Effective Protection with Global Value Chains*

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Abstract

The effective rate of protection (ERP) is a widely-used tool to evaluate the joint impact of input and output tariffs on the net protection offered to individual sectors. Nonetheless, the theoretical foundations underlying the traditional ERP are decades out of date. In this paper, we provide fresh guidance for policy analysts that seek to measure effective protection, with two core elements. First, we develop a new ERP concept, which measures how much tariffs shift the derived demand for real value added at the sector level. The ERP then has an intuitive interpretation: it is the effective subsidy to buyers of sectoral value added that replicates the impact of the tariff structure on demand. Second, we demonstrate how to compute our ERP index accounting for global value chain linkages. The result is an ERP for the GVC age, in which effective protection depends on the structure of value chains across countries and sectors, and the structure of all tariffs applied globally. We demonstrate the usefulness of our approach by analyzing how recent tariff changes have altered effective protection in the United States and the rest of the world.

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The world has taken a sudden turn toward protectionism, led by the United States. As a result, there is renewed attention to quantifying the impacts of tariffs. One approach is to simulate tariff scenarios in fully-fledged quantitative general equilibrium models. A second approach is to build indexes of trade policy restrictiveness. When these indexes are motivated by economic theory, they shed light on the mechanisms through which tariffs work, without specifying and solving a full equilibrium model. They also provide useful summary statistics for policy analysis.[1]

The Effective Rate of Protection (ERP) is a venerable workhorse in the policy analysis toolkit. Corden (1966) is widely credited with developing the ERP formula and popularizing the concept.[2] The Corden-style index takes the form: $ERP_s = \frac{t_s - \sum_{s'} a_{s's} t_{s'}}{1 - \sum_{s'} a_{s's}}$, where s and s' denote sectors (industries) and $a_{s's}$ represents the share of input from industry s' in the costs of down stream industry s. Naturally, applying tariffs on the output of a given sector raises that sector's effective protection, while increasing tariffs on the sector's inputs lowers its effective protection. The ERP thus seeks to measure the net protection offered to each sector by the combination of applied output and input tariffs.

Although Corden's original ERP index has intuitive appeal, it was much maligned (and then refined) by trade theorists, on the grounds that it was unclear exactly what it measured. Beyond this theoretical critique, a modern trade economist may find it striking that the ERP is silent about how global value chains (GVCs) matter for quantifying the burden of trade barriers. Traditional ERP calculations study the effects of only a country's own tariffs, taking only domestic input linkages into account, and taking both export and import prices as given. Moreover, the traditional ERP does not allow for differences in tariffs across trading partners, which may have dramatically different degrees of GVC integration. As a result, the traditional ERP index is silent about the role of cross-border input linkages in transmitting the impact of tariffs across countries.

In this paper, we take a fresh look at the ERP for a modern audience, putting GVC linkages at center stage. We make four main contributions. First, we leverage a value-added perspective on trade to refine the theory underlying the measurement of effective protection.[3]

[1]Trade restrictiveness indexes, developed by Anderson and Neary (2005) to measure the welfare effects of policy, have been widely used in policy work. Estefania-Flores et al. (2022) develop a new policy index at the IMF, which aggregates binary indicators for tariff and non-tariff policy instruments that impede trade.

[2]While Corden (1966) is the leading reference, the concept appears in several roughly contemporaneous papers, including Balassa (1965), Johnson (1965), and Basevi (1966), as well as in other earlier applied work cited by Corden himself. Additional theoretical papers that develop the concept are Corden (1969, 1971), Anderson and Naya (1969), Ethier (1972), and Bhagwati and Srinivasan (1973). See Greenaway and Milner (2003) and Bhagwati, Panagariya and Srinivasan (1998) for surveys of the ERP literature, which we discuss further below.

[3]Bems and Johnson (2017) used a similar "demand for value added" approach to measure real effective exchange rates, by combining global input-output data with value-added price indexes (i.e., GDP defla-

Specifically, we direct attention to how tariffs affect the derived demand for real value added produced by each industry. When a tariff change raises demand for a given industry's real value added, we say that the effective protection of that industry has increased. Our effective protection index then measures the magnitude of the vertical shift in the inverse demand curve for industry value added. As such, the index can be re-interpreted as the (direct) tax/subsidy on the consumption of sectoral value added that is equivalent to the (indirect) final and input tariffs in terms its impact on demand for value added from a given industry.

Our approach provides a firm grounding for ERP measurement in economic theory, in the same spirit as important work by Anderson and Neary (2005) on measuring trade restrictiveness. At the same time, it is important to keep in mind that the ERP is not a sufficient statistic for welfare, unlike Anderson and Neary's index. Rather, the index we develop dovetails with our understanding of Corden's original aims. Reflecting on the ERP concept, Corden (2016) writes: "the effective rate of protection... is the proportional increase in the effective price [of real value added] made possible by tariffs." The ERP we define makes this concept precise, by converting the impact of tariffs into an equivalent tax/subsidy applied to real value added, which changes the price buyers would be willing to pay for real value added from a given sector.[4]

The ERP we develop is also related to the "distributional effective rate of protection" concept introduced by Anderson (1998), which computes the uniform import tariff that replicates the impact of the actual tariff structure on returns to specific factors in a given industry. We share a common interest in applying economic theory to ERP measurement with Anderson, but our ERP concept differs in several respects. First, we calculate direct value-added tax/subsidy equivalents, rather than equivalent import tariffs. More importantly, our definition is built using demand for value added as the organizing concept, so it applies outside the specific factors context that underpins Anderson's approach. As a result, our ERP sheds light on the directional impacts of tariff changes for resource allocation, which Bhagwati and Srinivasan (1973) highlighted as a central goal of the ERP literature.

To operationalize our ERP concept, we develop an explicit index number formula to measure it, via linearization and manipulation of a standard gravity-type trade framework. The resulting formula sharpens intuition about how input-output linkages shape the ERP, along with elasticities of substitution along different production and demand margins. This consti-

tors) and nominal exchange rates. Patel, Wang and Wei (2019) extend this approach with many sectors. Blanchard, Bown and Johnson (2025) also deploy value-added concepts to study optimal trade policies.

[4]Our focus on the demand side is deliberate: by defining the ERP as a shift in the demand curve for value added, we avoid the need to specify the structure or parameters of supply. This sidesteps the more demanding task of calculating equilibrium price changes—an exercise that typically requires stronger assumptions and more data. The resulting index isolates the directional pull of trade policy on resources and is readily implementable without a full general equilibrium model.

tutes our second contribution, and provides a practical guide for constructing ERP indexes using world input-output tables, together with elasticities which must be calibrated by the researcher. Extending this linear-approximate approach, we describe how exact ERP index numbers can be constructed, and we compare the exact and linear-approximate approaches via numerical analysis.

We first develop the new ERP concept in a small open economy with input-output linkages across many domestic sectors. Then, we extend it to a multi-country setting with global value chains. The resulting ERP is suitable for a world with GVCs – this is our third contribution. Here again we compute the value-added tax/subsidy equivalent, but now for the entire global structure of final and input tariffs, whose impacts propagate via GVC linkages. We study that propagation, with insights that are useful for analyzing bilateral and multilateral trade policy. For example, we demonstrate raising tariffs vis-à-vis GVC partners may reduce demand for domestic value added, amounting to an effective tax on domestic value added via 'roundabout' GVC linkages.

The fourth contribution of the paper is to apply the framework to evaluate the impact of recent changes in tariffs by the United States. We set parameters in the model using OECD Inter-Country Input-Output Tables and elasticity estimates obtained from the literature. We also construct tariff scenarios that capture key features of the tariffs imposed by U.S. President Trump in 2025. Because the policy environment is fluid, and the model framework we deploy is stylized, we view these as informative thought exercises, rather than a definitive policy evaluation.

Our first quantitative application builds intuition for the myriad channels through which tariffs impact demand for value added in a small open economy (one country, with multilateral policy changes), where we can decompose the channels through which final goods versus input tariffs work. We find that recent US tariff tariff changes have widely divergent effects across sectors, in part due to the structure of network linkages. We identify important backward spillovers from tariffs on imported autos into primary materials sectors, like metals and plastics. We also show how tariffs on steel are propagated forward to dampen effective protection in downstream manufacturing.

Feeding in a set of recent tariff changes that capture the salient features of US policy, we find effective protection increases, as expected in a partial equilibrium setting. More surprisingly, expected protection rises more in upstream (input-supplying) sectors than downstream ones, on average. These results imply the relative resource pull associated with recent tariff changes may be directing U.S. economic activity away from downstream sectors like machinery and shipbuilding, toward upstream sectors like metals. Further, these results suggest cost pressures arising from input tariff increases have attenuated the increase in effective protection resulting from the tariff changes; this drag on industrial activity due to input tariffs is readily apparent for motor vehicles, among other manufacturing sectors.

Our second application extends the ERP index to a multi-country environment, where we account for foreign and domestic trade policy and the structure of bilateral GVC linkages. Where as the traditional ERP aggregates policies imposed by the importing country, the ERP with global value chains (GVC-ERP) depends on policy in all countries. Feeding bilateral Trump tariffs through the framework, we find that effective protection has tended to increase more in upstream sectors. Moreover, we decompose changes in effective protection into components coming from final goods versus input tariffs, sectoral tariffs on autos and metals, and tariffs on China versus the rest of the world. The Chinese tariffs alone have a sizable impact on effective protection in many sectors, especially in Textiles and Electrical Equipment. We also illustrate how effective protection abroad changes as a result of the US tariffs. Mexico and Canada see increased effective protection, as if US tariff changes have subsidized value added originating in Mexico and Canada. Looking forward, we also present results for hypothetical intensification of the trade war, looking at the effects of US-China escalation and a breakdown in USMCA trade.

The paper proceeds as follows. In Section 1, we lay out the general framework for measuring effective protection. Section 2 implements the framework for the small open economy, and provides an initial numerical analysis of recent tariff changes. Section 3 extends the framework and numerical analysis to the multi-country setting.

1 Conceptual Framework

Tariffs influence production and resource allocation through many channels. Tariffs are (obviously) direct taxes on final goods and imported inputs, which alter production costs and sourcing decisions. The direct effects of tariffs are then propagated through upstream and downstream production network linkages to producers and sectors beyond those directly engaged in importing. Ultimately, tariffs translate into shifts in demand for primary factors of production – both domestic and foreign – and factor prices, triggering further adjustments.

Given this complexity, a natural question arises: is a given sector ultimately "helped" or "hurt" by tariffs? More precisely, are tariffs on net providing market signals that pull resources into a given sector, or are they pushing them out? As we discuss further below, this resource allocation question was the central motivation for early efforts to measure effective protection, according to Bhagwati and Srinivasan (1973). But because tariffs have complex effects through many distinct channels, it is a hard question to answer cleanly.

Our core contribution is to show how measuring changes in the derived demand for real value added provides a useful answer to that simple question. To fix ideas, we use the term "real value added" to mean the quantity of output net of intermediate inputs. In the model we describe below, this will correspond to real GDP at the sector level.[5] Since real value added is produced from primary factors (e.g., capital and labor), changes in demand for real value added translate into changes in the derived demands for those factors. In turn, the demand for real value added is itself derived from producers' decisions about how to combine factors and intermediate inputs in production.

Our end goal is to define and measure how the level of demand for value added from a given sector is influenced by tariffs. That is, are tariffs (or tariff changes) raising or lowering demand for real value added from a particular sector? If tariffs raise demand, then they will tend to pull resources into that sector; if they lower demand, they will push them out. Importantly, these directional implications hold regardless of how the supply side adjusts. While an increase in demand may lead to higher prices, higher quantities, or both in equilibrium—depending on the shape and potential shift in supply— it always implies movement along the supply curve. One need not solve for the full supply response to assess the direction of the resource pull; knowing whether demand has shifted up or down is enough.

There is a second, distinct way to understand why evaluating the change in demand for value added is informative about the directional effects of tariff policy on resource allocation. As every economist knows, taxes imposed on buyers depress demand, while subsidies to buyers increase demand. As such, when you tax something you get less of it; when you subsidize it, you get more. From this perspective, an alternative way to think about the tariff structure is that it imposes an effective net tax on, or provides an effective subsidy to, purchasers of real value added from particular sectors. Thus, one can characterize the net impact of tariffs in terms of the effective tax/subsidy they impose on buyers of real value added from each sector.[6]

To formalize this intuition, we define the effective rate of protection (ERP) as the ad-valorem subsidy to, or tax on, buyers of real value added that would replicate the shift in demand caused by the observed tariff change.[7] Let v_s denote the quantity of real value added

[5]In the perfectly competitive, constant-returns-to-scale economy framework we adopt below, real value added corresponds to double-deflated real GDP, as measured by national statistical authorities. In models with distortions (e.g., markup pricing), then measured real GDP may diverge the theoretical concept of real value added. This is a measurement issue, not a conceptual problem.

[6]Note that the ultimate incidence of this tax – whether it borne by suppliers or buyers in equilibrium – depends not only on how demand shifts, but the nature of supply. Questions of incidence are interesting, but outside the scope of this paper.

[7]One subtlety is that we must choose a specific point along the demand curve at which to measure the effective tax/subsidy – i.e., at which point to evaluate the shift. As is clear below, we elect to evaluate the implied tax/subsidy at initial equilibrium values, consistent with the standard practice of log-linearizing around the initial baseline in applied economic analysis.

used in production by sector s, and let p^v_s denote its price. Let $\mathbf{p}^v_{s'}$ be the vector of value-added prices in all other sectors $s' \neq s$, and let $\boldsymbol{\tau}$ denote the vector of tariffs on final goods and intermediate inputs. We define the sectoral demand for value added as a function of these variables, along with other determinants (e.g., foreign prices, aggregate consumption) that are collected in the demand shifter D for notational convenience:

$$v_s = D_s(p^v_s; \mathbf{p}^v_{s'}, \boldsymbol{\tau}, D), \tag{1}$$

where $D(\cdot)$ is a standard demand function, with the demand for a sector's own value added being strictly decreasing in its own price. It proves convenient to re-write the demand relation in terms of the inverse demand curve:

$$p^v_s = P^D_s(v_s; \mathbf{p}^v_{s'}, \boldsymbol{\tau}, D). \tag{2}$$

We define the *effective rate of protection* (ERP) as the *ad valorem shifter* δ_s applied to the initial price of value added, p^v_{s0}, that replicates the upward shift in the inverse demand curve for real value added caused by a change in the tariff schedule, evaluated at the initial resource allocation, v_{s0}.

Definition 1 (Exact ERP). *Given the inverse demand curve $p^v_s = P^D_s(v_s; \mathbf{p}^v_{s'}, \boldsymbol{\tau}, D)$, described above, together with an initial tariff vector $\boldsymbol{\tau}_0$ and a new tariff vector $\boldsymbol{\tau}_1$, the ERP is implicitly defined by:*

$$(1+\delta_s) \cdot P^D_s(v_{s0}; \mathbf{p}^v_{s'0}, \boldsymbol{\tau}_0, D_0) = P^D_s(v_{s0}; \mathbf{p}^v_{s'1}, \boldsymbol{\tau}_1, D_1),$$

where $(1+\delta_s)$ is the ERP index. Correspondingly, δ_s is the implicit ad-valorem subsidy to buyers of value added from sector s.

This expression defines the ERP as the vertical shift in the inverse demand schedule induced by tariffs. A positive ERP implies an upward shift in inverse demand, reflecting an increase in the price buyers are willing to pay for a given quantity of value added; a negative ERP implies a downward shift.[8] If $\delta_s > 0$, the ERP constitutes an implicit subsidy to the *buyer side* of the value-added market in sector s, which reproduces the impact on factor demand of the change in tariffs from $\boldsymbol{\tau}_0$ to $\boldsymbol{\tau}_1$. If $\delta_s < 0$, the implied subsidy is negative, which we interpret as a tax.

[8] Alternatively, we can express the same logic in terms of the demand function itself. The ERP is the value of δ_s that satisfies: $D_s(p^v_{s0}; \mathbf{p}^v_{s'0}, \boldsymbol{\tau}_0, D_0) = D_s((1+\delta_s)p^v_{s0}; \mathbf{p}^v_{s'1}, \boldsymbol{\tau}_1, D_1)$. This alternative definition implies that the ERP can be interpreted as the increase in the value added in a given sector that would unwind the effects of the tariff change, returning demand for value added to its value before the change was implemented.

Figure 1 illustrates this logic in the case where tariffs increase the demand for value added in sector s. The ERP corresponds to the upward shift in the inverse demand curve under the original tariff regime needed to replicate the new willingness to pay for the original quantity of value added.

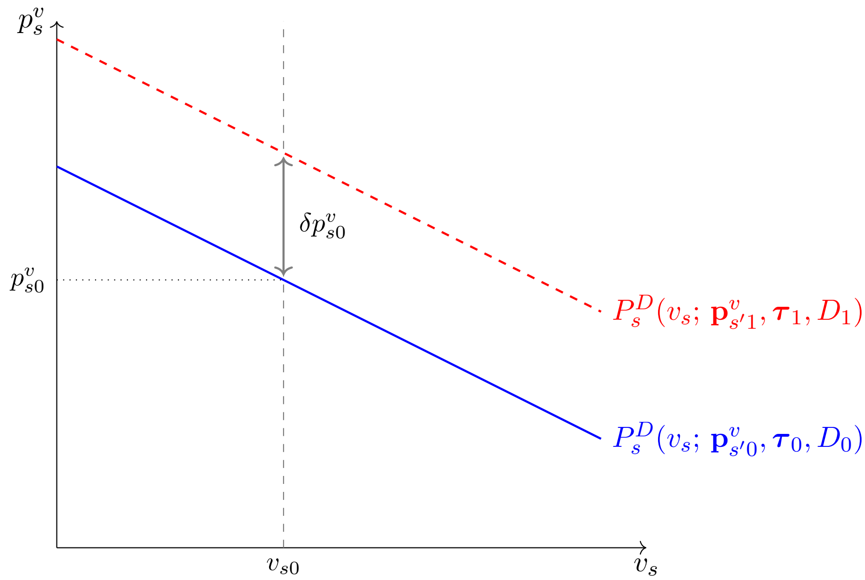


Figure 1: Tariff-induced upward shift in inverse demand for value added in sector s

A last point to note is that this definition allows tariffs to potentially have direct effects on demand for value added, as well as indirect effects that operate through equilibrium responses of the price of value added in other sectors ($\mathbf{p}^v_{s'0}$ vs. $\mathbf{p}^v_{s'1}$) and other determinants of demand (D_0 vs. D_1). Proceeding step by step below, we first show how to construct the ERP for a small open economy, which takes foreign prices and the demand schedule for its exports as given. In that context, we will discuss how to account for changes in value-added prices in outside sectors. We then provide an ERP index for large open economies embedded in a multi-country setting with GVCs, which allows for endogenous changes in foreign variables (e.g., prices, output levels, consumption patterns, etc.) in response to tariffs. Together, the collected formulas provide a rich description of the sector-specific push/pull of trade policy through the lens of derived demand in the context of potentially complex GVC linkages.

Comments on the Classic ERP Literature Our exposition of the ERP concept is closely related to concepts in the early ERP literature, but with a few important distinctions. Consistent with the discussion above, one similarity is that we seek to summarize the push/pull forces induced by tariffs for resource allocation. This approach resonates with the problem statement by Bhagwati and Srinivasan (1973): "The task of the theory of effective protection may then be conceived essentially as one of examining, in a model allowing imported inputs, the question whether it is possible to devise a 'price' of value-added, which can be used as an index to rank different activities such that, in exact analogy with the nominal tariff theory, the change in the 'quantity' of value-added can be correctly predicted."

To be clear, we are not computing the price of value added per se as an equilibrium object. Instead, we measure the shift in the demand schedule. Naturally, this approach is consistent with measuring the change in the willingness to pay for real value added from the sector, which places bounds on the change in equilibrium prices made possible by tariffs. Further, it is generally sufficient to predict the direction of quantity changes – i.e., to summarize the impulse provided by trade policy on the allocation of domestic resources – which is the stated goal in Bhagwati and Srinivasan (1973). But importantly, because it is defined in terms of the shift in the derived demand schedule, our ERP does not require a full specification of factor supply.

Related to this discussion, Corden (1966) articulates the goal somewhat differently: our reading is that he seeks to measure the realized change in the price of real value added. He is able to do that due to the combination of three strong assumptions: he restricts substitution in production (imposing a Leontief production function); he assumes that the economy is "small" and takes both its export and import prices as given; and he assumes that domestic output is perfectly substitutable with domestically produced goods, which locks down domestic prices.[9] Given these powerful assumptions, Corden (1966) is able to compute the realized equilibrium price of real value added induced by tariffs. We discuss a variant of his approach and how it compares to ours in context below.

2 An ERP Index for a Small Open Economy

With core concepts laid out above, we turn to implementation of the ERP index for a small open economy. For illustration purposes, we keep the framework as simple as possible, in two senses. First, we adopt a standard, competitive multi-sector framework with input-output linkages. This serves to explain key concepts, which extend to more elaborate models. Second, we do not specify a complete model up front; we focus only on the elements of the model needed to define our ERP index.

We describe the elements of this model in Section 2.1. In Section 2.2 we linearize the

[9]Later contributions by Corden (1969), Leith (1968), Bhagwati and Srinivasan (1973), and others then relax the Leontief assumption, but maintain the remaining two assumptions. Bhagwati and Srinivasan (1973) provide a clear discussion of how the Corden (1966) index emerges via a first-order approximation when the production function is not Leontief in primary factors and inputs. The early literature also devotes extensive attention to how to treat non-traded goods, whose prices are not pinned down by world prices.

framework to write down an explicit index number formula for the ERP. We discuss how tariffs directly enter the linear index in Section 2.3, and we apply the index to analyze tariff changes numerically in Section 2.4. Section 2.5 extends the basic approach to account for price changes in outside sectors and non-linear effects.

2.1 Model Framework

We assume there are S sectors, and two countries (home H and foreign F). Home is small in the sense that it takes foreign prices as given on the import side. On the export side, it faces a downward sloping demand curve for its exports, where the level of foreign demand for Home exports is exogenous.[10]

The output of each sector is produced from primary factors and intermediate inputs. We assume that primary factors may be combined into a composite factor, and that the production function is separable in this common factor and a composite input. Together, these assumptions imply that the quantity of real value added produced by each sector is well defined, as discussed in Sims (1969) and Bhagwati and Srinivasan (1973). To make this concrete, let $V_t(s) = Z_t(s)L_t(s)$ be real value added, where $L_t(s)$ is a composite primary factor used by sector s (potentially a bundle of labor, land, capital, etc.) and $Z_t(s)$ is the productivity of that factor in producing real value added. Then, we assume that real value added is combined with a composite input $M_t(s)$ to produce output via a CES production function:

$$Q_t(s) = \left(\alpha(s)^{1/\gamma(s)} V_t(s)^{(\gamma(s)-1)/\gamma(s)} + \left(1-\alpha(s)^{1/\gamma(s)}\right) M_t(s)^{(\gamma(s)-1)/\gamma(s)}\right)^{\gamma(s)/(\gamma(s)-1)} \quad (3)$$

Competitive producers then minimize costs, taking prices of real value added and the composite inputs as given, which yields the demand for value added:

$$V_t(s) = \alpha(s) \left(\frac{P_t^V(s)}{P_{Ht}(s)}\right)^{-\gamma(s)} Q_t(s), \quad (4)$$

where $P_t^V(s)$ is the price of real value added in sector s and $P_{Ht}(s)$ is the price of gross output. Naturally, demand for real value added declines in its own price relative to the price of output, due to substitution, and increases with overall sectoral output – i.e., as the scale of production rises.

[10] This small open economy definition differs from one in which sectoral prices are pinned down via arbitrage with exogenous world prices. The setup we consider mimics a standard approach in the macroeconomics literature, building on Galí and Monacelli (2005). In the trade literature, Caliendo and Feenstra (2024) and Demidova et al. (2024) have recently provided limit arguments in gravity-type trade models that yield a similar setup. Flam and Helpman (1987) is another early paper that used a similar "small country" structure.

The output of each sector is sold to consumers as a final good, downstream users as intermediate inputs, and foreign buyers as exports. Consumers have constant elasticity preferences (CES) across sectors, and CES preferences between home and foreign goods within each sector. The composite input that firms use in production is a CES composite of composite input goods sourced from upstream sectors, and within each sector the composite input good is a CES composite of home and foreign inputs. On the export side, producers face a downward sloping foreign demand for exports.

Because most of these assumptions are entirely standard, we skip derivations and proceed to summarize the elements needed to characterize demand for value added in Table 1. It should be obvious that demand for real value added is inherently non-linear, not only because conditional input demand in Equation 4 is non-linear, but also because both the level of output and the output price are non-linear functions of the price of that sector's real value added. All together, the system of equations in Table 1 can be used to define exact ERP indexes that account for these non-linearities. Rather than proceeding down that route immediately, we first linearize and manipulate equilibrium conditions in the model to develop an index number formula that approximates the exact ERP. We then discuss how to compute the exact ERP in Section 2.5.2, where we also compare the approximate versus exact approaches.

2.2 Linear Approximation to Form the ERP Index

We provide linear approximations to the equilibrium conditions in Table 2. The hat notation denotes log deviation from an initial equilibrium, and values in that initial equilibrium are indicated by date 0. Further, all prices are measured relative to the domestic price level, so prices in Table 2 should be interpreted as deviations in relative (real) prices from the initial equilibrium.[11]

[11] Some technical notes are worth clarifying here. First, we have redefined the export equation slightly, to simplify matters. In Table 1, exports are written as: $X_t(s) = \left(\frac{P_{Ht}(s)}{P_t^*(s)}\right)^{-\eta_X(s)} X_t^*(s)$, where $P_t^*(s)$ is the foreign import price level (equivalently, an export demand shifter). In what follows, we set $P_t^*(s) = P_F(s)$, which restricts the foreign import price level to be equal to the price of Home's imports in that sector. This is innocuous, in that any gaps between $P_t^*(s)$ and $P_F(s)$ would then be subsumed into the exogenous demand shifter $X_t^*(s)$. Second, the resulting foreign price that appears in log-linear export demand is $\hat{p}_{Ft}$, which corresponds to the log deviation in the price of imported foreign goods relative to the domestic price level. We treat this real value as exogenous. One way to interpret this is to note that $P_{Ft}(s)/P_{Ct} = (P_{Ft}(s)/P_{Ct}^*)(P_{Ct}^*/P_{Ct})$. So taking $P_{Ft}(s)/P_{Ct}$ as exogenous is equivalent to treating both the real price of foreign goods $(P_{Ft}(s)/P_{Ct}^*)$ and the real exchange rate (P_{Ct}^*/P_{Ct}) as given. Since we do not specify a full model to pin down either aggregate consumption or real exchange rates, this approach is consistent with the partial equilibrium emphasis of traditional approaches to ERP construction. Third, consistent with this discussion, note that we drop the equation describing the aggregate consumer price index from the system; it is not needed to characterize demand for real value added, given the partial equilibrium set up.

Table 1: Model Elements for Small Open Economy

Consumption	$C_t(s)=\zeta(s)\left(\frac{P_{Ct}(s)}{P_{Ct}}\right)^{-\vartheta}C_t$ $C_{Ht}(s)=\nu(s)\left(\frac{P_{Ht}(s)}{P_{Ct}(s)}\right)^{-\eta_C(s)}C_t(s)$
Inputs	$V_t(s)=\alpha(s)\left(\frac{P_t^V(s)}{P_{Ht}(s)}\right)^{-\gamma(s)}Q_t(s)$ $M_t(s)=(1-\alpha(s))\left(\frac{P_{Mt}(s)}{P_{Ht}(s)}\right)^{-\gamma(s)}Q_t(s)$ $M_t(s',s)=\alpha(s',s)\left(\frac{P_t(s',s)}{P_{Mt}(s)}\right)^{-\kappa}M_t(s)$ $M_{Ht}(s',s)=\xi(s',s)\left(\frac{P_{Ht}(s')}{P_t(s',s)}\right)^{-\eta_M(s')}M_t(s',s)$
Prices	$P_{Ht}(s)=\left(\alpha(s)P_{Vt}(s)^{1-\gamma(s)}+(1-\alpha(s))P_{Mt}(s)^{1-\gamma(s)}\right)^{1/(1-\gamma(s))}$ $P_{Mt}(s)=\left(\sum_{s'}\alpha(s',s)P_t(s',s)^{1-\kappa}\right)^{1/(1-\kappa)}$ $P_t(s',s)=\left(\xi(s',s)P_{Ht}(s')^{1-\eta_M(s')}+(1-\xi(s',s))(\tau_{Mt}(s')P_{Ft}(s'))^{1-\eta_M(s')}\right)^{1/(1-\eta_M(s'))}$ $P_{Ct}=\left(\sum_s\zeta(s)P_{Ct}(s)^{1-\vartheta}\right)^{1/(1-\vartheta)}$ $P_{Ct}(s)=\left(\nu(s)P_{Ht}(s)^{1-\eta_C(s)}+(1-\nu(s))\left(\tau_{Ct}(s)P_{Ft}(s)\right)^{1-\eta_C(s)}\right)^{1/(1-\eta_C(s))}$
Output	$Q_t(s)=C_{Ht}(s)+\sum_{s'}M_{Ht}(s,s')+X_t(s)$ $X_t(s)=\left(\frac{P_{Ht}(s)}{P_t^*(s)}\right)^{-\eta_X(s)}X_t^*(s)$

Table 2: Log-Linearization of Model Elements for Small Open Economy

Consumption	$\hat{c}_t(s)=-\vartheta\hat{p}_{Ct}(s)+\hat{c}_t$ $\hat{c}_{Ht}(s)=-\eta_C(s)\left(\hat{p}_{Ht}(s)-\hat{p}_{Ct}(s)\right)+\hat{c}_t(s)$
Inputs	$\hat{v}_t(s)=-\gamma(s)\left(\hat{p}_{Vt}(s)-\hat{p}_{Ht}(s)\right)+\hat{q}_t(s)$ $\hat{m}_t(s)=-\gamma(s)\left(\hat{p}_{Mt}(s)-\hat{p}_{Ht}(s)\right)+\hat{q}_t(s)$ $\hat{m}_t(s,s')=-\kappa\left(\hat{p}_t(s,s')-\hat{p}_{Mt}(s')\right)+\hat{m}_t(s')$ $\hat{m}_{Ht}(s,s')=-\eta_M(s')\left(\hat{p}_{Ht}(s)-\hat{p}_t(s,s')\right)+\hat{m}_t(s,s')$
Prices	$\hat{p}_{Ht}(s)=\left(\frac{P_{V0}(s)V_0(s)}{P_{H0}(s)Q_{H0}(s)}\right)\hat{p}_{Vt}(s)+\left(\frac{P_{M0}(s)M_0(s)}{P_{H0}(s)Q_{H0}(s)}\right)\hat{p}_{Mt}(s)$ $\hat{p}_{Mt}(s)=\sum_{s'}\left(\frac{P_0(s',s)M_0(s',s)}{P_{M0}(s)M_0(s)}\right)\hat{p}_t(s',s)$ $\hat{p}_t(s',s)=\left(\frac{P_{H0}(s')M_{H0}(s',s)}{P_0(s',s)M_0(s',s)}\right)\hat{p}_{Ht}(s')+\left(\frac{\tau_{M0}(s')P_{F0}(s')M_{Ft}(s',s)}{P_0(s',s)M_0(s',s)}\right)\left(\hat{\tau}_{Mt}(s')+\hat{p}_{Ft}(s')\right)$ $\hat{p}_{Ct}(s)=\left(\frac{P_{H0}(s)C_{H0}(s)}{P_{C0}C_0}\right)\hat{p}_{Ht}(s)+\left(\frac{\tau_{C0}(s)P_{F0}(s)C_{F0}(s)}{P_{C0}C_0}\right)\left(\tau_{Ct}(s)+\hat{p}_{Ft}(s)\right)$
Output	$\hat{q}_t(s)=\left(\frac{P_{H0}(s)C_{H0}(s)}{P_{H0}(s)Q_0(s)}\right)\hat{c}_{Ht}(s)+\sum_{s'}\left(\frac{P_{H0}(s)M_{H0}(s,s')}{P_{H0}(s)Q_0(s)}\right)\hat{m}_{Ht}(s,s')+\left(\frac{P_{H0}(s)X_0(s)}{P_{H0}(s)Q_0(s)}\right)\hat{x}_t(s)$ $\hat{x}_t(s)=-\eta_X(s)\widehat{p}_{Ht}(s)+\eta_X(s)\hat{p}_{Ft}(s)+\hat{x}_{Ft}(s)$

Getting to the index number formula then requires stacking and manipulating these standard equations. To explain this procedure and notation step-by-step, demand for value added can be written as:

$$\hat{\mathbf{v}}_t = -\gamma\hat{\mathbf{p}}_{Vt} + \gamma\hat{\mathbf{p}}_{Ht} + \hat{\mathbf{q}}_t, \tag{5}$$

where the bold typeface represents a vector with individual sector-level elements – e.g., $\mathbf{q}_t \equiv [\hat{q}_t(1), \cdots, \hat{q}_t(S)]$, and γ is a diagonal matrix with sector-specific elasticities of substitution (i.e., $\{\gamma(s)\}$) on the diagonal.

With CES, demand for value added in each sector depends on the sector's own value-added price, the price of its gross output, and its output level, per Equation 4. We seek to characterize demand for value added directly in terms of value added prices, exogenous variables (especially trade costs), and macro-level aggregates, however, so we substitute for $\hat{\mathbf{p}}_t$ and $\hat{\mathbf{q}}_t$ using remaining elements of the system.

2.2.1 Demand for Gross Output

Stacking the market clearing conditions, demand for gross output depends on demand for output by consumers, by downstream domestic producers, and export buyers in each sector:

$$\hat{\mathbf{q}}_t = \mathbf{S}_C\hat{\mathbf{c}}_{Ht} + \mathbf{S}_M\mathbb{M}_{Ht} + \mathbf{S}_X\hat{\mathbf{x}}_t, \tag{6}$$

where $\mathbf{S}_C$, $\mathbf{S}_M$, and $\mathbf{S}_X$ are matrices collecting the share of output in each sector allocated to each end use, and $\mathbb{M}_{Ht} = [\hat{m}_{Ht}(1,1), \hat{m}_{Ht}(1,2), \cdots, \hat{m}_{Ht}(S,1), \cdots, \hat{m}_{Ht}(S,S)]^T$ is a $S^2 \times 1$ column vector.[12]

Demand for home goods by home consumers is given by:

$$\hat{\mathbf{c}}_{Ht} = -\boldsymbol{\eta}_C\left(\hat{\mathbf{p}}_{Ht} - \hat{\mathbf{p}}_{Ct}\right) + \hat{\mathbf{c}}_t, \tag{7}$$

$$\text{with} \quad \hat{\mathbf{c}}_t = -\vartheta\hat{\mathbf{p}}_{Ct} + \iota\hat{c}_t \quad \text{and} \quad \hat{\mathbf{p}}_{Ct} = \mathbf{W}_{CH}\hat{\mathbf{p}}_{Ht} + \mathbf{W}_{CF}\left(\hat{\boldsymbol{\tau}}_{Ct} + \hat{\mathbf{p}}_{Ft}\right), \tag{8}$$

where $\boldsymbol{\eta}_C \equiv diag\left(\eta_C(s)\right)$ and ι is a column vector of ones. The matrices $\mathbf{W}_{CH} = diag\left(w_{CH}(s)\right)$ and $\mathbf{W}_{CF} = diag\left(w_{CF}(s)\right)$ collect expenditure shares on home and foreign goods in each sector. Similarly, demand for exports can be written as:

$$\hat{\mathbf{x}}_t = -\boldsymbol{\eta}_X\hat{\mathbf{p}}_{Ht} + \boldsymbol{\eta}_X\hat{\mathbf{p}}_{Ft} + \hat{\mathbf{x}}_{Ft}, \tag{9}$$

[12] $\mathbf{S}_C$ is a diagonal matrix with elements $\left\{\frac{C_{H0}(s)}{Q_0(s)}\right\}$ on the diagonal, $\mathbf{S}_M$ is a $S \times S^2$ matrix where each block diagonal element contains shares $\left\{\frac{M_{H0}(s,s')}{Q_0(s)}\right\}$ for industry s, and $\mathbb{M}_{Ht}$ is a column vector (defined above), and $\mathbf{S}_X$ is a diagonal matrix with elements $\left\{\frac{X_0(s)}{Q_0(s)}\right\}$.

where $\hat{\mathbf{x}}_{Ft}$ collects export demand shifters.[13]

Demand for home inputs by downstream domestic buyers requires some effort to obtain. Total input demand by each sector is:

$$\hat{\mathbf{m}}_t = -\boldsymbol{\gamma}\hat{\mathbf{p}}_{Mt} + \boldsymbol{\gamma}\widehat{\mathbf{p}}_{Ht} + \hat{\mathbf{q}}_t \tag{10}$$

$$\text{with} \quad \hat{\mathbf{p}}_{Mt} = \mathbf{W}_M \mathbb{P}_{Mt} \quad \text{and} \quad \mathbb{P}_{Mt} = \mathbf{W}_{MH}\hat{\mathbf{p}}_{Ht} + \mathbf{W}_{MF}\left[\hat{\tau}_{Mt} + \hat{\mathbf{p}}_{Ft}\right], \tag{11}$$

where $\mathbb{P}_{Mt} = [\hat{p}_t(1,1), \cdots, \hat{p}_t(1,S), \cdots, \hat{p}_t(S,1), \cdots, \hat{p}_t(S,S)]'$ is a vector of sector-to-sector input prices, $\mathbf{W}_M$ is a $S \times S^2$ matrix of corresponding input expenditure weights, and the matrices $\mathbf{W}_{MH}$ and $\mathbf{W}_{MF}$ record expenditure shares on home and foreign goods for each sector-to-sector input flow.[14]

Demand for inputs from the home country then depends on total input demand, along with relative prices.

$$\mathbb{M}_{Ht} = -\bar{\boldsymbol{\eta}}_M \hat{\mathbf{p}}_{Ht} + \tilde{\boldsymbol{\eta}}_M \mathbb{P}_{Mt} + \mathbb{M}_t \tag{12}$$

$$\text{with} \quad \mathbb{M}_t = -\kappa \mathbb{P}_{Mt} + \kappa \mathbf{T}\hat{\mathbf{p}}_{Mt} + \mathbf{T}\hat{\mathbf{m}}_t, \tag{13}$$

where $\mathbb{M}_t = [\hat{m}_t(1,1), \cdots, \hat{m}_t(1,S), \cdots, \hat{m}_t(S,1), \cdots, \hat{m}_t(S,S)]'$ is a $S^2 \times 1$ column vector of input purchases, and $\mathbb{M}_{Ht} = [\hat{m}_{Ht}(1,1), \cdots, \hat{m}_{Ht}(1,S), \cdots, \hat{m}_{Ht}(S,1), \cdots, \hat{m}_{Ht}(S,S)]'$ is an $S^2 \times 1$ vector of domestic inputs purchased by domestic buyers. The matrix $\mathbf{T} = \iota_{S\times 1} \otimes \mathbf{I}_{S\times S}$ selects the destination-sector element of the vectors that it multiplies. The parameter matrices are defined as: $\boldsymbol{\eta}_M \equiv diag\left(\eta_M(s)\right)$, $\bar{\boldsymbol{\eta}}_{\mathbf{M}} \equiv \mathbf{I}_{S\times S} \otimes \boldsymbol{\eta}_M \iota_{S\times 1}$, and $\tilde{\boldsymbol{\eta}}_{\mathbf{M}}$ is a $(S^2 \times S^2)$ matrix with block elements $\boldsymbol{\eta}_M$ along its diagonal.[15]

Equations 6-13 represent a system in which demand for output ($\hat{\mathbf{q}}_t$) depends on output prices ($\hat{\mathbf{p}}_{Ht}$ and $\hat{\mathbf{p}}_{Ft}$), trade costs ($\hat{\boldsymbol{\tau}}_{Mt}$ and $\hat{\boldsymbol{\tau}}_{Ct}$), export demand shifters ($\hat{\mathbf{x}}_{Ft}$), and aggregate consumption ($\hat{c}_t$). Note there is also an input-output loop here, where output is demanded as an input by home producers, and that level of demand is itself a function of output. Isolating $\hat{\mathbf{q}}_t$ results in pre-multiplying terms on the right side by the matrix $\left[\mathbf{I} - \mathbf{S}_M\mathbf{T}\right]^{-1}$. To interpret this operation, we note that we can re-write this matrix inverse in terms of the more conventional Leontief inverse of the domestic input-output matrix: $\left[\mathbf{I} - \mathbf{S}_M\mathbf{T}\right]^{-1} = \mathbf{Y}^{-1}\left[\mathbf{I} - \mathbf{A}_D\right]^{-1}\mathbf{Y}$, where $\mathbf{Y}$ is a matrix with the value of sectoral output $(P_{H0}(s)Q_0(s))$

[13] Here we have assumed that the home export goods competes against foreign goods in the export market, which carry the same prices as importable foreign goods. Any deviations from this assumption would be picked up in the demand shifter, which is exogenous, so presents no substantive complications.

[14] $\mathbf{W}_{MH}$ and $\mathbf{W}_{MF}$ are matrices with column vectors $\mathbf{w}_H(s,\cdot)$ and $\mathbf{w}_F(s,\cdot)$ as blocks along the main diagonal.

[15] In terms of notation, $\iota_{S\times 1}$ is a $S \times 1$ vector of ones and $\mathbf{I}_{S\times S}$ is a $S \times S$ identity matrix, and $\otimes$ denotes a Kroneker product.

along the diagonal, and $\mathbf{A}_D$ is the domestic use matrix, where each (s', s) element equals the share of domestic inputs from sector s' used by sector s in total output of sector s. The Leontief inverse operation here translates changes in downstream demand into changes in direct and indirect upstream output.

2.2.2 Output Prices

So far, we have linked demand for value added to gross output prices, trade costs, and other elements of downstream demand. Now, we want to swap value-added prices for gross output prices, to describe demand for value added directly in terms of prices of value added. To do so, we combine expressions for $\hat{p}_{Ht}(s)$, $\hat{p}_{Mt}(s)$, and $\hat{p}_t(s', s)$, to solve for domestic prices as a function of value-added prices, foreign prices, and input tariffs:

$$\hat{\mathbf{p}}_{Ht} = \left[\mathbf{I} - \mathbf{A}'_D\right]^{-1} \mathbf{S}_V \hat{\mathbf{p}}_t^V + \left[\mathbf{I} - \mathbf{A}'_D\right]^{-1} \mathbf{A}'_I \left[\hat{\boldsymbol{\tau}}_{Mt} + \hat{\mathbf{p}}_{Ft}\right]. \tag{14}$$

The matrix $\mathbf{A}_D$ is a domestic use matrix, defined above. The matrix $\mathbf{A}_I$ is an import use matrix, with (s', s) element equal the share of imported inputs from sector s' used by sector s in total output of sector s. The matrix $\mathbf{S}_V$ is a diagonal matrix with elements $\frac{P_{V0}(s)V_0(s)}{P_0(s)Q_0(s)}$, and $\hat{\boldsymbol{\tau}}_{Mt}$ and $\hat{\mathbf{p}}_{Ft}$ are $S \times 1$ vectors of changes in input tariffs and foreign prices for imported goods.

Equation (14) has a familiar cost-push logic. The price of output in each sector depends on the price of real value added in all upstream sectors (including its own sector), where these price increases spill forward through the Leontief inverse of the domestic use matrix: $[\mathbf{I} - \mathbf{A}'_D]^{-1}$. The output price also depends on the price of imported inputs in all upstream sectors, where tariffs map directly into unit costs (and output prices) via the import use matrix $\mathbf{A}'_I$, and then these cost changes are propagated via domestic input-output linkages (the Leontief inverse of the domestic use matrix again). Lastly, note that prices are entirely determined on the supply side, due to the dual assumptions of perfect competition and constant returns to scale.

2.2.3 Demand for Value Added

With these results in hand, take Equation (5), substitute for demand for gross output using Equation (6) and the related equation block, and then swap out the output prices using Equation (14). In reduced form, demand for value added takes the form:

$$\hat{\mathbf{v}}_t = -\Sigma \hat{\mathbf{p}}_{Vt} + \mathbf{R}_{V1}\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}\hat{\boldsymbol{\tau}}_{Mt} + \mathbf{R}_{V3}\hat{\mathbf{p}}_{Ft} + \mathbf{R}_{V4}\iota\hat{c}_t + \mathbf{R}_{V5}\mathbf{x}_{Ft}, \tag{15}$$

where the matrices are defined in Appendix A.

The matrix $\mathbf{\Sigma}$ collects the own and cross-price elasticities of demand for value added with respect to changes in the price of value-added in each sector. The own price elasticities $\sigma(s,s) = \left|\frac{\partial \hat{v}_t(s)}{\partial \hat{p}_{Vt}(s)}\right|$ are on the diagonal of this matrix. The matrices $\mathbf{R}_{V1}$ and $\mathbf{R}_{V2}$ capture the direct response of demand for value added in each sector to tariff changes, including both the sector's own tariffs as well as those in outside sectors. We provide a detailed intuitive discussion as to how these elements are structured in Section 2.3.

Lastly, we note that foreign prices ($\hat{\mathbf{p}}_{Ft}$), foreign export demand shifters ($\hat{\mathbf{x}}_{Ft}$), and aggregate domestic consumption ($\hat{c}_t$) also influence demand for domestic real value added. The role of consumer demand is straightforward: an increase total real consumption filters backward through the domestic value chain into higher demand for output from individual sectors, and thus demand for real value added. Foreign prices operate along the same margins as changes in tariffs discussed above, plus they influence export demand.

As a next step toward the ERP index, focus on demand for value-added in sector s:

$$\hat{v}_t(s) = -\Sigma(s)\hat{\mathbf{p}}_{Vt} + \mathbf{R}_{V1}(s)\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}(s)\hat{\boldsymbol{\tau}}_{Mt} + \mathbf{R}_{V3}(s)\hat{\mathbf{p}}_{Ft} + \mathbf{R}_{V4}(s)\iota\hat{c}_t + \mathbf{R}_{V5}(s)\mathbf{x}_{Ft}, \quad (16)$$

where the (s) notation here denotes the s$^{\text{th}}$ row the corresponding vector or matrix. Then, invert this demand curve to write price as a function of quantity demanded:

$$\hat{p}_{Vt}(s) = \mathbf{\Omega}_t(s) + \frac{1}{\sigma(s,s)}\left[\mathbf{R}_{V1}(s)\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}(s)\hat{\boldsymbol{\tau}}_{Mt}\right] - \frac{1}{\sigma(s,s)}\hat{v}_t(s), \quad (17)$$
$$\text{with} \quad \mathbf{\Omega}_t(s) = -\sum_{s'\neq s}\frac{\sigma(s,s')}{\sigma(s,s)}\hat{p}_{Vt}(s') + \frac{1}{\sigma(s,s)}\left[\mathbf{R}_{V3}(s)\hat{\mathbf{p}}_{Ft} + \mathbf{R}_{V4}(s)\iota\hat{c}_t + \mathbf{R}_{V5}(s)\hat{\mathbf{x}}_{Ft}\right].$$

The $\mathbf{\Omega}_t(s)$ term collects non-tariff determinants of the level of demand into a single object, and $\sigma(s,s') = -\frac{\partial \hat{v}_t(s)}{\partial \hat{p}_{Vt}(s')}$ depends on the cross-price elasticity of demand for value-added in sector s with respect to value-added prices in sector s', for which the sign is generally ambiguous in our framework.[16]

2.2.4 Effective Protection

We now define the Approximate ERP.

Definition 2 (Approximate ERP)**.** *In the log-linear framework presented above, the effective*

[16]For further discussion, see Appendix A.

rate of protection for sector s is given by:

$$ERP(s) = \mathbf{\Omega}_t(s) + \frac{1}{\sigma(s,s)}\left[\mathbf{R}_{V1}(s)\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}(s)\hat{\boldsymbol{\tau}}_{Mt}\right].$$

This ERP index measures the amount by which the demand curve for real value added from sector s shifts due to changes in tariffs. It is decomposed here into direct and indirect channels via which tariffs influence demand for value added at the sector level. The direct effects appear in $\frac{1}{\sigma(s,s)}\left[\mathbf{R}_{V1}(s)\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}(s)\hat{\boldsymbol{\tau}}_{Mt}\right]$. These are direct effects in the following sense: they capture the amount by which demand changes holding foreign variables, macroeconomic variables, and the price of real value added in outside sectors fixed at baseline values.[17] Indirect effects then arise through these channels, embedded in $\mathbf{\Omega}_t(s)$.

Among the indirect effects, recall that we have adopted small open economy assumptions in this section (which we will relax later). Given this, holding foreign prices and export demand levels constant (i.e., $\hat{\mathbf{p}}_{Ft} = \hat{\mathbf{x}}_{Ft} = \mathbf{0}$) is straightforward to motivate. Changes in value-added prices in outside sectors ($s' \neq s$) are a secondary source of indirect effects, from the perspective of sector s. While we will abstract from them in our initial discussion, we illustrate how endogenous changes in value-added prices for outside sectors can be incorporated into the ERP in Section 2.5.1.

An important point for interpretation is that we also abstract from indirect macroeconomic (general equilibrium) effects of tariff changes. That is, in computing the ERP formula, we set $\hat{c}_t = 0$. Further, in the background, the assumption that import prices are measured in real terms implies that we also hold the real exchange rate constant. (This will be made explicit in the multicountry setting below.) The motivation for abstracting from the macro-effects of tariffs on consumption and real exchange rates is that our objective is to measure effective protection at the *sector level*, where partial equilibrium forces loom largest. Nonetheless, one could re-activate macro-effects by making additional assumptions about the nature of the macro-equilibrium, to pin down $\hat{c}_t$ and real exchange rates as a function of the tariffs. In many models, an increase in the Home tariff for a small open economy leads to a decline in consumption, along with a real exchange rate appreciation, both of which ultimately reduce demand for domestic value added. This reduction would have differential effects across sectors, depending on each sector's sensitivity to macro-conditions. These effects are interesting in their own right, and generally work against finding an increase in demand for value added after the imposition of tariffs; nonetheless, we set them aside for future work.

As a final point, the ERP index defined here measures both of the effective protection

[17] That is, $\mathbf{\Omega}_t(s) = 0$ if $P_{Ft}(s) = P_{F0}(s)$, $X_{Ft}(s) = X_{F0}(s)$, $C_t = C_0$, and $P_{Vt}(s') = P_{V0}(s')$.

concepts defined in Section 1; the concepts coincide because the approximate demand curve is linear. This ERP is the proportional increase in the price of the sector's own value added that would be equivalent to the tariff change in its impact on demand for value added. It is also the proportional decrease in the price of the sector's own value added that would hold demand for value added at the same level it obtained prior to the change in tariffs. We will reinforce this intuition in examples below.

2.3 Parsing the Direct Effects of Tariff Changes

In Definition 2, the direct effects of tariff changes are controlled by $\mathbf{R}_{V1}(s)$, $\mathbf{R}_{V2}(s)$, and $\sigma(s,s)$. To advance quickly, we relegate discussion of $\sigma(s,s)$ to Appendix A, because it plays a secondary role in the quantitative results. Here we build up intuition for the main channels through which tariffs propagate to demand for value added, using both algebra and data.

2.3.1 Direct Impact of Final Goods Tariffs

Tariffs on final goods directly enter the Approximate ERP in the form: $\frac{1}{\sigma(s,s)}\mathbf{R}_{V1}(s)\hat{\boldsymbol{\tau}}_{Ct}$, where $\mathbf{R}_{V1}(s)$ is the s^{th} row of the matrix $\mathbf{R}_{V1}$, which takes the form:

$$\begin{aligned}\mathbf{R}_{V1} &= \overbrace{[\mathbf{I}-\mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_C}^{\hat{\mathbf{c}}_H\mapsto\hat{\mathbf{q}}}\overbrace{(\boldsymbol{\eta}_C-\vartheta\mathbf{I})\,\mathbf{W}_{CF}}^{\hat{\boldsymbol{\tau}}_C\mapsto\hat{\mathbf{c}}_H}\\ &= \mathbf{S}_C(\boldsymbol{\eta}_C-\vartheta\mathbf{I})\,\mathbf{W}_{CF}+\underbrace{\mathbf{Y}^{-1}\left[[\mathbf{I}-\mathbf{A}_D]^{-1}-\mathbf{I}\right]\mathbf{Y}\mathbf{S}_C(\boldsymbol{\eta}_C-\vartheta\mathbf{I})\,\mathbf{W}_{CF}}_{\text{backward propagation via input linkages}},\end{aligned} \tag{18}$$

To interpret this, note that final goods tariffs ($\hat{\boldsymbol{\tau}}_{Ct}$) only affect demand for value added ($\hat{\mathbf{v}}_t$) through demand for gross output ($\hat{\mathbf{q}}_t$). We break down the effects into two channels.

First, the $(\boldsymbol{\eta}_C-\vartheta\mathbf{I})\,\mathbf{W}_{CF}$ term maps $\hat{\boldsymbol{\tau}}_{Ct}$ into $\hat{\mathbf{c}}_{Ht}$. Recalling that $\mathbf{W}_{CF}$ has the final goods import shares for each sector on the diagonal, this matrix maps own-sector import tariffs into the composite price of consumption in each sector, where the composite price increases more when the import share is large. Then, $(\boldsymbol{\eta}_C-\vartheta\mathbf{I})$ controls how much consumption of domestic goods changes in response, where $\eta_C(s)$ regulates the strength of expenditure switching between home and foreign goods.[18] Thus, a first takeaway is that raising final goods tariffs has a larger impact on demand for home goods when either the import share of sectoral consumption is large, or the elasticity of substitution between home and foreign goods ($\eta_C(s)$) is large.

[18]The elasticity ϑ controls how consumers switch expenditure across sectors. Cross-sector elasticities tend to be smaller than within-sector elasticities ($\eta_C(s)>\vartheta$ in general), so expenditure switching between home and foreign goods is stronger than expenditure switching across sectors.

Second, the matrix $[\mathbf{I}-\mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_C$ describes how tariff-induced changes in demand for domestic final goods ($\hat{\mathbf{c}}_{Ht}$) are passed backward through the domestic production network. That is, how $\hat{\mathbf{c}}_{Ht}$ maps to demand for gross output ($\hat{\mathbf{q}}_t$). To isolate this backward propagation, we re-write the $\mathbf{R}_{V1}$ in Equation 18.[19] The first term records the direct impact of tariffs on demand, which depends on the share of output dedicated to final use (in $\mathbf{S}_C$). The second term measures the role of backward linkages, where the (s,s') entries in $\mathbf{Y}^{-1}\left[[\mathbf{I}-\mathbf{A}_D]^{-1}-\mathbf{I}\right]\mathbf{Y}\mathbf{S}_C$ reflect the share of sector s output that is required (directly or indirectly) to produce sector s' final goods. The second takeaway is that demand for value-added from sector s depends not only on its own final goods tariffs, but also final goods tariffs in downstream industries. The strength of this "pass back" tariff effect depends on how much of sector s's output is ultimately allocated (directly or indirectly) to satisfying the demand for those domestic final goods.

2.3.2 Direct Impact of Input Tariffs

Tariffs on inputs enter the Approximate ERP in the form: $\frac{1}{\sigma(s,s)}\mathbf{R}_{V2}(s)\hat{\boldsymbol{\tau}}_{Mt}$, where $\mathbf{R}_{V2}(s)$ is the s^{th} row of the matrix following matrix:

$$\mathbf{R}_{V2} = \overbrace{[\mathbf{I}-\mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_M}^{\hat{\mathbf{m}}_H\mapsto\hat{\mathbf{q}}}\ \overbrace{\mathbf{R}_{M2}}^{\hat{\boldsymbol{\tau}}_{Mt}\mapsto\hat{\mathbf{m}}_H} + \overbrace{\gamma}^{\hat{\mathbf{p}}_H\mapsto\hat{\mathbf{v}}}\ \overbrace{[\mathbf{I}-\mathbf{A}'_D]^{-1}\mathbf{A}'_I}^{\hat{\boldsymbol{\tau}}_{Mt}\mapsto\hat{\mathbf{p}}_H}$$
$$+ \underbrace{[\mathbf{I}-\mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C1}+\mathbf{S}_M\mathbf{R}_{M1}-\mathbf{S}_X\boldsymbol{\eta}_X]}_{\hat{\mathbf{p}}_H\mapsto\hat{\mathbf{c}}_H,\hat{\mathbf{m}}_H,\hat{\mathbf{x}}_H\mapsto\hat{\mathbf{q}}}\underbrace{[\mathbf{I}-\mathbf{A}'_D]^{-1}\mathbf{A}'_I}_{\hat{\boldsymbol{\tau}}_{Mt}\mapsto\hat{\mathbf{p}}_H}, \quad (19)$$

where $\mathbf{R}_{C1}$ and $\mathbf{R}_{M1}$ map prices to demand for domestic final goods and inputs respectively, and $\mathbf{R}_{M2}$ maps input tariffs into input demand, with definitions in the appendix.

The first term captures the protective effects of tariffs. Similar to the discussion above for final goods, there are two channels here. First, sectors that produce the inputs on which tariffs are imposed obviously receive protection, where $\hat{\boldsymbol{\tau}}_{Mt}\mapsto\hat{\mathbf{m}}_H$. Then, the benefits of increased demand for domestic inputs are passed backward through the domestic production network to upstream sectors whose output is directly or indirectly used to produce those protected inputs.

The second term term measures how input tariffs lead domestic producers to substitute away from produced inputs toward using more domestic value-added inputs (factor inputs)

[19] To follow the algebra, recall that $[\mathbf{I}-\mathbf{S}_M\mathbf{T}]^{-1} = \mathbf{Y}^{-1}[\mathbf{I}-\mathbf{A}_D]^{-1}\mathbf{Y}$. Then, we apply the standard input-output property: $[\mathbf{I}-\mathbf{A}_D]^{-1}-\mathbf{I}=\mathbf{A}_D+\mathbf{A}_D^2+\cdots$, where the sequence $\mathbf{A}_D+\mathbf{A}_D^2+\cdots$ captures the role of input linkages. Further, note these backward linkages are controlled by the Leontief inverse of the domestic use matrix ($\mathbf{A}_D$). Holding the total use matrix $\mathbf{A}=\mathbf{A}_D+\mathbf{A}_I$ constant, a more closed economy (with $\mathbf{A}_D\to\mathbf{A}$) will have larger backward spillovers. Put differently, some of the increase in demand for domestically-produced goods leaks abroad through demand for imported inputs.

in production. The matrix $[\mathbf{I} - \mathbf{A}'_D]^{-1}\,\mathbf{A}'_I$ maps input tariff changes into output prices ($\hat{\mathbf{p}}_H$). Recall that input tariff changes are propagated forward through the domestic production network, from upstream sectors where tariffs are imposed to into prices for downstream sectors that use them (directly or indirectly).[20] The elasticity between real value added and produced inputs ($\boldsymbol{\gamma}$) then translates the output price change into demand for domestic factors.

The third term reflects how tariff-induced cost shocks change demand for gross output, along three margins. First, tariffs on imported inputs raise the cost of domestic goods relative to foreign goods. This leads to substitution toward imports in consumption and input use, as well as a reduction in export demand, which all reduce demand for domestic output and thus value added.[21] Second, tariffs have heterogeneous effects across sectors, based on their total exposure to imported input cost changes. This triggers cross-sector substitution in both consumption and input use, generally away from sectors whose costs rise more. Third, increased costs trigger substitution away from input use toward value added, which reduces demand for gross output. Together these substitution effects alter demand for gross output (via $\hat{\mathbf{c}}_H$, $\hat{\mathbf{m}}_H$, and $\hat{\mathbf{x}}_H$), and these changes again filter backward through the domestic production network.[22]

Some takeaways from this discussion follow. First, input tariffs raise effective protection for import-competing input producers, as well for their upstream suppliers. For example, the services industry will receive protection from tariffs applied to plastics (an input producing sector), because the plastics sector uses services in production. Second, input tariffs raise effective protection by increasing the cost of intermediates, which triggers substitution toward primary factors in all sectors. Third, on the flip side, input tariffs reduce effective protection for downstream sectors that directly or indirectly rely on imported inputs. For example, the effective protection of services will fall with input tariffs applied to goods imports, because goods are used in production of services. All together, the net impact of input tariff changes on demand for value added is ambiguous, so we turn to numerical analysis below.

[20] The matrix $\mathbf{A}'_I$ contains cost shares for imports (as a share of gross output), such that sectors with larger imported input cost shares will experience larger direct cost shocks due to tariff increases. Then, $[\mathbf{I} - \mathbf{A}'_D]^{-1}$ controls downstream propagation of those cost shocks.

[21] In the background, the $\mathbf{R}_{C1}$ and $\mathbf{R}_{M1}$ matrices depend on the elasticity of substitution between home and foreign goods in final goods and input usage: $\boldsymbol{\eta}_C$ and $\boldsymbol{\eta}_M$, respectively. Further, the export demand elasticity $\boldsymbol{\eta}_X$ obviously appears in the formula as well.

[22] Cross-sector substitution effects can be shut down by setting $\kappa = \vartheta = 0$. Substitution between value added and input use is eliminated when $\gamma = \mathbf{0}$. With these restrictions, only substitution effects between home and foreign goods remain.

2.4 Numerical Analysis of the Direct Effects of Tariffs

Building on the discussion above, we turn to numerical analysis of the direct impact of tariffs on effective protection. We use data for the United States and its trading partners along with calibrated parameters to operationalize the framework, as described in Section 2.4.1. We first study the mechanics of changes in final goods and input tariffs for particular sectors to build intuition in Section 2.4.2. We then look at the impact of tariff changes that mimic US policy in 2025 in Section 2.4.3.

2.4.1 Parameterizing the Framework

We use data for the United States from the OECD Inter-Country Input-Output tables (2025 edition, regular ICIO) to set values for all the shares needed in the formulas. In this, we aggregate all non-US countries to form the foreign composite.[23]

We set trade elasticities in $\boldsymbol{\eta}_C$, $\boldsymbol{\eta}_M$, and $\boldsymbol{\eta}_X$ based on Fontagnè, Guimbard and Orefice (2022), which estimates disaggregated trade elasticities as the elasticity of bilateral trade to tariffs in a gravity regression.[24] Specifically, we use elasticities estimated for OECD ICIO sectors, separately for final goods and inputs, to populate $\boldsymbol{\eta}_C$ and $\boldsymbol{\eta}_M$. We then use elasticities that are estimated by pooling data across end uses to set $\boldsymbol{\eta}_X$, since we do not distinguish final goods versus inputs on the export side in the framework. For reference, the median elasticity for final goods is about 5.2, while the median elasticity for inputs is higher at 6.5. These elasticities lie within the range of standard values used by the literature; they also vary across sectors in meaningful economic ways, as documented by Fontagnè et al. Moreover, because the elasticities are estimated principally off cross-sectional variation in tariffs, they are appropriate for studying the long run effects of tariffs.

Turning to calibration of substitution elasticities, we set the elasticity across sectors in consumption (ϑ) to 0.5, which aligns with standard values in the macroeconomic structural change literature. We set both the elasticity of substitution across sectors in input use (κ) and the elasticity of substitution for value added versus inputs to 0.5, where we constrain $\gamma(s)$ to be the same in all sectors. This low scope for substitution in cross-sector input use,

[23] The OECD input-output tables record cross-border transactions at basic prices (essentially, the price that producers receive, rather than purchasers' prices), and we use these data to calculate expenditure shares in construction of price indexes. This amounts to an implicit assumption that tariffs are zero in the pre-tariff baseline equilibrium. Since US tariffs are generally low, this assumption is tolerable. To allow for non-zero baseline tariffs would require collecting the appropriate tariff data, adjusting import prices, and recalibrating the baseline equilibrium.

[24] We use the datasets for "ICIO classification by type of product" and "ICIO classification" that are available for download at `https://sites.google.com/view/product-level-trade-elasticity`. For sectors and end uses for which estimates are not available (principally services industries, for which import shares are near zero), we assign an elasticity of 6.

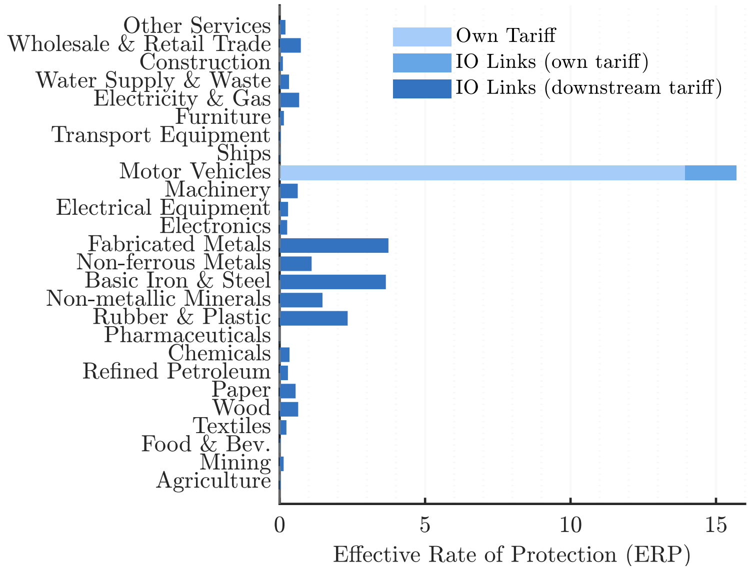


Figure 2: Direct ERP for 10% increase in final tariffs in the motor vehicles sector.

and between inputs and value added in production, is broadly consistent with elasticities estimated by Atalay (2017) and Peter and Ruane (2025).[25]

2.4.2 The Mechanics of Final Goods and Input Tariffs

As an initial exercise, consider raising the tariff on (only) imports of final motor vehicles, from zero to 10%. Using the model, we compute the direct effect of this tariff change across sectors and report the results in Figure 2. While the direct impact of final goods tariffs is necessarily to raise effective protection for motor vehicles, note that the effective rate of protection conferred by a 10% final goods tariff may differ from 10%, because the tariff change is converted into its value-added subsidy equivalent. Indeed, we find that a 10% tariff on imported motor vehicles is equivalent to a subsidy of over 15% for buyers of value added from the motor vehicles sector.

The impact of the tariff on finished automobiles evidently raises effective protection for the auto sector. In the figure, we use the algebra in Equation 18 to decompose the impact of the auto tariff on the auto sector into components that come via the direct protective

[25] Atalay (2017) argues for a very low cross-sector elasticity of substitution in input use, near 0.1, based on estimates using annual variation in sector-level input-output data for the United States and other developed countries. While Peter and Ruane (2025) find a higher (> 1) elasticity for substitution among manufacturing inputs using longer run variation in Indian firm level data, they also estimate relatively low (≤ 1) elasticities of substitution across energy, materials, and services. The framework could be extended to allow for this nesting structure, though we do not pursue this avenue. Both papers also find estimated elasticities for substitution between real value added (composite primary factors) and inputs that are less than one, near the value we use.

effect of the tariff for downstream producers (labeled Own Tariff) and the propagation of the tariff through input linkages to suppliers of intermediate automotive goods (labeled IO Links (own tariff)). Then, the auto tariff also spills backward through the value chain to provide an effective subsidy to upstream sectors, such as rubber, plastics, iron, and steel, as shown. As one might expect, metals, minerals, rubber and plastic are the biggest upstream beneficiaries of an increase in US auto tariffs.

To illustrate the full role of backward linkages, we consider a second exercise in which we impose a uniform 10% tariff on final goods in *all* goods-producing sectors at once. In Figure 3, we again plot the impact on each sector broken down into the direct effect of its own final goods tariff, the IO propagation of each sector's own final goods tariff on itself, and the upstream propagation of downstream final goods tariffs imposed in other sectors. Despite the fact that the tariff change is uniform across all goods sectors, there are widely different outcomes across sectors. Effective protection rises in downstream industries, which supply larger shares of their output to final end use, largely due to the direct effects of those sector's final goods tariffs (e.g., in autos, furniture, electrical equipment, etc.). Input-output linkages propagate the final goods tariffs upstream, in two senses. First, like the autos example, tariffs on imported final goods lead to large indirect effects in upstream sectors, like metals, minerals, paper, wood, agriculture, and rubber/plastics sectors. Second, final goods tariffs also raise the effective protection received by services sectors. Though the magnitudes appear modest here, recall that the subsidy equivalent is expressed in percentage terms. So, because services base value is larger, the implied dollar equivalent subsidy would also be sizable.

Turning to input tariffs, we compute the direct ERP for hypothetical input tariff changes and the decomposition of the three channels identified in Equation (19). Recall that decomposition isolates the protective effects of input tariffs from two substitution channels: substitution between real value added and inputs, and goods market substitution that results from the cost shocks induced by input tariffs.

In Figure 4, we plot results for a 10% increase in tariffs for the steel, iron, precious, and non-ferrous metals sectors. Here the impact of the tariff change is concentrated in the sectors in which it is imposed, due to the direct protective effects for metals producers, and the limited backward propagation of those shocks due to the lack of upstream linkages for those sectors. Further, note that the direct substitution of value added for inputs is generally limited in response to the shocks, as substitution elasticities are low in that dimension. Most importantly, one sees the effect of the the tariff-induced cost shock in the figure, where many sectors experience decreased effective protection as a result of higher input costs. This is true not only in the targeted sectors themselves, but also in downstream manufacturing sectors

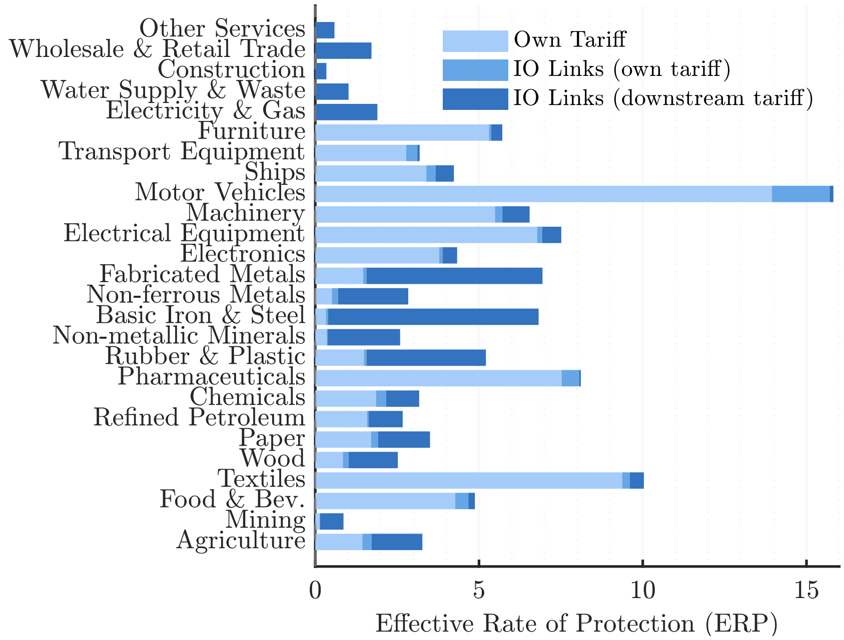


Figure 3: Direct ERP for 10% increase in final tariffs in all goods sectors.

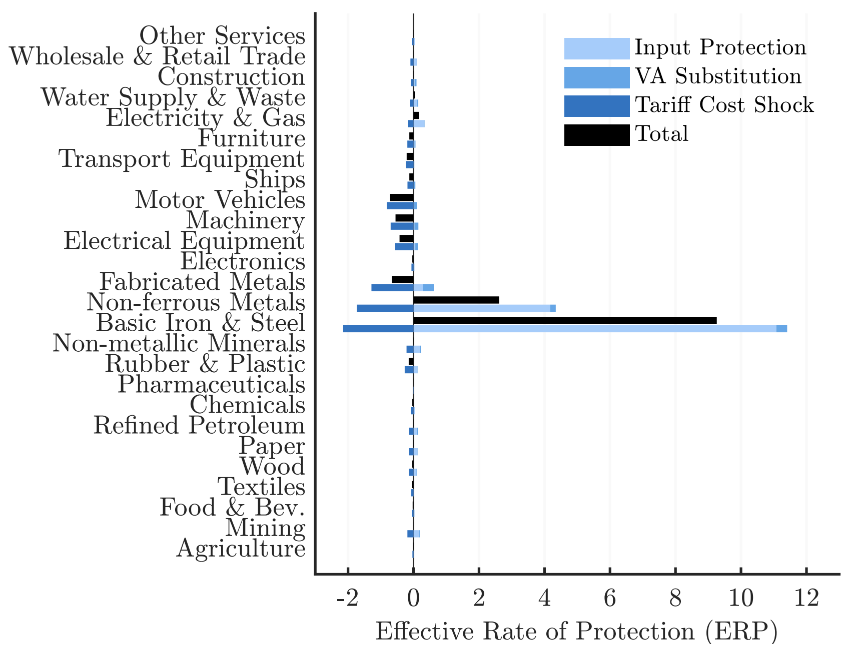


Figure 4: Direct ERP for 10% increase in input tariffs in the steel & iron sector.

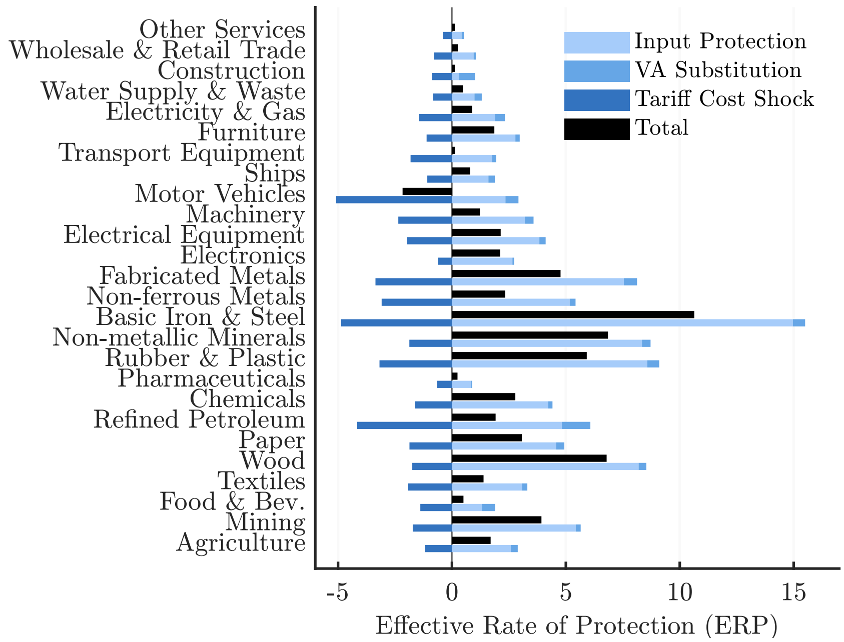


Figure 5: Direct ERP for 10% increase in input tariffs in all goods sectors.

(motor vehicles, machinery, etc.). In those downstream sectors, these effects are large enough to result in negative effective protection – i.e., an inward shift in demand for value added from those sectors. Nonetheless, these spillovers are relatively small in magnitude, since metals are a relatively small share of costs in those downstream sectors.

When tariffs are increased for all inputs, however, the role of input linkages in propagating the tariff cost shock is stronger. We plot the same decomposition of channels for a 10% input tariff imposed in all goods sectors in Figure 5. Looking across sectors, the protective effects of input tariffs are helt most strongly in input producing industries, as is sensible. In contrast, the costs of the input tariffs are borne widely throughout the economy, as costs rise in all sectors. This cost channel attenuates the increase in effective protection due to increased input tariffs. For example, these effects turn drive effective protection to zero in transport equipment, and effective protection turns negative in the motor vehicles sector.

Like for final goods, we note that while the input tariff change is uniform across sectors, differences in trade, sectoral linkages, and elasticities together lead to large differences in the effective protection that results. With this thought in mind, we now turn to consider the combined effects of simultaneous increases in final goods and input tariffs.

2.4.3 Combined Tariffs on Final Goods and Inputs

Now we consider changes in final goods and input tariff simultaneously. To set the size of the tariff changes, we appeal to a stylized depiction of the multilateral policy adopted by the Trump administration in 2025. To discipline this exercise, we measure pre-shock tariffs using US MFN rates applied in 2023.[26] We then construct post-Trump multilateral tariffs by interpreting the executive orders, which have two key components.[27]

First, tariffs on goods sectors were raised through the imposition of "reciprocal tariffs."[28] These were announced in April, then paused, then revised (partly through negotiation, partly through additional unilateral actions), and then finally imposed in their current form in August (with further minor revisions in September). While the reciprocal tariffs were heterogeneous across partners, which we address below, the mean reciprocal tariff across US trade partners is approximately 15%. These reciprocal tariffs were "stacked" on top of the US MFN (column 1) tariff; for example, if the US MFN tariff rate is 3%, then the applied tariff would be 18%. Because the reciprocal tariffs were stacked on top of all sectors and end uses, tariffs have largely risen by a uniform amount under this policy. This implies that the degree of tariff escalation (the gap between final and input tariffs) was largely preserved by this policy change. Further, in proportional terms, the size of the log tariff shock ($\hat{\tau}(s)$) is nearly uniform uniform across sectors and end uses, at about $\hat{\tau}(s) \approx 0.14$.[29]

Second, tariffs for particular sectors were raised above this baseline level.[30] In the auto sector, the default tariff was raised to 25% on both finished vehicles and parts. For steel and aluminum (and derivative products), tariffs were raised to 50%. For semi-fininshed copper and derivative products, the copper content of covered products was also taxed at 50%. Unlike the reciprocal tariffs, these are absolute tariff levels (not stacked on top of US MFN

[26] We obtain tariffs from the UNCTAD TRAINS database, via the World Bank's WITS interface (we use the ad valorem equivalent MFN applied tariff). We then concord HS 6-digit categories to ICIO sectors and end uses using the "Bilateral Trade in Goods by End-use (BTiGE): HS to end-use conversion key" [OECD (2025)]. We take simple averages across HS categories in aggregating the data to ICIO sectors.

[27] In principle, one can directly use published tariff schedules, though these are hard to parse without recourse to the narrative record. However, recall that tariff changes were not implemented in a uniform way at the multilateral level, so one also confronts thorny issues regarding aggregation for this multilateral exercise. Moreover, the sectors for which tariffs were changed do not correspond exactly to the sector nomenclature in the OECD ICIO data. Our intent is to use stylized tariff changes to illustrate the ERP theory, rather than provide a definitive quantification of the effects of the tariffs. So stylized interpretation of the policy changes in helpful in this regard. We found the following background document helpful: Congressional Research Service (2025).

[28] It should be noted that tariffs on services sectors are unchanged in the model, held at zero.

[29] Literally speaking, the log tariff shock ($\hat{\tau}(s)$) is inversely related to the initial tariff level. Since initial tariffs were low in the United States in most sectors, and the log-transformation is non-linear, the resulting $\hat{\tau}(s)$ shocks are nearly identical across sectors.

[30] Naturally, tariffs are not imposed in services industries, so the tariff shock pertains to goods only. In these ICIO data, these are sectors 1-7 and 9-27 (sector 8 is mining support service activities).

tariffs). Interpreting these changes, we raise tariffs to these levels in three corresponding ICIO sectors.[31] Because steel, aluminum, and copper are all intermediate inputs, and auto tariffs apply to parts in additional to finished vehicles, these sector-specific changes lead to de-escalation in US tariffs. For steel, aluminum, and copper sectors, baseline MFN tariffs were near zero. For motor vehicles, baseline tariffs were nearly 8% for finished autos (largely due to the 25% tariff on light trucks), only 2% on auto parts. As a result, the log tariff changes were $\hat{\tau} \approx 0.15$ for finished autos, $\hat{\tau} \approx 0.2$ for auto parts, and $\hat{\tau} \approx 0.4$ for metals imports (which are predominantly inputs in practice).

In Figure 6, we plot the direct effects of the tariffs using the Approximate ERP formula. The black bars in the figure correspond to the total direct ERP, and the second set of blue bars present a decomposition into effects coming from auto tariffs, metals tariffs, and other tariffs.[32] In general, effective protection is positive, which is not surprising – tariffs generally raise effective protection. Nonetheless, we find it striking how heterogeneous the outcomes are, given the fact that the increase in tariffs is nearly identical across sectors, outside of metals and autos. Effective protection evidently increases most in basic iron and steel production, and note that the metals tariff spills over negatively into sectors like motor vehicles, machinery, and electrical equipment. Then, effective protection on motor vehicles also increases, and this tariff spills over positively into demand for metals.

In Figure 7, we present results for the final goods tariffs and input tariffs imposed by the Trump administration separately. These mimic the prior figures, now with realistic tariff changes in all sectors. Consistent with our observation about the largely uniform nature of the tariff changes at the multilateral level across sectors, we note that the role of the different propagation channels largely mimics the "tariffs on all goods" scenarios discussed above.

2.5 Extensions: Indirect Effects and Non-Linearities

Thus far, we focused on describing the "direct effects" of tariff changes in the Approximate ERP, which was defined using a log-linear approximation. In this section, we extend this analysis in two ways. First, in both Definition 1 and Definition 2, the effective rate of protection for a given sector depends on how value-added prices adjust in outside sectors, in response to tariff changes. While we have ignored these "indirect effects" thus far, we now study how taking them into account matters alters the ERP. Second, while the Approximate

[31] Autos corresponds to sector 24 (CPA 29: manufacture of motor vehicles, trailers and semi-trailers) in the ICIO data. Steel is sector 18 (CPA 24A: manufacture of basic iron and steel). Aluminum and copper are included in sector 19 (CPA 24B: manufacture of basic precious and other non-ferrous metals).

[32] In each case, the respective shock includes both changes in input and final goods tariffs in that sector group. For example, the 'auto tariff' results in the figure include both tariffs on finished autos and auto parts.

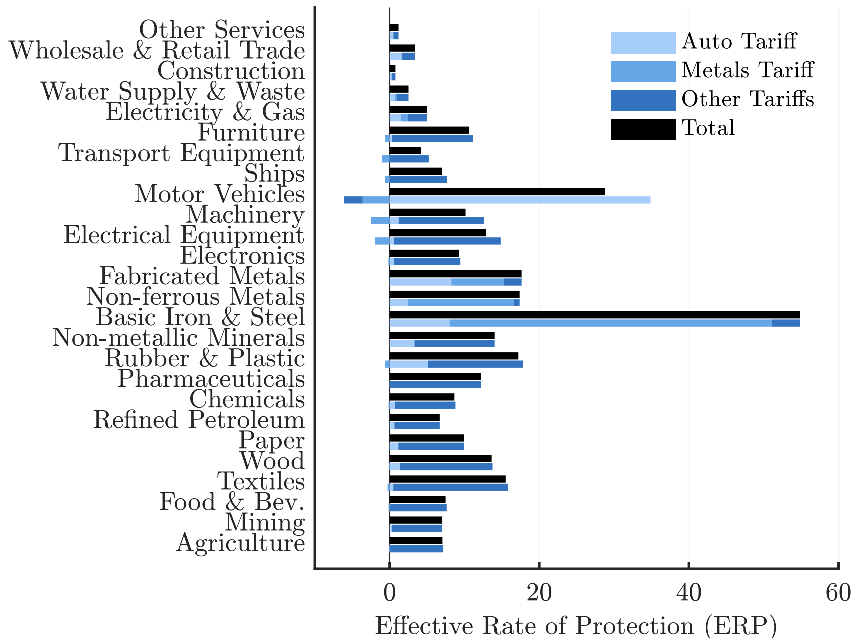
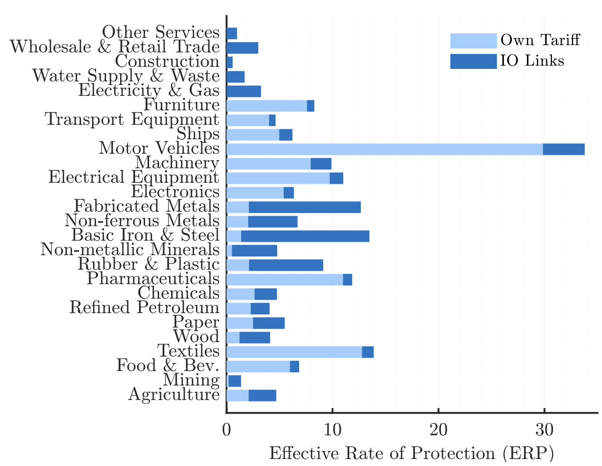
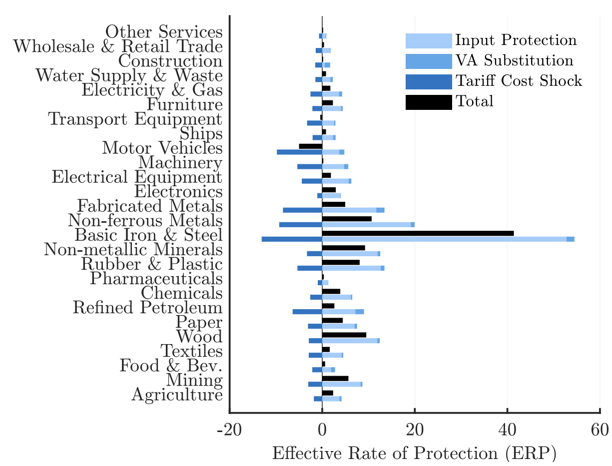


Figure 6: Direct ERP all tariffs imposed by the Trump administration.



(a) Final goods tariffs.



(b) Input tariffs.

Figure 7: ERP decomposition for tariffs imposed by the Trump administration.

ERP (Definition 2) relies on a log-linear approximation, the Exact ERP (Definition 1) can be computed for our framework. We now also take potential non-linear effects of tariff changes into account, by computing the Exact ERP and comparing it to the Approximate ERP.

2.5.1 Accounting for Changes in Value-Added Prices in Outside Sectors

To allow for value-added prices in outside sectors ($s' \neq s$) to adjust endogenously following a tariff change, we must make additional assumptions about supply functions in those outside sectors. We consider two alternative formulations, one with inelastic supply and one where supply responds to prices. To make the discussion concrete, let us specify a simple production function for real value added: $V_t(s) = K(s)^{\alpha(s)} L_t(s)^{1-\alpha(s)}$, where $K(s)$ is capital, $L_t(s)$ is labor, and $\alpha(s) \in (0,1)$. In what follows, we always treat capital as a sector-specific factor in fixed supply (hence he lack of subscript on it). Then, consider two alternatives for labor.

The first case is one where labor is also sector specific, again in fixed supply, so $L_t(s) = L(s)$. This obviously implies that the supply of real value added is fixed, in inelastic supply: $V_t(s) = V(s)$. The second case is one where labor is perfectly mobile across sectors. In this case, labor is paid the wage W_t, which is an aggregate equilibrium object. Cost minimization implies that labor demand is $L_t(s) = (1-\alpha(s)) p_t^V(s) V_t(s)/w_t$, so then the supply curve for real value added is:

$$V_t(s) = K(s) \left(\frac{1-\alpha(s)}{W_t} P_{Vt}(s) \right)^{(1-\alpha(s))/\alpha(s)} . \tag{20}$$

Here the supply curve for real value added is upward sloping, where $(1-\alpha(s)/\alpha(s)$ is the elasticity of $V_t(s)$ with respect to $P_{Vt}(s)$.

2.5.2 Exact vs. Approximate ERP

Starting from the non-linear economic framework in Table 1, we now use "exact hat algebra" to re-write it in Table 3, where $\hat{X}_t = \frac{X_t}{X_0}$ is the ratio of the value of a given variable in the post-shock equilibrium relative to its initial equilibrium value.[33] To trace out the demand curve for real value added from sector s, one computes how $V(s)$ changes as $P^V(s)$ is exogenously varied. Inverting this logic, one can alternatively trace out the inverse demand curve: how $P^V(s)$ changes as $V(s)$ varies, which captures willingness to pay for alternative amounts of real value added. This sets the stage for computing the exact ERP.

To compute the Exact ERP, we evaluate the shift in the indirect demand curve due to tar-

[33] As in Table 2, all prices are "real" prices, normalized relative to the domestic price level. If one takes tariffs $\{\hat{\tau}_C(s), \hat{\tau}_M(s)\}$, prices $\{\hat{P}_V(s), \hat{P}_F(s), \hat{P}_t^*(s)\}$, and quantities $\{\hat{C}, \hat{X}^*(s)\}$ as given, the $9S + 3S^2$ equations in Table 3 can be solved for $6S + 2S^2$ quantities $\{\hat{C}(s), \hat{C}_H(s), \hat{V}(s), \hat{Q}(s), \hat{M}(s), \{\hat{M}(s', s), \hat{M}_H(s', s)\}, \hat{X}(s)\}$, plus $3S + S^2$ prices $\{\hat{P}_H(s), \hat{P}_M(s), \hat{P}_C(s), \{\hat{P}(s', s)\}\}$.

Table 3: Exact-Hat Formulation of the Model Elements for Small Open Economy

Consumption	$\hat{C}_t(s) = \left(\hat{P}_{Ct}(s)\right)^{-\vartheta}\hat{C}_t$ $\hat{C}_{Ht}(s) = \left(\frac{\hat{P}_{Ht}(s)}{\hat{P}_{Ct}(s)}\right)^{-\eta(s)}\hat{C}_t(s)$
Inputs	$\hat{V}_t(s) = \left(\frac{\hat{P}_{Vt}(s)}{\hat{P}_{Ht}(s)}\right)^{-\gamma(s)}\hat{Q}_t(s)$ $\hat{M}_t(s) = \left(\frac{\hat{P}_{Mt}(s)}{\hat{P}_{Ht}(s)}\right)^{-\gamma(s)}\hat{Q}_t(s)$ $\hat{M}_t(s',s) = \left(\frac{\hat{P}_t(s',s)}{\hat{P}_{Mt}(s)}\right)^{-\kappa}\hat{M}_t(s)$ $\hat{M}_{Ht}(s',s) = \left(\frac{\hat{P}_{Ht}(s')}{\hat{P}_t(s',s)}\right)^{-\eta(s')}\hat{M}_t(s',s)$
Prices	$\hat{P}_{Ht}(s) = \left(\left(\frac{P_{V0}(s)V_0(s)}{P_{H0}(s)Q_0(s)}\right)\hat{P}_{Vt}(s)^{1-\gamma(s)} + \left(\frac{P_{M0}(s)M_0(s)}{P_{H0}(s)Q_0(s)}\right)\hat{P}_{Mt}(s)^{1-\gamma(s)}\right)^{1/(1-\gamma(s))}$ $\hat{P}_{Mt}(s) = \left(\sum_{s'}\left(\frac{P_{M0}(s')M_0(s',s)}{P_{Mt}(s)M_0(s)}\right)\hat{P}_t(s',s)^{1-\kappa}\right)^{1/(1-\kappa)}$ $\hat{P}_t(s',s) = \left(\left(\frac{P_{H0}(s')M_{H0}(s',s)}{P_{M0}(s')M_0(s',s)}\right)\hat{P}_{Ht}(s')^{1-\eta(s')} + \left(\frac{\tau_{M0}(s')P_{F0}(s')M_{F0}(s',s)}{P_{M0}(s')M_0(s',s)}\right)\left(\hat{\tau}_{Mt}(s')\hat{P}_{Ft}(s')\right)^{1-\eta(s')}\right)^{1/(1-\eta(s'))}$ $\hat{P}_{Ct}(s) = \left(\left(\frac{P_{H0}(s)C_{H0}(s)}{P_{C0}(s)C_0(s)}\right)\hat{P}_{Ht}(s)^{1-\eta(s)} + \left(\frac{\tau_{C0}(s)P_{F0}(s)C_{F0}(s)}{P_{C0}(s)C_0(s)}\right)\left(\hat{\tau}_{Ct}(s)\hat{P}_{Ft}(s)\right)^{1-\eta(s)}\right)^{1/(1-\eta(s))}$
Output	$\hat{Q}_t(s) = \left(\frac{P_{H0}(s)C_{H0}(s)}{P_{H0}(s)Q_0(s)}\right)\hat{C}_{Ht}(s) + \sum_{s'}\left(\frac{P_{H0}(s)M_{H0}(s,s')}{P_{H0}(s)Q_0(s)}\right)\hat{M}_{Ht}(s,s') + \left(\frac{P_{H0}(s)X_0(s)}{P_{H0}(s)Q_0(s)}\right)X_t(s)$ $\hat{X}_t(s) = \left(\frac{\hat{P}_{Ht}(s)}{\hat{P}^*_t(s)}\right)^{-\eta_X(s)}\hat{X}^*_t(s)$

iffs, holding the quantity of real value added used in sector s constant. Recalling Definition 1, this corresponds to the following price ratio: $P^D_s(v_{s0}; \mathbf{p}^v_{s'1}, \boldsymbol{\tau}_1, D_1)/P^D_s(v_{s0}; \mathbf{p}^v_{s'0}, \boldsymbol{\tau}_0, D_0)$. As above, in what follows we hold exogenous and macro-factors constant: $D_1 = D_0$.[34] Pending an assumption about how to handle $\mathbf{p}^v_{s'1}$ versus $\mathbf{p}^v_{s'0}$ – to be discussed next – the exact ERP is the value of $\hat{P}_V(s)$ consistent the tariff change and holding the quantity of real value added in sector s constant at v_{s0}, which is enforced by setting $\hat{V}(s) = 1$.[35]

2.5.3 Results

Building on the discussion above, we now present results comparing six alternative versions of the ERP.

The first is the Approximate ERP, in which we hold value-added prices in outside sectors constant, by setting $\hat{p}_V(s') = 0$ for $s' \neq s$ in Definition 2. This simply measures the "direct effects" of tariff changes in the log-linear framework, which were discussed above.

The second and third versions are then the Approximate ERP again, but taking into account endogenous changes in value-added prices in outside sectors: $\hat{p}_V(s') \neq 0$ for $s' \neq s$.

[34] Translating this to the specific context in Table 3, this implies the restrictions $\hat{X}^*(s) = \hat{P}_F(s) = \hat{P}^*(s) = 1$ for all $s \in S$ and $\hat{C} = 1$.

[35] To be clear, holding real value added for sector s constant in this way does not imply that we are restricting the true supply of real value added in sector s; rather it is a choice about the point at which to evaluate the shift in the indirect demand curve.

The second and third version differ, based on whether we assume that real value added in outside sectors is inelastically supplied or not. In the inelastic case, then we solve for $\hat{p}_V(s')$ given $\hat{v}(s') = 0$ for $s' \neq s$. In the upward sloping supply scenario, the supply curve $\hat{v}_t(s') = \left(\frac{1-\alpha(s')}{\alpha(s')}\right) \hat{p}_{Vt}(s')$ is added to the equilibrium system and used to solve for $\{\hat{p}_{Vt}(s')\}_{s' \neq s}$, and then these are used in Definition 2 to compute the Approximate ERP. We present more detailed discussion of the required steps in Appendix B.

The fourth, fifth, and sixth versions are different flavors of the Exact ERP. In the fourth version, we focus on "direct effects" in the non-linear setting, by setting $\hat{P}_V(s') = 1$, which is analogous to the first version above. The fifth version then solves for the Exact ERP under the assumption of inelastically supplied real value added in outside sectors: $\hat{V}(s') = 1$ for $s' \neq s$. The sixth solves for the Exact ERP under the upward sloping supply assumption.

We compare these six alternatives for the same Trump tariff exercise we used above, where we feed in all US tariff changes simultaneously. The models are all parameterized using the same data and structural parameters discussed previously. The one new parameter is the elasticity of supply of real value added, governed by the Cobb-Douglas parameter $\alpha(s)$. We set to $\alpha(s) = 0.4$ in all sectors, roughly motivated by capital shares in value added.

The results of this comparison are in Table 4. The main result is that the approximate ERP which includes "direct effects" of tariffs only is highly correlated with alternatives, both those that take changes in value-added prices in outside sectors into account in the linear approximate framework, as well as fully non-linear alternatives. The correlation of the sectoral ERPs with the approximate, direct benchmark exceeds 0.96 in all cases, so we conclude that the simplest ERP does a remarkably good job of capturing the impacts of the tariffs in the extended frameworks considered here.

3 An ERP Index with Multi-Country GVCs

We now generalize the ERP to a multi-country context. Section 3.1 briefly describes the extended setup. In Section 3.2, we define the approximate ERP in this setting and discuss important features of it. Section 3.3 applies the definition to study the impacts of tariff changes in 2025.

3.1 Framework Setup

The multi-country framework extends the small open economy in two ways. First, import prices for each country are pinned down by foreign unit costs, which depend on both domestic and foreign value-added prices, as well as trade costs. Second, each country's exports are

Table 4: Comparing Approximate and Exact ERPs, allowing for changes in value-added prices in outside sectors.

	Approximate ERP			Exact ERP		
Sector	Direct Effects	Inelastic Supply	Upward-Sloping Supply	Direct Effects	Inelastic Supply	Upward-Sloping Supply
Agriculture	7.1	5.4	6.1	8.8	6.8	7.4
Mining	7.0	5.0	5.8	8.8	6.6	7.2
Food & Beverage	7.5	5.8	6.5	9.1	7.1	7.6
Textiles	15.5	13.3	14.1	19.3	16.8	17.4
Wood	13.6	10.4	11.6	14.2	10.6	11.4
Paper	9.9	6.3	7.6	9.6	5.7	6.6
Refined Petroleum	6.7	2.5	4.1	7.1	2.4	3.4
Chemicals	8.7	6.2	7.2	8.9	5.7	6.5
Pharmaceuticals	12.2	11.9	12.0	8.6	8.2	8.3
Rubber & Plastic	17.2	10.1	12.9	17.9	11.4	13.0
Non-metallic Minerals	14.0	9.4	11.3	14.3	10.1	11.2
Basic Iron & Steel	54.9	44.6	48.9	70.7	55.7	59.6
Non-ferrous Metals	17.4	12.3	14.4	19.0	10.9	13.0
Fabricated Metals	17.6	7.2	11.7	17.7	7.9	10.8
Electronics	9.3	8.4	8.7	11.1	10.2	10.4
Electrical Equipment	12.9	9.7	10.9	15.6	11.5	12.6
Machinery	10.1	6.0	7.8	12.8	7.7	9.2
Motor Vehicles	28.8	23.5	25.8	24.2	18.1	19.8
Ships	7.0	5.6	6.1	5.1	3.8	4.1
Transport Equipment	4.2	2.6	3.2	3.1	1.2	1.7
Furniture	10.6	8.8	9.5	12.8	10.7	11.2
Electricity & Gas	5.0	0.7	2.3	4.4	0.7	1.5
Water Supply & Waste	2.5	0.3	1.1	2.2	0.4	0.7
Construction	0.8	0.1	0.3	0.7	0.1	0.2
Wholesale & Retail Trade	3.4	0.4	1.6	2.6	0.4	0.9
Other Services	1.2	0.0	0.5	1.0	0.0	0.2

Table 5: Log-Linearization of Model Elements for the Multi-country Economy

Consumption	$\hat{c}_i(s) = -\vartheta \hat{p}_{Ci}(s) + \hat{c}_t$ $\hat{c}_{ji}(s) = -\eta_C(s)\left(\hat{\tau}_{Cji}(s) + \hat{r}_{i/j} + \hat{p}_j(s) - \hat{p}_{Ci}(s)\right) + \hat{c}_i(s)$
Inputs	$\hat{v}_i(s) = -\gamma(s)\left(\hat{p}_{Vi}(s) - \hat{p}_i(s)\right) + \hat{q}_i(s)$ $\hat{m}_i(s) = -\gamma(s)\left(\hat{p}_{Mi}(s) - \hat{p}_i(s)\right) + \hat{q}_i(s)$ $\hat{m}_i(s', s) = -\kappa\left(\hat{p}_i(s', s) - \hat{p}_{Mi}(s)\right) + \hat{m}_i(s)$ $\hat{m}_{ji}(s', s) = -\eta_M(s')\left(\hat{\tau}_{Mji}(s') + \hat{r}_{i/j} + \hat{p}_j(s') - \hat{p}_i(s', s)\right) + \hat{m}_t(s', s)$
Prices	$\hat{p}_i(s) = \left(\frac{P_{Vi}(s)V_i(s)}{P_{Hi}(s)Q_{Hi}(s)}\right)\hat{p}_{Vi}(s) + \left(\frac{P_{Mi}(s)M_i(s)}{P_{Hi}(s)Q_{Hi}(s)}\right)\hat{p}_{Mi}(s)$ $\hat{p}_{Mi}(s) = \sum_{s'}\left(\frac{P_i(s',s)M_i(s',s)}{P_{Mi}(s)M_i(s)}\right)\hat{p}_i(s', s)$ $\hat{p}_i(s', s) = \sum_j\left(\frac{\tau_{Mji}(s')\psi_{Mji}P_j(s')M_{ji}(s',s)}{P_i(s',s)M_i(s',s)}\right)\left(\hat{\tau}_{Mji}(s') + \hat{r}_{i/j} + \hat{p}_j(s)\right)$ $\hat{p}_{Ci}(s) = \sum_j\left(\frac{\tau_{Cji}(s')\psi_{Cji}(s)P_j(s)}{P_{Ci}C_i}\right)\left(\hat{\tau}_{Cji}(s) + \hat{r}_{i/j} + \hat{p}_j(s)\right)$
Output	$\hat{q}_i(s) = \sum_j\left(\frac{P_i(s)\psi_{Cij}(s)C_{ij}(s)}{P_i(s)Q_i(s)}\right)\hat{c}_{ij}(s) + \sum_j\sum_{s'}\left(\frac{P_i(s)\psi_{Mij}(s)M_{ij}(s,s')}{P_i(s)Q_i(s)}\right)\hat{m}_{Hi}(s, s')$

determined by foreign demand for final goods and inputs. Because foreign input use increases with foreign production, which in turn depends on domestic demand for both final goods and inputs, the resulting framework features a closed global input-output system. Within this system, the reduced-form demand for each country's output then depends on relative value-added prices, trade costs, and real final expenditure in all countries.

The elements of the multi-country framework are familiar, so we proceed directly to present the linearized components of it in Table 5 . Most of the notation is a straightforward adaptation of that used for the small open economy. Because we need to keep track of bilateral goods flows and bilateral trade costs, the subscript ij notation is ordered such that i is the source and j is the destination.[36] Further, prices in each country are normalized relative to their own-country consumption price level; this normalization is implicit in the table, so all prices should be interpreted as real values.[37] Then, $\hat{r}_{i/j}$ is the bilateral real exchange rate $\hat{r}_{i/j} \equiv \hat{p}_{Cj} - \hat{p}_{Ci}$, so $\hat{r}_{i/j} > 0$ is a real depreciation for country i relative to country j.

[36]For example, $\hat{c}_{ji}(s)$ is the log change in final goods shipments from country j to country i, and $\hat{\tau}_{ji}(s)$ is the tariff applied by country i on final goods from country j.

[37]For example, $\hat{p}_{Ci}(s)$ is the price of the composite consumption good for sector s relative to the aggregate price level in country i, $\hat{p}_i(s)$ is the price of gross output for sector s relative to the aggregate price level in country i, and so on.

3.2 Demand for Value Added and the GVC ERP

The equations in Table 5 can be stacked and manipulated to define demand for value added, and we provide details on various steps in Appendix C. Here we highlight how some key results differ from the small open economy, and then define the approximate ERP in this global setting.

To start, prices for gross output again reflect marginal costs, where the solution can be written as:

$$\hat{\mathbf{p}} = [\mathbf{I} - \mathbf{A}']^{-1} \mathbf{S}_V \hat{\mathbf{p}}_V + [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \square \hat{\boldsymbol{\tau}}_M] \iota + [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \square \hat{\mathbf{r}}] \iota. \tag{21}$$

Here $\mathbf{A}'$ is the transpose of the global input-output matrix, which captures input linkages across all countries and sectors, $\hat{\mathbf{p}}_V$ is the global vector of value-added prices in all countries and sectors. The matrix $\hat{\boldsymbol{\tau}}_M$ includes bilateral, sector-level changes in tariffs on imported inputs, with block elements $\hat{\boldsymbol{\tau}}_{Mij}$ that pertain to tariffs imposed by j on imports from i in all sectors. The $\square$-notation signifies the block Hadamard (entry-wise) product: for block matrices X and $\mathbf{Y}$, then $(\mathbf{X} \square \mathbf{Y})_{mn} = \mathbf{X}_{mn}\mathbf{Y}_{mn}$. So then, $[\mathbf{A}' \square \hat{\boldsymbol{\tau}}_M] \iota$ measures the direct impact of changes in input tariffs on downstream output prices. The full matrix $[\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \square \hat{\boldsymbol{\tau}}_M] \iota$ captures the total impact of upstream tariffs on downstream prices, taking both direct and indirect input linkages into account. The final term is a related calculation for changes in real exchange rates, which are embedded in the matrix $\hat{\mathbf{r}}$. Again focusing on partial equilibrium effects, we will take real exchange rate changes as given (set to zero) in constructing the ERP.

Comparing Equation 21 to its counterpart in the small open economy (Equation 14), note first that output prices in country i depend both on domestic and foreign value-added prices, as foreign value-added prices partially determine imported input costs. In the analysis to follow, we will hold value-added prices in outside domestic sectors and all foreign sectors as given as we analyze the direct effects of tariff changes.

A second point to note is that each country's output price depends on *all* upstream input tariffs, not just its own tariffs. That is, the price of output from sector s in country i depends on the tariffs imposed by imported inputs by country i, as in the small open economy, and the tariffs imposed by foreign countries ($j \neq i$). The reason is that foreign tariffs raise unit costs for output produced abroad, which is then used (directly or indirectly) as an input by country i.[38] Furthermore, note also that we disaggregate tariffs imposed on a bilateral

[38]The aggregation of tariffs that appears here is closely related to measurement of "cumulative tariffs," as discussed in Miroudot, Rouzet and Spinelli (2013), Muradov (2017), and Johnson (2018). Jaax, Miroudot and van Lieshout (2023) apply the cumulative tariff concept to study recent reorganization in GVCs.

basis, because bilateral tariffs will have different propagation patterns due to heterogeneity in GVC linkages across sectors and countries.

Turning to demand for value added, the conditional demand schedule features the same substitution and scale effects as in the small open economy. Producers substitute toward using more domestic value added when the cost of produced inputs increases relative to primary (value-added) factors; equivalently, when the output price increases all else equal, which introduces a first expenditure-switching role for input tariffs. Then, demand again rises with total production, as before.

Not surprisingly, the mapping from tariffs to total production levels is more complicated, as both final goods and input tariff changes are propagated through the global input-output structure. There are again direct protective effects of tariffs, which propagate backward through input linkages, as well as substitution channels induced by changes in gross output prices due to tariffs. As in the discussion of Equation 21, we make note that tariffs in *all countries* influence demand for value added. Own tariffs have effects that are similar to the SOE in a direct sense, but global input-output linkages propagate these tariffs to have complex higher order effects. Foreign tariffs affect demand for value added because they directly affect demand for exports, but also because they feed into foreign unit costs, which determine domestic unit costs through input linkages.

In the end, demand for value added can be written in reduced form as follows:

$$\hat{\mathbf{v}} = -\boldsymbol{\Sigma}\hat{\mathbf{p}}_V + \mathbf{R}_{V1}\left(\boldsymbol{\tau}_C\right) + \mathbf{R}_{V2}\left(\boldsymbol{\tau}_M\right) + \mathbf{R}_{V3}\left(\hat{\mathbf{r}}\right) + \mathbf{L}\mathbf{S}_C\mathbf{R}_{C4}\left[\hat{\mathbf{c}} \otimes \mathbf{1}_S\right]. \tag{22}$$

The matrix $\boldsymbol{\Sigma}$ is a collection of own-price and cross-price elasticities of demand, which are a function of the global input-output data and structural parameters. The objects $\mathbf{R}_{V1}(\cdot)$, $\mathbf{R}_{V2}(\cdot)$, and $\mathbf{R}_{V3}(\cdot)$ are (respectively) linear functions of bilateral final goods tariffs, input tariffs, and real exchange rates. And then the final term maps changes in final demand in all countries into demand for output and value-added. Skipping the details here, we provide the full formulas for all the elements of this expression in Appendix C. Using this derived demand for value added, we now define the (approximate) GVC ERP.

Definition 3 (Approximate GVC-ERP)**.** *In the global log-linear framework presented above, the effective rate of protection for sector s in country i is given by:*

$$ERP_i(s) = \boldsymbol{\Omega}_i(s) + \frac{1}{\sigma_i(s,s)}\left[\mathbf{R}_{V1(i,s)}\left(\boldsymbol{\tau}_C\right) + \boldsymbol{R}_{V2(i,s)}\left(\boldsymbol{\tau}_M\right)\right],$$

Here $\sigma_i(s,s)$ is again the own-price elasticity of demand for value added: if $\boldsymbol{\Sigma}_{ii}$ is the ii block of matrix $\boldsymbol{\Sigma}$, defined above, then $\sigma_i(s,s)$ is the ss element of that block. Then,

$\mathbf{R}_{V1(i,s)}(\boldsymbol{\tau}_C)$ and $\mathbf{R}_{V2(i,s)}(\boldsymbol{\tau}_M)$ are the country i, sector s element of the corresponding vector. The object $\boldsymbol{\Omega}_i(s)$ again includes changes in value-added prices in outside sectors and general equilibrium effects.

3.3 Application of the GVC-ERP to Trade War Tariffs

3.3.1 Parameters and Tariff Scenarios

We again turn to the OECD ICIO tables to parameterize the framework, now for 80 countries (including the United States) and composite rest of the world region. We also set elasticity parameters to the same values as in the small open economy setup.

To form the tariff shock scenario, we obtain applied bilateral tariffs from the CEPII-MAcMap-HS6 database for 2022, which include preferential applied tariff rates, and we aggregate these to sectors and end uses to match the ICIO data.[39] To construct the Trump tariffs scenario, we proceed in several steps.

First, we obtain bilateral reciprocal tariffs that entered into force on August 7 from Executive Order 14326.[40] As noted in the discussion above, these tariffs are stacked on the US MFN (column 1) tariffs, so we also again use MFN tariff rates (source described above) to construct the updated applied bilateral tariff. Then, we modify the bilateral tariffs to reflect other executive orders, which alter bilateral rates with particular country groups. These include the Brazil tariffs, the China tariffs, the India tariffs, and the bilateral agreement reached with the EU. Further, we assume that imports from Canada and Mexico receive the pre-existing (2022) applied tariff rates, consistent with most trade continuing to receive USMCA tariff preferences.

Second, we then adjust sector tariffs in autos, steel, aluminum, and copper – following the discussion in Section 2.4.3 above. Here we take into account side deals reached with the United Kingdom, Japan, and South Korea for autos, as well as exemptions to auto tariffs for Mexico and Canada under the USMCA. We also account for preferential access granted to the United Kingdom for steel and aluminum.

To summarize the tariff shock briefly, $0.1 \leq \hat{\tau} \leq 0.2$ for most countries and goods. Brazil, China, India, and South Africa all receive substantially larger tariff increases. Across sectors, steel and aluminum again stand out. Due to exemptions from the auto tariffs for various countries, including the many countries in the EU, the realized tariff shock in the auto sector is not an outlier overall relative to other sectors.

[39] We concord and aggregate these tariffs to ICIO sectors and end uses, using the same OECD correspondence cited above. See Guimbard et al. (2012) and https://www.cepii.fr/CEPII/en/bdd_modele/bdd_modele_item.asp?id=12 for data.

[40] Published here: https://federalregister.gov/d/2025-15010.

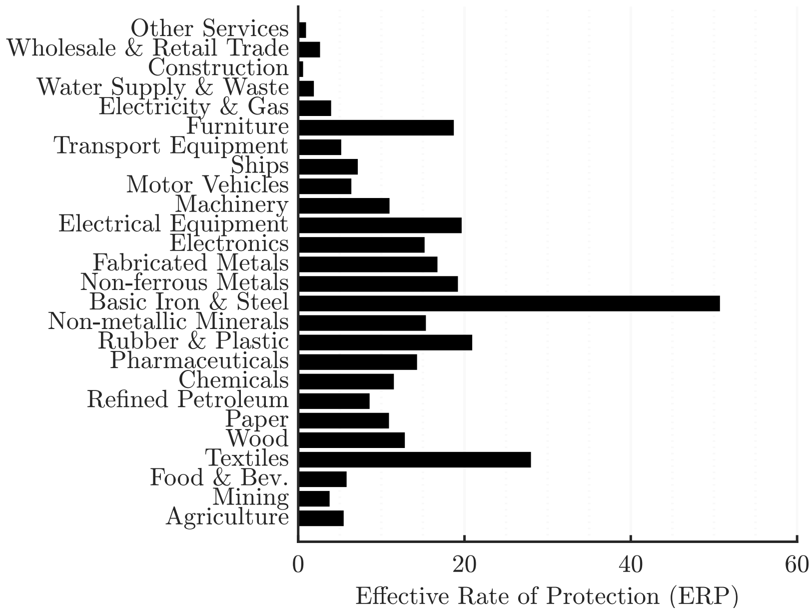
3.3.2 Results

In Figure 8, we plot the the direct impacts of the tariffs imposed by the Trump administration in the multicountry framework, which accounts for bilateral variation in both policies and trade patterns. The overall ERP impacts are included in Figure 8a. In Figure 8b, we decompose the overall ERP effects in several ways. First, we decompose effects arising from final goods versus input tariffs. Second, we decompose the effects of sectoral tariffs, splitting up the impacts of tariffs in autos, metals (aluminum, steel, and copper), and other sectors. Third, we compute the impact of the China tariffs separately from tariffs on other countries.

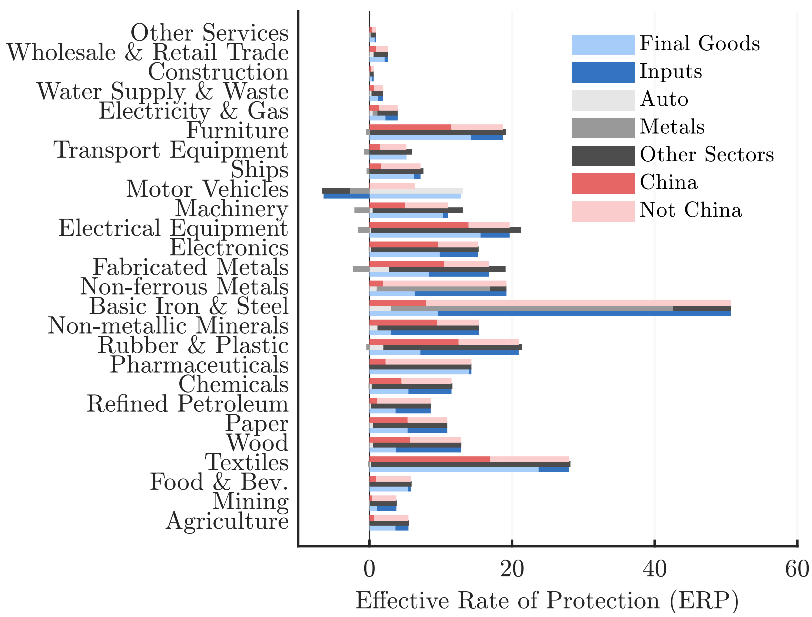
Looking across the economy, there are large differences in the effective protection generated by sector. The tariffs confer the largest effective subsidies to Iron & Steel, which is not surprising given the very high sectoral tariff imposed. Interestingly, Textiles ranks second in terms of effective protection received, where tariffs levied against China have their largest ERP impact. The China tariffs also account for a major share of the effective protection in rubber and plastics, electrical equipment, and furniture, which all receive substantial net subsidies.

On the flip side, motor vehicles receive a surprisingly small effective subsidy. One reason is the US's major auto trade partners (Mexico, Canada, EU, Japan, and South Korea) received preferential carve outs from the sectoral tariff. Another reason is that tariffs were imposed both on imported automotive inputs and upstream sectors supplying inputs to the auto industry, which serve as an effective value-added tax on the auto sector. To put numbers to this, the ERP generated by final goods tariffs alone for autos is just over 14%, a sizable subsidy. Then, input tariffs shave almost half of this away, lowering the overall ERP to 7.75%. Reading this result together with the ERPs in other sectors, it implies that textiles, steel, and many other industries have received more effective protection than motor vehicles, suggesting that the tariffs may may pull resources out of US automotive production on net.

Recalling that the multi-country framework generates ERPs for all countries at the same time, we present two brief results to illustrate how this works. In Figure 9a, we illustrate effective protection for Mexico induced by US tariffs. The interesting result is that Mexico appears to gain effective protection due to US tariff increases – that is, demand for Mexican value added increases in most sectors (outside metals). Though we do not show the results for brevity, similar results hold for Canada. One reason is that the USMCA is largely still in tact, which implies that the US preference margin for Mexico's exports has increased, re-directing demand to Mexico. A second reason is that Mexico benefits from backward spillovers from the US. As US tariffs raise demand for US-produced goods, this is passed back through the GVC to Mexico. The end result is that US tariffs "look like" a large value-added subsidy for sectors like Textiles, Electronics, and Electrical Equipment.



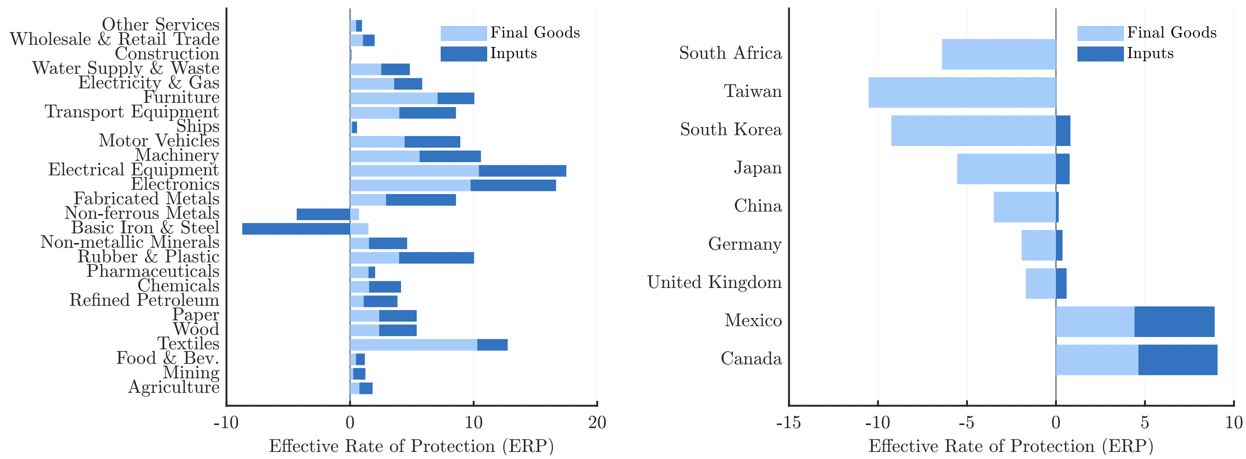
(a) Direct Tariff Impact in Approximate ERP



(b) Decomposition of Tariff Impacts

Figure 8: United States ERP for tariffs imposed by the Trump administration in the multi-country framework.

Drilling down to the sector-level, we plot outcomes for the motor vehicles sector in Figure 9b. This illustrates how countries are affected differently by the US tariffs. Echoing the result above, effective protection for the Canadian and Mexican auto sector rises, while it falls in other major US automobile trade partners. An interesting point to note is that this effect is entirely due to US tariffs on finished autos, rather than tariffs on imported auto parts. In particular, tariffs on imported auto parts modestly raise effective protection for Japan and South Korea (like Mexico and Canada, with stronger effect). This likely reflects the how input tariffs work as a cost shock for US producers.



(a) Direct Tariff Impacts for Mexico

(b) Direct Impacts of Motor Vehicle Tariffs

Figure 9: Foreign ERPs for tariffs imposed by the Trump administration in the multicountry framework.

Turning to hypotheticals, we plot the change in the direct impact of tariffs for the United States in the approximate multi-country ERP for two different scenarios in Figure 10. The first scenario is a breakdown in North American integration, simulated as a reversion to trade war tariffs imposed by the US on Canada and Mexico of 25%, along with reciprocal trade war tariffs by Canada and Mexico on the US at the same level. The effects of a USMCA trade war are most evident in the auto sector, where US effective protection for autos increases, in large part to the exemptions from the US multilateral auto tariffs currently granted to Mexico and Canada. In contrast, net effective protection would fall in non-ferrous metals, refined petroleum, and other sectors reflecting retaliatory tariffs by Mexico and Canada on US exports. Somewhat surprisingly, however, the overall effects are unexpectedly (to us) small relative to the baseline Trump tariffs. In a sense, this means that the US has little gain – in terms of effective protection – from a North American trade war.

The second scenario is a flare up in the US-China trade war, with reciprocal US-China

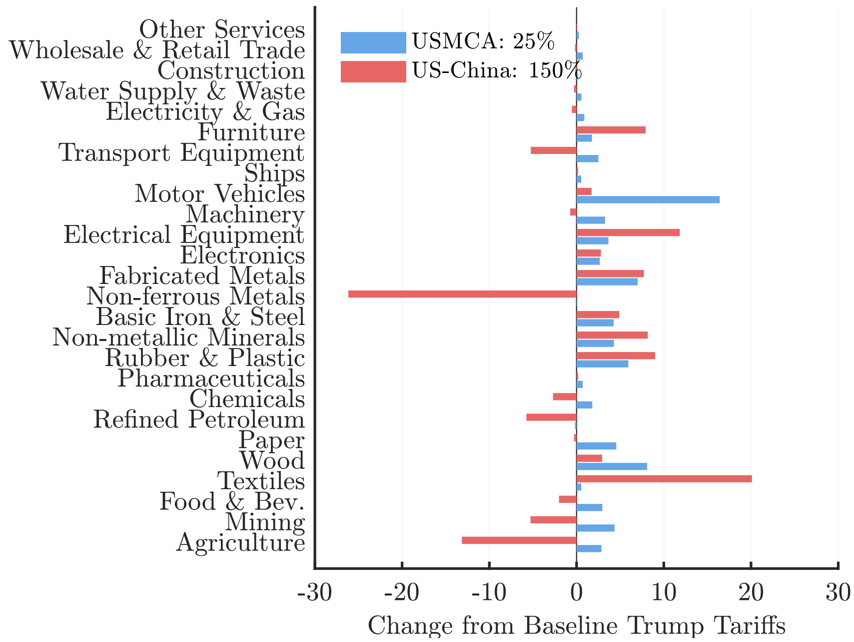


Figure 10: Change in United States ERP for counterfactual trade war scenarios.

tariffs set to 150%. As a note, this US-China trade war leads to such extreme (though potentially realistic!) tariffs that the log-linear approximation likely understates the effects by a healthy amount, but it serves to give a sense of direction. The joint impact of the tariffs would be to raise effective protection for the US in some sectors, while lowering it in others. Textiles gains effective protection, while non-ferrous metals and agriculture suffer. Both results are intuitive, in that they capture the role of US tariffs protecting domestic sectors that compete with China, and Chinese tariffs hitting exports from the US to China. Since effective protection rises in some sectors, while falling in others, the net change in effective protection for the US from escalating depend on how the winners and losers are traded off against one another. And of course, one should remember that general equilibrium effects and consumer welfare are absent from this analysis.

4 Conclusion

In this paper, we have advanced a new measure of the effective rate of protection, which captures the effective subsidy to buyers of real value added from a given industry. Put differently, the ERP measures the shift in the demand curve for sectoral value added. We describe how to implement this basic idea in two multi-sector settings: a single country (small open economy) that trades final goods and inputs, and a multi-country global economy (a collection of large open economies) that are connected through global value chains.

Applying the framework to study tariffs imposed by the Trump administration in 2025, we show value chain linkages play an important role in transmitting the impacts of the shock. We emphasize that even uniform tariffs have very different impacts across sectors due to heterogeneity in elasticities, openness, and the input-output network structure. This is a reminder that the optimal design of tariffs is a complicated and messy business: simple formulas are unlikely to be optimal.

More generally, we hope the framework will provide useful guidance for trade policy analysis. While we study construction of the ERP in standard structural gravity-type models, we see scope for extending the approach to richer settings. For example, one could incorporate pro-competitive effects of trade, where markup adjustment shapes resource allocation across sectors and demand for factors. Another important extension could be to allow for increasing returns to scale in downstream industries, which would amplify the costs of input tariffs, as in Antràs et al. (2024).

We also think it worthwhile to build out the supply side of the model along other dimensions. With additional structural assumptions to describe the supply curve for value added by industry, then one can extend the value-added approach to discuss the incidence of the effective value-added subsidies conferred by tariffs. Finally, we again emphasize that the effective rate of protection does not incorporate important general equilibrium effects, or the consumer-side welfare consequences of tariffs; therefore, it must not be misinterpreted as a full welfare analysis of tariff policy.

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A Log-Linear Demand for Value Added in the Small Open Economy

This appendix provides algebraic details underlying ERP definition for the small open economy. The stacked system of equations are included in Table 6.

Table 6: Stacked Model Elements for Small Open Economy

Consumption	$\hat{\mathbf{c}}_t = -\vartheta \hat{\mathbf{p}}_{Ct} + \iota \hat{c}_t$ $\hat{\mathbf{c}}_{Ht} = -\boldsymbol{\eta}_C \left(\hat{\mathbf{p}}_{Ht} - \hat{\mathbf{p}}_{Ct}\right) + \hat{\mathbf{c}}_t$
Inputs	$\hat{\mathbf{v}}_t = -\gamma \hat{\mathbf{p}}_{Vt} + \gamma \hat{\mathbf{p}}_{Ht} + \hat{\mathbf{q}}_t$ $\hat{\mathbf{m}}_t = -\gamma \hat{\mathbf{p}}_{Mt} + \gamma \hat{\mathbf{p}}_{Ht} + \hat{\mathbf{q}}_t$ $\mathbb{M}_t = -\kappa \mathbb{P}_{Mt} + \kappa \mathbf{T} \hat{\mathbf{p}}_{Mt} + \mathbf{T} \hat{\mathbf{m}}_t$ $\mathbb{M}_{Ht} = -\bar{\boldsymbol{\eta}}_M \hat{\mathbf{p}}_{Ht} + \tilde{\boldsymbol{\eta}}_M \mathbb{P}_{Mt} + \mathbb{M}_t$
Prices	$\hat{\mathbf{p}}_{Ht} = \mathbf{S}_V \hat{\mathbf{p}}_t^V + \mathbf{S}_M \hat{\mathbf{p}}_{Mt}$ $\hat{\mathbf{p}}_{Mt} = \mathbf{W}_M \mathbb{P}_{Mt}$ $\mathbb{P}_{Mt} = \mathbf{W}_{MH} \hat{\mathbf{p}}_{Ht} + \mathbf{W}_{MF} \left[\hat{\tau}_{Mt} + \hat{\mathbf{p}}_{Ft}\right]$ $\hat{\mathbf{p}}_{Ct} = \mathbf{W}_{CH} \hat{\mathbf{p}}_{Ht} + \mathbf{W}_{CF} \left(\hat{\boldsymbol{\tau}}_{Ct} + \hat{\mathbf{p}}_{Ft}\right)$
Output	$\hat{\mathbf{q}}_t = \mathbf{S}_C \hat{\mathbf{c}}_{Ht} + \mathbf{S}_M \mathbb{M}_{Ht} + \mathbf{S}_X \hat{\mathbf{x}}_t$ $\hat{\mathbf{x}}_t = -\boldsymbol{\eta}_{\mathbf{X}} \hat{\mathbf{p}}_{Ht} + \boldsymbol{\eta}_{\mathbf{X}} \hat{\mathbf{p}}_{Ft} + \hat{\mathbf{x}}_{Ft}$

Recalling Equation (6), we need demand for consumption of domestic goods by home consumers, demand for home inputs by domestic firms, and export demand to obtain demand for gross output. We will write this demand for output as a function of $\hat{\mathbf{p}}_{Ht}$, $\hat{\mathbf{p}}_{Ft}$, $\hat{\boldsymbol{\tau}}_{Ct}$, $\hat{\boldsymbol{\tau}}_{Mt}$, and $\hat{c}_t$.

Start with consumption, plug in the expressions for $\hat{\mathbf{c}}_t$ and $\hat{\mathbf{p}}_{Ct}$, and collect terms to obtain:

$$\hat{\mathbf{c}}_{Ht} = \mathbf{R}_{C1} \hat{\mathbf{p}}_{Ht} + \mathbf{R}_{C2} \left(\hat{\tau}_{Ct} + \hat{\mathbf{p}}_{Ft}\right) + \iota \hat{c}_t, \tag{23}$$

$$\text{where} \quad \mathbf{R}_{C1} \equiv -\left[\boldsymbol{\eta}_C - \left(\boldsymbol{\eta}_C - \vartheta \mathbf{I}\right) \mathbf{W}_{CH}\right] \tag{24}$$

$$\text{and} \quad \mathbf{R}_{C2} \equiv \left(\boldsymbol{\eta}_C - \vartheta \mathbf{I}\right) \mathbf{W}_{CF}. \tag{25}$$

For inputs, substitute for $\mathbb{P}_{Mt}$, $\hat{\mathbf{p}}_{Mt}$, and $\hat{\mathbf{m}}_t$ in the expression for $\mathbb{M}_{Ht}$ to obtain:

$$\mathbb{M}_{Ht} = \mathbf{R}_{M1} \hat{\mathbf{p}}_{Ht} + \mathbf{R}_{M2} \left[\hat{\tau}_{Mt} + \hat{\mathbf{p}}_{Ft}\right] + \mathbf{T} \hat{\mathbf{q}}_t \tag{26}$$

$$\text{where} \quad \mathbf{R}_{M1} \equiv -\bar{\boldsymbol{\eta}}_{\mathbf{M}} + \tilde{\boldsymbol{\eta}}_{\mathbf{M}} \mathbf{W}_{MH} - \kappa \mathbf{W}_{MH} + \kappa \mathbf{T} \mathbf{W}_M \mathbf{W}_{MH} + \mathbf{T} \gamma \left[\mathbf{I} - \mathbf{W}_M \mathbf{W}_{MH}\right] \tag{27}$$

$$\text{and} \quad \mathbf{R}_{M2} \equiv \tilde{\boldsymbol{\eta}}_{\mathbf{M}} \mathbf{W}_{MF} - \kappa \mathbf{W}_{MF} + \kappa \mathbf{T} \mathbf{W}_M \mathbf{W}_{MF} - \mathbf{T} \gamma \mathbf{W}_M \mathbf{W}_{MF} \tag{28}$$

Demand for exports is the same as in the text: $\hat{\mathbf{x}}_t = -\boldsymbol{\eta}_X \hat{\mathbf{p}}_{Ht} + \boldsymbol{\eta}_X \hat{\mathbf{p}}_{Ft} + \hat{\mathbf{x}}_{Ft}$.

Recalling that $\hat{\mathbf{q}}_t = \mathbf{S}_C \hat{\mathbf{c}}_{Ht} + \mathbf{S}_M \mathbb{M}_{Ht} + \mathbf{S}_X \hat{\mathbf{x}}_t$, we substitute for $\hat{\mathbf{c}}_{Ht}$, $\mathbb{M}_{Ht}$, and $\hat{\mathbf{x}}_t$, rearrange and collect terms to obtain:

$$\hat{\mathbf{q}}_t = [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1} - \mathbf{S}_X\boldsymbol{\eta}_X]\hat{\mathbf{p}}_{Ht} + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_C\mathbf{R}_{C2}\hat{\tau}_{Ct} + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_M\mathbf{R}_{M2}\hat{\tau}_{Mt} + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C2} + \mathbf{S}_M\mathbf{R}_{M2} + \mathbf{S}_X\boldsymbol{\eta}_X]\hat{\mathbf{p}}_{Ft} + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\iota\hat{c}_t + \mathbf{S}_X\hat{\mathbf{x}}_{Ft}] \quad (29)$$

Note that this expression is written in terms of gross output prices ($\hat{\mathbf{p}}_{Ht}$), for which we will substitute below.

Recall that demand for real value added is: $\hat{\mathbf{v}}_t = -\gamma\hat{\mathbf{p}}_{Vt} + \gamma\hat{\mathbf{p}}_{Ht} + \hat{\mathbf{q}}_t$. We provide the expression for $\hat{\mathbf{p}}_{Ht}$ in terms of $\hat{\mathbf{p}}_{Vt}$ in Equation (14). Use that plus the solution for $\hat{\mathbf{q}}_t$ above to substitute and obtain the reduced form demand for value added:

$$\hat{\mathbf{v}}_t = -\Sigma\hat{\mathbf{p}}_{Vt} + \mathbf{R}_{V1}\hat{\boldsymbol{\tau}}_{Ct} + \mathbf{R}_{V2}\hat{\boldsymbol{\tau}}_{Mt} + \mathbf{R}_{V3}\hat{\mathbf{p}}_{Ft} + \mathbf{R}_{V4}\iota\hat{c}_t + \mathbf{R}_{V5}\mathbf{x}_{Ft} \quad (30)$$

where the matrices are defined as:

$$\boldsymbol{\Sigma} \equiv \boldsymbol{\gamma}\left[\mathbf{I} - [\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{S}_V\right] - [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1} - \mathbf{S}_X\boldsymbol{\eta}_X][\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{S}_V \quad (31)$$

$$\mathbf{R}_{V1} \equiv [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_C\mathbf{R}_{C2} \quad (32)$$

$$\mathbf{R}_{V2} \equiv [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_M\mathbf{R}_{M2} + \left[\gamma + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1} - \mathbf{S}_X\boldsymbol{\eta}_X]\right][\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{A}'_I \quad (33)$$

$$\mathbf{R}_{V3} \equiv [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C2} + \mathbf{S}_M\mathbf{R}_{M2} + \mathbf{S}_X\boldsymbol{\eta}_X] + \left[\gamma + [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1} - \mathbf{S}_X\boldsymbol{\eta}_X]\right][\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{A}'_I \quad (34)$$

$$\mathbf{R}_{V4} \equiv [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_C \quad (35)$$

$$\mathbf{R}_{V5} \equiv [\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_X \quad (36)$$

Some comments on these matrices are helpful for interpretation.

Starting with $\boldsymbol{\Sigma}$, the first term captures the net impact of $\hat{\mathbf{p}}_{Vt}$ on demand for value added, through relative prices conditional on output. That is, it captures the substitution effects embedded in $\boldsymbol{\gamma}(\hat{\mathbf{p}}_{Vt} - \hat{\mathbf{p}}_{Ht})$.[41] Because we pull the negative sign outside of the matrix, the diagonal elements of $\boldsymbol{\gamma}\left[\mathbf{I} - [\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{S}_V\right]$ are all positive, while off-diagonal elements are always negative. The second term captures the impact of $\hat{\mathbf{p}}_{Vt}$ on demand for output, as downstream consumption, input purchases, and exports changes in response to output price changes. Note it has a similar structure to the mapping from $\hat{\mathbf{p}}_H \mapsto \hat{\mathbf{c}}_H, \hat{\mathbf{m}}_H, \hat{\mathbf{x}}_H \mapsto \hat{\mathbf{q}}$ in Equation 19, only here it the final term captures the mapping from domestic value-added prices to gross output prices: $\hat{\mathbf{p}}_V \mapsto \hat{\mathbf{p}}_H$ through $[\mathbf{I} - \mathbf{A}'_D]^{-1}\mathbf{S}_V$. Following the discussion in the text, an increase in domestic value-added prices tends to trigger substitution away from

[41] The own-price effect effect is standard: holding constant $P_H(s)$ and $Q(s)$, an increase in $P_V(s)$ decreases demand for value added in sector s, with an elasticity $\gamma(s)$. Due to the roundabout nature of production, the increase in $P_V(s)$ also raises $P_H(s)$, which dampens effective own-price elasticity. Cross-price elasticities are then always positive, as an increase in $P_V(s')$ increases $P_H(s)$, so triggers substitution that raises demand for value added in sector s.

home goods, so this term is often negative. As a result, the off-diagonal elements of $\mathbf{\Sigma}$ are shaped by two off-setting forces, and thus are generally near zero.

Turning to $\mathbf{R}_{V1}$ and $\mathbf{R}_{V2}$, we discuss these in detail in the main text, but make a few additional comments here. In the main text, we present results for parameter values such that $\eta(s) > \vartheta$, meaning that protection in downstream sectors translates into increased demand for value added in upstream sectors. However, it is theoretically possible that $\eta(s) < \vartheta$ for some sectors, in which case the corresponding entries in $\mathbf{R}_{C2}$ and thus $\mathbf{R}_{V1}$ would be negative. That is, protection of final goods actually reduces demand for value added in upstream sectors, as that protection triggers expenditure switching away from those goods. More work on estimating the relevant elasticities would therefore be useful, but is outside the scope of this paper.

As for input tariffs, special cases help simply interpretation of results. Looking at the first term in $\mathbf{R}_{V2}$, the restrictions $\kappa = 0$ and $\gamma = \mathbf{0}$ imply that $\mathbf{R}_{M2} = \tilde{\boldsymbol{\eta}}_{\mathbf{M}}\mathbf{W}_{MF}$. Then $\mathbf{R}_{M2}$ maps the change in the input price index due to tariffs (as measured by $\mathbf{W}_{MF}$) into demand for domestic inputs (via $\tilde{\boldsymbol{\eta}}_{\mathbf{M}}$), and then passing that demand back through the production network, through $[\mathbf{I} - \mathbf{S}_M\mathbf{T}]^{-1}\mathbf{S}_M$.[42] The second term in $\mathbf{R}_{V2}$ captures the impacts of the increase in output prices due to rising input tariffs on demand for value added. The output price changes trigger substitution, but they also directly alter demand for value added (conditional on output). See the main text for discussion.

B Endogenous Value-Added Prices in Outside Sectors

In this section, we provide details about how to account for price changes for real value added in outside sectors when computing the Approximate ERP.[43]

Starting from Equation (5), set variables held constant at initial values to zero:

$$\hat{\mathbf{v}}_t = -\Sigma\hat{\mathbf{p}}_{Vt} + \mathbf{R}_{V1}\hat{\tau}_{Ct} + \mathbf{R}_{V2}\hat{\tau}_{Mt}. \tag{37}$$

Then rearrange rows of these matrices so that the sector of interest (s) is in the first row of each matrix/vector, and partition the matrices as follows:

$$\begin{bmatrix}\hat{v}_t(s)\\ \hat{\mathbf{v}}_t^{\neg s}\end{bmatrix} = -\begin{bmatrix}\sigma(s,s) & \Sigma(s,\neg s)\\ \Sigma(\neg s,s) & \Sigma(\neg s,\neg s)\end{bmatrix}\begin{bmatrix}\hat{p}_{Vt}(s)\\ \hat{\mathbf{p}}_{Vt}^{\neg s}\end{bmatrix} + \begin{bmatrix}\mathbf{R}_{V1}(s)\\ \mathbf{R}_{V1}(\neg s)\end{bmatrix}\hat{\tau}_{Ct} + \begin{bmatrix}\mathbf{R}_{V2}(s)\\ \mathbf{R}_{V2}(\neg s)\end{bmatrix}\hat{\tau}_{Mt} \tag{38}$$

Inelastic Real Value Added Suppose that real value added is inelastically supplied in outside sectors, such that $\hat{\mathbf{v}}_t^{\neg s} = 0$ in equilibrium. Then, we can use demand for value added in outside sectors together with this supply restriction to solve for $\hat{\mathbf{p}}_{Vt}^{\neg s}$ as a function of $\hat{p}_{Vt}(s)$

[42] Related to the discussion of consumption above, when κ and γ are not zero, one needs to take patterns of cross-sector substitution into account as well, and the direction of the substitution effects will depend on relative values of these parameters together with various expensiture shares.

[43] The concepts underlying computation of the Exact ERP are similar; we do not present them here, as there is little additional insight to be gained from them.

and trade costs. This yields:

$$\hat{\mathbf{p}}_{Vt}^{\neg s} = -\left[\Sigma(\neg s, \neg s)\right]^{-1} \Sigma(\neg s, s)\hat{p}_{Vt}(s) \\ + \left[\Sigma(\neg s, \neg s)\right]^{-1} \mathbf{R}_{V1}(\neg s)\hat{\tau}_{Ct} + \left[\Sigma(\neg s, \neg s)\right]^{-1} \mathbf{R}_{V2}(\neg s)\hat{\tau}_{Mt} \quad (39)$$

Demand for value added in sector s is:

$$\hat{v}_t(s) = -\sigma(s, s)\hat{p}_{Vt}(s) - \Sigma(s, \neg s)\hat{\mathbf{p}}_{Vt}^{\neg s} + \mathbf{R}_{V1}(s)\hat{\tau}_{Ct} + \mathbf{R}_{V2}(s)\hat{\tau}_{Mt}, \quad (40)$$

Plug the solution for $\hat{\mathbf{p}}_{Vt}^{\neg s}$ from Equation (39) into Equation (40) to write $\hat{v}_t(s)$ as a function of $\hat{p}_{Vt}(s)$, $\hat{\tau}_{Ct}$, and $\hat{\tau}_{Mt}$ taking endogenous movements in the prices of real value added in outside sectors into account. The resulting demand curve can be used to define the effective rate of protection as in the main text.

Upward Sloping Supply of Real Value Added Referring to the upward sloping supply curve for real value added in Section 2.5.1, the approximate (log-linear) supply curve is:

$$\hat{v}_t(s) = \frac{1 - \alpha(s)}{\alpha(s)} \left(\hat{p}_{Vt}(s) - \hat{w}_t\right). \quad (41)$$

Again holding wages fixed at their baseline equilibrium value, this implies that in outside sectors we have:

$$\hat{\mathbf{v}}_t^{\neg s} = \left(\mathbf{I} - \alpha^{\neg s}\right) \left(\alpha^{\neg s}\right)^{-1} \hat{\mathbf{p}}_{Vt}^{\neg s} \quad (42)$$

where α is a diagonal matrix with $\alpha(s)$ along the diagonal and $\alpha^{\neg s}$ is the sub-matrix with entries for outside sectors only.

The supply curve in Equation (42) can be combined with the demand for these outside sectors from Equation (38) to solve for $\hat{\mathbf{p}}_{Vt}^{\neg s}$ as a function of $\hat{p}_{Vt}(s)$ and trade costs. The result can be inserted again into Equation (40) to write $\hat{v}_t(s)$ as a function of $\hat{p}_{Vt}(s)$, $\hat{\tau}_{Ct}$, and $\hat{\tau}_{Mt}$, and the resulting demand curve can be used to define the effective rate of protection taking endogenous movements in the prices of real value added in outside sectors into account. Note that when $\alpha(s) = 1$ in all sectors, the solution is effectively the same as in the inelastic factor supply case discussed above.

C Log-Linear Demand for Value Added in the Multi-country Framework

In this appendix, we lay out the steps to obtain demand for value added in the multi-country framework. To start, we present the elements of the framework in Table 7, which are then log-linearized to produce Table 5. We then stack the sector-level elements on a country-by-country basis in Table 8. The matrix notation for the stacked elements follows the same scheme as the small open economy, with country indexes added here, so we do not repeat all the definitions here. In terms of indexing, there are $s \in \{1, \ldots S\}$ sectors and $i, j \in \{1, \ldots, D\}$ countries.

Table 7: Model Elements for the Multi-country Economy

Consumption	$C_i(s) = \zeta_i(s)\left(\frac{P_{Ci}(s)}{P_{Ci}}\right)^{-\vartheta} C_i$ $C_{ji}(s) = \left(\frac{\tau_{Cji}(s)\psi_{Cji}(s)P_j(s)}{P_{Ci}(s)}\right)^{-\eta(s)} C_i$
Inputs	$V_i(s) = \alpha_i(s)\left(\frac{P_i^V(s)}{P_{it}(s)}\right)^{-\gamma(s)} Q_i(s)$ $M_i(s) = (1-\alpha_i(s))\left(\frac{P_{Mi}(s)}{P_i(s)}\right)^{-\gamma(s)} Q_i(s)$ $M_i(s', s) = \alpha_i(s', s)\left(\frac{P_i(s',s)}{P_{Mi}(s)}\right)^{-\kappa} M_i(s)$ $M_{ji}(s', s) = \left(\frac{\tau_{Mji}(s')\psi_{Mji}P_j(s')}{P_i(s',s)}\right)^{-\eta(s')} M_i(s', s)$
Prices	$P_i(s) = \left(\alpha_i(s)P_{Vi}(s)^{1-\gamma(s)} + (1-\alpha_i(s))P_{Mi}(s)^{1-\gamma(s)}\right)^{1/(1-\gamma(s))}$ $P_{Mi}(s) = \left(\sum_{s'} \alpha_i(s', s)P_i(s', s)^{1-\kappa}\right)^{1/(1-\kappa)}$ $P_i(s', s) = \left(\sum_j \xi(s', s)\left(\tau_{Mt}(s')\psi_{Mji}P_j(s')\right)^{1-\eta(s')}\right)^{1/(1-\eta(s'))}$ $P_{Ci} = \left(\sum_s \zeta_i(s)P_{Ci}(s)^{1-\vartheta}\right)^{1/(1-\vartheta)}$ $P_{Ci}(s) = \left(\sum_j \left(\tau_{Ct}(s')\psi_{Cji}(s)P_j(s)\right)^{1-\eta(s)}\right)^{1/(1-\eta(s))}$
Output	$Q_i(s) = \sum_j C_{ij}(s) + \sum_{s'} M_{ij}(s, s')$

Table 8: Stacked Model Elements for the Multi-country Economy

Consumption	$\hat{\mathbf{c}}_i = -\vartheta\hat{\mathbf{p}}_{Ci} + \iota\hat{c}_i$ $\hat{\mathbf{c}}_{ij} = -\boldsymbol{\eta}_C\left(\hat{\boldsymbol{\tau}}_{Cij} + \iota\hat{r}_{j/i} + \hat{\mathbf{p}}_i\right) + \boldsymbol{\eta}_C\hat{\mathbf{p}}_{Cj} + \hat{\mathbf{c}}_j$
Inputs	$\hat{\mathbf{v}}_i = -\boldsymbol{\gamma}\hat{\mathbf{p}}_{Vi} + \boldsymbol{\gamma}\hat{\mathbf{p}}_i + \hat{\mathbf{q}}_i$ $\hat{\mathbf{m}}_i = -\boldsymbol{\gamma}\hat{\mathbf{p}}_{Mi} + \boldsymbol{\gamma}\hat{\mathbf{p}}_i + \hat{\mathbf{q}}_i$ $\mathbb{M}_i = -\kappa\mathbb{P}_{Mi} + \kappa\mathbf{T}\hat{\mathbf{p}}_{Mi} + \mathbf{T}\hat{\mathbf{m}}_i$ $\mathbb{M}_{ij} = -\bar{\boldsymbol{\eta}}_M\left(\hat{\boldsymbol{\tau}}_{Mij} + \iota\hat{r}_{j/i} + \hat{\mathbf{p}}_i\right) + \tilde{\boldsymbol{\eta}}_M\mathbb{P}_{Mj} + \mathbb{M}_j$
Prices	$\hat{\mathbf{p}}_i = \mathbf{S}_{Vi}\hat{\mathbf{p}}_{Vi} + \mathbf{S}_{Mi}\hat{\mathbf{p}}_{Mi}$ $\hat{\mathbf{p}}_{Mi} = \mathbf{W}_{Mi}\mathbb{P}_{Mi}$ $\mathbb{P}_{Mi} = \sum_j \mathbf{W}_{Mji}\left[\hat{\boldsymbol{\tau}}_{Mji} + \iota\hat{r}_{i/j} + \hat{\mathbf{p}}_j\right]$ $\hat{\mathbf{p}}_{Ci} = \sum_j \mathbf{W}_{Cji}\left(\hat{\boldsymbol{\tau}}_{Cj} + \iota\hat{r}_{i/j} + \hat{\mathbf{p}}_j\right)$
Output	$\hat{\mathbf{q}}_i = \sum_j \mathbf{S}_{Cij}\hat{\mathbf{c}}_{ij} + \mathbf{S}_{Mij}\mathbb{M}_{ij}$

As in the small open economy, we start by solving for gross output prices. Plugging expressions for $\hat{\mathbf{p}}_{Mi}$ and $\mathbb{P}_{Mi}$ into the expression for $\hat{\mathbf{p}}_i$ we obtain:

$$\hat{\mathbf{p}}_i = \mathbf{S}_{Vi}\hat{\mathbf{p}}_{Vi} + \sum_j \mathbf{S}_{Mi}\mathbf{W}_{Mi}\mathbf{W}_{Mji}\left[\hat{\boldsymbol{\tau}}_{Mji} + \iota\hat{r}_{i/j} + \hat{\mathbf{p}}_j\right]. \tag{43}$$

Stacking this expression across countries and manipulating the result to isolate prices yields:

$$\hat{\mathbf{p}} = \mathbf{S}_V \hat{\mathbf{p}}_V + [\mathbf{A}' \,\Box\, \hat{\boldsymbol{\tau}}_M]\, \iota + [\mathbf{A}' \,\Box\, \hat{\mathbf{r}}]\, \iota + \mathbf{A}'\hat{\mathbf{p}}, \tag{44}$$

where $\hat{\mathbf{p}} \equiv \text{vec}([\hat{\mathbf{p}}_1, \cdots, \hat{\mathbf{p}}_D])$ and $\hat{\mathbf{p}}_V \equiv \text{vec}([\hat{\mathbf{p}}_{V1}, \cdots, \hat{\mathbf{p}}_{VC}])$ are $SD \times 1$ vectors of prices, ι is a conformable vector of ones. The matrix $\mathbf{A}'$ is the transpose of the global input-output matrix, defined as follows:

$$\mathbf{A}' = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1D} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{D1} & \cdots & \mathbf{A}_{DD} \end{bmatrix}' = \begin{bmatrix} \mathbf{S}_{M1}\mathbf{W}_{M1}\mathbf{W}_{M11} & \cdots & \mathbf{S}_{M1}\mathbf{W}_{M1}\mathbf{W}_{MD1} \\ \vdots & \ddots & \vdots \\ \mathbf{S}_{MD}\mathbf{W}_{MD}\mathbf{W}_{M1D} & \cdots & \mathbf{S}_{MD}\mathbf{W}_{MD}\mathbf{W}_{MDD} \end{bmatrix}, \tag{45}$$

where $\mathbf{A}_{ji}$ is the $S \times S$ direct use matrix for country i for inputs from j. Then, the trade cost and real exchange rate matrices are defined as:

$$\hat{\boldsymbol{\tau}}_M = \begin{bmatrix} \hat{\boldsymbol{\tau}}_{M11} & \cdots & \hat{\boldsymbol{\tau}}_{MD1} \\ \vdots & \ddots & \vdots \\ \hat{\boldsymbol{\tau}}_{M1D} & \cdots & \hat{\boldsymbol{\tau}}_{MDD} \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{r}} = \begin{bmatrix} \iota\hat{r}_{1/1} & \cdots & \iota\hat{r}_{1/C} \\ \vdots & \ddots & \vdots \\ \iota\hat{r}_{C/1} & \cdots & \iota\hat{r}_{CC} \end{bmatrix}, \tag{46}$$

where $\hat{\boldsymbol{\tau}}_{Mji}$ is the $S \times 1$ vector of tariffs applied by i on imported inputs from j, and $\hat{r}_{i/j}$ is the bilateral real exchange rate (defined in the main text). By construction, $\hat{\boldsymbol{\tau}}_{Mii} = \mathbf{0}$ and $\hat{r}_{i/i} = 0$.

Recall demand for gross output from country i is given by: $\hat{\mathbf{q}}_i = \sum_j \mathbf{S}_{Cij}\hat{\mathbf{c}}_{ij} + \mathbf{S}_{Mij}\mathbb{M}_{ij}$. Stacking across countries, we write this in compact notation:

$$\hat{\mathbf{q}} = \mathbf{S}_C \mathbb{C} + \mathbf{S}_M \mathbb{M}, \tag{47}$$

where $\hat{\mathbf{q}} \equiv \text{vec}([\hat{\mathbf{q}}_1, \cdots, \hat{\mathbf{q}}_D])$ is the vector of output in all countries and sectors, $\mathbf{S}_C$ is a block matrix with blocks $\begin{bmatrix} \mathbf{S}_{C11} & \mathbf{S}_{C12} & \cdots & \mathbf{S}_{C1D} \end{bmatrix}$ along the diagonal and zeros elsewhere, and $\mathbf{S}_M$ is a similar matrix for inputs. The vectors $\mathbb{C}$ and $\mathbb{M}$ collect bilateral final goods and input shipments, and can be defined as follows:

$$\mathbb{C} = \text{vec}\left(\begin{bmatrix} \hat{\mathbf{c}}_{11} & \hat{\mathbf{c}}_{21} & \cdots & \hat{\mathbf{c}}_{D1} \\ \hat{\mathbf{c}}_{12} & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{c}}_{1D} & \hat{\mathbf{c}}_{2D} & \cdots & \hat{\mathbf{c}}_{2DD} \end{bmatrix} \right) \quad \text{and} \quad \mathbb{M} = \text{vec}\left(\begin{bmatrix} \hat{\mathbf{m}}_{11} & \hat{\mathbf{m}}_{21} & \cdots & \hat{\mathbf{m}}_{D1} \\ \hat{\mathbf{m}}_{12} & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{m}}_{1D} & \hat{\mathbf{m}}_{2D} & \cdots & \hat{\mathbf{m}}_{2DD} \end{bmatrix} \right) \tag{48}$$

Bilateral final goods shipments can then be written as:

$$\begin{aligned} \mathbb{C} = {} & [[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}] \otimes \mathbf{I}_{S\times S}]\, [\hat{\mathbf{c}} \otimes \mathbf{1}_S] + [[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}] \otimes [\boldsymbol{\eta} - \mathbf{I}\vartheta]]\, \hat{\mathbf{p}}_C \\ & - [[\mathbf{I}_{D\times D} \otimes \mathbf{1}_D] \otimes \boldsymbol{\eta}]\, \hat{\mathbf{p}} - [\mathbf{I}_{D^2\times D^2} \otimes \boldsymbol{\eta}]\, vec\,(\hat{\boldsymbol{\tau}}_C) - [\mathbf{I}_{D^2\times D^2} \otimes \boldsymbol{\eta}]\, vec\,(\hat{\mathbf{r}})\,. \end{aligned} \tag{49}$$

And the vector of consumption price indexes is:

$$\hat{\mathbf{p}}_C = \mathbf{W}_C \hat{\mathbf{p}} + [\mathbf{W}_C \,\Box\, [\hat{\boldsymbol{\tau}}_C + \hat{\mathbf{r}}]]\, \iota, \tag{50}$$

where $\mathbf{W}_C$ is populated by $\mathbf{W}_{Cji}$, where each row-column block collects the share of final goods from j purchased by country i in total final spending for each sector on the diagonal. Plugging the price index back into the expression for $\mathbb{C}$ and collecting terms gives us:

$$\mathbb{C} = \mathbf{R}_{C1}\hat{\mathbf{p}} + \mathbf{R}_{C2} vec\left(\hat{\boldsymbol{\tau}}_C\right) + \mathbf{R}_{C3}\left[\mathbf{W}_C \,\square\, \hat{\boldsymbol{\tau}}_C\right]\iota \\ + \mathbf{R}_{C2} vec\left(\hat{\mathbf{r}}\right) + \mathbf{R}_{C3}\left[\mathbf{W}_C \,\square\, \hat{\mathbf{r}}\right]\iota + \mathbf{R}_{C4}\left[\hat{\mathbf{c}} \otimes \mathbf{1}_S\right] \quad (51)$$

where the composite matrices are defined as follows:

$$\mathbf{R}_{C1} \equiv -\left[\left[\mathbf{I}_{D\times D} \otimes \mathbf{1}_D\right] \otimes \boldsymbol{\eta}\right] + \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \left[\boldsymbol{\eta} - \mathbf{I}\vartheta\right]\right]\mathbf{W}_C \quad (52)$$

$$\mathbf{R}_{C2} = -\left[\mathbf{I}_{D^2\times D^2} \otimes \boldsymbol{\eta}\right] \quad (53)$$

$$\mathbf{R}_{C3} = \left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \left[\boldsymbol{\eta} - \mathbf{I}\vartheta\right] \quad (54)$$

$$\mathbf{R}_{C4} \equiv \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S\times S}\right]. \quad (55)$$

On the input side, we combine and stack the elements of input demand to arrive at:

$$\mathbb{M} = -\left[\mathbf{I}_{D^2\times D^2} \otimes \bar{\boldsymbol{\eta}}_{\mathbf{M}}\right]\left[vec\left(\hat{\boldsymbol{\tau}}_M\right) + vec\left(\hat{\mathbf{r}}\right)\right] - \left[\mathbf{I}_{D\times D} \otimes \mathbf{1}_D\right] \otimes \bar{\boldsymbol{\eta}}_{\mathbf{M}}\hat{\mathbf{p}} \\ + \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \tilde{\boldsymbol{\eta}}_{\mathbf{M}}\right]\mathbb{P}_M + \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right]\begin{bmatrix}\mathbb{M}_1\\ \vdots \\ \mathbb{M}_D\end{bmatrix} \quad (56)$$

where the last term is sector-to-sector input demand organized by country. Then, these sector-to-sector demands are given by:

$$\begin{bmatrix}\mathbb{M}_1\\ \vdots \\ \mathbb{M}_D\end{bmatrix} = -\kappa\mathbb{P}_M + \kappa\left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\right]\hat{\mathbf{p}}_M - \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\boldsymbol{\gamma}\right]\hat{\mathbf{p}}_M + \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\boldsymbol{\gamma}\right]\hat{\mathbf{p}} + \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\right]\hat{\mathbf{q}} \quad (57)$$

Further, sector-to-sector input price indexes are:

$$\mathbb{P}_M = \bar{\mathbf{W}}_M\hat{\mathbf{p}} + \left[\bar{\mathbf{W}}_M \square \left[\hat{\boldsymbol{\tau}}_M + \hat{\mathbf{r}}\right]\right]\iota, \quad (58)$$

where $\bar{\mathbf{W}}_M$ is a matrix of input use shares with block elements $\mathbf{W}_{Mji}$ in position ij. Further, note that $\hat{\mathbf{p}}_M = \mathbf{W}_M\mathbb{P}_M$, where $\mathbf{W}_M$ is a block diagonal matrix with matrices $\mathbf{W}_{Mi}$ along the diagonal. Putting all these pieces together and collecting terms we arrive at:

$$\mathbb{M} = \mathbf{R}_{M1}\hat{\mathbf{p}} + \mathbf{R}_{M2}\left[\bar{\mathbf{W}}_M \square \left[\hat{\boldsymbol{\tau}}_M + \hat{\mathbf{r}}\right]\right]\iota + \mathbf{R}_{M3}\left[vec\left(\hat{\boldsymbol{\tau}}_M\right) + vec\left(\hat{\mathbf{r}}\right)\right] + \mathbf{R}_{M4}\hat{\mathbf{q}} \quad (59)$$

with the underlying matrices defined as:

$$\mathbf{R}_{M1} = -\left[\mathbf{I}_{D\times D} \otimes \mathbf{1}_D\right] \otimes \bar{\boldsymbol{\eta}}_{\mathbf{M}} + \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \tilde{\boldsymbol{\eta}}_{\mathbf{M}}\right] \bar{\mathbf{W}}_M$$
$$- \kappa \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \bar{\mathbf{W}}_M$$
$$+ \kappa \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\right] \mathbf{W}_M \bar{\mathbf{W}}_M$$
$$- \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\boldsymbol{\gamma}\right] \mathbf{W}_M \bar{\mathbf{W}}_M$$
$$+ \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\boldsymbol{\gamma}\right] \quad (60)$$
$$\mathbf{R}_{M2} = \left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \tilde{\boldsymbol{\eta}}_{\mathbf{M}} - \kappa \left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2} \quad (61)$$
$$+ \kappa \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\right] \mathbf{W}_M \quad (62)$$
$$- \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\boldsymbol{\gamma}\right] \mathbf{W}_M \quad (63)$$
$$\mathbf{R}_{M3} = -\left[\mathbf{I}_{D^2\times D^2} \otimes \bar{\boldsymbol{\eta}}_{\mathbf{M}}\right] \quad (64)$$
$$\mathbf{R}_{M4} = \left[\left[\mathbf{1}_D \otimes \mathbf{I}_{D\times D}\right] \otimes \mathbf{I}_{S^2\times S^2}\right] \left[\mathbf{I}_{D\times D} \otimes \mathbf{T}\right] \quad (65)$$

Then, we have all the elements to define demand for gross output and prices for gross output spelled out above. And demand for real value added is simply: $\hat{\mathbf{v}} = -\tilde{\boldsymbol{\gamma}}\hat{\mathbf{p}}_V + \tilde{\boldsymbol{\gamma}}\hat{\mathbf{p}} + \hat{\mathbf{q}}$, where where $\tilde{\boldsymbol{\gamma}} = \mathbf{I}_{D\times D} \otimes Diag(\boldsymbol{\gamma})$ is a $DS \times DS$ diagonal matrix. Putting the pieces together, we can define demand for value added as follows:

$$\hat{\mathbf{v}} = -\boldsymbol{\Sigma}\hat{\mathbf{p}}_V + \mathbf{R}_{V1}\left(\boldsymbol{\tau}_C\right) + \mathbf{R}_{V2}\left(\boldsymbol{\tau}_M\right) + \mathbf{R}_{V3}\left(\hat{\mathbf{r}}\right) + \mathbf{L}\mathbf{S}_C\mathbf{R}_{C4}\left[\hat{\mathbf{c}} \otimes \mathbf{1}_S\right] \quad (66)$$

where the various terms are defined as:

$$\boldsymbol{\Sigma} = \tilde{\boldsymbol{\gamma}}\left[\mathbf{I} - \tilde{\boldsymbol{\gamma}}\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\mathbf{S}_{\mathbf{V}}\right] - \mathbf{L}\left[\mathbf{S}_{\mathbf{C}}\mathbf{R}_{\mathbf{C1}} + \mathbf{S}_{\mathbf{M}}\mathbf{R}_{\mathbf{M1}}\right]\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\mathbf{S}_{\mathbf{V}} \quad (67)$$
$$\mathbf{R}_{V1}\left(\boldsymbol{\tau}_C\right) = \mathbf{L}\mathbf{S}_C\left[\mathbf{R}_{C2}vec\left(\hat{\boldsymbol{\tau}}_C\right) + \mathbf{R}_{C3}\left[\mathbf{W}_C \,\square\, \hat{\boldsymbol{\tau}}_C\right]\iota\right] \quad (68)$$
$$\mathbf{R}_{V2}\left(\boldsymbol{\tau}_M\right) = \tilde{\boldsymbol{\gamma}}\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\left[\mathbf{A}' \,\square\, \hat{\boldsymbol{\tau}}_M\right]\iota + \mathbf{L}\left[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1}\right]\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\left[\mathbf{A}' \,\square\, \hat{\boldsymbol{\tau}}_M\right]\iota \quad (69)$$
$$+ \mathbf{L}\mathbf{S}_M\left[\mathbf{R}_{M2}\left[\bar{\mathbf{W}}_M \square \hat{\boldsymbol{\tau}}_M\right]\iota + \mathbf{R}_{M3}vec\left(\hat{\boldsymbol{\tau}}_M\right)\right] \quad (70)$$
$$\mathbf{R}_{V3}\left(\hat{\mathbf{r}}\right) = \tilde{\boldsymbol{\gamma}}\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\left[\mathbf{A}' \,\square\, \hat{\mathbf{r}}\right]\iota + +\mathbf{L}\left[\mathbf{S}_C\mathbf{R}_{C1} + \mathbf{S}_M\mathbf{R}_{M1}\right]\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\left[\mathbf{A}' \,\square\, \hat{\mathbf{r}}\right]\iota \quad (71)$$
$$+ \mathbf{L}\mathbf{S}_C\left[\mathbf{R}_{C2}vec\left(\hat{\mathbf{r}}\right) + \mathbf{R}_{C3}\left[\mathbf{W}_C \,\square\, \hat{\mathbf{r}}\right]\iota\right] \quad (72)$$
$$+ \mathbf{L}\mathbf{S}_M\left[\mathbf{R}_{M2}\left[\bar{\mathbf{W}}_M \square \hat{\mathbf{r}}\right]\iota + \mathbf{R}_{M3}vec\left(\hat{\mathbf{r}}\right)\right], \quad (73)$$

where $\mathbf{L} = \left[\mathbf{I} - \mathbf{S}_M\mathbf{R}_{M4}\right]^{-1}$ is an auxiliary matrix, which is is a transformation of the Leontief inverse of the global input-output matrix.