TECHNICAL ASSISTANCE REPORT

TONGA

Liquidity Management and Forecasting

October 2025

Prepared By

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Glossary

AFs	Autonomous Factors
AIC	Akaike Information Criterion
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
CBT	Central Bank Transparency
CDS	Credit Default Swap
CiC	Currency in Circulation
ELA	Emergency Liquidity Assistance
ER	Exchange Rate
ETS	Exponential Smoothing
FRFA	Fixed-rate Full Allotment
FX	Foreign Exchange
IMF	International Monetary Fund
IRC	Interest Rate Corridor
LFU	Liquidity Forecasting Unit
MAE	Mean Absolute Error
MCM	Monetary and Capital Market Department
MDF	Marginal Deposit Facility
MIS	Mean Interval Score
MLF	Marginal Lending Facility
MLMO	Main Liquidity Management Operation
MPO	Monetary Policy Operation
ME	Mean Error
MOU	Memorandum of Understanding
MPS	Monetary Policy Statement
NFA	Net Foreign Assets
NLP	Natural Language Processing
NRBT	National Reserve Bank of Tonga
OMO	Open Market Operations
RMP	Reserve Maintenance Period
RMSE	Root Mean Squared Error
RR	Reserve Requirements

SLA	Service Level Agreement
SLF	Standing Lending Facility
SRD	Statutory Reserve Deposit
TA	Technical Assistance
TBATS	Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend and Seasonal component
TOP	The Pa'anga
TSA	Treasury Single Account
UIP	Uncovered Interest Parity
US	United States
USD	United States Dollar
VRFA	Variable Rate Fixed Amount

Preface

At the request of the National Reserve Bank of Tonga (NRBT), a Monetary and Capital Markets (MCM) Department conducted a virtual Technical Assistance (TA) mission from July 14–18, 2025 (Virtual leg) and from September 1–5, 2025 (On-site leg), to assist the authorities with implementing the MCM liquidity management and forecasting framework. The mission team was led by Roger McLeod (MCM) and comprised Luyao Liu (MCM), and Bernard J. Laurens (Short-term Expert). All members participated in both legs of the mission, apart from Bernard J. Laurens who participated in the virtual leg only.

The mission met with the following NRBT officials: Tatafu Moeaki, Governor; Ungatea Latu, Deputy Governor; Daniel Taumoepeau, Assistant Governor Policy; Mefilina Tohi, Chief Manager, Economics and Research; Nanuma Fakatava, Chief Manager Financial Markets; Kueni Fifita Chief Manager Financial Systems; Vasi Taufaloa, Senior Manager Finance.

The mission wishes to thank the NRBT for their cooperation, productive discussions, and their hospitality.

Executive Summary

The NRBT has started the process of modernizing its monetary operational framework to strengthen the role of interest rates and enhance monetary policy transmission. This process is being guided by the IMF TA recommendations in the context of capacity building missions (i.e., January 2025 MCM TA mission—final report June 2025). The January 2025 mission recommended that the NRBT gradually implements the proposed monetary policy implementation framework, which would involve two complementary workstreams to: (i) develop the analytical capacity to determine the appropriate policy rate; and (ii) develop the operational capacity to align short-term money market rates, as they emerge, with the NRBT's policy rate.

As guided by the January 2025 TA recommendations, the NRBT has focused on enhancing the key building blocks of monetary policy implementation. These include: (i) the design of the currency basket; (ii) the modification of the Interest Rate Corridor, including the design of the standing facilities and open market operations (OMOs); (iii) the design of the SRD instrument; (iv) the design of a short-term liquidity monitoring and forecasting framework; (v) the activation of collateral policies to support NRBT's monetary operations; and (vi) the execution of the recommended NRBT communications framework. Notwithstanding the NRBT's progress in modernizing its operational framework, some components require further IMF TA assistance. The NRBT has set an initial policy rate at two percent but has requested guidance in setting an appropriate rate that can sustain the exchange rate peg. In addition, NRBT's liquidity forecasting framework produces large discrepancies between forecasts and actual outcomes. As such, NRBT's forecasting capacity needs to be enhanced to effectively implement monetary policy, particularly relating to the calibration of its OMOs.

Against this background, this mission focused on the provision of two key building blocks. First, the mission provided advice on developing an analytical framework to appropriately set the policy rate. Second, the mission focused on institutionalizing the MCM liquidity forecasting framework as a tool to steer short term monetary rates toward the policy rate. In this context, the mission facilitated the continuity of the reform program recommended by the January 2025 mission. These reforms will ensure the NRBT adequately bolsters the resilience of its exchange rate peg by establishing effective market-based implementation of monetary policy.

Central to the choice of an appropriate policy rate is the question of whether the central bank plans to maintain or relax capital and exchange controls. In this context, there are two approaches that NRBT could undertake. First, the NRBT could set a policy rate derived from an Uncovered Interest Parity (UIP) condition augmented with a suitable risk premium. This would be the appropriate method if the NRBT plans to relax capital and exchange controls. This would also prevent arbitrage opportunities and could engender economic benefits. Second, NRBT could develop a reaction function based solely on its domestic conditions, i.e., on its macroeconomic forecasts of inflation. However, this approach relies on maintaining capital controls.

This mission provided guidance on setting a policy rate based on the UIP condition augmented with an estimated risk premium. This guidance is provided with the assumption that the NRBT intends to relax controls. However, NRBT's ultimate decision on the maintenance or relaxation of capital controls should be made in consultation with the area department of the IMF. To this end, the mission provided guidance on the analytical underpinning of estimating the risk premium. However, given the unavailability of data to estimate the risk premium using a market basket approach, the mission provided guidance on gauging an appropriate risk premium through a process of gradually adjusting the risk premium as part of the implementation of the policy rate. Once the policy rate is established and the interbank market is reactivated, the NRBT should then employ the model-based approach to forecast changes in the risk premium as a function of financial and macroeconomic fundamentals.

The mission also supported the enhancement of NRBT's capacity to manage and forecast liquidity. Currently, the NRBT conducts a short-term liquidity forecasting exercise with a view to determining how much estimated excess liquidity in the system should be mopped up monthly. This exercise, however, produces large discrepancies between NRBT forecasts and actual outcomes. To significantly upgrade the NRBT's liquidity forecasting capacity, the mission institutionalized the statistical forecasting framework developed by MCM. The mission also provided guidance on implementing optimal institutional arrangements to support this framework. It is essential that the NRBT understands and anticipates the behaviors of both its monetary and non-monetary policy counterparties. The ability to effectively manage and forecast liquidity is crucial in calibrating OMOs. In the meantime, the NRBT discretionary liquidity absorbing monetary operations can be conducted with the issuance of NRBT Notes at fixed-rate full allotment (FRFA) auctions.

The mission conducted several workshops to institutionalize the MCM liquidity forecasting framework. The MCM framework comprises 12 forecasting models across three categories: exponential smoothing, Autoregressive Integrated Moving Average (ARIMA), Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend and Seasonal component (TBATS), and volatility models. It features an out-of-sample performance testing system grounded in four criteria that evaluate accuracy, bias, and confidence intervals. The framework enables model selection, forecast generation, and forecast reconciliation to enhance forecast accuracy. The mission thoroughly explained these components to NRBT staff, conducted staff training that employed simulations, and provided code packages for forecasting autonomous factors and estimating reserve demand.

The mission recommended that the NRBT start conducting liquidity forecasts. A standalone Liquidity Forecasting Unit (LFU) should be established to manage and execute this process. Developing a comprehensive daily liquidity database should also be pursued to adequately underpin the liquidity forecasting process. In addition to this, data availability of the government account daily transactions should be addressed in the near-term. A cadre of dedicated econometricians should be sought to apply the MCM methodology for forecasting liquidity. All autonomous factors and net liquidity should be forecasted with the process continually refined to lower the modeling risk.

The implementation of the MCM framework will be useful in both the current application of FRFA and the future adoption of variable rate fixed amount (VRFA) allotment. NRBT should fully utilize the framework and make use of the reconciliation strategy to forecast the sum of the autonomous factors with an estimation of the confidence interval. Even under the current process of conducting auctions of one-week NRBT Notes at FRFA, the MCM liquidity forecasting framework should still be applied to publish the forecast to inform counterparties' bidding at the FRFA operations. To further bolster the effectiveness of these processes, the structure of NRBT's analytical balance sheet should be revised and made available at a daily frequency. The mission provided guidance, including simulations, on estimating commercial bank's own demand for reserves as part of the MCM framework. By calculating the difference between the liquidity supply and the commercial banks' own demand for reserves, the resulting liquidity gap guides the allotment decisions for the volume of OMOs under VRFA NRBT Notes auctions.

The mission also assessed the implementation status of the January 2025 TA recommendations, with a view to assisting its implementation. The NRBT plans to absorb the excess liquidity by issuing NRBT Notes at the current policy rate of two percent, in line with the recommendations of the January 2025 TA. Spurring the development of the interbank market remains a crucial objective for monetary policy implementation. In this context, NRBT has intensified its engagement with the commercial banks, and has implemented key interbank market infrastructural requirements. The mission provided guidance to unblock potential impediments and further incentivize interbank trading.

Recommendations

Table 1. Key Recommendations

Recommendations and Authority Responsible for Implementation	Priority	Timeframe 1/
Policy Rate Setting		
Develop an implementation plan for the policy rate, including a gradual sequencing for determining its appropriate level.	High	Near-term
Forecast the risk premium using a model-based approach. Integrated risk premium forecasts in monetary policy decisions.	Low	Medium-term
Liquidity Management and Forecasting		
Establish a standalone Liquidity Forecasting Unit.	High	Near-term
Develop a comprehensive daily liquidity database that will adequately underpin the liquidity forecasting process.	High	Near-term
Enhance the methodological process by incorporating expert judgment and granular knowledge of the NRBT experts.	Low	Medium-term
Monitor the daily flows to and from the government account held at the NRBT and systematically capture daily balances.	High	Near-term
Forecast each autonomous factor and net liquidity position individually using the IMF framework, applying dynamic selection and averaging to enhance accuracy and lower modeling risk.	High	Near-term
Collect market intelligence and exchange information during weekly committee meetings to adjust for the baseline statistical forecasts generated from the forecasting framework.	High	Near-term
Publish the liquidity forecast.	High	Medium-term

1/ Near term: < 12 months; Medium term: 12 to 24 months.

Introduction

1. **The de jure and de facto monetary policy objective of the NRBT is to maintain internal and external price stability.** Internal price stability is defined as the maintenance of inflation below a reference rate (currently set at five percent) while external price stability is the stability of Tonga's currency, the Pa'anga (TOP). The Tongan pa'anga is pegged to a basket of currencies, that includes the US dollar, New Zealand dollar, Fijian dollar, and the Australian dollar. Tonga also employs capital flow management measures. The implementation of this monetary policy arrangement, however, lacked many components of an effective operational framework. Such absences include: (i) a clear operational target; (ii) an interest rate that would incentivize savings in Tongan pa'anga, and an active interbank market; and (iii) other components such as liquidity forecasts, standing facilities, and open market instruments. To address these absences, a January 2025 IMF TA mission produced several recommendations to modernize the framework and safeguard the exchange rate peg.

2. **Following the January 2025 mission, the NRBT started the process of modernizing its monetary operational framework to strengthen the role of interest rates and enhance monetary policy transmission.** The January 2025 mission recommended that the NRBT gradually implements an improved framework, which would involve two complementary workstreams to: (i) develop the analytical capacity determine an appropriate policy rate; and (ii) develop the operational capacity to determine and assess actions in the money market to align short-term money market rates with the NRBT's policy rate.

3. **As guided by the January 2025 TA recommendations, the NRBT has focused on enhancing the key building blocks of monetary policy implementation.** These include: (i) the design of the currency basket; (ii) the modification of the Interest Rate Corridor, including the design of the standing facilities and OMOs; (iii) the design of the SRD instrument; (iv) the design of a short-term liquidity monitoring and forecasting framework; (v) the activation of collateral policies to support NRBT's monetary operations; and (vi) the execution of the recommended NRBT communications framework. Notwithstanding the NRBT's progress in modernizing its operational framework, some components require further IMF TA assistance. The NRBT has set an initial policy rate at two percent but has requested guidance in setting an appropriate rate that can sustain the exchange rate peg. In addition, NRBT's liquidity forecasting framework produces large discrepancies between forecasts and actual outcomes, and as such, NRBT's forecasting capacity needs to be enhanced to effectively implement monetary policy.

4. **Against this background, this mission was tasked with bridging this gap by continuing the reform program.** In so doing, the focus of this mission is to assist the NRBT in setting an appropriate policy rate and in implementing an active liquidity management framework. To assist with the latter, the mission sought to institutionalize the MCM liquidity forecasting model within NRBT. Implementing a modernized approach to monetary policy with appropriate instruments to absorb excess liquidity is crucial in restarting the interbank market, making it possible for the NRBT to conduct short-term liquidity management operations aimed at steering short-term interbank rates, as they emerge, towards the monetary policy rate.

5. **Central to the choice of an appropriate policy rate is whether the central bank plans to maintain or relax capital and exchange controls.** In this context, there are two approaches that the NRBT could undertake. First, the NRBT could set a policy rate derived from a UIP condition augmented with a suitable risk premium. This would be the appropriate method if the NRBT plans to relax capital and exchange controls. Second, NRBT could develop a reaction function based

solely on its domestic conditions, i.e., on its macroeconomic forecasts of inflation. However, this is an approach that relies on maintaining capital controls. This mission provided guidance on applying the first approach.

6. **The remainder of this report is arranged as follows**: Section I describes the policy rate setting and suggests some recommendations for estimating the risk premium, including a phased approach for implementing the policy rate. Section II discusses the design of the liquidity forecasting framework and Section III discusses the implementation status of the January 2025 IMF TA recommendations.

I. Monetary Policy: Institutional Framework and Policy Rate Setting

A. Current Situation

7. **The NRBT has started the process of modernizing the institutional framework for monetary policy by focusing on strengthening the role of interest rates to enhance monetary policy transmission.** This process is being guided by the IMF TA recommendations in the context of capacity building missions (i.e., January 2025 MCM TA mission—final report June 2025). The January 2025 mission focused on the following building blocks: (i) the design of the currency basket; (ii) the Interest Rate Corridor (IRC); (iii) the design of the Statutory Reserve Deposit (SRD) instrument; (iv) the design of a short-term liquidity monitoring and forecasting framework; (v) the collateral policies to support liquidity providing monetary operations; and (vi) NRBT communications.

8. **The NRBT has gradually implemented the proposed monetary policy operational framework.** This process involves two complementary workstreams to: (i) develop the analytical capacity to determine an appropriate policy rate; and (ii) develop the operational capacity to determine and assess actions in the money market to align short-term money market rates with the NRBT's policy rate. The January 2025 mission focused on point (ii) to set the conditions for the emergence of an interbank market in Tonga, while this mission focuses on point (i), and on implementing the MCM liquidity forecasting tool within NRBT, to complete the modernization process.

9. **To clarify the monetary policy objectives, amendments to the NRBT Act are also being undertaken.** A key amendment includes establishing price stability as the core priority with the primary focus on low and stable inflation. Other amendments being pursued include establishing financial stability as the second objective and developing provisions for the NRBT to proactively facilitate and nurture private sector growth. NRBT's current monetary policy objectives are also reflected in its governance arrangements for monetary policy formulation and implementation. The NRBT Board approves the Monetary Policy Statement (MPS), which outlines the bank's monetary policy stance to maintain internal and external price stability.

10. **The NRBT has maintained its exchange rate peg to a weighted currency basket, which has been the main anchor for achieving price stability**. The currency basket comprises the US dollar, New Zealand dollar, Fijian dollar, and the Australian dollar. There is a ±five percent monthly adjustment limit, with the US dollars used as an intervention currency. Tonga also employs capital flow management measures. The January 2025 mission provided guidance on

the optimization of the currency basket weights, which is also in the process of being implemented by the NRBT.

11. **Other recommendations from the January 2025 mission have already been implemented, with further progress anticipated.** The IRC has been modernized and announced to the public (see Table 2)**.** The policy rate, which has been set at an initial two percent level, now serves as a benchmark for the issuance of NRBT Notes and other relevant monetary instruments. Notwithstanding this progress in implementation of a market-based operational framework, the policy rate level has not been derived using an analytical approach that reflects the country's exchange rate peg. Moreover, the NRBT has not operationalized an active liquidity management framework to align interbank rates, as they emerge, to the policy rate.

Table 2. NRBT Interest Rate Corridor

<table>
<tr><th colspan="2" rowspan="2">Categories of Monetary Policy Operations</th><th colspan="2">Types of Instruments</th><th rowspan="2">Maturity</th><th rowspan="2">Frequency</th><th rowspan="2">Procedures</th></tr>
<tr><th>Liquidity Absorption</th><th>Liquidity Provision</th></tr>
<tr><td rowspan="2">OMOs (NRBT discretion)</td><td>Main liquidity management operation-MLMO</td><td>Issuance of NRBT Notes</td><td>Repo operations 1/</td><td>7-day</td><td>Weekly</td><td>Fixed-rate, full allotment or variable rate fixed amount tenders</td></tr>
<tr><td>Fine-tuning OMOs</td><td>Deposits with NRBT</td><td>Repo operations 1/</td><td>Overnight</td><td>As needed</td><td>Policy rate serves as maximum/minimum bid rate on absorbing/providing operations</td></tr>
<tr><td rowspan="2">Standing Facilities (Banks' discretion)</td><td>Marginal lending facility-MLF</td><td></td><td>Overnight repo operations</td><td>Overnight</td><td colspan="2">IRC Upper bound (policy rate+200 bps).
Collateral: NRBT Notes, Government of Tonga bills and bonds</td></tr>
<tr><td>Marginal deposit facility-MDF</td><td>Overnight deposits</td><td></td><td>Overnight</td><td colspan="2">IRC Lower bound (policy rate–200 bps)</td></tr>
<tr><td colspan="7">1/ Collateral similar as for MLF.
Source: Mission's elaborations.</td></tr>
</table>

12. **The NRBT is committed to absorbing the structural excess liquidity to support interbank market development.** As such, the NRBT plans to execute the excess liquidity absorption at the current policy rate of 2.0 through FRFA issuance of NRBT notes. The challenge faced by NRBT is to determine an appropriate level of the policy rate going forward, and to ensure this is derived from an analytical approach that is compatible with the country exchange rate peg. The weak transmission of monetary policy, which was historically not anchored in market-determined rates, underscores the necessity for a more proactive policy rate setting in alignment with the UIP condition augmented with a risk premium. It also calls for the absorption of all structural excess liquidity once the commercial bank's own demand for reserves is considered.

B. Recommendation

Calibrating a Policy Rate

13. **NRBT should increase its reliance on prices and market principles instead of capital and monetary control**. While capital and exchange controls can effectively manage cross border flow of funds, these controls may also significantly reduce foreign investment as investors perceive a significant risk when they are unable to exit their investments and/or repatriate returns. Likewise, capital and exchange controls have the impact of limiting—in a non-market-oriented fashion—the ability of domestic economic agents to invest and/or consume abroad. The restrictions also unavoidably imply a large and expensive operational burden on all agents involved in imports and exports. Capital and exchange controls may also invite the emergence of a shadow industry to circumvent them. This reduces the perceived fairness and effectiveness of the financial system and leads to a drag on the economy. Financial intermediation and the efficient allocation of financial resources could also be enhanced further with the gradual relaxation of controls.

14. **The large build-up of excess liquidity was driven by the accumulation of foreign exchange (FX) reserves, and the inflow of donor funds, which remained unsterilized.** In this context, NRBT could assess options for relaxing controls (in consultation with the area department of the IMF) to allow for specifically targeted outflows that would reduce the financial burden of liquidity absorption, without jeopardizing the sufficiency of international reserves. At a later stage, the NRBT could consider a more general relaxation of controls (in consultation with the area department of the IMF) when an appropriately determined policy rate has been implemented.

15. **Under a pegged exchange rate regime, the intermediate objective of monetary policy is to maintain the exchange rate peg to ultimately support price stability and financial stability goals**. Annex I provides a conceptual framework for liquidity management under a pegged exchange rate arrangement and perfect capital mobility. While Tonga does not have perfect capital mobility, the conceptual framework provides a basis for understanding the adjustment process in the FX market in response to interest rate adjustments and liquidity shocks. Importantly, the conceptual framework also incorporates friction that arises inter alia from risk premia, counterparty limits, liquidity and capital prudential regulations, and FX transactions costs, which are relevant considerations for Tonga.

16. **Under perfect capital mobility, the success of the sustainability of the peg hinges on ensuring that interest rates satisfy the UIP condition** i.e., that interest rates levels are consistent with balancing foreign exchange flows. Given that capital controls are employed by NRBT, the NRBT could set policy rates independent of its anchor currencies.

17. **There are therefore two approaches that the NRBT could undertake to establish a policy rate.** First, the NRBT could set a policy rate derived from a UIP condition augmented with a suitable risk premium. This would be the appropriate method if the NRBT plans to relax capital and exchange controls. This would also prevent arbitrage opportunities and could engender economic benefits. Second, NRBT could develop a reaction function based solely on its domestic conditions, i.e., on its macroeconomic forecasts of inflation. However, this is an approach that relies on maintaining capital controls. This mission provided guidance on applying the first approach, i.e., setting a policy rate based on the UIP condition augmented with an estimated risk premium.

18. **Against this background, the mission's guidance on an appropriate policy rate proceeded under the assumption that the capital and exchange controls will be relaxed**. However,

NRBT's ultimate decision on the maintenance or relaxation of capital controls should be made in consultation with the area department of the IMF. It should also be noted that the previous mission's guidance on the monetary operational framework, as well as the guidance from this mission on liquidity management, remains the same whether the capital and exchange controls are relaxed or maintained. These two workstreams are to ensure that the NRBT establishes appropriate policy instruments, spurs the development of the interbank market, and aligns the interbank rate with an established policy rate.

19. **A UIP-based policy rate in Tonga should incorporate the equivalent interest rates of the US, Australia, New Zealand, and Fiji.** The interest rates should be combined using the weights of the currency basket. A risk premium for the Tonga should then be added to this weighted interest rate. Monetary policy decisions would thus be driven from rate decisions of the U.S. Federal Reserve, the Reserve Bank of Australia, the Reserve Bank of New Zealand and the Reserve Bank of Fiji, as follows:

$$i_t - i_t^* \approx (E_t(s_{t+1}) - s_t)/s_t + risk\ premium$$

Where i_t is the domestic interest rate, i_t^* is the foreign interest rate, $i_t - i_t^*$ is the differential, and s_t is the spot exchange rate. The NRBT should set its policy rate in reference to the anchor countries' policy rates and adjust the rate as the risk premia changes.[1] In the case of Tonga, the foreign interest rate is a weighted sum of the interest rates of all the currencies in the currency basket, with the weights derived from the currency basket.

20. **By applying the current currency basket weights and the current level of the international policy interest rates, the UIP interest rate for Tonga is derived as 3.65 percent**. The next step for the NRBT would be to estimate the risk premium and add this to this calculated UIP interest rate, as shown in the above equation. While the policy rate should reflect the anchor currencies in a UIP augmented with a risk premium (hereafter referred to as simply the UIP condition), the level of the risk premium may also reflect objectives for FX reserves to the extent that it doesn't cause any undue harm to the sustainability of the peg. If an increase (decrease) in FX reserves is desired, the premium should be set above (below) the estimated risk premium.

Table 3: International Policy Rates

Country	Policy Rates	Level
United States	Effective Fed Funds Rate	4.33%
New Zealand	Official Cash Rate	3.00%
Australia	Cash Rate Target	3.60%
Fiji	Overnight Policy Rate	0.25%

Source: IMF Staff Calculations. Note: Policy rates are based on data on September 5, 2025.

21. **The challenge for the NRBT is to be able to identify fundamental changes in the exchange rate equilibrium, which includes determining the changes in the risk premium.** To determine the adequate level of the risk premium, the different components of the risk premium, i.e., the sovereign risk premium and the currency risk premium, should both be estimated. The appropriate risk premium will thus be a sum of all the components, which are then used to adjust the spread between the domestic and foreign rates accordingly. Leaning against the UIP

[1] While discretionary changes should be avoided, any discretionary crawl in the exchange rate should also be incorporated in the calculated policy rate.

condition will put pressure on the peg. Leaning against the lowering of a UIP condition would lead to the accumulation of large FX reserves which puts added pressure on the NRBT to absorb the liquidity induced by this event. Leaning against the increase of a UIP condition derived policy rate would lead to FX outflows that would induce depreciation pressures on the exchange rate, and in the worst case, undermine the peg.

22. **In the following paragraphs, the mission outlines a framework for determining the appropriate risk premium level and introduces a different framework for forecasting the risk premium**. The market-based approach is used for estimating the current level of the risk premium. However, to anticipate how shocks could affect the risk premium in a forward-looking manner, a different framework is required, i.e., a model-based approach. The model-based approach is required because the market-based approach relies on market data (sovereign risk and currency risk indices) that are not forecasted into the future. Thus, a model-based approach is used to model the risk premium on macroeconomic variables, which can be robustly forecasted.

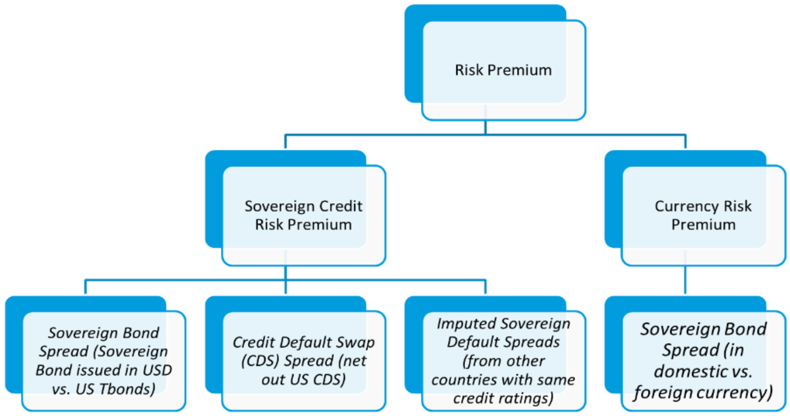
23. **While both frameworks can be combined to evaluate the policy rate in a forward-looking manner, it is important to understand that both frameworks solve different problems and should be applied independently.** The market-based approach is used for determining the level of the risk premium to assist with assessing its appropriateness. It can also be used in a system where balancing FX flows is not possible either due to capital controls or the lack of monetary policy implementation. The model-based approach is not a substitute to the market-based approach. The model-based approach cannot be used to assess the level of the risk premium but rather should be used to forecast changes in the risk premium for forward-looking decision making. Moreover, the model-based approach relies on the existence (and implementation) of an appropriate risk premium and at an adequate data length to enable empirical modeling. Therefore, the market-based approach assesses the level of an appropriate risk premium at any stage of monetary policy implementation, while the model-based framework forecasts the changes in the risk premium, which is only applicable at the later stage where monetary policy implementation is at an advanced phase.

Estimation of the Risk Premia Using a Market-based Approach

24. **The market-based approach for risk premium estimation focuses on indicators that could capture the sovereign risk and currency risk premia.** The sovereign risk spreads can be estimated using three methods. First, the sovereign bond spread method calculates the spread between the yield between the government bonds for the home country issued in USD and the corresponding yield for the US Treasury Bond under the same remaining maturity. Second, the Credit Default Swap (CDS) spread method calculates the spread as the difference between the home country's CDS spread minus the USD CDS spread**.** Third, the imputed sovereign default method assumes that countries with the same credit rating have similar default risk profiles.

25. **A market-based method can also be employed to estimate the currency risk premium (See Figure 1).** The premium is estimated based on the difference between the yields on sovereign bond issuances in domestic currency vs. sovereign bond issuances in USD for the same remaining maturity. In the event of misaligned maturity, normalized yields can be calculated then converted to an overnight yield.

Figure 1: The Market-based Approach for Estimating the Risk Premium



Source: IMF staff.

26. **To implement this market-based approach, this involves analyzing data from the secondary market, but this data is unavailable for Tonga given the lack of adequate market development.** This data is typically sourced from platforms such as Bloomberg for secondary market trading. For Tonga, there is limited market development and as such this data is unavailable, which prevents the implementation of the market-based approach as described above. As such, this method should be applied once data becomes available. At the current juncture, the NRBT should implement the policy rate by applying the current rate of two percent with gradual adjustments in the rate until the UIP interest rate is reached, after which this rate should be adjusted by incrementally adding a risk premium as needed. This process is described in the following section.

Forecasting the Risk Premium

27. **The risk premium can be forecasted using a predictive model for the spread between domestic rates and international rates.** A predictive model provides a forward-looking method for ascertaining the risk premium, by modeling spreads as a function of risk factors. In modeling the risk premium there are two main steps: identifying an appropriate proxy for the risk premium to model, and identifying which factors explain its movements. These are two separate regression modelling tasks, using a similar methodology. See Annex II for further details on this methodology.

Implementing the Policy Rate

28. **A situation with risk free domestic interest rates significantly above zero is without precedence in Tonga, and as such a gradual policy rate implementation is needed.** While the NRBT has established a policy rate of two percent, its transmission to the market, via excess liquidity absorption, is not complete. Thus, the implementation of the policy rate should be carefully communicated and implemented in a gradual fashion to allow economic agents time to adjust and reduce the risk of large shocks to the economy.

Recommendation: **Develop an implementation plan to outline the introduction of a policy rate and a gradual sequencing for determining its appropriate level.**

29. **The implementation plan may comprise the following elements.**

- **The objectives and the main operational elements of the reform should be laid down in a publicly available document.** The document should be published on the NRBT website, and all stakeholders should be invited to comment and attend dedicated meetings at NRBT. The document could have a lead time of, say, one year before NRBT would be aiming to implement the policy rate (i.e. applying it to its reformed instruments to steer the marginal value of liquidity) and starting the process of adjusting the rate as necessary. The document would also contain information on the intention to implement the reform gradually, thereby facilitating the formation of prices in the banking system during the transition.

- **Specific attention may be needed to clarify and safeguard the financial implications of the reform.** Remunerating bank reserves consistently with the UIP condition would at the present level of interest rates in anchoring countries imply large financial transfers from NRBT to the banking sector. This may require establishing an agreement with the government, whereby the government will indemnify any resulting damage to the financial resources available to NRBT. The government, in turn, may also need to revisit revenue expectations to reflect a situation where it will see less income generated via the NRBT balance sheet.

- **In a first step, the policy rate would be set at the current two percent.** This is the minimum rate that would allow the NRBT to start with the current interest rate corridor of +/- 200 basis points, as discussed below.[2] Thereafter the policy rate would be increased by 50 basis points on a quarterly basis until it reaches a level around the weighted average rate prevailing in the anchoring countries. The hikes would be implemented almost mechanically unless new information would suggest otherwise, on predefined dates which would all be aligned with a policy meeting schedule of NRBT. In this process, NRBT would carefully monitor financial and economic conditions to see whether the hikes in interest rates are implying undue tightening in monetary policy and/or whether they have undue implications for financial stability. A set of quantitative and qualitative indicators could be developed and reviewed systematically. These variables could include estimates of inflation expectations, NRBT's inflation forecasts, market interest rates, credit growth, and other market indices

- **In the second step, the policy rate could then be fine-tuned in steps of 25 basis points, on a quarterly basis, until it reaches a level that also incorporates a reasonable risk premium**. With the latter being highly uncertain, an experimental approach would be unavoidable, involving careful monitoring of FX inflows. A reasonable risk premium could also be gauged from countries with similar characteristics that apply pegged exchange rate with free capital mobility. During this phase, NRBT would at each policy meeting mechanically adjust its policy rate to changes in the weighted average (policy) rate observed in the anchoring countries, while at the same time it would adjust the spread it adds on top of that. Risk premium increases smaller than 25 basis points could be contemplated, but this may give the impression of "excessive fine-tuning" in view of the significant uncertainties and the inability of banks to do arbitrage.

- **In a final step, capital controls could be relaxed slightly to allow some arbitrage flows with NRBT standing ready to counter them via the policy rate.** As NRBT gains empirical experience with the interest rate elasticity of arbitrage flows, capital controls could be relaxed further (in consultation with the area department of the IMF) and NRBT could adjust the spread between its policy rate and the weighted average rate of the anchoring countries in

[2] This rate is also in line with the current policy rate of the NRBT.

steps as small as five or 10 basis points. Such adjustments would be driven by estimated changes in the risk premium and objectives for the stock of Net Foreign Assets (NFA).

- **During the final step of the implementation of the policy rate, NRBT should integrate risk premium forecasts into monetary policy decisions.** The forecasts can assist with anticipating adjustments in the risk premium while closely monitoring FX inflows, as part of the NRBT monetary policy decision process.

Recommendation: Forecast the risk premium using a model-based approach. Integrate risk premium forecasts in monetary policy decisions.

30. **Forecasting the risk premium should be done once the requisite data becomes available and at the acceptable data length.** The modeling of the risk premium should be underpinned by the activation of the policy rate and the interbank market. The resulting market data should then be managed effectively to apply the proposed methodology and forecast the risk premium.

31. **In the initial stages of market based monetary policy transmission, the level of the spread may be subject to imperfections that may prevent accurate capturing of the level of the risk premium (as a proxy).** However, once the influence of key fundamentals that impact the spread has been captured empirically in the model, the model can be used to effectively forecast changes in the risk premium in a reasonably accurate manner. As such, this methodology can be combined with the market-based approach to evaluating the risk premium in a forward-looking manner.

II. Liquidity Forecasting Framework

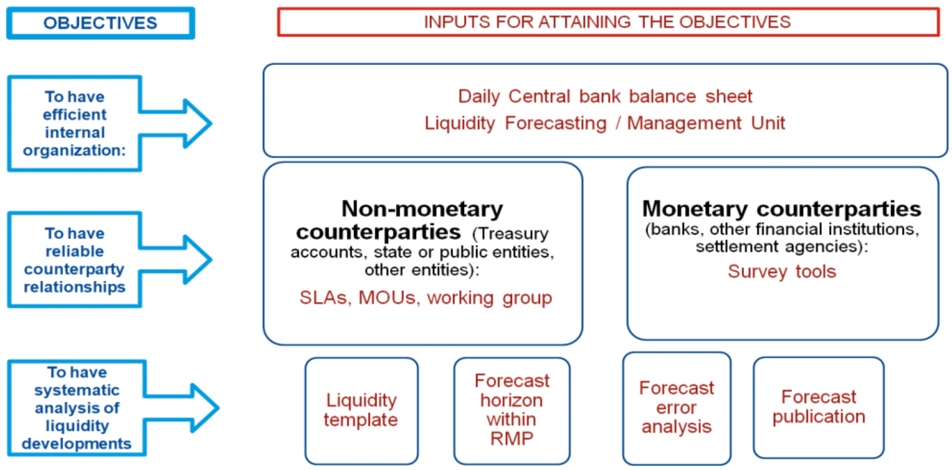
A. Institutional Arrangements

32. **NRBT does not have in place comprehensive institutional arrangements to support liquidity management and forecasting**. While the NRBT has established communication lines with its monetary and non-monetary counterparts, this is not done on a routine basis. Moreover, the NRBT does not have a standalone Liquidity Forecasting Unit. Notwithstanding, the NRBT has hired one staff member dedicated solely to the management and forecasting of liquidity.

33. **The NRBT collects daily data on all the autonomous factors of liquidity, but improvements in data management are warranted**. The NRBT does not systematically record daily balances of all its autonomous factors of liquidity. While this information is available and can be reconciled, it is a highly inefficient process that leads to errors.

34. **The NRBT has allocated additional resources to establish routine liquidity management and forecasting.** NRBT has conducted organizational adjustments such as the hiring of a new staff member, as well as the training of existing staff members, dedicated to liquidity forecasting. This is an important modification that reduces implementation risks and expedites the transition to a modernized operational framework.

Figure 2. Objectives and Outcomes of Liquidity Forecasting



Source: IMF staff.

35. Short-term liquidity forecasting is essential for NRBT's operations. It is also important that key autonomous factors are identifiable from the balance sheet. Net FX assets, currency in circulation, and government account balances drive autonomous factors (AF) volatility (see Figure 3). By adopting forecasts with statistical models, this will improve the anticipation of liquidity shifts, market understanding, contingency planning, and monetary policy responsiveness.

Figure 3. Stylized Central Bank Balance Sheet (NRBT, June 30, 2023)

Assets		Liabilities
1 Foreign Assets	3	Foreign Currency Liabilities
Short/Long Term Investments		Accrued Interest
Accrued Interest		Demand Deposits
International Monetary Fund (IMF)		IMF
	4	Statutory Reserve Deposits
2 Other Assets	5	Demand Deposit
	5.1	International/Domestic banks
	5.2	Government of Tonga
	6	Payable to Government
	7	Currency in Circulation
	8	Debt Securities Issued
	9	Other Liabilities

Assets	Liabilities
Net Foreign Assets (NFA=1-3)	Currency in Circulation(CiC=7)
Other Items Net (OIN =2-9)	Government Account Balance (SAB=5.2+6)
	Banks reserves (R=4)
	Net Claims on Deposit Money

Autonomous Factors (AF)

CB Policy Tools (MPO)

Source: NRBT, IMF staff.

36. **The forecast can be based on the methodology described in the next section.** The IMF MCM TA handbook chapter on Liquidity Forecasting—Part I: The Institutional Arrangements, provides useful references on the institutional and organization arrangements.[3]

Recommendations

37. **Recommendation: Establish a standalone Liquidity Forecasting Unit. The establishment of a standalone LFU is a critical institutional reform aimed at enhancing the accuracy and timeliness of liquidity forecasts.** The LFU should operate as a standalone entity with a clear mandate encompassing information gathering, oversight, and analytical support for monetary operations. Internally, the LFU should utilize inputs from departments such as research, FX operations, and payment systems. The LFU should coordinate with the Ministry of Finance, state-owned enterprises, and financial institutions. To ensure robust governance, the LFU must maintain secure databases, develop publication standards, and implement regular reporting protocols to inform decision-making by the Liquidity Management Committee. While the NRBT has already hired and trained staff for liquidity forecasting, the NRBT should also executing organizational adjustments to implement this recommendation.

Recommendation: Develop a comprehensive daily liquidity database that will adequately underpin the liquidity forecasting process.

38. **To activate the liquidity forecasting framework, a phased transition plan is essential.** The first step involves initiating the daily calibration of data on AFs, which forms the foundation of short-term liquidity forecasts. Establishing structured processes for producing AF forecasts is critical. Regular reviews comparing realized versus forecasted AFs, along with assessments of forecast errors and their market impact will help refine the forecasting models. These insights should be systematically presented to the Liquidity Management Committee or Operations Committee, ensuring that decisions on monetary policy operations (MPOs) are grounded in up-to-date and accurate reserve forecasts. This structured approach will support the central bank's objective of maintaining reserve stability and enhancing the effectiveness of implementation of monetary policy.

B. Liquidity Forecasting Methodology

39. **NRBT does not forecast daily changes in liquidity and has not developed a methodology to implement this process.** Currently, the NRBT forecast excess liquidity, but this is only done monthly. Moreover, large discrepancies are found between the actual and forecasted values.

40. **The mission introduced the IMF liquidity forecasting framework for projecting the main AFs on the central bank balance sheet.** The methodological framework includes a variety of advanced statistical models to predict the daily level of AFs. The model pool includes Naïve (random walk) as a benchmark, seasonal Naïve, ARIMA, seasonal ARIMA, ARIMA with regression, Exponential Smoothing (ETS), ETS with regression, seasonal ETS, Trigonometric seasonality, Box-Cox transformation, Autoregressive Moving Average (ARMA) errors (TBATS), and volatility models. ETS and ARIMA model family can be augmented by adding regressors in the same fashion as conventional regression modeling. Technical details on the statistical models

[3] See Lattie, C, G. Barbakadze, G. Della Valle, T. Silva (2025): Liquidity Forecasting—Part I: The Institutional Arrangements in Monetary and Capital Markets Department: Technical Assistance Handbook)

are presented in Annex III. The IMF MCM TA handbook chapter on Liquidity Forecasting—Part II: The Statistical Component also provides the overview of the proposed modelling approach.[4]

41. **The mission applied the liquidity forecasting methodology to the time series of the major autonomous factors on the NRBT balance sheet.** These autonomous factors include net foreign assets, currency in circulation, and government accounts balance. The training on the statistical time series modelling and liquidity forecasting tool in R was delivered to the NRBT staff during the mission.

42. **The selected models with regressors can account for calendar and seasonality effects.** For example, some of the effects include: (i) holidays that impact consumer behavior and cash demand; (ii) fluctuations associated with payroll cycles; (iii) variations in economic activity based on different days of the week; etc. These effects can be modeled using binary indicators or parabolic indicators to capture the leading and trailing impacts of special events. This ensures that the models account for calendar and seasonal effects, providing more accurate and reliable short-term forecasts.

43. **The forecast performance was evaluated for forecast accuracy and bias.** This was done for the statistical and the final adjusted forecast separately to identify the contribution of each to the total forecast errors, and to indicate potential improvements. The Root Mean Squared Error (RMSE), and the Mean Absolute Error (MAE) as defined below are suggested as accuracy metrics:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2},$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \widehat{y_i}|,$$

where y_i is the actual value, (y_i)ˆis the forecast for the same period, and n is the number of periods considered. The errors should be measured across forecasts of the same horizon, e.g., one period ahead, and then further averaged to provide an overall performance measurement. Both RMSE and MAE provide the errors in the scale and units of the time series and cannot be used to summarize across different time series. The RMSE is sensitive to extreme errors (outliers), while the MAE is more robust. The forecasting framework also contains one indicator of bias Mean Error (ME), and one indicator assessing how well the prediction intervals capture the real distribution of the observed data (Mean Interval Score, MIS).

Recommendations

Recommendation: Enhance the methodological process by incorporating expert judgment and granular knowledge of the NRBT experts.

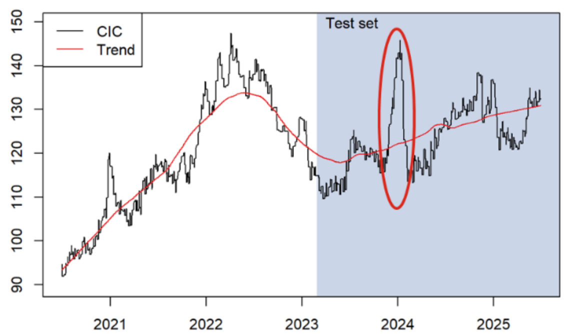
44. **The NRBT should have dedicated staff fine-tune the liquidity forecasting framework and analyze market developments on an annual basis**. The methodology of the forecasts should incorporate expert judgment and granular knowledge of the NRBT liquidity experts.

[4] See Chen, Z., N. Kourentzes, R. Lafarguette., A. Panagiotelis, and R. Veyrune. (2022) Liquidity Forecasting—Part II: The Statistical Component in Monetary and Capital Markets Department: Technical Assistance Handbook

C. Liquidity Forecasting Application

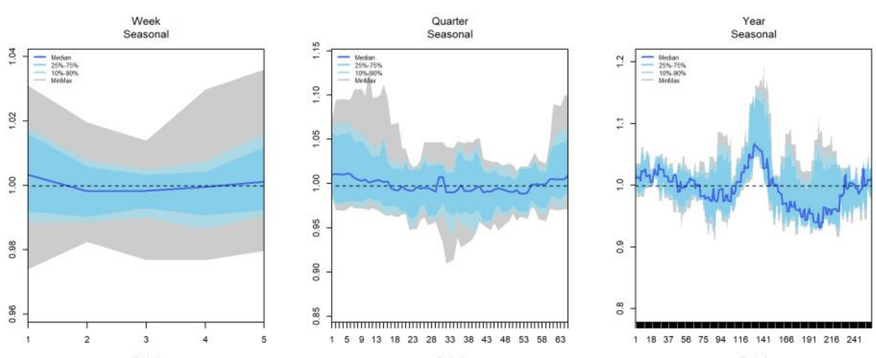
45. **The mission forecasted total currency in circulation (CiC) with two components: banknotes and coins.** While CiC generally follows an upward trend, a sharp spike followed by a decline is observed between late 2023 and early 2024, as observed in Figure 4. This temporary surge reflects the central bank's issuance of the new banknote series, which initially boosted demand for physical currency as financial institutions, and the public exchanged old notes for the new series. The subsequent decline represents the withdrawal of the old banknotes from circulation, as the replacement process settled. In addition, the analysis identifies distinct seasonal patterns in CiC at the weekly, quarterly, and annual frequencies (Figure 5).

Figure 4. Historical Trend of Currency in Circulation (TOP Million)



Source: NRBT, IMF staff calculations.

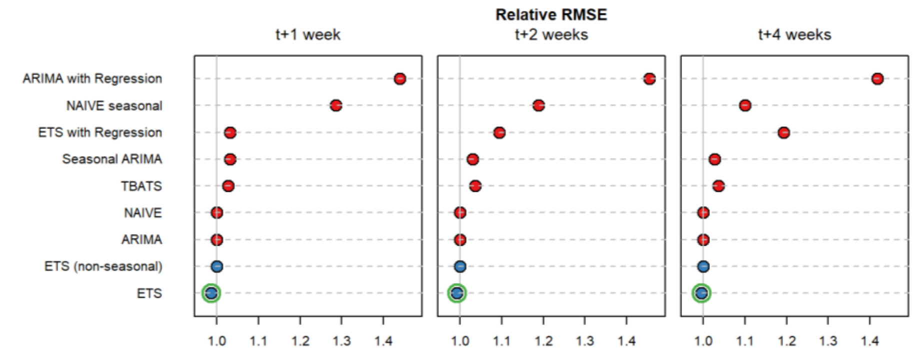
Figure 5. Seasonality Analysis of Currency in Circulation



Source: NRBT, IMF staff calculations.
Notes: Y axis is detrended time series.

46. **The model validation fits the data from 2024-12-17 to 2025-06-02.** Among the candidate specifications, the ETS model demonstrated the lowest out-of-sample forecast errors and marginally better performance relative to alternative models. However, the differences across models were not statistically significant when compared with the benchmark Naïve random walk forecast. For transparency, the best forecast at each horizon is indicated with a green circle. Where a Naïve benchmark forecast is available, model errors are reported relative to this, with a value of one indicating performance equal to the random walk. Forecasts less accurate than the Naïve benchmark should not be considered reliable.

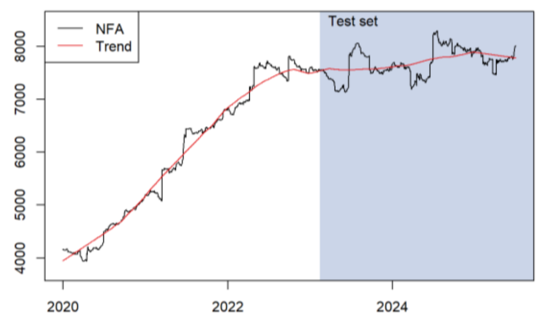
Figure 6. Predictive Accuracy of Forecasting Models for CiC



Source: IMF staff calculations.

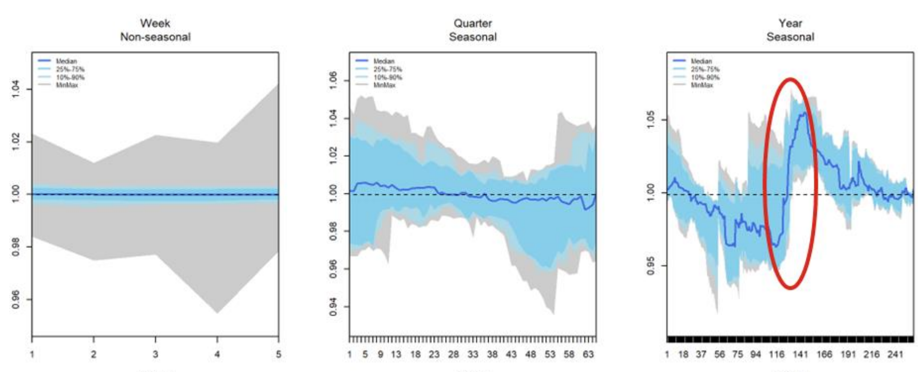
47. **NFAs have exhibited an upward trend, accompanied by pronounced seasonal fluctuations in Figure 7.** Since late 2022, however, the growth has leveled off, reflecting the resumption of foreign debt service after the Covid pandemic and higher financing needs post natural disasters. As shown in Figure 8, seasonal increases are evident around mid-year, driven by budget disbursements in June and donor support at the start of the fiscal year in July.

Figure 7. Historical Trend of Net Foreign Assets (TOP Million)



Source: NRBT, IMF staff calculations.

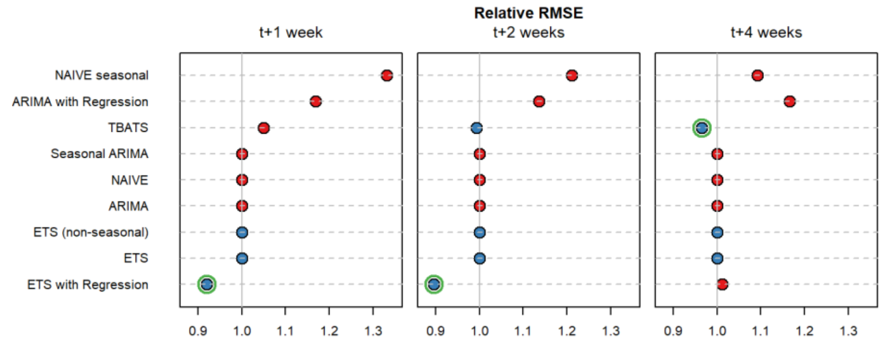
Figure 8. Seasonality Analysis of Net Foreign Assets



Source: NRBT, IMF staff calculations.
Notes: Y axis is detrended time series.

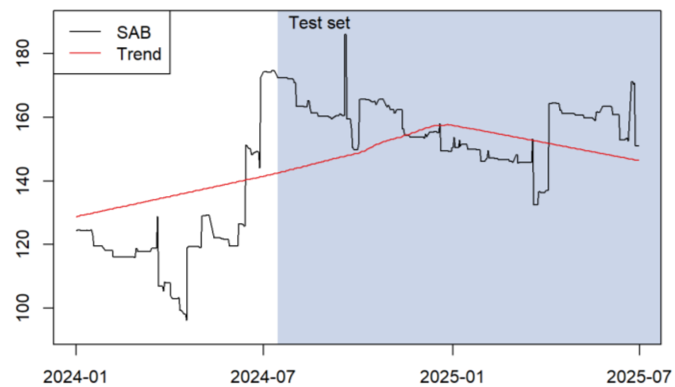
48. ETS with regression gave the best NFA forecasts at one- and two-week horizons. On the other hand, the TBATS had the lowest RMSE at four weeks under the same testing period from 2024-12-17 to 2025-06-02 (see Figure 9).

Figure 9. Predictive Accuracy of Forecasting Models for NFA



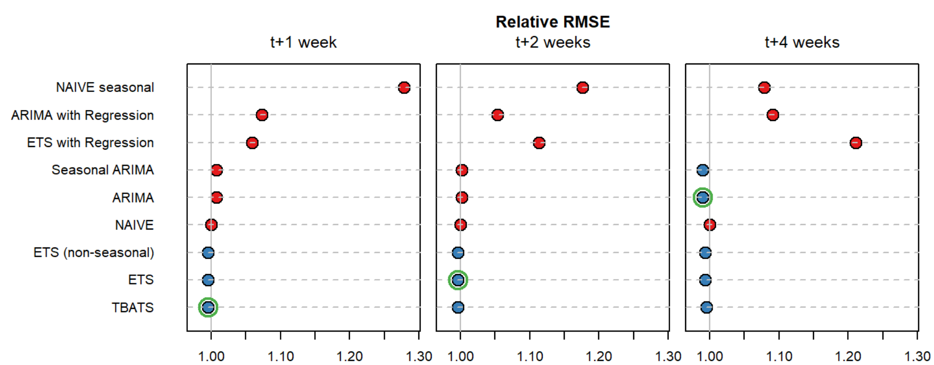
Source: IMF staff calculations.

Figure 10. Historical Trend of Government Account Balance at the NRBT (TOP Million)



Source: NRBT, IMF staff calculations.

Figure 11. Predictive Accuracy of Forecasting Models for SAB



Source: IMF staff calculations.

49. **The daily balance and flows to and from the government account balances held at the NRBT are not adequately captured and monitored by the NRBT.** Transactions with the government accounts held outside of the NRBT are not captured daily, as this is currently done on a weekly basis. The mission created a daily balance that captures this information using daily receipts and payments, however, there is no systematic process in place to routinely capture this information and create a daily balance. Figure 10 also shows that the constructed data series is erratic, with no discernible trend or seasonality. This leads to uncertainty and reduces the value for statistical forecasting. Model evaluation in Figure 11 indicates no significant difference between the chosen model and the Naïve benchmark.

Recommendations

Recommendation: Monitor the daily flows to and from the government account held at the NRBT and systematically capture daily balances.

50. **NRBT should ensure that the daily transactions to and from the government account balances at the NRBT are monitored and captured systematically.** Daily government account balances should be adequately captured for forecasting this series in the liquidity forecasting framework. A systematic process would improve data management, leading to increased certainty of the trend analysis and the strengthening of the forecasting process.

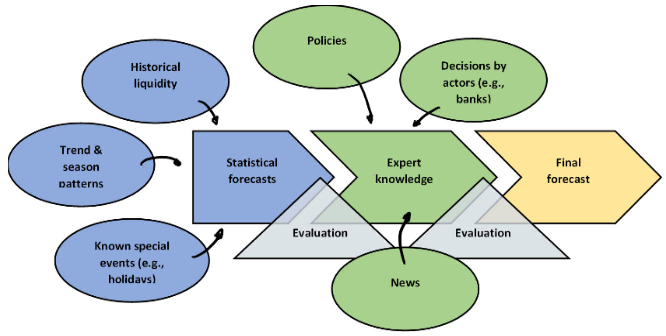
51. **Forecasts for 'other items net' should also be included in the total liquidity forecast as soon as this data is available.** For now, these forecasts can be assumed to be constant, since they reflect the residuals on the central bank's balance sheet. If net other items change significantly, however, the central bank may need to separate an additional autonomous factor to represent this series.

D. Liquidity Table and Calibration

52. **As the excess liquidity has not yet been absorbed, the NRBT does not apply a liquidity table to conduct its calibration of OMOs.** Having said that, the NRBT should ensure that this capacity is developed in anticipation of the liquidity absorption and the reactivation of the interbank market. To this end, the mission provided extended guidance for establishing a liquidity table and for OMO calibration.

53. **The mission also provided guidance, including simulations, on estimating the demand for reserves as part of the MCM framework.** Given that interbank rates are not yet available, the estimation of the reserve demand curve cannot be implemented immediately, but this should be promptly implemented as the data emerges. The mission provided a detailed explanation of this process. By calculating the difference between the liquidity supply and the demand for reserves, the resulting liquidity gap guides the allotment decisions for the volume of OMOs.

Figure 12. Incorporating Statistical Forecasts to Final Forecast



Source: IMF staff.

Recommendations

Recommendation: Forecast each autonomous factor and net liquidity position individually using the IMF framework, applying dynamic selection and averaging to enhance accuracy and lower modeling risk.

54. **The mission recommends that the NRBT estimates the model and start forecasting daily autonomous factors.** The model should be run on a weekly basis**.** This approach will ensure that the NRBT has accurate and timely data to identify the liquidity supply.

Recommendation: Collect market intelligence and exchange information during weekly committee meetings to adjust for the baseline statistical forecasts generated from the forecasting framework.

55. **NRBT should adjust baseline statistical forecasts generated from the forecasting framework (see Figure 12).** Additionally, the mission suggests evaluating the model selection periodically (e.g., monthly or quarterly) to ensure the best model is used, incorporating the latest data inputs.

56. **Furthermore, it is useful to have a forecast evaluation framework, where the accuracy of the forecasts is measured periodically.** This framework should include a detailed liquidity table that captures discrepancies between forecasted and actual liquidity levels. By systematically monitoring these forecast errors, the NRBT can identify patterns and potential sources of inaccuracies, allowing for continuous improvement of the forecasting model. This iterative process will enhance the precision of forecasts, ensuring that the NRBT can make more informed decisions regarding open market operations and other monetary policy tools. A robust liquidity monitoring framework will also contribute to greater transparency and accountability in the NRBT's liquidity management practices, fostering confidence among market participants and supporting overall financial stability.

Recommendation: Publish the liquidity forecast.

57. **Once the NRBT has applied the MCM framework to the NRBT balance sheet and has refined the accuracy of the model, the forecasts should be published.** Liquidity forecasts are essential under both FRFA and VRFA operations. Under FRFA operations, NRBT should publish the forecast to inform counterparties' bidding at the FRFA operations. Under VRFA operations, forecasts would be used to calibrate the OMO operations, and these forecasts would be

published. As such, the establishment of an LFU, along with an enhancement of the capacity to forecast liquidity, would be important in the near term.

III. Monetary Policy Implementation Status

58. **This mission serves as a follow-up to the January 2025 mission.** As such, the mission assessed the implementation status of the recommendations of the January 2025 mission. In this context, Table 4 provides a detailed assessment of the implementation status. This mission also provided further guidance to the NRBT based on the progress achieved in implementing the TA recommendations. Figure 12 shows the sequencing of measures that outline all steps to be taken for issuing NRBT Notes. The sequencing should start with conducting preparatory measures, which leads to the issuance of NRBT Notes at FRFA (Stage 1). Subsequently, the NRBT would develop the capacity to forecast liquidity, which leads to the issuance of NRBT Notes at VRFA Stage 2).

59. **The NRBT has made good progress in implementing the recommendations.** The progress in implementation reflects the NRBT's strong commitment to modernizing its operational framework as guided by the January 2025 TA. All recommendations programmed for the near term have been completed and all other recommended actions have started (see Table 4).

60. **Of note, there are two recommendations that may require further discussions to remove potential impediments (see Table 4).** First, the operationalization of the ELA framework may require technical assistance on the implementation of sound valuation methodologies for collateral valuation, (including risk mitigation measures), as well as guidance on the development of the ELA policy and framework. Second, the implementation of a Treasury Single Account (TSA) framework (as part of the movement of Government accounts to the NRBT) has not progressed. The NRBT should have further discussions with the Government of Tonga and assist with setting up a study on its implementation.

Figure 13. Sequenced Measures for Launching Weekly Issuance of Seven-day NRBT Notes

Stage 1. Launching weekly issuance of 7-day NBRT Notes at fixed rate and full allotment tenders

Stage 2. Launching weekly issuance of 7-day NBRT Notes at variable rate, fixed allotment tenders

	Preparatory measures		Ongoing measures, including preparatory measures for Stage 2	
Analytical and Forecasting Capacity	Develop NBRT capacity to assess the level of the NRBT Policy Rate	**Stage 1**	Ongoing strengthening of NRBT analytical capacity to assess the level of the NRBT Policy Rate	**Stage 2**
	NBRT Departments provide analytical information to the Monetary Policy Committee-MPC on the adequate level of the NRBT Policy Rate			
	Start developing a short-term liquidity monitoring and forecasting framework based on the autonomous factors of bank liquidity		The ST liquidity monitoring and forecasting framework is robust and allows ***calibration*** of weekly issuance of 7-day NRBT Notes	
Decision Making Process	MPC recommends level of Policy Rate to NRBT Board of Directors		Monetary Policy Committee recommends a level of the NRBT Policy Rate	
	NBRT Board of Directors sets the level of the NRBT Policy Rate in a dedicated Resolution to be published		NBRT Board of Directors sets the level of the NRBT Policy Rate in a dedicated Resolution to be published	
	MPC recommends a structure for a mid-corridor interest rate corridor (IRC): Main liquidity management operation-MLMO; Marginal Lending Facility; Marginal Deposit Facility, width of the IRC		MPC recommends changes to the width of the IRC as needed.	
	NBRT Board of Directors adopts IRC framework in a dedicated Resolution		NBRT Board of Directors adopts changes in the width of the IRC as needed	
Communication	NRBT Resolutions are published in a timely fashion		NRBT Resolutions are published in a timely fashion	
	NRBT relevant departments hold meetings with the banks to explain the new monetary policy implementation framework		NRBT relevant departments hold periodic meetings with the banks to discuss issues related to monetary policy implementation	
	The Monetary Policy Statement-MPS contains detailed information on the new monetary policy implementation framework		Periodic MPS provide information on outcomes of monetary policy implementation and information on changes to the framework	
Technical Measures	Set up Liquidity Unit responsible for developing ST liquidity monitoring and forecasting framework		Ongoing analysis of the differences between the liquidity forecasts and actual outcomes	
	Set up a Liquidity Committee to discuss the forecasts prepared by the Liquidity Unit (weekly meetings ahead of 7-day NRBT Notes issuances)			
	Engage in discussion with Treasury for exchange of cash flow forecasts		Ongoing exchange of information on government cash flow forecasts	
	Engage in discussion with the Treasury related to the deployment of the Treasury Single Account-TSA framework		Continue discussion with the Treasury on the deployment of the TSA until the framework is fully implemented	
Supporting Measures	Collect daily data on interbank market transactions		Ongoing collection of data on interbank market transactions	
	Collect data on banks' deposit and lending rates		Continue collection of data on banks' deposit and lending rates	

Table 4: Implementation Status of TA Recommendations.

Recommendation	Priority	Timeline	Action Taken
		Monetary Operational Framework	
Consider a gradual modernization of monetary policy implementation by setting up an active liquidity management framework to allow the NRBT to steer short-term money market rates to its announced policy rate.	High	Near-term	1. The NRBT Board approved a budget as well as the creation of the liquidity monitoring unit. Recruitment for new LFU staff has been completed. 2. Discussions with banks to introduce fixed-rate full-allotment operations are ongoing. 3. NRBT has announced the new monetary policy operational framework in the August Monetary Policy Statement. 4. NRBT will be issuing NRBT Notes in September 2025. 5. The government is looking into implementing a TSA framework. 6. The process of revamping the reporting template for the interbank market is ongoing. 7. IMF has provided guidance on a standard framework for determining the level of the policy rate 8. Institutional changes for monetary policy decision making are ongoing. TORs for the MPC (all NRBT relevant departments) need to be formalized. There is, however, no need for NRBT Law amendments.
Start monetary policy modernization by Introducing a conventional mid-rate IRC with fixed rate and full allotment tenders.	High	Near-term	1. NRBT Board has approved the introduction of a fixed-rate full allotment framework. 2. Standing lending facility (SLF) was already available. The NRBT Board has approved the IRC with a zero rate on deposits, and +200 bp for the SLF. 3. Currently, the bidding process is paper based, but improvements are ongoing. IT department is working on a modernized framework.
Move to mid-rate IRC with variable rates and fixed allotment tenders once a liquidity monitoring and	Medium	Medium-term	Ongoing.

forecasting framework is fully operational.			
Engage discussions with the banks to assess what specific actions the NRBT could take to support interbank market activities.	High	Medium-term	Completed.
Adopt a wide IRC at the start**.**	High	Near-term	Completed.
Strengthen liquidity monitoring and forecasting by developing a capacity to analyze and forecast the autonomous factors of liquidity (F) on a weekly basis.	High	Medium-term	1. The LFU is being established and adequately staffed. 2. This mission has provided guidance on liquidity forecasting.
NRBT should recommend to the Treasury to stop placing time deposits in the commercial banks, and to engage in a study to assess the feasibility of setting a TSA framework.	High	Near-term	1. Contacts with the government were made. The government is considering implementing the TSA, but a feasibility study has not yet been planned. 2. The government has agreed to move some of their deposit account funds to the NRBT. 3. There are some risks to implementation: election in November and the Minister who agreed may no longer be there.
Develop operational guidelines for the activation of an ELA framework.	High	Medium-term	1. Nothing specific has been done. The NRBT Act needs to be amended, which was already programmed as part of the current revamping of NRBT Act. However, this process will need to wait until a new Government is appointed.
Engage in discussions with the Treasury for the exchange of cash flow forecasts.	High	Near-term	1. Ongoing. No meeting has occurred between the NRBT and the government's technical staff. NRBT is attempting to develop an MOU with the government.
Rely on the SRD to adjust the structural liquidity position. Consider remunerating the SRD. Consider introducing averaging over the reserve maintenance period and linking noncompliance penalty to the policy rate.	High	Near and Medium-term	1. The NRBT has financial capacity concerns for remunerating the SRD, but Board approval has been achieved for absorbing the excess liquidity. 2. SRD averaging will be implemented within the recommended timeframe.
Collateral Policy			
Remove the preference order for monetary policy collateral and undertake necessary actions to	Medium	Medium-term	1. Completed.

standardize the mobilization procedures for assets.			
Develop sound valuation methodologies to mark collateral to market.	High	Medium-term	1. Ongoing.
Coordinate with the Government of Tonga to issue bonds (consistent with the Government's financing priorities) at maturities that could help calibrate a term-structure sovereign yield curve.	Medium	Medium-term	1. No progress. Discussions have started with the Government.
Develop and implement risk mitigation measures, including haircuts that capture key financial risks such as market, liquidity, and credit risk.	High	Medium-term	1. No progress. NRBT plans to request ELA TA, which will cover risk mitigation techniques.
Communication			
Develop a formalized institutional policy for communications along with key complementary frameworks.	High	Near-term	1. Ongoing. Institutional policy has been drafted.
Develop a Financial Stability Report.	High	Medium-term	1. Completed. The Financial Stability Report has been finalized (with assistance from IMF) and approved for publication. Further support may still be requested from IMF going forward.
Clearly communicate, following international best practice, the implementation of the proposed monetary operational framework.	High	Near-term	1. Completed.
Expand outreach and adopt multitiered communication for monetary policy communication.	Medium	Medium-term	1. Ongoing. New communication instruments are being developed.
Enhance the effectiveness of communication by: (i) improving readability indices of monetary policy decisions by reducing the wording complexity; (ii) producing more focused monetary policy statements in terms of topics; and (iii) integrating more forward-looking elements in monetary policy statements.	Medium	Near and Medium-term	1. Completed. The guidance provided in the TA report has been used to gauge all communications.

Conclusion

61. **The implementation of the mission's recommendations will support effective monetary operations and transmission.** To significantly upgrade the NRBT's liquidity forecasting capacity, the mission institutionalized the statistical forecasting framework developed by MCM. The mission also provided guidance on implementing optimal institutional arrangements to support the NRBT's liquidity forecasting function. It is essential that the NRBT understands and anticipates the behaviors of both its monetary and non-monetary policy counterparties. Once the recommendations are implemented, the NRBT will be able to effectively forecast liquidity and calibrate OMOs effectively.

62. **The mission recommends that the NRBT start the process of establishing a formal liquidity forecasting capacity while relying on OMOs with FRFA issuance and NRBT Notes.** A standalone LFU should be established to manage and execute this process and pave the way for VRFA issuance of NRBT Notes. Developing a comprehensive daily liquidity database should be pursued to adequately underpin the liquidity forecasting process. The NRBT should apply the MCM methodology for forecasting liquidity. All autonomous factors and net liquidity should be forecasted with the process continually refined to lower the modeling risk under FRFA operations. Once the NRBT is confident in publishing the forecasts, the forecast could be used for calibrating OMOS under VRFA.

63. **The MCM framework enables model selection, forecast generation, and forecast reconciliation to enhance forecast accuracy.** The mission thoroughly explained these components to NRBT staff, conducted staff training that employed simulations, and provided code packages for forecasting autonomous factors and estimating reserve demand. Once the framework is implemented, this would enhance monetary transmission by steering interbank rates, as they emerge, to an established policy rate.

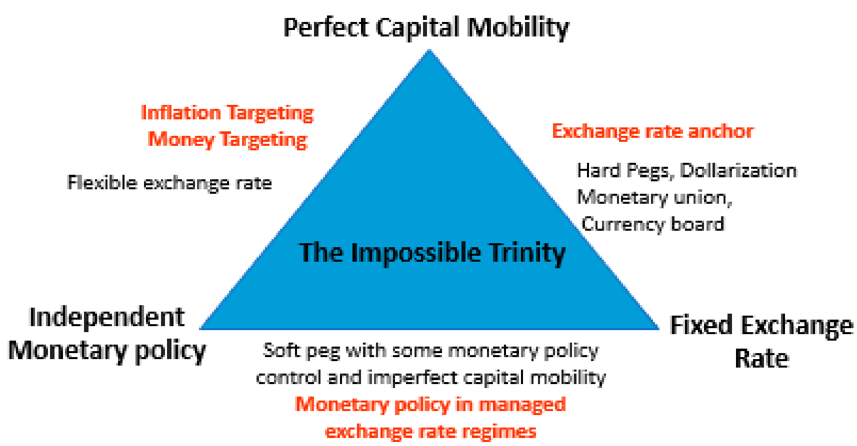
64. **Guidance was provided to enable the NRBT to establish an appropriately determined policy rate to support the peg**. The implementation of this approach will provide adequate market-based support for the sustainability of the exchange rate peg. Once this framework is gradually implemented, this will safeguard the stability of market conditions during the implementation phase, thereby ensuring a smooth transition towards a UIP derived policy rate. The mission also noted that as an alternative, the NRBT could apply a reaction function based on domestic conditions, but this would involve maintaining the capital controls in place.

65. **Follow-up missions would allow IMF staff to support NRBT with the implementation of other supporting measures**. Future TA can also be requested to assist with: the estimation of the demand for reserves in the liquidity framework; central bank stress testing for evaluating policy solvency; and a Central Bank Transparency (CBT) review.

Annex I. Conceptual Framework for Liquidity Management under a Pegged Exchange Rate Arrangement and Perfect Capital Mobility

Under a fixed exchange arrangement, the ultimate objective of monetary policy is price stability, and the intermediate objective is exchange rate stability. The exchange rate plays the role of nominal anchor as the central bank fixes the price of the local currency against another currency or a currency basket and does not control the volumes of its foreign exchange (FX) interventions. Under the currency board rule, the central bank can issue the local currency liabilities only against FX reserves, whereas under fixed exchange rate regimes, the central bank is usually required to maintain a minimum statutory coverage ratio.

Figure 1. Monetary Policy, Exchange Rate, and Capital Mobility

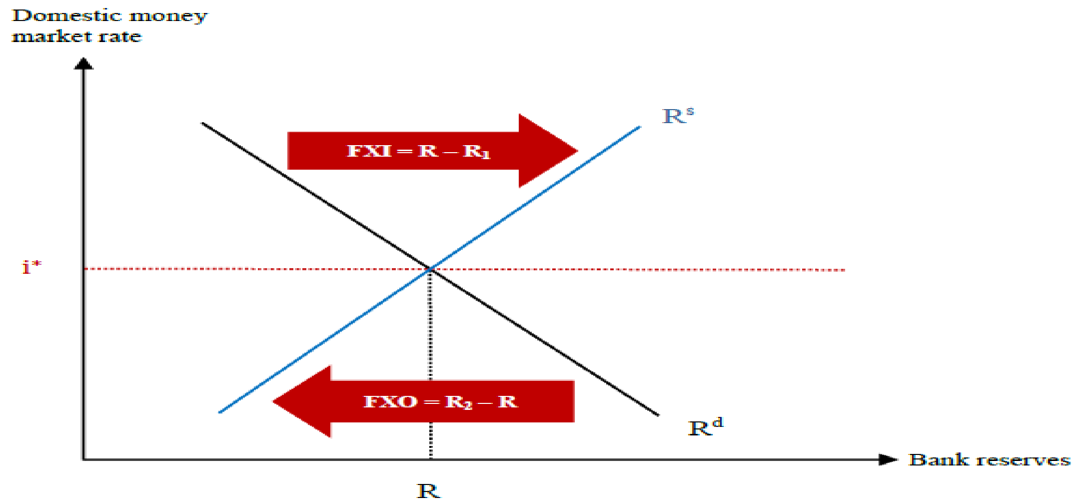


Source: IMF.

In theory, under a fixed exchange rate arrangement and perfect capital mobility, liquidity management relies essentially on FX flows. In a system where banks can buy and sell FX freely without any attached costs[1] to converting local currency in FX and conversely, and without any country and counterparty risk, any decrease in banks reserves at the central bank at a level below equilibrium will lead to an immediate increase in domestic short-term rates that immediately trigger FX inflows which will exactly *offset* the shortage bringing the level of bank reserves back to equilibrium. Similarly, any increase in bank reserves will trigger FX outflows which will offset the increase in bank reserves bringing it back to the equilibrium level. Hence, in the absence of frictions, any shortage or excess in bank reserves is offset by FX flows in a seamless fashion and does not rely on the central bank intervention in the money markets.

[1] Neither in term of transaction costs, bid ask spread, nor in FX settlement value dates.

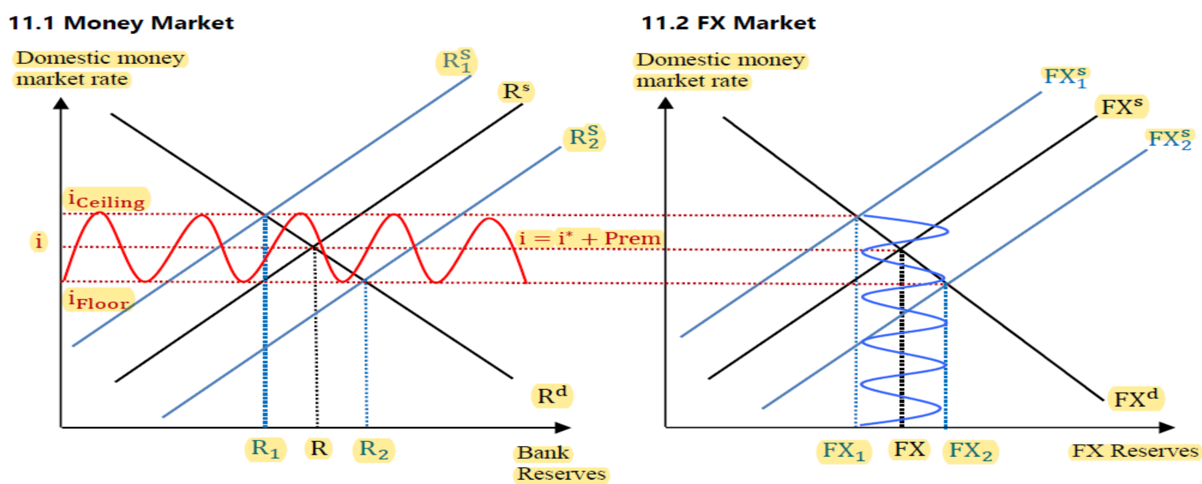
Figure 2. Automatic Adjustment through Capital Flows under the Assumption of No Frictions



Source: El Hamiani M. and Veyrune R. (2019).

Nevertheless, the presence of frictions in the money and FX markets hampers the automatic liquidity adjustment and lead to excessive volatility of the domestic interest rate around the level implied by the Uncovered Interest Rate Parity (UIP) condition. Frictions arise inter alia from risk premia, counterparty limits, liquidity and capital prudential regulations, and FX transactions costs. They impact the speed of adjustment to various types of liquidity shocks such as Term of Trade shocks, changes in an autonomous factor of liquidity affecting the supply of reserves or change in the demand for reserves. As a result, the domestic money market rate, FX reserves, and bank reserves at the central bank will revert to their initial equilibrium position. However, in the meantime, volatility in all these variables may have been large depending on the size of the shock.

Figure 3. Money and Foreign Exchange Markets Dynamics in Response to Various Liquidity Shocks



Source: El Hamiani M. and Veyrune R. (2019).

Ultimately, friction costs and other impairments of seamless liquidity adjustment could un-anchor short-term money market rates and even jeopardize financial stability. Volatile liquidity conditions can increase the precautionary demand for reserves, leading to an increase in funding costs and lending rates. They can also undermine market development and endanger financial stability. In the case of adverse Term of Trade shocks or sudden capital outflows, higher domestic interest rates may not trigger enough capital flows in the short-term to offset the liquidity shocks. In extreme circumstances of sizeable liquidity shortfalls, domestic interest rates may have to remain high, potentially leading to liquidity pressures in the banking sector, which could also affect banks' solvency. On the other hand, if capital outflows do not respond to low domestic short-term rates, excess reserves may become persistent, which deters the development of the money market and, in turn, limits the market capacity to absorb idiosyncratic liquidity shocks.

Hence, there is a need for central bank intervention to stabilize short-term interest rates at the appropriate level which would keep the foreign reserves at the central bank stable over the medium term. Under a pegged exchange rate and no capital controls, the specific challenges for liquidity management are to: (i) identify fundamental changes in the exchange rate equilibrium without absorbing them; and (ii) identify and absorb the volatility that does not represent a fundamental change in the exchange rate equilibrium. Hence, the central bank's liquidity management aims at reducing interest rate volatility, by providing room for intertemporal smoothing of liquidity shocks. The central bank can either opt for a market-based policy rate in line with the interest rate setting or can set its policy rate in line with the UIP condition. The choice between the two options depends on whether the risk premia can be estimated and whether the market or the central bank is in the best position to determine the allotment amount.

Under a market-based interest rate setting, the central bank will implement a floating interest rate corridor centered around a variable rate and fixed allotment main open market operation. In presence of structural excess liquidity, the central bank issues a fixed amount of debt securities which rate is allowed to fluctuate with the demand and supply of reserves in the money market reflecting the capital flows. The fixed amount is usually derived from the forecasts of the autonomous factors and the estimation of the demand for reserves. As this may still lead to high-interest rate volatility, frequent operations and reserve requirement averaging are needed to smooth minor and temporary liquidity shocks. The reserve requirement is higher than in other arrangements due to large and less predictable autonomous factors swings. Standing facilities are set at a margin of the open market operation average rate. The spread between the liquidity management facilities, if they are introduced, is usually rather large as it should be greater than the friction costs to avoid preventing the automatic adjustment.

Alternatively, the central bank will set the interest level that is balancing the FX flows in line with the implied level from the UIP or with the expected policy rate of the country to which it is tied to under a forward-looking approach. Under this set-up, a misalignment between the interest rate set by the central bank and the equilibrium interest rate could occur. If the policy rate is set too low, the peg could become under pressure if counterparties use the standing credit facility to purchase FX against domestic currency collateral and speculate against the currency. If the rate is set too high, the central bank balance sheet inflates, which represents a fiscal cost, and excess liquidity will hamper market development. The central bank can implement a mid-corridor with a fixed-rate full-allotment open market operation or a floor-rate corridor set at the policy rate. Same considerations for the width of the interest rate corridor, frequency of operations, reserve requirement level and averaging apply.

Annex II. Model-based Approach for Forecasting the Risk Premium

1. **The modelling of the risk premium relies essentially on the identification of the "push" and "pull" factors that are driving the FX flows for the country.** FX flow dynamics can largely be explained by multiple factors which could be split between country-specific determinants (pull factors) such as economic activity, level of indebtedness, currency and political stability, regulatory framework, and other common global determinants (push factors) such as geopolitical conditions, liquidity and market access, perceptions of risk, and global economic conditions, among others. Not all these factors are quantifiable in a reasonably objective way, nor are all equally relevant to Tonga's Islands' case.

2. **In modeling the risk premium there are two main steps: identifying an appropriate target variable for the risk premium to model (Table 1), and identifying which factors explain its movements.** The analysis can be repeated at different data frequencies; monthly data was used for our analysis. Daily data could be used, and testing for seasonal differences would be advisable to account for any daily or weekly seasonality.

3. **The mission developed a collection of relevant push and pull factors, which serve as regressors to model the risk premium.** The regressors considered in this analysis are shown in Table 2. All regressors are differenced as needed to become stationary.

Table 2. Regressors for Modelling Risk Premium

	Factors	Frequency
1	VIX	Daily
2	Oil prices	Daily
3	Exchange rate (USD/TOP)	Daily
4	Total government revenues	Quarterly
5	Net Foreign Assets	Monthly
6	Inflation	Monthly
7	Nominal GDP	Quarterly

Source: IMF staff.

The full regression model is:

$$r_t = \beta_0 + \sum_{i=1}^{l} \beta_i r_{t-i} + \sum_{k=1}^{p} \sum_{j=1}^{l} \beta_{l+i(k-1)} z_{k,t-l} + \eta_t, \quad (2)$$

where β_0 is the intercept, the term $\sum_{i=1}^{l} \beta_i r_{t-i}$ accounts for up to l autoregressive lags of r_t, the term $\sum_{k=1}^{p} \sum_{j=1}^{l} \beta_{l+i(k-1)} z_{k,t-l}$ accounts for up to l lags of each of the p regressors listed above, and η_t is the error term of the model. The full model in (2) is simplified iteratively by removing at each step the variable that improves the AIC of the model the most. This process is repeated until the AIC cannot be improved further, or until only the β_0 remains in the model. Moreover, it is strongly recommended that the resulting regression is validated in terms of economic theory with respect to the included lags and coefficient values.

4. Once the target variable has been established, this is modelled using a collection of relevant variables to the risk premium. These explanatory variables can be collected at any sampling frequency. If those are in a higher frequency than the target variable, then these can be subsampled to the same frequency. For instance, a daily series can be sampled only at a monthly rate to produce a monthly series. If the higher frequency time series is very irregular in its sampling, it is preferable to use monthly averages of that series. If a variable is sampled at a lower frequency than the target, for example at a quarterly rate, then this variable is linearly interpolated to the monthly level.

5. Once the model is estimated, the forecasting procedure can be commenced. This will involve saving the coefficients from the estimated model and projecting the risk premium (proxied by the interest rate spread) using future values of the regressors. Future values of the regressors could be derived from scenario-based assumptions.

Annex III. Technical Details on the IMF Liquidity Forecasting Framework

1. **The Naïve model (random walk) is often used as the simplest forecast benchmark.** The Naïve forecast assumes that the time series has no structure, while at the same time requires no parameter estimation or any other modelling choices. The forecast is generated as:

$$\hat{y}_{t+h} = y_t,$$

Where y_t is the observation at time t, $\hat{y}_{t+h}$ is the forecast for period $t+h$, and h is the forecast horizon. The intuition behind the Naïve forecast is that all forecasted values are equal to the last observation, and therefore there is no additional information to model. Arguably, this is an inappropriate model to forecast liquidity, but it does make it a useful benchmark. More complex modelling approaches are often not transparent or intuitive enough in what they do. Therefore, at a minimum, they must outperform such a simple forecast. A helpful modification of the Naïve is its seasonal counterpart, where instead of repeating the last observation, the last seasonal period is repeated:

$$\hat{y}_{t+h} = y_{t-s+h},$$

Where s is the seasonal period, corresponding to the number of days in the week (5 without the weekend).

2. **Each observation in a time series contains both structure and noise.** The structure makes up the part of the series that can be modelled and used to inform our forecasts. The noise part is inherently random and unforecastable. Let y_i be an observation of a time series at period i, and:

$$y_i = \mu_i + \varepsilon_i,$$

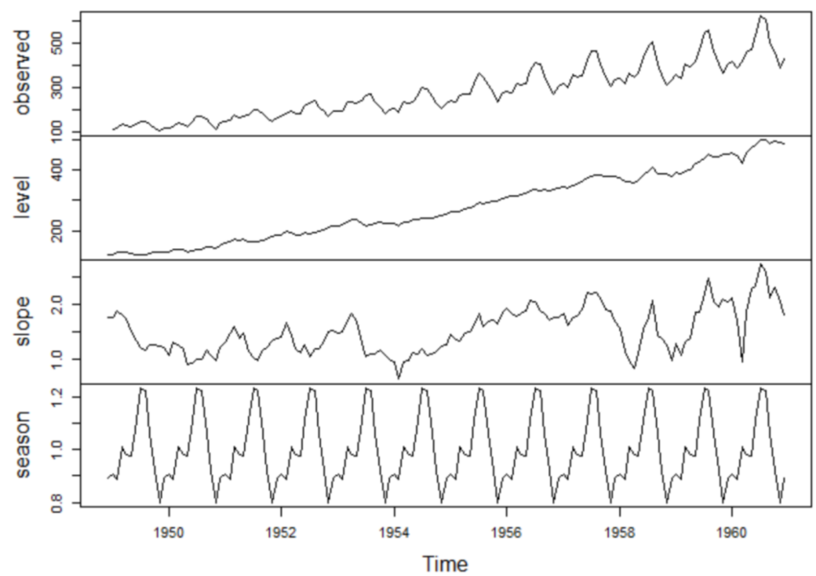
Where μ_i denotes the structure of the time series and ε_i the randomness. In forecasting, the main challenge is to identify a model that can separate the structure from noise, as well as to correctly characterize the patterns in the structure (e.g., slope, seasonality). If the forecasting model is appropriate for the data, the noise part should have no patterns and be random. Therefore, the noise can only be characterized in terms of the statistical distribution it follows. Usually, it is assumed to follow the normal distribution, and therefore $\varepsilon \sim N(0, \sigma^2)$. In other words, the noise is normally distributed with zero mean and standard deviation σ. A well-specified model should provide a function for μ_i as well as an estimated $\hat{\sigma}$. Note that neither μ_i nor ε_i are observable, and therefore it is the task of the modeler to specify a forecasting model that clearly separates them from the observed y_i.

3. **ETS models operate by modelling the time series as a collection of patterns, namely level, trend, and seasonality.** Usually, ETS is framed within a state-space model, where each component of the time is a state, and together they produce the forecast $\hat{y}_i$, as:

$$\hat{y}_i = f(\hat{\mu}_i, \varepsilon_i)$$
$$\hat{\mu}_i = g(level_i, slope_i, season_i)$$

The functions $f(\cdot)$ and $g(\cdot)$ can be either additive, multiplicative, or have some mixed form. Figure 1 below provides an example of the decomposition of a time series into separate components by exponential smoothing. Observe that the level, slope, and season components together can explain most of the time series, with any unexplained part attributed to the noise component. The level tracks the local mean of the time series, while the slope models how the level increases or decreases over time (e.g., a slope of +2 suggests an upward movement by two units per period). Finally, the season component models any periodic patterns in the data. Not all time series require all components to be modelled, as some may be absent.

Figure 1: An Example of the Decomposition



Source: IMF mission's calculation.

In the fully additive case, the model becomes:

$$\hat{y}_i = \hat{\mu}_i + \varepsilon_i$$
$$\hat{\mu}_i = level_i + slope_i + season_i$$

Each of the states ($level_i$, $slope_i$, and $season_i$) is structured similarly. For example, the additive $level_i$ is:

$$level_i = level_{i-1} + \alpha e_{i-1},$$

Where α is a smoothing parameter between 0 and 1 and e_{i-1} is the previous period error. Intuitively, this equation suggests that the current level estimate is updated by α times the last error. Given that the error is the difference between the actuals (y_i) and the forecast $(\hat{y}_i)$ for the case of the ETS that has only an additive level, the model can be written in two alternative forms to help explain its function:

$$\hat{y}_i = \hat{\mu}_i + \varepsilon_i$$
$$\hat{\mu}_i = level_i$$
$$level_i = level_{i-1} + \alpha e_{i-1}$$

Or equivalently:

$$\hat{y}_i = \hat{\mu}_i + \varepsilon_i$$
$$\hat{\mu}_i = level_i$$
$$level_i = \alpha \cdot actuals_{i-1} + (1 - \alpha) level_{i-1}$$

The second set of equations suggest that the smoothing parameter α decides by how much to update the previous level with the last observed actuals. Noting that $0 < \alpha < 1$, a percentage contribution interpretation becomes possible. For example, if $\alpha = 0.2$, the last estimated level is updated by 20 percent of the last observation. All other states operate similarly, requiring an additional parameter for each additional state, and a model may have any of these states on their own or together.

4. **A useful way to see how ETS operates is to consider the equations across time, which makes each component an exponentially weighted moving average.** For example, using $\alpha = 0.2$, the most current actual is weighted by 0.2, while the one before is weighted by $\alpha(1 - 0.2)^2$. The calculation becomes apparent if we replace (in the equations above) the $level_{i-1}$ with its respective state equation:

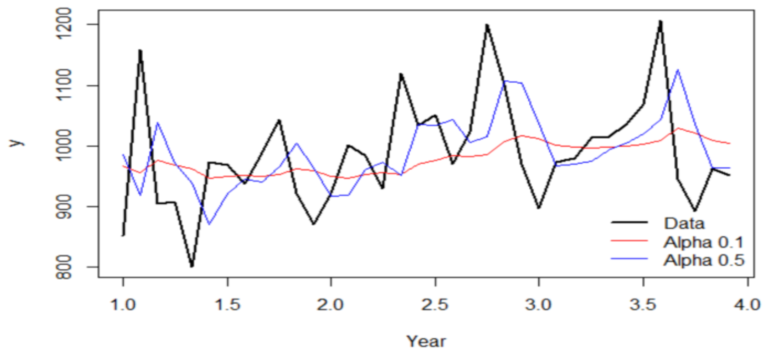
$$level_i = \alpha \cdot actuals_{i-1} + (1 - \alpha) level_{i-1}$$
$$level_{i-1} = \alpha \cdot actuals_{i-2} + (1 - \alpha) level_{i-2}$$

$$\rightarrow$$
$$level_i = \alpha \cdot actuals_{i-1} + (1 - \alpha)(\alpha \cdot actuals_{i-2} + (1 - \alpha) level_{i-2})$$

$$\rightarrow$$
$$level_i = \alpha \cdot actuals_{i-1} + \alpha \cdot actuals_{i-2} + (1 - \alpha) level_{i-2} - \alpha^2 \cdot actuals_{i-2} - \alpha(1 - \alpha) level_{i-2}$$
$$level_i = \alpha(1 - \alpha)^0 \cdot actuals_{i-1} + \alpha(1 - \alpha) actuals_{i-2} + (1 - \alpha)^2 level_{i-2}$$

and so on. More generally the jth previous observation is weighted by $\alpha(1 - \alpha)^{j-1}$. All weights will by construction be between 0 and 1, and sum up to 1, forming a weighted moving average. Therefore, each state in ETS models is a component of the time series (level, slope, or season) and achieves that by filtering the noise by using long averages. As the noise is randomly distributed, a sufficiently long average will tend to cancel out the positive and negative "errors," leaving the underlying structure. This is exemplified in Figure 2 below, where the models for $\alpha = 0.1$ and $\alpha = 0.5$ are presented for a simulated time series (with a mean of 1,000) and additive normally distributed noise (with a standard deviation of 100). Observe that the model with $\alpha = 0.1$ is closer to the underlying mean, using a long average of historical values and therefore canceling out the noise. On the other hand, with $\alpha = 0.5$ the model is very reactive to noise, giving the wrong impression of additional fluctuations in the underlying time series structure (where all these fluctuations are due to the unforecastable noise). Naturally, in this case, a simulated time series was used with a known underlying data generating process. In practice, setting the appropriate parameters is a more challenging task as the underlying structure is unknown.

Figure 2: Comparison of the ETS Models with Various Constants (Alpha)



Source: IMF mission's calculations.

5. **The smoothing parameter for each component defines how reactive that component is to new information.** It is helpful to consider the extremes of 0 and 1. A 0-value smoothing parameter suggests that the component (e.g., level) is not updated at all by the observed data. On the other hand, a smoothing parameter of 1 suggests that the component is fully updated by the last observation and does not retain any underlying structure. More generally, low parameters can be interpreted as long-weighted moving averages that are resilient to increased noise and outliers. In contrast, higher parameter values produce more reactive components that quickly adjust to new data but may also overreact to irregularities. Although the parameters could be set manually for simple models (e.g., level-only ETS), numerical optimization is typically preferable. This is especially true for models with more parameters, which can automatically identify reasonable parameters for both simple and complex models.

6. **For numerical optimization, an appropriate loss function needs to be specified, typically based on quadratic errors.** To do this, the errors for the in-sample data that were used to fit the model are recorded. Quadratic errors, as summarized in the Mean Squared Error, track the mean of a time series. The numerical optimization provides the smoothing parameters—one corresponding to each state of the model—minimizes the number of errors.

7. **The appropriate ETS model can be identified using a suitable information criterion, such as the Akaike Information Criterion (AIC).** The intuition behind such metrics is that they attempt to balance how well a model fits the data against the complexity of the model, as captured by its various parameters. A model without enough complexity—in the case of ETS, one without the appropriate states that capture the level, slope, and seasonality in a time series—will underfit the time series and provide poor forecasts. On the other hand, a model with superfluous complexity will overfit the data, which means that it will attempt to model the normally unforecastable noise and thus mistakenly model non-existing patterns in the time series structure. In general, a more complex model (i.e., a model with more parameters) is more flexible to fit better with the in-sample data, and therefore potentially overfit. Overfit models can provide substantially inaccurate forecasts. A simplified view of the AIC is:

$$AIC = 2\sqrt{MSE} + 2k,$$

Where k is the number of model parameters. For ETS. k relates to the number of states/components in the model. The first half of the equation improves as the model fit becomes better, minimizing the errors between the in-sample observations and the model output. This typically correlates with the model having more parameters. The second half of the equation is minimized when the number of parameters is as small as possible, which typically happens when the model underfits, and therefore has larger in-sample errors. The model with the lowest AIC is preferable because it forces a balance between model fit and model complexity. This results in selecting models that can forecast well.

8. **To include regressors, the model can be augmented by adding them to the description of μ_t in the same fashion as with conventional regression modeling.** Additional details about ETS can be found in Hyndman et al. (2008)[2] and Ord, Fildes, and Kourentzes (2017).[3]

9. **The ARIMA family of models is a flexible class of models used for time series forecasting in a wide range of settings.** In general, the ARIMA model is defined as:

$$(1-\phi(B))(1-B)^d y_t = (1+\theta(B))\epsilon_t$$

Here B is the backshift operator that lags a variable, i.e., $By_t = y_{t-1}$,$By_t = y_{t-2}$, etc. The order of differencing d is typically equal to 1 (or in rare cases 2) for nonstationary series and 0 for stationary series. The term $\left(1-\phi(B)\right) = 1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p$ is known as the autoregressive (AR) polynomial (or order p) and the term $\left(1+\theta(B)\right) = 1+\theta_1 B+\theta_2 B^2+\cdots+\theta_p B^p$ is known as the moving (MA) polynomial (or order q). The term ϵ_t is a random "noise" or "innovation" term. The nomenclature ARIMA (p,d,q) is used to describe an ARIMA model. For example, an ARIMA model with $p=2$, $d=1$ and $q=2$ would be referred to as an ARIMA (2,1,2) model.

For all ARIMA models, the order of $p, d, q, P, D, and\ Q$ must be made. The estimation is done using the stepwise algorithm of Hyndman and Kandakar (2008):[4]

i. Find d using the KPSS (Kwiatkowski–Phillips–Schmidt–Shi) test.

ii. Estimating four initial models and choose the best.

iii. Expand the candidate model set by considering models that have p or q differing from the current best by 1.

iv. Iterate until no improvement is made.

The criterion for selection is the Akaike Information Criterion corrected for small sample size (AIC). The algorithm implemented in auto-ARIMA function within the forecast package in the R software environment. The same algorithms can be modified and applied to seasonal ARIMA and seasonal ARIMA using the regression described below.

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[2] Hyndman, R., A. B. Koehler, J. K. Ord, and R. D. Snyder. 2008. *Forecasting with Exponential Smoothing: The State Space Approach*. Berlin: Springer Science and Business Media.

[3] Ord, K., R. Fildes, and N. Kourentzes. 2017. *Principle of Business Forecasting*, 2nd ed. New York: Wessex Press.

[4] Hyndman, R. J., and Y. Khandakar. 2008. "Automatic Time Series Forecasting: The Forecast Package for R." *Journal of Statistical Software* 27 (3): 1-22.

An important extension to ARIMA models is seasonal ARIMAs (SARIMA), which allows for the modelling of patterns that repeat themselves every m observations. In general, SARIMA takes the form:

$$(1-\phi(B))(1-\Phi(B^m))(1-B)^d(1-B^m)^D y_t = (1+\theta(B))(1+\theta(B^m))\epsilon_t$$

Where P, D, and Q are the orders of the seasonal AR component, seasonal differencing, and seasonal MA component. The nomenclature ARIMA(p,d,q)(P,D,Q)[m] is used to describe such models, for instance an ARIMA(1,0,0)(0,1,1)[5] model would be equivalent to:

$$y_t = y_{t-5} + \phi(y_{t-1} - y_{t-6}) + \epsilon_t - \epsilon_{t-5}$$

Seasonal ARIMA models of this form are only capable of explicitly capturing one form of seasonality. Fortunately, the ARIMA model can easily incorporate covariates by extending its equation in the same manner as with conventional regression modelling. Additional details for ARIMA models can be found in Ord, Fildes, and Kourentzes (2017).

10. **Seasonality can be modelled using indicator variables.** This approach is particularly well suited when the length of the seasonal pattern is short and when the pattern is not necessarily smooth. For example, flexible day of week effects can be modelled using only four variables of the form:

$$D_t^{(Sun)} = \begin{cases} 1 \text{ if day t is a Sunday,} \\ 0 \text{ otherwise.} \end{cases}$$

Similar dummies can be defined for Mon, Tue, Wed, Thur. These indicators are then included in a vector of covariates x'_t and the ARIMA model has the same specification as before, but with y_t replaced by $y_t - x'_t\beta$. A similar modification occurs for ETS. A similar approach is used to encode holidays and special events. Structural breaks can be encoded using a continuous indicator:

$$\mathrm{D}_t = \begin{cases} 1 \text{ if t occurs after the structural break,} \\ 0 \text{ otherwise.} \end{cases}$$

11. **Daily time series can exhibit multiple seasonal cycles that must be accounted for in the modeling.** These include day in the week, day in the month, and day in the year, corresponding to different cyclicities in the data. This substantially complicates the creation of forecasts, as many models typically incorporate a single seasonal periodicity. Three elements are of interest in modelling multiple seasonality: the length of the seasonal cycles, their encoding, and the efficiency of the latter, as we aim for parsimonious models. To resolve questions raised by the first element, one counts how many days there are in each periodicity. For example, there are five days in the week (without weekends). However, day in the month seasonality is more challenging as months have a different number of days. To overcome this, quarterly seasonality is used, as a quarter contains a fixed number of weeks, and by extension days.

12. **The multiple seasonal cycles are encoded using trigonometric indicator variables.** Given the length of a season of s periods, $s/2$ pairs of trigonometric variables are constructed, with $i = 1, \ldots, s/2$:

$$d_i = cos\left(\frac{2i\pi t}{s}\right),$$

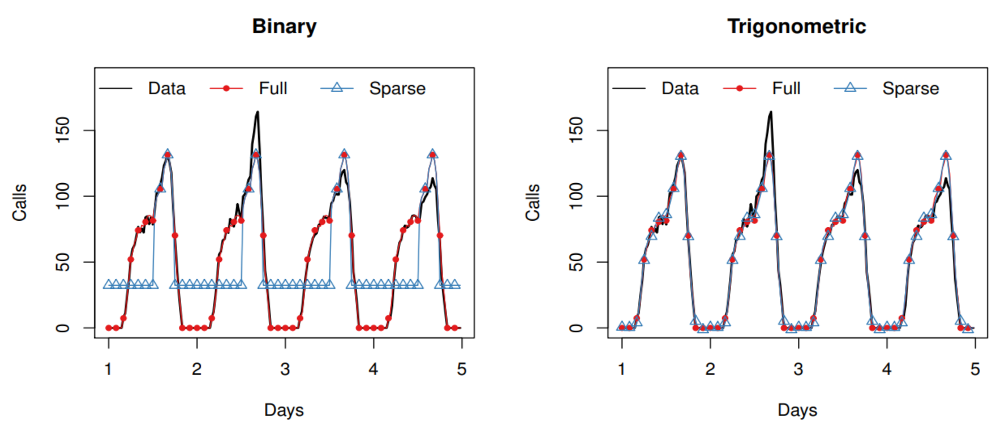
$$d_{i+s/2} = sin\left(\frac{2i\pi t}{s}\right),$$

Where $t = 1, \dots, n$ (with n being the sample size). When s is an odd number, $s/2$ is rounded up to the closest integer. This encoding is mathematically equivalent to using s binary indicator variables, in which case each binary indicator would encode the level of a particular day in the season. Note that in some cases one of the indicators will correspond to a constant, resulting in $s - 1$ informative indicator variables. One major advantage of the trigonometric representation is that it can encode complex seasonal effects, such as leap years. This is not possible with binary indicator encoding. For binary encoding, the number of trigonometric pairs is calculated as the floor of the true s. For example, for the day in the year we will have 260/2 cosines and 260/2 sines (five-day week years contain 260 days).

13. **To obtain parsimony, redundant seasonal indicators are filtered.** To do this, the time series trend is removed using a centered moving average (Ord, Fildes, and Kourentzes 2017). The centered moving average simply calculates the average of all values within a season that effectively models the trend in the time series. This is subtracted from the data, and the residuals are then modelled with different trigonometric indicator variables as explanatory variables using a regression model. However, to eliminate less informative inputs and help obtain a sparse representation, a lasso regression is used. Lasso regression is tasked to find a good compromise between how well the model fits the data and its complexity as measured by the number of parameters. The idea is that models with more parameters (and therefore input variables) are better able to model the observations, but can potentially overfit, capturing the randomness in the time series instead of just the structure. More details about the lasso regression can be found at Ord, Fildes, and Kourentzes (2017) and Kourentzes and Sagaert (2018).[5]

14. **An advantage of trigonometric indicator variables is that they provide an efficient sparse approximation of seasonal patterns.** Figure 3 below exemplifies this. Using all indicators, binary and trigonometric variables (for integer s) provide the same output. However, when terms are eliminated, binary encoding omits all seasonal information for that period, while the trigonometric encoding merely provides a smoother approximation of the seasonal profile.

Figure 3: Exemplification of the Advantages of Trigonometric Variables



Source: IMF mission's calculations.

[5] Kourentzes, N., and Y. R. Sagaert. 2018. "Incorporating Leading Indicators into Sales Forecasts." *Foresight: The International Journal of Applied Forecasting* 48: 24-40.

15. **The TBATS model incorporates many of the features of the models already introduced.** With TBATS, seasonality and trend are handled via ETS (using trigonometric terms for the former), a Box-Cox transformation is used, and ARIMA innovations are incorporated. This allows seasonality to change over time. A particularly attractive feature of the TBATS model is its ability to handle multiple calendars. Additional details about the TBATS model can be found in de Livera, Hyndman, and Snyder (2011).[6]

16. **Volatility models are appropriate for forecasting series with high volatility.** Normally, these models will be applied to forecast Net Foreign Assets. Three classes of models are fitted.

17. **The most popular family of conditional volatility models is the GARCH model.** The variance is modeled as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \, \sigma_{t-i}^2 + \sum_{i=1}^{p} \alpha_i \, e_{t-i}^2$$

The specification of the exponential GARCH model (eGARCH) is given by:

$$\log\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \log\sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \, g(\epsilon_{t-i}).$$

Where $g(\epsilon_t) = \theta\epsilon_t + \lambda\big(|\epsilon_t| - E(|\epsilon_t|)\big)$. An advantage of this specification is its asymmetry since the sign and magnitude of innovations have different effects on the variance.

The GJR (Glosten-Jagannathan-Runkle)-GARCH specification is given by:

$$\sigma_t^2 = \omega + \delta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2 + \phi\epsilon_{t-1}^2 I_{t-1}$$

Where $I_{t-1} = 0$ if $\epsilon_{t-1} \geq 0$ and $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$. Like eGARCH, this specification allows for asymmetric effects.

[6] De Livera, A. M., R. J. Hyndman, and R. D. Snyder. 2011. “Forecasting Time Series with Complex Seasonal Patterns Using Exponential Smoothing.” *Journal of the American Statistical Association* 106 (496): 1513-1527.