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Sovereign Defaults, External Debt, and Real Exchange
Rate Dynamics

by Tamon Asonuma

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I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

Research Department

Sovereign Defaults, External Debt, and Real Exchange Rate Dynamics

Prepared by Tamon Asonuma¹

Authorized for distribution by Atish Rex Ghosh

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Abstract

Emerging countries experience real exchange rate depreciations around defaults. In this paper, we examine this observed pattern empirically and through the lens of a dynamic stochastic general equilibrium model. The theoretical model explicitly incorporates bond issuances in local and foreign currencies, and endogenous determination of real exchange rate and default risk. Our quantitative analysis replicates the link between real exchange rate depreciation and default probability around defaults and moments of the real exchange rate that match the data. Prior to default, interactions of real exchange rate depreciation, originated from a sequence of low tradable goods shocks with the sovereign's large share of foreign currency debt, trigger defaults. In post-default periods, the resulting output costs and loss of market access due to default lead to further real exchange rate depreciation.

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I. INTRODUCTION

Emerging market economies experience real exchange rate depreciations around default events. We first empirically examine this stylized fact. The theoretical part of the paper explores interactions between the real exchange rate and default decisions in a stochastic general equilibrium model featuring defaultable debt. Our quantitative theoretical analysis, using Argentina’s default episode in 2001 for calibration purposes, goes some way to explain the observed links between real exchange rate depreciation and default probability.

In the empirical section, we present new stylized facts on real exchange rate dynamics around sovereign defaults. For 18 sovereign debt default and restructuring episodes over 1998–2013, we find an empirical link between real exchange rate depreciation and default risk (default decision): In the period prior to default, the real exchange rate depreciation increases the burden of debt service payments and ultimately triggers default. In the post-default period, the country’s announcement of default, or restructuring, leads to further real exchange rate depreciation due to loss in market access associated with default or restructuring. Our panel regression results, using these episodes, also confirm the observed link. Motivated by this stylized fact, we aim to explain the pattern of currency depreciations often observed in the lead-up to and aftermath of sovereign defaults.

The theoretical part of this paper attempts to explore interactions between the real exchange rate and default decision in a standard dynamic model of defaultable debt, where a sovereign debtor borrows from a “representative” foreign creditor through issuances of one-year bonds in local and foreign currencies. Both the country and the foreign creditor are risk-averse and subject to tradable and nontradable goods volume shocks. As in international real business cycle literature where neither nominal rigidity nor money is included for instance, Gourio and others (2013), the real exchange rate is defined as units of home consumer price indices (CPI) against one unit of foreign CPI where the price of tradable goods is the numeraire.² It interacts with both the lending choices of the creditor and the prices of the two debt instruments issued by the sovereign—incorporating default risk, which increases with level of debt.

Prior to default, the debtor sovereign, receiving a sequence of low income shocks in the tradable sector, accumulates more debt and becomes more exposed to real exchange rate depreciation due to increased default risk. Since a majority of debt is denominated in foreign currency (as observed in data), the real exchange rate depreciation increases the burden of payments in terms of local currency, increasing default probability and ultimately triggering the default decision by the sovereign. Once the sovereign declares default, it suffers output

²Na and others (2014) and Moussa (2013) incorporate the nominal wage rigidities in a conventional sovereign debt model to explain the interaction between defaults and devaluations.

costs associated with default and loses access to markets.³ By achieving financial autarky, the sovereign opts to have higher consumption of traded goods, indicating lower marginal utility of consumption, which leads to higher prices of nontraded goods and a higher overall price level relative to that of the foreign creditor. This drives an equilibrium depreciation of the real exchange rate, and is one plausible explanation of the observed patterns in the data.

The model is calibrated to the case of Argentina's default in 2001. Our quantitative exercise successfully replicates both business cycle and non-business cycle moments that match with the data. Most importantly, our model generates real exchange rate moments consistent with what we observe in the Argentine data, particularly a higher average real exchange rate in the post-default period than in the pre-default period and correlations with bond spreads and output. The model helps explain the observed real exchange rate dynamics around defaults.⁴

We embed the real exchange rate dynamics and currency denomination of debt in a dynamic, sovereign debt model with endogenous default. This part of the model builds on the recent quantitative analysis of sovereign debt such as Aguiar and Gopinath (2006), Arellano (2008), and Tomz and Wright (2007), all of which are based on the classical setup of Eaton and Gersovitz (1981). To account for the creditor's willingness to avoid real exchange rate risks and demanding an excess risk premium as observed in the real world, we depart from the conventional risk-neutral investor assumption. Instead, we assume a "representative" risk-averse creditor who faces exogenous income shocks, as in Borri and Verdelhan (2009) and Lizarazo (2013). Our model also amends the traditional debt issuance in domestic currency and considers that a sovereign issues external bonds in both local and foreign currencies, as observed in the data.

The rest of the paper is structured as follows. Following the literature review, Section II overviews stylized facts and empirical analysis on real exchange rate dynamics and sovereign defaults. We provide our dynamic stochastic general equilibrium model in Section III. Recursive equilibrium of the model is defined in Section IV. Quantitative analysis of the model is shown in Section V. A short conclusion summarizes the discussion. A computation algorithm is presented in Appendix I.

³The current paper neither deals with bailouts nor distinguishes between restructurings that are well-designed/implemented vs. those that are not. Thus, its conclusions should not be seen as an implicit critique of all restructurings given that in many cases, restructurings may be the least worst course of action, as noted in IMF (2014).

⁴Our analysis is not specific to Argentina and can be applied to any emerging countries experiencing sovereign defaults and exchange rate depreciations. We have obtained similar quantitative results and implications from exercises on other countries.

A. Literature review

Our paper builds on some strands of existing literature. First, this paper is related to the literature of sovereign debt and defaults, which extends a classical model of Eaton and Gersovitz (1981) and applies quantitative analysis. Arellano (2008) and Aguiar and Gopinath (2006) explore the connection between endogenous default, interest rates and income fluctuations in a model of sovereign debt and generate empirical regularities in emerging markets. Similarly, Gumus (2013) analyzes currency denomination of debt on default risk and interest rates in small open economy model with sovereign debt. Arellano and Heathcote (2010) explore what determines credit limits and how these vary across exchange rate regimes in a sovereign debt model.^{5,6} Introducing nominal wage rigidities and endogenous labor dynamics, Na and others (2014) and Moussa (2013) analyze sovereign decision of defaults and devaluation. This paper differs in that we mainly focus on interactions between default choices and real exchange rate dynamics.

The second grouping of literature deals with sovereign debt and risk-averse investors. Borri and Verdelhan (2009), Lizarazo (2013), and Presno and Puzo (2011) study the case of risk-averse lenders and show that risk aversion allows the model to generate spreads larger than default probabilities, as observed in emerging markets. Borri and Verdelhan (2009) consider risk aversion with external habit preference, whereas Lizarazo (2013) assumes decreasing absolute risk aversion (DARA). On the contrary, Presno and Puzo (2011) introduce fears about model misspecification for the lenders. Moreover, Gu (2015) replicates a further endogenous decline in output through a terms-of-trade channel. What distinguishes this current paper with these studies is that we incorporate real exchange rate determination together with bond prices.

Lastly, this paper also contributes to the literature on currency composition of external debt. Jeanne (2003) claims that unpredictable monetary policy increasing the uncertainty in the future real value of domestic currency debt may induce sovereigns to dollarize their liabilities. Bussiere, Fratzcher and Koeniger (2004) link the exchange rate uncertainty in foreign currency debt to solvency of debt and the choice of debt maturity, and Chamon and Hausmann (2004) explore the interplay between an individual borrower's choices for liability denomination through the effect on optimal monetary response of the central bank. On the contrary, Eichengreen, Hausmann and Panizza (2004) consider that an inability to borrow

⁵Jahjah and others (2013) empirically analyze how exchange rate policy affects the supply and pricing of sovereign bonds in developing countries.

⁶The theoretical work sovereign debt models restructurings between a sovereign debtor and its creditors to explain stylized evidence on debt restructurings, for instance, Bulow and Rogoff (1989), Benjamin and Wright (2009), Kovrijnykh and Szentes (2007), Bi (2008), Bai and Zhang (2010), D'Erasmus (2010), Yue (2010), Pitchford and Wright (2012), Hatchondo and others (2014), Asonuma (2012), and Asonuma and Trebesch (2016).

abroad in domestic currency (“original sin”) is owing to structure and operation of the international financial system together with weakness of policies and institutions.^{7 8 9} This paper complements existing studies by explaining how behavior of foreign creditors, avoiding the real exchange rate risk, leads to lending in foreign currency rather than local currency.¹⁰

II. STYLIZED FACTS

In this section, we provide an observed evidence and empirical analysis on real exchange rate dynamics and sovereign defaults. From recent sovereign default and restructuring episodes, we demonstrate an empirical link between real exchange rate depreciation and a sovereign’s default choices. Our results on cross-sectional analysis also support the observed link.

A. Real Exchange Rate Dynamics around Defaults/Restructurings

Figure 1 displays fluctuations of real exchange rates against the US dollar in quarterly frequency before and after defaults and announcements of restructurings from 18 episodes in 1998–2013.^{11 12} Following definitions of preemptive and post-default restructurings in Asonuma and Trebesch (2016), we set t at the time of defaults for post-default restructuring cases and at the time of announcements of restructurings for preemptive episodes.^{13 14} Real

⁷Burger and Warnock (2006) stress that by improving policy performance and strengthening institutions, emerging economies may develop the local currency bond market, reduce their currency mismatch and lessen the likelihood of future crises.

⁸Aghion, Bacchetta, and Banerjee (2004) propose the following: borrowers can use unsecured debt in domestic currency as collateral to obtain a loan in foreign currency. This reduces the interest rate on foreign currency debt since in the case of a crisis, the loss is partially transferred to lenders in domestic currency.

⁹In addition, Corsetti and Mackowiak (2004) show how monetary and fiscal policies, including maturity and currency denomination of debt, interact to determine the dynamic response of the economy and magnitude of devaluation and inflation.

¹⁰We relate our paper to the literature on portfolio allocation between an emerging market economy and an advanced economy as in Devereux and Sutherland (2009) and Tille and Van Wincoop (2010), which examine determinants of optimal risk-sharing allocations. Coeurdacier and Gourinchas (2013) consider international portfolio with real exchange rate and nonfinancial risks that account for observed levels of equity home bias.

¹¹We exclude episodes of default/debt restructurings of external debt held by official creditors because of the absence of precise data on defaults and announcements of restructurings. Moreover, for default/restructuring of external debt held by private creditors, cases of Antigua and Barbuda, Serbia and Montenegro, and Iraq are not included due to a lack of quarterly data on both nominal exchange rates and CPI. The case of Greece is not included in our sample due to the absence of nominal exchange rate against the euro associated with Euro zone membership.

¹²Real exchange rate against the U.S. is more likely to be a representative of real effective exchange rate (REER) at the time of debt distress.

¹³Asonuma and Trebesch (2016) introduce a new typology of two types of sovereign debt restructurings: those implemented prior to a unilateral payment default, which they term “preemptive”, and those where the

(continued...)

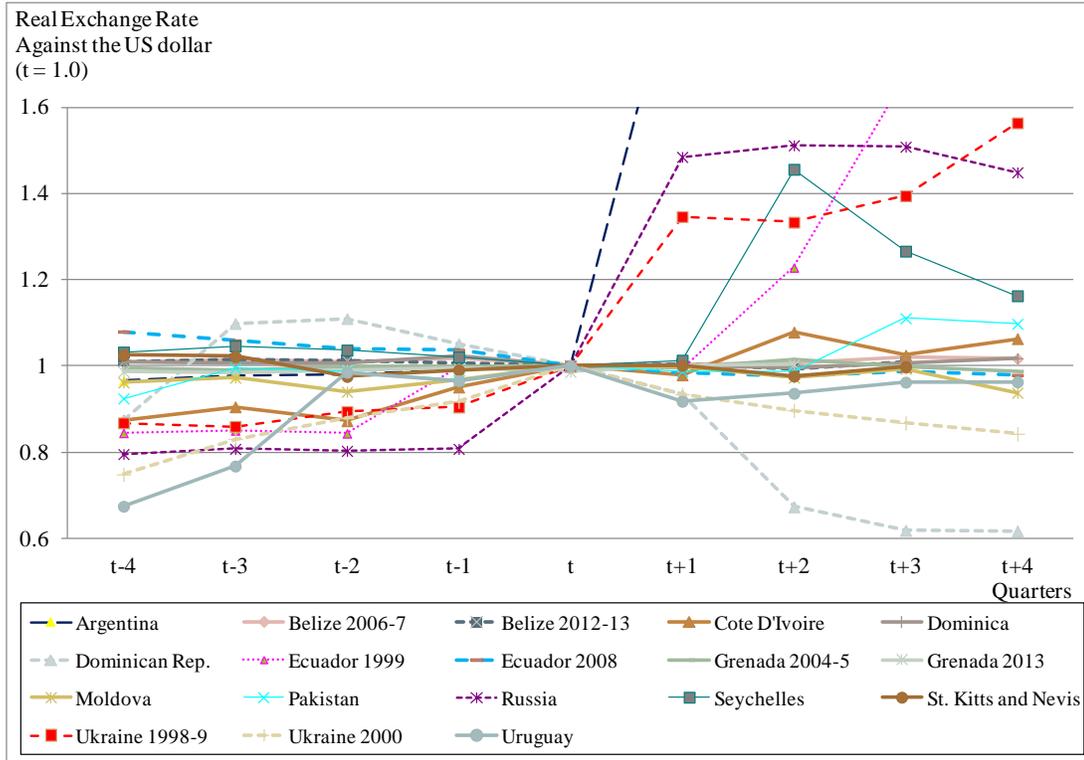
exchange rates are normalized with respect to levels at the time of defaults or announcements of restructurings. We observe an empirical link between the real exchange rate depreciation and default risk (default choice): on the one hand, the real exchange rate depreciation increases the burden of debt service payments for a debtor sovereign and helps trigger default; on the other hand, the country's announcement of default or restructuring leads to further real exchange rate depreciation. Exceptions are Ecuador 2008 for the pre-default period and the Dominican Republic and Ukraine 2000 for the post-default period.¹⁵

government defaults first and then starts to renegotiate its debt later on, which are termed as “post-default” cases.

¹⁴An alternative approach is to use selective default (SD) ratings on foreign currency debt by Standard and Poor's. We have a smaller sample of 12 default/restructuring episodes than our original sample of 18 episodes. This sample also excludes episodes of defaults/restructurings of debt held by official creditors. As expected, with the smaller sample, we also replicate similar real exchange rate dynamics before/after downgrading to selective default ratings.

¹⁵A number of factors could explain why the real exchange rate against the US dollar behaved exceptionally in these cases. Some of explanations as follows: The Ecuador 2008 episode can be treated as exceptional since it announced in December 2008 that the government missed an interest payment of \$30.6 million on its \$510 million of 12-percent global bonds due in 2012. Prior to the announcement, the authorities made statements in November 2008 that the 2012 and the 2030 securities were “illegal” and “illegitimate”. Therefore, default was considered to be triggered by political decision rather than solvency or liquidity concerns. The Dominican Republic in 2004–05 is also considered an outlier since it announced its debt restructuring on private debt in April 2004, following restructuring on official sector debt. However, its debt restructuring had proceeded in two separate approaches. For bank loans, though the sovereign started its negotiation with creditors in August 2004, it missed its payments in February 2005 and launched the final exchange offer in June 2005, which was completed later in October 2005. On the contrary, for external bonds, after negotiation was initiated in January 2005, the sovereign launched the final exchange offer in April 2005 and completed the exchange later in May 2005. In the case of Ukraine 2000, the real exchange rate had been on a depreciation trend for 9 quarters until its peak in January 2000 (depreciated by 95 percent compared to the level in November 1997) due to the first default/restructuring in 1998–99. Therefore, in the post-default period, we observed a slight rebound of the real exchange rate (appreciated by 15 percent compared to the level in January 2000).

Figure 1. Real Exchange Rates Against the US Dollar Before and After Defaults/Restructurings¹



Sources: Asonuma and Trebesch (2016); IMF IFS.

¹Exchange rates at end of period.

B. Empirical Analysis of the Link

With our sample of 18 episodes, empirical analysis attempts to examine a relationship between real exchange rate depreciations and default probability (default choice) in both pre-default and post-default periods.¹⁶

First, in pre-default periods, we explore whether the lagged exchange rate depreciations lead to an increase in default probability. Our sample is in quarterly frequency, and each episode covers periods from 5 quarters before to the time of defaults/announcement of restructurings. To proxy for default probability, we use credit ratings on foreign currency debt, which are transformed into discrete forms following Sy (2002).^{17 18 19} One advantage of this approach is to capture the high degree of variation in default probability. We use lagged annual exchange rate depreciations to reflect the magnitude of depreciations over last 4 quarters towards defaults or announcement of restructurings. Given the possibility of an endogeneity problem, we apply a two-step generalized method of moment (GMM) estimation using both U.S. GDP deviation from the trend and the U.S. Treasury bill rate as instruments for lagged real exchange rate depreciation. These instruments have enough explanatory power, as shown in high Adj – R² reported in Table AIII in Appendix II. Our specification is the following:

$$\text{Rating}_t = \text{ER}_{t-1}\beta + Z_{t-1}\gamma + Z_t\gamma' + \varepsilon_{1,t} \quad (1)$$

where ER_{t-1} are estimates of lagged real exchange rate depreciations, Z_{t-1} and Z_t are vectors of other explanatory variables at time $t - 1$ and t , respectively.

Our choice of control variables has been guided by the literature on sovereign debt crises and is especially close to Kohlscheen (2009) and Dreher and Walter (2010). We include GDP growth rates, debt service-to-GDP, an indicator of institutional quality, and 1-year LIBOR

¹⁶Details and sources of variables used in empirical analysis are reported in Appendix II.

¹⁷The alternative approach is to use a binary variable showing default and non-default choices and to apply a probit estimation. This method also provides results similar to Table 1, with a smaller degree of significance. The rationale of current approach, i.e., using credit ratings as proxy for default probability is to capture variations in default probability driven by exchange rate depreciation prior to the actual default at t (periods from $t-5$ to $t-1$). In contrast, using the binary choice of default limits us to capture increase in default probability at t driven by exchange rate depreciation in previous period.

¹⁸Sy (2002) convert S&P's and Moody's ratings to numerical values using a linear scale from 0 to 20 with S.D. and CC/Ca ratings corresponding to values of 0 and 1, respectively, and AAA/Aaa ratings being assigned a value of 20.

¹⁹To account for possibility of non-linearity of default probability with corresponding defaulting ratings (below B3 or B-), we use non-linear credit rating series which all ratings below B3 or B- are set to 1 for robustness check. We confirm that regression results are robust if we use non-linear credit ratings as shown in Table AIII in Appendix II.

rates in baseline specification, which are found to be key factors in the sovereign debt crisis literature.^{20,21} Including debt service-to-GDP does not raise an issue of multi-collinearity since lagged exchange rate depreciation captures an estimated increase in next-period debt service while lagged debt service-to-GDP reflects next-period debt service forecasts without a further exchange rate depreciation.

Baseline GMM fixed regression results (2nd column) confirm that the real exchange rate depreciations (lagged) increase default probability: depreciations over last 4 quarters entered with lagged, lead to lower levels of ratings implying higher default probability/default choice—an annual exchange rate depreciation of 1 percent increases default probability proxied by 0.23 notches in credit ratings.²² In line with empirical findings in the sovereign debt crisis literature, default probability is high if GDP growth is low and debt burden (debt service-to-GDP) is high. This is consistent with findings in theoretical literature of sovereign debt and defaults using one-period bonds. An indicator of institutional quality is entered with a counter-intuitive sign because country-specific influence has been largely captured by fixed effect. The results are robust if we apply least square fixed effect estimation (3rd column). In this case, the indicator of institutional quality is significant and with an expected sign.

Table 1. Regression Results on Pre-Default Period

Dependent variable: Ratings	(A) Baseline	(B) Fixed effect
Estimation	2-step GMM Fixed effect	Least Square Fixed-effect
Real exchange rate, lagged	-0.23* (0.12)	-0.04** (0.02)
GDP growth rate, lagged	0.21* (0.12)	0.07 (0.04)
Debt service-to-GDP, lagged	-0.15* (0.08)	-0.15*** (0.05)
Institutional quality ¹	-0.12 (0.21)	0.18*** (0.06)
LIBOR 1-year	0.69*** (0.22)	0.83*** (0.13)
Sample	48	48
Root MSE	1.48	0.90

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

¹Institutional quality is the quarterly average of monthly PRC composite political risk ratings from 1985–2012 with 100 and 0 as the highest and lowest indices respectively.

²⁰As none of external official debt restructurings was completed prior to defaults or announcements of restructurings, there was no impact of external official debt restructurings on sovereigns' decision of defaults or announcement of restructurings on external private debt.

²¹Given that our sample is restricted to sovereigns under debt crisis (defaults or restructurings), we follow closely conventional specifications used in sovereign debt crisis literature. In contrast, for samples including both debt crisis periods and tranquil periods, a traditional specification on credit ratings in sovereign debt literature might be a desirable approach to take.

²²Given that default choice of the sovereign—dependent variable in our dataset—is highly correlated with lagged exchange rate (explanatory variables) and highly heteroskedastic, GMM fixed effect regression is appropriate in our regression analysis. See Cizek and others. (2014).

Next, for the post-default period, we analyze whether default choices of sovereigns influence real exchange rate depreciation in subsequent periods. Our sample is in quarterly frequency, and each episode covers periods from time of defaults or announcements of restructurings to 5 quarters after. Ratings on sovereign bonds are treated as indicators of default choices.²³ The same method of a two-step GMM regression is taken using credit ratings of other emerging countries in other regions with a similar size and degree of openness and quality of institution as instruments for lagged default probability. High Adj – R² in Table AII2 in Appendix II confirms high explanatory power of these instruments. We apply the following specification:

$$ER_t = \text{Rating}_{t-1}\beta + Z_{t-1}\gamma + Z_t\gamma' + \varepsilon_{2,t} \quad (2)$$

where Rating_{t-1} are estimates of lagged ratings, Z_{t-1} and Z_t are vectors of other explanatory variables at time $t - 1$ and t respectively.

For choice of control variables, we follow the literature on determinants of real exchange rates, especially Maeso-Fernandez and others (2001) and IMF (2006). The set of explanatory variables in baseline specification includes GDP growth rate differential, real interest rate differential, net foreign assets-to-GDP and real oil price shock, which are considered to be dominant determinants of real exchange rates in the literature. In addition to these variables, we also include an indicator of an IMF program and 1-year LIBOR rates because real exchange rates are also influenced by the conditionality under an IMF program and global liquidity.

From baseline pooled regression results, we see that sovereigns' default choices, expressed as lower credit ratings, result in real exchange rate depreciations: defaults denoted by lower levels of ratings, entered with lagged, lead to real exchange rate depreciation shown by a higher level of subsequent real exchange rates. Similar to what the literature on determinants of real exchange rates has explained, real exchange rates depreciate when GDP growth rates and real interest rates are lower than those of partner countries and the sovereign reduces net foreign assets. An increase in real oil prices, considered as a terms of trade shock, leads to depreciation in real exchange rates since deterioration of the terms of trade of a country should result in a real exchange rate depreciation of that country. On the contrary, neither the IMF program nor LIBOR rates have significant influence over real exchange rates. Obtained results are robust, even if we apply the pooled regression with global liquidity proxied by LIBOR rates and fixed effect regression.

²³Using a binary variable showing default and non-default choices as one of the explanatory variables is an alternative approach. We obtain similar results shown in Table 2 with less significance.

Table 2. Regression Results on Post-Default Period

Dependent variable: Real Exchange Rate	(A) Baseline	(B) w. Global Liquidity	(C) Fixed Effect Regression
Estimation	2-step GMM	2-Ssep GMM	2-step GMM
Pooled / Fixed effect	Pooled	Pooled	Fixed effect
Ratings, lagged	-0.25*** (0.03)	-0.26*** (0.03)	-0.11*** (0.04)
GDP growth differential, lagged	-0.02** (0.009)	-0.02** (0.009)	-0.017** (0.009)
Real interest rate differential, lagged	-0.004*** (0.002)	-0.004*** (0.002)	-0.009** (0.002)
Net foreign assets-to-GDP, lagged	-0.05*** (0.013)	-0.05*** (0.014)	-0.053*** (0.02)
Real oil price shock dummy, lagged ¹	2.67 (1.62)	3.04* (1.57)	1.55* (0.90)
IMF program	1.36 (1.10)	1.53 (1.07)	0.49 (0.67)
LIBOR 1-year	-	0.07 (0.05)	0.13* (0.07)
Constant	0.72 (1.06)	0.43 (1.02)	0.77 (0.55)
Sample	32	32	32
Root MSE	0.30	0.30	0.18

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

¹Real oil price shock is an indicator showing the world oil price index deflated by the U.S. Producer Price Index (PPI) for countries heavily dependent on oil prices, while 0 is given for those less dependent on oil prices.

III. MODEL ENVIRONMENT

A. Intuition

Our theoretical model attempts to explore interactions between real exchange rate and default decision in a standard dynamic model of defaultable debt, where a sovereign debtor borrows from a “representative” foreign creditor through bond issuances in local and foreign currencies. Both the country and the foreign creditor are risk-averse and subject to tradable and nontradable goods shocks. The real exchange rate is defined as units of the sovereign’s CPI against one unit of the creditor CPI where the price of tradable goods is the numeraire. It interacts with the prices of two debt instruments issued by the sovereign—incorporating default risk which increases with level of debt.

Prior to default, the sovereign, receiving a sequence of low income shocks in tradable sector, tends to accumulate more debt and is more exposed to real exchange rate depreciation due to increased default risk. Since a majority of debt is denominated in foreign currency as observed in data, the real exchange rate depreciation increases the burden of payments in terms of local currency, increasing default probability and ultimately proceeding to the default decision of the sovereign.

Once the sovereign declares default, it suffers output costs associated with default and loses access to the markets. By achieving financial autarky, the sovereign opts to have higher consumption of traded goods, indicating lower marginal utility of consumption, which leads to higher prices of nontraded goods and a higher overall price level relative to that of foreign creditors. Thus, it results in a further depreciation of the real exchange rate. This mechanism drives the equilibrium depreciation of the real exchange rate around defaults, and it is a plausible explanation of the observed patterns in the data.

B. General Points

The basic structure of the model is in line with previous work, extending the sovereign debt model of Eaton and Gersovitz (1981).²⁴ Our model embeds real exchange rate dynamics and currency denomination in a two-country framework. We consider a risk-averse sovereign and a representative risk-averse creditor. Their preferences are shown by following utility functions:

²⁴Our incomplete market assumption of capital market under the two-country framework follows Benigno and Thoenissen (2008) and Chari, Kehoe and McGrattan (2002).

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad E_0 \sum_{t=0}^{\infty} (\beta^*)^t u(c_t^*)$$

where $0 < \beta < 1$ is a discount factor of the sovereign, and $0 < \beta^* < 1$ is a discount factor of the creditor. c_t and c_t^* denote consumptions of borrower and lender in period t , and $u(\cdot)$ is one-period utility function, which is continuous, strictly increasing and strictly concave, and satisfies the Inada conditions. A discount factor of the sovereign rejects both pure time preference and probability that the current sovereignty will survive into next period, whereas a discount factor of the creditor shows only pure time preference. An assumption of a risk-averse creditor is in line with the behavior of investors in emerging financial markets, who prefer to avoid real exchange rate risks.²⁵

All information on income processes of two parties and bond issuances is perfect and symmetric. In each period, the sovereign starts with total debt b_t , a fraction dominated in local currency αb_t , and the remaining denominated in foreign currency $(1 - \alpha)b_t$. We provide a brief explanation of fixed share of foreign currency debt in Section III.C.

Both the sovereign and creditor receive stochastic endowment streams of tradable goods y_t^T , y_t^{T*} , and nontradable goods y_t^N , y_t^{N*} . We denote y_t , a column vector of four income processes: $y_t = [y_t^T, y_t^{T*}, y_t^N, y_t^{N*}]$. It is stochastic, drawn from a compact set $Y = [y_{\min}^T, y_{\max}^T] \times [y_{\min}^{T*}, y_{\max}^{T*}] \times [y_{\min}^N, y_{\max}^N] \times [y_{\min}^{N*}, y_{\max}^{N*}] \subset \mathbb{R}_+^4$. $\mu(y_{t+1}|y_t)$ is probability distribution of a vector of shocks y_{t+1} conditional on previous realization y_t . Both sovereign and creditor consume not only nontradable goods, but also two types of tradable goods endowed in each country. They export their own endowed tradable goods and import tradable goods endowed in the counterpart's country. When the sovereign repays its debt and issues new debt, it can import tradable goods endowed in the creditor's country more than its exports of tradable goods, i.e., having the current account deficit. On the contrary, when the sovereign defaults, it only imports tradable goods endowed in the creditor's country equal to its exports of tradable goods, i.e., having the current account balanced.

The representative creditor is risk-averse. As mentioned above, it is also subject to stochastic income shocks and opts to smooth its consumption through lending/borrowing to the sovereign. The risk-averse creditor prefers to avoid real exchange rate risks and opts to issue

²⁵Lizarazo (2013) explains that assumption of risk-averse creditors seems to be justified by characteristics of the investors in emerging financial markets. These investors are both individuals and institutional investors such as banks, mutual funds, hedge funds, pension funds and insurance companies. For individual investors, it is straightforward to assume that these agents are risk-averse. For institutional investors, risk aversion may follow from two sources: regulations over the composition of their portfolio and the characteristics of the institutions' management. Regarding the first source, these entities face restrictions on asset allocations; for instance, banks are regulated by capital adequacy ratio. Regarding the second source, for each class of institutional investor, managers ultimately make the portfolio allocation decisions. These managers can also be treated as risk-averse managers.

bonds in their local currency. A large fraction of external debt denominated in foreign currency, shown in Section III.C, also reflects behavior of a risk-averse creditor. Risk-averseness, rather than risk-neutrality, is necessary for determination of real exchange rate depending not only on the sovereign's but also the creditor's income shocks.

The international capital market is incomplete. The sovereign and creditor can borrow and lend only via one-period, zero-coupon bonds indexed to their consumer price index (CPI), and there are two types of bonds the sovereign (creditor) issues: bonds denominated in local and foreign currency. b_{t+1} (b_{t+1}^*) denotes the amount of bonds to be repaid next period whose set is shown by $B = [b_{\min}, b_{\max}] \subset \mathbb{R}$ where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound at $b_{\min} < -y_{\max}^T / \underline{r}^*$, which is the largest debt that the sovereign could repay. The upper bound b_{\max} is the high level of assets that the sovereign may accumulate.²⁶ We assume $q^i(b_{t+1}, y_t)$ for $i \in \{H, F\}$ to be the price of bonds with asset position b_{t+1} and a vector of income shocks y_t . We assume that q^H and q^F are denominated in local and foreign currency, respectively. Both bond prices are determined in equilibrium.

As in conventional international real business cycle model where neither money or nominal rigidities are included for instance, Gourio et al, (2013), We define the current real exchange rate e_t as units of home CPI against one unit of foreign CPI: $e_t = p_t / p_t^*$. An increase in e_t means an increase in units of domestic CPI relative to one unit of foreign CPI, i.e., a depreciation of domestic currency. The real exchange rate is also determined in equilibrium together with bond prices.

We assume that the creditor always commits to repay its debt. However, the sovereign is free to decide whether to repay its debt or to default. If the sovereign chooses to repay its debt, it will preserve its status to issue bonds next period. On the contrary, if it chooses not to pay its debt, it is then subject to both exclusion from the international capital market and direct output costs. The sovereign suffers symmetric output costs on tradable and nontradable goods. This assumption is consistent with none of the empirical findings justifying asymmetric output costs across tradable and nontradable sectors in the literature of costs of sovereign defaults.²⁷ We consider that the debtor defaults total external debt (b_t). Defaulting total external debt is supported by evidence on recent external debt restructurings, where

²⁶ b_{\max} exists when the interest rates on the sovereign's savings are sufficiently low compared to the discount factor, which is satisfied as $(1 + \underline{r}^*)\beta < 1$.

²⁷Though it is within the manufacturing goods sector, not across the tradable and nontradable goods sectors, Borensztein and Panizza (2010) find that a more export-oriented industry would see its severe growth drop relative to a less export-oriented industry in each year in which the sovereign is in default.

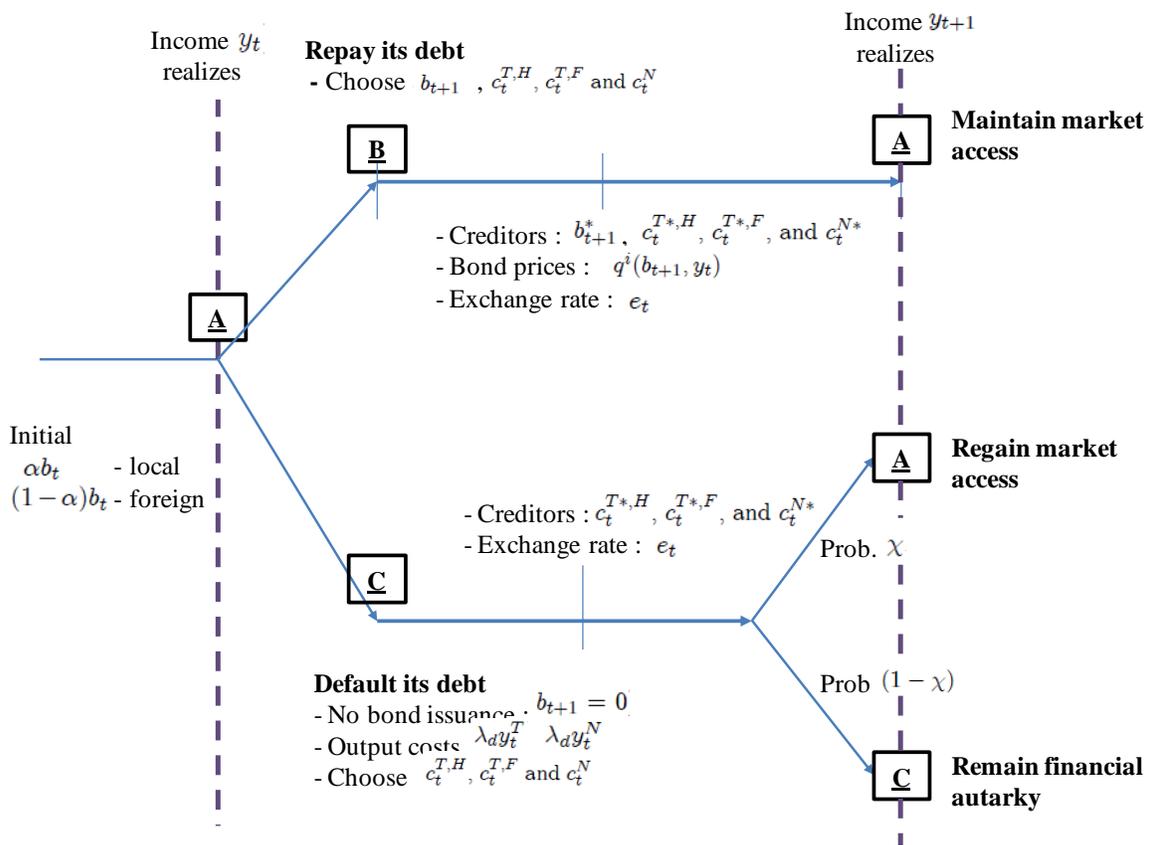
sovereigns default on both local and foreign currency debt issued at the international market.²⁸

When a default is chosen, the sovereign will be in temporary autarky. After being excluded from the market for one period, with exogenous probability χ , it will regain access to the market. Otherwise, it will remain in financial autarky next period.

C. Timing of the Model

Figure 2 summarizes the timing of decisions within each period.

Figure 2. Timing of the Model



1. The sovereign starts current period with total debt b_t , comprised of local and foreign currency debt αb_t and $(1 - \alpha)b_t$. We are in node (A).
2. A vector of income shocks y_t realizes. The sovereign decides whether to pay its debt or to default after observing its income.

²⁸There are only a few episodes where sovereigns apparently differentiate creditors of foreign currency and local currency external debt.

3. In node (B) (payment node), if payment is chosen, we move to the upper branch of a tree. The sovereign chooses level of next-period debt (b_{t+1}) and consumption $c_t^{T,H}$, $c_t^{T,F}$, and c_t^N . Default risk is determined and the creditor also choose b_{t+1}^* and consumption $c_t^{T*,H}$, $c_t^{T*,F}$, and c_t^{N*} . Bond prices and the real exchange rate are determined in equilibrium. We move back to node (A).
4. In node (C) (default node), if default is chosen, we move on to the lower branch of a tree. The sovereign cannot raise funds in the international capital market this period ($b_{t+1} = 0$), and suffers output costs $\lambda_d p_t^{T,H} y_t^T$ and $\lambda_d p_t^N y_t^N$. The sovereign chooses consumption $c_t^{T,H}$, $c_t^{T,F}$, and c_t^N . The creditor also choose consumption $c_t^{T*,H}$, $c_t^{T*,F}$, and c_t^{N*} . The real exchange rate is determined in equilibrium. With exogenous probability χ , we move back to node (A) and the sovereign regains its market access. Otherwise, we return to node (C) and the sovereign remains financial autarky.
5. A vector of income shocks y_{t+1} realizes.

D. Fixed Share of Foreign Currency Debt

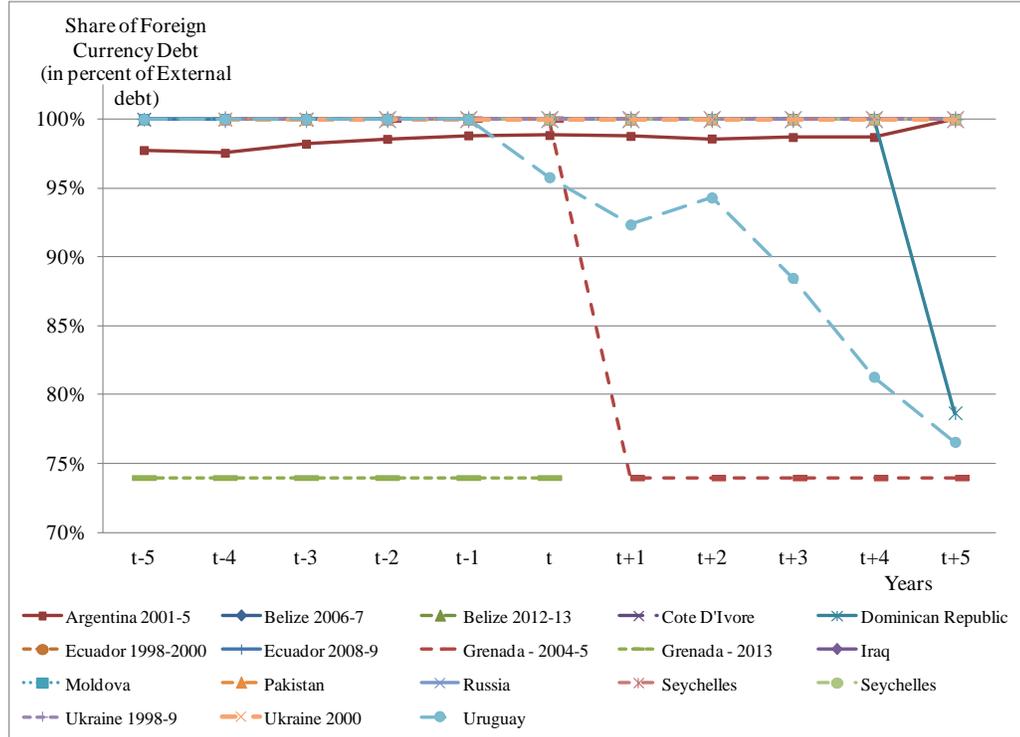
We explain, succinctly, a rationale of assumption on share of foreign currency debt. Figure 3 shows shares of foreign currency debt in annual frequency before and after defaults and announcement of restructurings for 18 episodes from 1999–2013.²⁹ We compute shares of foreign currency debt for 18 episodes using data of international bond issuance from Bloomberg and Dealogic.³⁰ A majority of countries, which have experienced defaults or restructurings recently, had a large fraction of their external debt, close to 100 percent, denominated in foreign currency both before and after defaults and announcement of restructurings. Even among four exceptional cases, three episodes, such as the Dominican Republic in 2004–05, Grenada in 2004–05 and Uruguay in 2003, witnessed a decrease in share of foreign currency denominated debt only after defaults or announcements of restructurings. In addition, these countries seldom changed shares of foreign currency debt, as shown in limited variations over the sample period in Figure 3.³¹ These clearly support our assumption of fixed and high shares of foreign currency debt.

²⁹Data on external bond issuances for Dominica and St. Kitts and Nevis are not available.

³⁰Due to a lack of currency denomination data on loans from Dealogic, our computed shares are based only on bond issuances.

³¹Small variance in share of foreign currency denominated debt over the sample period reported in Table A1 in Appendix C also supports our assumption.

Figure 3. Share of Foreign Currency Debt Before and After Defaults or Announcements of Restructurings



Sources: Asonuma and Trebesch (2016); Bloomberg and Dealogic.

IV. RECURSIVE EQUILIBRIUM

In this section, we define the stationary recursive equilibrium of the model. Our framework incorporates three key features: (1) optimal behavior of a risk-averse foreign creditor, (2) two types of external bonds denominated in local and foreign currencies, and (3) endogenous determination of real exchange rate in equilibrium.

A. Sovereign Country's Problem

The country's problem is to maximize the expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (3)$$

A consumption basket c_t is defined by the CES aggregates of consumption shown as

$$c_t = \left[\omega^{\frac{1}{\kappa}} (c_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (c_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}} \quad (4)$$

where c_t^T and c_t^N are consumptions of tradable and nontradable goods, and κ is the elasticity of intratemporal substitutions between these goods. The tradable component is, in turn, comprised of local and foreign-endowed goods in the following manner:

$$c_t^T = \left[v^{\frac{1}{\theta}} (c_t^{T,H})^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} (c_t^{T,F})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (5)$$

where $c_t^{T,H}$ and $c_t^{T,F}$ are consumptions of traded goods endowed in the country and the creditor's country, respectively. θ is the elasticity of intratemporal substitution between traded goods endowed in the country and the creditor's country.

Corresponding to the CES bundles of consumption goods, we have an isomorphic price index:

$$p_t = [\omega(p_t^T)^{1-\kappa} + (1-\omega)(p_t^N)^{1-\kappa}]^{\frac{1}{1-\kappa}} \quad (6)$$

where p_t^T and p_t^N are prices of traded and nontraded goods. The price of tradable goods is the numeraire ($p_t^T = 1$). The tradable good price is, in turn, comprised of prices of local and foreign-endowed goods:

$$1 = \left[v(p_t^{T,H})^{1-\theta} + (1-v)(e_t p_t^{T,F*})^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (7)$$

where $p_t^{T,H}$ and $p_t^{T,F*}$ are prices of traded goods endowed in each country.

Let $V(b_t, y_t)$ be the lifetime value function of the country that starts the current period with initial assets b_t and a vector of income shocks y_t . Given sovereign bond prices $q^i(b_{t+1}, y_t)$ for $i = H, F$, and the real exchange rate e_t , the country solves its optimization problem.

If the country decides to pay its debt, it chooses its next-period assets (b_{t+1}) and current consumption after paying back its initial debt. On the contrary, if the country defaults, it will not be able to issue bonds in the current period. It simply chooses current consumption.

Given the option to default, $V(b_t, y_t)$ satisfies

$$V(b_t, y_t) = \max[V^R(b_t, y_t), V^D(y_t)] \quad (8)$$

where $V^R(b_t, y_t)$ is the value associated with paying debt:

$$V^R(b_t, y_t) = \max_{c_t^{T,H}, c_t^{T,F}, c_t^N, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, y_{t+1}) d\mu(y_{t+1}, y_t) \quad (9)$$

$$\begin{aligned} \text{s.t.} \left(\begin{array}{l} p_t^{T,H} c_t^{T,H} + e_t p_t^{T,F*} c_t^{T,F} \\ + p_t^N c_t^N \end{array} \right) + p_t \left[\begin{array}{l} q^H(b_{t+1}, y_t) \alpha + \\ e_t q^F(b_{t+1}, y_t) (1-\alpha) \end{array} \right] b_{t+1} = \\ \left(\begin{array}{l} p_t^{T,H} y_t^T \\ + p_t^N y_t^N \end{array} \right) + p_t \left[\begin{array}{l} \alpha + \\ e_t (1-\alpha) \end{array} \right] b_{t+1} \end{aligned}$$

s.t. (4) and (5)

and $V^D(y_t)$ is the value, which the country decides to default, shown as

$$V^D(y_t) = \max_{c_t^{T,H}, c_t^{T,F}, c_t^N, b_{t+1}} u(c_t) + \beta \left[\begin{array}{l} \chi \int_Y V(0, y_{t+1}) d\mu(y_{t+1}, y_t) \\ + (1 - \chi) \int_Y V^D(y_{t+1}) d\mu(y_{t+1}, y_t) \end{array} \right] \quad (10)$$

$$\text{s.t. } p_t^{T,H} c_t^{T,H} + e_t p_t^{T,F*} c_t^{T,F} + p_t^N c_t^N = (1 - \lambda_d) p_t^{T,H} y_t^T + (1 - \lambda_d) p_t^N y_t^N$$

s.t. (4) and (5)

where $V(0, y_{t+1})$ is its value next period with no initial debt. $\lambda_d p_t^{T,H} y_t^T$ and $\lambda_d p_t^N y_t^N$ express output costs, which the country suffers due to a default. When the country decides the next-period assets, it also takes into consideration impacts of the real exchange rate, which is determined by optimality conditions of the sovereign debtor and the creditor.

The country's default policy can be characterized by default set $D(b_t) \subset Y$. The default set is a set of income vectors y 's for which default is optimal given the debt position b_t .

$$D(b_t) = \{y_t \in Y: V^R(b_t) < V^D(b_t)\} \quad (11)$$

In the case where the country chooses to pay its debt, we obtain the following optimality conditions:

$$\frac{c_t^T}{c_t^N} = \left(\frac{\omega}{1-\omega} \right) \left(\frac{1}{p_t^N} \right)^{-\kappa} \quad (12)$$

$$\frac{c_t^{T,H}}{c_t^{T,F}} = \left(\frac{v}{1-v} \right) \left(\frac{p_t^{T,H}}{e_t p_t^{T,F*}} \right)^{-\theta} \quad (13)$$

$$\begin{aligned} & [q^H(b_t, y_t) \alpha + e_t q^F(b_{t+1}, y_t) (1 - \alpha)] \\ &= \int_Y \beta \frac{u'(c_{t+1})}{u'(c_t)} \mathbb{I}_{\text{Non-Default}} [\alpha + e_t (1 - \alpha)] d\mu(y_{t+1}, y_t) \end{aligned} \quad (14)$$

On the contrary, if the country chooses to default, we have equation (12) and (13), not (14).

B. The Foreign Creditor's Problem

The foreign creditor is also risk-averse and behaves competitively at the market. The problem is maximizing its expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} (\beta^*)^t u(c_t^*) \quad (15)$$

Its consumption basket c_t^* is similar to that of the country:

$$c_t^* = \left[(\omega^*)^{\frac{1}{\kappa}} (c_t^{T*})^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (c_t^{N*})^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}} \quad (16)$$

where c_t^{T*} and c_t^{N*} are consumptions of traded and nontraded goods. Tradable goods consumption is composed of consumptions of two tradable goods: $c_t^{T*,H}$ and $c_t^{T*,F}$:

$$c_t^{T*} = \left[(v^*)^{\frac{1}{\theta}} (c_t^{T*,H})^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} (c_t^{T*,F})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (17)$$

Corresponding to the CES bundles of consumption goods, we have an isomorphic price index:

$$p_t^* = [\omega (p_t^{T*})^{1-\kappa} + (1-\omega) (p_t^{N*})^{1-\kappa}]^{\frac{1}{1-\kappa}} \quad (18)$$

where p_t^T and p_t^N are prices of traded and nontraded goods. The price of tradable goods is the numeraire ($p_t^T = 1$). The tradable good price is, in turn, comprised of prices of local and foreign-endowed goods:

$$p_t^{T*} = \left[v^* \left(\frac{p_t^{T,H}}{e_t} \right)^{1-\theta} + (1-v^*) (p_t^{T,F})^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (19)$$

If the country repays its debt, the creditor also decides its assets for next period (b_{t+1}^*) and current consumption $c_t^{T*,H}$, $c_t^{T*,F}$, and c_t^{N*} subject to its budget constraint, such as

$$\begin{aligned} & \left(\frac{p_t^{T,H}}{e_t} c_t^{T*,H} + p_t^{T*,F} c_t^{T*,F} \right) + p_t^* \left[\frac{q^H(b_{t+1}, y_t)}{e_t} \alpha + \right] b_{t+1}^* \\ & \quad + p_t^{N*} c_t^{N*} \quad \left[q^F(b_{t+1}, y_t) (1-\alpha) \right] \\ & = \left(p_t^{T*,F} y_t^{*T} \right) + p_t^* \left[\frac{1}{e_t} \alpha + \right] b_{t+1}^* \\ & \quad + p_t^{N*} y_t^{N*} \quad \left[(1-\alpha) \right] \end{aligned} \quad (20)$$

Then, we obtain the following optimality conditions:

$$\frac{c_t^{T^*}}{c_t^{N^*}} = \left(\frac{\omega^*}{1-\omega^*} \right) \left(\frac{p_t^{T^*}}{p_t^{N^*}} \right)^{-\kappa} \quad (21)$$

$$\frac{c_t^{T^*,F}}{c_t^{T^*,H}} = \left(\frac{v}{1-v} \right) \left(\frac{p_t^{T^*,F}}{p_t^{T^*,F}/e_t} \right)^{-\theta} \quad (22)$$

$$\begin{aligned} & \left[\frac{q^H(b_t, y_t)}{e_t} \alpha + q^F(b_{t+1}, y_t)(1 - \alpha) \right] \\ &= \int_Y \beta \frac{u'(c_{t+1}^*)}{u'(c_t^*)} \mathbb{I}_{\text{Non-Default}} \left[\frac{1}{e_t} \alpha + (1 - \alpha) \right] d\mu(y_{t+1}, y_t) \end{aligned} \quad (23)$$

If the country defaults, the creditor maximizes its utility by choosing current consumption $c_t^{T^*,H}$, $c_t^{T^*,F}$, and $c_t^{N^*}$ subject to its budget constraint:

$$\frac{p_t^{T^*,H}}{e_t} c_t^{T^*,H} + p_t^{T^*,F} c_t^{T^*,F} + p_t^{N^*} c_t^{N^*} = p_t^{T^*,F} y_t^{*T} + p_t^{N^*} y_t^{N^*} \quad (24)$$

Then, we have we have equation (21) and (22), not (23).

C. Bond Prices and Real Exchange Rate

Bond prices indexed to the sovereign's and creditor's CPI $q^i(b_t, y_t)$ for $i = H, F$ are functions of the next-period assets and a vector of income shocks. If the country chooses to pay its debt, the creditor receives payoffs equal to the face value of bonds, which is normalized to 1. If the country chooses to default, payoffs are zero. We derive bond price functions for both the sovereign's and the creditor's Euler equations, which take into account the sovereign's decision of paying its debt and defaulting.

$$\begin{aligned} & [q^H(b_t, y_t)\alpha + e_t q^F(b_{t+1}, y_t)(1 - \alpha)] \\ &= \int_Y \beta \frac{u'(c_{t+1})}{u'(c_t)} \mathbb{I}_{\text{Non-Default}} [\alpha + e_t(1 - \alpha)] d\mu(y_{t+1}, y_t) \end{aligned} \quad (25)$$

$$\begin{aligned} & \left[\frac{q^H(b_t, y_t)}{e_t} \alpha + q^F(b_{t+1}, y_t)(1 - \alpha) \right] \\ &= \int_Y \beta \frac{u'(c_{t+1}^*)}{u'(c_t^*)} \mathbb{I}_{\text{Non-Default}} \left[\frac{1}{e_t} \alpha + (1 - \alpha) \right] d\mu(y_{t+1}, y_t) \end{aligned} \quad (26)$$

The real exchange rate is defined as relative CPI between the sovereign and the creditors as

$$e_t = \frac{p_t}{p_t^*} \quad (27)$$

D. Market Clearing Conditions for Bonds and Goods

If the country repays its debt in the current period, market clearing conditions for tradable goods and nontradable goods are

$$c_t^{T,H} + c_t^{T^*,H} = y_t^T \quad (28)$$

$$c_t^{T,F} + c_t^{T^*,F} = y_t^{T^*} \quad (29)$$

$$c_t^N = y_t^N \quad (30)$$

$$c_t^{N^*} = y_t^{N^*} \quad (31)$$

On the contrary, in the case of default, the following are market clearing conditions for tradable goods and nontradable goods endowed in the country.

$$c_t^{T,H} + c_t^{T^*,H} = (1 - \lambda_d)y_t^T \quad (28')$$

$$c_t^N = (1 - \lambda_d)y_t^N \quad (30')$$

Market clearing condition for bonds is

$$\pi b_t + (1 - \pi)b_t^* = 0 \quad (31)$$

E. Recursive Equilibrium

We define a stationary recursive equilibrium of the model.

Definition: A **recursive equilibrium** is a set of functions for, (A) the country's value function $V(b_t, y_t)$, consumption, $c_t^{T,H}(b_t, y_t)$, $c_t^{T,F}(b_t, y_t)$, $c_t^N(b_t, y_t)$; asset position $b_{t+1}(b_t, y_t)$, default set $D(b_t)$; (B) foreign creditor's consumption, $c_t^{T^*,H}(b_t, y_t)$, $c_t^{T^*,F}(b_t, y_t)$, $c_t^{N^*}(b_t, y_t)$; asset position $b_{t+1}^*(b_t, y_t)$; and (C) bond prices $q^i(b_{t+1}, y_t)$ for $i = H, F$, and the real exchange rate $e_t(b_{t+1}, y_t)$ such that

[1]. Given bond prices and the real exchange rate, the country's value function $V(b_t, y_t)$; consumption, $c_t^{T,H}(b_t, y_t)$, $c_t^{T,F}(b_t, y_t)$, $c_t^N(b_t, y_t)$; asset position $b_{t+1}(b_t, y_t)$; default set $D(b_t)$ satisfy the country's optimization problem.

[2]. Given bond prices and the real exchange rate, the creditor's consumption $c_t^{T*,H}(b_t, y_t)$, $c_t^{T*,F}(b_t, y_t)$, $c_t^{N*}(b_t, y_t)$; asset position $b_{t+1}^*(b_t, y_t)$ satisfy the creditor's optimization problem.

[3]. Bond prices $q^i(b_{t+1}, y_t)$ for $i = H, F$, and the real exchange rate $e_t(b_{t+1}, y_t)$ satisfy optimality conditions of two parties.

[4]. Market clearing conditions for goods and bonds are satisfied.

In equilibrium, default probability $p^*(b_{t+1}, y_t)$ is related to the sovereign's default decision in the following manner:

$$p^*(b_{t+1}, y_t) = \int_{D^*(b_{t+1})} d\mu(y_{t+1}, y_t) \quad (33)$$

Risk-free interest rate is defined as

$$\frac{1}{1 + r^*(b_{t+1}, y_t)} = \beta^* \int_Y \frac{u'(c_{t+1}^*)}{u'(c_t^*)} d\mu(y_{t+1}, y_t) \quad (34)$$

We define total spreads for domestic and foreign currency debt evaluated by the creditor's side as follows:

$$s^H(b_{t+1}, y_t) = \frac{e_t}{q^H(b_{t+1}, y_t)} - 1 - r^*(b_{t+1}, y_t) \quad (35)$$

$$s^F(b_{t+1}, y_t) = \frac{1}{q^F(b_{t+1}, y_t)} - 1 - r^*(b_{t+1}, y_t) \quad (36)$$

V. QUANTITATIVE ANALYSIS

This section provides quantitative analysis of model. Our major findings can be summarized as follows. First, at the steady state distribution, we show that at any level of tradable goods, the real exchange rate tends to depreciate sharply when the sovereign defaults. Moreover, the real exchange rate depreciates when the sovereign receives a low tradable goods shock. Second, our simulation exercise uses Argentine default in 2001 and replicates both business cycle and non-business cycle regularities, including moments of the real exchange rates in both pre-default and post-default periods. Lastly, most importantly, the model generates the link between real exchange rate dynamics and default choices (default probability) around default.

A. Parameters and Functional Forms

We use most of the parameters and functional forms specified in previous work. There are three new elements in the model associated with a two-country, four-goods set-up: (i) relative size of the sovereign, (ii) weights on consumption of home-endowed tradable goods, and (iii) share of domestic currency debt.

The following utility functions are used in numerical simulation:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \quad u(c_t^*) = \frac{c_t^{*1-\sigma} - 1}{1-\sigma} \quad (37)$$

where σ expresses degree of risk aversion. We set σ equal to 2, which is commonly used in real business cycle analysis for advanced and emerging markets. The creditor's discount factor is set to $\beta^* = 0.982$ to replicate the risk-free interest rate of 1.7%.³² The elasticity of substitution between tradable and nontradable consumption is taken from Gonzales and Neumeyer (2003) where they estimate the elasticity for Argentina to be equal to 0.48. We assume an elasticity of substitution between tradable goods endowed in the sovereign and the creditor country θ , of 2, as in Benigno and Thoenissen (2008). Weights of tradable goods consumption and home-endowed tradable goods consumptions are set to $\omega = 0.51$, $\omega^* = 0.5$, and $\nu = \nu^* = 0.5$ in order to have the price of tradable goods at steady-state distribution ($p^T = 1$).

The probability of re-entry to credit markets after defaults is set at $\chi = 0.282$, which is consistent with observed evidence regarding the exclusion from credit markets of defaulting countries mentioned in Gelos and others (2011). Output loss parameter λ_d is assumed to be 2% following Sturzenegger (2004)'s estimates.

We assume each exogenous endowment stream y_t^i for $i = \{T, N, T^*, N^*\}$ follows a log-normal AR(1) process where innovations to the shocks are allowed to be correlated:

$$\log(y_t^i) = \log(\bar{y}^i) + \rho_y^i (\log(y_{t-1}^i) - \log(\bar{y}^i)) + \epsilon_y^i \quad (38)$$

where \bar{y}^i is the mean income, $E[\epsilon_y^i] = 0$ for $i = \{T, N, T^*, N^*\}$ and the variance-covariance matrix of the error terms is the following:

$$E[\epsilon' \epsilon] = \begin{bmatrix} \sigma^T & \sigma^{TN} & \sigma^{TT^*} & \sigma^{TN^*} \\ \sigma^{TN} & \sigma^N & \sigma^{NT^*} & \sigma^{NN^*} \\ \sigma^{TT^*} & \sigma^{NT^*} & \sigma^{T^*} & \sigma^{T^*N^*} \\ \sigma^{TN^*} & \sigma^{NN^*} & \sigma^{T^*N^*} & \sigma^{N^*} \end{bmatrix} = \begin{bmatrix} 0.0027 & 0.0019 & 0 & 0 \\ 0.0019 & 0.0019 & 0 & 0 \\ 0 & 0 & 0.006 & 0.002 \\ 0 & 0 & 0.002 & 0.002 \end{bmatrix} \quad (39)$$

³²Similarly, Lizarazo (2013) set the creditor's discount rate as $\beta^* = 0.98$ to generate the international interest rate of 1.7%.

where $\epsilon = [\epsilon_y^T, \epsilon_y^N, \epsilon_y^{T*}, \epsilon_y^{N*}]'$. Auto-correlation coefficients and the variance-covariance matrix are computed from the quarterly real GDP data of Argentina from 1993Q1 to 2011Q4 (sovereign) and of the U.S. from 1988Q1 to 2011Q4. Sector-level GDP data are seasonally adjusted and are taken from the Ministry of Economy and Production (MECON) and the U.S. Bureau of Economic Analysis (BEA). The sectoral classification into tradable and nontradable goods follows the traditional approach adopted in real business cycle literature. The tradable goods sector comprises “manufacturing” and the primary sectors, whereas the nontradable goods sector is composed of remaining sectors. The data are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600. Each shock is then discretized into a finite state Markov chain by using a quadrature procedure in Hussey and Tauchen (1991) from their joint distribution. We obtain estimated coefficients such as $\rho^T = 0.59$ and $\rho^N = 0.70$ for Argentina and $\rho^{T*} = 0.59$ and $\rho^{N*} = 0.67$ for the U.S.

For remaining country-specific parameters, size of the sovereign relative to that of the creditor is set to 0.025 to reflect the ratio of US dollar GDP of Argentina to that of the U.S. over the period 1993–2012. Sturzenegger and Zettelmeyer (2006) report that Argentina experienced 6 defaults in 1820–2004. We specify the sovereign’s discount factor $\beta = 0.86$ (Argentina) to replicate the average default frequency of 3.4%. The share of domestic currency debt is set at 0.01 based on the average share over the period 1996–2006 from Bloomberg and Dealogic. Table 3 summarizes the model parameters. Our computation algorithm is shown in Appendix A.

Table 3. Model Parameters

Parameter	Value	Source
General		
Risk aversion	$\sigma = 2$	RBC literature
Elast. of sub. b/w c^T and c^N	$\kappa = 0.48$	Gonzales and Neumeyer (2003)
Elast. of sub. b/w $c^{T,H}$ and $c^{T,F}$	$\theta = 2$	Benign and Thoenissen (2008)
Weight of $c^{T,H}$ and $c^{T*,F}$ in CES	$\omega = 0.51$ $\omega^* = 0.5$	Computed
Weight of $c^{T,H}$ and $c^{T*,F}$ in CES	$u = u^* = 0.5$	Computed
Output cost	$\lambda_d = 0.02$	Sturzenegger (2004)
Discount rate—creditor	$\beta^* = 0.982$	Computed
Autoreg. of income—creditor	$\rho^{T*} = 0.49,$ $\rho^{N*} = 0.67$	Computed—U.S. BEA
Sovereign specific		
Autoreg. of income—sovereign	$\rho^T = 0.59,$ $\rho^N = 0.70$	Computed—MECON
Relative size of sovereign	$\pi = 0.025$	IMF WEO
Discount rate—sovereign	$\beta = 0.86$	Computed
Share of domestic currency debt	$\alpha = 0.01$	Bloomberg / Dealogic

B. Numerical Results on Equilibrium Properties

In this subsection, we cover the equilibrium properties of the model. Figure 4 shows that the default probability at mean level of tradable goods is weakly increasing with respect to the level of total debt. Furthermore, default probability is weakly increasing respect to level of tradable goods. These two findings are consistent with recent quantitative analysis of sovereign debt—as in Aguiar and Gopinath (2006), Arellano (2008) and Yue (2010)—that the sovereign is more likely to default when it has accumulated its debt and a bad income shock.

Figure 4. Default Probability

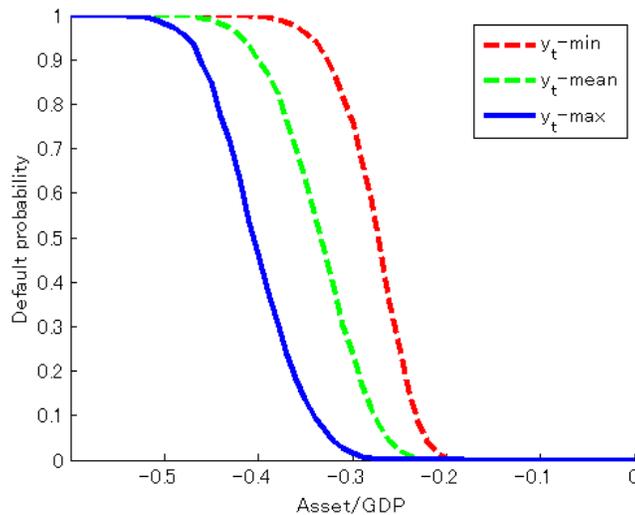


Figure 5. Real Exchange Rates

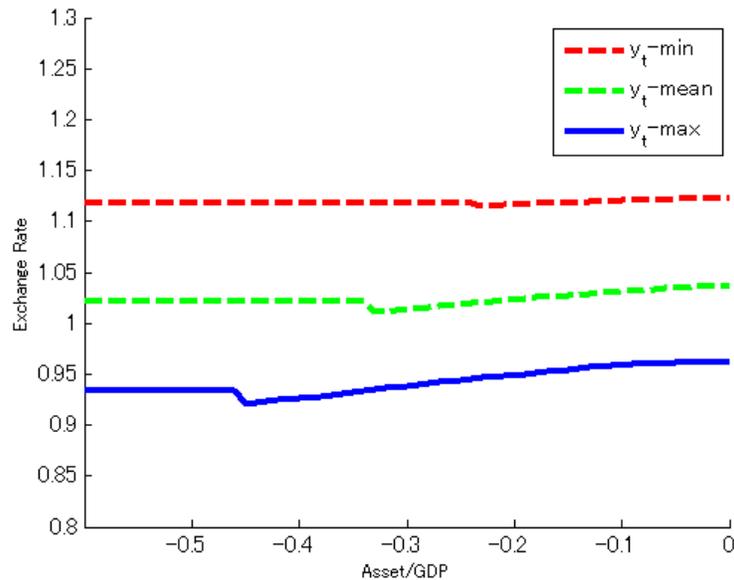


Figure 5 displays that, at a given level of tradable goods below threshold of debt/GDP ratio where the sovereign opts to default, a low level of real exchange rate meaning appreciation is associated with a high current debt/GDP ratio. With a fixed tradable goods income shock,

higher current debt leads to lower consumption of tradable goods (with less tradable goods endowment left for consumption) indicating higher marginal utility of consumption of tradable goods. This, in turn, results in both the lower price of nontradable goods and the lower overall price. On the contrary, when the sovereign defaults at current debt, the real exchange rate tends to depreciate. By defaulting debt obligations, i.e., no debt payments, the sovereign enjoys higher consumption of tradable goods, indicating lower marginal utility of consumption of tradable goods, which leads to both a higher price of nontraded goods and a higher overall price level.

Moreover, the level of the real exchange rate is high implying depreciation when the sovereign has a low level of traded goods. With a low level of traded goods, the sovereign tends to accumulate higher debt which leads to an increase in default probability. Then, the real exchange rate depreciates and is associated with an increase in default probability. Price functions for newly-issued debt and debt level are shown in Appendix C.

C. Simulation—Argentina

We conduct 1000 rounds of simulations, with 2000 periods per round and then extract the last 200 periods to analyze features evaluated at the steady-state distribution. In the last 200 periods, we choose 40 observations before and after a default event to compare with moments in data for Argentina. The second column in Table 4 and 5 summarizes moments of data.³³ Output data are seasonally adjusted from the MECON for 1993Q1–2001Q3 and 2001Q4–2011Q4. Trade balance is calculated as ratio to GDP. Argentina’s external debt data are from the IMF WEO for 1993–2001 and 2002–11. We calculate two measures of the sovereign’s indebtedness; the first measure is the average external debt to GDP ratio. We also compute the ratio of the country’s debt service (including short-term debt) to its GDP for Argentina. Bond spreads are from the J.P. Morgan’s Emerging Market Bond Index (EMBI) Global for Argentina for 1998Q1–2001Q3 and 2001Q4–2011Q4. Real exchange rate is computed based on monthly Argentina nominal exchange rates against the US dollar, Argentina CPI, and U.S. CPI from IMF IFS for 1993Q1–2001Q3 and 2001Q4–2011Q4. We compare our simulation results with those of Aguiar and Gopinath (2006) and Arellano (2008).

³³See also Arellano (2008) and Yue (2010) for similar treatment of simulation.

Table 4. Business Cycle Statistics for Argentina

	Data	Model	A and G (2006)	Arellano (2008)
<i>Before Default</i>				
Consumption Std./Output Std.	1.14	1.65	1.06	1.10
Trade Balance/Output Std Dev.(%)	0.38	2.96	0.21	0.26
Corr. (Trade Balance/GDP, Output)	-0.87	-0.10	-0.18	-0.25
<i>After Default</i>				
Consumption Std./Output Std.	1.14	1.65		
Trade Balance/Output Std Dev.(%)	0.40	3.05		
Corr. (Trade Balance/GDP, Output)	-0.92	-0.04		

Sources: Aguiar and Gopinath (2006); Arellano (2008); MECON.

As is obvious in Table 5, the model matches business cycle statistics in data in both pre-default and post-default periods. Our model replicates volatile consumption and trade balance/GDP volatility, both of which are prominent features of emerging economy business cycle models as in Aguiar and Gopinath (2007) and Neumeyer and Perri (2005). Trade balance/output standard deviation in the model is much higher than that of data because trade balance in our model also includes variations of imports merely driven by real exchange rate fluctuations. Moreover, it also generates a negative correlation between trade balance and output.

On non-business cycle statistics, the model shows relations among bond spreads, debt/GDP ratio and output, as in the data in both pre-default and post-default periods. Bond spreads are positively correlated with debt/GDP ratio, but negatively correlated with output. This is because default probability is high, leading to higher spreads when debt/GDP ratio is high and output is low. Our simulation also reproduces similar levels of average bonds spreads and volatility of spreads in both pre-default and post-default periods, though simulated moments in post-default periods are closer to those in the data. However, we see some deviations of average debt/GDP ratio from the total debt service/GDP ratio in data in both pre-default and post-default periods.

What makes our model unique compared to previous studies is that our model generates four new statistics of the real exchange rate which match with the data. Among four moments, it is noteworthy that the current model replicates a higher average real exchange rate in the post-default period than in the pre-default periods, as observed in the data. We also explain that the real exchange rate negatively correlates with output, but positively correlates with spreads in both the pre-default and post-default periods. Simulated real exchange rate volatility is 9.6%, close to data (5.0%) in the pre-default period, whereas it is 9.4%, much lower than data (27.6%) in the post-default period.

Table 5. Non-Business Cycle Statistics for Argentina¹

	Data	Model	A. and G. (2006)	Arellano. (2008)
Target Statistics				
Default Probability	3.3	3.4	0.92	3.0
Non-Target Statistics				
<i>Before Default¹</i>				
Average Debt/GDP ratio	45.4 / 8.0	13.5	-	5.95
Corr. (Spreads, Output) ²	-0.62	-0.19	-0.29	-0.29
Average Bond Spreads. (%) ²	7.6	5.9	-	3.58
Bond Spreads Std Dev. (%) ²	2.7	5.8	8.00	6.38
Corr. (Debt/GDP, Spreads) ²	0.92 / 0.93	0.38		-
Average Real Exchange Rate ³	0.95	0.98		
Real Exchange Rate Std Dev. (%)	4.7	9.4		
Corr. (Exchange, Output)	-0.56	-0.18		
Corr. (Exchange, Spreads) ²	0.62	0.84		
<i>After Default²</i>				
Average Debt/GDP ratio ⁴	75.3 / 19.8	11.7	-	-
Corr. (Spreads, Output) ⁴	-0.73	-0.16	-	-
Average Bond Spreads. (%) ⁴	6.7 / 22.9	6.2	-	-
Bond Spreads Std Dev. (%) ⁴	4.0 / 23.1	5.9	-	-
Corr. (Debt/GDP, Spreads) ⁴	0.95 / 0.83	0.32	-	-
Average Real Exchange Rate ³	2.23	1.05		
Real Exchange Rate Std Dev. (%)	31.5	9.5		
Corr. (Exchange, Output)	-0.65	-0.20		
Corr. (Exchange, Spreads) ⁴	0.55	0.74		

Sources: Aguiar and Gopinath (2006); Arellano (2008); Datastream; IMF IFS and WEO; MECON.

¹Spreads corresponds to spreads on foreign currency bonds.

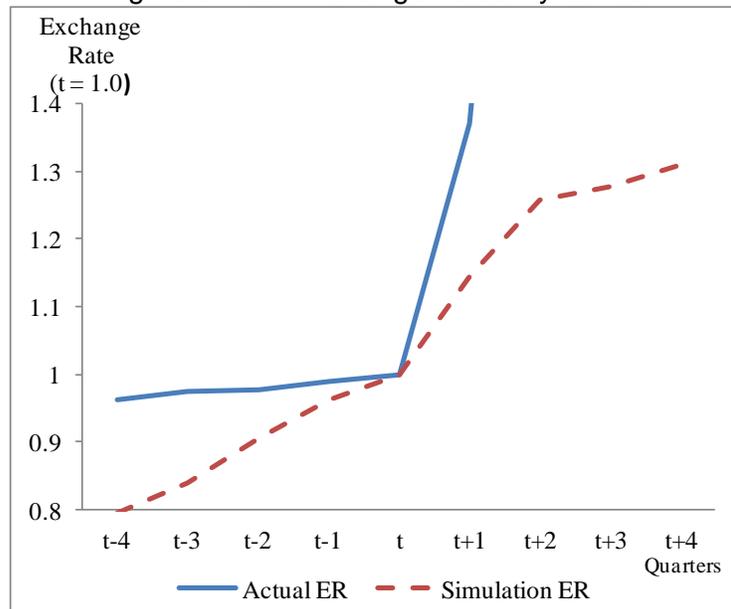
²Data for spreads are from 1997Q1 to 2001Q4 for Argentina.

³Over 10 quarters.

⁴Excluding autarky periods.

Figure 6 contrasts the simulated process with the actual dynamics of the real exchange rate of Argentina before and after default. We replicate two features of real exchange rate movements around defaults. In the model, before defaults, the sovereign receiving a series of low traded goods shocks, tends to accumulate more debt and faces real exchange rate depreciation. Since a majority of debt is denominated in foreign currency, this, in turn, increases the burden of payments in terms of local currency, increasing default probability and forcing the sovereign to default. Once the sovereign declares default, it suffers output costs due to default and loses access to the market. By defaulting, the sovereign enjoys higher consumption of traded goods, indicating a lower marginal utility of consumption, which leads to both a higher price of nontraded goods and a higher overall price level. Associated with an increase in the domestic CPI relative to that of the creditors, this results in a further depreciation of the real exchange rate. This mechanism drives the equilibrium depreciation of real exchange rate in the model and it is a plausible explanation of observed pattern in the data.

Figure 6. Real Exchange Rates Dynamics



Sources: Author's computation; and IMF IFS.

D. Comparison with the Model of a Risk-neutral Creditor

To understand the role of a creditor's risk aversion, we contrast moments of bond spreads to those under a conventional sovereign debt model with a risk-neutral creditor, as in Aguiar and Gopinath (2006) and Arellano (2008). As reported in Table 8, average bond spreads in the current model are higher and closer to the data than those in a model with a risk-neutral creditor in both pre- and post-default periods. Moreover, the current model generates higher standard deviations of bond spreads than the model with a risk-neutral creditor. These are associated with high average bond spreads, since we assume no spreads when the sovereign is in autarky. What drives a large difference in average bond spreads is the risk aversion of the creditor. Both average bond spreads and the standard deviation in the current model are higher and closer to the data than those in a traditional sovereign debt model with a risk-neutral creditor. In a standard model with a risk-neutral creditor, bond spreads do not include any spread premia since bond prices are simply determined by default probability. On the contrary, in the current model with the risk-averse creditor, bond prices are determined by interaction between stochastic discount factors and expected payoff, as shown in equation (25) and (26). Risk premia, due to risk aversion of the creditor, are included in bond spreads and increase spreads close to the data.

Table 6. Statistics for Bond Spreads

	Data	Baseline	Model – Risk-neutral creditor
<i>Before Default</i>			
Average Bond Spreads. (%)	7.6	5.9	1.1
Bond Spreads Std Dev. (%)	2.7	5.8	2.4
<i>After Defaults/Restructurings</i>			
Average Bond Spreads. (%)	6.7 / 22.9	6.2	0.8
Bond Spreads Std Dev. (%)	4.0 / 23.1	5.9	2.0

Sources: Author's computations; and Bloomberg.

E. Policy Implication - Share of Domestic Currency Debt

In this subsection, we explore how share of domestic currency debt in total external debt influences exchange rate dynamics.

Table 7 shows how default probability, average debt and real exchange rate moments change under different values of share of domestic currency debt α leaving other parameters at their benchmark values. Higher share of domestic currency debt leads to lower default probability and lower average real exchange rate in post-default periods; if the sovereign issues more debt in domestic currency ($\alpha = 0.5$), one-percentage real exchange rate depreciation does not increase much burden of payments in terms of local currency, reducing default probability. The higher share of domestic currency debt also results in higher average debt. On the contrary, standard deviations of real exchange rate are similar to those under a smaller share of domestic currency debt ($\alpha = 0.01$),

Table 7. Share of Domestic Currency Debt and Key Non-Business Cycle Statistics

	Share of domestic currency debt		
	$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.01$
Default Probability	3.05	3.2	3.4
<i>Before Default</i>			
Average Debt/GDP ratio	17.9	15.3	13.5
Average Real Exchange Rate	0.965	0.973	0.98
Real Exchange Rate Std Dev (%)	9.8	9.7	9.4
<i>After Default</i>			
Average Debt/GDP ratio	15.6	13.5	11.7
Average Real Exchange Rate	1.037	1.04	1.05
Real Exchange Rate Std Dev. (%)	9.8	9.7	9.5

Source: Author's calculation.

The results above suggest insightful policy implications on currency denomination of external debt. The sovereign debtor would receive benefits of balancing the currency composition of external debt by issuing more domestic currency debt. If it does so, it would see smaller risks of an increase in debt burden in local currency driven by the exchange rate depreciation. Consequently, the sovereign becomes more patient to repay the debt, i.e., less willing to default. This enables the sovereign debtor to accumulate larger debt in both pre-default and post-default periods than the baseline case. Even if it defaults due to bad tradable goods shocks, it could reduce further exchange rate depreciation because an increase in consumption on tradable goods associated with defaults is mitigated owing to a smaller increase in debt payments in local currency.

Transferring exchange rate risks from the sovereign debtor to foreign creditors could also potentially benefit foreign creditors. The standard effect of issuing more domestic currency debt is that foreign creditors charge higher premium on domestic currency debt. This is simply because the creditors who are risk averse, perceive more risks of a decrease in debt payments in their currency. On the contrary, they reduce spreads on domestic currency debt since they see smaller default probability on domestic currency debt than the baseline case. This is clearly beneficial for creditors that they perceive more certainty on payments on domestic currency debt as the debtor could succeed transferring repayment risks due to

exchange rate depreciations. When the latter effect dominates the former, foreign creditors find accepting transferred exchange rate risks desirable.

VI. CONCLUSION

Emerging countries experience real exchange rate depreciations around default events. This paper attempts to explore this observed evidence within a dynamic stochastic general equilibrium model in which bond issuance in local and foreign currencies is explicitly embedded and the real exchange rate and default risk are determined endogenously. Our quantitative analysis using data of Argentina, replicates a link between real exchange rate depreciation and default probability before and after defaults.

In the model, before default, the sovereign, receiving a series of low tradable goods shocks, tends to accumulate more debt and faces real exchange rate depreciation which is defined as units of home consumer price indices (CPI) against one unit of foreign CPI. Since a majority of debt is denominated in foreign currency, this, in turn, increases the burden of payments in terms of local currency, increasing default probability and forcing the sovereign to default. Once the sovereign declares default, it suffers output costs due to default and loses access to the market. By defaulting, the sovereign prefers to have a higher consumption of traded goods indicating lower marginal utility of consumption, which leads to both a higher price of nontraded goods and a higher overall price level. Thus, the default ends up with a further depreciation of the real exchange rate. This mechanism drives the equilibrium depreciation of real exchange rate in the model, and it is a plausible explanation of the observed pattern in the data. So far, we have analyzed the endogenous real exchange rate dynamics before and after the default in the framework, where income processes are exogenous and output cost is fixed. It will be possible to consider interactions between real exchange rate depreciation and output costs due to default as in Mendoza and Yue (2012). This might be a potential area future research could explore.

Appendix I. Computation Algorithm

The procedure to compute the stationary equilibrium distribution of the model is the following.

- (i) First, we set grids on the space of asset holdings as $B = [-0.6, \dots, 0]$. The limits of the asset space are set to ensure that the limits do not bind in equilibrium.
- (ii) Second, we set finite grids on the space of endowments of both the sovereign and the creditor. The limits of each endowment space are big enough to include large deviations from the average value of shocks. We approximate stochastic income processes given by equation (38) using a discrete Markov chain of equally-spaced grids. Moreover, we calculate the transition matrix based on the probability distribution $\mu(y_{t+1}|y_t)$.
- (iii) Third, we set the initial value of the real exchange rate ($e_0 = 1$, and $e_0^D = 1$).
- (iv) Fourth, we set the initial value for equilibrium bond price. We use the risk-free bond price ($q_0 = \bar{q} = (1 + r)^{-1}$) for the baseline value for equilibrium bond price.
- (v) Fifth, given the baseline equilibrium bond price ($q_0^H = \bar{q}$) and real exchange rate ($e_0 = 1$), we solve for the country's and its creditor's optimization problems. This procedure finds the value function as well as default decisions. In order to solve the limit of the finite-horizon problem, we solve backwards. We start with the problem of last period. Then, we solve the last two-period problem. We keep iterating the process until we obtain the converged value function. We first guess the value function ($V^0, V^{D,0}$) and iterate it using the Bellman equation to find the fixed value ($V^*, V^{D,*}$), given the baseline bond price and real exchange rate. By iterating the Bellman function, we also derive the optimal asset policy function (b'). In addition, we obtain default choices, which require comparison of values of defaulting and non-defaulting choices. By contrasting these two values, we calculate a default set. Based on the derived default set, we also evaluate the default probability using a transition matrix.
- (vi) Sixth, using a default set in step (v) and bond price equations (25) and (26), we compute the new bond price (q_1^H and q_1^F). Then, we iterate (v) to have a fixed value of the equilibrium bond price.
- (vii) Seventh, using the default set in step (v) and equation (12), (13), (21), (22) and (27), we calculate the new real exchange rate (e_1, e_1^D). Then we iterate step (v) and (vi) to have a fixed value of equilibrium real exchange rate.

Appendix II. Data and Empirical Analysis in Section II

Table All1. Details and Sources of Macroeconomic Variables

Variable	Frequency	Source
Real exchange rate	Monthly	IMF IFS
Credit ratings	Monthly	S&P, Moody's
GDP growth rate	Yearly	IMF WEO
Debt service-to-GDP	Yearly	IMF WEO
Institutional quality	Monthly	PRC
IMF program	Monthly	IMF
LIBOR 1-year	Monthly	Bank of England
GDP growth rate differential	Yearly	IMF WEO
Real interest rate differential	Yearly	IMF IFS
Net foreign asset-to-GDP	Yearly	IMF IFS
Real oil price shock	Yearly	IMF WEO

We briefly explain how instruments used for regression analysis are appropriate. First, for the pre-default period, we use both U.S. GDP deviation from the trend and the U.S. Treasury bill rate as instruments for lagged real exchange rates. These variables, entered with a lag, are correlated with sovereigns, real exchange rates against the US dollar, but not with default probability of sovereigns. Moreover, in the first-stage regression, these variables are significant at 1 percent, and the specification fits the data well, shown by high Adj – R^2 and small root mean square errors (MSE).

Table All2. 1st-stage Regression Results for the Pre-Default Period

Dependent variable: Real exchange rate, lagged	(A) Baseline
Estimation	Least square
U.S. GDP deviation from trend, lagged	-22.62*** (5.13)
U.S. Treasury rate, lagged	0.27*** (0.02)
Sample	106
Adj – R^2	0.71
Root MSE	0.58

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

Next, for the post-default period, ratings of foreign countries and sovereigns' institutional quality are considered to be appropriate instruments for ratings, which are proxy for default choice / default probability. Sovereigns' ratings are correlated with ratings on foreign countries, since the method by how credit rating agencies assess other countries with similar economic size and macro framework influence sovereigns' ratings. In addition, institutional quality is one of the factors which credit rating agencies use to judge sovereigns' ratings. As the first-stage regression results clearly show, they are significant at 5 percent and 1 percent, respectively, and have high explanatory power (high Adj – R^2).

Table AII3. 1st-stage Regression Results for the Post-Default Period

Dependent variable: Credit ratings, lagged	(A) Baseline
Estimation	Least square
Credit ratings on foreign countries, lagged	-0.27** (0.12)
Institutional quality, lagged	0.10*** (0.018)
Sample	60
Adj – R ²	0.75
Root MSE	2.08

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

For robustness check for pre-default period regressions, we apply specification without global liquidity. As expected, our main result remains robust: exchange rate depreciation in the previous period significantly increases default probability (3rd column in Table AII4). When we also drop debt service-to-GDP from the vector of explanatory variables, exchange rate depreciation in the previous period continues to negatively influence default probability (4th column in Table AII4). To account for possibility of non-linearity of default probability with corresponding defaulting rates (below B3 or B-), we use non-linear credit rating series which defaulting rates (below B3 or B-) are symmetrically set to 1. Our regression results are robust if we use non-linear credit ratings (5th column in Table AII4).

Table AII4. Robustness Check on Pre-Default Period Regressions

Dependent variable: Ratings	(A) Baseline	(B) w/o Global Liquidity	(C) Omitting debt service- to-GDP	(D) Truncated ratings
Estimation	2-step GMM Fixed effect	2-step GMM Fixed-effect	2-step GMM Fixed effect	2-step GMM Fixed effect
Real exchange rate, lagged	-0.23* (0.12)	-0.39*** (0.15)	-0.19*** (0.06)	-0.26* (0.15)
GDP growth rate, lagged	0.21* (0.12)	0.23 (0.18)	-0.08 (0.08)	0.21* (0.12)
Debt service-to-GDP, lagged	-0.15* (0.08)	-0.16 (0.12)	-	-0.18** (0.08)
Institutional quality ²	-0.12 (0.21)	-0.40 (0.25)	-0.23 (0.14)	-0.23 (0.24)
LIBOR 1-year	0.69*** (0.22)	-	-	1.10*** (0.27)
Sample	48	48	56	48
Root MSE	1.48	2.45	2.12	1.48

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

¹Institutional quality is the quarterly average of monthly PRC composite political risk ratings from 1985–2012 with 100 and 0 as the highest and lowest indices respectively.

For post-default period regressions, we take a similar approach with the ordinary least squares (OLS) and fixed effect (FE) estimators. Even in these two specifications, default or announcement of restructurings obviously leads to further exchange rate depreciation (3rd and 4th columns in Table AII5). To capture influence of uncertainty on total debt sustainability by market participant, we include a dummy for external official debt restructurings, i.e., Paris Club restructurings. Our baseline results continue to hold even if we control the negative

market perception triggered by completion of external official debt restructuring (5th column in Table AII5) or the negative influence of banking crisis (6th column in Table AII5).³⁴

Table AII5. Regression Results on Post-Default Period

Dependent variable: Real Exchange Rate	(A) Baseline	(B) OLS	(C) Fixed Effect Regression	(D) Official Debt Restructurings	(E) Banking Crisis
Estimation	2-step GMM	OLS	Least Square	2-step GMM	2-step GMM
Pooled / Fixed effect	Pooled	Pooled	Fixed effect	Pooled	Pooled
Ratings, lagged	-0.25*** (0.03)	-0.11*** (0.02)	-0.037* (0.019)	-0.34*** (0.08)	-0.25*** (0.03)
GDP growth differential, lagged	-0.02** (0.009)	-0.04** (0.009)	-0.013 (0.008)	-0.06*** (0.015)	-0.02** (0.01)
Real interest rate differential, lagged	-0.004*** (0.002)	-0.004 (0.003)	-0.007*** (0.002)	-0.0007 (0.004)	-0.004*** (0.002)
Net foreign assets-to- GDP, lagged	-0.05*** (0.013)	-0.03** (0.011)	-0.026** (0.012)	-	-0.05*** (0.014)
Real oil price shock dummy, lagged ¹	2.67 (1.62)	-0.19 (0.20)	0.11* (0.95)	-0.27 (0.70)	2.67 (1.61)
IMF program	1.36 (1.10)	-0.16 (0.11)	0.20 (0.22)	-0.07 (0.38)	1.38 (1.10)
External Official Debt Restructuring ²	-	-	-	-0.58 (0.44)	-
Banking Crisis ³	-	-	-	-	-0.02 (0.09)
Constant	0.72 (1.06)	1.77 (0.19)	1.96*** (0.26)	2.53 (0.55)	0.72 (1.05)
Sample	32	52	52	53	32
Root MSE	0.30	0.29	0.21	0.52	0.30

Note: *, **, *** denote significance at 10%, 5%, and 1% respectively.

¹Real oil price shock is an indicator showing the world oil price index deflated by the U.S. Producer Price Index (PPI) for countries heavily dependent on oil prices, while 0 is given for those less dependent on oil prices.

²External official debt restructuring is a dummy variable indicating 1 for episodes which external official debt restructuring was completed during over the post-default period.

³Following Laeven and Valencia (2013), banking crisis dummy is set at 1 if there is a systemic banking crisis and 0 otherwise.

³⁴See Asonuma and others. (2015) which explore sovereign debt restructuring strategies and financial sector instability based on a large sample of restructurings.

Appendix III. Share of Foreign Currency Debt

Table AIII1. Average and Variance of Shares of Foreign Currency Debt

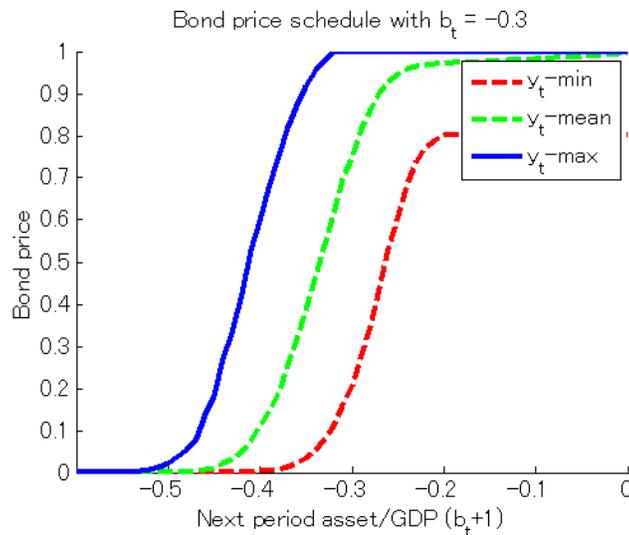
	Before / after default / restructuring		After default / restructuring	
	Average	Variance	Average	Variance
Argentina 2001–05	99%	0%	98%	0%
Belize 2006–07	100%	0%	100%	0%
Belize 2012–03	100%	0%	100%	0%
Cote D'Ivoire	100%	0%	100%	0%
Dominican Rep.	98%	1%	98%	0%
Equator 1998–2000	100%	0%	100%	0%
Ecuador 2008–09	100%	0%	100%	0%
Grenada 2004–05	84%	2%	84%	0%
Grenada 2013	74%	0%	74%	0%
Iraq	100%	0%	100%	0%
Moldova	100%	0%	100%	0%
Pakistan	100%	0%	100%	0%
Russia	100%	0%	100%	0%
Seychelles	100%	0%	100%	0%
Ukraine 1998–99	100%	0%	100%	0%
Uruguay	100%	1%	99%	0%

Appendix IV. Figures and Calibrated Moments for Bond Prices

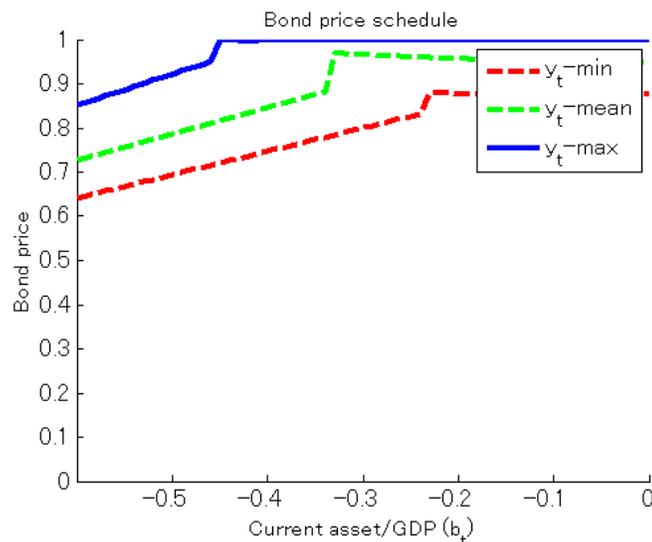
The left panel of Figure AIV1 shows the bond price schedule with a different level of new debt issuance given initial debt of 0.3. Bond price is weakly decreasing with the level of new bond issuance, since it incorporates default probability, which is weakly increasing with respect to the level of new bond issuance. Next, the bond price schedule with the initial level of debt is presented in the right panel of Figure AIV1. At each level of debt, bond price is computed based on an optimal amount of bond issuance. This figure clearly shows that bond price is weakly decreasing with level of initial debt because expected default probability is higher for the high level of initial debt.

Figure AIV1. Bond Prices

(A) Bond Price with Current Debt Fixed at 0.3



(B) Bond Price with Current Debt



We compare simulated moments of spreads on domestic and foreign currency bonds in the post-default periods with data.³⁵ The current model replicates a common feature that average spreads on domestic currency bonds are higher than those of foreign currency bonds, incorporating real exchange rate fluctuations as we see in the data. Average bond spreads for domestic currency bonds in our model are three times as high as those of foreign currency bonds. In addition, we also generate much more volatile domestic currency bonds than foreign currency bonds as observed in the data. Both real exchange rate fluctuations and investor risk aversion interact and produce high and volatile domestic currency bond spreads.

Table AIV2. Statistics for Bond Spreads in Post-default Periods

	Data ¹	Model
<i>After Default</i>		
Domestic currency debt		
Average Bond Spreads. (%)	26.4	20.3
Bond Spreads Std Dev. (%)	15.7	10.5
Foreign currency debt		
Average Bond Spreads. (%)	6.7 / 22.9	6.2
Bond Spreads Std Dev. (%)	4.0 / 23.1	5.9

Sources: Author's calculations; and Bloomberg.

¹For domestic currency debt, data are from 2009M1 to 2011M5.

³⁵Given the lack of spreads data on domestic currency bonds before 2009M1, we focus on moments of spreads in post-default periods.

Appendix V. Sensitivity Analysis—Income Processes and Elasticity of Substitution

Table AV1 reports key moment statistics under different values of the standard deviation of endowments σ^T and σ^N elasticity of substitution between tradable and nontradable goods and discount rate, leaving other parameters at their benchmark values.

Lower discount rates yield both a higher default probability and a higher level of debt. When the sovereign is less patient and willing to increase current consumption, the sovereign tends to accumulate more debt resulting in frequent defaults. Real exchange rate moments are similar to those in the baseline case.

When tradable and nontradable goods are highly substitutable ($\kappa = 2.5$), the sovereign can smooth volatility of consumption of tradable goods by substituting with nontradable goods. This results in a smaller change in marginal substitution of consumption, leading to a smaller change in the real exchange rate. Therefore, real exchange rate depreciation and volatility are smaller than those under the benchmark case.

Volatile income processes for both traded and nontraded goods result in higher default probability and lower level of debt. Due to an increase in volatility of income realization, the sovereign tends to default more frequently with a lower level of debt. Through positive correlation between the real exchange rate and spreads, volatile income processes for both traded and nontraded goods relate to high standard deviations of the real exchange rate in both pre-default and post-default periods.

Table AV1. Sensitivity Analysis for Income Processes and Elasticity of Substitution

	Baseline	δ		κ	
		0.80	0.96	0.8	2.5
Default Probability	3.4	3.8	1.6	1.8	0.5
<i>Before Default</i>					
Average Debt/GDP ratio	13.5	16.6	3.0	21.3	33.2
Average Real Exchange Rate ¹	0.98	0.98	1.02	0.98	0.99
Real Exchange Rate Deviations (%)	9.4	9.3	8.8	5.8	1.9
<i>After Default</i>					
Average Debt/GDP ratio ²	11.7	14.4	3.1	19.1	32.1
Average Real Exchange Rate ¹	1.05	1.05	1.05	1.02	1.00
Real Exchange Rate Deviations (%)	9.5	9.5	8.6	5.9	1.9
	Baseline	σ^T		σ^N	
		0.02	0.058	0.02	0.058
Default Probability	3.4	1.9	3.9	2.0	4.7
<i>Before Default¹</i>					
Average Debt/GDP ratio	13.5	16.7	12.7	15.0	13.0
Average Real Exchange Rate ¹	0.98	0.98	0.98	0.98	0.97
Real Exchange Rate Deviations (%)	9.4	7.6	9.9	8.1	10.3
<i>After Default</i>					
Average Debt/GDP ratio ²	11.7	15.1	11.1	13.3	11.3
Average Real Exchange Rate ¹	1.05	1.03	1.05	1.04	1.05
Real Exchange Rate Deviations (%)	9.5	7.6	9.8	8.1	10.3

Source: Author's calculation.

¹Over 10 quarters.

²Excluding autarky periods.

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