

Please Note:

- You have 60 minutes to complete
- Calculators are not allowed
- Please show all your work and write your answers in the designated space

 $Thank\ you$

Country:	
Reference Number:	
Name:	

Problem 1. Compute the following

$$\frac{100}{2} + 5$$

Answer:

Problem 2. Suppose that

$$Y = K^{\alpha} L^{1-\alpha}$$

Which of the following is true?

Answer:

Problem 3. Suppose that

$$y = x^2 - 2ax$$

Which of the following is equivalent to this equation:

Problem 4. Solve for x:

$$\frac{1}{x} = \frac{3}{1-x}$$

Answer:

Problem 5. Note that e is Euler's constant. Solve for x:

$$\frac{e^{6x-10}}{e^x} = 1$$

Answer:

Problem 6. Solve for x:

$$\frac{(1+x)^2}{1+x} = 3$$

Problem	7.	Solve	for	x
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$$2\ln(1+x) - \ln(1+x) = \ln(3)$$

Answer:

Problem 8. Find a first order Taylor expansion (nearby x = 0) of

$$f(x) = \ln(1+x)$$

Using this, approximate ln(1 + .02)

Answer:

Problem 9. Suppose there is a box containing 50 red balls and 50 green balls. A ball is removed and then set aside (and not returned to the box). Which of the following is true?

Problem 10. Suppose that we have the set of numbers:

$$X = [87, 63, 40, 76, 2, 28, 6, 30, 25, 49]$$

The average value of this set of numbers is $\bar{x} = 40.6$. Compute the following, where $x_i \in X$,

$$\sum_{i=1}^{10} x_i.$$

Answer:

Problem 11. Suppose that we have the set of numbers:

$$X = [87, 63, 40, 76, 2, 28, 6, 30, 25, 49]$$

The average value of this set of numbers is $\bar{x} = 40.6$. Compute the following, where $x_i \in X$,

$$\sum_{i=1}^{10} \left(x_i - \bar{x} \right)$$

Answer:

Problem 12. Compute the derivative of the function

$$f\left(x\right) = x \ln\left(x\right) + 1$$

Problem 13. Suppose that

$$y(t) = \frac{x(t)}{z(t)}$$

where x(t) and z(t) are functions of time

$$x(t) = x_0 e^{g_x t}, \ z(t) = z_0 e^{g_z t}$$

where g_x and g_z are constants. Compute

$$g_y \equiv \frac{d \ln y(t)}{dt}$$

Answer:

Problem 14. Suppose that

$$\int_{-\infty}^{\infty} f(x)dx = 1$$
$$\int_{-\infty}^{0} f(x)dx = 0.2$$

Compute

$$\int_0^\infty f(x)dx$$

Problem 15. Note that

$$\Phi(X) \equiv \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}(z^2)} dz.$$

Evaluate the following integral:

$$Y \equiv \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{\frac{-1}{2} \left(\frac{x-\mu}{\sigma}\right)^2} dx \tag{1}$$

Answer:

Problem 16. Find the value of x in which

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{\frac{-1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}, \quad \sigma > 0$$

is maximized:

Problem 17. Suppose that $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$. Compute AA^T :

Answer:

Problem 18. Suppose that

$$\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right] = \left[\begin{array}{cc} 1 & 2 \\ 3 & 5 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right].$$

Write y_1 as a function of and x_1 and x_2 :

Answer:

Problem 19. If possible, solve for the values of x_1 , x_2 , and x_3 in which

$$\begin{bmatrix} 1 & 5 & 10 \\ 2 & 10 & 20 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

 $\textbf{Problem 20.} \ \, \textbf{Evaluate the limit (use L'Hôpital's rule)}$

$$\lim_{x \to \infty} x \ln \left(1 + \frac{1}{x} \right)$$