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Financial Constraints and the Effectiveness of Green Financial Policies

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ABSTRACT: This paper analyzes the effectiveness of green financial policies—green credit policies and free emissions allowances—at improving emission efficiency while supporting output. We develop a heterogeneous-firm model with financial constraints and endogenous adoption of cleaner capital. The model matches key targeted and untargeted moments from granular micro-data, including the facts that more financially constrained firms are less productive, more emission intensive, and respond less to carbon pricing. In counterfactual simulations in our model, credit policies without green bias raise output but also raise emissions, as firms become more capital and energy intensive. In contrast, well-targeted green credit policies—focusing on frontier technologies—cut emissions while boosting output. In the presence of financial frictions, free emissions allowances offset the output costs of carbon pricing, breaking the usual irrelevance of permits allocation.

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1 Introduction

Many governments have introduced financial-sector interventions designed to steer credit and investment toward cleaner technologies and to ease firms' transition to lower emissions while supporting growth. Such 'green financial policies' include measures that increase the supply of preferential or targeted credit for green investment, adjustments to collateral or risk-weighting rules, and the way permits are allocated within emissions trading systems. Such policies have gained prominence because of the political challenges of raising carbon taxes (Blanchard et al., 2023) and because financial constraints prevent firms from investing in cleaner technologies. Examples include Europe's complement of carbon pricing with large-scale green lending supported by the European Investment Bank (EIB) and the upcoming shifts in the European Central Bank (ECB)'s collateral policy.[1] Despite the growing use of green financial policies, the literature offers little evidence about their effectiveness.

In this paper, we examine the effectiveness of two key green financial policies: (i) policies that expand credit or guarantees for green investment ("green credit policy") and (ii) policies governing the distribution of emissions permits ("free permits policy"). We begin by showing that less financially constrained firms are more productive and, in turn, greener—they emit less relative to value added. Motivated by these facts, we develop a heterogeneous-firm general equilibrium model with endogenous vintage adoption under financial constraints, calibrated to match key empirical patterns.[2] The model also matches new evidence that less constrained firms reduce emissions by more when carbon prices rise. Counterfactual simulations show that broad relaxations of financial constraints raise emissions by increasing GDP and capital-energy intensity. Targeted green credit policies can lower emissions while raising output, but only when support is focused on the appropriate frontier vintages. Finally, in the presence of financial frictions, allocating free emissions permits to firms can offset the output costs

[1]The EIB's website points to financing for €550 billion in new investments to support the green transition since 2021. The ECB plans to introduce a "climate factor" in valuing assets pledged as collateral in 2026. The ECB's website also details initiatives under consideration for bank supervision, and regulation. Central banks in China and Japan have also introduced special lending facilities for banks to facilitate loans for green investments.

[2]Different vintages of capital vary in quality: newer vintages produce more output holding inputs fixed, and generate lower emissions holding energy consumption fixed (Capelle et al., 2024). An example of newer capital that greens the production process and also increases productivity is the transition from fossil-fuel boilers to industrial heat pumps, which cut both emissions and energy consumption per unit of heat produced.

of carbon pricing.

We draw on verified data on emissions for European manufacturing firms in energy-intensive industries subject to carbon pricing. Emissions data from 2005-2021 for 3,200 manufacturing firms are obtained from the European Union (EU) Emission Trading Scheme (ETS).[3] We merge the emissions data with balance sheet and income statement data from Orbis (Letout, 2021). Crucially, this approach allows us to use verified data on emissions for both listed and private firms. The latter are smaller and more likely to face binding financial constraints (Beck et al., 2005). In 2021, firms in our sample emitted 315 Megatons of greenhouse gas equivalents—about 40% of European manufacturing emissions.

We present two new stylized facts which show that financial constraints shape environmental performance at the firm level: firms that need to operate close to their borrowing capacity tend to be less productive and dirtier. We construct net worth—capturing unused borrowing capacity—as in Ottonello and Winberry (2024). We examine the relationship between net worth, productivity, and emission efficiency within the same 4 digit SIC industry and country.[4] Firms with higher net worth are more productive. And more productive firms have smaller environmental footprints relative to the scale of their operations: they have lower emissions relative to value added than within-industry peers.

Motivated by these stylized facts, we propose a tractable continuous time heterogeneous-firm general equilibrium model of investment across different capital vintages, subject to financial constraints. Firms vary on two dimensions in the model: stochastically varying productivity, and net worth accumulated through retained earnings. Importantly, firms face leverage limits that depend on their net worth as in Ottonello and Winberry (2024). The model includes endogenous entry and exit. Key features of the production function include decreasing returns to scale—pinning down optimal scale for each firm—and complementarity between energy and capital. Firms make endogenous dynamic decisions about investments in different vintages of physical capital. A key ingredient of the model is that more efficient vintages embed technologies that enhance productivity, cut energy consumption, and reduce emissions

[3]The EU ETS covers manufacturing firms engaged in energy-intensive activities such as steel and metal production. Emissions data includes both carbon dioxide and other greenhouse gases, reported in equivalent terms, when directly generated at the covered plant (Scope 1).

[4]This allows us to make granular comparisons. For example, we treat forging of raw steel as a distinct industry from the transformation of steel into specific products.

(Capelle et al., 2024).

The key tradeoff that firms face in the model is between investing in more efficient capital vintages and scaling up their capital stock. On one hand, firms prefer better vintages because they are more productive and help cut energy consumption and emissions. On the other hand, better vintages are also more expensive. This generates a pecking order of vintages. Constrained firms—young firms with low net worth—are smaller than their optimal scale as the leverage constraint binds tightly. These firms choose to use less efficient vintages because these allow them to operate at a larger capital stock scale. As firms accumulate net worth and lessen their financial constraint, they progressively upgrade to better, more productive vintages. Unconstrained firms can operate at their optimal scale with the best vintage.

We carefully calibrate the model to allow for quantitative counterfactual analysis. We match important moments of the empirical distributions of firm size and productivity, as well as evidence from the literature on the extent to which capital embedded technologies help cut emissions holding energy consumption fixed (Capelle et al., 2024). We target data on energy consumption and average leverage for European manufacturing firms. Finally, we match past levels of carbon prices.

Our model quantitatively matches our motivating stylized facts as well as additional untargeted empirical evidence on the impact of carbon pricing on firms' emissions by levels of net worth. In the simulations from the steady state distribution in the model, firms with higher net worth are more productive, and more productive firms have lower emission intensities. To further validate the model, we show empirically that firms' ability to adjust to exogenous changes in carbon prices depends on their level of net worth. We use a series of exogenous shocks to ETS carbon prices constructed by Känzig (2023) following the high frequency monetary policy literature (e.g., Gurkaynak, Sack and Swanson, 2005; Nakamura and Steinsson, 2018). Firms with higher net worth—which we interpret as less financially constrained following the model—are able to cut emissions by more than peers in narrowly defined industries in response to exogenous increases in carbon prices. Regression results utilizing simulated data from the model quantitatively match this empirical finding. The tight link of the model with the data on untargeted moments offers strong validation for the model.

Counterfactual simulations from the calibrated model yield three key insights. First, holding carbon prices fixed, relaxing financial constraints raises—rather than lowers—emissions in general equilibrium. The impact of relaxed financial constraints on total emissions is theoretically ambiguous. On one hand, looser financial constraints allow some firms to upgrade to newer, more efficient vintages and reduce emissions relative to firm size. On the other hand, looser constraints allow firms to increase their capital intensity, which increases energy consumption due to the complementarity between capital and energy. This increases emissions relative to firm size. Moreover, firms—and the economy—grow, raising emissions holding emission intensities fixed. Quantitatively, this final channel dominates: GDP grows significantly when financial constraints are eased. Eliminating financial constraints in our calibrated model results in a 34% increase in total emissions, largely due to a 32% increase in GDP.

Second, well-targeted green credit policies can help improve environmental performance in our model. We consider policies—such as public lending or guarantees as currently deployed in Europe—that specifically support green investments. In the context of the model, we consider policies that relax financial constraints only for firms that operate with sufficiently green vintages of capital. When appropriately targeted, such green credit policies can both raise output and lower emissions. A 20% relaxation of the borrowing constraint for investments in the greenest vintage in use by unconstrained firms raises GDP by 4% and lowers emissions by 2%.[5]

However, targeting green credit policies is not straightforward in our model. If coverage is extended to vintages greener than those used by many firms but not at the frontier, green credit policies become less effective. Providing coverage for just one additional vintage below the frontier generates an increase in total emissions. Moreover, green credit policies interact with carbon pricing. Increases in carbon prices lead unconstrained firms to upgrade to greener vintages. This means that green credit policies must be adjusted to be more stringent and offer support only for the new frontier vintage in use.

Third, allocating free emissions permits to firms to offset the cost of carbon pricing can be beneficial in a model that incorporates financial frictions and endogenous entry. Consistent with prior work, carbon pricing is highly effective at cutting emissions in our model, but with significant economic cost. Doubling the price of carbon from the baseline level of €30 to €60 cuts emissions by 45%, but also reduces steady state output

[5]Green credit policies are not part of the first best allocation, which would rely on carbon prices to cut emissions and financial sector policies to address financial constraints. However, if it is politically difficult to raise carbon prices significantly, there may be room to include green credit policies in the second best policy mix.

by nearly 7%. In the presence of financial constraints, allocating free permits in the context of an emissions trading system can significantly shift outcomes. They can help constrained firms retain earnings and grow out of their borrowing constraints faster, while they also incentivize additional entry of new firms. Indeed, when free permits are allocated to firms, emissions can be cut by 41% with no reduction in output relative to the baseline. Green financial policies can therefore be complementary with carbon pricing: achieving a 41% emission cut with carbon pricing alone requires increasing the carbon price to €53, which reduces output by 6%. More broadly and in contrast to a frictionless environment, the presence of financial constraints and endogenous entry make the distribution of free permits relevant for the equilibrium allocation.

Literature and contributions. This paper presents evidence on how financial constraints, and policies aimed at alleviating them, matter for aggregate emissions and the effectiveness of carbon pricing. Our theoretical and empirical results make three main contributions to the literature.

First, we consider how financial constraints shape endogenous technological adoption in a general equilibrium macro model of firm dynamics. Our model combines a Hopenhayn (1992) framework, in which firms face decreasing returns to scale and exogenous idiosyncratic productivity shocks, with financial constraints as in Khan and Thomas (2013) and Ottonello and Winberry (2024). The key theoretical novelty here is to embed into this framework a technological choice of vintages as in Capelle et al. (2024). The interaction of financial constraints with endogenous vintage choice gives rise to a trade-off between upgrading to newer vintages and scaling up the capital stock, similar to complementary work by Lanteri and Rampini (2025). The pecking order of vintages—a key and novel result of this paper—is the outcome of this trade-off. Importantly, we incorporate a production function for emissions that accounts for newer capital vintages in a way that is consistent with empirical evidence presented by Capelle et al. (2024).

Second, we contribute to a growing theoretical literature on how the macroeconomic impact of environmental policies depends on firm-level heterogeneity by incorporating financial constraints that slow technological adoption.[6] Prior work has largely considered carbon pricing and investment subsidies when firms differ in their use of technolo-

[6]See Bilal and Stock (2025) for a comprehensive recent review of the wider literature on macroeconomic aspects of climate change.

gies and are subject to real investment frictions but not financial constraints (Capelle et al., 2024; Finkelstein Shapiro and Metcalf, 2023; Klenow et al., 2024). A nascent body of work considers the impact of financial constraints on emissions (Aghion et al., 2025; Lanteri and Rampini, 2025; Kim, 2023).

To our knowledge, ours is the first paper to evaluate the general equilibrium impact of green financial policies on aggregate emissions. We find that the effect of financial constraints on total emissions is theoretically ambiguous. Looser financial constraints can allow firms to make upgrades that improve efficiency and reduce emissions, as in Lanteri and Rampini (2025), but also stimulate economic growth, leading to greater input consumption and emissions. Quantitatively, GDP growth dominates and thus policies that broadly relax financial constraints raise total emissions. However, financial policies that specifically support green investments have the potential to simultaneously raise output and lower emissions. At the same time, achieving policy goals through green financial policies is challenging, as they need to be appropriately targeted and the target depends on features of the economy, including the level of carbon pricing.

Third, we present new empirical evidence using micro data that less financially constrained firms are better able to adapt to carbon pricing. Specifically, using verified data on emissions for a large group of both public and private firms, we consider how emissions respond to exogenous carbon price shocks (Känzig, 2023). Important prior work offers evidence on the role of financial constraints (Accetturo et al., 2022; Fang et al., 2023; De Haas et al., 2024; Costa et al., 2024) and on the impact of carbon pricing (Martin et al., 2014b,a; Colmer et al., 2024; Martinsson et al., 2024) individually. However, there is little prior empirical evidence considering both financial constraints and carbon prices.[7]

The remainder of this paper is structured as follows. Section 2 describes the data and presents stylized facts. Sections 3, 4, and 5 discuss the model set up, properties,

[7] Berthold et al. (2023) examine how stock prices differentially react to exogenous carbon price shocks for dirtier European firms based on self-reported and potentially imputed data on emissions, also drawing on work by Känzig (2023). Their focus on stock prices means that they cannot consider unlisted firms. Carradori et al. (2023) show that more levered firms covered by the EU ETS are able to deliver smaller cuts in emissions. However, they do not consider exogenous variation in carbon prices. Kaldorf and Shi (2024) show that less financially constrained firms cut emissions by less in response to carbon prices across a panel of 33 countries. They also use self-reported and potentially imputed emissions data and do not consider exogenous carbon price shocks. Xu and Kim (2022) consider how financially constrained firms respond to more stringent environmental regulations across a broad range of pollutants and do not focus on greenhouse gas emissions.

and calibration, respectively. Section 6 validates the model by showing that it is consistent with new empirical evidence. Section 7 discusses results from counterfactual simulations of our calibrated model. Section 8 concludes.

2 Stylized Facts

Do financial constraints shape firms' productivity and environmental performance? In this section, we provide novel stylized facts suggesting that they do. Using cross-sectional data on the universe of firms in the European Emissions Trading System (ETS), we show that financially constrained firms seem to lack the capacity to raise funds to undertake the investments necessary to become more productive and greener.

2.1 Data sources

We combine firm-level data on emissions, balance sheets, and income statements for more than 3200 manufacturing firms, headquartered in 30 European countries covered under the EU Emissions Trading Scheme between 2005 and 2021.

Emissions data. Data on verified annual emissions and emissions allowances at the installation level are from the EU Emissions Trading System (EU ETS).[8] The EU ETS employs a cap-and-trade approach (Schmalensee and Stavins, 2017), setting a limit on the total amount of GHG emissions that can be emitted by all participating installations, and allowing firms to trade allowances for such emissions. Each allowance permits the holder to emit one tonne of CO2 equivalent. Companies must acquire these allowances either through auctions, free allocations, or by purchasing them to other firms. The trading of allowances creates a market price for carbon emissions.

We aggregate emissions data to the firm level to match the aggregation of other data sources. We calculate firm-level emissions by adding the emissions from all installations associated with each firm. The system records and reports CO2 equivalent Scope 1

[8] Installations are stationary technical units where activities covered by the EU ETS are carried out. The inclusion of installations in the EU ETS is primarily determined by sector-specific capacity thresholds. For instance, installations which manufacture glass, including glass fiber, are covered by the EU ETS if they possess a production capacity above 20 tonnes per day. Firms may own multiple installations, and it is possible that only a subset of these are covered by the EU ETS.

(direct) emissions.[9]

Emissions for each installation are verified under an 'ETS Compliance Cycle'. This annual compliance procedure for monitoring, reporting and verification applies to all operators—the entities operating each account and installation—covered under the ETS. Data for each year must be verified by an accredited verifier within March of the following year.[10] Subsequently, the operators must surrender an equivalent number of allowances for that year. If they fail to do so, operators face a €100 penalty for each tonne of excess emissions.[11] We can therefore draw on reliable firm-level data on emissions.

Balance sheet and income data. We gather annual firm-level balance sheet and income statement data from ORBIS, a comprehensive global database covering both public and private firms. The presence of private firms is particularly important for our analysis, as these firms are generally smaller than publicly listed ones and more likely to be financially constrained (Beck et al., 2005).

Merging and cleaning. We merge firm-level EU ETS and ORBIS data utilizing a publicly available crosswalk from the EU Joint Research Center (Letout, 2021). We keep observations where there is available information on both emissions and financials, and focus on firms in the manufacturing sector. Our final dataset covers 85% of overall ETS manufacturing emissions on average.

Construction of variables. We measure a firm's environmental performance through its emission intensity, which we define as the ratio of its emissions and value added.

We measure the extent to which a firm is financially unconstrained through its net worth, defined as the sum of tangible fixed and current assets minus total debt.[12] Net worth captures the difference between assets which can be used as collateral and the existing debt stock—unused borrowing capacity. Ottonello and Winberry (2024) argue

[9] The EU ETS currently covers approximately 40% of overall emissions across the EU, primarily focusing on power generation and manufacturing sectors.

[10] The European Commission provides templates to be used for annual emission, verification, and other reports.

[11] Payment of the penalty does not negate the surrender obligation.

[12] Tangible fixed assets are defined as all tangible assets on the books of the firm such as buildings and machinery. Current assets are the sum of short term receivables from debtors and customers, trade inventories and other current assets, defined as short term receivables from all other sources, short term investments and cash balance.

that firms with low net worth face more binding financial constraints. Importantly, using net worth as a measure of financial constraint is consistent with the model of section 3.

As we do not have information on product-level prices, we estimate TFP from revenue data as in Asker et al. (2014). This approach uses input expenditure shares to inform estimates of production functions.

2.2 Results

We find that firms with higher levels of net worth, which face looser financial constraints, are more productive (Figure 1 panel a). Moreover, firms with higher productivity tend to have lower emission intensities (panel b). These relationships hold within industry, country, and year.[13] These results suggest that financial constraints shape firms' productivity and environmental performance. Implementing policies that relax firms' financial constraints may thus allow firms to tackle the investments necessary to become more productive, and such investments may also result in lower emissions per unit of output, improving environmental performance.

3 A Model of Firm's Technological Adoption and Financial Constraints

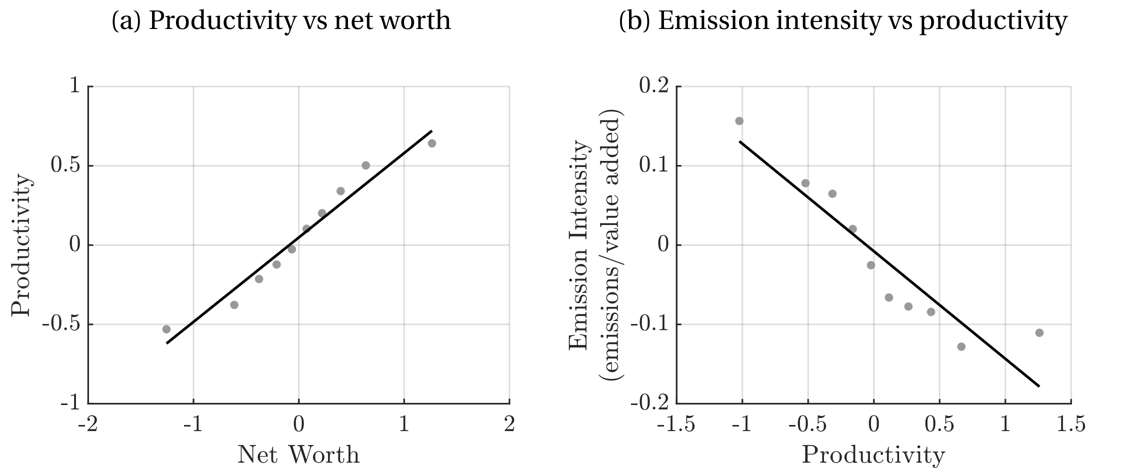
To interpret this evidence, we develop a small open-economy heterogeneous-firm general equilibrium model in which firms choose investment and capital vintage subject to financial constraints. In the following sections, we will use a calibrated version of the model to assess the aggregate effects of green financial policies. We first describe the household's problem, then the firm's and finally close with the general equilibrium conditions.

3.1 Household

Time is continuous, indexed by t and runs to infinity. There is no aggregate uncertainty. The representative household has Greenwood–Hercowitz–Huffman (GHH) preferences

[13] In Appendix, Figure A1 shows that results are robust to residualizing against firm and year fixed effects. Results also hold when measuring emission intensity as emissions over revenue.

Figure 1: More productive firms are less financially constrained and have lower emission intensity



Notes: The figure displays firm-level binned scatter plots of productivity against net worth (panel a) and of emission intensity (emissions over value added) against productivity (panel b). All variables are in logs, residualized against industry × country × year fixed effects and standardized. Only manufacturing firms are included.

over consumption c_t and labor ℓ_t (Greenwood et al., 1988):

$$u(c_t, \ell_t) = \frac{1}{1-\psi_1}\left(c_t - \psi_2 \frac{\ell_t^{1+\psi_3}}{1+\psi_3}\right)^{1-\psi_1} \tag{1}$$

where ψ_1 governs the degree of intertemporal substitution, ψ_2 captures the household's taste for leisure and ψ_3 is the inverse Frisch elasticity of labor supply.

The household's objective is to maximize the integral of discounted flow utility

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} u(c_\tau, \ell_\tau)\, d\tau \tag{2}$$

where ρ denotes the discount rate.

The household faces a budget constraint. Her earnings depend on her supply of labor to firms, ℓ_t, and the real wage per unit of time, w_t. She can save in a risk-free bond b_t with instantaneous real rate of return r_t^*. Finally, she receives a transfer $\mathscr{D}_t$, which includes the dividends distributed by incumbent firms, the net worth of exiting firms, net of the costs paid by starting firms, the initial net worth of entering firms, and the receipts of the carbon price. We describe these terms in more detail below. The household's financial wealth evolves as follows

$$\dot{b}_t = r_t^* b_t + w_t \ell_t - c_t + \mathscr{D}_t \tag{3}$$

where we normalize the price of final goods to 1.

The optimal intratemporal and intertemporal decisions give the following household labor supply schedule and Euler equation:

$$\ell_t = \left(\frac{w_t}{\psi_2}\right)^{\frac{1}{\psi_3}} \quad \text{and} \quad \frac{\dot{c}_t - \psi_2 \ell_t^{\psi_3} \dot{\ell}_t}{c_t - \psi_2 \frac{\ell_t^{1+\psi_3}}{1+\psi_3}} = \frac{1}{\psi_1}(r_t^* - \rho). \tag{4}$$

3.2 Final good producers

There is an endogenous mass F_t of final good producers which supply the final good to the household. Firms are heterogeneous and characterized by two state variables: the exogenous component of their total factor productivity z ("exogenous productivity" for short) and their net worth a. The exogenous productivity follows a stochastic process while net worth evolves endogenously and deterministically. We denote $f_t(z,a)$ the mass of firms over these two state variables at time t. By definition, the integral of the mass of firms over the entire state space amounts to the total mass of firms in the economy $\int f_t(z,a)d(z,a) = F_t$.

Technology. Firms produce using a production function that combines capital k_v of vintage v, energy n and labor ℓ. The set of capital vintages $\mathcal{V} = \{v_0, v_1, v_2, ...\}$ is countable, exogenous and ordered with $v_{i+1} > v_i$. The production function is given by

$$y = z\left[v\tilde{k}_v\right]^{\kappa} \ell^{\lambda} \tag{5}$$

$$\tilde{k}_v = \left((1-\alpha)k_v^{1-1/\epsilon} + \alpha n^{1-1/\epsilon}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{6}$$

where α is the energy intensity and $\epsilon > 0$ is the elasticity of substitution between capital and energy. This nesting structure is commonly used in the literature estimating elasticities of substitution.[14] In addition, the technology features decreasing returns to scale, $\kappa + \lambda < 1$, so that firms have an optimal size.

[14] See for example Koetse et al. (2008) for a meta-analysis.

The log exogenous productivity, $\log(z)$, follows an Ornstein-Uhlenbeck process, which is the continuous-time analogue of the discrete time AR(1) process:

$$d\log(z) = \mu(z)dt + \sigma_z dW_t \tag{7}$$

$$\text{with} \quad \mu(z) = \theta(E(\log(z)) - \log(z)) \tag{8}$$

where dW_t is a Brownian motion, θ governs the degree of mean reversion, $\mu(z)$ its deterministic drift, σ_z its dispersion and $E(\log(z))$ is the unconditional mean of the process.

In addition, firms emit greenhouse gases. We assume that the production function of emissions depends linearly on the amount of energy consumed, n, and decreases with the quality of the vintage used, v:

$$e = \phi v^{-\gamma} n \tag{9}$$

where ϕ parametrizes the average emission per unit of energy consumption in the economy and γ is the elasticity of emissions over energy to vintages. Capelle et al. (2024) showed that this production function for emissions provides an accurate description of the data in the cross-section of firms. It captures the idea that more productive vintages are also more emission-efficient for a given level of energy consumption, consistent with empirical evidence.

Firms need to purchase their stock of capital of a given vintage at a price q_{vt} which depends on the quality of the vintage. Intuitively, more productive and newer vintages are more expensive $v < v' \Rightarrow q_{vt} < q_{v't}$. Firms fund their capital investment with a mix of net worth a and debt b. In continuous time, the balance sheet accounting identity is given by

$$q_{vt} k_v \equiv a + b. \tag{10}$$

Financial constraints. Importantly, firms face two external financing constraints. First they are subject to a leverage limit

$$q_{vt} k_v \leq \chi a \tag{11}$$

where $\chi \geq 1$ is an exogenous parameter. When $\chi = 1$ firms must fully self-finance. By contrast, when $\chi \to +\infty$ capital markets are perfect and external finance becomes unconstrained. This constraint is a common feature of models with moral hazard and limited commitment and captures the fact that, to ensure a certain minimum skin in the game, the amount of capital available to a firm is limited by its equity (Buera et al., 2011; Midrigan and Xu, 2014; Ottonello and Winberry, 2024).

Second they cannot obtain external equity finance, which means that a can only be increased through retained earnings. The law of motion of net worth is given by

$$\dot{a} = r_t^* a + \pi_t(z, a) - d_t. \tag{12}$$

where $\pi_t(z, a)$ is the equilibrium profit of a firm with productivity z and net worth a and d_t denotes dividends which are distributed in a lump-sum fashion to the household. The term $r_t^* a$ corresponds to the share of the cost of capital that is self-financed. We derive this expression below when we define profits $\pi_t(z, a)$.[15]

Profit maximization. At each instant, firms face a static profit maximization problem. They make four decisions regarding their inputs. They choose how much labor to hire, ℓ, how much energy to use, n, how much capital to use, k_v, and which capital vintage to use, v. Taking as given the wage w_t, the price of energy m_t, the price of carbon τ_t, and the vintage-specific capital prices q_{vt} and user costs r_{vt}, the firm's static problem is:

$$\pi_{it}(a, z) = \max_{v, k_v, \ell, n} y - w_t \ell - m_t n - \tau_t e - r_{vt} k_v \tag{13}$$

subject to the technological constraints (5), (6), (9) and the financial constraint (11). The user cost, r_{vt}, of capital is the key determinant of the optimal choice of capital. It is given by

$$r_{vt} = q_{vt}\left(r_t^* + \delta - \frac{\dot{q}_{vt}}{q_{vt}}\right) \tag{14}$$

[15]While investment frictions are important in shaping the response of capital to policies, the focus of this paper is on financial frictions. A complementary analysis incorporating investment adjustment costs is developed in Capelle et al. (2024).

where r_t^* is the interest rate, δ the exogenous rate of depreciation and $\frac{\dot{q}_{vt}}{q_{vt}}$ is the rate of change of the price of capital of vintage v.[16]

Firms' objectives. Firms maximize the discounted sum of future dividends $\int_t^{+\infty} e^{-r_\tau^* \tau} d_\tau d\tau$. Firms choose the path of dividends d_t to maximize the value of the firm. When doing so, they trade off paying dividends in the present against using their earnings to accumulate net worth, which can lead to higher dividends in the future. In recursive form, the value function of a firm with productivity z and net worth a is the solution to the following Hamilton-Jacobi-Bellman (HJB) equation

$$(\rho+\zeta)V_t(z,a) = \max_d d + \frac{\partial V_t}{\partial a}(r_t^* a_t + \pi_t(z,a) - d) + \mu(z)\frac{\partial V_t}{\partial z} + \frac{\sigma_z^2}{2}\frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t}{\partial t} \tag{15}$$

where the term $\frac{\partial V_t}{\partial a}$ corresponds to the deterministic drift in net worth, $\frac{\partial V_t}{\partial z}$ to the deterministic drift in exogenous productivity, $\frac{\partial^2 V_t}{\partial z^2}$ to the volatility of exogneous productivity, and $\frac{\partial V_t}{\partial t}$ to deterministic aggregate changes in prices and policies.

3.3 Entry, exit and firms' dynamics

Exit. Firms die randomly at rate ζ per unit of time. When they die, their net worth is transferred to the household in a lump-sum fashion.

Entry. There are potential entrepreneurs who can start a company with net worth $\underline{a}$ after paying a fixed cost κ_e in terms of the final good and before learning about their productivity draw $z \sim H(z)$. $H(.)$ denotes the cdf of the initial productivity distribution, where $h(.)$ is the pdf. The flow of entrants at time t, p_t, is a function of the profit

[16] To see where this expression comes from, consider a firm investing in capital k_{vt} for a small time interval dt. It has to purchase its capital stock $q_{tv}k_{vt}$, then it has to pay the interest rate r_t^* on its debt $q_{tv}k_{vt} - a$, and it can resell the undepreciated capital stock at a market value $q_{vt+dt}k_{vt}$:

$$q_{tv}k_{vt} + r_t^* \times dt \times (q_{vt}k_{vt} - a) - (1-\delta \times dt) \times q_{vt+dt} \times k_{vt}$$

$$\text{which gives} \quad k_{vt}\left(dt \times q_{vt} \times \left(r_t^* + \frac{q_{vt+dt}}{q_{vt}} \times \delta\right) - (q_{vt+dt} - q_{vt})\right) - r_t^* \times dt \times a.$$

Dividing by dt and taking the limit $dt \to 0$, we obtain $q_{vt}\left(r_t^* + \delta - \dot{q}_{vt}/q_{vt}\right)k_{vt} - r_t^* a$. The first term corresponds to the user cost of capital times the capital stock, and the second corresponds to the interest charges saved on the share of the capital funded with net worth. The latter term shows up in the law of accumulation of net worth (12).

opportunities going forward relative to the fixed cost of starting a business

$$p_t = p_\infty \exp\left\{\xi\left(\int_z \frac{v_t(\underline{a}, z)}{\kappa_e} dH(z) - 1\right)\right\} \tag{16}$$

with $\xi > 0$ is the elasticity of entrants to profits opportunities and p_∞ is the entry rate in the final steady state.

This formulation lies between two extreme assumptions made in the literature: on the one hand that entry is infinitely elastic to profit opportunities as in Hopenhayn (1992) and on the other hand that the mass of entrepreneurs is exogenous as in Moll (2014). In the long run, this formulation converges to the standard free entry condition $\int v_t(\underline{a}, z) dH(z) = \kappa_e$ with infinite elasticity. But in the short run, the elasticity is finite and parametrized by ξ. As $\xi \to +\infty$, the short-term elasticity converges to the long-term elasticity and becomes infinite. We adopt this generalized specification for firm entry to obtain a more realistic transition path following policy changes.

Firms' dynamics and the Kolmogorov-forward equation. The mass of firms $f_t(z, a)$ evolves according to the following Kolmogorov-Forward equation, capturing firm exit, net worth accumulation, new entrants, and stochastic productivity shocks:

$$\begin{aligned}\frac{\partial f(z,a)}{\partial t} &= -\zeta f(z,a) - \frac{\partial}{\partial a}[f(z,a)(r_t^* a_t + \pi(z,a) - d(z,a))] \\ &\quad + \mathbb{1}_{a=\underline{a}} p_t h(z) - \frac{\partial}{\partial z}[\mu(z) f(z,a)] + \frac{\partial}{\partial z^2}[\sigma_z^2 f(z,a)].\end{aligned} \tag{17}$$

3.4 Capital goods producers

Competitive capital-good producers transform final output into vintage-specific capital goods with linear technology:

$$i_v = \iota_v y \tag{18}$$

where the productivity ι_v is allowed to be vintage-specific, reflecting the fact that some vintages are more costly to produce than others. At the aggregate level, the quantity of capital of each vintage, K_{vt}, follows the law of motion of capital with depreciation rate δ

$$\dot{K}_{vt} = -\delta K_{vt} + I_{vt}. \tag{19}$$

where I_{vt} is the aggregate investment in vintage v.

3.5 General equilibrium

Markets clearing. We consider a small-open economy which trades freely capital goods, final goods, energy and risk-free bonds freely with the rest of the world. The price of energy m_t and the risk-free interest rate r_t^* are thus exogenous. Labor is not tradable, and thus the labor market has to clear domestically. Denoting ℓ_t the household's labor supply at time t and $\ell_t^d(z,a)$ the labor demand of a firm with productivity z and net worth a, the labor market clearing condition is given by

$$\ell_t = \int \ell_t^d(z,a) f_t(z,a) d(z,a) \tag{20}$$

While the goods market clears globally, the domestic resource constraint is given by:

$$Y_t = c_t + I_t + NX_t \quad \text{with} \quad I_t = \sum_v \iota_{vt}^{-1}(\dot{K}_{vt} + \delta K_{vt}) + p_t \kappa_e \tag{21}$$

where NX_t denotes net exports.

Consistent with our focus on the European Union, we assume that firms import energy from the rest of the world.

The markets for emissions permits also needs to clear. The government supplies permits, in aggregate quantity S_t. Firms demand permits for their emissions, e_{ft}. The equilibrium on the market determines the price of emissions τ_t.

$$S_t = \int e_t(z,a) f_t(z,a) d(z,a) \tag{22}$$

The government chooses the path of permits S_t to implement its desired path of carbon price, τ_t.

Equilibrium definition. An equilibrium is a path of household policy rules $\{c_t, \ell_t, b_t\}_{t=0}^{\infty}$, firm's policy rules and values $\{\ell_t^d(z,a), k_{vt}(a,z), v_t(a,z), n_t(z,a), d_t(z,a), V_t(z,a)\}_{t=0}^{\infty}$, firms distributions, transfers, flow of entrants and wages, $\{f_t(z,a), \mathscr{D}_t, p_t, w_t\}_{t=0}^{\infty}$ such that (i) given prices, c_t and ℓ_t maximize the household' utility subject to their budget constraint; (ii) given prices, $\ell^d(z,a)$, $k_{vt}(a,z)$, $v(a,z)$, $d(z,a)$ maximize the firm's value function subject to the financial constraint and the value $v_t(a,z)$ is solution to the HJB equation (15); (iii) the labor and emissions permits markets clear, free entry holds and the distribution of firms is consistent with the Kolmogorov-forward equation (17).

4 Equilibrium Properties and Pecking Order of Vintages

This section characterizes the firms' optimal behaviors in equilibrium. We first derive their optimal input choices and then highlight the central trade-off that financially constrained firms face between scaling up capital and adopting higher-quality vintages. We show that this trade-off generates a pecking order of vintage adoption: financially-constrained firms adopt less productive and dirtier vintages. Details on the derivations can be found in Appendix A2.

4.1 Optimal energy, labor and capital decisions

Capital goods producers. We begin with the optimality conditions of firms producing capital goods. Given that their technology is linear and they are atomistic, the prices of capital goods are pinned down by the inverse of the productivity parameters:

$$q_v = \iota_v^{-1} \tag{23}$$

and they make zero profit in equilibrium.

Final good firms - demand for labor and energy. We now turn to the final good firms. We start with their choice of energy and labor. For tractability and clarity, we illustrate the firms' input choices under the special case of Leontief preferences ($\epsilon = 0$). In the quantitative version of the model which we calibrate in the following section, we go back to a general CES formulation and calibrate ϵ to a realistic value. But given the low degree of substitution between capital and energy in our calibrated model, the Leontief function is a reasonable approximation of the general CES function.

Assumption 1 (Leontief technology)**.** *In this section we assume that energy and capital are perfect complement, $\epsilon = 0$: $\tilde{k} = \min\left((1-\alpha)k_v, \alpha n\right)$.*

Under Assumption 1, the optimal level of energy consumption is the ratio of the stock of capital over the energy efficiency of its vintage and the optimal choice of labor is given by the traditional equalization of the marginal product of labor with its marginal cost:

$$n = \frac{\alpha}{1-\alpha} k_v \quad \text{and} \quad \ell = \lambda \frac{y}{w_t}. \tag{24}$$

Final good firms - demand for capital, output and profits. The optimal choice of capital depends on whether the firm is financially constrained. Let Λ denote the multiplier on the financing constraint (11). The first-order condition for capital is:

$$\kappa \frac{y}{k} = r_{vt} \left(1 + \Lambda \left(r_t^* + \delta\right)^{-1}\right) + \frac{\alpha}{1-\alpha} \left(m_t + \tau_t v^{-\gamma}\right) \tag{25}$$

where we used the fact that in equilibrium $\frac{\dot{q}_{vt}}{q_{vt}} = 0$ since ι_v is time-invariant.

When the firm is unconstrained ($\Lambda = 0$), it chooses capital to equate the marginal product of capital, $\kappa y/k$, with its marginal cost. The marginal cost includes both the user cost of capital, r_{vt}, and the cost of energy, $\frac{\alpha}{1-\alpha}\left(m_t + \tau_t v^{-\gamma}\right)$, given the Leontief technology. The following lemma provides a closed-form expression for profits in the unconstrained case; expressions for output and capital are reported in Appendix A2.

Lemma 1 (Unconstrained Profits)**.** *Assume the firm is financially unconstrained and uses vintage v. At the optimum, its profits are given by*

$$\pi_t(v,z) = \left(z \left(v \frac{\kappa}{r_{vt} + \frac{\alpha}{1-\alpha}\left(m_t + \tau_t v^{-\gamma}\right)} \right)^{\kappa} \left(\frac{\lambda}{w_t}\right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} [1 - \lambda - \kappa]. \tag{26}$$

In contrast, when the constraint binds ($\Lambda > 0$), firms operate below their unconstrained capital level. As a result, the marginal product of capital $\kappa y/k$ is higher than its marginal cost $r_{vt} + \frac{\alpha}{1-\alpha}\left(m_t + \tau_t v^{-\gamma}\right)$ as the firm cannot be as capital-intensive as it would like to be.

Lemma 2 (Constrained Capital, Output and Profits)**.** *Consider a financially-constrained firm that uses vintage v. Its capital stock is given by $k_v = \frac{\chi a_t}{q_{vt}}$, its output by $y = \left[z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda}{w_t}\right)^{\lambda} \right]^{\frac{1}{1-\lambda}}$ and profits by*

$$\pi_t = \left[z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda}{w_t}\right)^{\lambda} \right]^{\frac{1}{1-\lambda}} [1-\lambda] - \chi a_t \left(r_t^* + \delta + \frac{\alpha}{1-\alpha} \frac{m_t + \tau_t v^{-\gamma}}{q_{vt}} \right) \tag{27}$$

4.2 The trade-off between the quantity of capital and vintage quality

We now turn to the firm optimal vintage decision. We highlight the trade-off faced by financially constrained firms between scaling up capital and adopting higher-quality vintages.

Unconstrained firms. Taking the log of profits and keeping only the term that depends on v, the optimal vintage in this case is the one that maximizes the following expression

$$\log\left(\frac{v}{q_v}\frac{1}{\left(r_t^*+\delta\right)+\frac{\alpha}{1-\alpha}\frac{m_t+\tau_t v^{-\gamma}}{q_v}}\right) \tag{28}$$

When choosing their optimal vintage, unconstrained firms balance higher productivity (v) and emission-efficiency ($v^{-\gamma}$) to save on energy and carbon cost, with the higher price of higher-quality vintages, (q_v). The optimal vintage choice trades off these different forces. The choice of vintage that maximizes the objective (28) is independent of the firm's characteristics, including its productivity. We impose some regularity conditions to ensure that an interior optimal vintage exists.

Assumption 2. *Define* $T(v) = \log\left(\iota_{vt} v \frac{1}{(r_t^*+\delta)+\iota_{vt}\frac{\alpha}{1-\alpha}(m_t+\tau_t v^{-\gamma})}\right)$. $T(v)$ *is continuous, first strictly increasing then strictly decreasing.*

This assumption means that for low v, increases in v result in efficiency gains ($v^{-\gamma}$) that outweigh price increases $q_v = \iota_{vt}^{-1}$. The reverse is true for sufficiently large values of v. A necessary condition for this assumption to hold is that the term $v/q_v = v\iota_{vt}$—the inverse of the cost of more productive vintages relative to their productivity—decreases at least for high enough values of v, given that the other term $\frac{1}{(r_t^*+\delta)+\frac{\alpha}{1-\alpha}\frac{m_t+\tau_t v^{-\gamma}}{q_v}}$ is increasing in v. A direct implication of these assumptions is that there exists a unique optimal vintage v^* common to all firms.

Lemma 3. *Under Assumption 2, there exists a unique* v^* *such that* $\pi_t^U(v^*, z) \geq \pi_t^U(v, z)$ *for all* v *and all* z.

The optimal vintage depends only on the price of energy m_t, the interest rate r_t^*, the carbon price τ, and the productivity of the capital production technology of all vintages $\{\iota_v\}_v$.

Constrained firms. Unlike financially unconstrained firms, the optimal vintage decision of a constrained firm considers the fact that the choice of vintage affects the maximum amount of capital that the firm can invest in. Because higher quality vintages are more expensive, they force firms with limited ability to leverage to buy less capital to stay within their constraint. This effect is all the more relevant when the firm is far from its optimal level of capital.

To see this, consider a firm with productivity z and net worth a that contemplates the difference in profits between two consecutive vintages $v < v'$:

$$\left[z\left(\chi a_t\right)^{\kappa}\left(\frac{\lambda}{w_t}\right)^{\lambda}\right]^{\frac{1}{1-\lambda}}[1-\lambda]\left[\left(\frac{v'}{q_{v't}}\right)^{\frac{\kappa}{1-\lambda}}-\left(\frac{v}{q_{vt}}\right)^{\frac{\kappa}{1-\lambda}}\right]-\chi a_t\frac{\alpha}{1-\alpha}\left(\frac{m_t+\tau_t(v')^{-\gamma}}{q_{v'}}-\frac{m_t+\tau_t v^{-\gamma}}{q_v}\right) \tag{29}$$

Each line corresponds to one distinct side of the trade-off between vintage quality and the scale of investment in capital that financially constrained firms face. The vintage quality side of the trade-off is captured by the second line $\frac{\alpha}{1-\alpha}\left(\frac{m_t+\tau_t(v')^{-\gamma}}{q_{v'}}-\frac{m_t+\tau_t v^{-\gamma}}{q_v}\right)$. This term is always negative, because higher quality vintages are at the same time more productive and more emission-efficient (have lower emissions per unit of energy consumption). This force pushes firms to adopt higher quality vintages because they can save on costs.

On the other hand, higher quality vintages are also more expensive, and they force firms with limited ability to leverage to buy less capital to stay within their constraint. In turn, this lowers their output and eventually their profits given that the firm is below its optimal level of capital. This second force is captured by the first line, and for it to hold however, the term $\left[\left(\frac{v'}{q_{v't}}\right)^{\frac{\kappa}{1-\lambda}}-\left(\frac{v}{q_{vt}}\right)^{\frac{\kappa}{1-\lambda}}\right]<0$ needs to be negative. If it were positive, then the most productive vintage (highest v) would always be strictly preferred for any level of wealth a. For this term to be negative, in turn, the productivity-price ratio should decrease with v, $v'/q_{v'} < v/q_v$, *i.e.* the cost of more productive vintages grows quicker than their productivity. We formalize this assumption below:

Assumption 3. *Consider any pair of consecutive vintages $v < v'$. Then*

$$\iota_v v > \iota_{v'} v'. \tag{30}$$

4.3 Pecking order of vintage adoption

Under Assumption 3, firms follow a pecking order of vintages: as they grow and accumulate net worth, they upgrade to more productive vintages until they settle on their optimal unconstrained vintage v^*. Rewriting Equation (29) and dividing by χa_t gives

$$\frac{\alpha}{1-\alpha}\left(\frac{m_t+\tau_t(v')^{-\gamma}}{q_{v'}}-\frac{m_t+\tau_t v^{-\gamma}}{q_v}\right)-(\chi a_t)^{\frac{\kappa}{1-\lambda}-1}\left[z\left(\frac{\lambda}{w_t}\right)^{\lambda}\right]^{\frac{1}{1-\lambda}}[1-\lambda]\left[\left(\frac{v}{q_{vt}}\right)^{\frac{\kappa}{1-\lambda}}-\left(\frac{v'}{q_{v't}}\right)^{\frac{\kappa}{1-\lambda}}\right].$$

The term on the right hand side is continuous and strictly decreasing with a_t, it goes to infinity as a approaches zero and to 0 as a goes to infinity. This equation thus defines a threshold $\bar{a}_v$ at which a firm is indifferent between v and v', as formalized in the following lemma.

Lemma 4. *Under Assumption 3, for any t, z and any pair of consecutive vintages $v < v'$, there exists $\bar{a}_t(z, v') > 0$ such that*

$$a \leq \bar{a}_t(z, v') \iff \pi_t^C(z, a, v) \geq \pi_t^C(z, a, v'). \tag{31}$$

That is, for net worth values lower than the threshold $\bar{a}_t(z, v')$, it is more profitable for the firm to utilize the lower productivity vintage v for production. The net worth threshold $\bar{a}_t(z, v')$ depends on the firm's productivity type z. Firms with lower z have a lower threshold of adopting a given vintage, i.e., $z < z' \Rightarrow \bar{a}_t(z, v') < \bar{a}_t(z', v')$. This is because firms with lower productivity have a smaller optimal quantity of capital, so they face weaker incentives to distort their vintage decision to increase their capital stock.

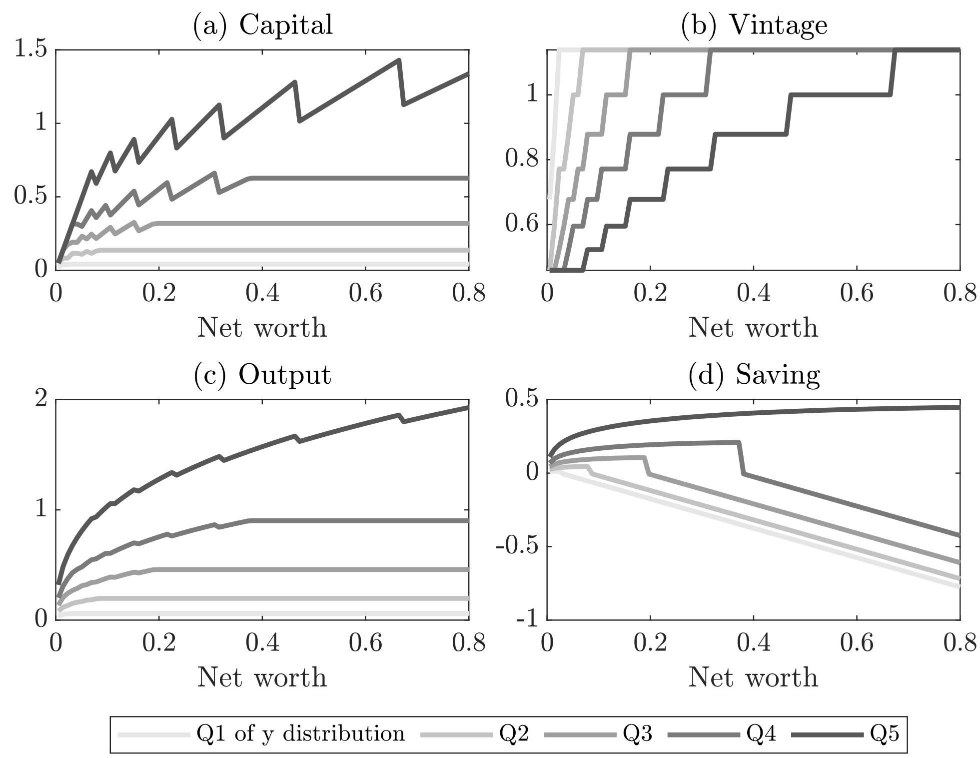
The thresholds of net worth also depend on time through changes in wages and capital goods prices. Increases in wages lower all thresholds such that firms adopt a given vintage at lower levels of net worth. The intuition is that when wages increase, firms' optimal size and thus level of capital decrease, which alleviates the cost of the borrowing constraint.

The previous results lead to the following proposition which says that there is a pecking order of vintages through which firms move as they grow and accumulate net worth, until they settle on the most productive vintage v^*.

Proposition 1 (Pecking order of vintages)**.** *Under the same assumptions as lemma 4, for any z and t, there exists a strictly increasing sequence $\bar{a}_t(z, v_0) < \bar{a}_t(z, v_1) .... < \bar{a}_t(z, v^*)$ such that a firm uses vintage $v \leq v'$ if and only if $a \in [\bar{a}_t(z, v), \bar{a}_t(z, v')]$.*

Figure 2 shows firms' equilibrium policy rules for the capital stock k, the quality of the capital vintage v, total output and the level of savings as a function of their net worth a for five different quantiles of the productivity z-distribution. These policy rules come from the quantitative model we calibrate in the next section. Consistent with Proposition 1, there exists a pecking order of vintages: given an exogenous productivity z, lower-net-worth firms adopt inferior vintages and they upgrade to higher quality vintages as they save and accumulate net worth. Delaying the adoption of higher quality vintages is optimal because it allows firms to buy more units of capital while they are still financially constrained. The fact that the size of the capital stock and output shrinks when firms upgrade shows the trade-off between vintage quality and the scale of investment in capital that financially-constrained firms face.

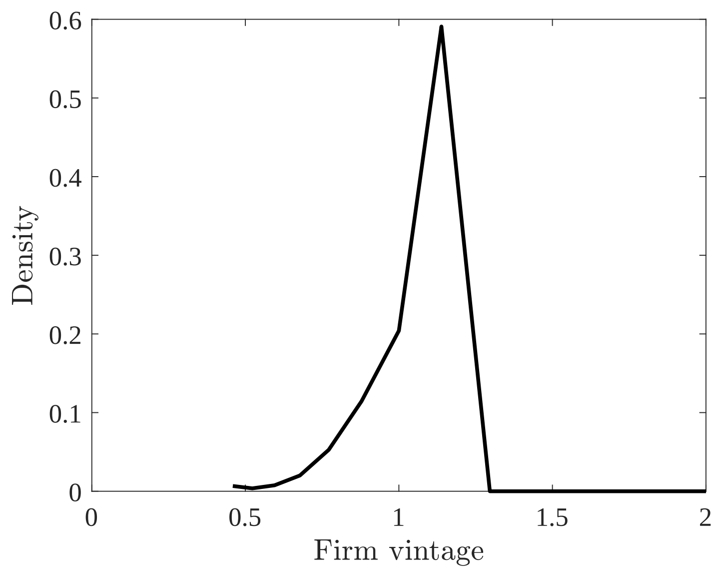
Figure 2: Policy rules in steady state



Notes: This graph shows the policy rules in the market equilibrium for the optimal scale of capital investment (top left), vintage quality (top right), output (bottom left) and saving (bottom right). The x-axis is the firm net worth (a). Different shades of gray correspond to different values of firm TFP z. The values of z are the quintiles of the cumulative distribution of firms output. Given that the distribution of firm size is highly skewed, we select the quantiles so that each quantile accounts for 20% of aggregate output.

Figure 3 shows the equilibrium distribution of vintages in the calibrated model. The distribution of vintages is left-skewed. A large fraction of firms, which are not financially constrained, use the same vintage. There is a long tail of financially constrained firms using vintages that are of inferior quality relative to the vintage used by financially unconstrained firms. In Appendix Figure A6 we also report the stationary distributions of net worth a, exogenous productivity z, output y and emissions e.

Figure 3: Distribution of vintages



Notes: This figure plots the stationary distribution of firms across vintages. Approximately 60% of firms are financially unconstrained and adopt the same vintage. There is a long tail of financially constrained firms using vintages that are of inferior quality relative to the vintage used by financially unconstrained firms. No firm uses a vintage of a higher quality that the one chosen by financially unconstrained firms.

5 Calibration

We now calibrate the model to match important moments of the empirical distribution of firms' sizes and productivity, the greenness of capital vintages of different qualities, the firms' energy consumption and the average leverage of firms in Europe. Unlike the previous section, we allow for general substitution between capital and energy using a CES production function. Some parameters are externally calibrated from the literature or data, while others are internally estimated to match targeted moments.

5.1 External calibration

We begin by calibrating a number of parameters that can be set externally. These parameters are listed in Table 1. The discount rate $\rho = 0.04$ and capital depreciation rate $\delta = 0.05$ are set to standard values commonly used in the macroeconomic literature. Similarly, the interest rate is set to 4%.

The emissions elasticity with respect to vintage, γ, follows Capelle et al. (2024). We scale their estimate (0.036) by the ratio of vintage dispersion in their data (1.11) to that in our model (0.15), yielding $\gamma = 0.27$.

We set the elasticity of substitution between capital and energy to $\epsilon = 0.2$, consistent with European firm-level estimates reviewed in Koetse et al. (2008) (Table 4 and 5) and consistent with Hassler et al. (2021).

For the elasticity of output to labor, we follow Ottonello and Winberry (2024) and set it to $\lambda = .64$, which is also consistent with the labor share in the European Union. The wage elasticity of labor is set to 2.5, within the range used in the literature (Smets and Wouters, 2007; Hall, 2009; Rogerson and Wallenius, 2009; Keane and Rogerson, 2012; Mui and Schoefer, 2024). To calibrate the elasticity of entry to profit opportunities—a parameter that matters for the speed of the transition—we follow Gutiérrez and Philippon (2019) who estimate that in the recent decades it has been around .1. The mean reversion of firm TFP is set to .1, consistent with firm-level empirical estimates in European countries (Asker et al., 2014).

We put structure on the productivity-price change over vintages. We assume that the rate of improvement of productivity v across vintages is constant and denoted g_v. In addition, we assume the ratio of productivity to price for successive vintages declines at a constant rate, governed by $u < 1$, such that:

$$\iota_{v'} \times v' = u \times \iota_v \times v \tag{32}$$

We set the value for the growth rate of vintage productivity to 11.9%, which is close to values used in the literature analyzing the contribution of improvements in capital goods to long-run growth (Greenwood et al., 1997). For the parameter u we choose a value below but close to one, consistent with a pecking order of vintages. We set $u = 0.9$ implying an increase in the price/productivity ratio of 10% between vintages released one year apart.

Table 1: Externally calibrated parameters

Parameter	Description	Value	Source
ρ	Discount rate	0.04	Standard
δ	Depreciation rate of capital	0.05	Standard
r^*	Interest rate	4%	Standard
g_v	Growth rate of vintage productivity	11.9%	Capelle et al. (2024)
$g_v - g_p$	Vintage productivity/price growth	.9	Benchmark
γ	Emissions elasticity to vintage	0.27	Capelle et al. (2024)
ϵ	Elasticity of substitution (k, n)	.2	Koetse et al. (2008); Hassler et al. (2021)
λ	Labor elasticity of output	0.64	Ottonello and Winberry (2024)
ψ_3	Wage elasticity of labor	2.5	Standard (See notes)
m	Price of energy	1	Normalization
ξ	Elasticity of entry	.1	Gutiérrez and Philippon (2019)
θ	Mean reversion of $\log(z)$	.1	Asker et al. (2014)
τ	Price of carbon	30	ETS and authors' calculations

Notes: Macroeconomic estimates of the Frisch elasticity generally range between values of 1 to 4 (Smets and Wouters, 2007; Hall, 2009; Rogerson and Wallenius, 2009; Keane and Rogerson, 2012; Mui and Schoefer, 2024).

We calibrate the price of a ton of carbon in the ETS market to its level before 2020, at €30 a ton. We normalize the price of energy to 1. Finally we assume that the productivity distribution of new entrants, h, is the same as the stationary distribution of the exogenous productivity of incumbents.

5.2 Internal calibration

Other model parameters are estimated to match empirical moments of the distribution of firms' input and output. Table 2 provides a full list.

We briefly summarize our calibration strategy here. Although no single moment pins down one parameter, we explain which moment is most informative about which parameter. The cost of entry parameter κ_e is tightly connected to the cost of entry relative to GDP in the data. The parameter is set to .145, which results in an entry cost of 3.4% of annual income per capita, the cost of starting a new business in the Europan Union according to the World Bank. The exit rate in the data is directly connected to the probability of exit in the model.

The energy intensity α is disciplined by the share of cost spent on energy, which is around 5% of total costs. The average emissions per unit of energy (in tons per gigajoule) is informative about ϕ, the scale parameter in the firms' emissions technology.

Table 2: Internally calibrated parameters

Parameter	Description	Value
κ_e	Cost of entry	.145
ζ	Exit rate	.025
α	Energy intensity	.06
ϕ	Average emissions per unit of energy	.06
σ_z	Dispersion of productivity	.2
κ	Capital elasticity of output	.145
χ	Maximum leverage ratio	2.15
ψ_2	Labor preference parameter	2.7056

The standard deviation of the productivity process σ_z is directly related to the firm size variance in the data. The covariance of output and TFP directly informs the elasticity of output to capital κ. The leverage constraint parameter χ has a first-order effect on the average leverage of firms, which is about 35% in the data. Finally, the labor preference parameter ψ_2 is set to be consistent with a labor supply normalized to $\ell = \left(\frac{w}{\psi_2}\right)^{1/\psi_3} = 1$.

Table 3 shows that the model performs well in matching the targeted aggregate and distributional moments.

Table 3: Targeted moments

Moment	Data	Model	Source
Cost of entry relative to GDP	3.4%	3.4%	World Bank (WDI)
Exit rate	2.5%	2.5%	Eurostat
Energy cost (% of total cost)	3.1%	2.9%	Eurostat
Average emissions per unit of energy	.06	.06	Orbis, ETS, own calculations
Firm size variance	2.66	2.55	Orbis, own calculations
Covariance of output and TFP	.57	.55	Orbis, own calculations
Average leverage of corporations	35%	35%	Eurostat

Notes: Data on entry cost is from the World Bank, on exit rate is from the Bundesbank, on energy cost is from the European Commission, and on firms' leverage is from the European Systemic Risk Board.

6 Validation

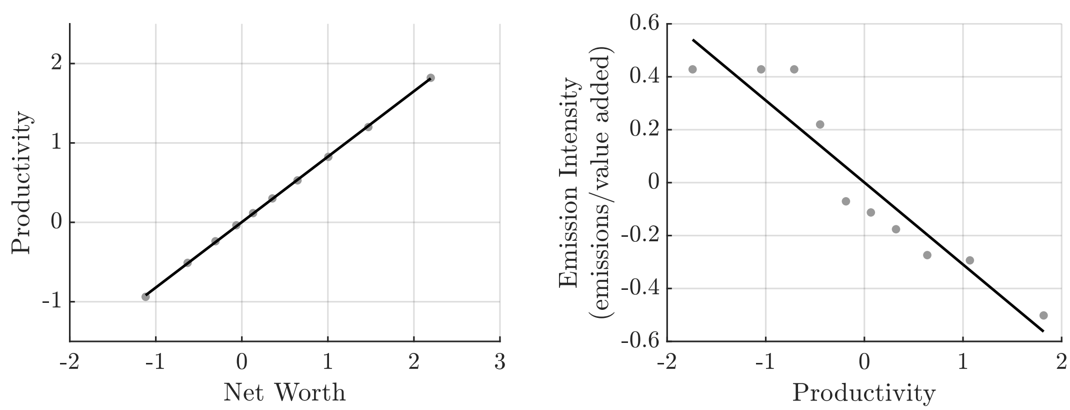
We now present results that validate the calibrated model. First, we show that the model is broadly consistent with the untargeted stylized facts presented in Section 2. Second, we show that it is consistent with novel causal evidence on the differential emissions response of firms to exogenous changes in carbon prices by levels of net worth.

6.1 Cross-sectional relationships

Recall that the stylized facts presented in Section 2 showed that in the cross-section of European firms, firms with higher net worth are more productive and firms that are more productive have lower emission intensities. We reproduce the same scatter plots using the model's implied stationary distribution of net worth (a), productivity (zv^{κ}) and emission intensity (e/y). All variables are in logs and standardized.

Figure 4: More productive firms are less financially constrained and have lower emission intensity

(a) Productivity vs net worth (b) Emission intensity vs productivity



Notes: The figure displays binned scatter plots of firm-level productivity against net worth (panel a) and of emission intensities (emissions over value added) against productivity (panel b), in the stationary distribution of the calibrated model. All variables are in logs and standardized.

Like in the data, Figure 4 panel (a) shows a positive relationship between net worth and productivity and panel (b) shows a negative relationship between emission intensity and productivity. The order of magnitudes of these cross-sectional relationships are also consistent with their empirical counterparts.

In the model, the positive relationship between net worth (a) and productivity (zv^{κ}) in panel (a) reflects two forces. It reflects the pecking order of vintages: as firms accumulate net worth they upgrade to more productive vintages, i.e. vintages with higher v. But it also reflects the fact that firms with higher exogenous productivity (z) have a larger optimal size and accumulate more net worth to reach that size. The negative relationship between productivity (zv^{κ}) and emission intensity (e/y) in panel (b) reflects the fact that when firms upgrade to a better vintage, they raise their productivity and they reduce their emissions per unit of energy.

6.2 Differential response to carbon price shocks by net worth

A central implication of the model is that net worth and thus financial constraints condition how firms respond to policies, in particular to carbon pricing. Firms with larger net worth can finance the adoption of cleaner, more efficient vintages when carbon prices rise, whereas firms with lower net worth are constrained and find it more challenging. We now show that the differential response of firms to exogenous changes in carbon prices based on their level of net worth is similar in the data and in the model. This provides a direct validation of the model's core mechanism and supports the use of this model to evaluate green financial policies.

Data on carbon prices and carbon price shocks. We obtain daily carbon spot prices for EU ETS allowances from Bloomberg. To match the annual frequency of the emissions and firm financials data, we calculate annual averages of the daily carbon prices.

To address issues of endogeneity and simultaneity, we instrument carbon prices with exogenous carbon price shocks following Känzig (2023). The time series for carbon price shocks comes from this paper and is constructed using changes in the European Union carbon allowance (EUA) futures price on days when regulation shifts expectations of the supply of emission allowances. Given the frequency of our other variables, we aggregate the daily shocks at the annual frequency. This approach isolates exogenous variation in carbon prices in the spirit of the literature using shifts in interest rates within narrow windows around monetary policy announcements (e.g., Gurkaynak, Sack and Swanson, 2005; Nakamura and Steinsson, 2018).

Empirical strategy. We use instrumental variables local projections (Jordà et al., 2015) for our baseline specification:

$$\Delta^h \ln(e_{j,t+h}) = \alpha_h \widehat{\Delta \ln(\tau_t) \times D_{j,t-1}} + \beta_h \mathbb{X}_{j,t} + \mu^h_{t,i(j),k(j)} + \epsilon_{j,t+h} \tag{33}$$

where $\Delta^h \ln(e_{j,t+h})$ represents the long difference $\ln(e_{j,t+h}) - \ln(e_{j,t-1})$ of emissions for firm j; h indicates the yearly horizon, with $h = 0, ..., 3$; $\Delta \ln(\tau_t)$ is the yearly change in the log of the average carbon price in year t and the variable $D_{j,t-1}$ is a dummy variable which takes the value of 1 if the firm has high net worth and thus faces loose financial constraints and 0 if it has low net worth and thus faces tight financial constraints. Firms are classified as high net worth if their net worth lies above the 75th percentile of the distribution within its industry, country, and year, and low net worth otherwise.

The key explanatory variable of interest is $\widehat{\Delta \ln(\tau_t) \times D_{j,t-1}}$, which is the instrumented interaction between the change in carbon prices and financial constraints, using carbon price shocks from Känzig (2023) as the instrument. Our main coefficient of interest is α_h, which captures the cumulative differential percentage point response in emissions e in year $t+h$ to a one percentage point increase in the instrumented carbon price from $t-1$ to t if the firm is classified as high net worth in $t-1$, relative to firms classified as low net worth.

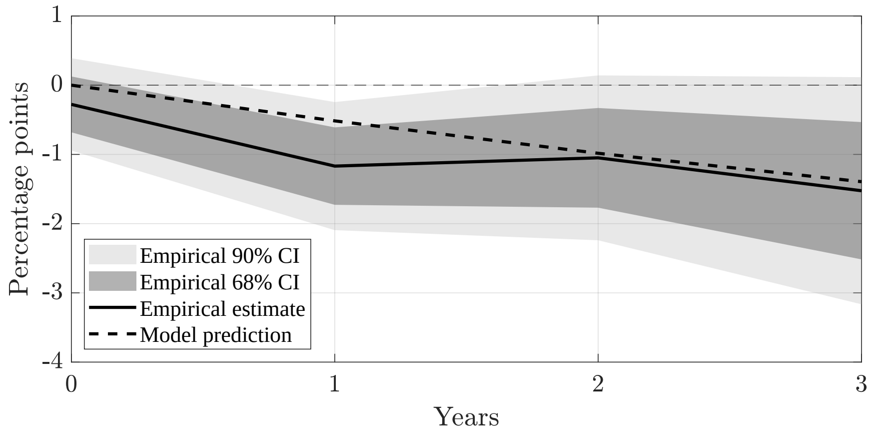
Finally, the set of controls $\mathbb{X}_{j,t}$ includes the dummy variable $D_{j,t-1}$, two lags of the dependent variable, and two lags of the interaction between the dummy variable and the dependent variable: $\mathbb{X}_{j,t} = \sum_{l=1}^{2} \gamma_{h,l} \Delta \ln(e_{j,t-l}) \times D_{j,t-1} + \sum_{l=1}^{2} \delta_{h,l} \Delta \ln(e_{j,t-l}) + \zeta_h D_{j,t-1}$. We also include a set of year-specific fixed effects for j's industry $i(j)$ and country $k(j)$, denoted by $\mu_{t,i(j),k(j)}$. Standard errors are clustered at the time level (Almuzara and Sancibrián, 2024).

Results. The solid line in Figure 5 shows the empirical results. We find that firms with higher net worth cut more emissions in response to higher carbon prices. The figure plots the differential emissions response, in percentage points, to a 13% increase in the price of carbon—corresponding to the average annual percentage change of the EU carbon price from 2010 to 2023. Quantitatively, we find that firms with higher net worth reduce emissions by an additional 1.5 percentage points relative to lower net worth firms three years after the shock.

The dashed line illustrates a quantitatively similar differential response from firms in the model. The consistency of this untargeted set of moments with their empirical counterpart provides support for our model.[17]

[17] Figure A2 extends the model's differential impulse response over a 20-year horizon. The response converges to above 3 percentage points, underscoring its quantitative relevance.

Figure 5: Cumulative emissions differential response to carbon price shocks for firms with higher net worth



Notes: The solid line shows the cumulative emissions differential empirical impulse response to carbon price shocks for firms with higher net worth relative to firms with lower net worth, estimated using Equation (33). A firm is classified as high net worth if its net worth exceeds the 75th percentile of the distribution within its industry, country, and year. Only manufacturing firms are included. The empirical impulse response is shown over a 3-year horizon, with confidence intervals (CIs) at 68% and 90% levels. Standard errors are clustered at the year level. The dashed line performs the same exercise in the model.

Carbon leakage. One concern with our results is that they may be driven by carbon leakages across and within firms. If our results were driven by shifts in production across firms, we would observe a decline in the use of other inputs. Appendix Figure A3 provides evidence that there is no differential decline in the number of employees nor in material costs, suggesting that the empirical differential decline in emissions is not driven by carbon leakage to other firms.

Carbon leakage may also occur within firms. We only observe emissions for the installations covered by the EU ETS. Therefore, if firms shift production from EU ETS-covered installations to those that are not covered, we would observe a decline in emissions, even if the aggregate emissions of the firm do not fall. Appendix Figure A4 addresses this concern by showing that the results are robust to only including firms for which the number of recorded installations in Orbis does not exceed the number recorded under the EU ETS. Our results are also robust to adding firm-level controls (see Figure A5).

7 Quantitative Assessment of Green Financial Policies

We now use the calibrated model to analyze the effectiveness of two green financial policies: (i) policies that expand credit or guarantees for green investment ("green credit policies") and (ii) policies governing the distribution of emissions permits ("permit allocation"). We highlight three important results. First, untargeted credit policies—those applying to all capital qualities—increase output but also emissions. Second, green-biased credit policies can boost output and reduce emissions but they need to target the best technologies in use and adjust to the level of carbon pricing. Third, free permits policies can fully offset the output costs of carbon pricing.

First-best policy mix. Green financial policies are not part of the first-best policy mix. The first-best allocation would require financial sector policies addressing financial constraints and a carbon price aligning private abatement costs with the social cost of carbon. In this allocation, there would be a complete separation of objectives and there is no need for credit policies to be green or for free permits policies. However, carbon pricing is often constrained by political economy considerations. There may thus be scope for green credit policies to complement carbon pricing (Döttling and Rola-Janicka, 2025; Heider and Inderst, 2022).

Scope of counterfactual analysis. While we do not explicitly model political constraints, they motivate this section's focus on evaluating the macroeconomic implications of two green financial policies. We conduct a positive investigation of the aggregate trade-offs and mechanisms, and leave the optimal policy analysis to future research, as it would require taking into account the implementation costs of green financial policies. These costs include the risks of moral hazard, the fiscal costs that may require distortionary taxation, as well as the potential distortions from subsidizing specific investments and risks of greenwashing in the absence of clear and enforceable taxonomies. Recognizing these challenges, the next subsections examine how green financial policies policies operate in general equilibrium and the conditions under which they can effectively support the green transition.

7.1 Green credit policies

This subsection analyzes the effectiveness of green credit policies in improving emission-efficiency while supporting output. We begin with a stark counterfactual in which we completely lift the borrowing limit faced by firms. We then turn to more realistic policies that selectively relax financing constraints for cleaner investments. Unless stated otherwise, we hold the price of carbon fixed at its baseline value (€30).

7.1.1 Lifting all frictions

To simulate the removal of financial constraints, we set the leverage limit to infinity, *i.e.* $\chi \to \infty$. This experiment isolates the effects of financial constraints on firm behavior, aggregate emissions and output. While extreme, it is a useful benchmark for the more realistic credit policies that we study next.

Qualitatively, the impact on emissions is ambiguous and depends on the interplay of opposing channels. On one hand, financial constraints make it difficult for firms to adopt greener, more efficient—but more expensive—technologies. On the other hand, financial constraints lower the average scale of production, thus reducing energy consumption, and tilt the optimal input mix away from capital which is complementary to energy use. The first effect implies that financial constraints could raise emissions while the latter two go in the opposite direction.

To quantify these channels, we decompose aggregate emissions E into three terms as shown in Equation (34). The first term is average emissions per unit of energy E/N, which isolates the first channel; the second is energy consumption per unit of output N/Y, which isolates the second channel; the third corresponds to output Y.

$$E = \frac{E}{N} \times \frac{N}{Y} \times Y \tag{34}$$

Quantitatively, the final channel—expanded output—dominates. Table 4 shows the percentage changes in total emissions and in each of the three components in formula (34) when financial constraints are completely removed. We find that overall emissions increase by 34.4%. There is a 2.5% reduction in emissions per unit of energy, reflecting adoption of greener capital vintages. There is also a 4.8% increase in energy per unit of output, due to the increase in capital intensity allowed by the relaxation of the leverage constraint and the complementarity between capital and energy. The main component driving the change in emissions is the 31.5% expansion of output. Appendix Table A1 shows the levels of several important macroeconomic variables in the steady-state equilibria both with and without financial constraints.

Table 4: Impact of removing financial constraints on key variables.

Output	Energy / Output	Emissions / Energy	Emissions	TFP
31.53%	4.77%	-2.50%	34.36%	20.66%

These quantitative results show that while relaxing financial constraints does support adoption of greener technologies—reducing emissions per unit of energy consumed—it also increases output and capital intensity, resulting in higher aggregate emissions. The increase in output is driven by improvements in productivity and firm entry. As the last column of Table 4 shows, aggregate TFP increases by 20.7%. In other words, financial constraints are barriers to green technology adoption, but lifting them need not help with reducing aggregate emissions.

The increase in output and in TFP generated by the removal of financial constraints in our model is consistent with earlier work looking at the costs of misallocation (Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014). Reviewing the literature, Buera et al. (2015) find that relaxing financial constraints leads to an increase in aggregate TFP between 20 and 30%. For example, Midrigan and Xu (2014) finds that overall financial frictions can reduce TFP by 25% in Korea. In terms of GDP, Buera et al. (2011) finds a large increase of 40%. Ottonello and Winberry (2024) find an increase in GDP of 23% at a fifty years horizon in the U.S.[18] In our model, relaxing financial constraints lead to an 20.66% increase in TFP and a 31.53% increase in GDP.

7.1.2 Realistic green credit policies

Modeling green credit policies. Many green credit policies aim at giving incentives to firms to green their production processes, through public loans, guarantees and interest rate subsidies. In this section, we examine whether there is scope for policies supporting

[18] Results from Ottonello and Winberry (2024) are not directly comparable to our own because in their model financial constraints also prevent the efficient allocation of resources to innovation, which negatively affects growth. Like most other cited papers, we abstract from this feature.

firms borrowing to invest in greener technologies.[19] To do so, we we consider credit policies that relax the leverage constraint for firms with sufficiently green capital stocks.

We consider simple functional forms for $\chi(.)$ that depend on the firm's capital vintage

$$\chi(v) = \chi + \omega_\chi \times \mathbb{1}(v \geq \underline{\omega}_\chi) \tag{35}$$

where ω_χ governs the generosity of the policy, i.e., the degree to which it relaxes the borrowing constraint faced by firms, and where $\underline{\omega}_\chi$ governs the degree of green bias. We label the policy as green biased if and only if $\underline{\omega}_\chi > 0$.

On the importance of targeting greener technology. Figure 6 illustrates how equilibrium GDP, emissions, emissions per unit of energy (E/N) and energy consumption per unit of output (N/Y) evolve, relative to the baseline, for different degrees of generosity of the green credit policy ω_χ. The x-axis corresponds to the degree of relaxation of the financial constraint parameter χ, as a fraction of its baseline level ($\chi = 2.2$).

In panel (a), the degree of green bias of the policy is held fixed at $\underline{\omega}_\chi = 0$, corresponding to an unbiased credit policy in which all capital vintages are covered by the policy. In panel (b), we consider a green biased policy. Specifically, we consider $\underline{\omega}_\chi = 1.1$—the value of the greenest vintage used by unconstrained firms in equilibrium (see Figure 3)—so that only capital vintages at least as good as the optimal are covered. The carbon price, τ, remains fixed at €30.

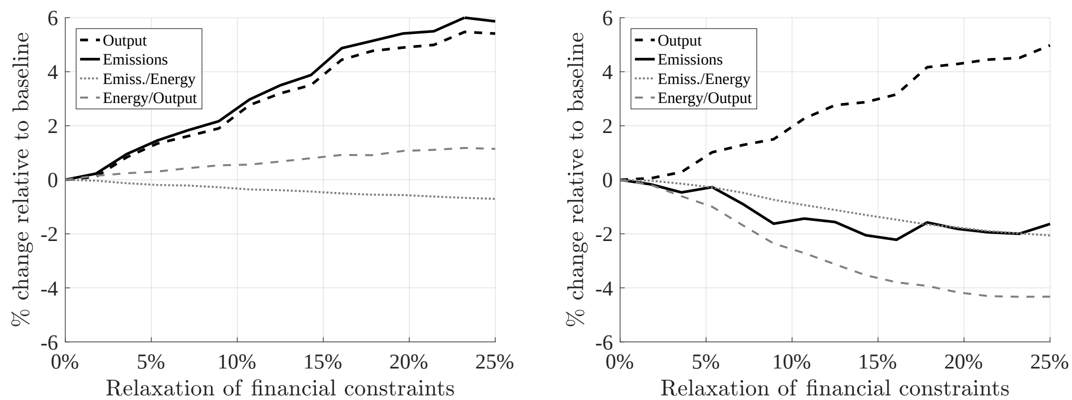
Unbiased policies do not lead to a reduction in aggregate emissions (Panel a). Although capital efficiency improves—as indicated by a decline in E/N—this effect is more than offset by increases in capital intensity (E/Y) and output growth. Consequently, emissions rise more rapidly than GDP. This finding is consistent with the estimates we obtained when fully lifting financial constraints, as shown in Table 4.

In contrast, well-targeted green-biased credit policies can reduce emissions while simultaneously supporting output growth (Panel b). Targeting greener vintages enhances both capital efficiency—lowering emissions per unit of energy—and capital productivity—reducing energy intensity per unit of output. For example, a 20% relaxation of the constraint in panel (b) lowers emissions by 2% and raises GDP by 4%.

[19] Solving for the second-best optimal mix of policies is beyond the scope of the paper, which would require a detailed modeling of the costs of financial policies and carbon pricing. Instead, we simply compare the properties of different policy mixes.

Figure 6: Effect of green credit policies

(a) Unbiased credit policy ($\underline{\omega}_{\chi} = 0$) (b) Green-biased credit policy ($\underline{\omega}_{\chi} = 1.1$)



Notes: Panel (a) shows the impact on emissions, output and the decomposition terms of relaxing the borrowing constraint parameter χ by up to 25%, which corresponds to an increase in the parameter ω_{χ}. The degree of green bias of the policy is held fixed at $\underline{\omega}_{\chi} = 0$, meaning that the policy does not include any green bias. Panel (b) shows the same impact of relaxing the borrowing constraint parameter χ by up to 25%, fixing the degree of green bias of the policy at $\underline{\omega}_{\chi} = 1.1$, which corresponds to the greenest vintage used by unconstrained firms in equilibrium. In both panels, the carbon price is fixed at €30.

Extending coverage to lower-quality vintages leads the outcomes to converge toward those observed under an unbiased policy (Panel a of Figure 6). Appendix Figure A7 illustrates the effects of a less stringent green-biased policy, which also covers the second-best vintage used in the baseline steady state. While the policy improves both capital efficiency and productivity, these gains are insufficient to offset the associated increase in output, so emissions continue to rise.

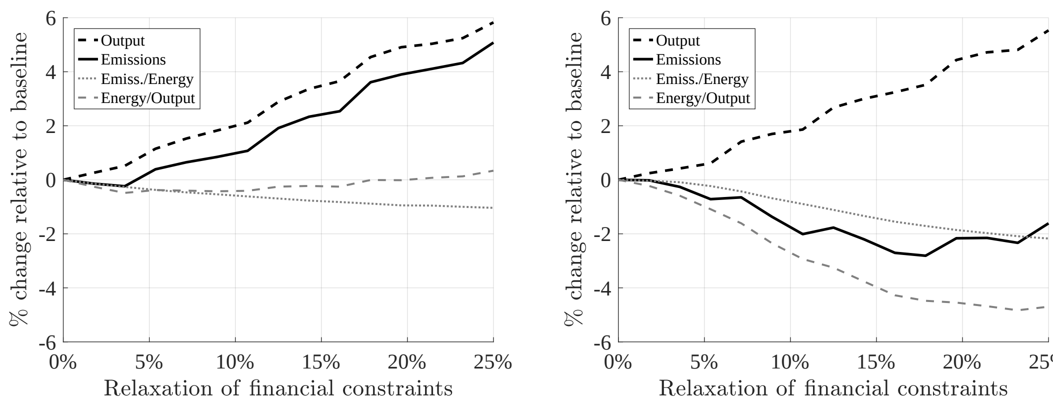
These results suggest that for credit policies to deliver higher output and lower emissions, they need to be very well-targeted, which could be challenging in practice.

Adjusting green credit policies to carbon prices. We now assess the interaction between carbon pricing and green-biased credit policies. We repeat the previous exercise under a higher carbon price, $\tau = €60$. Panel (a) of Figure 7 shows, under a carbon price of $\tau = €60$, the impact of different degrees of relaxation of the policy when targeted to cover vintages at least as good as the greenest vintage ($\underline{\omega}_{\chi} = 1.1$) used by unconstrained firms in the equilibrium with a lower carbon price $\tau = €30$.

We find that, in contrast to the results in the previous exercise (Panel b of Figure 6),

Figure 7: Effect of green credit policies

(a) Green-biased credit policy ($\underline{\omega}_\chi = 1.1$) (b) Green-biased credit policy ($\underline{\omega}_\chi = 1.4$)



Notes: Panel (a) shows the impact on emissions, output and the decomposition terms of relaxing the borrowing constraint parameter χ by up to 25%, which corresponds to an increase in the parameter ω_χ. The degree of green bias of the policy is held fixed at $\underline{\omega}_\chi = 1.1$, which corresponds to the greenest vintage used by unconstrained firms in equilibrium when $\tau = €30$. Panel (b) shows the same impact of relaxing the borrowing constraint parameter χ by up to 25%, fixing the degree of green bias of the policy at $\underline{\omega}_\chi = 1.4$, which corresponds to the greenest vintage used by unconstrained firms in equilibrium when $\tau = €60$.

the targeted green-bias policy no longer reduces emissions. Although both the emission intensity of capital and the energy intensity of output decline, the associated increase in output driven by the policy more than offsets these efficiency gains, leading to higher overall emissions. The reason why the same credit policy does not lead to the same reduction in emissions is that a higher carbon price incentivizes firms to adopt cleaner vintages, thereby shifting the optimal level of technological adoption above what was optimal with a lower carbon price of $\tau = €30$.

To be effective at reducing emissions, the degree of green bias of the credit policy needs to be recalibrated. It should be made more stringent to target the new and higher optimal vintage implied by the higher carbon price. In that case, the policy is able to simultaneously raise output and reduce emissions (Panel b). These results highlight that carbon pricing and green credit policies interact: the effectiveness of the latter depends critically on its alignment with the incentives created by the former.

7.2 Free permits policies to alleviate the cost of carbon pricing

We now analyze the implications of allocating free permits to firms. We start with simulating the impact of an increase in the price of carbon in the absence of free permits. We then introduce a free permits policy that partially or completely offsets the loss in profits implied by higher carbon prices. We show how increasing degrees of generosity of this policy shape the macroeconomic impacts of the carbon price increase.

7.2.1 Carbon price impact without free permits policies

We start by simulating a realistic increase in the price of carbon in Europe from €30 to €60. This provides a useful benchmark for our analysis of free permits policies that follows. This calibration of the increase in the price of carbon is consistent with the average EU carbon permit price in 2024, which was €61.30 per ton[20]. Table 5 shows the implied percentage change in key aggregate variables from the initial to the final steady state equilibrium.

Table 5: Impact of increasing carbon price from €30 to €60 on key variables.

Output	Energy / Output	Emissions / Energy	Emissions
-6.73%	-37.09%	-6.59%	-45.19%

Consistent with previous evidence, we find that carbon pricing is highly effective at reducing emissions. A doubling the price of carbon leads to a reduction of more than 45% in total emissions, achieved mainly through reductions in energy per output, but also through reduction in emissions per energy and through a decline in output. This substantial abatement thus comes at a significant economic cost: output declines by approximately 6.7% relative to the baseline steady state. These GDP losses are within the range of empirical estimates for Europe (Metcalf and Stock, 2023; Kapfhammer, 2023; Känzig, 2023). Känzig (2023) finds the largest economic costs: a carbon price increase leading to a reduction in emissions by 45% would decrease aggregate output by 18%. Our estimates are also consistent with other estimates of the long-run costs of carbon prices derived in calibrated models (Goulder and Hafstead, 2017).[21]

[20] Sourced from `https://carbonpricingdashboard.worldbank.org/`

[21] Our estimate is higher than recent model-based estimates by Finkelstein Shapiro and Metcalf (2023) and Capelle et al. (2024). This body of prior work focuses on real investment frictions, while we focus on

7.2.2 Introducing free permits policies

Current carbon pricing schemes in Europe include free permits policies. Specifically, since the creation of the EU Emissions Trading System (ETS), incumbents firms have been given free permits based on their past emissions.

These transfers play an important role in our environment because of financial constraints. Free permits act as transfers that recycle carbon price revenues back to firms. This in turn allows them to retain more earnings and grow quickly out of their borrowing limits. In other words, free permits policies indirectly alleviate firms' financing constraints, reducing the need for external borrowing and mitigating the investment costs associated with higher carbon prices. The expectation of free permits for potential entrepreneurs also supports entry of new firms.

Modeling free permits policies. We extend the model to allow for the distribution of free permits s_t that offsets the cost of carbon pricing. The firm's flow profit is given by

$$\pi_t(a,z) = \max_{v,k_v,\ell,n} y - w_t\ell - m_t n - \tau_t e - r_{vt}k_v + \tau_t s_t \tag{36}$$

where $\tau_t s_t$ stems from the sale of free permits. The allocation of permits is a function of the firms' emissions e, the change in carbon pricing τ_t relative to its initial level $\underline{\tau} = 30$, as well as the degree of generosity of the free permits policy α:

$$s_t = \alpha\left(1 - \frac{\underline{\tau}}{\tau_t}\right)e. \tag{37}$$

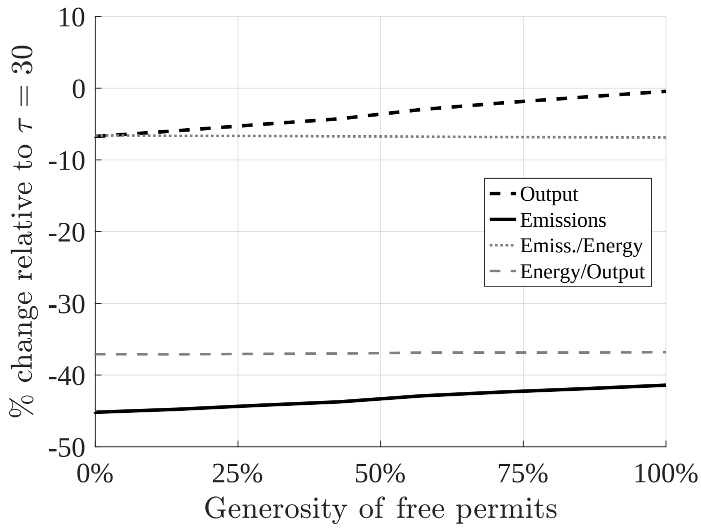
A generous free permits policy with $\alpha = 1$ completely offsets the reduction in profits implied by the increase in the price of carbon, $\tau_t - \underline{\tau}$. Importantly, firms take the path of these permits, s_t, as exogenous. The firms' problem is otherwise the same as in the baseline model.

Results. We now explore how the impact of an increase in the carbon price from €30 to €60 is affected by the degree of generosity of the free permits policy, α. Figure 8 shows the impact of the increase of the carbon price on output and emissions with a free permits policy of increasing degree of generosity $\alpha \in [0,1]$, where 0 corresponds to

financial constraints. This result highlights that while real investment frictions can rationalize a negligible negative impact of carbon prices on GDP, financial constraints cannot.

the case with no free permit and 1 to the case where all carbon price revenues are given back to firms.

Figure 8: Impact of increasing the carbon price from €30 to €60 as a function of the generosity of permits allocation, α



Notes: This figure shows the impact on emissions, output, emission over energy and energy over output of increasing the generosity of the free permits policy after an increase in the carbon price from €30 to €60. The x-axis corresponds to different values of α from 0 to 1, reflecting the fraction of the carbon price that the firms receive back.

Without free permits, $\alpha = 0\%$, the impact of increasing the carbon price is the same as reported in Table 5. A more generous free permits policy, however, allows firms to accumulate net worth and reach their optimal size more quickly, which leads to aggregate TFP gains. In addition, a more generous free permits policy also raises the value of starting a business for potential entrants which raises the entry rate in equilibrium. As a result, both output and emissions increase, with the magnitude of the effect rising monotonically with the generosity of the policy.

Importantly, the combination of a higher carbon price and a free permits policy can achieve more desirable outcomes than either policy alone. For instance, raising the carbon price to €60 while providing a 50% rebate can reduce emissions by over 41% without causing any output loss relative to the €30 price scenario. By contrast, achieving the same emissions reduction solely through the carbon price would require a price of €53, which would lower GDP by more than 6%.

These findings show that free permits policies can improve the overall effectiveness of carbon pricing. In a model with financial frictions and endogenous entry, free permits policies can have significant positive effects on aggregate output.

8 Conclusion

Many jurisdictions have introduced or are considering green financial sector policies—motivated by both political challenges with carbon pricing and financial constraints that prevent investments in frontier technologies. However, the literature offers little evidence about the effectiveness of green financial policies.

We propose a tractable heterogeneous-firm general equilibrium model of investment across different capital vintages subject to financial constraints. Firms in the model face a key tradeoff: they can invest in more efficient capital vintages, but this may reduce their ability to scale their capital stock up towards optimal size.

The model quantitatively matches key patterns in the data. As in the data, less financially constrained firms in the model are more productive, and more productive firms are greener. Simulated results from the model also match our empirical finding that less financially constrained firms are able to cut emissions by more when carbon prices increase.

Counterfactual simulations from our calibrated model lead to three important insights. First, holding carbon pricing fixed, relaxing financial constraints raises total emissions due to the general equilibrium dynamics of output in our model. Second, well-targeted green credit policies can both raise output and lower emissions. However, such policies are only effective in our model when targeting frontier technologies. Third, in contrast to a frictionless environment, the presence of financial constraints and endogenous entry make the distribution of free permits relevant for the equilibrium allocation: distributing free permits to firms largely eliminates the output costs of carbon pricing.

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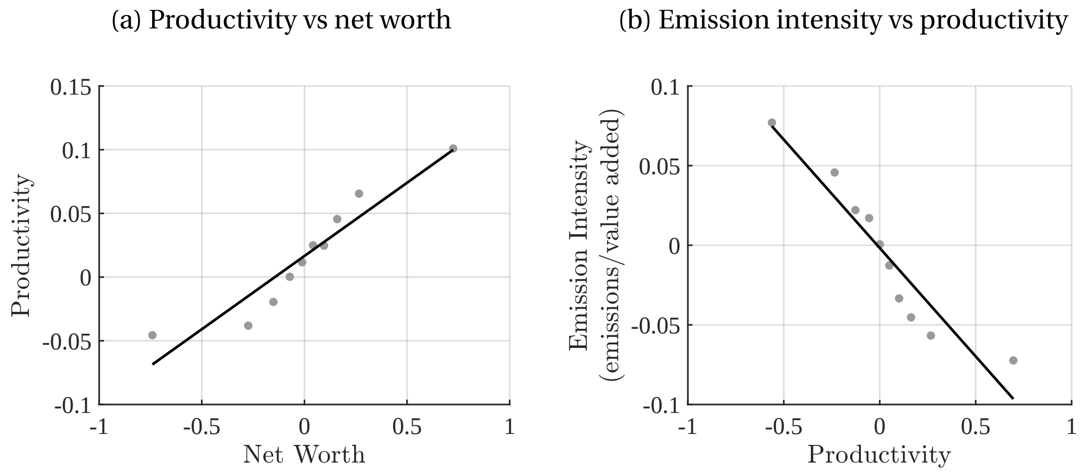
A1 Additional Figures and Tables

A1.1 Stylized facts

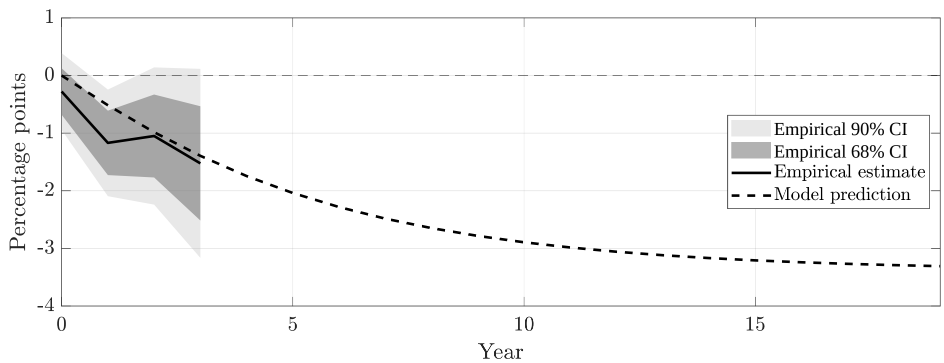
Figure A1: More productive firms are less financially constrained and have lower emission intensity



Notes: The figure displays firm-level binned scatter plots of productivity against net worth (panel a) and of emission intensity (emissions over value added) against productivity (panel b). All variables are in logs, residualized against firm + year fixed effects, and standardized. Only manufacturing firms are included.

A1.2 Differential response to carbon price shocks

Figure A2: Cumulative emissions differential response to carbon price shocks for firms with higher net worth

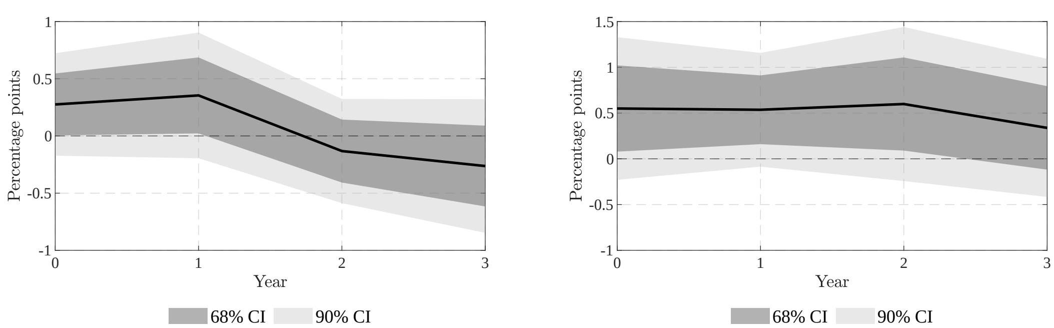


Notes: The solid line shows the cumulative emissions differential empirical impulse response to carbon price shocks for firms with higher net worth relative to firms with lower net worth, estimated using Equation (33). A firm is classified as high net worth if its net worth exceeds the 75th percentile of the distribution within its industry, country, and year. Only manufacturing firms are included. The empirical impulse response is shown over a 3-year horizon, with confidence intervals (CIs) at 68% and 90% levels. Standard errors are clustered at the year level. The dashed line performs the same exercise in the model for a longer horizon.

Figure A3: Cumulative differential response to carbon price shocks for firms with higher net worth

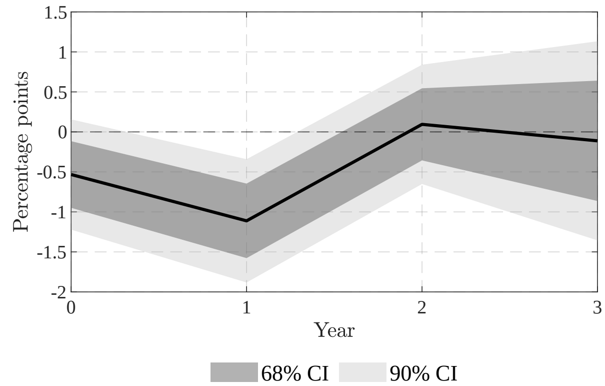
(a) Number of employees

(b) Material costs



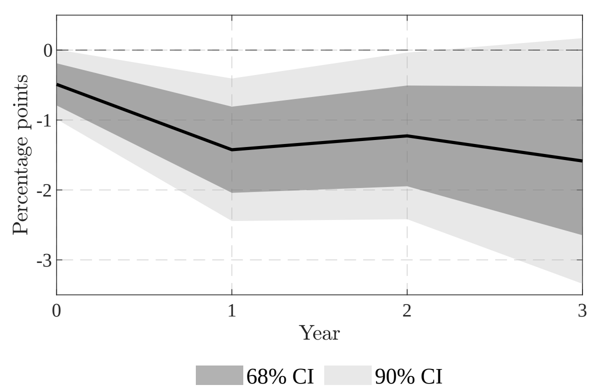
Notes: This figure shows cumulative differential impulse responses for firms with high net worth to carbon price shocks, relative to firms with low net worth, estimated using Equation (33). A firm is classified as high net worth if its net worth exceeds the 75th percentile of the distribution within its industry, country, and year. Only manufacturing firms are included. Impulse responses are shown over a 3-year horizon, with confidence intervals (CIs) at 68% and 90% levels. Standard errors are clustered at the year level.

Figure A4: Cumulative differential emissions response to carbon price shocks for firms with higher net worth, for firms whose number of recorded installations in Orbis does not exceed the number recorded under the EU ETS



Notes: This figure shows cumulative differential impulse responses for firms with high net worth to carbon price shocks, relative to firms with low net worth, estimated using Equation (33). A firm is classified as high net worth if its net worth exceeds the 75th percentile of the distribution within its industry, country, and year. Only manufacturing firms for which the number of recorded installations in Orbis does not exceed the number recorded under the EU ETS. Impulse responses are shown over a 3-year horizon, with confidence intervals (CIs) at 68% and 90% levels. Standard errors are clustered at the year level.

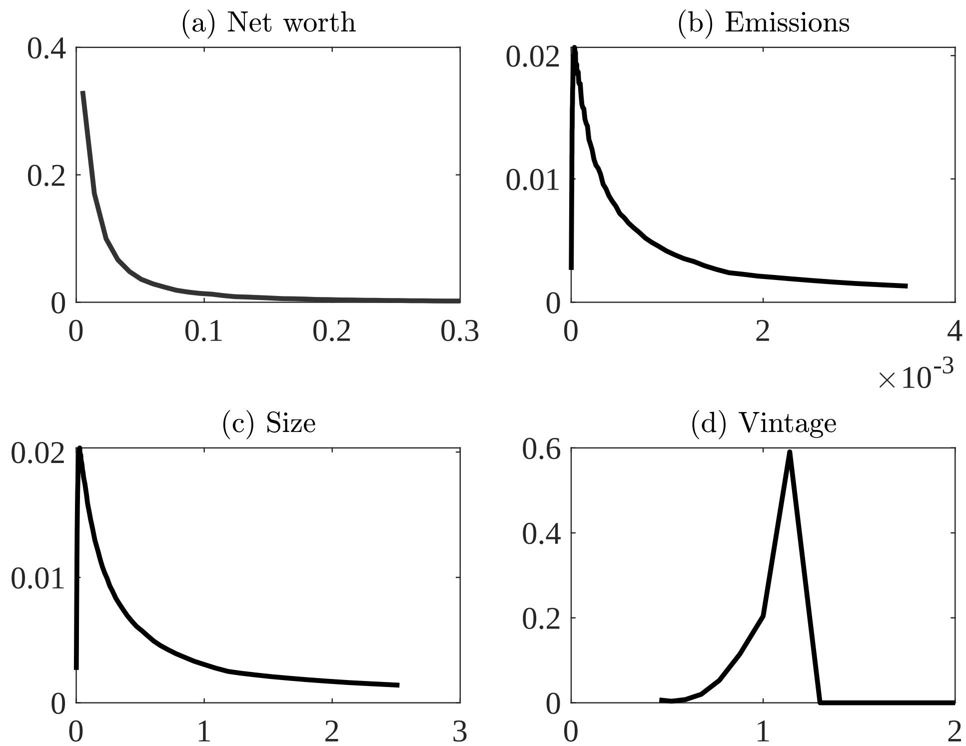
Figure A5: Cumulative emissions differential response to carbon price shocks for firms with higher net worth, adding firm controls



Notes: This figure shows cumulative differential impulse responses for firms with high net worth to carbon price shocks, relative to firms with low net worth, estimated using Equation (33). A firm is classified as high net worth if its net worth exceeds the 75th percentile of the distribution within its industry, country, and year. Firm controls include the lag of the cost of employees, of cash flow, and of EBIT. Only manufacturing firms are included. Impulse responses are shown over a 3-year horizon, with confidence intervals (CIs) at 68% and 90% levels. Standard errors are clustered at the year level.

A1.3 Model: stationary distributions

Figure A6: Equilibrium distributions of net worth, emissions, output and vintages



Notes: This graph shows the equilibrium densities in the market equilibrium over net worth (panel a), emissions (panel b), output (panel c) and vintages (panel d).

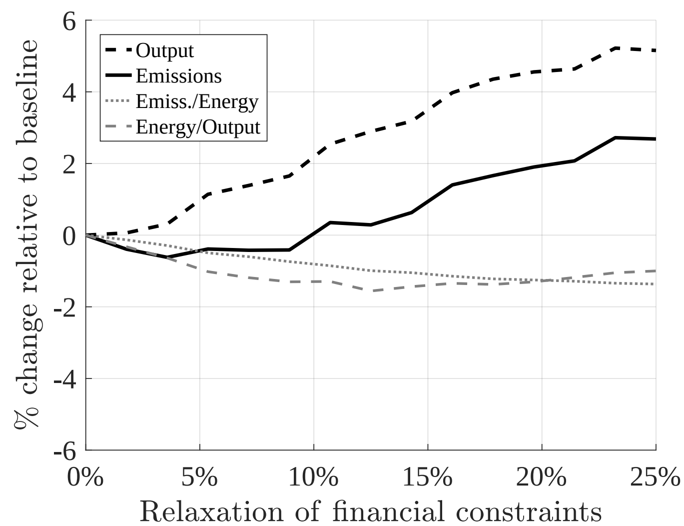
A1.4 Model counterfactuals

Table A1: Steady-state counterfactual comparison

	(1) With financial constraints	(2) No financial constraints	(3) % change
Welfare	-0.46	-0.28	39.13
Output (Y)	4.24	5.57	31.37
Emissions (E)	0.57	0.76	33.33
Energy (N)	9.54	13.12	37.56
Labor (L)	1.00	1.07	7.00
Capital (K)	2.93	3.87	32.08
TFP	3.63	4.38	20.66
Var (log y)	2.55	3.07	20.39
Mass of new firms (p)	0.92	3.24	252.17

Notes: The carbon tax is set to τ = €30 in both counterfactuals. Welfare is the household utility evaluated at the steady-state equilibrium, abstracting from the damages implied by emissions. Values N and E are multiplied by 100. TFP is defined as $Y/(K^{\kappa}L^{\lambda})$. "With financial constraints" refers to the baseline calibrated model's equilibrium and "No financial constraints" refers to the counterfactual steady-state equilibrium with $\chi \to +\infty$. Column (3) reports the percentage change in the reported variables between the model with and without financial constraints.

Figure A7: Effect of low green-biased financial policies



Notes: This figure shows the impact on emissions, output and the decomposition terms of relaxing the borrowing constraint parameter χ by up to 25%, which corresponds to an increase in the parameter ω_χ. The degree of green bias of the policy is held fixed at $\underline{\omega}_\chi = 0.99$, which corresponds to the second greenest vintage used in equilibrium.

A2 Model Derivations

A2.1 Capital and labor

To derive the optimal choice of capital and labor, we write down the Lagrangian associated with the profit maximization problem (13) for a given vintage, v. Denoting Λ the Lagrange multiplier associated with the financial constraint (11), the FOCs are given by

$$(\ell) \quad \lambda \frac{y}{\ell} = w_t \tag{38}$$

$$(k_v) \quad \kappa \frac{y}{k} = r_{vt} \left(1 + \Lambda \left(r^* + \delta - \frac{\dot{q}_{vt}}{q_{vt}}\right)^{-1}\right) + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v} \tag{39}$$

There are two cases. Either the firm is constrained, or it is not. If the firm is unconstrained then we can substitute in the production function:

$$y = z \left(v \frac{\kappa y}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \right)^{\kappa} \left(\frac{\lambda y}{w_t} \right)^{\lambda} \tag{40}$$

This pins down an optimal size as long as $\kappa + \lambda < 1$

$$y = \left(z \left(v \frac{\kappa}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} \tag{41}$$

And profits are given by

$$\pi_t(v,z) = \left(z \left(v \frac{\kappa}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} [1 - \lambda - \kappa] \tag{42}$$

The optimal level of capital in the unconstrained case is given by

$$k_v = \left(\frac{\kappa}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \right)^{\frac{1-\lambda}{1-\lambda-\kappa}} p^{\frac{1}{1-\kappa-\lambda}} \left(z v^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} \tag{43}$$

$$= \frac{\kappa}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \left(z \left(v \frac{\kappa}{r_{vt} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} \tag{44}$$

We summarize these results in the following lemma

Lemma 5 (Unconstrained Capital, Output and Profits)**.** *Assume the firm is financially unconstrained and uses vintage v. The optimal level of capital is given by*

$$k_v = \frac{\kappa}{r_{vt} + \frac{\alpha}{1-\alpha}(m_t + \tau_t v^{-\gamma})} \left(z \left(v \frac{\kappa}{r_{vt} + \frac{\alpha}{1-\alpha}(m_t + \tau_t v^{-\gamma})} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} \tag{45}$$

with $\kappa + \lambda < 1$. Output is given by

$$y = \left(z \left(v \frac{\kappa}{r_{vt} + \frac{\alpha}{1-\alpha}(m_t + \tau_t v^{-\gamma})} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} \tag{46}$$

and profits are given by

$$\pi_t(v, z) = \left(z \left(v \frac{\kappa}{r_{vt} + \frac{\alpha}{1-\alpha}(m_t + \tau_t v^{-\gamma})} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right)^{\frac{1}{1-\kappa-\lambda}} [1 - \lambda - \kappa]. \tag{47}$$

In the case where the firm is constrained, we have $k_v = \frac{\chi a_t}{q_{vt}}$ and output and labor are given by

$$y = z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda y}{w_t} \right)^{\lambda} \iff y = \left[z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right]^{\frac{1}{1-\lambda}} \tag{48}$$

$$\ell = \left[z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right) \right]^{\frac{1}{1-\lambda}} \tag{49}$$

And profits are given by

$$\pi_t(v,z) = \left[z \left(v \frac{\chi a_t}{q_{vt}} \right)^{\kappa} \left(\frac{\lambda}{w_t} \right)^{\lambda} \right]^{\frac{1}{1-\lambda}} [1-\lambda] - \chi a_t \left(r^* + \delta - \frac{\dot{q}_{vt}}{q_{vt}} + \frac{m_t + \tau_t v^{-\gamma}}{\eta_v q_{vt}} \right) \tag{50}$$

A2.2 CES technology

In this subsection, we solve for the optimal input decision, the effective cost of capital and the reduced-form expression for output when the technology is CES. We use these expressions in the code for the quantitative version of the model.

We start with solving the subproblem of the optimal mix of energy and capital. The problem of the firm is to minimize costs of energy and capital:

$$\begin{aligned} \min_{n,k} \quad & (m + \tau_e v^{-\gamma}) \times n + r_v \times k_v \\ s.t. \quad & \left(\alpha^{\frac{1}{\epsilon}} n^{1-\frac{1}{\epsilon}} + (1-\alpha)^{\frac{1}{\epsilon}} k_v^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \geq \bar{\bar{k}} \end{aligned}$$

for some constant $\bar{\bar{k}}$.

Denoting the λ the lagrange multiplier associated with the technological constraint.

$$\begin{aligned} m + \tau_e v^{-\gamma} + \lambda \left(1 - \frac{1}{\epsilon} \right) \alpha^{\frac{1}{\epsilon}} n^{-\frac{1}{\epsilon}} \left(\alpha^{\frac{1}{\epsilon}} n^{1-\frac{1}{\epsilon}} + (1-\alpha)^{\frac{1}{\epsilon}} k_v^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} &= 0 \\ r_v + \lambda \left(1 - \frac{1}{\epsilon} \right) (1-\alpha)^{\frac{1}{\epsilon}} k_v^{-\frac{1}{\epsilon}} \left(\alpha^{\frac{1}{\epsilon}} n^{1-\frac{1}{\epsilon}} + (1-\alpha)^{\frac{1}{\epsilon}} k_v^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} &= 0 \end{aligned}$$

Taking the ratio gives

$$\begin{aligned} \frac{m + \tau_e v^{-\gamma}}{r_v} &= \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1}{\epsilon}} \left(\frac{n}{k_v} \right)^{-\frac{1}{\epsilon}} \\ \Rightarrow n &= k \left(\frac{m + \tau_e v^{-\gamma}}{r_v} \right)^{-\epsilon} \frac{\alpha}{1-\alpha} \end{aligned}$$

Substituting for n in the sum of capital and energy cost, we obtain the effective cost of capital

$$(m+\tau_e v^{-\gamma}) \times k\left(\frac{m+\tau_e v^{-\gamma}}{r_v}\right)^{-\epsilon}\frac{\alpha}{1-\alpha}+r_v \times k_v = \left(\left(\frac{m+\tau_e v^{-\gamma}}{r_v}\right)^{-\epsilon}\frac{\alpha}{1-\alpha}+r_v\right)k_v$$

Substituting for n in the production function, we obtain

$$\begin{aligned} y &= z\,(v\Omega_v k_v)^{\kappa}\,\ell^{\lambda} \\ \Omega_v &= \left(\alpha^{\frac{1}{\epsilon}}\left(\left(\frac{m+\tau_e v^{-\gamma}}{r_v}\right)^{-\epsilon}\frac{\alpha}{1-\alpha}\right)^{1-\frac{1}{\epsilon}}+(1-\alpha)^{\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\alpha\left(\left(\frac{m+\tau_e v^{-\gamma}}{r_v}\right)^{-\epsilon}\frac{1}{1-\alpha}\right)^{1-\frac{1}{\epsilon}}+(1-\alpha)^{\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

A2.3 Households

The household's objective is to maximize the integral of discounted utility

$$U_t = \int_t^{\infty} e^{-\rho(\tau-t)}\,u(c_\tau,\ell_\tau)\,d\tau \qquad (51)$$

where ρ is the discount rate.

The household faces a budget constraint. Her earnings depend on her supply of labor to firms ℓ_t at a wage w_t per unit of time. She can also save in a risk-free bond b_t with instantaneous real rate of return r_t^*. Finally, she receives a transfer $\mathscr{D}_t$, which includes the dividends distributed by incumbent firms, the net worth of exiting firms, net of the entry costs paid by starting firms, and the receipts of the carbon price. We describe these terms in more detail below. The household's financial wealth evolves as follows

$$\dot{b}_t = r_t^* b_t + w_t \ell_t - c_t + \mathscr{D}_t \qquad (52)$$

where we normalize the price of final goods to 1.

The optimal intratemporal and intertemporal decision give the household labor supply schedule and the Euler equation below

$$\ell_t = \left(\frac{w_t}{\psi_2}\right)^{\frac{1}{\psi_3}} \quad \text{and} \quad \frac{\dot{c}_t - \psi_2 \ell_t^{\psi_3} \dot{\ell}_t}{c_t - \psi_2 \frac{\ell_t^{1+\psi_3}}{1+\psi_3}} = \frac{1}{\psi_1}(r_t^* - \rho). \tag{53}$$

PUBLICATIONS

Financial Constraints and the Effectiveness of Green Financial Policies
Working Paper No. WP/2025/269