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Unconventional Policy Instruments in the New Keynesian Model

by Zineddine Alla, Raphael A. Espinoza, and Atish R. Ghosh

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Abstract

This paper analyzes the use of unconventional policy instruments in New Keynesian setups in which the 'divine coincidence' breaks down. The paper discusses the role of a second instrument and its coordination with conventional interest rate policy, and presents theoretical results on equilibrium determinacy, the inflation bias, the stabilization bias, and the optimal central banker's preferences when both instruments are available. We show that the use of an unconventional instrument can help reduce the zone of equilibrium indeterminacy and the volatility of the economy. However, in some circumstances, committing not to use the second instrument may be welfare improving (a result akin to Rogoff (1985a) example of counterproductive coordination). We further show that the optimal central banker should be both aggressive against inflation, and interventionist in using the unconventional policy instrument. As long as price setting depends on expectations about the future, there are gains from establishing credibility by using any instrument that affects these expectations.

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I. Introduction

Since the 2008 global financial crisis, central banks around the world have been forced to rethink their monetary and financial stability frameworks. Concerns about both financial stability and the risk of deflation have led central banks to use a variety of policy instruments, from macro-prudential tools to balance sheet operations, including credit policy, quantitative easing, and foreign exchange intervention (the latter, especially in emerging markets). As a result, old questions about the appropriate objectives of monetary policy, the desirability of targeting asset prices or other financial stability measures, and the instruments that should be in the central bank's toolkit, have re-emerged. These questions had seemed settled by the success that inflation-targeting central banks enjoyed during the so-called "Great Moderation." For instance, in his volume *Interest and Prices*, Woodford (2003), argues that central banks should *only* target the inflation of the basket of goods whose prices are updated the least frequently (because volatility in these prices is what distort most relative prices).[1] On the other hand, the crisis highlighted frictions other than nominal rigidities, in particular those that originate in financial intermediation. Acknowledging the need to include financial frictions in the standard framework, the literature has investigated the benefits of other policy regimes, starting from flexibilizing (e.g., Woodford (2012)) to more radical rethinking (Giavazzi and Giovannini (2012)) of inflation targeting.

In the standard model, however, the policy interest rate is sufficient to achieve economic stability because the inflation target and output at its first best level coincide – often called *divine coincidence* (Blanchard and Gali (2007)). Optimal monetary policy then consists of indexing the real interest rate on the natural rate of interest.[2] But when additional elements are added to the model, this "divine coincidence" often breaks down and the conduct of monetary policy becomes more challenging. These elements could be reduced-form, exogenous, cost-push shocks, as commonly included in New Keynesian Phillips Curves. Central banks then face a trade-off between reducing output volatility and inflation volatility (Taylor (1979), Clarida et al. (1999)). Or there could be frictions beyond the nominal rigidities already included in the New Keynesian Model. In models with real wage rigidities, stabilizing inflation and the output gap is not optimal (Blanchard and Gali (2007)). In models where interest rates affect marginal costs, standard policy rules may lead to indeterminacy (Surico (2008)) and monetary policy is inefficient (the output gap and inflation both fluctuate following productivity or demand shocks, see Ravenna and Walsh (2006)). Or there could be limits to the efficacy of standard interest rate policy – for instance, because of the zero-lower bound (Eggertsson and Woodford (2003)), because of risk premia in international capital markets (Farhi and Werning (2014)), or because of disruptions in the process of financial intermediation (Curdia and Woodford (2010)).

[1]For this reason, asset prices, which adjust at high frequency and thus reflect the market view of relative prices, should not be part of the inflation measure that guides monetary policy decisions.
[2]The interest rate that would prevail at the flexible allocation.

In each of these circumstances, it is natural to ask how a secondary, unconventional, policy instrument can alleviate the challenges faced by policymakers. Different instruments have been discussed, depending on the source of the friction: capital controls can lean against volatile capital flows when there are shocks to risk premia (Farhi and Werning (2014)); fiscal policy can support monetary policy if it is constrained (Correia et al. (2013)); quantitative easing can help reduce credit spreads that hamper financial intermediation (Curdia and Woodford (2011)); macroprudential policy can help resolve financial instability or aggregate demand externalities (e.g., De Paoli and Paustian (2013), Farhi and Werning (2013)). This literature has also touched upon the capacity of monetary policy alone to do the job (Woodford (2012)), and the need for coordination of the different policy instruments (Svensson (2014)).

In many of these papers, despite the diversity of circumstances considered, the formal models often boil down to an extended New Keynesian Model, where the linearized expected Investment Saving (IS) curve and Phillips curve are affected by the "friction," by the new instrument, and where (the quadratic approximation of) the welfare function directly includes the unconventional policy instrument (typically penalizing its use). That is the general problem we study. Our objective is to provide a unified framework to draw general results on the use of additional policy instruments. We show that additional policy instruments can be useful in ruling out equilibrium indeterminacy and in reducing welfare losses after exogenous shocks or in the presence of a distorted steady state, although under some circumstances, committing *not* to use the unconventional instrument may be welfare improving. We also establish that the inflationary bias and the stabilization bias are mitigated if the central bank aggressively uses the secondary instrument. Finally, we characterize the optimal preferences for the central bank governor in cases where societal preferences would result in indeterminacy.

Our contribution is to present a unifying framework in which to cast New Keynesian Models with multiple instruments and to derive general results that are applicable to a wide variety of models. Section 2 presents the analytical framework, which is a general linear New Keynesian Model, and discusses how it relates to different strands of the literature. Section 3 analyzes equilibrium determinacy, and characterizes the stabilization bias and the inflationary bias. Section 4 discusses the optimal preferences (over inflation and over the use of the second instrument) of the central bank to mitigate the inflationary bias and the stabilization bias, given the weights in the social welfare function. Section 5 concludes by discussing some of the policy implications of our analysis.

II. Analytical Framework

A. The Extended New Keynesian Framework

We want to analyze the optimal use of an unconventional policy instrument, denoted θ, in a general framework that comprises a Phillips curve, an expected IS curve and a quadratic loss function. Since Kydland and Prescott (1977), we know that as long as the dynamic system features expected terms, optimal policy under commitment is not time-consistent. We then consider a purely discretionary framework in which expected values of future variables are taken as given, and analyze the ways in which a central bank can reinforce its credibility.

Our approach is general enough to encompass various candidates for the unconventional instrument θ: public spending as in Gali and Monacelli (2008); fiscal policy (Alla (2016)); capital controls (Farhi and Werning (2014)); foreign exchange intervention (Alla et al. 2016)), quantitative easing (Curdia and Woodford (2011)) or macroprudential policy (e.g., De Paoli and Paustian (2013), Farhi and Werning (2013)).

The Dynamic Equations

A fairly general model is one in which the New Keynesian Phillips Curve (NKPC) and the IS curve take the following forms:

$$\pi_{H,t} = \Phi(\pi^e_{H,t+1}, y_t, y^e_{t+1}, i_t, \theta_t, \theta^e_{t+1}, u_t) \ , \ y_t = \Psi(y^e_{t+1}, \pi_{H,t}, \pi^e_{H,t+1}, i_t, \theta_t, \theta^e_{t+1}, v_t)$$

Where π_H is domestic inflation y is the output gap i is the policy interest rate θ is the unconventional policy instrument, u and v are exogenous shocks, and Φ and Ψ are linear functions.

Note that this formulation is more general than the standard New Keynesian Model. In particular, in the standard model, there is no additional instrument ($\partial\Phi / \partial\theta_t = \partial\Phi / \partial\theta^e_{t+1} = \partial\Psi / \partial\theta_t = \partial\Psi / \partial\theta^e_{t+1} = 0$) and the interest rate does not enter the NKPC ($\partial\Phi / \partial i_t = 0$), and some of the coefficients in Φ and Ψ are constrained. Model modifications that change these coefficients are not minor as they can affect essential results in the monetary policy literature (equilibrium determinacy, for instance). Substituting for the interest rate[3] in the Phillips Curve,[4] we can summarize the model's dynamics by:

$$\pi_{H,t} = k_\pi \pi^e_{H,t+1} + k_y y_t + k_{y^e} y^e_{t+1} + k_\theta \theta_t + k_{\theta^e} \theta^e_{t+1} + u_t \tag{1}$$

It is important to note that our results would apply to any optimal control problem, not just models where the state variables are output and inflation. The only important ingredient is that the unconventional instrument and its expected value affect the variable the central bank wants to stabilize.

[3]This substitution is only possible if the interest rate enters the Phillips curve. If not, the NKPC directly takes the form of equation (1).

[4]We choose to keep the Phillips Curve because it is the relevant dynamic equation in the standard New Keynesian model in which the interest rate can control output in the IS curve. However, this is without loss of generality.

Let us describe briefly how the aforementioned secondary instruments would affect economic dynamics. In Farhi and Werning (2014), capital controls introduce a wedge between domestic and foreign consumption levels, and thus impact domestic output and domestic consumption asymmetrically in an open economy framework. Capital controls then enter the IS curve since they affect consumption choices. In addition, since an increase in domestic consumption increases the real wage at which domestic households supply labor, capital controls also affect firms' marginal costs and thus enter the Phillips Curve.

In Alla et al. (2016), sterilized foreign exchange interventions generate an endogenous risk premium that increases foreign investors' rate of return (i.e. the effect of intervention is via the portfolio balance channel). This risk premium allows the central bank to exert some control over domestic consumption, independently from output. This channel is similar to the one modelled in Farhi and Werning (2014).

Alla (2015) also analyzes the optimal VAT and labor tax (fiscal devaluation) paths following a variety of exogenous macroeconomics shocks. The VAT affects domestic consumption, and thus both the inflation rate and output dynamics. The labor tax, paid by firms, on top of wages, affects the firms' marginal cost and thus enters the Phillips Curve linearly, in a way to could be described as an "endogenous cost-push shock."

Finally, Woodford (2012), allowing for heterogeneous households whose marginal utilities of income differ, models the difference between these two marginal utilities as an endogenous state variable representing a financial friction. This variable measures the distortion in the allocation of expenditure due to credit frictions, and is positively related to leverage and output (in a non-linear way). This variable impacts the IS curve, since a worsening of the financial friction affects aggregate demand. It also enters the Phillips Curve since changes in this financial friction shift the relationship between aggregate real expenditure and the marginal utility of income.

The Objective Function and the Intertemporal Constraint

The objective is to minimize the welfare loss function:

$$\sum_{t=0}^{\infty} \beta^t \left[\alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \right]$$

where α_π and α_θ are the weights on inflation and on the unconventional instrument, respectively, with the weight on the output gap normalized to unity. The first two terms are standard in the New Keynesian framework, and stand for the distortionary costs due to variations in inflation and output. Since the secondary instrument affects allocations, its distortive effect must also be costly (from a welfare perspective in the quadratic approximation of the social welfare function; else, an extreme use of this instrument would not be costly from a welfare point of view, a situation that is implausible for a tool that affects macroeconomic dynamics.

Since we are in a discretionary framework, the central banker's problem boils down to minimizing the current term of the above expression:

$$\alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \tag{2}$$

If the unconventional policy instrument has budgetary implications (for the Treasury, the central bank, or the country as a whole), one may need to take into account an intertemporal budget constraint of the form:[5]

$$\sum_{t=0}^{\infty} \beta^t \theta_t = 0 \tag{3}$$

In many models, this constraint can be derived endogenously; see for instance Farhi and Werning (2014) or Alla et al. (2016). Although our analysis will take into account the government's (or central bank's) inability to commit to specific future policies (on asset purchases, the deficit, etc.), we assume it can commit not to default. Unless the government can commit not to default, the intertemporal budget constraint means that the second instrument could not be used.[6] We define Γ as the Lagrange multiplier associated with the intertemporal budget constraint (3); Γ can be set to 0 if this constraint is not relevant to the specific problem under study.

B. The Rationale for Unconventional Policy Instruments

Breaking the Divine Coincidence

Since our purpose is to analyze the relevance and implications of additional policy instruments, we consider two cases where monetary policy alone cannot perfectly stabilize the economy.
The first case, which we refer to as *exogenous shocks*, represents any model element that leads to additive factors in the Phillips or the IS curve and breaks the divine coincidence. Since monetary

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[5]A No-Ponzi condition would imply that the discounted value of the instrument is smaller than a given value. This value is normalized to zero since a non-zero target would imply a steady-state deviation of $\overline{\Theta}$ in the non-linearized budgetary equation (θ is the log-deviation of the secondary instrument). For the same reason, the constraint is an equality.

[6] In a purely discretionary framework, the government would promise at each period to reimburse the current period deficit with future revenues: $\theta_t = -\sum_{s=t}^{\infty} \beta^{s-t} \theta_s$. However, this promise, renewed each period, is not credible since it omits past deficits. The only solution consistent with rational expectations is then $\theta_t = 0$:
$\theta_t = -\sum_{s=t+1}^{\infty} \beta^{s-t} \theta_s = -\beta \sum_{s=t+1}^{\infty} \beta^{s-(t+1)} \theta_s = 0$. Such an intertemporal constraint thus requires a commitment not to default.

policy is then insufficient to ensure perfect stabilization, the secondary instrument can help smooth economic fluctuations. These exogenous shocks have appeared in recent models, for instance: risks to financial intermediation in the version of the model of Curdia and Woodford (2009) where leverage is exogenous (see Woodford (2012)); risks premia in models of capital flows that describe the investors preferences for the government bonds from a given country (Farhi and Werning (2014)). Because they break the divine coincidence, they are important for much of our analysis. Exogenous shocks that affect the welfare criterion (by moving the stabilization targets) have similar implications and can also be analyzed within our framework.

We also introduce *financial frictions* in a reduced form, as any model component that implies that the (domestic or foreign) interest rate enters the Phillips curve. This is a fairly general definition of a financial friction, since balance sheet valuation, costs of working capital, etc., can all be related to the interest rate. The cost channel of monetary policy is operative when firms' marginal costs are affected directly by the interest rate, and is well documented in empirical studies (Tillman (2008) shows that the cost channel adds significantly to the explanation of inflation dynamics). Such a financial friction can lead to monetary policy indeterminacy (see Surico (2008)) and reduces the efficacy of monetary policy as a stabilization tool (Ravenna and Walsh (2006)).

Forward-Looking Determinants of Inflation

The presence of such a financial friction is not, however, required for our analysis. What matters is the presence of the expected terms y_{t+1}^{e} and θ_{t+1}^{e} in the Phillips Curve.[7] There is no empirical consensus on the role of expectations in the new Keynesian Phillips Curve (Mavroeidis et al. (2014)), but this has not prevented the macroeconomic literature from restricting itself to future inflation or future output gap as the sole forward-looking determinant of inflation. This assumption can lead to strong policy prescriptions. For instance, Clarida et al. (1999)'s argument in favor of a conservative central banker is based on the assumption that only inflation expectations influence price-setting behaviors entering the Phillips Curve:

> *"If price-setting depends on expectations of future economic conditions, then a central bank that can credibly commit to a rule faces an improved short-run trade-off between inflation and output. [...] The solution under [a rule][8] in this case perfectly resembles the solution that would obtain for a central bank with discretion that assigned to inflation a higher cost than the true social cost."*

We investigate in this paper how other instruments, whose expected values could also affect current inflation, should be used by central banks.

[7] One such case is when the interest rate enters the Phillips Curve by substituting it using the IS curve, but this is not the only situation where this could happen.
[8] Clarida et al. (1999) use the word "solution under commitment," but this is meant as a synonym for solution under a rule rather than the "first-best" commitment solution.

C. An Example

At this stage it may help intuition to provide an example of such an extended New Keynesian Model, in the context of an open economy where the firms' marginal costs depend on the current interest rate, as in Ravenna and Walsh (2006). Capital controls, modeled along the lines of Farhi and Werning (2014), are available to the central bank as an unconventional instrument. The policy problem at any date t is to minimize the quadratic loss function:

$$\min_{\{i_t, \pi_{H,t}, y_t, \theta_t\}} \alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \tag{4}$$

subject to the Phillips Curve and the IS curve:

$$\pi_{H,t} = \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta^\pi \theta_t + \kappa_{\theta^e}^\pi \theta_{t+1}^e + \kappa_f i_t + u_t$$

$$y_t = y_{t+1}^e - (i_t - \pi_{H,t+1}^e - \rho) + \kappa_\theta^y \theta_t + \kappa_{\theta^e}^y \theta_{t+1}^e$$

and to the intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \beta^t \theta_t = 0$$

where θ_t represents the wedge, due to capital controls, between foreign and domestic (exchange-rate adjusted) consumption levels ; see Farhi and Werning (2014) for details. Substituting for the interest rate in the Phillips curve by using the IS curve leads to the dynamic equation:

$$\pi_{H,t} = (\beta + \kappa_f)\pi_{H,t+1}^e + (\kappa_y - \kappa_f) y_t + \kappa_f y_{t+1}^e + (\kappa_\theta^\pi + \kappa_f \kappa_\theta^y)\theta_t + (\kappa_{\theta^e}^\pi + \kappa_f \kappa_{\theta^e}^\pi)\theta_{t+1}^e + u_t$$

The financial friction introduces the expected terms y_{t+1}^e and θ_{t+1}^e in the dynamic behavior of inflation. We will show how this affects several of the main results in the monetary policy literature. Note, in addition, that the financial friction also increases the forward coefficient on inflation, because any increase in expected inflation would lower the real interest rate, and thus requires a hike in the nominal interest rate to stabilize output. This increase in the interest rate affects firms' costs and thus contemporaneous inflation.

Finally, the intertemporal budget constraint represents a no-Ponzi condition on the country's net foreign asset position. Capital controls, by imposing a wedge between domestic and foreign interest rates, distort the path of domestic consumption and thus the trade balance. The present value of this distortion must be zero.

III. The Need for Unconventional Policy Instruments

A. Equilibrium Determinacy

We first analyze the conditions under which equilibrium determinacy is guaranteed under discretionary policy. To do so, we solve the maximization problem and substitute optimal policies in equation (1) to assess the dynamics of π_H. The first-order conditions for y_t and θ_t are respectively:

$$y_t = -\alpha_\pi k_y \pi_{H,t} \tag{5}$$

$$\theta_t = -\frac{\alpha_\pi}{\alpha_\theta} k_\theta \pi_{H,t} - \frac{\Gamma}{\alpha_\theta} \tag{6}$$

Domestic inflation thus obeys the following law of motion:

$$\pi_{H,t} = \frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \pi_{H,t+1} - \frac{\left(\frac{k_\theta + k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \Gamma \tag{7}$$

Equation 5 shows that the optimal policy is to choose a positive level of inflation together with a negative output gap (or a negative level of inflation with a positive output gap)—otherwise, if the output gap and the inflation were positive, the central bank could reduce both by increasing the interest rate. In other words, the central bank "leans against the wind," engineering a contraction if inflation is excessive. Similarly, for a given Lagrange multiplier, the unconventional instrument is used to moderate inflation. However, as the budget constraint becomes tighter, the use of the secondary instrument is restrained ($|\theta|$ falls). We also use equation (7) to determine the conditions for equilibrium determinacy.

Proposition 1. Equilibrium Determinacy under Discretionary Policy

Equilibrium determinacy is ensured when the Blanchard-Kahn condition is satisfied, i.e., when:[9]

[9] and $\alpha_\pi < \frac{k_\pi + 1}{k_y(k_{y^e} - k_y) + (k_\theta / \alpha_\theta)(k_\theta^e - k_\theta)}$ if $k_y(k_{y^e} - k_y) + (k_\theta / \alpha_\theta)(k_\theta^e - k_\theta) > 0$. If the expected impact of output and of the second instrument on inflation is greater than the current impact, optimal policy might result in expected inflation being stabilized more effectively than current inflation, leading to indeterminacy. While such a situation may appear counterintuitive (in particular, inflation would change sign at each date), it can be avoided by ensuring that the weight on inflation, α_π, is not too high: the expected impact is then offset by the indexation of current inflation on expected inflation, ensuring determinacy. We omit this condition in the rest of this section.

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \dfrac{k_\theta(k_\theta + k_\theta^e)}{\alpha_\theta}} \tag{8}$$

Proof: The proof simply consists in applying the Blanchard-Kahn condition to equation (7), i.e., verifying that:

$$\left| \frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right| < 1$$

What are the conditions under which the model leads to indeterminacy? In the standard New Keynesian Model, $k_{y^e} = k_\theta = k_\theta^e = 0$ and $k_\pi = \beta < 1$. This implies that the denominator of the right hand side of (8) is positive, that its numerator is negative, and, since $\alpha_\pi > 0$, the Blanchard-Kahn condition is satisfied.[10] Equilibrium determinacy is thus guaranteed. In the general model, however, there are parametrizations for which the Blanchard-Kahn condition could be violated. An important situation where this could happen is when the financial friction is non-negligible ($k_f > 1 - \beta$ in problem (4)), and more generally when current inflation is strongly determined by expected inflation, in which case, the numerator in (8) becomes positive.

To understand the role of the second instrument in ensuring determinacy, it is useful to first understand the determinacy condition when the second instrument is not used. This is found by adding a constraint $\theta_t = 0$ to the minimization problem (2) or, alternatively, by assuming that the cost of using the secondary instrument is infinite, i.e., $\alpha_\theta \to +\infty$ (so that $\theta \to 0$). The determinacy condition is then:

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e})}$$

Denoting by X_y the recession engineered by the central bank when inflation is 1 percent (i.e., $X_y = \alpha_\pi k_y > 0$, from equation 5), the first condition for equilibrium determinacy is:

$$(k_y + k_{y^e})X_y > k_\pi - 1$$

Intuitively, determinacy requires that the central bank's optimal decision is to engineer recessions

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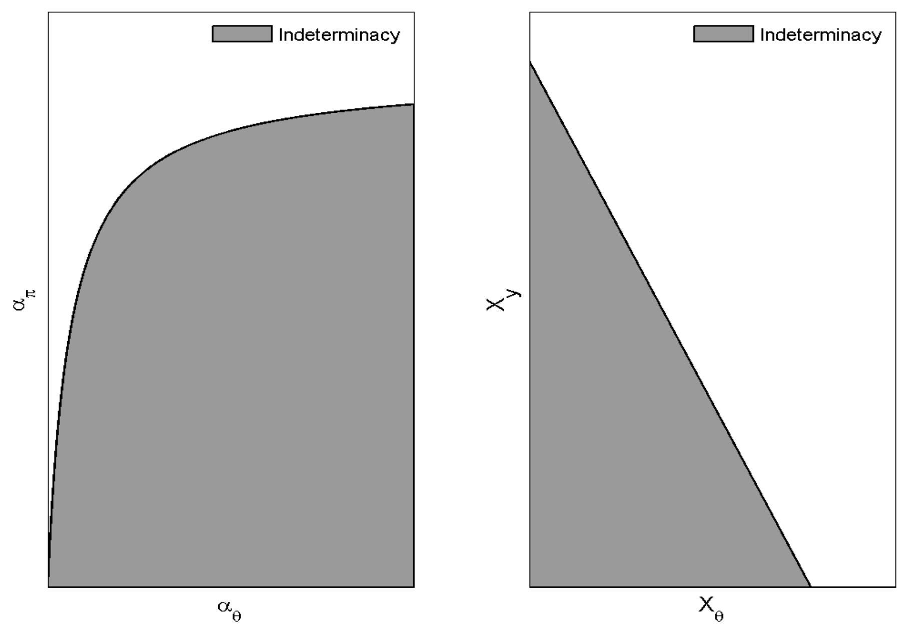
[10]Moreover, the second condition, detailed in the previous footnote, does not apply since $k_{y^e} = k_{\theta^e} = 0$.

such that the total impact on today's inflation, $1+(k_y+k_{y^e})X_y$, is greater than the dynamic impact of expected inflation on today's inflation, k_π; this ensures that current inflation is low, ruling out multiple equilibria.

However, with a financial friction, the decision to increase the interest rate also affects the marginal cost in the Phillips curve (k_π increases; this is akin to the cost channel of monetary policy). The recession must thus be deeper, or the sensitivity of inflation to the output gap higher, to ensure marginal costs are sufficiently reduced. If the weight on inflation in the loss function is too low, the recession engineered by the central bank may be insufficient to offset the impact of the financial friction on inflation. Current inflation may then be too high, resulting in multiple equilibria.

Figure 1. Optimal Policy Determinacy Condition



Source: Authors' calculations.

We now reintroduce the second instrument, and define $X_\theta = \frac{\alpha_\pi}{\alpha_\theta} k_\theta > 0$ as the marginal increase in the optimal use of the unconventional instrument for a decrease in the level of inflation. The determinacy condition becomes:[11]

$$(k_y + k_{y^e})X_y + (k_\theta + k_\theta^e)X_\theta > k_\pi - 1 \tag{9}$$

The rationale is as before. The optimal use of the new instrument (and its use in period $t+1$) can

[11]A second condition is $\alpha_\pi < \frac{k_\pi + 1}{k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta)}$ if $k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta) > 0$.

mitigate current inflation, the more so if the effect of the instrument on today's inflation is high (i.e., k_θ and k_θ^e are high) and if the central bank uses this instrument aggressively (if X_θ is high). Figure 1, shows the zone of indeterminacy provided by conditions (8) and (9). When the use of the unconventional policy instrument comes at no cost ($\alpha_\theta = 0$, see left-hand chart), or when the new instrument has a strong effect on inflation (X_θ is high, see right-hand chart) the risk of indeterminacy is eliminated, even if the central bank is not very willing to engineer recessions. The downward sloping frontier in the right-hand chart clarifies the trade-off: for given impacts of the interest rate and conventional policy instruments, the central bank must either be willing to engineer large recessions or to be activist with the second instrument.

B. Optimal Stabilization Policy Following Real Shocks

In this section, we analyze the complementarity of policy tools by focusing on optimal stabilization policy after exogenous shocks. We assume that the model parameters are such that equilibrium determinacy is guaranteed. We thus focus on how the unconventional instrument is used in presence of a cost-push shock. Our objective is to find theoretical results, which is why we consider cost-push shocks that enter the Phillips Curve linearly; this allows us to obtain closed-form solutions. These cost-push shocks, which are common in the literature, can also capture financial disruption, as in e.g., Curdia and Woodford (2010). However, our results would stand for more general exogenous shocks that distort the Phillips Curve or the loss function.

Proposition 2. Optimal Policy Following Cost-Push Shocks

Following a cost-push shock with autoregressive process $u_t = \rho_u^t u_0$, the optimal paths of inflation, output, and of the unconventional instruments are:

$$\pi_{H,t} = \frac{1}{D(\rho_u)} u_0 \rho_u^t - \frac{k_\theta + k_\theta^e}{\alpha_\theta D(1)} \Gamma; \quad y = -X_y \pi_{H,t}; \quad \theta_t = -X_\theta \left[\rho_u^t - \frac{1-\beta}{1-\beta\rho} \right] u_0 \tag{10}$$

Where:[12]

$$D(\rho_u) = 1 - \rho_u k_\pi + \alpha_\pi \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\alpha_\theta} \right]$$

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[12] Note that:

$$D(\rho_u) > 0 \Leftrightarrow \alpha_\pi > (\rho_u k_\pi - 1) \left\{ k_y (k_y + \rho_u k_f) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\alpha_\theta} \right\}^{-1}$$

$$\Leftrightarrow 1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right) > \rho_u \left[k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right) \right]$$

which is always true since the last inequality is verified for $\rho_u = 1$ in the Blanchard-Kahn condition (8) (and if the last bracket is negative, then the result is trivial).

Proof: The proof consists of iterating forward equation (7):

$$\pi_{H,t} = \sum_{i=0}^{\infty} \left(\frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right)^i \frac{1}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \left(u_0 \rho_u^{t+i} - \frac{k_\theta + k_\theta^e}{\alpha_\theta} \Gamma \right)$$

We solve for the Lagrangian multiplier Γ by using the intertemporal constraint and the first-order condition $\theta_t = -X_\theta \pi_{H,t} + \frac{\Gamma}{\alpha_\theta}$, yielding:

$$\Gamma = \frac{\alpha_\theta (1-\beta)}{1-\rho\beta} \frac{X_\theta}{D(\rho_u) \left(1 - X_\theta \frac{k_\theta + k_\theta^e}{D(1)} \right)} u_0$$

Using the unconventional instrument enables the central bank to stabilize inflation and output more efficiently. The impact of the unconventional instrument is captured by the term $\frac{\alpha\pi}{\alpha_\theta} k_\theta (k_\theta + \rho_u k_\theta^e)$ in $D(\rho_u)$. This formula is intuitive: the stabilization power of the second instrument is increasing in its current impact on inflation (coming from both current and expected actions), and is decreasing in the cost of using it.

As long as this term is positive, the impact of the cost-push shock on the economy is minimized thanks to the availability of the unconventional policy instrument.[13] However, if the impact of the expected use of the instrument more than offsets the impact of its current use ($k_\theta (k_\theta + \rho_u k_\theta^e) < 0$), then it is preferable to commit not to use the secondary instrument. In that case, the availability of the secondary instrument makes the economy more volatile, and a commitment not to use that instrument may be welfare improving. This result is akin to that of Rogoff (1985a), who argues that international monetary policy coordination could affect inflation expectations and worsen the trade-off faced by central banks.[14]

The use of the unconventional instrument is however constrained by the intertemporal budget constraint (equation (3)). A tighter budget constraint (a higher Γ in absolute value in equation (10)) reduces the ability of policymakers to stabilize the economy.

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[13]This is always the case, for instance, if the future unconventional instrument does not enter the Phillips curve and the IS curve (in which case $k_\theta^e = 0$).

[14]Rogoff's result may seem counter-intuitive inasmuch as the central bank, under coordination, could always choose the same policies as it would under the Nash equilibrium. Thus, by revealed preference, it would appear that the central bank could never be worse off under cooperation than under the Nash equilibrium. Likewise here, since the central bank could always choose not to use the second instrument, it would appear that its availability could never make the central bank worse off. In both examples, the revealed preference argument breaks down because of the presence of forward-looking private agents, together with the inability of the central bank to commit to future policies.

IV. Central Banker's Preferences

We analyzed above optimal policy assuming the central banker's and the social preferences coincide. However, the central bank's inability to commit to future policies restricts the space of feasible allocations, reduces its ability to stabilize the economy, and worsens social welfare. Kydland and Prescott (1977) and Barro and Gordon (1983) first showed how discretionary policy could lead to inefficient levels of inflation when the central bank targets a positive output gap (the *inflationary bias*). If the central bank cannot commit to future policies, it should thus target inflation more aggressively and tolerate a larger output gap in the current period in order to reduce inflation expectations, thus improving the trade-off characterized by the forward-looking Phillips Curve (Rogoff (1985b)). Clarida et al., (1999) extend this result by showing that even when the output objectives are realistic and the steady-state is efficient, the central bank could improve its short-run trade-offs by assigning to inflation a higher cost than the true social cost (*the stabilization bias*).

We investigate in this section which central banker's preferences (with respect to the weights on inflation and on the unconventional policy instrument in the loss function) minimize the welfare losses due to the stabilization bias and to the inflationary bias. Although alternative design strategies for central banks have been proposed (in particular in Walsh (1995) and in Svensson (1997)), we focus on preference weights for simplicity. We first explore the stabilization bias, and then present similar results for the inflationary bias (that may be seen as a particular case featuring a permanent shock).

A. The Stabilization Bias

If the weight that the central banker assigns to inflation is $\tilde{\alpha}_\pi$ and the weight on the unconventional instrument is $\tilde{\alpha}_\theta$, the central banker's objective is (using Proposition 2):[15]

$$W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right) = \frac{\tilde{\alpha}_\pi + \tilde{\alpha}_\pi^2 k_y^2 + \alpha_\theta \frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2} k_\theta^2 \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}}{\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta)^2} \frac{u_0^2}{1-\beta\rho_u^2}$$

where:

$$\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta) = 1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right]$$

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[15]In this section, we make the assumption that when the intertemporal constraint apply, its impact on optimal policy for inflation and output is small compared to the time-varying components of policy, i.e., $\left|\frac{1}{D(\rho_u)} u_0\right| >> \left|\frac{k_\theta + k_\theta^e}{\alpha_\theta D(1)} \Gamma\right| \Leftrightarrow 1 >> \left|\left(\frac{1-\beta}{1-\rho\beta}\right)\left(k_\theta + k_\theta^e\right) X_\theta \left(1 - k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + k_y^e) + \frac{k_\theta (k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta}\right]\right)^{-1}\right|$. This assumption is valid for shocks that are transitory (where ρ is sufficiently small). It is also easy to justify in a micro founded framework (Alla et al. 2016)).

The central banker who should be appointed is the one whose preferences are:

$$\left\{\tilde{\alpha}_{\pi}^{opt},\tilde{\alpha}_{\theta}^{opt}\right\} = \text{argmin } W\left(\tilde{\alpha}_{\pi},\tilde{\alpha}_{\theta}\right) \tag{11}$$

under the constraint that his preferences lead to equilibrium determinacy, i.e.,

$$\tilde{\alpha}_{\pi}^{opt} > \frac{k_{\pi}-1}{k_y(k_y+k_{y^e})+\frac{k_{\theta}(k_{\theta}+k_{\theta}^e)}{\tilde{\alpha}_{\theta}^{opt}}}$$

Proposition 3 presents the solution assuming that social preferences remain in the area where equilibrium determinacy is guaranteed. Proposition 4, in the next section, will present the solution for the "dual" problem of minimizing the social cost function when the initial social preferences would be in an area of indeterminacy.

Proposition 3. A Conservative and Interventionist Central Banker

If the social preferences are such that equilibrium determinacy is guaranteed:

(i) The central banker's optimal preferences cannot induce equilibrium indeterminacy;

(ii) When the shock is not highly persistent $\left(\rho_u < \frac{1}{k_{\pi}}\right)$, the central banker's preferences that minimize welfare losses are:

$$\tilde{\alpha}_{\pi} = \frac{1+\rho_u(k_y^e/k_y)}{1-\rho_u k_{\pi}}\alpha_{\pi};\tilde{\alpha}_{\theta} = \frac{1+\rho_u(k_y^e/k_y)\beta(1-\rho_u)^2}{1+\rho_u(k_{\theta}^e/k_{\theta})(1-\beta\rho_u)^2}\alpha_{\theta}$$

(iii) If the shock is sufficiently persistent $\left(\rho_u > \frac{1}{k_{\pi}}\right)$, then the optimal preferences are:

$$\tilde{\alpha}_{\pi} = +\infty; \tilde{\alpha}_{\theta} = \frac{1+\rho_u(k_y^e/k_y)\beta(1-\rho_u)^2}{1+\rho_u(k_{\theta}^e/k_{\theta})(1-\beta\rho_u)^2}\alpha_{\theta}$$

(iv) When the intertemporal budget constraint (3) for the unconventional instrument θ is not applicable, the optimal weight for this instrument is:

$$\tilde{\alpha}_{\theta} = \frac{1+\rho_u(k_y^e/k_y)}{1+\rho_u(k_{\theta}^e/k_{\theta})}\alpha_{\theta}$$

Proof: See section A.1 in the Appendix.

Corollary 1. *Optimal preferences*

Using Proposition 3, it is possible to show how the optimal central bank's preferences deviate from social preferences:

(i) $\tilde{\alpha}_\pi \geq \alpha_\pi$

(ii) $\tilde{\alpha}_\pi$ is increasing in the persistence of the shocks and in the effect of future output[16] on current inflation, k_y^e;

(iii) $\tilde{\alpha}_\theta < \alpha_\theta$ if $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$;

(iv) $\tilde{\alpha}_\theta$ is decreasing in the persistence of the shock if $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$.

Proposition 3, first shows that the optimal central banker always improves credibility and economic stability in the following sense: if the social preferences are such that equilibrium determinacy is guaranteed, then determinacy is also guaranteed under optimal preferences. In addition, determinacy may be obtained under the optimal preferences even when the social preferences are in the indeterminacy area. In other words, when an unconventional instrument is available, the optimal central banker uses it to improve its short-run trade-off and in doing so, she reduces the possibility of indeterminacy.

Proposition 3 and its corollary also show that the weight given to inflation by the optimal central banker is higher than social preferences ($\tilde{\alpha}_\pi \geq \alpha_\pi$). The advantage of appointing a "conservative central banker" even when the target for the output gap is zero was first explained in Clarida et al. (1999); because inflation depends on future output gaps, the central bank has always an incentive to promise strong future actions against inflation before reneging on its promises. Since, under rational expectations, the private sector anticipates this, inflation will be higher under discretionary policy than under commitment. A Rogoff conservative central banker can mitigate this bias. This result is valid in our more general framework.[17] In addition, the more persistent the shock, or the stronger the effect of future output on inflation, the more averse to inflation the central banker should be (if the shocks are one-off, i.e. $\rho_u = 0$, then $\tilde{\alpha}_\pi = \alpha_\pi$ because expected inflation is always zero and thus is unaffected by the commitment technology). The objective is indeed to tackle anticipations of

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[16] Which is equal to the financial friction coefficient for the example presented in subsection 2.3

[17] Our results for $\tilde{\alpha}_\pi$ are the same as those in Clarida et al. (1999) when $k_y^e = 0$.

inflation, and inflation expectations create inflation today (and the more so the higher k_π, for instance in presence of a financial friction). For very persistent shocks, when inflation is strongly influenced by expected inflation, the minimization problem (11) does not have an interior solution, and the optimal central banker is Mervyn King (1997)'s "inflation nutter", as he cannot accept any deviation of inflation from his target. Finally, the optimal weight on inflation does not depend on the presence of the second instrument: indeed, the central banker's weight on inflation does not depend on the cost of using this instrument (α_θ) or on its impact in the IS curve or Phillips Curve.

Proposition 3 also determines what the optimal preferences for the unconventional instrument should be. The central bank should use the secondary instrument more actively than if it were following social preferences ($\tilde{\alpha}_\theta < \alpha_\theta$) if $(k_\theta^e / k_\theta > k_y^e / k_y)$. This condition is one where the effect of future unconventional policy on inflation (relative to current policy) is larger than the effect of future conventional policy (relative to current policy).[18] This would be the case in the model of capital controls presented in section 2.3, for instance. Using the unconventional instrument aggressively enables the central banker to tackle expectations of high inflation, thus improving the short-run trade-off he faces. The optimal central banker should then not only be conservative, but also more interventionist with instruments whose future use affects substantially current economic conditions. Note also that the extent of deviation from social preferences for the use of θ (i.e., the ratio $\tilde{\alpha}_\theta / \alpha_\theta$) appears to be independent of the cost of inflation α_π.

Finally, findings (ii) and (iv) in Proposition 3 show that even when the shocks are one-off, the optimal weight for θ can be different from that of the social preferences because of the budget constraint (as mentioned earlier, the optimal weight for inflation $\tilde{\alpha}_\pi$ is equal to α_π when facing one-off shocks because there is no stabilization bias: expected inflation is always zero independently from the policymaker's credibility). The difference between the solutions for $\tilde{\alpha}_\pi$ and for $\tilde{\alpha}_\theta$ comes from the intertemporal budget constraint, $\sum_{t=0}^{\infty} \beta^t \theta_t = 0$. Because of this constraint, even after one-off shocks, $\sum_{t=1}^{\infty} \beta^t \theta_t \neq 0$ since $\theta_0 \neq 0$. Since the unconventional instrument is used in the future even for one-off shocks, how it is used is important for today's inflation and thus it is possible to improve the inflation-output trade-off by choosing a central banker whose preferences differ from the social preferences.

B. The Stabilization Bias when Optimal Preferences Trigger Multiple Equilibria

The previous results were found assuming that under optimal preferences, equilibrium determinacy is guaranteed. But if this not the case, who should be appointed as central banker? Assuming that the social costs of indeterminacy are large enough that it needs to be ruled out altogether, the

[18] Output is the reference since its weight is normalized to 1 in the objective function.

problem can be formalized as follows:

$$\left\{\tilde{\alpha}_{\pi}^{copt}, \tilde{\alpha}_{\theta}^{copt}\right\} = \operatorname{argmin} W\left(\tilde{\alpha}_{\pi}, \tilde{\alpha}_{\theta}\right) \tag{12}$$

subject to:

$$\tilde{\alpha}_{\pi}^{copt} > \frac{k_{\pi} - 1}{k_y (k_y + k_{y^e}) + \frac{k_{\theta}(k_{\theta} + k_{\theta}^e)}{\tilde{\alpha}_{\theta}^{copt}}}$$

and knowing that:

$$\tilde{\alpha}_{\pi}^{opt} \leq \frac{k_{\pi} - 1}{k_y (k_y + k_{y^e}) + \frac{k_{\theta}(k_{\theta} + k_{\theta}^e)}{\tilde{\alpha}_{\theta}^{opt}}}$$

Proposition 4. Optimal Preferences in Situations of Equilibrium Indeterminacy

If the optimal preferences described in Proposition 3 are indeterminate, then the optimal constrained choice $\left\{\alpha_{\pi}^{copt}, \alpha_{\theta}^{copt}\right\}$:

(i) Is located on the determinacy frontier ;

(ii) Features a higher weight on inflation $\alpha_{\pi}^{copt} > \alpha_{\pi}^{opt}$;

(iii) Features a lower weight on the unconventional instrument $\alpha_{\theta}^{copt} < \alpha_{\theta}^{opt}$ if, and only if, $(k_{\theta}^{e} / k_{\theta} > k_{y}^{e} / k_{y})$.

Proof: See section A.2 in the Appendix.

Proposition 4, shows that the optimal preferences are located on the determinacy frontier, to be as close as possible to social preferences. In addition, the optimal, constrained, choice always reinforces the central bank credibility, in the sense that it features a higher inflation weight, and a lower weight on the use of the unconventional instrument if and only if the effect of the future use of the instrument on today's inflation is strong enough. The intuition is similar to that of Proposition 3. If the central banker has an instrument whose future use matters a lot, he should be more interventionist with this instrument, even though the constraint on determinacy forces him to adopt “second-best”' preferences.

C. The Inflationary Bias

Finally, we undertake a similar analysis to solve for the optimal central banker's preferences if the social welfare objective function targets a level of output $\overline{y}$ that is higher than its steady state value (i.e., in presence of the traditional inflationary bias).

The social welfare loss is:

$$\min_{\{\pi_{H,t},y_t,\theta_t\}} \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\alpha_\pi \pi_{H,t}^2 + (y_t - \overline{y})^2 + \alpha_\theta \theta_t^2\right]$$

Subject to:

$$\pi_{H,t} = k_\pi \pi_{H,t+1}^e + k_y y_t + k_{y^e} y_{t+1}^e + k_\theta \theta_t + k_{\theta^e}\theta_{t+1}^e$$

and the intertemporal budget constraint:

$$\sum_{t=0}^{\infty}\beta^t \theta_t = 0$$

Proposition 5. The inflationary bias

Assume that the optimal preferences are determinate. We distinguish models in which there is an intertemporal budget constraints from models where this constraint does not apply.

(i) Assume that the instrument is not constrained by the intertemporal budget constraint (the constraint (3) does not apply),

a. If current inflation depends weakly on expected inflation $\left(k_\pi < 1\right)$, the central banker's preferences that minimize welfare losses are:

$$\tilde{\alpha}_\pi = \frac{1+(k_y^e / k_y)}{1-k_\pi}\alpha_\pi ; \tilde{\alpha}_\theta = \frac{1+(k_y^e / k_y)}{1+(k_\theta^e / k_\theta)}\alpha_\theta$$

b. If current inflation strongly depends on expected inflation $\left(k_\pi > 1\right)$, the central banker's preferences become:

$$\tilde{\alpha}_\pi = +\infty ; \tilde{\alpha}_\theta = \frac{1+(k_y^e / k_y)}{1+(k_\theta^e / k_\theta)}\alpha_\theta$$

(ii) Assume that the instrument is constrained intertemporally:

a. It is optimal not to use it, i.e., $\theta_t = 0$ (its weight is then irrelevant).

Proof: Similar to the proof of Proposition 3.

Since the problem is formally similar to that of the stabilization bias, the results and intuitions developed for Proposition 3 carry over. Item (ii) in Proposition 5 shows that the effect of the budget constraint on policy (captured by the term $(1-\beta)/(1-\beta\rho_u)$ in equation (10) is crucial. If the budget constraint is applicable to the problem at hand, when the shock is permanent ($\rho_u = 1$), the optimal use of the unconventional instrument would be constant, which is only possible if $\theta_t = 0$ given that $\sum_{t=0}^{\infty} \beta^t \theta_t = 0$. Intuitively, the intertemporal budget constraint on the second instrument, if applicable, means that any use today must be paid back by the opposite use in the future; hence, there is no purpose in using the instrument in the face of a permanent shock.

V. Conclusion

According to the Tinbergen principle, a policymaker needs as many (independent) instruments as (independent) objectives in order to reach his bliss point. In New Keynesian Models where there is divine coincidence, the twin objectives of zero inflation and zero output gap coincide, and one instrument (conventional monetary policy) is sufficient to stabilize the economy perfectly. In practice, policymaking is almost always more challenging than this result would imply because divine coincide does not hold well; this situation is often captured in theoretical models by the presence of cost-push shocks. The optimal response when the policy interest rate policy is the only available instrument is then to maintain a positive output gap as long as inflation stays below target. The depth of the global crisis, however, has forced central banks to explore the use of new instruments, either because the interest rate was constrained (by the zero-lower bound, by fixed currency arrangements) or because new objectives arose (for financial stability, for asset prices, for the balance of payments or for the exchange rate). These additional instruments, chosen according to availability and the central bank's specific objectives, have included balance sheet operations (quantitative easing), sterilized FX intervention, macroprudential policy, fiscal devaluations, and other measures.

The theoretical literature followed suit in justifying the use of such instruments in microfounded models. But the literature is yet to arrive at a consensus on when and how to use these instruments, and how to coordinate their use with the central bank's traditional tool, the policy interest rate.[19] The purpose of this paper has been to contribute to this literature by addressing the issue of instruments and objectives in a general but tractable framework of discretionary policy, and to examine how some of the key results in the monetary policy literature (determinacy, inflationary bias, discretionary bias, conservative central banker) carry over to a situation in which the central bank has additional instruments available. We establish that such additional instruments are useful in ensuring equilibrium determinacy and reducing economic volatility in presence of cost-push shocks, although under some specific parameterizations it is possible that committing *not* to use the unconventional instrument is optimal.

We also examined whether the intuition of Rogoff (1985b)'s conservative central banker holds in a

[19] A case in point is that of the central bank of Sweden, which split over the decision to use interest rate policy to reduce risks to financial stability (Svensson (2011), Svensson (2014)).

model with several instruments. We find that if the future use of the unconventional instrument has relatively more importance for inflation than the future output gap, then the optimal central banker is more interventionist with the instrument than social preferences would imply. In addition, we investigated how a conservative central banker could reduce the risk of equilibrium indeterminacy.

Extensions to our framework could include incorporating an explicitly stochastic setup (though in many situations, models are linearized and stochastic exogenous shocks do not change the results). More important, therefore, may be to allow for non-linear dynamics. This is particularly relevant for financial stability problems, characterized by abrupt transitions and regime-switching (Woodford (2012)). Finally, since the policy implications of these models depend on the coefficients that capture the effects of current and future instruments on current inflation, an important task for empirical analysis is to improve our knowledge of the shape of the Phillips Curve, and in particular the impact of unconventional instruments on economic activity and inflation.

Appendix

A.1. Proof of Propositions 3

Planning Problem

The central banker has to solve the following problem to determine his optimal preferences:

$$W\left(\tilde{\alpha}_\pi,\tilde{\alpha}_\theta\right)=\frac{\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\right]^2}\frac{u_0^2}{1-\beta\rho_u^2}$$

subject to

$$\tilde{\alpha}_\pi>\frac{k_\pi-1}{k_y(k_y+k_{y^e})+k_\theta(k_\theta+k_\theta^e)/\tilde{\alpha}_\theta}$$

We assume that the constraint is satisfied for the social preferences. We verify *ex post* that the constraint is also satisfied for the optimal preferences. We denote:

$$\tilde{W}(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta})=\frac{\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\right]^2}\frac{u_0^2}{1-\beta\rho_u^2}=\frac{N}{D^2}\frac{u_0^2}{1-\beta\rho_u^2}$$

Where:[20]

$$N=\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}\ ,\quad D=1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]$$

Optimal preferences

We then compute the partial derivatives:

[20]Since determinacy is ensured, $D>0$.

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \tilde{\alpha}_\pi} = 2\frac{\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2}\right]\tilde{\alpha}_\pi D - \left[k_y\left(k_y + \rho_u k_y^e\right) + \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]N}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

$$= 2\frac{\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2}\right]\left[1-\rho_u k_\pi\right]\tilde{\alpha}_\pi - \left[k_y\left(k_y + \rho_u k_y^e\right) + \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\alpha_\pi}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

We need to consider two cases:

- if $\rho_u k_\pi < 1$, there is an interior point where the partial derivative $\partial \tilde{W}(\tilde{\alpha}_\pi, (1/\tilde{\alpha}_\theta))/\partial\tilde{\alpha}_\pi$ is equal to zero.
- if $\rho_u k_\pi > 1$, this derivative is negative for any value of $\{\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\}$, the optimal solution is then $\tilde{\alpha}_\pi = +\infty$. The welfare loss converges to a finite value since it is bounded from below by zero.

The second partial derivative can be expressed as follows:

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} = 2\frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} D - \left(k_\theta + \rho_u k_\theta^e\right)N\right]}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

$$= \frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta}\left[1-\rho_u k_\pi + k_y\left(k_y + \rho_u k_y^e\right)\tilde{\alpha}_\pi\right] - \left(k_\theta + \rho_u k_\theta^e\right)(\alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2)\right]}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

If $1 > \rho_u k_\pi$, this second derivative necessarily admits an interior cancellation point. Let us first consider this case.

In this situation, each partial derivative cancels and changes signs in one point (for a given value of the other parameter). There is thus only one interior point in which the two derivatives cancel simultaneously. Since the also change signs in this point (from being negative to positive), this is the global minimum.

Using the partial derivatives formulations with N and D, we see that this interior point verifies:

$$\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2}\right]\tilde{\alpha}_\pi D = \left[k_y\left(k_y + \rho_u k_y^e\right) + \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]N$$

$$\frac{\alpha_\theta k_\theta^2 \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta}D = \left(k_\theta + \rho_u k_\theta^e\right)N$$

By dividing the two equations, we find that:

$$\tilde{\alpha}_\theta^{opt} = \frac{\left(k_y + \rho_u k_y^e\right)k_\theta}{k_y\left(k_\theta + \rho_u k_\theta^e\right)}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1+\rho_u\frac{k_\theta^e}{k_\theta}}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta;$$

We then substitute for $\tilde{\alpha}_\theta^{opt}$ in any of the above equations, and find that the optimal choice for inflation is:

$$\tilde{\alpha}_\pi^{opt} = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1-\rho_u k_\pi}\alpha_\pi;$$

If $1 < \rho_u k_\pi$, we saw that the optimal choice for the inflation coefficient is $\tilde{\alpha}_\pi^{opt} = +\infty$. Using the second equality for the partial derivative[21] $\partial\tilde{W}(\tilde{\alpha}_\pi,(1/\tilde{\alpha}_\theta))/\partial(1/\tilde{\alpha}_\theta)$, we find that the optimal choice for $\tilde{\alpha}_\theta^{opt}$ satisfies:[22]

$$\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi) = \left\{\frac{\alpha_\theta k_\theta \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\tilde{\alpha}_\pi\left(1-\rho_u k_\pi + k_y(k_y+\rho_u k_y^e)\tilde{\alpha}_\pi\right)\right\}\left\{(k_\theta+\rho_u k_\theta^e)(\alpha_\pi + k_y^2\tilde{\alpha}_\pi^2)\right\}^{-1}$$

Hence:

$$\lim_{\tilde{\alpha}_\pi\to+\infty}\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi) = \frac{1+\rho_u(k_y^e/k_y)\beta(1-\rho_u)^2}{1++\rho_u(k_\theta^e/k_\theta)(1-\beta\rho_u)^2}\alpha_\theta$$

The coefficient for the unconventional tool is then unchanged, and the optimal choice is:

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[21]Since it cancels out only once $\tilde{\alpha}_\pi$ is sufficiently large.

[22]Formally, for any couple $\{\tilde{\alpha}_\pi,\tilde{\alpha}_\theta\}$ we have: $\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta}\right) > \tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)$

Then, denoting $g(\tilde{\alpha}_\pi) = \tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)$, we have:

$$g'(\tilde{\alpha}_\pi) = \frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial\tilde{\alpha}_\pi} + \frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial\frac{1}{\tilde{\alpha}_\theta}}\frac{\partial\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}}{\partial\tilde{\alpha}_\pi} = \frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial\tilde{\alpha}_\pi}$$

by the definition of $\frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,(\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi))^{-1}\right)}{\partial(1/\tilde{\alpha}_\theta)}$. We then see that e.g.: $\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta}\right) > \tilde{W}\left(+\infty,\frac{1}{\tilde{\alpha}_\theta(+\infty)}\right)$.

$$\tilde{\alpha}_{\pi}^{opt} = +\infty \quad , \quad \tilde{\alpha}_{\theta}^{opt} = \frac{1+\rho_u \frac{k_y^e}{k_y}}{1+\rho_u \frac{k_\theta^e}{k_\theta}} \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \alpha_\theta;$$

Optimal Preferences Determinacy

Finally, let us prove that if the determinacy constraint is satisfied for the social preferences $\{\alpha_\pi, \alpha_\theta\}$, then it is also satisfied for the optimal preferences chosen by the central banker.

Given that the frontier is concave (see Figure 1 and equation (8)), and since $\alpha_\pi^{opt} \geq \alpha_\pi$, we see that if $\alpha_\theta^{opt} \leq \alpha_\theta$, then the optimal preferences are also determined.

We then consider the case when the unconventional instrument is less forward-looking than output (i.e., $(k_y^e / k_y) > (k_\theta^e / k_\theta)$), potentially inducing an optimal cost that is higher than the social cost. The slope of the optimal deviation is then:

$$S = \frac{\alpha_\pi^{opt} - \alpha_\pi}{\alpha_\theta^{opt} - \alpha_\theta} = \frac{(k_y^e / k_y) + k_\pi}{\left((k_y^e / k_y) - (k_\theta^e / k_\theta)\right)\left(1-\rho_u k_\pi\right)} \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{\alpha_\pi}{\alpha_\theta}$$

We want to compare this slope to the frontier derivative for $\alpha_\theta = \tilde{\alpha}_\theta$. Since the frontier is strictly concave, if S is greater than its derivative, the optimal preferences are in the determinacy area. Figure 2 illustrates the proof.

The frontier can be parametrized as follows:

$$\tilde{\alpha}_\pi^{fr}\left(\tilde{\alpha}_\theta\right) = \frac{a\tilde{\alpha}_\theta}{1+b\tilde{\alpha}_\theta}$$

where $a = (k_\pi - 1)\left(k_\theta(k_\theta + k_\theta^e)\right)^{-1}$ and $b = k_y(k_y + k_{y^e})\left(k_\theta(k_\theta + k_\theta^e)\right)^{-1}$. Its derivative for $\tilde{\alpha}_\theta = \alpha_\theta$ is then equal to $D = a\left(1+b\tilde{\alpha}_\theta\right)^{-2}$.

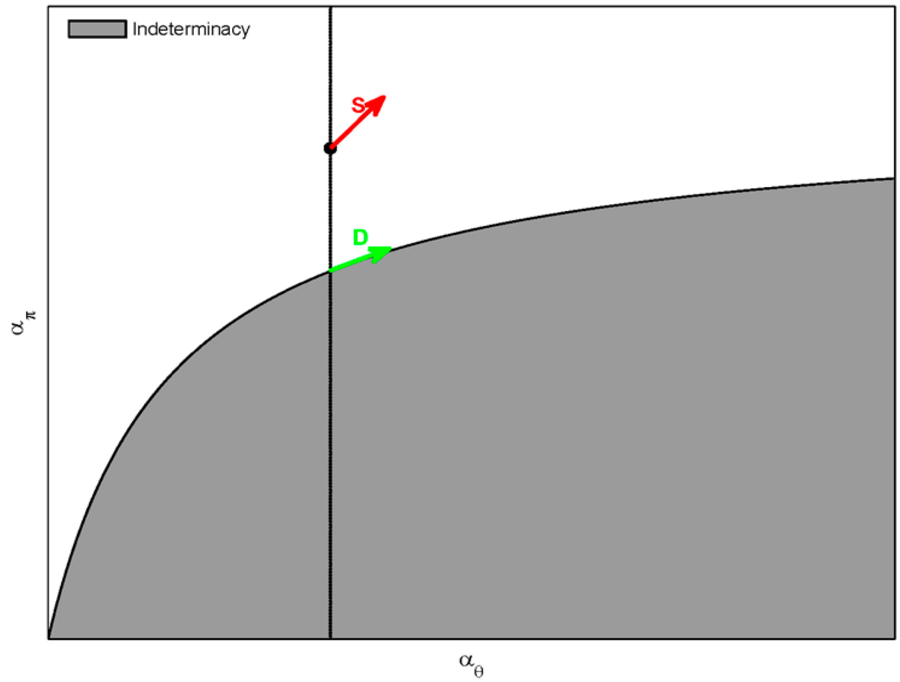
Since the social preferences are located above the determinacy frontier, we have:

$$\alpha_\pi \geq \frac{a\alpha_\theta}{1+b\alpha_\theta}$$

We finally get that:

$$S \geq \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{a}{1+b\tilde{\alpha}_\theta} \geq \frac{a}{\left(1+b\tilde{\alpha}_\theta\right)^2} = D$$

Figure 2. Optimal Preferences Determinacy



Source: Authors' calculations.

This proves that if the social preferences are determinate, then so are the optimal preferences. In this sense, the optimal central banker preferences strengthen its credibility.

A.2 Proof of Proposition 4

We consider that the optimal choice, as defined in section A.1, leads to indeterminacy, e.g.:

$$\tilde{\alpha}_{\pi}^{opt} \leq \frac{k_{\pi}-1}{k_{y}(k_{y}+k_{y^{e}})+k_{\theta}(k_{\theta}+k_{\theta}^{e})/\tilde{\alpha}_{\theta}^{opt}} \tag{13}$$

The determinacy constraint (8) assumes that the inflation weight should be strictly above the frontier. However, we show below that the solution to the problem that includes the border is unique, and located on the border.

It is then easy to see that the solution to the strict inequality problem will be in the neighborhood of the above point (there would be no solution *per se*, but a sequence converging to this point). We will then consider that the solution to the problem (12) is located on the border.

Solution location

Let us first prove that the solution to the constrained problem is located on the determinacy frontier. To that end, we reformulate the partial derivatives:

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \tilde{\alpha}_\pi} = 2\frac{k_y^2}{1-\rho_u k_\pi}\frac{\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt} + k_\theta\left(k_\theta + \rho_u k_\theta^e\right)\left(k_y\left(k_y + \rho_u k_y^e\right)\right)^{-1}\left[\left(\tilde{\alpha}_\theta^{opt} / \tilde{\alpha}_\theta\right)\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt}\right]}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

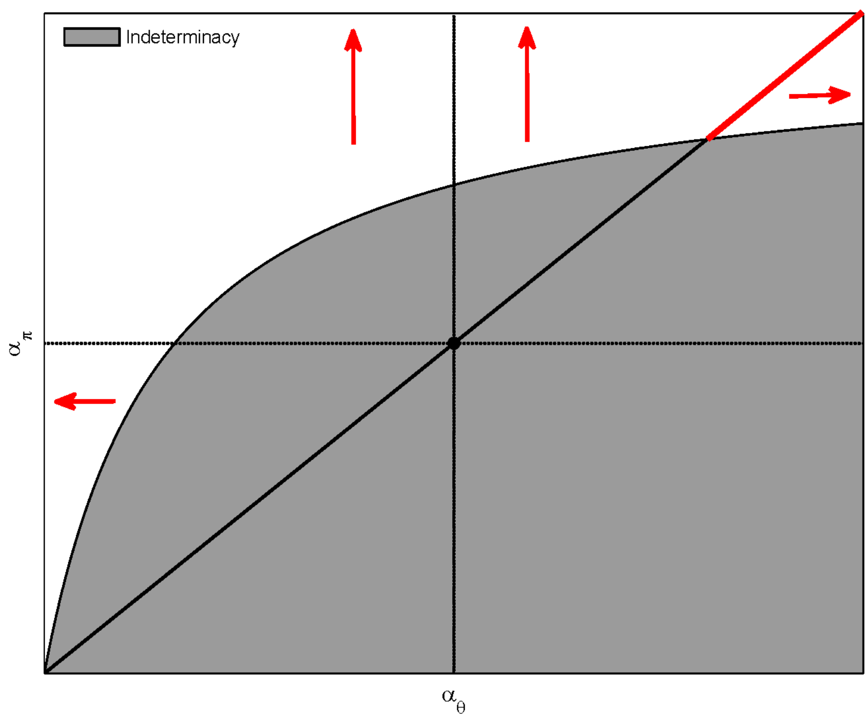
We then see that if $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$ and $\tilde{\alpha}_\pi > \frac{\tilde{\alpha}_\theta}{\tilde{\alpha}_\theta^{opt}}\tilde{\alpha}_\pi^{opt}$, the welfare loss is strictly increasing with $\tilde{\alpha}_\pi$. Similarly,

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} = 2\frac{\tilde{\alpha}_\pi k_\theta k_y\left(k_\theta + \rho_u k_\theta^e\right)}{k_y + \rho_u k_y^e}\frac{\left(\left(\tilde{\alpha}_\pi / \tilde{\alpha}_\theta\right)\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt} + k_y^2\tilde{\alpha}_\pi^2\tilde{\alpha}_\pi^{opt}\left[\left(\tilde{\alpha}_\theta^{opt} / \tilde{\alpha}_\theta\right) - 1\right]\right)}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

Since $\frac{\partial W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right)}{\partial \tilde{\alpha}_\theta} = -\tilde{\alpha}_\theta^2\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, (1/\tilde{\alpha}_\theta)\right)}{\partial (1/\tilde{\alpha}_\theta)}$, the welfare loss is strictly decreasing (resp. increasing) with $\tilde{\alpha}_\theta$ when $\tilde{\alpha}_\pi > \left(\tilde{\alpha}_\pi^{opt} / \tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\theta$ (resp. $\tilde{\alpha}_\pi < \left(\tilde{\alpha}_\pi^{opt} / \tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\theta$) and $\tilde{\alpha}_\theta < \tilde{\alpha}_\theta^{opt}$ (resp. $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$).

To get some intuition, let us represent graphically the above dynamics. The red arrows in Figure 3 represent the gradient of $W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right)$ along its partial derivatives.

Figure 3. Welfare Loss Variations in the Determinacy area



Source: Authors' calculations.

We then see that if the optimal preferences are located below the curve[23], starting from any point located above the frontier, it is optimal to move along a direction that brings you back to the frontier or to the red part of the line passing through the origin and the optimal point.

Along this line, denoting its slope $S^{opt} = \left(\tilde{\alpha}_\pi^{opt} / \tilde{\alpha}_\theta^{opt}\right)$ and the ratio $R = k_\theta\left(k_\theta + \rho_u k_\theta^e\right)\left(k_y\left(k_y + \rho_u k_y^e\right)\right)^{-1}$, the welfare loss can be expressed as follows:

$$W\left(\tilde{\alpha}_\pi\right) = \frac{\left(\alpha_\pi / k_y^2\right) + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^2}{\left[\left(\alpha_\pi / k_y^2\right) + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^2}$$

This function derivative is:

$$W'\left(\tilde{\alpha}_\pi\right) = \frac{\left(\left(\alpha_\pi / k_y^2\right) + RS^{opt}\tilde{\alpha}_\pi^{opt}\right)\left(\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt}\right)}{\left[\left(\alpha_\pi / k_y^2\right) + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^3}$$

We see that the welfare loss is strictly increasing along this ray for $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$. It is then optimal to get back to the frontier on the red part of the line too.

We then proved that the solution to the problem featuring a lower or equal sign is located on the determinacy border.

Solution Determination

Since the solution of the constrained problem is located on the determinacy frontier, using the frontier parametrization introduced in Appendix A.1, the optimal parameters are linked by the following relation:

$$\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} = a - b\tilde{\alpha}_\pi \tag{14}$$

Using the above notations, the welfare loss can then be expressed as follows:

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[23]Since the constraint frontier is concave and the optimal point is located below the frontier, the line passing through the origin and the optimal point cuts the frontier once for $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$.

$$W\left(\tilde{\alpha}_\pi\right)=\frac{\left(\alpha_\pi / k_y^2\right)+R\tilde{\alpha}_\theta^{opt}\left(a-b\tilde{\alpha}_\pi\right)^2+\tilde{\alpha}_\pi^2}{\left[\left(\alpha_\pi / k_y^2\right)+\tilde{\alpha}_\pi^{opt}\left[\tilde{\alpha}_\pi+R\left(a-b\tilde{\alpha}_\pi\right)\right]\right]^2}=\frac{\left(\alpha_\pi / k_y^2\right)+R\tilde{\alpha}_\theta^{opt}a^2-2abR\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi+\left(1+Rb^2\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^2}{\left[\left(\alpha_\pi / k_y^2\right)+aR\tilde{\alpha}_\pi^{opt}+\tilde{\alpha}_\pi^{opt}\left(1-bR\right)\tilde{\alpha}_\pi\right]^2}$$

Its derivative is then equal to:

$$W'\left(\tilde{\alpha}_\pi\right)=\frac{\left[\left(\alpha_\pi / k_y^2\right)\left(1+Rb^2\tilde{\alpha}_\theta^{opt}\right)+aR\tilde{\alpha}_\pi^{opt}\left(1+b\tilde{\alpha}_\theta^{opt}\right)\right]\tilde{\alpha}_\pi-\left[\left(\alpha_\pi / k_y^2\right)\left[\tilde{\alpha}_\pi^{opt}+bR\left(a\tilde{\alpha}_\theta^{opt}-\tilde{\alpha}_\pi^{opt}\right)\right]+a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}\right]}{\left[\left(\alpha_\pi / k_y^2\right)+aR\tilde{\alpha}_\pi^{opt}+\tilde{\alpha}_\pi^{opt}\left(1-bR\right)\tilde{\alpha}_\pi\right]^3}$$

The cancellation point, that corresponds to the constrained optimal, is then unique and defined by:

$$\tilde{\alpha}_\pi^{copt}=\frac{\left(\alpha_\pi / k_y^2\right)\left[\tilde{\alpha}_\pi^{opt}+bR\left(a\tilde{\alpha}_\theta^{opt}-\tilde{\alpha}_\pi^{opt}\right)\right]+a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}}{\left(\alpha_\pi / k_y^2\right)\left(1+Rb^2\tilde{\alpha}_\theta^{opt}\right)+aR\tilde{\alpha}_\pi^{opt}\left(1+b\tilde{\alpha}_\theta^{opt}\right)}$$

We want to compare this constrained optimal to the unconstrained optimal choice. After some algebra, we get:

$$\tilde{\alpha}_\pi^{copt}-\tilde{\alpha}_\pi^{opt}=\frac{R\left(b\left(\alpha_\pi / k_y^2\right)+a\tilde{\alpha}_\pi^{opt}\right)\left[a\tilde{\alpha}_\theta^{opt}-\left(1+b\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^{opt}\right]}{\left(\alpha_\pi / k_y^2\right)\left(1+Rb^2\tilde{\alpha}_\theta^{opt}\right)+aR\tilde{\alpha}_\pi^{opt}\left(1+b\tilde{\alpha}_\theta^{opt}\right)}$$

Since the optimal preferences are indeterminate, following equation (13), we have:

$$a\tilde{\alpha}_\theta^{opt}>\left(1+b\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^{opt}$$

The optimal constrained inflation choice is then always above the optimal unconstrained point.

We now want to determine the location of the constrained optimum for the unconventional instrument. Using the frontier equation (14) and the above formula for$\tilde{\alpha}^{copt}$, we get after some algebra:[24]

$$\tilde{\alpha}_\theta^{copt}=\frac{\frac{\alpha_\pi}{k_y^2}\left[\tilde{\alpha}_\pi^{opt}+bR\left(a\tilde{\alpha}_\theta^{opt}-\tilde{\alpha}_\pi^{opt}\right)\right]+a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}}{\frac{\alpha_\pi}{k_y^2}\left[a+b(bR-1)\tilde{\alpha}_\pi^{opt}\right]+a^2R\tilde{\alpha}_\pi^{opt}}$$

Finally, we compute the difference between the constrained optimal and the unconstrained optimal

[24]Since $a>b\tilde{\alpha}_\pi^{opt}$, the denominator is always strictly positive.

for the unconventional instrument:

$$\tilde{\alpha}_{\theta}^{copt} - \tilde{\alpha}_{\theta}^{opt} = \frac{\frac{\alpha_{\pi}}{k_y^2}(bR-1)\left[a\tilde{\alpha}_{\theta}^{opt} - \left(1+b\tilde{\alpha}_{\theta}^{opt}\right)\tilde{\alpha}_{\pi}^{opt}\right]}{\frac{\alpha_{\pi}}{k_y^2}\left[a+b(bR-1)\tilde{\alpha}_{\pi}^{opt}\right]+a^2 R\tilde{\alpha}_{\pi}^{opt}}$$

$bR = \left(1+\rho_u \frac{k_y^e}{k_y}\right)\left(1+\rho_u \frac{k_{\theta}^e}{k_{\theta}}\right)^{-1}$ is simply the ratio of the forward-looking impacts of output and the unconventional instrument. When it is smaller (resp., larger) than one—i.e., the unconventional instrument is more (resp., less) forward-looking than output—the constrained optimum uses this instrument more (resp., less) aggressively.

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