Related to this issue is the possibility that negative shocks may hit the economy during the phase of delayed consolidation. If the country cannot delay consolidation any further, the increase in multipliers due to the shock would considerably increase the contractionary effects of the fiscal consolidation.

Other factors

More generally, a decision on the optimal fiscal stance in any specific situation will need to take into account a range of considerations beyond simply the effects on the PV of GDP via the aggregate demand effects that are the focus of this paper. Such considerations could include, for example, effects on external balance or direct positive effects that certain public investments might have on potential GDP, especially if these effects are large relative to the costs of financing such investment (IMF, 2014). For these and other reasons, the analysis in this paper should not be construed as supporting any specific fiscal path in any specific country.

That said, the results in this paper indicate that the widespread focus on the absolute size of the fiscal multiplier as a key factor affecting the desirability of fiscal stimulus may be misplaced—the way in which the multiplier varies across the cycle may instead be more crucial. We hope that our analysis will encourage further empirical investigation in that regard.

[15] Note that this argument that does not necessarily rely on the country having fiscal room to postpone consolidation only once. The simple presence of a constraint on the debt-to-GDP ratio, no matter how loose, implies that the country moves closer to the constraint each time it delays consolidation and thus has less fiscal space to further ease fiscal policies to offset possible negative shocks.

APPENDIX A

In this appendix we briefly summarize the model by DeLong and Summers (2012) that we used to trace the interest rate thresholds in Figure 3.

In this model, the time-1 fiscal stimulus S generates a contemporaneous increase in GDP by $S\mu$, where μ is the fiscal multipliers. Furthermore, the stimulus permanently increases potential GDP from time 2 onwards by $S\mu h$, with the parameter h capturing the strength of the hysteresis effect.

Regarding the implications for the stock of debt and fiscal balances, the stimulus increases public debt by $S(1-\tau\mu)$, where τ is the elasticity of the fiscal balance to GDP. Instead of requiring a subsequent phase of consolidation to bring the debt-to-GDP ratio back to the level it would have been at without the stimulus, DeLong and Summers only require the government to pay the growth-adjusted interest rates on the new debt. Therefore, from time 2 onwards the government has to collect additional taxes by $S(1-\tau\mu)(r-g)$, where r and g are respectively the real interest rate on government debt and the GDP real growth rate.

By increasing potential GDP, the fiscal stimulus increases future fiscal revenues by $\tau S\mu h$, so that the government has to increase taxes only to cover the difference. This generates a contractionary effect on GDP equal to $\xi S((1-\tau\mu)(r-g)-\tau\mu h)$, where ξ is the distortionary impact of taxation, which we set to 0.5, as in DeLong and Summers.

The partial derivative of the PV of GDP with respect to the fiscal stimulus is thus given by:

$$\frac{\partial PV}{\partial S} = \mu + \frac{\mu\, h}{1/\beta - G} - \xi\frac{(1-\tau\mu)(r-g)-\tau\mu h}{1/\beta - G}$$

Setting this derivative to zero, we can solve for the interest rate threshold shown in Figure 3:

$$\mathrm{r} = g + \mu\frac{1/\beta - G + h(1-\xi\tau)}{\xi(1-\tau\mu)}$$

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