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Amortization Requirements May Increase Household Debt: A Simple Example

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Research Department

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Abstract

Debt amortization requirements have been suggested as a way to reduce household indebtedness. However, a closer look reveals that amortization requirements may create incentives for both borrowers and lenders to borrow and lend *more* rather than less. Suppose that a household plans to finance a given housing purchase through a preferred future mortgage path. If that mortgage path violates a new amortization requirement, the household can still achieve its preferred mortgage path, net after savings, by initially borrowing more, investing the excess borrowing in a savings account, and fulfilling the amortization requirement by withdrawals from the savings account over time. This is obvious, if the savings interest rate equals the mortgage rate, because then the excess borrowing is costless. But even if the savings interest rate is less than the debt interest rate, so that the excess borrowing is costly, there remains a strong incentive to initially borrow more than without an amortization requirement. Furthermore, under these circumstances, it is profitable and quite riskless for banks to let borrowers borrow more and invest the excess borrowing in a savings account in the bank, giving lenders an incentive to lend more, not less, than without amortization requirements. Thus, amortization requirements as a way of reducing household indebtedness may be counterproductive.

Contents

1 Introduction

Debt amortization requirements, that borrowers are required to repay their debt in regular installments over time, have been proposed as a way to reduce household indebtedness (for Sweden, see Finansinspektionen (2015a, p. 2), Sveriges Riksbank (2014) and International Monetary Fund (2015)). However, such a proposal should take into account what new incentives amortization requirements may create for borrowers and lenders. Indeed, it is shown in this paper that amortization requirements may create incentives for both borrowers and lenders to borrow and lend *more* rather than less.

In the Swedish case, perhaps somewhat surprisingly, the justification given for the endeavor to reduce household indebtedness is not that there are substantial or increasing risks that banks may suffer credit losses on household debt. As stated by Finansinspektionen, the Swedish Financial Supervisory Authority (Finansinspektionen (2015a, p. 5)):

> Highly indebted households may pose a risk, both to financial stability and the Swedish economy as a whole. If households cannot repay their debts, the firms granting loans to households, primarily banks and credit market companies ('mortgages firms'), incur losses which may jeopardise financial stability. However, Finansinspektionen considers that the vast majority of households with loans collateralised by homes ('mortgages') have sound resilience to economic shocks. Furthermore, Swedish borrowers compared with, for example, those in the United States have a far-reaching payment liability for their mortgages. On the whole, the risk of households not managing to pay their mortgages, and loan-granting firms suffering substantial credit losses, is thereby currently limited.

Indeed, Finansinspektionen's annual Mortgage Market Report (Finansinspektionen (2015b)) shows in detail, using stress tests with individual borrower data, that Swedish borrowers have substantial debt-service capacity and considerable resilience to disturbances in the form of increased mortgage rates, housing price falls, and income losses due to unemployment. Furthermore, new Swedish borrowers on average have a loan-to-value (LTV) ratio of about 70 percent, implying a substantial loss-absorbing capacity, in particular in an international comparison. Old borrowers have even larger debt-service and loss-absorption capacity. Lending standards are high and are closely monitored in the Report.

Instead, the justification given is the presumption that indebted household, in spite of the vast majority of them having "sound resilience to shocks," would reduce their consumption substantially in economic downturns (Finansinspektionen (2015a, p. 2)):

> International research suggests that households with a relatively high loan-to-value ratio (LTV) are more inclined to significantly change their consumption behaviour in the

event of economic shocks, which in turn can create and aggravate economic downturns. This is because such households may be sensitive to shocks, such as higher interest rates, a drop in house prices or loss of income. A rising share of new mortgage holders in Sweden take out mortgages that exceed 50 % of the value of the home... At the same time, interest-only mortgages are common for those with LTVs of between 50 and 70 %. Finansinspektionen therefore considers that the macroeconomic risks associated with household indebtedness are currently escalating. Increased mortgage amortisation will ensure that these relatively highly leveraged households reduce their leverage over time, thereby reducing the risks.

This paper will not scrutinize the justification given for amortization requirements.[1] A companion paper, Svensson (2016), will examine the impact of household debt on consumption response to a fall in housing prices. What is scrutinized here is instead the effect of amortization requirements on household indebtedness. The main result is that, for borrowers that are not restricted by LTV, loan-to-income (LTI), or debt-service-to-incomes (DSI) constraints, amortization requirements create incentives to borrow *more*, not less. Furthermore, amortization requirements may create incentives to lenders to lend more to those borrowers.

For example, suppose that a household wants to finance a given housing purchase through a preferred future mortgage path. In addition, suppose that a new amortization requirement would force the household to reduce its mortgage over time faster than it prefers. Then, a household unconstrained by LTV, LTI, or DSI caps can still achieve its preferred mortgage path, net of savings, by initially borrowing more, investing the excess borrowing into a safe assets, such as a savings account, and fulfilling the amortization requirement by withdrawals from the savings account over time. This is obvious, if the savings interest rate equals the mortgage interest rate, because then the excess borrowing is costless. However, even if the savings interest rate, more realistically, is less than the mortgage rate, so that the excess borrowing is costly, it is shown in this paper that there remains a strong incentive to initially borrow more with than without an amortization requirement. This is true even if the debt-savings interest-rate spread is substantial, several percentage points, and the excess borrowing is quite costly. The costly excess borrowing

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[1] The research on the experiences of Denmark, the U.K., and the U.S. during the last crisis does not support the presumption that higher debt generally leads to larger consumption fall in downturns. For Denmark, the results of Andersen, Duus, and Jensen (2015) suggest that the larger decline in spending among high-leverage households was the result of a "spending normalization pattern," where unsustainable overconsumption returned to normal consumption, rather than "deleveraging" and a causal effect of high debt levels suppressing household spending during the crisis. For the U.K., Bunn and Rostom (2014) show that the fall in consumption in connection with the crisis was mainly due to overconsumption of highly indebted households before the crisis, overconsumption that fell back to normal consumption after the crisis. For the U.S., the results of Mian and Sufi (2014) strongly indicate that the main reason for the fall in consumption was the collapse in lending standards and associated large expansion of credit to low-income households with totally inadequate debt-servicing capacity. In contrast, in Sweden, the historically high household savings rate indicates that there is little or no overconsumption. Furthermore, as shown in Finansinspektionen (2015b) and mentioned above, debt-service capacity and lending standards are high. This is further discussed in Svensson (2014).

means that demand for housing falls, but only by a rather small amount.

Furthermore, if the savings interest rate is less than mortgage rate, it is profitable and, as we shall see, quite riskless for banks to let borrowers borrow more from the bank and invest the excess borrowing in a savings account in the bank. Indeed, under these circumstances, lenders indeed have an incentive to lend more rather than less.

Households constrained by LTV, LTI, or DSI requirements are unlikely to be able to borrow more. Lenders may not be willing to extend further loans to such households. For such households, however, frequent refinancing will nevertheless be a way to reduce the impact of amortization requirements.

Hull (2015) constructs a quite detailed calibrated model of Swedish household debt contracts and simulates the steady-state effect of amortization requirements on Swedish household indebtedness. He finds that amortization requirements are largely ineffective at reducing household indebtedness in this calibrated model. In his models, borrowers are restricted by loan-to-income LTI and DSI constraints when mortgages are originated or refinanced but not at other times. Although amortization requirements mechanically tightens DSI constraints and puts downward pressure on indebtedness, refinancing allows borrowers to avoid DTI and DSI constraints over the lifecycle, for instance by refinancing when wages and income are high (for instance, in midlife) and by remaining in the same debt contract when income is lower (for instance, during retirement). The net effect on indebtedness is therefore small. In particular, if the critical but arguably doubtful DSI constraint is removed, and there are high penalties on refinancing, steady-state household indebtedness will be somewhat higher with amortization requirements than without.

This paper differs from Hull (2015) in that it focuses on the demand for loans, with and without amortization requirements, for a household that is not restricted by LTV, LTI or DSI constraints. In Sweden, Finansinspektionen has recommended a LTV cap of 85 percent, but, as mentioned, the average LTV ratio for new mortgages is only about 70 percent, meaning that most new mortgage borrowers are not constrained by the LTV cap. Also, according to Finansinspektionen (2015b), only a relatively small fraction of new borrowers are subject to LTI or DSI constraints. Indeed, any such constraints would in the Swedish case be imposed by the lender, as a way of maintaining strict lending standards and ensuring sufficient debt-service capacity. Lender incentives to supply additional loans are examined in this paper, and it is demonstrated that excess borrowing that is invested in a savings account to circumvent amortization requirements do not imply less debt-service capacity and would be profitable for the lender. This paper also differs from Hull (2015)

in that it is not restricted to steady-state analysis. Instead, the simple model used here allows the analysis of transitions to new amortization requirements, including the effect of announcements of future amortization requirements.

The paper is organized as follows: Section 2 sets up a simple example in the form of stylized model of owner-occupied housing. Section 3 sets up a benchmark case, without an amortization requirement, of a household's preferred debt path to finance a housing purchase. Section 4 shows that, with an amortization requirement, under the assumption of equal debt and savings interest rates, an unconstrained household can simply achieve its preferred net debt path through the costless strategy of initially borrowing more, investing the excess borrowing in a savings account, and then using withdrawals from the savings account to fulfill the amortization requirement. Indeed, under the assumption of equality of debt and savings interest rates, the household can achieve a preferred net debt of any form, rising, hump-shaped, or other, through the appropriate initial excess borrowing. However, if the savings rate of interest rate is less than the debt rate of interest, this strategy is no longer costless. Section 5 examines the household's optimal borrowing with an amortization requirement when the excess borrowing is costly. It shows that desired initial and average debt is still substantially higher than it would be without an amortization requirement, and that this is the case also for substantial spreads between the debt and savings interest rates. With costly excess borrowing, the demand for housing falls, but not by very much. Section 6 examines lender incentives and shows that lenders can increase profits by increasing lending without affecting the debt-service and loss-absorption capacity of the borrower. Section 7 discusses the consequences of refinancing and shows that the excess borrowing is smaller with refinancing but, for a moderate refinancing frequency, still substantial compared to the case without an amortization requirement. Section 8 presents some conclusions. Appendices A-E report debt service, housing cost and housing expenditures for the different cases examined, provide technical details and interpretations of the solutions presented, and discuss the constraints provided by LTV and DSI caps and "left to live on" (LTLO) tests.

2   A simple model of owner-occupied housing

Consider a household that lives for T periods, $t = 1, ..., T$, and has an intertemporal utility function of housing and non-housing consumption that is additively separable over time and has a period

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utility function that is the log of a Cobb-Douglas function,

$$u(c, h) = \sum_{t=1}^{t=T} \frac{1}{(1+\rho)^t} \ln(c_t^{1-\theta} h_t^\theta).$$ (2.1)

Here c and h denote the vectors $(c_1, c_2, ..., c_T)$ and $(h_1, h_2, ..., h_T)$, respectively, where c_t denotes non-housing consumption in period t, h_t denotes housing(-services) consumption, ρ satisfies $0 < \rho < 1$ and denotes the household's rate of time preference, and θ satisfies $0 < \theta < 1$ and denotes the relative value share of housing consumption in total consumption. One unit of housing is assumed to produce one unit of housing services each period, so h_t also denotes units of housing (such as the number of square meters of a residence). For simplicity, the household is assumed to have a constant (real, meaning measured in non-housing consumption units) labor income y in (the beginning of) each period $t = 1, ..., T$.

Suppose that the household cannot rent housing but has to buy and own its housing in order to consume its housing services. Suppose that the household enters period 1 without any housing and debt but with possibly some initial savings, and suppose that the household can borrow at the beginning of period 1 to finance a housing purchase. Suppose that the household can buy and sell housing and adjust its debt and savings at the beginning of each future period (all transactions are assumed to occur at the beginning of each period). Let $w_0 \geq 0$ denote the initial net worth, the (real) value, including interest, of the household's savings at the beginning of period 1 before any transactions.[2] Let $d_t \geq 0$ and $s_t \geq 0$ denote, respectively, the (real) debt and (real financial) savings, held after transactions at the beginning of each period $t = 1, ..., T$; let r^d denote the constant (real) interest rate paid on debt; and let r^s denote the constant (safe) (real) interest rates received on savings. Interest is paid and received at the beginning of next period. Let p denote the constant (real) price per housing unit. Assume that the maintenance and operating cost associated with housing (including a risk premium, as discussed in Englund (2011)) is equivalent to a fraction δ, satisfying $0 \leq \delta < 1$, of each housing unit and occur at the end of the period/beginning of next period. The household takes the interest rates, the housing price, and the housing and maintenance cost as given.

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[2] Without changing any results, we could more generally allow the household to start period 1 with some inherited housing (h_0) and debt (d_0) instead of just savings (s_0), without affecting the conclusions. In that case, the initial net worth before transactions in period 1 would be given by

$$w_0 = (1-\delta)ph_0 - (1+r^d)d_0 + (1+r^s)s_0.$$

Then the household's budget constraint at the beginning of period 1 is

$$c_1 + ph_1 + s_1 \leq y + d_1 + w_0. \tag{2.2}$$

The household's resources at the beginning of period 1 consists of its income y, its new debt d_1, and its net worth w_0. It spends these on consumption c_1, the purchase of housing ph_1, and possibly investment in a savings account s_1.

At the beginning of period $t = 2, ..., T$, the budget constraint is

$$c_t + ph_t + s_t \leq y + d_t + (1 - \delta)ph_{t-1} - (1 + r^d)d_{t-1} + (1 + r^s)s_{t-1}. \tag{2.3}$$

The household's resources at the beginning of period t consists of its income, its new debt, the value its old housing net of the operating and maintenance cost, and its savings including interest, less its old debt including interest. It uses these resources to finance its consumption, housing, and savings.

The household is assumed to leave a bequest, denoted $w_T \geq 0$. At the end of period T, the household dies and leaves its net worth in the form of value of the housing after the maintenance and operating cost and any savings including interest less debt including interest. This net worth satisfies the budget constraint

$$w_T \leq (1 - \delta)ph_T + (1 + r^s)s_T - (1 + r^d)d_T. \tag{2.4}$$

Because the marginal utility of non-housing and housing consumption are both positive, the budget constraints (2.2)-(2.4) will be fulfilled with equality.

3 A benchmark case

Assume that the savings interest rate is less than the debt interest rate,

$$r^s < r^d. \tag{3.1}$$

Then, absent any additional constraints, such as amortization requirements, the household would not simultaneously hold both savings and debt, but rather use any savings to reduce the debt. As long as debt is positive, savings will therefore be zero. The household will use its initial net worth and borrowing to finance its purchase of housing. Unless the initial net worth is large, the household will need to borrow to buy its desired housing, in which case the debt will indeed be

positive and savings will be zero. With positive debt and no additional restrictions on borrowing, the relevant interest rate for the household, the "shadow" interest rate, will equal the debt interest rate and be constant,[3]

$$r_t = r^d.$$

For simplicity, assume that the debt interest rate is equal to the rate of time preference,

$$r^d = \rho. \tag{3.2}$$

Then, the household prefers constant non-housing consumption, $c_t = \bar{c}, \ t = 1, ..., T$ (see (B.5) in appendix B).

Under the assumption that the bequest is equal to the initial net worth,

$$w_T = w_0, \tag{3.3}$$

it is easy to see that the household prefers constant housing $h_t = \bar{h}$ and constant debt $d_t = \bar{d}$, that is, an interest-only loan. In that case, the budget constraint (2.2) for period 1 is

$$\bar{c} + p\bar{h} = y + \bar{d} + w_0. \tag{3.4}$$

The budget constraint (2.3) for periods $2, ..., T$ can be written as

$$\bar{c} + \delta p\bar{h} + r^d\bar{d} = y. \tag{3.5}$$

By subtracting (3.5) from (3.4), we get

$$(1 + r^d)\bar{d} = (1 - \delta)p\bar{h} - w_0, \tag{3.6}$$

which satisfies the budget constraint (2.4).

Using (3.6) to substitute for $\bar{d}$ in (3.5) results in

$$\bar{c} + \bar{q}\bar{h} = y^p, \tag{3.7}$$

where

$$\bar{q} \equiv \frac{(r^d + \delta)p}{1 + r^d}, \tag{3.8}$$

$$y^p \equiv y + \frac{r^d w_0}{1 + r^d}. \tag{3.9}$$

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[3] See appendix B for the definition of the shadow interest rate.

Here, $\bar{q}$, is the benchmark "shadow" rent, the shadow price of housing services. It equals the foregone interest on the value of housing, $r^d p$, plus the maintenance and operating cost, δp. Because the the interest is paid and the maintenance and operating cost occurs at the beginning of next period, their present value requires discounting, that is, division by $1 + r^d$.

Furthermore, y^p is the household's permanent income, the sum of the labor income, y, and the permanent income, $r^d w_0/(1+r^d)$, from the initial net worth, w_0. This permanent income from the initial net worth comes in the form of lower debt interest payments, since the initial net worth by (3.6) reduces the required borrowing.[4]

With the budget constraint on the form (3.7) it follows that the preferred non-housing and housing consumption is given by

$$\bar{c} = (1 - \theta) \, y^p, \tag{3.11}$$

$$\bar{h} = \frac{\theta y^p}{\bar{q}}. \tag{3.12}$$

This exploits the fact that, with the Cobb-Douglas utility function assumed, the household's demand for non-housing and housing consumption is such that their shadow value shares of total consumption are given by, respectively, $1 - \theta$ and θ (where total consumption equals permanent income because the interest rate equals the rate of time preference, (3.2)).

Furthermore, from (3.6), (3.8), and (3.12), it follows that

$$\bar{d} = \frac{1 - \delta}{r^d + \delta} \, \theta y^p - \frac{w_0}{1 + r^d}, \tag{3.13}$$

In summary, the benchmark case is characterized by $(c_t, h_t, d_t, s_t, r_t, q_t) = (\bar{c}, \bar{h}, \bar{d}, 0, r^d, \bar{q})$ for $t = 1, ..., T$, where $(\bar{c}, \bar{h}, \bar{d})$ are given by (3.11)-(3.13) and $\bar{q}$ is given by (3.8).

Here, equation (3.11) is the household's benchmark demand for non-housing consumption as a function of its permanent income (y^p), equation (3.12) with (3.8) is the demand for housing as a function of permanent income, the price of housing, the debt interest rate, and the maintenance and

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[4] See appendix B for the definition of the shadow rent. We may futher note that (3.8) is consistent with the standard asset-pricing equation for housing. Assume that housing can be rented out to other households at a constant rent $\bar{q}$ paid at the beginning of each period. For the constant real interest rate, r^d, and taking into account the maintenance and operating cost of housing, $\delta p h$, the price of housing then satisfies the asset-pricing equation,

$$p_t = \bar{q} + \frac{1 - \delta}{1 + r^d} \, p_{t+1}. \tag{3.10}$$

The constant housing price $p_t = p$ satisfying (3.10) is

$$p = \frac{1 + r^d}{r^d + \delta} \, \bar{q},$$

which equals the present value at the real interest rate r^d of current and future rents $\bar{q}$ net of future depreciation δp and, furthermore, is the same equation as (3.8).

operating cost of housing (y^p, p, r^d, δ), and equation (3.13) is the demand for real debt as a function of permanent income, the debt interest rate, the maintenance and operating cost of housing, and initial net worth (y^p, r^d, δ, w_0).

It follows that a market for buying and selling owner-occupied housing, together with a market for loans, results in the same allocation of housing and non-housing consumption as a market for rented housing, if the rent, the debt interest rate, and the housing price satisfy (3.8) (and the permanent income from any initial net worth is taken into account). In particular, a household that enters period 1 without any initial housing can borrow and purchase housing. This way, the household can consume its optimal housing services from period 1 onward.

Let me take the period to be a year, so all rates are annual rates. As benchmark parameters, I choose $\theta = 0.3$, $\rho = r^d = 0.02$, $\delta = 0.05$, $y = 100$, $w_0 = w_T = 100$, and $p = 100$. Then, $y^p = 102.0$, $\bar{c} = 71.4$, $\bar{q}\bar{h} = 30.6$, $\bar{q} = 6.86$ $\bar{h} = 4.457$, $p\bar{h} = 445.7$, $\bar{d} = 317.1$, and $\bar{s} = 0.$[5]

In particular, we note that, whereas the value of the housing is 446, the household only borrows 317, 129 less than the value of the housing, corresponding to a loan-to-value ratio of 71 percent. At the beginning of each year, the household's housing expenditure from last year consists of interest on the debt, $r^d\bar{d} = 0.02 \cdot 317 = 6.3$, and the maintenance and operating cost represented by the depreciation $\delta p\bar{h} = 0.05 \cdot 446 = 22.3$. Annual housing expenditure from last year then sum to 28.6. Thus, we can say that the household prefers to pay the annual housing expenditure equal to 28.6 out of its labor income of 100, keeping 71.4 for non-housing consumption. This is consistent with the budget constraint (3.5). The housing cost of $\bar{q}\bar{h} = 30.6$ exceeds the housing expenditure $r^d\bar{d} + \delta p\bar{h} = 28.6$ by 2.0, which equals the opportunity cost of the initial net worth invested into the housing, $r^d w_0/(1 + r^d) = 2.0$. This is consistent with the budget constraint (3.7).
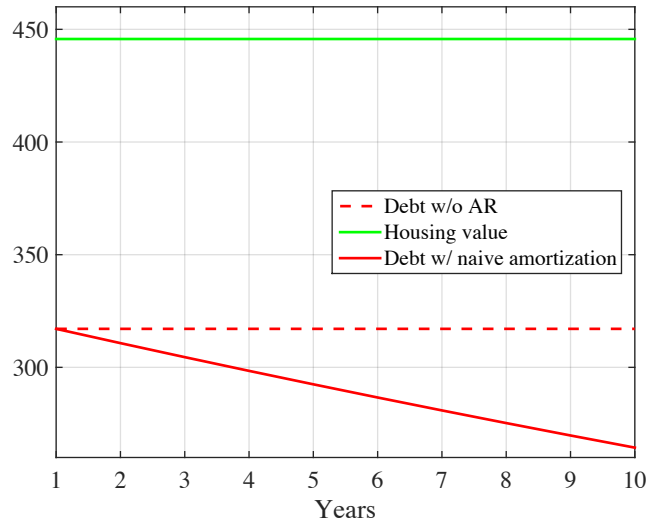
4   Amortization requirements

In the benchmark case above, the household can, subject only to the budget constraints (2.2)-(2.4), freely choose its debt level and increase or decrease it over time. This can be seen as a debt regime without amortization requirement. In this regime, for the assumed parameters, the household prefers a constant level of debt and housing.

Suppose now that a regulator instead imposes an amortization requirement (AR), such that the

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[5] The maintenance and operating cost is thus assumed to be a smaller proportion of the housing value than the assumption of $\delta = 0.07$ in Englund (2011), followed in Sørensen (2013) and Svensson (2013). This can be justified by housing prices now being higher than during previous sample periods.

Figure 4.1: The preferred debt path without amortization requirement (AR),

the value of the preferred housing, and the debt path with amortization requirement

that a naive advocate might expect

household is required to repay at least a fraction $1 - \alpha$ of the debt each period (where $0 < \alpha < 1$). That is, the debt level is required to fall over time according to the amortization requirement[6]

$$d_t \leq \alpha d_{t-1}, \quad t = 2, ..., T. \tag{4.1}$$

In the Swedish case, the purpose of such an amortization requirement is to reduce household demand for loans. To repeat part of the quote above: "Increased mortgage amortisation will ensure that these relatively highly leveraged households reduce their leverage over time, thereby reducing the risks." (Finansinspektionen (2015a, p. 2)) Let us see how this might or might not work. Figure 4.1 shows the situation for $T = 10$ years.

Here, a short period of 10 years should not, of course, be interpreted as literally the life time of a household. Rather, it is a stylized example of the financing of a household's 10-year housing project.

The dashed red line in the figure shows the households preferred constant debt level, $\bar{d} = 317$. The green line shows the value of the preferred housing, $p\bar{h} = 446$.

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[6] We can call this a *proportional* amortization, since the amortization is proportional to the current debt level. It simplifies the calcuations somewhat. We could also assume *linear* amortization, where the amortization is a given fraction, $1 - \alpha$, of the initial debt level,

$$d_t \leq d_{t-1} - (1 - \alpha)d_1, \quad t = 2, ..., T.$$

The difference between proportional and linear amortization does not matter for the results of this paper.

Figure 4.2: The debt path with and without amortization requirement (AR) and the value of the housing. Savings interest rate equal to debt interest rate.
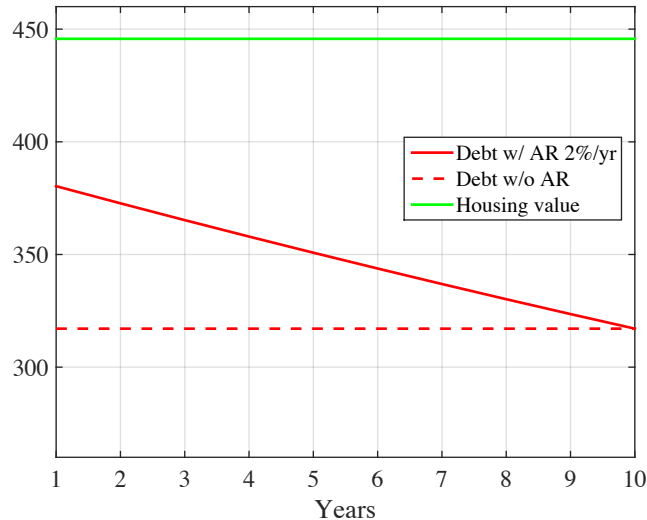
A naive advocate of amortization requirements might believe that, given a requirement such as (4.1), the household will starting amortizing from the preferred level 317 in year 1 and continue to amortize down to 264 in year 10, as shown by the solid red line in the figure 4.1 (drawn for 2 percent amortization per year, that is, for $\alpha = 0.98$). But such a debt path would make it impossible for the household to attain the housing consumption it prefers. What will the household then prefer to do, and what might the naive advocate be overlooking?

As we shall see, the amortization requirement will actually induce the household to borrow more in year 1 than needed to finance the housing purchase. This is obvious under the assumption that the savings interest rate equals the debt interest rate,

$$r^s = r^d. \tag{4.2}$$

Then it is costless for the household to borrow more than its preferred constant benchmark debt level $\bar{d}$ and invest the excess borrowing in savings. That is, the household is indifferent to gross debt and savings paths that result in the same preferred net debt $\bar{d}$, that is, satisfy

$$d_t - s_t = \bar{d}, \quad t = 1, ..., T. \tag{4.3}$$

This means that the household can choose a higher debt path than the benchmark debt, where the higher debt path is decreasing and satisfies the amortization requirement (4.1), and this way this still finance its housing purchase. The solid red line in figure 4.2 shows such a debt path, for

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Figure 4.3: A given debt path with and without amortization requirement (AR). Savings interest rate equal to debt interest rate
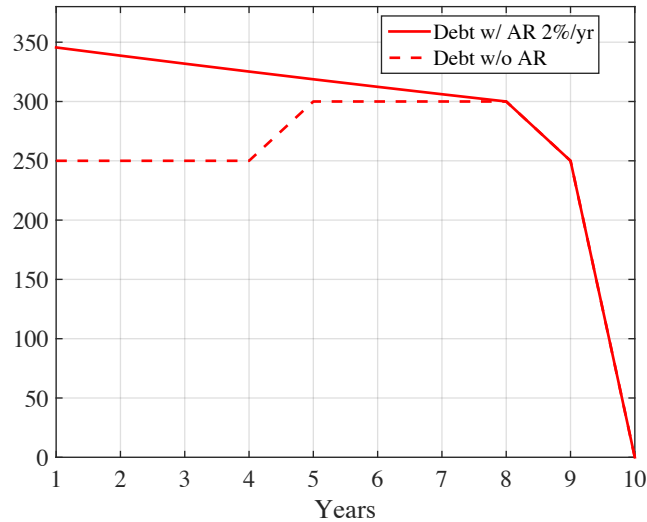
2 percent amortization per year. Thus, the household initially borrows 380 instead of 317, that is, 20 percent more $(1/0.98^9 = 1.20)$, corresponding to an initial LTV ratio of 85 percent instead of 71 percent. The excess borrowing of $380 - 317 = 63$ in year 1 is invested into a savings account. The savings are given by the vertical difference between the solid and dashed red lines. They are simply reduced over the 10 years to finance the amortization of the debt, until the debt level in year 10 reaches the preferred net debt level of 317.

This case, with the amortization requirement (4.1) fulfilled with equality, the assumption (4.2), and the debt path satisfying (4.3), represented by the solid red line in figure 4.2, I will call the benchmark case *with* an amortization requirement. The case with the debt level satisfying (3.13), represented by the dashed red line in figure 4.2, I will call the benchmark case *without* an amortization requirement.

Consider any preferred debt path of any shape, such as hump-shaped, increasing, or decreasing. Clearly, if such a debt path violates an amortization requirement such as (4.1), it is under the specified assumptions alway possible for the household to choose a higher debt path and a corresponding savings path so that the debt path fulfills the amortization requirement and the net debt path, debt net of savings, equals the preferred debt path. Figure 4.3 shows an example. Thus, under these circumstances, a binding amortization requirement would increase borrowing and household debt.

However, this is under the specific assumption that the savings interest rate is equal to the debt

interest rate, (4.2). If the savings rate of interest instead is less than the debt interest rate, the above strategy is costly. I will now examine that case.

5   Optimal borrowing when excess borrowing is costly

Consider then the problem of choosing non-negative (c_t, h_t, d_t, s_t) for $t = 1, , ..., T$ so as to maximize the intertemporal utility function (2.1), subject to the budget constraints (2.2)-(2.4) and the amortization requirement (4.1). Furthermore, assume that the savings interest rate is less than the debt interest rate,

$$r^s < r^d \qquad\qquad ((3.1) \text{ restated})$$

so there is a cost, due to the debt-savings interest-rate spread spread $r^d - r^s > 0$, for any excess borrowing invested in a savings account.[7]

Assume that housing services and number of housing units are restricted to be constant over time,

$$h_t = h. \qquad\qquad (5.1)$$

This can be interpreted as the household contemplating to buy a single home for a T-year period and wanting to choose the optimal size of this home.

Assume the same parameters as above, and in addition that $r^s = 0.01 < r^d = 0.02$, so there is a 1 percentage point spread between the debt and savings interest rates. As a comparison, in February 2016, the Swedish bank SBAB posted on its website (www.sbab.se) a mortgage rate of 1.46 percent and an interest rate of 0.80 percent on a savings account for its own customers, resulting in a debt-savings interest-rate spread of only 0.66 percentage points, which after the deduction a 30 percent capital-income tax equals a spread of only 0.46 percentage points. Thus, a spread no more than 1 percentage point may be quite realistic, although I will report results below for spreads up to 3 percentage points.

In figure 5.1, the solid red line shows the new debt path and the solid green line shows the the new housing value, with the amortization requirement. The dashed red and green lines show the corresponding benchmark debt level and housing value without amortization requirement. The red dotted line shows the benchmark debt path with amortization requirement (the solid line in figure 4.2). We see that the household still borrows much more than the benchmark without amortization requirement, but somewhat less than in the benchmark case with amortization requirement,

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[7] The household's optimization problem is discussed in detail in appendix D.

Figure 5.1: The debt path, the value of housing, and net debt with and without amortization requirement (AR). Savings interest rate less than debt interest rate.
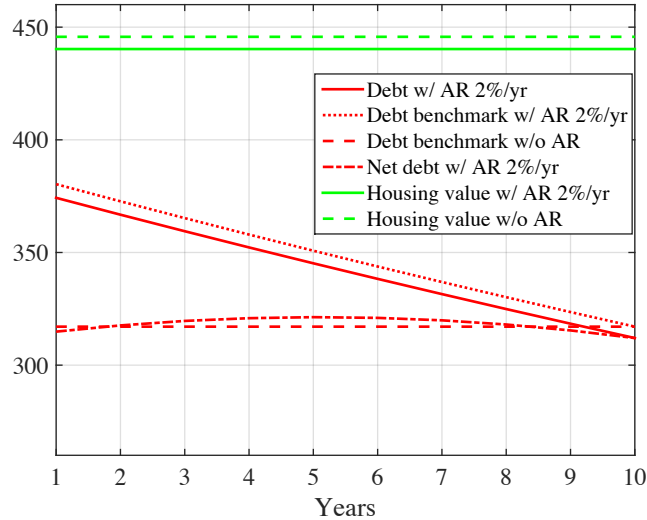
374 instead of 380, 1.6 percent less. But this is still 18 percent more than the benchmark level without amortization requirement, 317. Furthermore, the fact that there is a cost to excess borrowing makes the average housing cost somewhat higher and induces the household to choose somewhat less housing, with a value equal to 440 instead of 446. With the given price $p = 100$, the housing is 4.40 units instead of 4.46, 1.2 percent less. Furthermore, in year 10, the debt falls to a somewhat lower level than the benchmark level, 312 instead of 317, 1.6 percent less.

So, in spite of the lower savings interest rate, there is still a substantial excess borrowing compared to the benchmark case without amortization requirement. Debt and housing is thus just about 1 percent less than the benchmark case with amortization requirement.

Table 5.1 shows how the preferred initial debt (d_1) and the value of housing (ph), for given debt interest rate $r_d^d = 0.02$, depends on the savings interest rate (r^s) and thereby the debt-savings interest-rate spread $(r^d - r^s)$. We see that a lower savings rate of interest and an increasing spread have a rather modest effect on initial debt and the value of housing. The initial debt remains substantially above the benchmark debt of 317 without amortization requirement. For a large debt-savings interest-rate spread of 3 percentage points, the preferred initial debt is still 15 percent higher with than without an amortization requirement.[8]

Furthermore, as shown in some detail in appendices B and D.1, introducing a binding amor-

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[8] Appendix A reports debt service, housing cost, and housing expenditure for the different cases.

Table 5.1: Effect on debt and housing values of the savings interest rate, including benchmark cases (BM) with and without amortization requirement (AR)

	BM w/o AR	BM w/ AR	w/ AR		
Savings interest rate, %	2	2	1	0	−1
Debt-savings interest-rate spread, pp	0	0	1	2	3
Initial debt	317.1	380.3	374.3	368.5	364.0
Increase from benchmark w/o AR, %	0	19.9	18.0	16.2	14.8
Average debt	317.1	347.8	342.3	337.1	332.9
Increase from benchmark w/o AR, %	0	9.7	8.0	6.3	5.0
Housing value	445.7	445.7	440.3	435.2	431.1
Change from benchmark w/o AR, %	0	0	−1.2	−2.4	−3.3

tization requirement *lowers* the shadow interest rate on debt, because higher current debt eases future amortization constraints. A naive advocate might think that an amortization requirement *increases* the shadow debt interest rate and *reduces* the borrowing, but it is actually under these circumstances the other way around.

6 Lender incentives

Under the specified circumstances, it is rather obvious that lenders have an incentive to lend more and satisfy the borrower's demand for a larger loan. For completeness, let me nevertheless look at the numbers.

Without amortization requirements, a bank would lend the benchmark debt level, $\bar{d}$, and collect interest,

$$r^d \bar{d}, \quad t = 1, ..., T. \tag{6.1}$$

With benchmark values, $r^d = 0.02$ and $\bar{d} = 317$, the interest equals 6.34 per year.

With amortization requirements, under the assumption that the borrower invests the excess borrowing in a savings account with the bank, the bank would lend d_t and receive s_t in deposits, collecting the net interest

$$r^d d_t - r^s s_t, \quad t = 1, ..., T. \tag{6.2}$$

With benchmark values, $r^s = 0.01 < r^d = 0.02$ and $T = 10$ years, the average net interest collected equals 6.60 per year.

By adding and subtracting the term $r^d \bar{d} + r^s(d_t - \bar{d})$, we can write (6.2) as

$$r^d \bar{d} + (r^d - r^s)(d_t - \bar{d}) + r^s(d_t - s_t - \bar{d}). \quad t = 1, ..., T. \tag{6.3}$$

By subtracting (6.1) from (6.2), it follows that amortization requirements increases the bank's interest income by

$$(r^d - r^s)(d_t - \bar{d}) + r^s(d_t - s_t - \bar{d}), \quad t = 1, ..., T. \tag{6.4}$$

where the second term is small, because $s_t \approx d_t - \bar{d}$.[9]

From the point of view of the bank, does this affect the borrower's debt-service and loss-absorption capacity? Regarding debt-service capacity, because the amortization payments are financed through withdrawals from the savings account, the amortization payments do not reduce the debt-service capacity, so it is the debt service net of the amortization, that is, the net interest payment, that matters. The increase in the borrower's net interest payment is then given by (6.4), but that increase is modest and paid directly to the bank. From the point of view of the bank, the debt-service capacity of the borrower is in practice the same as without amortization requirements.

With regard to the borrower's loss absorption capacity, without amortization requirements, for the benchmark values, the value of the housing is 446 and the debt is 317, with an LTV ratio of 71 percent, thus with a substantial loss-absorbing capacity.

With amortization requirements, the value of the housing is 440 and the initial debt is 374, corresponding to a higher LTV ratio of 85 percent. However, the excess borrowing, the initial balance of the savings account, is 59, so the debt net of the savings account is only 315, corresponding to an LTV ratio of 72 percent From the point of view of the bank, the borrower's loss-absorption capacity is in practice the same as without amortization requirements.

Actually, the bank is financing the excess borrowing one-to-one with the borrower's deposit of the excess borrowing. The bank pays a lower deposit rate than its ledning rate, making a safe and cosy profit from the arrangement.

Clearly, from the point of view of the bank, increasing its lending increases its profits without reducing the borrower's debt-service and loss-absorption capacity. Thus, under the specified circumstances, not only does the borrower have an incentive to borrow more, the lender does also have an incentive to lend more.

7 Refinancing

Suppose that the household has the opportunity to refinance in periods $t = \tau_1, \tau_2, ..., \tau_n$, where $1 < \tau_1 < \tau_2 < ... < \tau_n < T$. This means that there is no amortization amortization requirement

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[9] Here, $d_t - s_t$ is net debt, shown as the dashed-dotted red line in figure 5.1, and $\bar{d}$ is the benchmark debt level without amortization, shown as the dashed red line in the figure. The difference is small.

Figure 7.1: The debt path, the value of housing, and net debt with and without amortization requirement (AR) when refinancing is possible. Savings interest rate less than debt interest rate.
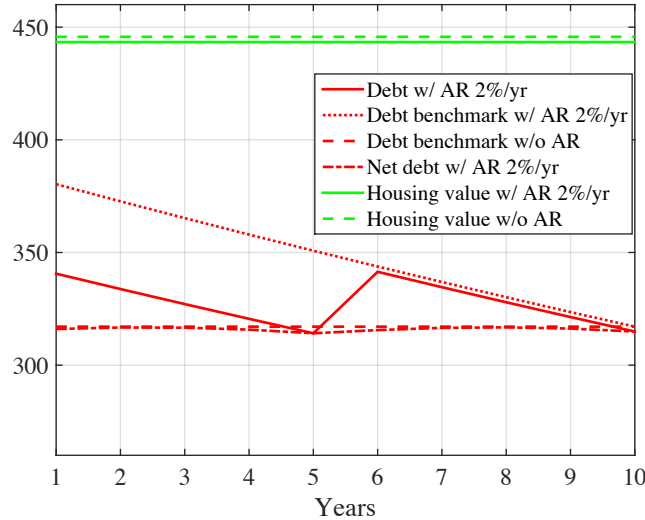
(4.1) for the periods $t = \tau_1, \tau_2, ..., \tau_n$.[10]

Figure 7.1 shows what the debt path looks like, with the benchmark parameters, $T = 10$ years, and an option to refinance in year 6, that is, after 5 years ($t = \tau_1 = 6$). Essentially, the 10 year housing project is split into two 5-year financing periods, where initial debt in each period starts at 341 and is reduced in 5 years to about 314. Average debt is 328. The value of the housing is 444, barely below the benchmark level. Initial and average debt is higher than the benchmark without amortization requirement, but not as high as without the refinancing possibility.

A naive advocate of amortization requirements as way to reduce household indebtedness might want to restrict refinancing, in order not to lessen the impact of the requirements and the presumed reduction in indebtedness. It is true that refinancing reduces the impact of the amortization requirements, but under the specified circumstances, it actually reduces the *increase* in indebtedness.

8   Conclusions

Thus, under the circumstances discussed in this paper, an amortization requirement gives households an incentive to initially borrow more than without an amortization requirement, invest the excess borrowing in a savings account, and then use withdrawals from the savings account over time to amortize the debt. This is obvious, if the savings interest rate is equal to the debt interest rate,

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[10] Appendix E provides some details on the refinancing case.

because then the excess borrowing is costless. But also if the savings interest rate is substantially lower than the debt interest rate, so the excess borrowing has a substantial cost, the household still prefers to initially borrow more than if there was no amortization constraint. The preferred average debt over time is then also substantially higher than without amortization requirement.

Advocates of amortization requirements might believe that amortization requirements would increase the "shadow" interest rate on borrowing. But because higher initial borrowing eases future amortization constraints, an amortization requirement actually *lowers* the initial shadow interest rate, making it more attractive to increase the initial borrowing.

That a household *prefers* to borrow more does not imply that the household always will *be able* to borrow more. That depends on whether the household is restricted by regulated or lender-imposed LTV, LTI, or DSI constraints. But not all borrowers are constrained by such caps. In Sweden, according to Finansinspektionen (2015b), a large majority of borrowers with loans collateralized by homes are apparently unconstrained, and such households may prefer to borrow more. Furthermore, with a positive spread between debt and savings interest rates, it is profitable for the lender to let the household borrow more and deposit the excess borrowing in a savings account. Also, using withdrawals from the savings account for amortization payments does not reduce the household's debt-service and loss-absorption capacity. Indeed, the lender is financing the household's excess borrowing one-to-one with the household's deposit in the savings account, making a safe and cosy profit from the arrangement. So, not only does the borrower have an incentive to borrow more with amortization requirements, under the specified circumstances, the lender does also have an incentive to lend more.

Thus, overall, through the mechanisms discussed in this paper, household debt may increase with amortization requirements. And not only initial debt may increase, but also average debt levels over time, for those households who can borrow more.

Various realistic complications do not seem to change this main results. For instance, the return on some assets is not safe, as assumed here, but risky. On the other hand, risky assets are likely to have higher returns than the debt interest rate, making the risk-adjusted cost of the excess borrowing not too different from the debt-savings interest-rate spreads considered here. And as shown, the results here go through even if there would be substantial spreads between the debt and savings interest rates. A rising or hump-shaped life-cycle income path would not change the result. Neither would uncertainty about future income, housing prices, or interest rates change the general mechanism that drives the results, namely that higher initial borrowing eases future amortization

constraints.

A general and rather obvious conclusion is that policies should not be proposed without a satisfactory analysis of what new incentives and changes in agents' behavior the policies may cause. Furthermore, a policy should not be proposed without a preceding thorough analysis of what the problem and market failure is, and why the particular policy proposal would be a solution to the problem or at least improve the situation.

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Appendix

A Debt service, housing cost, and housing expenditure

Gross debt service is the interest on the debt plus plus the amortization, $(r^d + 1 - \alpha)\,d_t$. *Net* debt service is defined as the interest on the debt less the interest on the savings, $r^d d_t - r^s s_t$. The housing *cost* is given by $q_t h$. The gross and net housing *expenditure* is, respectively, $(r^d + 1 - \alpha)d_t + \delta ph$ and $r^d d_t - r^s s_t + \delta ph$, respectively. Table A.1 shows, for the benchmark parameter values and $T = 10$, the results for the different cases examined.

Table A.1: The effect on gross and net debt service, housing cost, and gross and net housing expenditure of different savings interest rates

	BM w/o AR	BM w/ AR	w/ AR		
Savings interest rate, %	2	2	1	0	−1
Debt-savings interest-rate spread, pp	0	0	1	2	3
Gross debt service, initial	6.3	15.2	15.0	14.7	14.6
Gross debt service, average	6.3	13.9	13.7	13.5	13.3
Net debt service, initial	6.3	6.3	6.9	7.4	7.8
Net debt service, average	6.3	6.3	6.6	6.7	6.8
Housing cost, initial	30.6	30.6	32.0	33.4	34.7
Housing cost, average	30.6	30.6	30.6	30.6	30.7
Gross housing expenditure, initial	28.6	37.5	37.0	36.5	36.1
Gross housing expenditure, average	28.6	36.2	35.7	35.2	34.9
Net housing expenditure, initial	28.6	28.6	28.9	29.1	29.4
Net housing expenditure, average	28.6	28.6	28.6	28.5	28.3

B The shadow rent, the shadow interest rate, and some details of the optimal borrowing

It is practical to introduce two useful concepts, the shadow rent and the shadow interest rate. The shadow rent, q_t, is the shadow price in non-housing consumption units of a unit of housing services. It is defined as the (negative) marginal rate substitution of non-housing consumption for housing consumption, that is,

$$q_t \equiv -\left.\frac{dc_t}{dh_t}\right|_{u(c,h)=\text{const.}} = \frac{\partial u(c,h)/\partial h_t}{\partial u(c,h)/\partial c_t} = \frac{\theta}{1-\theta}\frac{c_t}{h_t}, \quad t = 1, ..., T. \tag{B.1}$$

Thus, the shadow rent in period t is proportional to the ratio of non-housing to housing con-

sumption, c_t/h_t. This is a convenient property of the Cobb-Douglas period utility function. It implies that the shadow value of housing consumption, $q_t h_t$, is proportional to non-housing consumption,

$$q_t h_t = \frac{\theta}{1-\theta} c_t, \tag{B.2}$$

and thus also proportional to the value of total consumption, $c_t + q_t h_t$,

$$q_t h_t = \theta(c_t + q_t h_t), \tag{B.3}$$

The shadow interest rate, r_t, can be defined from the (negative) intertemporal marginal rate of substitution of non-housing consumption in period t for non-housing consumption in period $t+1$, according to

$$\frac{1}{1+r_t} \equiv - \left. \frac{dc_t}{dc_{t+1}} \right|_{u(c,h)=\text{const.}} = \frac{\partial u/\partial c_{t+1}}{\partial u(c,h)/\partial c_t} = \frac{1}{1+\rho} \frac{c_t}{c_{t+1}}, \quad t = 1, ..., T, \tag{B.4}$$

The intertemporal marginal rate of substitution has this simple form due to the intertemporal utility function being a discounted sum of period utility functions that are the log of a Cobb-Douglas function, (2.1). It follows from (B.4) that non-housing consumption growth is related to the shadow interest rate and the rate of time preference according to the simple relation,

$$\frac{c_{t+1}}{c_t} = \frac{1+r_t}{1+\rho}. \tag{B.5}$$

Thus, non-housing consumption is rising, constant, or falling depending on whether the shadow interest rate is larger than, equal to, or less than the rate of time preference.

Determining what the shadow rent and interest is helpful in understanding the different cases with and without amortization requirement. Importantly, the shadow interest rate may deviate from the debt and savings interest rates, r^d and r^s, as we shall see.

Figure 5.1 shows the optimal debt path and housing value with an mortization requirement and a savings interest rate given by $r^s = 0.01 < r^d = 0.02$. The solid red line in figure B.1 shows the corresponding non-housing consumption, c_t, the solid green line shows the shadow value of housing consumption, $q_t h$, and the solid blue line shows the total shadow value of consumption, $c_t + q_t h$, where the shadow value of housing consumption and total consumption satisfy (B.2) and (B.3), with constant housing consumption, (5.1). The dashed lines shows the corresponding constant benchmark levels, at 71.4, 30.6, and 102.0, respectively. The dotted black line shows savings, which equal the difference between debt and net debt in figure 5.1.

Figure B.1: Non-housing consumption, the shadow value of housing consumption, the shadow value of total consumption (benchmark levels dashed), and savings. Savings interest rate less than debt interest rate.
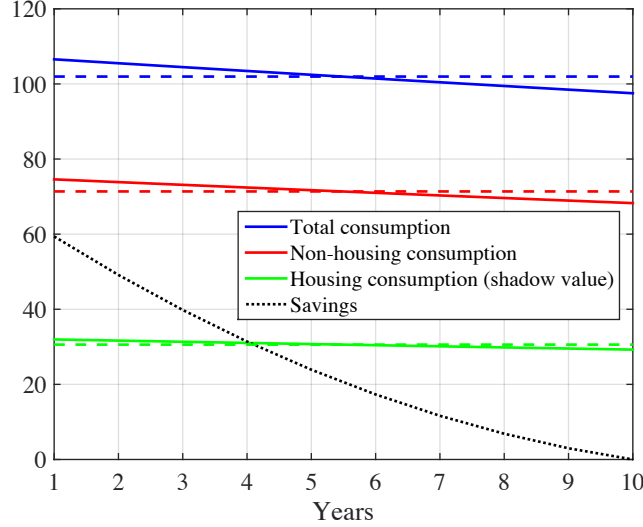
For constant housing consumption, the shadow rent by (B.1) and (5.1) satisfies

$$q_t = \frac{\theta}{1-\theta} \frac{c_t}{h}. \tag{B.6}$$
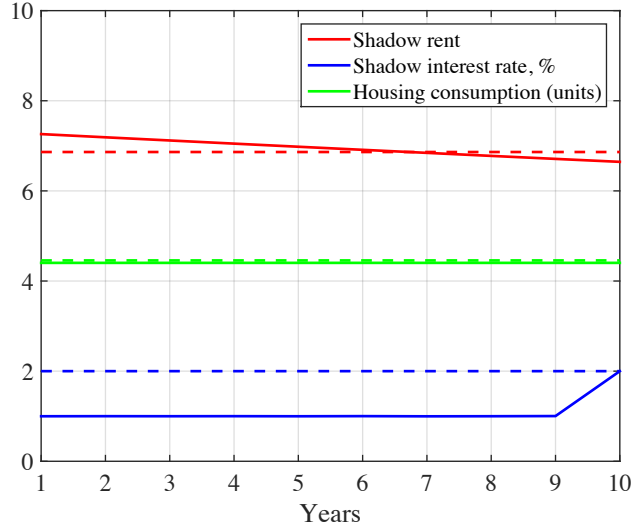
We see that non-housing consumption, the shadow value of housing consumption, and the shadow value of total consumption all fall over time. Non-housing consumption falls over time because the shadow interest rate, r_t, is less than $r^d = \rho$. In figure B.2, the solid blue line shows the shadow interest rate. Because savings are positive in periods 1–9, the shadow rate equals the savings interest rate in these periods,

$$r_t = r^s < r^b = \rho, \quad t = 1, ..., 9.$$

A shadow rate less than the rate of time preference means, by (B.5), that consumption is falling over time. Because consumption is falling over time, the shadow rent, given by (B.6), is also falling over time, as shown in figure B.2. From the figure, we see that the average shadow rent is slightly higher than the benchmark, which makes the constant housing consumption somewhat lower in units than the benchmark (barely visible in the figure B.1 but in different scale clearly visible for the housing value in figure 5.1).

Because the shadow rent is falling over time, the shadow value of housing consumption is falling over time even though housing consumption is constant, as shown in figure B.1. As a result the

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Figure B.2: The shadow rent, the shadow interest rates, and units of housing consumption (benchmark values dashed). Savings interest rate less than the debt interest rate.

shadow value of total non-housing and housing consumption is falling. Furthermore, in the early periods, it is larger than permanent income, 102. Thus, the lower shadow rate causes the household to dissave and consume more than its permanent income in early periods, making up for this by consuming less than permanent income in later years. This corresponds to the hump-shape of net debt in figure 5.1.

The low shadow interest rate is also the shadow interest rate on debt, taking into account the effect of debt in a particular period on the amortization constraints in the other periods. In particular, higher debt in period 1 will ease a binding amortization constraint for debt in period 2. This makes the shadow interest rate on debt in period 1 lower than the debt interest rate r^d, and induces the household to borrow more in period 1 (and in this case in all future periods except the last, period 10).[11]

C   Time-varying housing

Consider for completeness first the less realistic problem when housing h_t can vary freely over time. The problem, written with inequality constraints, is, with $c = \{c_t\}_{t=1}^T, h = \{h_t\}_{t=1}^T, d = \{d_t\}_{t=1}^T, s =$

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[11] See appendix D.1 for details.

$\{s_t\}_{t=1}^T$,

$$u(c,h) = \sum_{t=1}^{T} \left(\frac{1}{1+\rho}\right)^{t-1} \ln(c_t^{1-\theta} h_t^{\theta}) \tag{C.1}$$

$$c_1 + ph_1 - d_1 + s_1 \le y + w_0, \tag{C.2}$$

$$-(1-\delta)ph_{t-1} + (1+r^d)d_{t-1} - (1+r^s)s_{t-1} + c_t + ph_t - d_t + s_t \le y, \quad t = 2, ..., T, \tag{C.3}$$

$$-(1-\delta)ph_T + (1+r^d)d_T - (1+r^s)s_T \le -w_T, \tag{C.4}$$

$$-\alpha d_{t-1} + d_t \le 0, \quad t = 2, ..., T, \tag{C.5}$$

$$c_t, h_t, s_t, d_t \ge 0,$$

where the parameters satisfy $\rho \ge 0$, $p > 0$, $0 < \delta < 1$, $\alpha \le 1$, and $r^d \ge r^s > -1$.

C.1 Karush-Kuhn-Tucker conditions and Lagrange multipliers

The Lagrangian is

$$L(c, d, a, h; \lambda, \mu) = \sum_{t=1}^{T} \left(\frac{1}{1+\rho}\right)^{t-1} \ln(c_t^{1-\theta} h_t^{\theta})$$

$$+ \lambda_1[y + w_0 - c_1 + d_1 - s_1 - ph_1]$$

$$+ \sum_{t=2}^{T} \lambda_t[y + (1-\delta)ph_{t-1} - (1+r^d)d_{t-1} + (1+r^s)s_{t-1} - c_t + d_t - s_t - ph_t]$$

$$+ \lambda_{T+1}[-w_T - (1+r^d)d_T + (1+r^s)s_T + (1-\delta)ph_T]$$

$$+ \sum_{t=2}^{T} \mu_t[\alpha d_{t-1} - d_t].$$

The Karush-Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial c_t} = \left(\frac{1}{1+\rho}\right)^{t-1} \frac{1-\theta}{c_t} - \lambda_t = 0, \quad t = 1, ..., T, \tag{C.6}$$

$$\frac{\partial L}{\partial h_t} = \left(\frac{1}{1+\rho}\right)^{t-1} \frac{\theta}{h_t} - \lambda_t p + \lambda_{t+1}(1-\delta)p = 0, \quad t = 1, ..., T, \tag{C.7}$$

$$\frac{\partial L}{\partial d_t} = \lambda_t - \lambda_{t+1}(1+r^d) + \mu_{t+1}\alpha - \mu_t = 0, \quad t = 1, ..., T, \tag{C.8}$$

$$\frac{\partial L}{\partial s_t} = -\lambda_t + \lambda_{t+1}(1+r^s) \le 0, \quad t = 1, ..., T, \tag{C.9}$$

$$\frac{\partial L}{\partial s_t} s_t = [\lambda_t - \lambda_{t+1}(1+r^s)]s_t = 0, \quad t = 1, ..., T, \tag{C.10}$$

$$\frac{\partial L}{\partial \mu_t} = \alpha d_{t-1} - d_t) \ge 0, \quad t = 2, ..., T, \tag{C.11}$$

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$$\frac{\partial L}{\partial \mu_t} \mu_t = (\alpha d_{t-1} - d_t)\mu_t = 0, \quad t = 2, ..., T, \tag{C.12}$$

where $c_t, h_t, d_t, \lambda_t > 0$, $s_t \geq 0$, $\mu_t \geq 0$, and $\mu_1 = \mu_{T+1} = 0$.

Above, we have assumed $d_t > 0$, so (C.8) holds with equality. However, this need not necessarily be the case, for instance, for large initial net worth or for a prohibitively high debt interest rate. Then (C.8) should be replaced by

$$\frac{\partial L}{\partial d_t} = \lambda_t - \lambda_{t+1}(1 + r^d) + \mu_{t+1}\alpha - \mu_t \leq 0, \quad t = 1, ..., T, \tag{C.13}$$

$$\frac{\partial L}{\partial d_t} d_t = [\lambda_t - \lambda_{t+1}(1 + r^d) + \mu_{t+1}\alpha - \mu_t]d_t = 0, \quad t = 1, ..., T. \tag{C.14}$$

Note that, by (C.6), λ_t equals the marginal utility of consumption, $\partial u(c, h)/\partial c_t$. It then follows from the definition in (B.4) of the shadow interest rate, r_t, that the shadow rate of interest satisfies

$$\frac{1}{1 + r_t} \equiv \frac{\lambda_{t+1}}{\lambda_t}. \tag{C.15}$$

Furthermore, from the definition in (B.1) of the shadow rent, q_t, and from division of (C.7) by (C.6), it follows that the shadow rent satisfies

$$q_t = \left[1 - \frac{\lambda_{t+1}}{\lambda_t}(1 - \delta)\right] p. \quad t = 1, ..., T. \tag{C.16}$$

Combining (C.15) and (C.16), the shadow rent and the shadow interest rate satisfy

$$q_t = \frac{r_t + \delta}{1 + r_t}. \tag{C.17}$$

Furthermore, it follows from (C.8) that

$$\frac{1}{1 + r_t} = \frac{1 + (\alpha\mu_{t+1} - \mu_t)/\lambda_t}{1 + r^d}. \tag{C.18}$$

In particular, we have the relation

$$r_t \lessgtr r^d \text{ if and only if } q_t \lessgtr \bar{q} \text{ if and only if } (\alpha\mu_{t+1} - \mu_t) \gtrless 0,$$

where $\bar{q}$ denotes the benchmark constant shadow rent (3.8).

In particular, for $t = 1$, because $\mu_1 = 0$, if $\mu_2 > 0$, meaning that the restriction $d_2 \leq \alpha d_1$ is binding, we have $r_1 < r^d$. We would then expect this to translate into more borrowing than the benchmark in period 1, $d_1 > \bar{d}$. Similarly, for $t = T$, because $\mu_{T+1} = 0$, if $\mu_T > 0$, meaning that the restriction $d_T \leq \alpha d_{T-1}$ is binding, we have $r_T > r^d$. We would then expect less borrowing than the benchmark in period T.

Note that, if $s_t > 0$, by (C.10) we have $\lambda_{t+1}/\lambda_t = 1/(1+r^s)$, so then $r_t = r^s \leq r_d$. Thus the savings interest rate provides a lower bound for the shadow rate,

$$r_t \geq r^s.$$

Note that shadow interest rate can exceed the the debt interest rate, so the latter does not provide an upper bound for the shadow rate.

Furthermore, when $s_t > 0$ and thus $r_t = r^s$, we have that the μ-term above satisfies

$$(\alpha \mu_{t+1} - \mu_t)/\lambda_t = \frac{1+r^d}{1+r^s} - 1 = \frac{r^d - r^s}{1+r^s} \geq 0.$$

The effect on welfare of varying α can be determined as follows: Determine the dy equivalent of $d\alpha$, $dy/d\alpha$, which gives

$$\frac{\partial L}{\partial \alpha} = \sum_{t=2}^{T} \mu_t d_{t-1},$$

$$\frac{\partial L}{\partial y} = \sum_{t=1}^{T} \lambda_t,$$

$$\frac{dy}{d\alpha} = \frac{\partial L/\partial \alpha}{\partial L/\partial y} = \frac{\sum_{t=2}^{T} \mu_t d_{t-1}}{\sum_{t=1}^{T} \lambda_t}.$$

C.2 Writing the problem on a form suitable for Matlab

In order to apply the Matlab function `fmincon`, the problem is written on the form

$$\min_x f(x) \text{ such that } A x \leq b, \ x \geq 0.$$

The variables are $\{c_t, h_t, d_t, s_t\}_{t=1}^{T}$. Let the column 4-vector v_t satisfy $v_t = (c_t, h_t, d_t, s_t)'$, and let the column $4T$-vector x satisfy $x = (v_1', v_2', ..., v_T')'$. Then $(c_t, h_t, d_t, s_t) = (x_{1+4(t-1)}, x_{2+4(t-1)}, x_{3+4(t-1)}, x_{4+4(t-1)})$. Then we can write $f(x)$ as

$$f(x) = -\sum_{t=1}^{T} \left(\frac{1}{1+\rho}\right)^{t-1} [(1-\theta) \ln x_{1+4(t-1)} + \theta \ln x_{2+4(t-1)}].$$

Furthermore, the constraints (C.2)-(C.5) provide 4 sets of inequalities, where the constraints (C.2) and (C.4) each provide one inequality and the constraints (C.3) and (C.5) each provide $T-1$ inequalities. Let therefore the column $2T$-vector b satisfy $b = (\mathbf{1}_T' y, \mathbf{0}_T')'$, where $\mathbf{1}_T$ and $\mathbf{0}_T$ denote column T-vectors with each element equal to, respectively, unity and zero. Furthermore, modify b by adding w_0 to its first element and setting its $(T+1)$-th element equal to $-w_T$.

For the left side of (C.2)-(C.5), let the corresponding $2T \times 4T$ matrix $A = (A_{i,j})$ satisfy $A_{i,j} = 0$, except for $A_{1,j}$, corresponding to (C.2), satisfying

$$A_{1,1} = 1, \ A_{1,2} = p, \ A_{1,3} = -1, \ A_{1,4} = 1;$$

$A_{t,j}$, corresponding to (C.3), satisfying

$$A_{t,2+4(t-2)} = -(1-\delta)p, \ A_{t,3+4(t-2)} = 1 + r^d, \ A_{t,4+4(t-2)} = -(1+r^s),$$

$$A_{t,1+4(t-1)} = 1, \ A_{t,2+4(t-1)} = p, \ A_{t,3+4(t-1)} = -1, \ A_{t,4+4(t-1)} = 1, \quad t = 2, ..., T;$$

$A_{T+1,j}$, corresponding to (C.4), satisfying

$$A_{T+1,2+4(T-1)} = -(1-\delta)p, \ A_{T+1,3+4(T-1)} = 1 + r^d, \ A_{T+1,4+4(T-1)} = -(1+r^s);$$

and, finally, $A_{T+t,j}$, corresponding to (C.5), satisfying

$$A_{T+t,3+4(t-2)} = -\alpha, \ A_{T+t,3+4(t-1)} = 1, \quad t = 2, ..., T.$$

D   Constant housing

Consider next the problem when housing h is constant over the periods considered. Then we modify the problem by introducing the $T-1$ equality constraints (D.1).

$$-h_{t-1} + h_t = 0, \quad t = 2, ..., T. \tag{D.1}$$

D.1   The Karush-Kuhn-Tucker conditions and Lagrange multipliers

The Lagrangian is

$$
\begin{aligned}
L(c, d, a, h; \lambda, \mu, \nu) = & \sum_{t=1}^{T} \left(\frac{1}{1+\rho}\right)^{t-1} \ln(c_t^{1-\theta} h_t^\theta) \\
& + \lambda_1 [y + w_0 - c_1 + d_1 - s_1 - ph_1] \\
& + \sum_{t=2}^{T} \lambda_t [y + (1-\delta)ph_{t-1} - (1+r^d)d_{t-1} + (1+r^s)s_{t-1} - c_t - ph_t + d_t - s_t] \\
& + \lambda_{T+1} [-w_T - (1+r^d)d_T + (1+r^s)s_T + (1-\delta)ph_T] \\
& + \sum_{t=2}^{T} \mu_t [\alpha d_{t-1} - d_t] \\
& + \sum_{t=2}^{T} \nu_t [h_{t-1} - h_t].
\end{aligned}
$$

The Karush-Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial c_t} = \left(\frac{1}{1+\rho}\right)^{t-1}\frac{1-\theta}{c_t} - \lambda_t = 0, \quad t = 1, ..., T, \tag{D.2}$$

$$\frac{\partial L}{\partial h_t} = \left(\frac{1}{1+\rho}\right)^{t-1}\frac{\theta}{h_t} - \lambda_t p + \lambda_{t+1}(1-\delta)p + \nu_{t+1} - \nu_t = 0, \quad t = 1, ..., T, \tag{D.3}$$

$$\frac{\partial L}{\partial d_t} = \lambda_t - \lambda_{t+1}(1+r^d) + \mu_{t+1}\alpha - \mu_t = 0, \quad t = 1, ..., T, \tag{D.4}$$

$$\frac{\partial L}{\partial s_t} = -\lambda_t + \lambda_{t+1}(1+r^s) \leq 0, \quad t = 1, ..., T, \tag{D.5}$$

$$\frac{\partial L}{\partial s_t}s_t = [-\lambda_t + \lambda_{t+1}(1+r^s)]s_t = 0, \quad t = 1, ..., T, \tag{D.6}$$

where $c_t, h, d_t, \lambda_t > 0$, $s_t \geq 0$, $\mu_t \geq 0$, and

$$\mu_1 = \mu_{T+1} = \nu_1 = \nu_{T+1} = 0. \tag{D.7}$$

We note that summing over (D.3) and using (D.7) eliminates the Lagrange multipliers ν_t and results in

$$\frac{\partial L}{\partial h} = \sum_{t=1}^{T}\left(\frac{1}{1+\rho}\right)^{t-1}\frac{\theta}{h} - \lambda_1 p - \sum_{t=2}^{T}\lambda_t\delta p + \lambda_{T+1}(1-\delta)p = 0, \tag{D.8}$$

where h_t is replaced by the constant h. The condition (D.8) then replaces (D.3).

As for the case with time-varying housing, the shadow interest rate and the Lagrange multiplier λ_t satisfy (C.15). From (C.15) and (D.4) we get

$$\frac{1}{1+r_t} = \frac{1 + (\alpha\mu_{t+1} - \mu_t)/\lambda_t}{1+r^d}, \tag{D.9}$$

so

$$r_t \lessgtr r^d \text{ if and only if } \alpha\mu_{t+1} - \mu_t \gtrless 0.$$

In particular, for $t = 1$, because $\mu_1 = 0$, if $\mu_2 > 0$, meaning that the restriction $d_2 \leq \alpha d_1$ is binding, we have $r_1 < r^d$. We would then expect this to translate into more borrowing than the benchmark in period 1, $d_1 > \bar{d}$. Similarly, for $t = T$, because $\mu_{T+1} = 0$, if $\mu_T > 0$, meaning that the restriction $d_T \leq \alpha d_{T-1}$ is binding, we have $r_T > r^d$. We would then expect less borrowing than the benchmark in period T.

Furthermore, for $s_t > 0$, by (D.6) we have $\lambda_{t+1}/\lambda_t = 1/(1+r^s)$, which together with (C.15) implies that $r_t = r^s \leq r_d$. When $r^s < r^d$ and $s_t > 0$, which occurs with constant housing in section 5, we thus have $r_t = r^s < r^d$ in (D.9) and thus that $\alpha\mu_{t+1} - \mu_t > 0$. We can interpret this as the net effect of the amortization constraints being to increase the marginal utility with respect to d_t, which

is given by the left side of the equality (D.4), where a positive term $\alpha\mu_{t+1} - \mu_t$ serves to increase the left side, inducing an increase in d_t by effectively reducing the shadow rate on borrowing, r_t, below the debt rate of interest r^d. This way, a binding amortization restriction both reduces the shadow interest rate and increases the debt.

From (D.2), we have

$$q_t = \frac{\theta}{1-\theta}\frac{c_t}{h} = \left[1 - \frac{\lambda_{t+1}}{\lambda_t}(1-\delta)p\right] + \frac{\nu_{t+1} - \nu_t}{\lambda_t} = \frac{r_t + \delta}{1 + r_t}p + \frac{\nu_{t+1} - \nu_t}{\lambda_t}. \tag{D.10}$$

Thus, in this case, because housing can no longer be freely varied across periods, the shadow rent, q_t defined as (B.1) no longer satisfies (C.17). Instead it satisfies (D.10), where ν_t is the Lagrange multiplier for the constraint (D.1).

The term $\nu_{t+1} - \nu_t$ in (D.10) will be positive or negative depending on whether a freely chosen h_t would exceed or fall short of the chosen constant h, corresponding to the shadow rate being, respectively, greater or less than the shadow rate with variable housing, the first term on the right side of (D.10).

Furthermore, letting $\tilde{q}_t$ denote first term on the right side of (D.10), we can write the latter as

$$\lambda_t q_t = \lambda_t \tilde{q}_t + \nu_{t+1} - \nu_t.$$

Summing over this, using (D.7), and dividing by λ_1, we get

$$\sum_{t=1}^{T} \frac{\lambda_t}{\lambda_1} q_t = \sum_{t=1}^{T} \frac{\lambda_t}{\lambda_1} \tilde{q}_t.$$

Here the term λ_t/λ_1 is, by (C.15), the present value discount factors consistent with the the the shadow interest rates,

$$\frac{\lambda_t}{\lambda_1} = \prod_{\tau=1}^{t-1} \frac{1}{1 + r_\tau}.$$

That is, the present value of all future shadow rents, the present value shadow rent of constant housing from period t to T, is equal to the present value of the future variable-housing shadow rents.

D.2 Writing the problem on a suitable form for Matlab

The variables are $\{c_t, d_t, s_t\}_{t=1}^{T}$ and the constant h. Let the column 3-vector v_t satisfy $v_t = (c_t, d_t, s_t)'$, and let the column $(3T+1)$-vector x satisfy $x = (v_1', v_2', ..., v_T', h)'$. Then $(c_t, d_t, s_t, h) =$

$(x_{1+3(t-1)}, x_{2+3(t-1)}, x_{3+3(t-1)}, x_{3T+1})$, and we can write $f(x)$ as

$$f(x) = -\sum_{t=1}^{T} \left(\frac{1}{1+\rho}\right)^{t-1} [(1-\theta)\ln x_{1+3(t-1)} + \theta \ln x_{3T+1}].$$

Write the constraints for constant h as

$$c_1 - d_1 + s_1 + ph \leq y + w_0, \tag{D.11}$$

$$(1 + r^d)d_{t-1} - (1 + r^s)s_{t-1} + c_t - d_t + s_t + \delta ph \leq y, \quad t = 2, ..., T, \tag{D.12}$$

$$(1 + r^d)d_T - (1 + r^s)s_T - (1 - \delta)ph \leq -w_T, \tag{D.13}$$

$$-\alpha d_{t-1} + d_t \leq 0, \quad t = 2, ..., T, \tag{D.14}$$

The right side of (D.11)-(D.14) is then the same column $2T$-vector b that was defined above. The corresponding $2T \times (3T+1)$ matrix $A = (A_{i,j})$ satisfies $A_{i,j} = 0$, except for $A_{1,j}$, corresponding to (D.11), satisfying

$$A_{1,1} = 1, \ A_{1,2} = -1, \ A_{1,3} = 1, \ A_{1,3T+1} = p;$$

$A_{t,j}$, corresponding to (D.12), satisfying

$$A_{t,2+3(t-2)} = 1 + r^d, \ A_{t,3+3(t-2)} = -(1 + r^s), \ A_{t,1+3(t-1)} = 1, \ A_{t,2+3(t-1)} = -1,$$

$$A_{t,3+3(t-1)} = 1, \ A_{t,3T+1} = \delta p, \quad t = 2, ..., T;$$

$A_{T+1,j}$, corresponding to (D.13), satisfying

$$A_{T+1,2+3(T-1)} = 1 + r^d, \ A_{T+1,3+3(T-1)} = -(1 + r^s), \ A_{T+1,3T+1} = -(1 - \delta)p;$$

and, finally, $A_{T+t,j}$, corresponding to (D.14), satisfying

$$A_{T+t,2+3(t-2)} = -\alpha, \ A_{T+t,2+3(t-1)} = 1, \quad t = 2, ..., T.$$
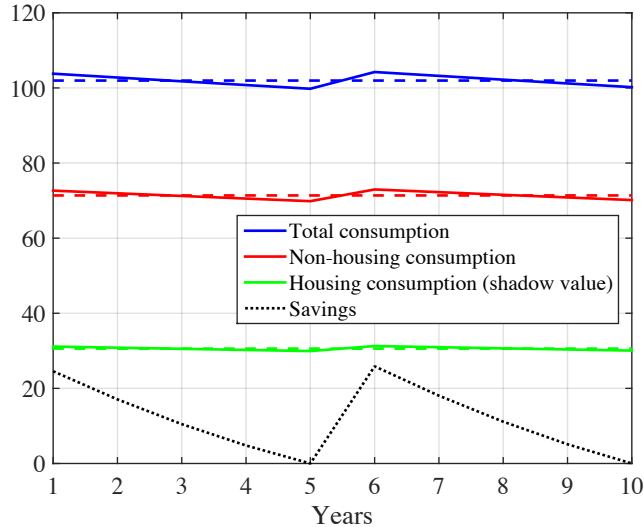
E   Refinancing

Refinancing means that there is no amortization constraint for the periods when refinancing is possible. This means that for each refinancing period τ_j, $j = 1, ..., n$, the corresponding rows $T + \tau_j$ in the matrix A and the vector b (specified in appendices C.2 and D.2) are deleted.

Figures 7.1 and E.1-E.2 show what this case look like, with the benchmark parameters and an option to refinance in year 6. The shadow interest rate in figure E.2, defined by (B.4) is high in

31

Figure E.1: Non-housing consumption, the shadow value of housing consumption, the shadow value of total consumption (benchmark levels dashed), and savings, when refinancing is possible. Savings interest rate less than debt interest rate.

year 5 because, as seen in figure E.1, non-housing consumption increases from year 5 to year 6, when the refinancing occurs.

Figure E.3 shows a case with $T = 20$ years split into four 5-year financing periods ($\tau_1 = 6$, $\tau_2 = 11$, and $\tau_3 = 16$).

F   Additional constraints on household borrowing

The discussion above is about a household's *demand* for loans, that is, what the household would like to borrow given its preferences, interest rates, budget constraints, and amortization constraints. The household's demand may of course face further restrictions, imposed by regulators or by lenders.

F.1   Loan to value

The household may, for instance, be constrained by a regulated LTV cap,

$$d_t \leq \ell \, p h_t, \quad t = 1, ..., T, \tag{F.1}$$

where $\ell > 0$ denotes a maximum LTV ratio allowed. For instance, in Sweden, Finansinspektionen (the Swedish Financial Supervisory Authority) has recommended mortgage lenders to apply an 85 percent LTV cap. Note that, with the numbers used in section 5, the initial debt rises from an LTV

Figure E.2: The shadow rent, the shadow interest rates, and units of housing consumption when refinancing is possible (benchmark values dashed)
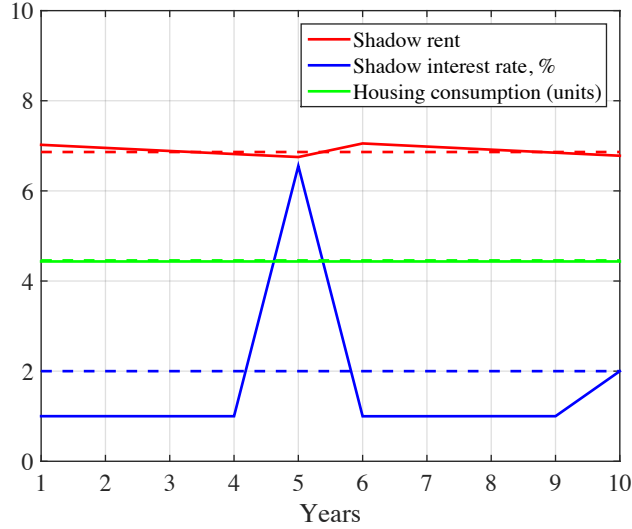

ratio of 71 percent without an amortization requirement to about 85 percent with an amortization requirement, just reaching but not being constrained by the Swedish LTV cap.

F.2 Debt service to income

As mentioned in appendix A, gross debt service is the interest on the debt plus plus the amortization, $(r^d + 1 - \alpha)\, d_t$. *Net* debt service can be specified as the interest on the debt less the interest on the savings, $r^d d_t - r^s s_t$. Table A.1 shows that benchmark gross and net debt and service without an amortization requirement equals 6.3 for the benchmark parameters, about 6 percent of permanent income, 102, and a about one fifth of the benchmark housing costs, 31. With an amortization requirement, gross debt service increases by the amortization payment, $(1 - \alpha)d_t$, 2 percent of the debt in benchmark case. This makes the initial and average gross benchmark debt service with amortization equal to 15.2 and 13.9, respectively.

However, since the amortization requirement in these cases is fulfilled through withdrawala from a savings account and does not affect the overall debt-service capacity of the household, it makes little sense to include it in a DSI restriction. Therefore, under these circumstances, net debt service seem to be the relevant item to include in such a constraint.

Figure E.3: The debt path, the value of housing, and net debt with and without amortization requirement (AR) when refinancing is possible. Savings interest rate less than debt interest rate.
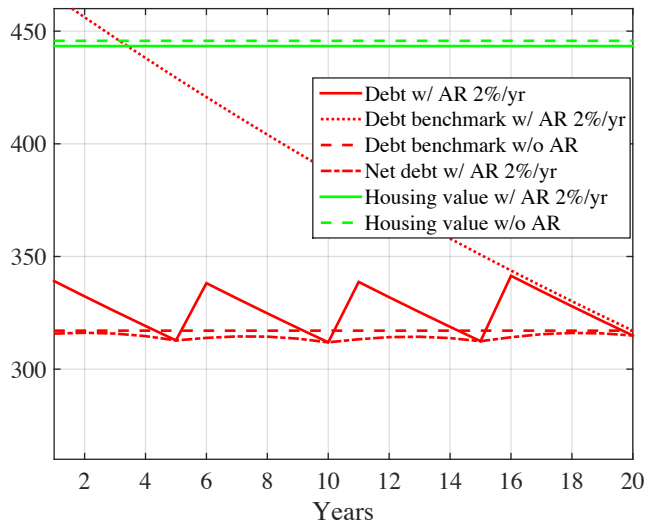
F.3 "Left to live on"

A household's borrowing may also be constrained by lending standards imposed by lenders, for instance, in the form of passing a so-called "left to live on" (LTLO) test. Suppose lenders use a maximum (real) interest rate, $r^{\max} > r^d$, larger than the lending rate, to calculate a maximum interest payment and requires that the household after paying debt service (equal to this maximum payment and amortization), plus the maintenance and operating cost (δph) out of its permanent income can afford at least a given minimum non-housing consumption, $c^{\min}$. This implies a constraint of the form

$$r^{\max} d_t + (1 - \alpha) d_t + \delta p \bar{h} + c^{\min} \le y^p, \quad t = 1, ..., T. \tag{F.2}$$

This implies the following constraint for the initial debt,

$$d_1 \le \frac{y^p - \delta p \bar{h} - c^{\min}}{r^{\max} + 1 - \alpha}. \tag{F.3}$$

Suppose the maximum (real) interest used is as high as 5 percent $(r^{\max} = 0.05)$ and that the amortization rate is 2 percent $(\alpha = 0.98)$; then we have $r^{\max} + 1 - \alpha = 0.07$. Furthermore, take the maintenance and operating cost to be $\delta p \bar{h} = 0.05 \cdot 446 = 22.3$, the minimum non-housing consumption to be half of the benchmark consumption, $c^{\min} = \bar{c}/2 = 35.7$, and recall that the benchmark permanent income $y^p = 102.0$. Then we get

$$d_1 \le 14.3 \, (y^p - \delta p \bar{h} - c^{\min}) = 628.$$

Thus, with the benchmark numbers of section 3 used in section 5, the initial borrowing with an amortization constraint falls much short of this LTLO constraint.

However, the above disregards the interest income from the excess borrowing invested into a savings account. A more relevant LTLO test would include that interest income, and replace (F.2) by

$$r^{\max} d_t - r^s s_t + (1 - \alpha) d_t = r^{\max} d_t - r^s (d_t - \bar{d}) + (1 - \alpha) d_t$$

$$= (r^{\max} - r^s + 1 - \alpha) d_t + r^s \bar{d}$$

$$\leq y^p - \delta p \bar{h} - c^{\min}, \quad t = 1, ..., T,$$

where I have used $s_t = d_t - \bar{d}$ (which holds approximately when $r^s < r^d$). This implies the following easier constraint for the initial debt,

$$d_1 \leq \frac{y^p - r^s \bar{d} - \delta p \bar{h} - c^{\min}}{r^{\max} - r^s + 1 - \alpha}. \tag{F.4}$$

For the benchmark values, (F.4) implies $d_1 \leq 680$. However, one might think that a higher real interest rate would also apply to the (safe) savings interest rate. If the savings interest rate would also be 3 percentage point higher, so the spread between debt and savings interest rates is only 1 percent instead of 4 percent, (F.4) implies the quite easy constraint $d_1 \leq 1361$.

If the excess borrowing is invested in a savings account with the lender, the lender can directly observe that the household has savings corresponding to the excess borrowing. It would then be obvious to the lender that the excess borrowing implies little risk.

In particular, since the lender can observe that the borrower has liquid savings that are used to amortize the loan, instead of taking into account the interest income on the savings corresponding to the excess borrowing, it would arguably be more relevant to drop the amortization term from the constraint (F.2), implying the following constraint for the initial debt,

$$d_1 \leq \frac{y^p - \delta p \bar{h} - c^{\min}}{r^{\max}}. \tag{F.5}$$

This implies the easy constraint $d_1 \leq 880$.

In these cases, because of the debt-savings interest-rate spread, the lender profits from the excess borrowing and has no incentive to prevent the household's excess borrowing in order to satisfy both its desired net debt level and the amortization constraint. Especially so, since the excess borrowing is matched one-to-one with savings and does not imply any increased risk.