

The Inflation Accelerator

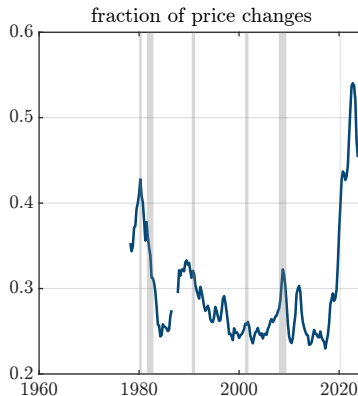
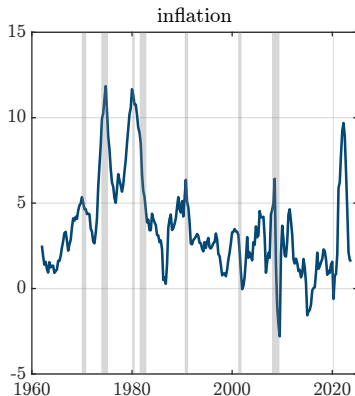
Andres Blanco Corina Boar Callum Jones Virgiliu Midrigan

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Motivation

- Slope of Phillips curve key ingredient in monetary policy analysis
- In sticky price models pinned down by fraction of price changes
- Data: fraction of price changes increases with inflation
 - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)

Evidence from the U.S.



- Source: Nakamura et al. (2018), Montag and Villar (2023). Fraction quarterly.
- Inflation computed using CPI without shelter (year-to-year changes).

► extensive margin decomposition

Motivation

- Slope of Phillips curve key ingredient in monetary policy analysis
- In sticky price models, key determinant: fraction of price changes
- Data: fraction of price changes increases with inflation
 - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)
- How does slope fluctuate in the time series?
 - answer using model that reproduces this evidence

Existing Models

- Time-dependent models
 - widely used due to their tractability
 - constant fraction of price changes
- State-dependent models
 - less tractable: state of the economy includes distribution of prices
- We develop tractable alternative with endogenously varying fraction
 - multi-product firms choose *how many*, but not *which*, prices to change
 - exact aggregation: reduces to one-equation extension of Calvo

Our Findings

- Our model predicts highly non-linear Phillips curve
 - slope fluctuates from 0.02 in 1990s to 0.12 in 1970s and 1980s
- Part of increase ($0.02 \rightarrow 0.04$) due to higher fraction of price changes
- Most increase due to feedback loop between fraction and inflation
 - *inflation accelerator*
 - inflation more sensitive to changes in fraction when inflation is high

Model

- Consumers: log-linear preferences + CIA constraint
 - so $W_t = P_t c_t = M_t$
 - $\log M_{t+1}/M_t = \mu + \varepsilon_{t+1}$ only aggregate shock (robust to Taylor rule etc.)
- Multi-product firms i sell continuum of goods k each
 - final good sector competitive:

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} dk di \right)^{\frac{\theta}{\theta-1}}$$

- demand for individual variety:

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \quad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} dk di \right)^{\frac{1}{1-\theta}}$$

- each produced with DRS technology $y_{ikt} = (l_{ikt})^\eta$

Firm Problem

- Real discounted flow profits of firm i

$$\frac{1}{P_t c_t} \int_0^1 (P_{ikt} y_{ikt} - \tau W_t l_{ikt}) dk = \left(\frac{P_{it}}{P_t} \right)^{1-\theta} - \tau \left(\frac{X_{it}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}}$$

- flow profits depend on two moments of its price distribution

$$P_{it} = \left(\int_0^1 (P_{ikt})^{1-\theta} dk \right)^{\frac{1}{1-\theta}} \quad \text{and} \quad X_{it} = \left(\int_0^1 (P_{ikt})^{-\frac{\theta}{\eta}} dk \right)^{-\frac{\eta}{\theta}}$$

- Firm chooses fraction of price changes n_{it} , cost $\frac{\xi}{2} (n_{it} - \bar{n})^2$ if $n_{it} > \bar{n}$
 - but not which, so history encoded in two state variables, P_{it-1} and X_{it-1}
 - e.g. $P_{it} = \left(n_{it} (P_{it}^*)^{1-\theta} + (1 - n_{it}) (P_{it-1})^{1-\theta} \right)^{\frac{1}{1-\theta}}$

Symmetric Equilibrium

- Let $p_t^* = P_t^*/P_t$, $x_t = X_t/P_t$, $\pi_t = P_t/P_{t-1}$
- Optimal reset price similar to Calvo, except n_t varies

$$(p_t^*)^{1+\theta(\frac{1}{\theta}-1)} = \frac{1}{\eta} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (y_{t+s})^{\frac{1}{\eta}} \prod_{j=1}^s (1 - n_{t+j}) (\pi_{t+j})^{\frac{\theta}{\eta}} \left. \vphantom{\sum_{s=0}^{\infty}} \right\} b_{2t}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^s (1 - n_{t+j}) (\pi_{t+j})^{\theta-1} \left. \vphantom{\sum_{s=0}^{\infty}} \right\} b_{1t}}$$

- Fraction of price changes

$$\xi(n_t - \bar{n}) = \underbrace{b_{1t} \left((p_t^*)^{1-\theta} - (\pi_t)^{\theta-1} \right)}_{\text{change price index}} - \underbrace{\tau b_{2t} \left((p_t^*)^{-\frac{\theta}{\eta}} - (x_{t-1})^{-\frac{\theta}{\eta}} (\pi_t)^{\frac{\theta}{\eta}} \right)}_{\text{reduce misallocation}}$$

Symmetric Equilibrium

- Inflation pinned down by the definition of price index

$$1 = n_t (p_t^*)^{1-\theta} + (1 - n_t) (\pi_t)^{\theta-1}$$

- Losses from misallocation

$$(x_t)^{-\frac{\theta}{\eta}} = n_t (p_t^*)^{-\frac{\theta}{\eta}} + (1 - n_t) (x_{t-1})^{-\frac{\theta}{\eta}} (\pi_t)^{\frac{\theta}{\eta}}$$

- Model reduces to one-equation extension of Calvo

– as $\xi \rightarrow \infty$, $n_t = \bar{n}$ so our model nests Calvo

- Unlike Calvo, important non-linearities so solve using global methods

– third-order perturbation accurate

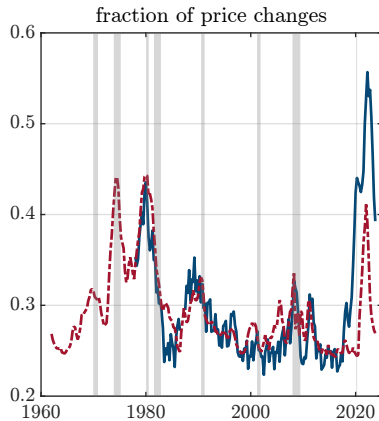
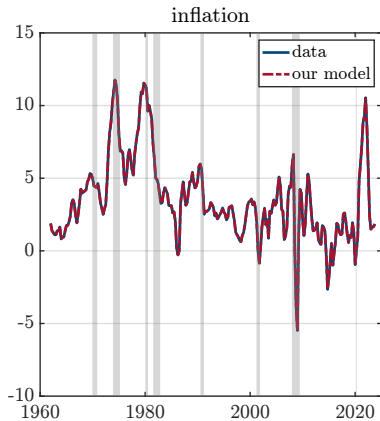
Parameterization

- Assigned parameters
 - period 1 quarter, $\beta = 0.99$, $\theta = 6$, $\eta = 2/3$
- Calibrated parameters
 - mean and standard deviation of money growth μ and σ
 - fraction of free price changes \bar{n} , price adjustment cost ξ
- Calibration targets

	Data	Model
mean inflation	0.035	0.035
s.d. inflation	0.027	0.027
mean fraction	0.297	0.297
slope of n_t on $ \pi_t $	0.016	0.016

Fraction of Price Changes

- Use non-linear solution to recover shocks that reproduce U.S. inflation



- Reproduces fraction well, except post-Covid
 - many price decreases due to sectoral shocks

► extensive margin model

Towards the Slope of the Phillips Curve

- First order perturbation around equilibrium point at each date t
 - hats denote deviations from equilibrium at that date

- Aggregate price index:

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\pi_t^{\theta-1}} \frac{\pi_t^{\theta-1} - 1}{\theta - 1}}_{\mathcal{M}_t} \hat{n}_t + \underbrace{\frac{1 - (1-n_t)\pi_t^{\theta-1}}{(1-n_t)\pi_t^{\theta-1}}}_{\mathcal{N}_t} \hat{p}_t^*$$

- Elasticity \mathcal{N}_t to reset price: identical to Calvo
 - increases with n_t , decreases with π_t (lower weight on new prices)
- Elasticity \mathcal{M}_t to frequency: zero if $\pi_t = 1$, increases with inflation

Intuition

- Why is inflation more sensitive to changes in n_t when inflation is high?

$$\mathcal{M}_t = \frac{1}{(1 - n_t) \pi_t^{\theta-1}} \frac{\pi_t^{\theta-1} - 1}{\theta - 1}$$

- Inflation \approx average price change \times fraction of price changes
 - $\pi_t = 1$: average price change = 0
 - so fraction inconsequential
 - π_t is high: average price change is large
 - so Δn_t increases inflation considerably

Inflation Accelerator

- Recall aggregate price index

$$\hat{\pi}_t = \mathcal{M}_t \hat{n}_t + \mathcal{N}_t \hat{p}_t^*$$

- elasticity \mathcal{M}_t increases with inflation, zero if $\pi_t = 1$

- Optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}_t \hat{\pi}_t + \mathcal{B}_t \hat{p}_t^* - \mathcal{C}_t \hat{x}_{t-1} + \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

- elasticities \mathcal{A}_t and \mathcal{B}_t also increase with π_t

- Feedback loop amplifies inflation response to changes in reset price

$$\hat{\pi}_t = \frac{\mathcal{M}_t \mathcal{B}_t + \mathcal{N}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{p}_t^* - \frac{\mathcal{M}_t \mathcal{C}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{x}_{t-1} + \frac{\mathcal{M}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

Slope of the Phillips Curve

- Let $\widehat{mc}_t = \frac{1}{\eta} \hat{y}_t$ aggregate real marginal cost

$$\hat{\pi}_t = \mathcal{K}_t \widehat{mc}_t + \dots$$

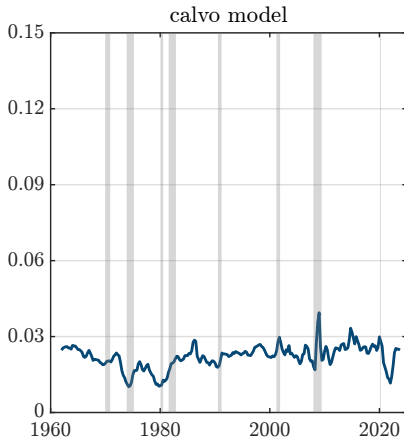
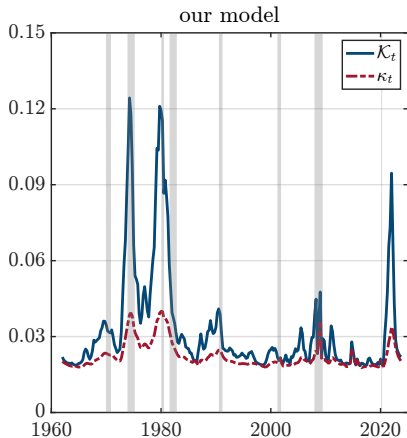
- Slope of the Phillips curve

$$\mathcal{K}_t = \underbrace{\frac{1}{1 + \theta \left(\frac{1}{\eta} - 1 \right)}}_{\text{complementarities}} \times \underbrace{\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}}_{\text{horizon}} \times \underbrace{\frac{\mathcal{M}_t \mathcal{B}_t + \mathcal{N}_t}{1 - \mathcal{M}_t \mathcal{A}_t}}_{\text{reset price}}$$

- Absent endogenous frequency response ($\mathcal{A}_t = \mathcal{B}_t = 0$)

$$\kappa_t = \frac{1}{1 + \theta \left(\frac{1}{\eta} - 1 \right)} \times \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \times \underbrace{\frac{1 - (1 - n_t) \pi_t^{\theta-1}}{(1 - n_t) \pi_t^{\theta-1}}}_{\mathcal{N}_t}$$

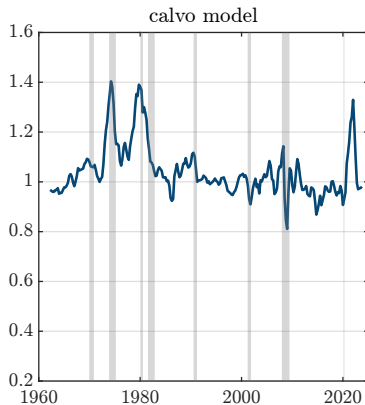
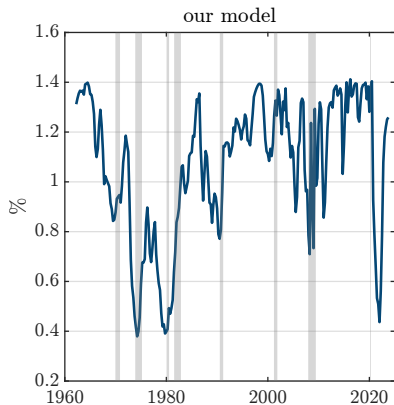
Time-Varying Slope of the Phillips Curve



Ranges from 0.02 to 0.12, mostly due to inflation accelerator

Sacrifice Ratio

- Calculate decline in annual output needed to reduce π by 1% over a year



Ranges from 0.4% to 1.4%, opposite of Calvo

Conclusion

- Data: fraction of price changes increases with inflation
- Developed tractable model consistent with this evidence
 - firms choose how many, but not which prices to change
 - reduces to one-equation extension of Calvo
- Implies slope of Phillips curve increases considerably with inflation
 - partly because more frequent price changes
 - primarily due to endogenous frequency response – *inflation accelerator*

Robustness

Eliminate Strategic Complementarities

- Set $\eta = 1$, recalibrate model

Targeted Moments

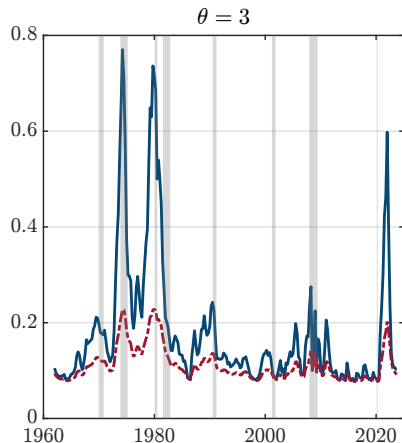
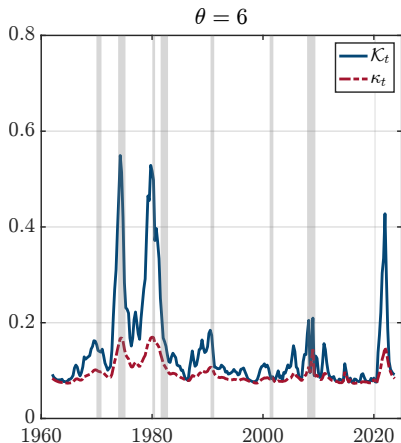
	Data	$\theta = 6$	$\theta = 3$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $ \pi_t $	0.016	0.016	0.016

Calibrated Parameters

	$\theta = 6$	$\theta = 3$
μ mean spending growth rate	0.035	0.035
σ s.d. monetary shocks	0.019	0.018
\bar{n} fraction free price changes	0.232	0.227
ξ adjustment cost	0.365	0.109

- Smaller price adjustment costs because less curvature in profit function

Slope of the Phillips Curve



Larger absent complementarities, but fluctuates as much

Taylor Rule

- Replace nominal spending target with Taylor rule

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i} \right)^{\phi_i} \left(\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\phi_y} \right)^{1 - \phi_i} u_t$$

- Two versions
 - u_t shocks iid
 - serially correlated with persistence ρ to match autocorrelation inflation
- Use Justiniano and Primiceri (2008) estimates
 - $\phi_i = 0.65, \phi_\pi = 2.35, \phi_y = 0.51$

Calibration of Economy with a Taylor Rule

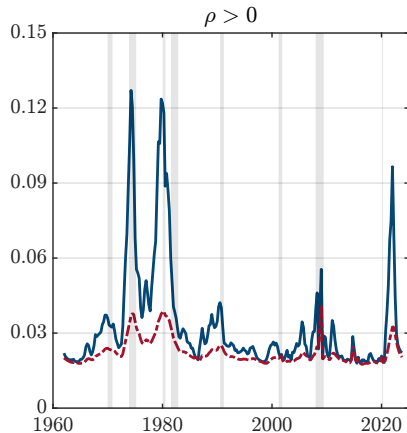
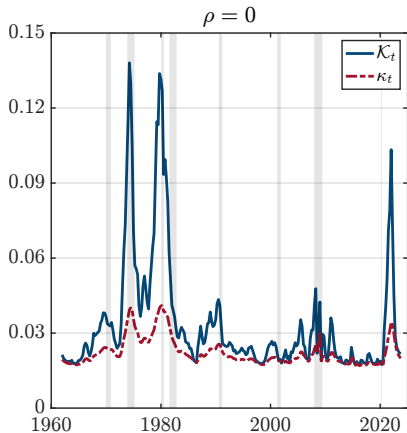
Targeted Moments

	Data	$\rho = 0$	$\rho > 0$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $ \pi_t $	0.016	0.016	0.016
autocorr. inflation	0.942	<i>0.913</i>	0.942

Calibrated Parameters

		$\rho = 0$	$\rho > 0$
$\log \pi$	inflation target	0.040	0.037
σ	s.d. monetary shocks $\times 100$	2.626	0.551
ρ	persistence money shocks	–	0.685
\bar{n}	fraction free price changes	0.241	0.241
ξ	adjustment cost	1.671	1.688

Slope of the Phillips Curve



Our results robust to assuming a Taylor rule

Losses from Misallocation

$$\begin{aligned}(X_{it+s})^{-\frac{\theta}{\eta}} &= n_{it+s} (P_{it+s}^*)^{-\frac{\theta}{\eta}} + (1 - n_{it+s}) n_{it+s-1} (P_{it+s-1}^*)^{-\frac{\theta}{\eta}} + \dots \\ &+ \prod_{j=1}^s (1 - n_{it+j}) \textcolor{red}{n}_{it} (\textcolor{red}{P}_{it}^*)^{-\frac{\theta}{\eta}} + \prod_{j=1}^s (1 - n_{it+j}) (1 - \textcolor{red}{n}_{it}) (X_{it-1})^{-\frac{\theta}{\eta}}\end{aligned}$$

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Steady-State Output and Productivity

$$y^{\frac{1}{\eta}} = \eta \frac{1 - \beta (1 - n) \pi^{\frac{\theta}{\eta}}}{1 - \beta (1 - n) \pi^{\theta - 1}} \left(\frac{n}{1 - (1 - n) \pi^{\theta - 1}} \right)^{\frac{1 + \theta (\frac{1}{\eta} - 1)}{\theta - 1}}$$

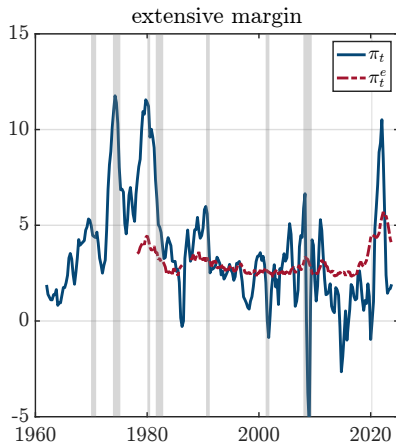
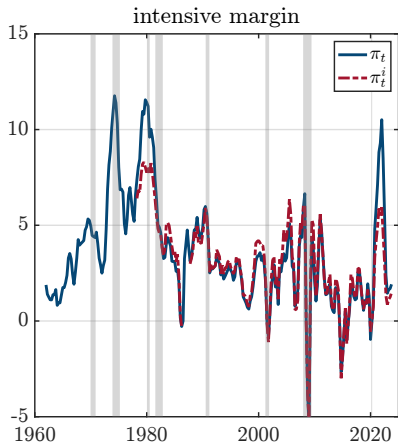
$$x^{\theta} = \left(\frac{1 - (1 - n) \pi^{\frac{\theta}{\eta}}}{n} \right)^{\eta} \left(\frac{1 - (1 - n) \pi^{\theta - 1}}{n} \right)^{-\frac{\theta}{\theta - 1}}$$

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Role of Extensive Margin

- Decompose $\pi_t = \Delta_t n_t$ into two components
 - Δ_t : average price change conditional on adjustment
 - n_t : fraction of price changes
- Isolate role of each using Klenow and Kryvtsov (2008) decomposition
 - intensive margin: $\pi_t^i = \Delta_t \bar{n}$
 - \bar{n} : mean fraction of price changes
 - extensive margin: $\pi_t^e = \bar{\Delta} n_t$
 - $\bar{\Delta}$: mean average price change

Role of Extensive Margin: Data

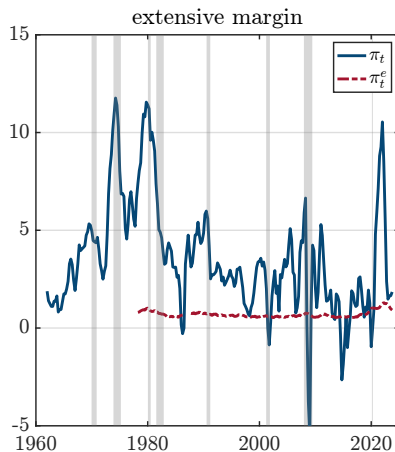
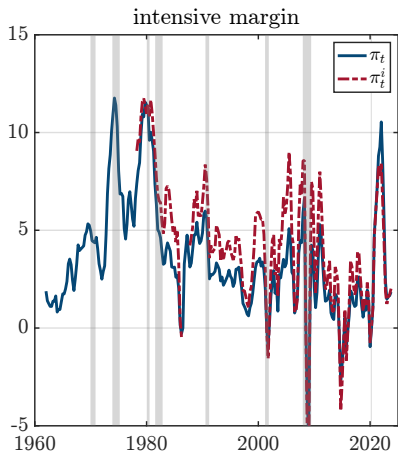


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Montag and Villar (2024)

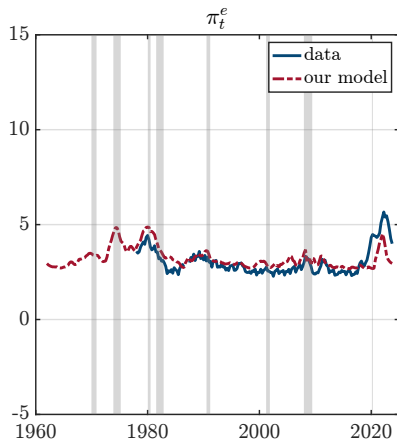
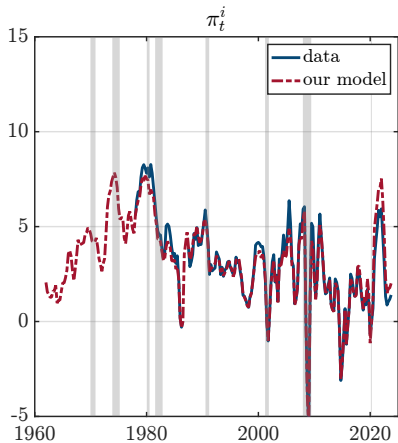
- Argue that extensive margin plays no role post Covid
- Same decomposition but set \bar{n} and $\bar{\Delta}$ equal to January 2020 values
 - due to seasonality, unusually large n and low Δ
- Illustrate fixing \bar{n} and $\bar{\Delta}$ at January 2020 values

Role of Extensive Margin using January 2020



► back

Role of Extensive Margin: Our Model



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