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Puttable and Extendible Bonds: Developing Interest Rate Derivatives for Emerging Markets

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Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

This paper analyzes the price stabilizing properties of puttable and extendible bonds, their potential to help develop interest-rate derivative markets, and their use by governments. Their stabilizing properties imply that, when bond prices fall, prices for puttable and extendible bonds fall by less. Their embedded options work as a cushion and replicate the trading gains from hedging long-term bonds with interest rate derivatives. These bonds can help develop interest-rate derivative markets in developing countries and eventually increase demand for long-term government bonds. Informal evidence from OECD countries suggests that these bonds were useful in the 1980s, when interest rates were volatile.

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I. INTRODUCTION

To tap financial markets, borrowers have a broad range of instruments from which to choose. Puttable and extendible bonds are one example.[2] These bonds give investors the right to sell them back at par to the borrower at a predetermined date or to extend their term beyond a specified date. They are common among private sector issuers in developed countries and in international financial markets, but not among public sector issuers both in developed and emerging markets.

In this paper, we analyze key characteristics of puttable and extendible bonds: the price stabilizing properties of the bonds, the potential of the bonds to start interest rate derivative markets, and the use of the bonds by governments. We argue that such bonds have interesting price stabilizing properties. Merton (1995) shows that puttable bonds are economically equivalent to a portfolio of discount bonds and a risk-free short-term bond. When discount bond prices fall, the put option kicks in and reduces the interest rate risk exposure incurred by investors. By contrast, when discount bond prices increase, the put option has no value and investors increase their risk exposure. Therefore, their price change is limited to a risk-free short-term bond when the put option is in-the-money and tracks a discount bond when the put option is out-of-the-money. We extend Merton's insight by examining other underlying bonds, such as coupon bonds and bonds subject to credit risk. We find that Merton's result remains intact: When prices of coupon bonds or bonds subject to credit risk fall, prices of puttable and extendible bonds written on these bonds fall less.

Due to these price stabilizing properties, we claim that puttable and extendible bonds can take advantage of the convexity gains from volatility in emerging markets. The embedded put and call options work as cushions and replicate the trading gains from convexity when investors hedge their long positions in long-term bonds by short selling short-term bonds and futures contracts. As IOSCO (2002) shows, these interest rate derivative instruments are not available to investors in emerging markets and most investors are hence deprived of benefiting from the convexity gains. Using puttable and extendible bonds would thus be a way to temporarily overcome obstacles in the development of long-term government bond markets.

We suggest that governments can consider using puttable and extendible bonds as part of a debt management strategy that also includes the development of interest rate derivative markets for government bonds.[3] The informal historical evidence from OECD countries shows that some countries used puttable and extendible bonds in the 1980s when short-term interest rates were high and volatile, the debt-to-GDP ratio was high, and bond futures and

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[2] Puttable bonds are also known as put bonds or retractable bonds.

[3] See De Broeck, Guillaume, and Van der Stichele (1998) for a discussion of the introduction of futures on government bonds in mature markets.

short selling bonds were not common. Favero and Giavazzi (2002), for example, mention the use of puttable bonds by Italy in the 1980s and also suggest them for lengthening the debt maturity in Brazil.

This does not mean that all government bonds should be puttable. We should caution that traders may be tempted to increase the price volatility of the underlying bond so that they increase the probability of exercising the option embedded in the bond. An analogous example is the stock options given to managers in the 1990s. Managers had an implicit interest in the increase of price volatility so that they could exercise their stock.[4] In addition, if all bonds were puttable or extendible, the exercise of the put or call option could lead to serious liquidity problems for governments.

Even though some would argue that the fiscal implication of early redemption in the case of puttable or extendible bonds is important, we do not think that it is significantly different from that generated when the government is obliged to buy back its option-free debt before maturity. The Brazilian experience in 2002 is an example of government concern for stability in the secondary markets for bonds and of intervention to buy back debt before its maturity. Therefore, the embedded put and call options make explicit what is already assumed to be an implicit guarantee that the government will buy back its paper. This should help reduce market jitters and avoid overshooting or overreaction of bond markets to price shocks.

We also believe that puttable and extendible bonds can signal to the private sector that the government is committed to macroeconomic stability and fiscal discipline since it is not in the government's interest for the private sector to exercise the embedded option. For example, when the government sells such bonds, it knows that they may be redeemed at sometime in the future when it is least convenient for the government. Governments would then need to make sure that, ex ante, their fiscal position is in order or that they have the fiscal resources to pay for the exercise of the options.

This paper contains four sections. The next section describes dynamic hedging and characterizes puttable and extendible bonds written on discount and coupon bonds with and without credit risk. Section III describes events in Brazil during 1999–2002 that led to a reduction in the share of outstanding discount bonds in the total stock of public debt and simulates puttable bonds written on the Brazilian C-Bond. Section IV looks at the international experience with the public issues of puttable and extendible bonds. The last section concludes with directions for future research.

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[4] See Tufano (1996) for the argument in the gold mining industry.

II. THE STABILIZING PROPERTIES OF PUTTABLE AND EXTENDIBLE BONDS

As noted, markets for short selling bonds and futures contracts are not available to investors in emerging markets, and investors are deprived of benefiting from the convexity gains.

How can puttable and extendible bonds replicate the convexity gains when investors hedge their long positions in long-term bonds by short selling short-term bonds and futures contracts? This section answers this question by analyzing dynamic hedging strategies and the stabilizing properties of puttable and extendible bonds. We also incorporate the credit risk aspects of bonds.

A. Hedging Long Positions in Long-Term Bonds

Let P_t dollars be invested in a portfolio consisting of a long-term bond, B_t^L, and a short-term bond, B_t^S:

$$P_t = \theta_1 B_t^L + \theta_2 B_t^S , \tag{1}$$

where B_t^L and B_t^S are the prices for the long-term and short-term bonds, and θ_1 and θ_2 are their respective number of units. The value of the portfolio P_t changes as time passes because of changes in B_t^L and B_t^S:

$$dP_t = \theta_1 dB_t^L + \theta_2 dB_t^S . \tag{2}$$

We assume that a one-factor interest rate model describes the dynamics of the short-term interest rate.[5] This assumption implies that there is only one source of randomness in equation (2) since the stochastic differential equations for the short- and long-term bonds have diffusion terms with the same Wiener process.[6] So we can eliminate all the randomness in equation (2) by selecting the appropriate quantities of each asset in the portfolio. We set them equal to:

$$\theta_1 = \frac{\sigma_S}{\left(\sigma_S - \sigma_L\right) B_t^L} P_t \text{ and } \theta_2 = -\frac{\sigma_L}{\left(\sigma_S - \sigma_L\right) B_t^S} P_t , \tag{3}$$

where σ_L and σ_S are the diffusion parameters for the long-term and short-term bonds, respectively.

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[5] See Neftci (2000a) and Rebonato (1998) for a review of one-factor interest rate models. Appendix I shows how the parameters describing the dynamics for discount bonds are a function of the parameters characterizing the dynamics for the short-term interest rate.

[6] See Neftci (2000b) for a justification for a one-factor model to describe the short-term structure of interest rates in emerging markets.

The positions in each asset are then long in the long-term bond and short in the short-term bond. This portfolio is then risk-free and should also yield a return equal to a risk-free investment to avoid arbitrage:

$$r_t \left(\theta_1 B_t^L + \theta_2 B_t^S \right) dt = \theta_1 dB_t^L + \theta_2 dB_t^S ,$$

(4)

where r_t is the risk-free short-term interest rate.

Dividing both sides of expression (4) by θ_1, we can rewrite it as:

$$r_t \left(B_t^L + \frac{\theta_2}{\theta_1} B_t^S \right) dt = dB_t^L + \frac{\theta_2}{\theta_1} dB_t^S .$$

(5)

In Appendix I we show that the relative quantity ratio θ_2/θ_1 can be approximated by the first derivative B_y^L, which measures how much the price for the long-term bond changes as its yield-to-maturity changes by 1 percentage point. This approximation allow us to rewrite expression (5) as:

$$r_t \left(B_t^L + B_y^L B_t^S \right) dt = dB_t^L + B_y^L dB_t^S ,$$

(6)

where B_y^L is also known as the hedge ratio.

Note that since there exists a negative and nonlinear relationship between the price of a bond and its yield-to-maturity, the hedge ratio B_y^L is negative and changes as the yield changes. If the yield increases, the hedge ratio B_y^L decreases and vice-versa. To keep the long-term bond hedged, we need to rebalance the number of units of the short-term bond in the portfolio as the yield changes. This rebalancing strategy is called a dynamic hedging strategy, requires no net investment, and is self-financing. In addition, the rebalancing strategy allows the investor to get a risk-free return on his portfolio that is generated from the trading gains (or convexity gains).

Figure 1 - Use of Interest Rate Derivatives by Emerging Markets
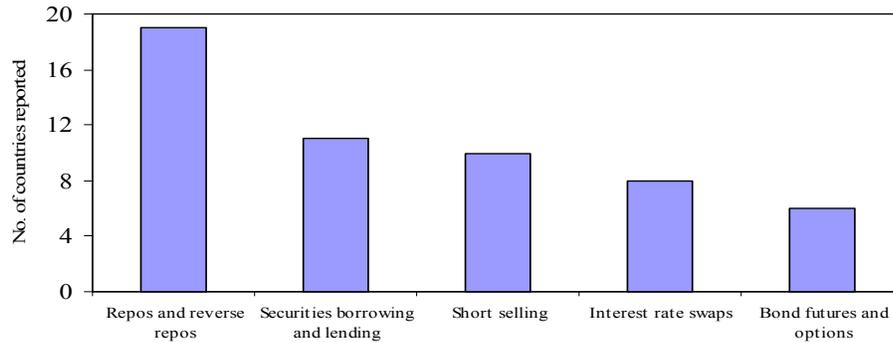
Source: IOSCO (2002).

The dynamic hedging strategy above seems a straightforward operation but cannot be easily performed in many emerging markets. It is effective only if we can short sell the short-term bond. As Figure 1, compiled from IOSCO (2002), shows, only 10 out of 22 emerging market countries allow short selling bonds.[7] Indeed, about the same number of countries allow securities borrowing and lending, which is a precondition for short selling. Therefore, few investors in emerging markets can perform dynamic hedging strategies and extract the convexity gains necessary to cover losses from price fluctuations on their bond holdings.

In addition, where short selling is allowed, investors might not be able to short sell because they are afraid of being caught in a short squeeze, that is, when it is costly to buy the collateral (bonds) to close out the short position. The majority of countries surveyed in IOSCO (2002) have repos and reverse repos, which can help investors acquire the bonds. However, if these instruments are not actively used because of tax treatment or other impediments and if liquidity in the secondary markets for the collateral is low, investors can be cornered. Short selling then becomes impractical and, as a consequence, dynamic hedging strategies based on them are unfeasible.

Investors can make dynamic hedging strategies feasible not only by short selling short-term bonds but also by using bond futures. In this case, investors sell an appropriate number of bond futures written on the underlying long-term bond to hedge one unit of the long-term bond. As the hedge ratio changes through time, the number of futures contracts also needs to be adjusted accordingly so that the portfolio remains delta-neutral or fully hedged. Even though the dynamic hedging strategy using bond futures also seems simple, few emerging markets have bond futures. Therefore, investors cannot benefit from the convexity gains that offset the losses investors incur in their long positions when bond prices fluctuate.

B. Puttable and Extendible Bonds Written on Discount and Coupon Bonds

Few governments in the developing world are aware that it is possible to mimic markets for short selling bonds and bond futures using puttable and extendible bonds. Merton (1995) suggests that governments can issue puttable and extendible bonds in addition to the usual "vanilla" instruments to mimic open market operations. In this section we use Merton's analysis of the stabilizing properties of puttable bonds written on discount bonds to suggest their use for mimicking markets for short selling short-term instruments.

Puttable bonds are bonds that give their holders the right to sell them back to the government for a fixed *strike price*, E, if the price of the underlying U-maturity bond, B_t, falls below the

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[7] The 22 countries in IOSCO (2002) are Argentina, Bangladesh, Brazil, Taiwan Province of China, El Salvador, Hungary, India, Indonesia, Kenya, Korea, Lithuania, Malaysia, Mauritius, Pakistan, Peru, Singapore, Slovenia, South Africa, Thailand, Trindad and Tobago, Tunisia, and Turkey.

strike price E at time T, where $T \leq U$. At time T, when the put option expires, puttable bonds have a price equal to:

$$F_T = Max(B_T, E).$$ (7)

A puttable bond, then, is equivalent to a portfolio of a straight bond and a put option on the underlying bond. Since the right to put back the bond to the government has positive time value, puttable bonds have a higher price and are issued at a lower discount or coupon than a straight bond. One can interpret the price of the put option as what the government charges the private sector for stabilizing financial markets and selling protection that is not widely available.

Similarly, extendible bonds give the holder the right to extend the term of the bond on a fixed date at a predetermined rate. So an extendible bond can be thought of as a short-term bond coupled with a call option to buy a long-term bond at a price E on the predetermined date T, while the puttable bond is a long-term bond coupled with a put option to sell the bond at a price E on the predetermined date T. In case of European options, due to put-call parity, the valuation of both bonds should be more or less equivalent.[8] As a consequence, the analysis that follows also applies to extendible bonds.

Since a puttable bond contains a derivative written on the underlying bond, the price F_t of a puttable bond depends on time t and the price of the underlying bond B_t :

$$F_t = F(B_t, t).$$ (8)

The functional form of F_t is found by using replicating trading strategies. Let P_t dollars be invested in a combination of the puttable bond F_t and the underlying discount bond B_t:

$$P_t = \theta_1 F_t + \theta_2 B_t,$$ (9)

where θ_1 and θ_2 are units of the puttable bond F_t and the underlying bond B_t, respectively.

The value of the portfolio P_t changes as time passes because of changes in F_t and B_t:

$$dP_t = \theta_1 dF_t + \theta_2 dB_t.$$ (10)

We also assume that one-factor interest rate models describe the dynamics of short-term interest rates in developing countries. So we can also eliminate the randomness in equation (10) by selecting the appropriate quantities of each asset in the portfolio, which are equal to:

$$\theta_1 = 1 \ and \ \theta_2 = -F_1,$$ (11)

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[8] See Kalotay and Abreo (1999).

where F_1 stands for the first derivative of the puttable bond F_t with respect to the underlying bond price B_t and is also known as the hedge ratio.[9] As we are short in the underlying bond and long in the puttable bond with the right proportions, random fluctuations cancel out and the portfolio is risk-free. To avoid arbitrage, it should then have a rate of return equal to a risk-free investment:

$$r_t P_t dt = dF_t - F_1 dB_t ,$$ (12)

By using equations (9) and (11) in equation (12) and rearranging terms in the resulting equation, we can note that puttable bonds are "economically equivalent" to a portfolio of F_1 units of the discount bond and $(F_t - B_t F_1)$ units of the risk-free short-term bond:

$$dF_t = F_1 dB_t + r_t \left(F_t - F_1 B_t \right) dt .$$ (13)

This economic equivalence results in a particular price dynamics for puttable bonds. Since $0 \le F_1 \le 1$ and F_t is twice continuously differentiable, with $F_{11} > 0$, as the price B_t for the discount bond falls, F_1 also falls.[10] This is equivalent to a repurchase of discount bonds by the government, which decreases the interest rate exposure incurred by investors. The reverse is also true. When the price B_t for the discount bond increases, F_1 also increases, which is equivalent to holding more discount bonds. Investors are thus more exposed to interest rate risk. Because of these characteristics, Merton (1995) describes puttable bonds as "the equivalent of a dynamic, 'open market,' trading operation without any need for actual transactions."

What is key to the price dynamics in equation (13) is the automatic cushion mechanism provided by the embedded put option. When discount bond prices fall due to interest rate increases, the value of the put option kicks in and offsets partially the capital loss. The put option then works as a cushion against capital losses. In the extreme case, when the underlying bond price has fallen enough and the put option is in-the-money, a puttable bond behaves like a short-term bond.[11] On the other hand, when discount bond prices rise, the put option is out-of-the-money and has little effect on puttable bond price changes. In this case, puttable bonds are similar to the underlying discount bonds, with their price changes limited to the price changes of discount bonds.

In equation (13), note that the second term on the right-hand side, $r_t(F_t - F_1 B_t)dt$, is similar to the risk-free return obtained from a dynamic hedging strategy with short selling or bond futures. When investors hold a puttable bond, they do not need to buy the underlying

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[9] See Appendix.

[10] See Hull (2000) and Merton (1990). The Appendix shows that the function F_t is found by solving a partial differential equation subject to some boundary conditions.

[11] See Fabozzi (2001).

discount bond and short sell short-term bonds or bond futures to hedge and manage interest-rate risk exposures. The puttable bond is able to mimic trading in these markets and replicate the *trading gains from convexity* (or *convexity gains*) used to offset losses on the long position in the discount bond. Since few emerging markets allow short selling, securities borrowing and lending, repos, reverse repos, and bond futures, puttable bonds would overcome restrictions in boosting demand for long-term government bonds.

In the case of coupon bonds, the no-arbitrage condition includes an extra term that captures coupon payments both on the underlying coupon and puttable bonds:

$$r_t P_t dt = dF_t - F_1 dB_t + \left(C_F + F_1 C_B \right) dt \,, \tag{14}$$

where C_B and C_F are the coupon payments for the underlying and puttable bonds, respectively.[12] We can rearrange the extra terms on the right hand-side in equation (15) and the economic interpretation of a puttable bond would remain the same. An investor holding a puttable bond with coupon payments has a net exposure (excluding coupon payments) equivalent to F_1 units of the underlying coupon bond and $(F_t \text{-} B_t F_1)$ units of the short-term bond:

$$dF_t = F_1 \left(dB_t - C_B dt \right) + \left[r_t \left(F_t - B_t F_1 \right) - C_F \right] dt \,. \tag{15}$$

Since the embedded put option has an implicit value for investors, governments can offer puttable bonds with lower coupons than the ones for underlying coupon bonds. The hybrid nature of a puttable bond with coupons implies that the price of a puttable bond is equal to the price of a coupon bond plus the put option written on the coupon bond. In this sense, puttable bonds are a cheaper source of financing for governments. The government essentially charges investors for the insurance policy provided.

C. Credit Risk Aspects

The previous analysis assumed that government bonds are exempt from credit risk. In this section we incorporate the credit risk aspect of government bonds into our analysis.

We assume that swap rates provide a set of default-free interest rates. Hull, Predescu, and White (2002) argue that even though U.S. treasury bonds do not have credit risk, other factors such as liquidity, taxation, and regulation affect them. They thus favor the use of swap rates as risk-free interest rates. We can extend their argument to emerging markets,

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[12] See Merton (1990). The Appendix shows that dynamics for discount and coupon bonds are slightly different. The latter includes the coupon rate in the drift component. The pricing formula for the puttable bond written on the coupon bond would also contain the coupon payment rate.

where a swap could be even less risky than a treasury bond because the counterparty is a highly rated financial institution.

Assuming different credit risk for different financial instruments, we extend the previous analysis to a puttable bond written on a risky bond. As in Duffie and Singleton (1999), we replace the default-free short-term interest rate in the pricing formula for our puttable bonds by an interest rate adjusted for the default risk:

$$B_t = E_t^Q \left(\exp\left(-\int_{s=t}^{T} \left(r_s + h_s L_s \right) ds \right) Max\left(B_T, E \right) \right), \qquad (16)$$

where E_t^Q is the expectation operator under the risk-neutral probabilities; h_t is the exogenous intensity or hazard rate process; and L_t is the exogenous loss process given by $L_t = 1 - R_t$, where R_t is the recovery process.[13] Equation (16) states that the price of a puttable bond is the present value of the put option payoff at time T, discounted at the default-adjusted rate $(r_t + h_t L_t)$ instead of the short-term interest rate r_t. If both the puttable and underlying bonds do not pay coupon and if the puttable bond at time t is a function of the underlying bond at time t, $F_t = F(B_t, t)$, then the no-arbitrage condition implies that:

$$dF_t = F_1 dB_t + \left(r_t + h_t L_t \right)\left(F_t - F_1 B_t \right) dt . \qquad (17)$$

As before, puttable bonds are equivalent to a portfolio of F_1 units of the underlying bond and $(F_t - F_1 B_t)$ units of a short-term bond adjusted by a premium for the credit risk incurred by bondholders. Price changes for puttable bonds would still be limited to the price changes for the underlying bond—which includes credit risk in its dynamics—when prices for the underlying bond go up, and would be limited to a default-adjusted short-term interest rate when prices for the underlying bond go down. With credit risk, puttable bonds would still be able to mimic markets for short selling short-term bonds and bond futures.

III. MIMICKING MARKETS FOR INTEREST RATE DERIVATIVES ON GOVERNMENT BONDS

In this section, we highlight events in Brazil that affected the government debt composition during 1999–2002. Our objective is to show what happens when markets for futures contracts and short selling bonds are missing. We then simulate a puttable bond written on Brazilian C-Bonds from January 1, 1997, to October 30, 2002 to show how price changes for puttable bonds would be less than price changes for C-Bonds and to mimic the convexity gains that

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[13] See the Appendix for the stochastic and partial differential equations for the discount bond.

mutual funds would have extracted if interest rate derivative markets had been used in 2002.[14]

A. The Brazilian Government Bond Markets

In November 1999 the Brazilian government embarked on a program to lengthen the term and duration of its debt securities and to reduce its financing costs.[15] The program included more than twenty measures to be implemented gradually. The measures were intended to allow better debt and cash management by the treasury, to introduce new market transactions with government securities, to improve secondary market liquidity, and to increase central bank consultations with the private sector.

In addition to the measures announced in 1999, the treasury also set objectives for debt management in 2000.[16] The treasury sought to minimize costs over the long run while limiting market and liquidity risks. It pursued this objective by lengthening the debt maturity in public auctions, gradually substituting indexed debt by discount bonds, developing a term structure of interest rates, and standardizing debt instruments. As a result of the partial implementation of the measures, the adoption of debt management objectives and strategies, and the decreasing interest rates after the 1999 Brazilian crisis, the average maturity and share of outstanding discount bonds in the total stock of public debt improved in 2000 as can be seen in Figures 2 and 3.[17]

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[14] C-Bonds are the most traded and liquid Brazilian Brady bonds in international financial markets.

[15] See Central Bank of Brazil (2002d).

[16] See National Treasury of Brazil (2001) and International Bank for Reconstruction and Development and International Monetary Fund (2001) for details on the design of debt management objectives and strategies.

[17] See Central Bank of Brazil (2001). In the first quarter of 2000 the Brazilian treasury was able to sell six-month and 1-year discount bonds in public auctions.

Figure 2. Brazil: Composition of Government Securities,
June 1994–December 2002
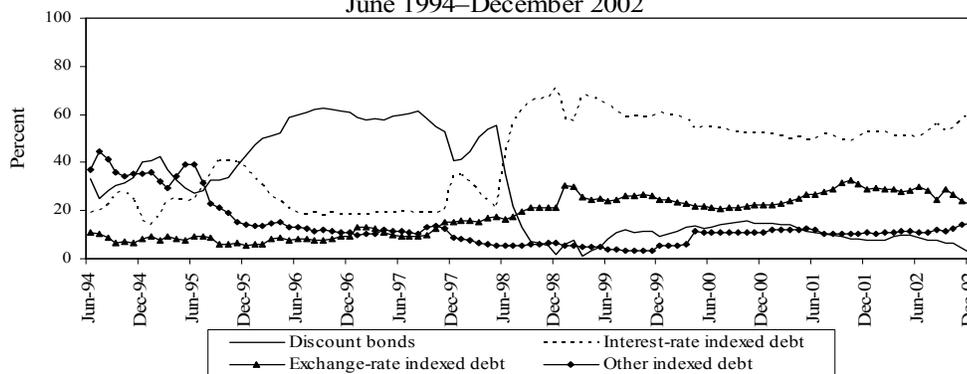
Source: Central Bank of Brazil.

However, the debt management improvements in 2000 were negatively affected by the Argentine crisis and the U.S. recession in 2001. With an increase in interest rates, the Brazilian treasury faced a dilemma in 2001: Whether to issue discount bonds with shorter maturities\ or to increase the share of indexed debt with longer maturities.[18] Given the high liquidity risk at the time, the treasury mainly relied on reducing the share of discount bonds in the total stock of public debt.[19]

Figure 3. Brazil: Average Maturity of Outstanding Discount
Bonds, July 1996–December 2002
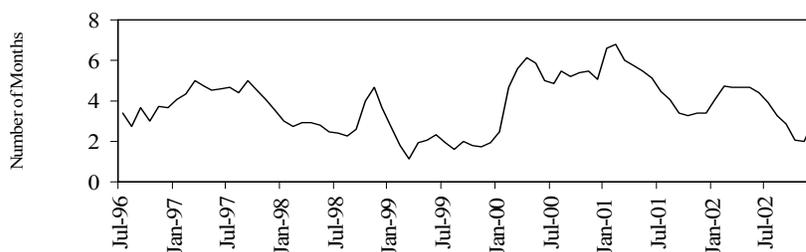
Sources: Central Bank of Brazil and National Treasury of Brazil.

After a calm first quarter, Brazilian financial markets suffered more turmoil in 2002. The uncertainty surrounding the presidential elections, the difficulties in financing the balance of payments during the year, and the implementation of mark-to-market accounting rules for mutual funds in May 2002 led to high levels of discounts on the indexed debt and an increased volatility in discount bond prices.[20] Investors reacted to this volatility in

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[18] See National Treasury of Brazil (2002).

[19] See Central Bank of Brazil (2002a).

[20] See Central Bank of Brazil (2002d).

government bond markets and the drop in earnings in mutual funds by fleeing from mutual funds—from May 2002 to August 2002, around R$ 55 billion was withdrawn.[21]

Authorities in Brazil responded to the run on mutual funds by shortening the debt maturity and loosening the fair-value accounting rules. In particular, the debt management strategy for discount bonds was to issue shorter maturities.[22] Given the high uncertainty in Brazilian financial markets, authorities also decided to exempt from the fair-value accounting rule assets with maturity of less than one year, making all other treasury securities unattractive.[23]

Even though the measures introduced in November 1999 acknowledged the important role of derivative instruments in increasing liquidity in secondary markets for government securities, the use of derivatives by the private sector did not occur. In December 1999, the central bank consolidated all laws concerning forward, repos (and reverse repos), and options on fixed income securities.[24] This consolidation was expected to create flexible trading conditions in government bond markets. However, except for repos (and reverse repos), IOSCO (2002) still listed Brazil as a country that did not allow short selling, securities borrowing and lending, bond futures and options, and interest rate swaps to enhance liquidity in government bond markets.[25]

Indeed, in the middle of the run on mutual funds in 2002, authorities in Brazil raised the possibility that the central bank would sell put options on bonds.[26] Authorities felt a need to introduce a derivative instrument to hedge the interest rate risk exposure by investors in government bonds. Puttable and extendible bonds contain embedded options that work as a cushion and offset losses due to the fall in bond prices. They would then help protect investors from the interest rate risk.

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[21] Figures are from the National Treasury of Brazil.

[22] See Central Bank of Brazil (2002d).

[23] See Securities and Exchange Commission of Brazil (2002).

[24] See Central Bank of Brazil (1999).

[25] One-day bond forward contracts are negotiated in the electronic system SIBEX run by the Rio de Janeiro Stock Exchange (BVRJ). Hybrid short-term interest-rate futures contracts are traded in the Brazilian Mercantile & Futures Exchange (BMF). The object of the futures contracts is the average interbank market interest rate. So futures contracts are an imperfect hedge for government bonds.

[26] See Central Bank of Brazil (2002b) and (2002e).

B. Some Weaknesses of Emerging Bond Markets

Consider a sovereign *default-free* discount bond with maturity U. Let its time t arbitrage-free price be denoted by $B(r_t, t; U)$. It can be shown that the price of this discount bond, without any embedded option, will satisfy the partial differential equation:[27]

$$B_2 - r_t B_t + \left(a(r_t,t) - b(r_t,t)\lambda_t \right) B_1 + \frac{1}{2}b(r,t)^2 B_{11} = 0,\qquad(18)$$

with the boundary condition:

$$B(r_U, U; U) = 1.\qquad(19)$$

Here the $a(r_t,t)$ and $b(r_t,t)$ are the drift and diffusion parameters in the stochastic differential equation for the short-term interest rate r_t, and λ is the market price of risk:

$$\frac{(\alpha - r_t)}{\sigma} = \lambda_t,\qquad(20)$$

where α and σ are the drift and diffusion parameters in the stochastic differential equation for the bond price.

This equation shows an important source of income for sovereign bond traders if B_{11} and/or $b(r_t,t)$ is large enough. In fact, in the above partial differential equation the first three terms can be regarded as the "daily" net cost of holding the bond position, apart from convexity gains. The last term can be interpreted as daily gains from convexity. These latter gains are due to volatility in the underlying short-term interest rate r_t.

What is important for us is that these daily gains have two components. One is the result of the earnings from passing time, and the other is due to volatility in the r_t. These latter gains are known as *convexity* or *Gamma* gains and play a significant role in markets for highly rated sovereign bonds.

Convexity gains basically suggest the following. If the bond's second derivative with respect to r_t and the interest rate volatility is high enough, then oscillations in the short rate will lead to significant trading gains for the long position holder. It may seem paradoxical, but a sudden increase in volatility may lead to an increase in the price of the long bond. In fact, this statement should be qualified. *Everything else being the same*, an increase in interest rate volatility will *increase* the bond price. The critical statement here is, ceteris paribus, because this means *credit risk* remains the same. There is no doubt that if the credit risk of a bond were to increase, the bond value would go down. It is possible that increased interest rate volatility increases credit risk and hence has a negative effect on the bond price. Thus, in the

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[27] See the Appendix.

discussion below we omit such effects in order to emphasize the effect of convexity on the bond price.

C. Measuring the Effect of C-Bond Volatility

An example of convexity gains is shown for Brazil using the daily returns of the Brazilian C-bond from January 1, 1997, to October 30, 2002.[28] We use the Vasicek model for the short-term interest rate and calculate the closed form formula for a discount bond having the same yield structure. The Vasicek model gives the price of a default-free pure discount bond as:

$$C(t,U) = \frac{\left(1 - \exp\left(-a(U - t)\right)\right)}{a}$$

$$A(t,U) = \exp\left(\frac{\left(C(t,U) - U\right)\left(a^2 d - 1/2 b^2\right)}{a^2} - \frac{b^2 C(t,U)^2}{4a}\right) \qquad (21)$$

$$B(t,U) = A(t,U)\exp\left(-C(t,U)r_t\right) \qquad ,$$

where a, d, and b are the parameters of the mean reverting dynamics assumed for the short-term rate r_t:

$$dr_t = a(d - r_t)dt + br_t dW_t. \qquad (22)$$

For the Brazilian C-bond we used the values:

$$b = 0.50, \ a = 0.90, \ d = 0.11, \qquad (23)$$

and evaluated the discount curve $B(r_t, t; U)$ at the mean interest rate 11 percent. We used two levels of volatilities. In one case (absolute) volatility was taken as 5 percent a year, and in the other, volatility was taken as 15 percent. The two discount curves are shown in Figure 4. We see that increased volatility leads to much higher long bond prices. In fact, the price of a 30-year bond would be about 30 percent higher in a more volatile environment. The price of a 10-year bond would increase by about 10 percent.

The critical point is this: in an environment where high volatility does not lead to credit risk changes, an increase in interest rate volatility would increase bond prices and is not detrimental to portfolios that are marked to market. But for this be true, there must be liquid tools and markets that bond traders can use to extract the Gamma gains expressed by the component $1/2 B_{11} b(r,t)^2$ in the partial differential equation shown above. This is done by taking positions in liquid interest rate futures or in short-term bonds. These positions are in

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[28] We use the Brazilian C-Bond for liquidity reasons and data availability.

the opposite direction to the position in the long bond. For example, a long bond *holder* will *short sell* B_1 units of the futures contract. Adjustments in this "delta-hedge" would extract the Gamma gains.

Figure 4. Discount Bond Prices as a Function of Interest Rate Volatility
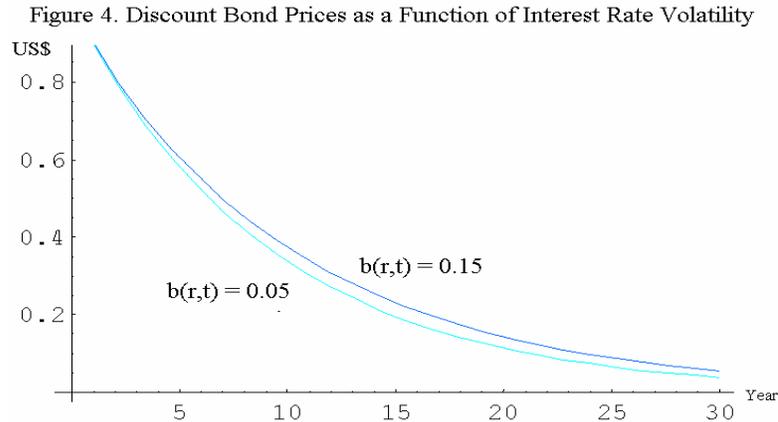
In emerging markets there are two major hurdles for extracting convexity gains. First, there are no markets for liquid interest rate derivatives. Second, interest rate volatility leads to significant increases in credit risk and lowers the bond's price. This implies that traders in emerging markets cannot benefit from the positive effects of volatility increases, yet suffer from the negative ones. Thus, changes in volatility have significantly different effects on the bond portfolios. A newly introduced mark-to-market requirement would lead a bond portfolio manager to lower his exposure to long-term interest rate movements. This increases bond market volatility and at the same time causes shortening of the maturities on new issues.

D. The Effect of Puttable Bonds

One advantage of puttable bonds is that the embedded option will *increase* in value when volatility increases. This increase in option's value would play the role of "Gamma trading" that is available to traders in mature markets and would balance out the negative effect of increased volatility on credit risk. Hence, puttable options can be used to introduce tradable convexity gains in emerging markets.

Figure 5. Brazil: C-Bond Price, January 1, 1997 – October 30, 2002
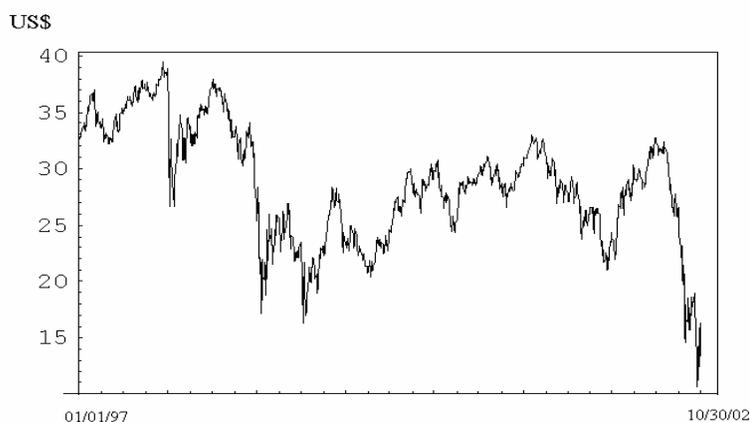
To see the effect of puttable bonds on Brazilian bond portfolios, we use daily returns data on the Brazilian C-Bond to calculate the price volatility of a 10-year discount bond with the *same* yield as the C-Bond. Figure 5 shows the explosive price volatility during the recent election and after the mark-to-market requirement in 2002, when the bond decreased by about 50 percent.

Figure 6. Brazil: Puttable Bond Price, January 1997 – October 30, 2002
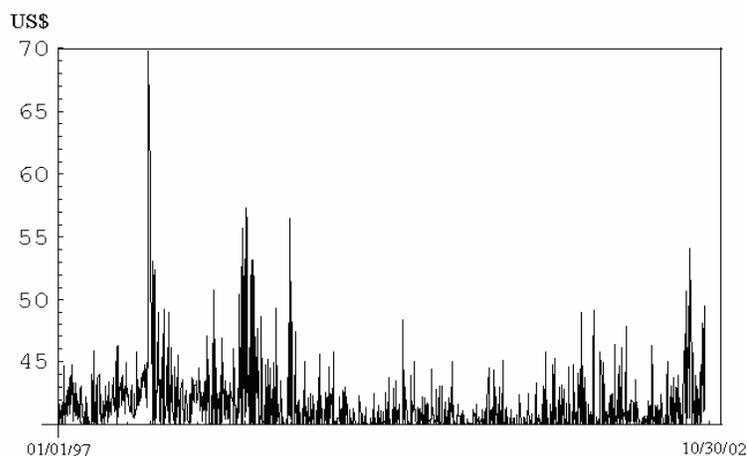
Using the Black-Scholes formula we then calculate an approximate value for the same bond that contains a 1-year put option with the strike price set at 40. Figure 6 shows how the addition of a put option in this fictitious bond would have stabilized bond portfolios in Brazil and, in fact, would have led to *higher* overall bond values in Brazil in the latest period.[29]

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[29] Of course a more detailed study would model the bond dynamics jointly with sovereign credit dynamics for Brazil and then simulate the effects of adding put options. But this

IV. EXPERIENCE WITH PUTTABLE AND EXTENDIBLE BONDS

What is the historical evidence on the use of puttable and extendible bonds by governments? Governments in OECD countries have used such bonds as a way to reduce their financing costs. Table 1 reports puttable and extendible bonds issued by Austria, Belgium, Canada, France, Germany, Italy, the Netherlands, the United Kingdom, and the United States. Note that, except for Belgium, most countries issued puttable and extendible bonds in the 1980s, when interest rates were high and volatile. Countries with high debt levels such as Belgium and Italy have used puttable and extendible bonds as well.

Table 1. Puttable and Extendible Bonds in Selected OECD Countries

Country	Type	Characteristics	Issue
Austria	Puttable 1/	Floating-rate notes	Currently in use
Belgium	Puttable	Floating-rate notes	Issued until 1991. New issues after 1994
	Extendible	State coupon bonds (*Bons d'État*); 5 years extendible to 7 years	Currently in use
Canada	Extendible	Medium-term coupon bonds	Issued during 1967–82, and traded between 1967–87
France	Extendible	Coupon bonds (*Emprunts à Fenêtre et Prorogeables*)	Issued during 1983–86
Germany	Puttable	Loans against borrowers' notes (*Schuldscheine*), typically 3 to 10 years	Currently being redeemed
Italy	Extendible	Treasury certificates with option (*CTO*), fixed coupon; 3- and 4-year securities extendible with 3 and 4 years more, respectively	Issued during 1988–92
Netherlands	Extendible	Coupon bonds	Issued until 1984
United Kindgom	Extendible	Coupon bonds and inflation-indexed bonds	From early 1980s, with last redemption in 1999
United States	Puttable	Three coupon bond issues in 1933	Last issue redeemed in 1962

Sources: Bank of England; Austrian National Bank; Deutsche Bundesbank, Belgian, Dutch, German, and Italian Ministries of Economy and Finance; Athanassakos (1996); Bliss and Ronn (1995); Board of Governors of the Federal Reserve System (1998); Branion (1995); McLean (1993); Missale (1999); Urich (1991).

Note: 1/ Publicly guaranteed bond.

Belgium, in particular, is an interesting case. The Belgian treasury issues state notes, which are long-term securities with fixed coupons. Only private individuals, nonprofit organizations, and public institutions have access to its primary market. Currently, there is one outstanding note that is extendible: the 5-year state note, which gives the right to the debt holder of extending the maturity for two extra years (to 7 years). Table 2 shows the evolution of state notes during 1996-2001. State notes rose from 0.64 percent and 0.50 percent of the long-term and total domestic government debt, respectively, in 1996 to 3.31 percent and 2.85 percent in 2001.

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would, by itself, be a major calibration effort. Our purpose here is to look simply at an example and get some rough idea about the impact of puttable bonds.

Table 2. Belgium: Outstanding State Notes, 1996–2001

| | State Notes | | |
	In millions of Euros	% of long-term bonds	% of domestic bonds
1996	1,106	0.64	0.50
1997	2,286	1.30	1.02
1998	3,880	2.12	1.73
1999	4,913	2.46	2.08
2000	6,119	2.93	2.52
2001	7,122	3.31	2.85

Source: National Bank of Belgium.

If we compare price movements for Belgian linear bonds (the underlying asset) and state notes (the extendible bond), we would expect Belgian state notes to move less than Belgian linear bonds when prices fall. Equation (15) above shows that an investor holding a state note with coupon payments has a net exposure (excluding coupon payments) of F_1 units of the underlying linear bond and $(F_t - B_t F_1)$ units of the short-term bond. Since $0 \leq F_1 \leq 1$ and F_t is twice continuously differentiable, with $F_{11} > 0$, as the price of the linear bond falls, F_1 also falls. This is equivalent to a repurchase of linear bonds by the government, which decreases the interest rate exposure incurred by investors. Table 3 shows some basic statistics for the price changes (from September 14, 2000, to October 25, 2002) of the 4.2 percent state note, issued in 1998, due September 4, 2003 and the 4 ¾ percent linear bond, issued in 1999, due September 28, 2005. The evidence suggests that price changes for the 4.2 percent state note were less than those for the 4 ¾ percent linear bond.

Table 3. Belgium: Descriptive Statistics for Price Changes of the 1998 State Note and the 1999 Linear Bond, September 14, 2000–October 25, 2002

	1998 State Note 1/	1999 Linear Bond 2/
Mean	0.01	0.01
Median	0.00	0.01
Maximum	0.43	1.71
Minimum	-0.36	-1.72
Std. Dev.	0.10	0.20
Skewness	-0.03	-0.19
Kurtosis	2.38	18.80
Observations	552	552

Source: Datastream.

1/ 4.2% State Note, due September 4, 2005.
2/ 4 3/4 linear bond, due September 14, 2005.

Emerging markets have also tapped international markets with puttable bonds. Table 4 shows that even though there was a decline in the issuance of puttable bonds by developing countries during 1997–2001, these bonds still remained an important source of financing for

developing countries. Therefore, puttable bonds have also been used by emerging markets to smooth price changes of their international debt.

Table 4. Sovereign Putable Bonds and Loans by Emerging Markets, 1997–2001

(In millions of U.S. dollars)					
	1997	1998	1999	2000	2001
Putable	3,052	4,064	2,543	1,295	2,062
Of which are loans	555	358	94	265	0
Number of transactions	22	12	12	7	3

Source: IMF (2002).

V. CONCLUSION

This paper analyzes the stabilizing role that puttable and extendible bonds can play in government securities markets. It finds that these bonds are less price sensitive than underlying bonds are to increases in short-term interest rates. When the prices of the underlying bonds fall, puttable and extendible bond prices fall less than those of the underlying bonds.

In addition to their stabilizing properties, puttable and extendible bonds can also mimic markets for short selling short-term bonds and futures. These derivatives instruments are important to investors since they allow them to extract the convexity gains that offset losses in long positions in long-term bonds. However, the derivatives markets that support the hedging of long-term bonds are absent in many emerging markets. Where absent, governments can therefore issue puttable or extendible bonds to help boost the demand for their long-term government bonds.

It appears to us that governments could issue puttable or extendible bonds as a transitory financing source until liquid and transparent interest rate derivative markets are established and as a debt strategy to move from indexed bonds to fixed-interest coupon bonds. Due to their lower price-sensitivity to short-term interest rate changes, puttable and extendible bonds can be an alternative to the issuing of indexed debt in periods of market turbulence. Indeed, historical evidence in OECD countries shows that many countries did use such bonds during the 1980s, when interest rates were high and volatile and countries did not have well established interest rate derivative markets.

PRICING GOVERNMENT BONDS

Discount bonds and hedging

We assume that bond prices with maturity at time U are a function of the short-term risk-free interest rate r_t and time t:

$$B_t = B\left(r_t,\ t;\ U\right). \tag{24}$$

The dynamics for the short-term r_t is given by the stochastic differential equation:

$$dr_t = a\left(r_t,\ t\right) dt + b\left(r_t,\ t\right) dW_t, \tag{25}$$

where the drift $a(r_t,t)$ and the diffusion $b(r_t,t)$ parameters are assumed to be known, and W_t is the standard Brownian motion. We also assume that the instantaneous variance, $b(r_t,t)^2$, is nonstochastic or a function of time.

In this paper, we rely heavily on the assumption that one-factor interest rate models describe movements in the term structure of interest rates in developing countries. This assumption implies that any change in short-term interest rates is followed by a parallel shift in the whole yield curve. This might indeed be a reasonable approximation in emerging markets, where the term structure is short. For pricing puttable or extendible bonds, one-factor interest rate models can be good approximations.

Applying Ito's Lemma to $B(r_t,\ t;\ U)$ and using equation (25) yields:

$$dB_t = \left[B_1 a\left(r_t,t\right) + B_2 + \frac{1}{2}B_{11}b\left(r_t,t\right)^2\right]dt + B_1 b\left(r_t,t\right)dW_t, \tag{26}$$

where B_i and B_{ii} indicate the first and second derivatives with respect to the *ith* argument, respectively. We can rewrite this stochastic differential equation for bonds, of all maturities, as:

$$dB_t = \alpha B_t dt + \sigma B_t dW_t, \tag{27}$$

where the drift component, αB_t, equals:

$$\alpha B_t = B_1 a\left(r_t,t\right) + B_2 + \frac{1}{2}B_{11}b\left(r,t\right)^2, \tag{28}$$

and the diffusion component, σB_t, is:

$$\sigma B_t = B_1 b\left(r_t,t\right). \tag{29}$$

Merton (1990) shows that bonds of different maturities offer the same risk premia adjusted by the corresponding volatility. So we can define the market price of interest rate risk, $\lambda(r_t,t)$, as:

$$\frac{(\alpha - r_t)}{\sigma} = \lambda_t,$$ (20')

and use it in expressions (28) and (29) to obtain a partial differential equation, which we solve for the bond price, B_t, as a function of the short-term interest rate, r_t. Note that expressions (28) and (29) contain an unknown drift αB_t. To replace αB_t in the equations, we first rewrite the market price of risk as:

$$\frac{B_t(\alpha - r_t)}{b(r_t,t)B_1} = \lambda_t,$$ (30)

which gives:

$$\alpha B_t = r_t B_t + b(r_t,t)B_1\lambda_t.$$ (31)

Then we substitute αB_t in expression (28) by the right-hand side of (31) and rearrange the resulting equation as:

$$B_2 - r_t B_t + \left(a(r_t,t) - b(r_t,t)\lambda_t\right)B_1 + \frac{1}{2}b(r,t)^2 B_{11} = 0.$$ (18')

This is a partial differential equation that can be solved for B_t subject to the boundary condition:

$$B(r_U,U;U) = 1.$$ (19')

To obtain the hedging strategy, let P_t dollars be invested in a portfolio of long-term bonds, B_t^L, and short-term bonds, B_t^S:

$$P_t = \theta_1 B_t^L + \theta_2 B_t^S,$$ (1')

where θ_1 and θ_2 are units of the long-term and short-term bonds, respectively. As time passes, the value of the portfolio P_t changes according to:

$$dP_t = \theta_1 dB_t^L + \theta_2 dB_t^S,$$ (2')

where we describe the dynamics for each bond by a similar stochastic differential equation as in (27).

Since we assume that the short-term interest rate is the only source of randomness in equation (2'), we can eliminate the randomness by selecting the appropriate quantities of each asset in the portfolio. We set them equal to:

$$\theta_1 = \frac{\sigma_S}{(\sigma_S - \sigma_L)B_t^L}P_t \text{ and } \theta_2 = -\frac{\sigma_L}{(\sigma_S - \sigma_L)B_t^S}P_t.$$ (3')

These positions in the two bonds are equivalent to being long in the long-term bond and short in the short-term bond. The portfolio is then risk-free and should also yield a return equal to a risk-free investment to avoid arbitrage:

$$r_t \left(\theta_1 B_t^L + \theta_2 B_t^S \right) dt = \theta_1 dB_t^L + \theta_2 dB_t^S . \tag{4'}$$

Dividing both sides of (4') by θ_1, we can rewrite it as:

$$r_t \left(B_t^L + \frac{\theta_2}{\theta_1} B_t^S \right) dt = dB_t^L + \frac{\theta_2}{\theta_1} dB_t^S . \tag{5'}$$

We can use (3') for θ_1 and θ_2 to write the ratio θ_2/θ_1 as:

$$\frac{\theta_2}{\theta_1} = -\frac{\sigma_L B_t^L}{\sigma_S B_t^S} , \tag{32}$$

which is equivalent to:

$$\frac{\theta_2}{\theta_1} = -\frac{B_1^L}{B_1^S} , \tag{33}$$

where B_1^L and B_1^S stand for the first derivative of the long-term and short-term bond prices with respect to the short-term risk-free interest rate, respectively. If B_t^S is the bond with the shortest maturity available in the market, we assume that $B_1^S = -1$. In this case, any change in the short-term interest rate is fully compensated by a negative price change for the short-term bond; that is, $\delta B_t^S = -\delta r_t$.

As mentioned above, one-factor interest rate models imply that any change in the short-term interest rate that affects the short end of the term structure also affects the long end in the same way. We can then approximate the changes in the short-term interest rate, δr_t, by the changes in the yield-to-maturity of the long-term bond, δy_t^L, and B_1^L by the first derivative B_y^L. This allows us to rewrite (33) as:

$$\frac{\theta_2}{\theta_1} = B_y^L , \tag{34}$$

and (5') as:

$$r_t \left(B_t^L + B_y^L B_t^S \right) dt = dB_t^L + B_y^L dB_t^S , \tag{6'}$$

where B_y^L is also known as the hedge ratio.

Puttable discount bonds

In the case of puttable bonds, their price depends only on the value of the underlying bond B_t:

$$F_t = F(B_t, t).$$ (8')

Using Ito's Lemma, we can find the stochastic differential equation for the puttable bond to equal:

$$dF_t = \left[F_1 \alpha B_t + F_2 + \frac{1}{2} F_{11} \sigma^2 B_t^2 \right] dt + F_1 \sigma B_t dW_t.$$ (35)

Recall that the portfolio P_t is invested in a combination of θ_1 units of the puttable bond and θ_2 units of the underlying discount bond B_t:

$$P_t = \theta_1 F_t + \theta_2 B_t.$$ (9')

The value of the portfolio P_t changes as time passes because of changes in $F(B_t, t)$ and B_t:

$$dP_t = \theta_1 dF_t + \theta_2 dB_t.$$ (10')

Replacing the expressions for dB_t and dF_t from (27) and (35) in equation (10') yields:

$$dP_t = \left[\theta_1 \left(F_1 \alpha B_t + F_2 + \frac{1}{2} F_{11} \sigma^2 B_t^2 \right) + \theta_2 \alpha B_t \right] dt + \left[\theta_1 F_1 + \theta_2 \right] \sigma B_t dW_t.$$ (36)

If we set $\theta_1 = 1$ units of the puttable bond and $\theta_2 = -F_1$ units of the underlying discount bond, then we eliminate all the randomness in the portfolio:

$$dP_t = \left[F_2 + \frac{1}{2} F_{11} \sigma^2 B_t^2 \right] dt.$$ (37)

To avoid arbitrage, the portfolio should yield the same rate of return as other risk-free assets:

$$dP_t = r_t (F_t - F_1 B_t) dt$$ (38)

and:

$$F_2 + \frac{1}{2} F_{11} \sigma^2 B_t^2 = r_t (F_t - F_1 B_t).$$ (39)

Rearranging terms in (39) yields:

$$F_2 - r_t F_t + r_t B_t F_1 + \frac{1}{2} \sigma^2 B_t^2 F_{11} = 0.$$ (40)

This equation is the Black-Sholes-Merton differential equation. The function $F_t = F(B_t, t)$ is found by solving equation (40) subject to the following boundary conditions:

$$F_t / B_t \leq 1 \quad as \quad B_t \to \infty,$$
$$F(0, t) = 0, \quad \quad \quad (41)$$
$$F(B_T, T) = \max(0, E - B_T).$$

Puttable bonds written on coupon bonds

When the underlying bond pays the coupon C_B, the drift component of the stochastic differential equation needs to be adjusted according to:

$$dB_t = (\alpha B_t - C_B) dt + \sigma B_t dW_t. \quad \quad \quad (42)$$

Hence, the drift of the stochastic differential equation for the puttable bond will also contain the coupon payments:

$$dF_t = \left[F_1 (\alpha B_t - C_B) + F_2 + \frac{1}{2} F_{11} \sigma^2 B_t^2 \right] dt + F_1 \sigma B_t dW_t. \quad \quad \quad (43)$$

The corresponding partial differential equation for the puttable bond with coupon C_F is:

$$F_2 - r_t B_t + (r_t B_t - C_B) F_1 + C_F + \frac{1}{2} \sigma^2 B_t^2 F_{11} = 0, \quad \quad \quad (44)$$

and is subject to the same boundary conditions as in (41).

Puttable bonds written on bonds subject to credit risk

If the intensity or hazard rate h_t and the loss process L_t are exogenous, the stochastic differential equation for the defaultable indexed bond is:

$$dB_t = (r_t + h_t L_t) B_t dt + \sigma B_t dW_t. \quad \quad \quad (45)$$

The corresponding partial differential equation is:

$$F_2 - (r_t + h_t L_t) F_t + (r_t + h_t L_t) B_t F_1 + \frac{1}{2} \sigma^2 B_t^2 F_{11} = 0, \quad \quad \quad (46)$$

subject to the same boundary conditions as in (41).

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