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Threshold Effects in the Relationship Between Inflation and Growth

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This paper re-examines the issue of the existence of threshold effects in the relationship between inflation and growth, using new econometric techniques that provide appropriate procedures for estimation and inference. The threshold level of inflation above which inflation significantly slows growth is estimated at 1–3 percent for industrial countries and 11–12 percent for developing countries. The negative and significant relationship between inflation and growth, for inflation rates above the threshold level, is quite robust with respect to the estimation method, perturbations in the location of the threshold level, the exclusion of high-inflation observations, data frequency, and alternative specifications. [JEL E31, 040]

High and sustained output growth in conjunction with low inflation is the central objective of macroeconomic policy. Not surprisingly, the question of the existence and nature of the link between inflation and growth has been the subject of considerable interest and debate. Although the debate about the precise relationship between these two variables is still open, the intensive research on this issue has uncovered some important results and a relatively wide consensus about some aspects of this relationship has been reached. In particular, it is generally accepted now that inflation has a negative effect on medium and long-term growth.[1] Inflation impedes efficient resource allocation by obscuring the signaling

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[1]See Barro (1991), Fischer (1983, 1993), Bruno and Easterly (1998), and Sbordone and Kuttner (1994). This link between low inflation and high growth has also been found by various regional studies, for example, by De Gregorio (1992) for Latin America, Hadjimichael, Ghura, and others (1995) for sub-Saharan Africa, and Fischer, Sahay, and Végh (1996) for transition economies.

role of relative price changes, the most important guide to efficient economic decisionmaking (Fischer, 1993).[2]

If inflation is inimical to growth, it readily follows that policymakers should aim at a low rate of inflation. But how low should inflation be? Should the target inflation be 10 percent, 5 percent, or for that matter, zero percent? More generally, at what level of inflation does the relationship between inflation and growth become negative?

These are the questions that several recent empirical studies have examined, focusing specifically on whether the relationship between inflation and long-run growth is a nonlinear one.[3] In other words, at some (low) rate of inflation, the relationship is positive or nonexistent, but at higher rates it becomes negative. If such a nonlinear relationship exists then it should be possible, in principle, to estimate the inflexion point, or threshold, at which the sign of the relationship between the two variables would switch. The possibility of such a nonlinear relationship was first identified by Fischer (1993), who noted the existence of a positive relationship at low rates of inflation and a negative one as inflation rose (which weakened as inflation increased). Sarel (1996) specifically tested for the existence of a structural break in the relationship between inflation and growth and found evidence of a significant structural break at an annual inflation rate of 8 percent. Below that rate, inflation does not have a significant effect on growth, or it may even show a slightly positive effect. For inflation rates greater than 8 percent, the effect is negative, statistically significant, and strong. Ignoring the existence of this threshold substantially biases the effect of inflation on growth. Ghosh and Phillips (1998), using a larger sample than Sarel's, find a substantially lower threshold effect at 2.5 percent annual inflation rate. They also find that inflation is one of the most important statistical determinants of growth. Christoffersen and Doyle (1998) estimate the threshold level at 13 percent for transition economies. Bruno and Easterly (1998) argue that the negative relationship between inflation and growth, typically found in cross-country regressions, exists only in high-frequency data and with extreme inflation observations. They find no cross-sectional correlation between long-run averages of growth and inflation in the full sample, but detect a negative effect of inflation and growth for inflation rates higher than 40 percent.[4] A useful discussion of previous work on this issue is given in Ghosh (2000).

This paper re-examines the nature of the relationship between inflation and growth, focusing specifically on the following questions:

- Is there a statistically significant threshold level of inflation above which inflation affects growth differently than at lower inflation rates?

[2]It has been argued that what matters for efficient resource allocation is not so much the level of inflation but its variance. However, to the extent that the variance of inflation is positively related to its level—see Bulkley (1984), Ball (1992), Grier and Perry (1996), and Ma (1998)—the latter does affect resource allocation. While theory seems to suggest that the variability of inflation should affect growth more than its level, empirical studies show the opposite result (see Fischer 1993).

[3]See, for example, Fischer (1993), Sarel (1996), Ghosh and Phillips (1998), Christoffersen and Doyle (1998), and Bruno and Easterly (1998).

[4]This finding has been confirmed in a separate study by Easterly (1996).

- Is the threshold effect similar across developing and industrial countries?
- Considering that the studies discussed above arrive at different threshold estimates, are these threshold values statistically different?
- How robust is the Bruno-Easterly finding that the negative relationship between inflation and growth exists only for high-inflation observations and for high-frequency data?

These questions are examined using new econometric methods for threshold estimation and inference.[5] There are two particular econometric issues related to the estimation and inference in models with threshold effects. First, the asymptotic distribution of the *t*-statistic on the threshold variable is nonstandard and requires bootstrap methods to compute its significance level. Second, methods need to be developed to conduct inference in the context of panel models with threshold effects.

I. Data Issues

The dataset includes 140 countries (comprising both industrial and developing countries) and generally covers the period 1960–98. Data for a number of developing countries, however, have a shorter span. Because of the uneven coverage, the analysis is conducted using unbalanced panels. The data come primarily from the World Economic Outlook (WEO) database for the following variables: the growth rate of GDP in local currency in constant 1987 prices, inflation computed as the growth rate of the CPI index, the initial income level measured as the five-year average of GDP per capita in 1987 PPP prices, gross domestic investment as a share of GDP, population growth, the growth rate of terms of trade, and the five-year standard deviation of terms of trade.

Figure 1 shows the relationship between real GDP growth and the logarithm of inflation (π).[6] The data have been smoothed out by reducing the full sample to five observations. The latter are the arithmetic means of five equal subsamples corresponding to increasing levels of inflation.

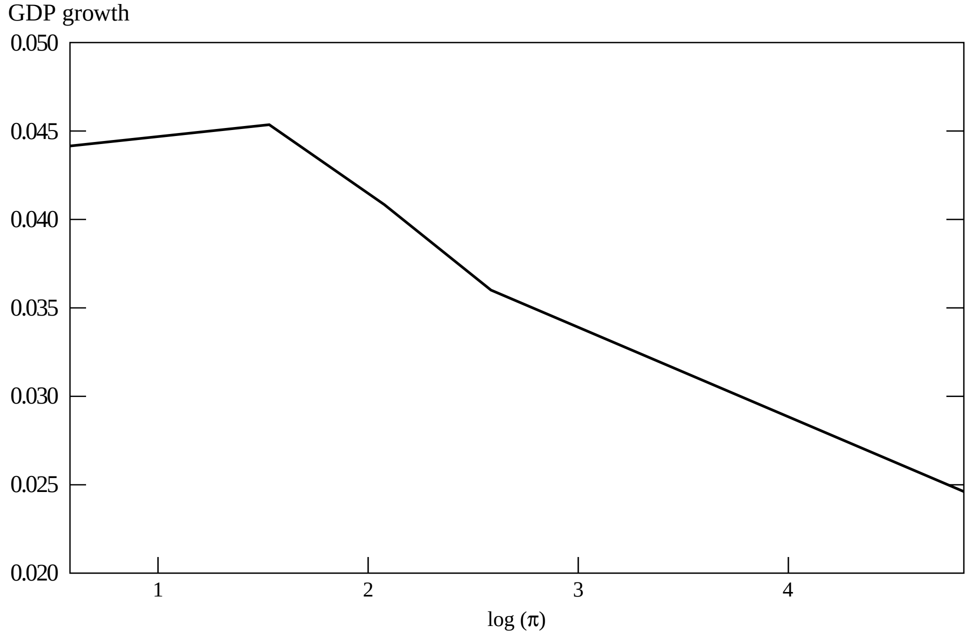
We can see from Figure 1 that the relationship between real GDP growth and the log of inflation is slightly positive for low levels of inflation and becomes negative for higher inflation levels, corroborating the findings of Ghosh and Phillips (1998). Note also that the negative effect of inflation on growth weakens somewhat at higher inflation rates, supporting Fischer's (1993) findings.

Should growth be related to the *level* or the *log* of inflation? The first panel in Figure 2 shows the distribution of inflation across the full sample of countries and time periods. It is clear that the distribution is highly skewed. A regression of real GDP growth on the level of inflation would give much weight to the extreme inflation observations, even though the bulk of the observations correspond to low and medium inflation rates. As suggested by Sarel (1996), the log transformation elim-

[5]These techniques have been developed by Chan and Tsay (1998), and Hansen (1999, 2000).

[6]The use of the log transformation obviously requires handling negative inflation observations (221 observations for annual data and only 9 observations for five-year averaged data). In the next section, a hybrid linear-log function that allows for negative inflation rates will be discussed.

Figure 1. Relationship Between Real GDP Growth and Inflation



inates, at least partially, the strong asymmetry in the inflation distribution (see second panel in Figure 2). In the class of nonlinear models, Ghosh and Phillips (1998) show that the log transformation provides the best fit. Finally, the log transformation can be justified by the fact that its implications are more plausible than those of a linear model. In particular, the linear model implies that *additive* inflation shocks will have identical effects on growth in low- and high-inflation economies, while the log model implies that *multiplicative* inflation shocks will have identical effects on low- and high-inflation economies. For example, in the linear model, an increase in inflation by 10 percentage points will have the same effect on growth in an economy with an initial inflation rate of 10 percent as in an economy with an initial inflation rate of 100 percent. In the log model, a doubling of the inflation rate in those two economies will have the same effect on growth.

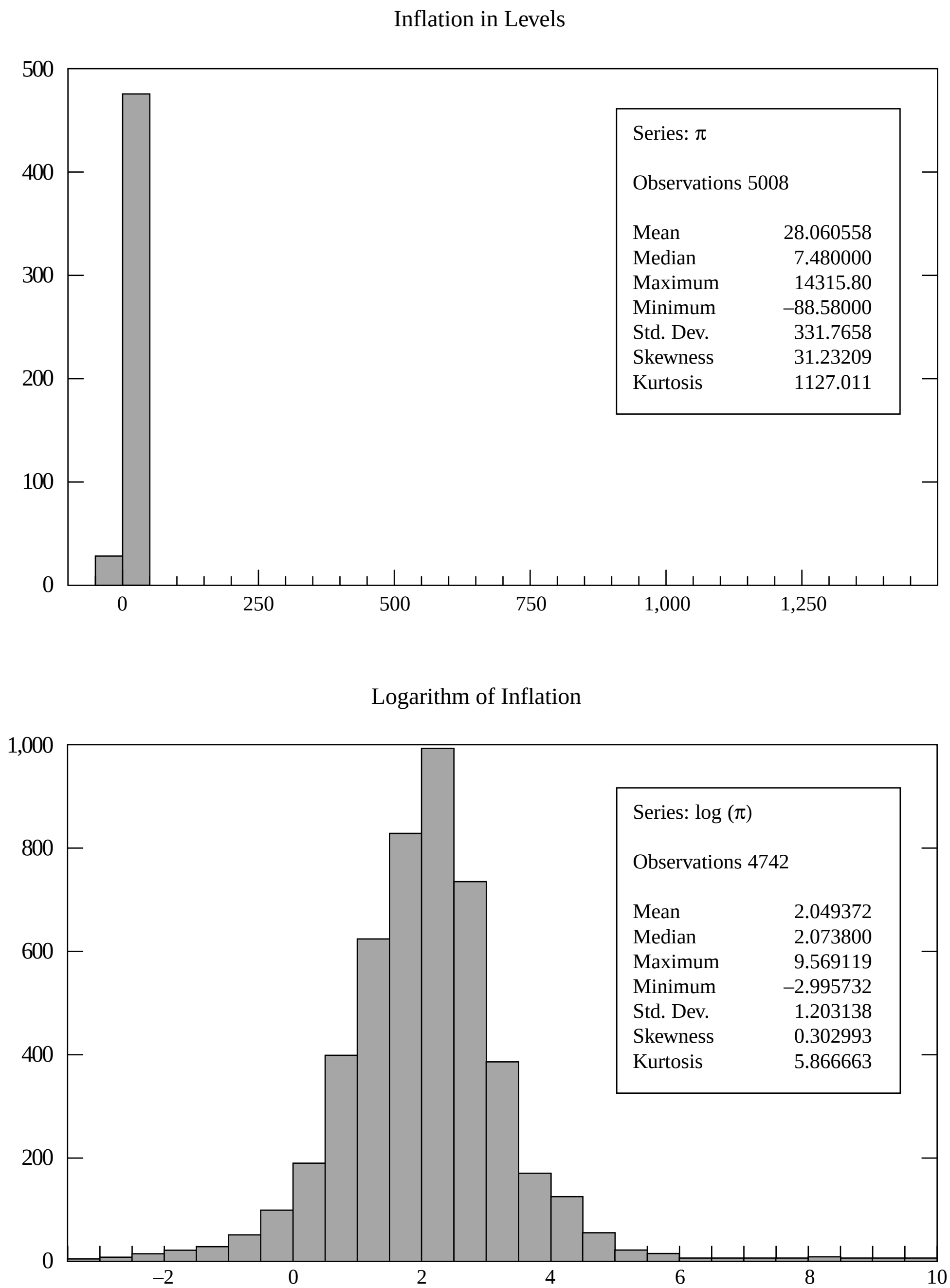
II. Model Specification and Estimation

To test for the existence of a threshold effect, the following model was estimated:

$$d\log(y_{it}) = \mu_i + \mu_t + \gamma_1(1 - d_{it}^{\pi^*})\left\{(\pi_{it} - 1)I(\pi_{it} \leq 1) + \left[\log(\pi_{it}) - \log(\pi^*)\right]I(\pi_{it} > 1)\right\}$$
$$+\gamma_2 d_{it}^{\pi^*}\left\{(\pi_{it} - 1)I(\pi_{it} \leq 1) + \left[\log(\pi_{it}) - \log(\pi^*)\right]I(\pi_{it} > 1)\right\} + \theta' X_{it} + e_{it} \quad (1)$$

$$d_{it}^{\pi^*} = \begin{cases} 1 \; if \; \pi_{it} > \pi^* \\ 0 \; if \; \pi_{it} \leq \pi^* \end{cases} \quad i = 1, \ldots, N; \quad t = 1, \ldots, T$$

Figure 2. Distribution of Inflation



where $d\log(y_{it})$ is the growth rate of real GDP,[7] μ_i is a fixed effect, μ_t is a time effect, π_{it} is inflation based on the CPI index, π^* is the threshold level of inflation, $d_{it}^{\pi^*}$ is a dummy variable that takes a value of one for inflation levels greater than π^* percent and zero otherwise, $I(\pi_{it} \leq 1)$ and $I(\pi_{it} > 1)$ are indicator functions—that is, functions that take the value of one if the term between parentheses is true, and zero otherwise—X_{it} is a vector of control variables which includes investment as a share of GDP (*igdp*), population growth ($d\log(pop)$), the log of initial income per capita ($\log(y_{i0})$), the growth rate of terms of trade ($d\log(tot)$), and the five-year standard deviation of terms of trade (σ_{tot}). The index "*i*" is the cross-sectional index, while "*t*" is the time-series index.

For reasons discussed above, the log of inflation is preferable to the level of inflation as explanatory variable in equation (1). However, the log function does not exist for negative inflation rates. Furthermore, the log function approaches minus infinity for inflation rates close to zero. Thus, the strategy adopted here is to specify a hybrid function of inflation which is linear for values of inflation rates below or equal to one and logarithmic for inflation rates greater than one. That function is:[8]

$$f(\pi_{it}) = (\pi_{it} - 1) I(\pi_{it} \leq 1) + \log(\pi_{\mathrm{it}}) I(\pi_{it} > 1) \tag{2}$$

The first term is simply the level of inflation, π_{it}, multiplied by an indicator function which disregards all observations with inflation rates above one, and thus $f(\pi_{it})$ is equal to $(\pi_{it} - 1)$ for $\pi_{it} \leq 1$. Similarly, the second term is the log of inflation, $\log(\pi_{it})$, multiplied by an indicator function which disregards all observations with inflation rates below or equal to one, and thus $f(\pi_{it})$ is equal to $\log(\pi_{it})$ for $\pi_{it} > 1$. We subtract one from the first term to allow $f(\pi_{it})$ to be continuous at unity, where the function changes from being linear in π_{it} to being log-linear in π_{it}. The function $f(\pi_{it})$ is also continuously differentiable. Consequently, $f(\pi_{it})$ allows us to take into account all observations, including observations with negative inflation rates. Finally, the subtraction of $\log(\pi^*)$ from $\log(\pi_{it})$ makes the relationship between growth and inflation, described by equation (1), continuous at the threshold level π^*.[9]

Note that X_{it} contains only the most important variables among the large set found in the empirical growth literature because very few of these variables pass the robustness tests in Levine and Renelt (1992) and Sala-i-Martin (1997). Furthermore, the model explicitly takes into account the individual country effects through μ_i and the time effect through μ_t. The effect of inflation on GDP growth is given by γ_1 for countries in which inflation is less than or equal to π^* percent, and γ_2 for countries with inflation rates higher than π^* percent.

[7]The growth rate of a variable x is computed as the first difference of $\log(x)$.

[8]We are grateful to the referee for suggesting this particular approach.

[9]Continuity of the relationship given by equation (1) is desirable, otherwise small changes in the inflation rate around the threshold level will yield different impacts on growth depending on whether inflation is increasing or decreasing.

In order to smooth out business cycle fluctuations and focus on the medium- and long-term relationship between inflation and growth, equation (1) has been estimated using five-year averages of the data in the panel of 140 countries and 39 annual observations each. Therefore, the time dimension reduces to eight observations: 1960–64, 1965–69, 1970–74, 1975–79, 1980–84, 1985–89, 1990–94, and 1995–98 (the last observation is an average over four observations only).[10] Potentially the dimension of the panel would be 140 x 8 = 1,120 observations. However, because of missing observations, the dimension of the *unbalanced* panel is smaller.

Estimation Method

If the threshold were known, the model could be estimated by ordinary least squares (OLS). Since π^* is unknown, it has to be estimated along with the other regression parameters. The appropriate estimation method in this case is non-linear least squares (NLLS). Furthermore, since π^* enters the regression in a non-linear and non-differentiable manner, conventional gradient search techniques to implement NLLS are inappropriate. Instead, estimation has been carried out with a method called *conditional least squares,* which can be described as follows. For any π^*, the model is estimated by OLS, yielding the sum of squared residuals as a function of π^*. The least squares estimate of π^* is found by selecting the value of π^* which minimizes the sum of squared residuals. Stacking the observation in vectors yields the following compact notation for equation (1):

$$d\log(Y) = X\beta_\pi + e, \quad \pi = \underline{\pi},...,\overline{\pi} \tag{3}$$

where $\beta_\pi = (\mu_i\ \mu_t\ \gamma_1\ \gamma_2\ \theta')'$ is the vector of parameters and X is the corresponding matrix of observations on the explanatory variables. Note that the coefficient vector β is indexed by π to show its dependence on the threshold level of inflation, the range of which is given by $\underline{\pi}$ and $\overline{\pi}$. Define $S_1(\pi)$ as the residual sum of squares with the threshold level of inflation fixed at π. The threshold estimate level π^* is chosen so as to minimize $S_1(\pi)$, that is:

$$\pi^* = \underset{\pi}{\operatorname{argmin}}\left\{S_1(\pi), \pi = \underline{\pi},...,\overline{\pi}\right\} \tag{4}$$

Inference

It is important to determine whether the threshold effect is statistically significant. In equation (1), to test for no threshold effects amounts simply to testing the null hypothesis H_0: $\gamma_1 = \gamma_2$. Under the null hypothesis, the threshold π^* is not identified, so classical tests, such as the *t*-test, have nonstandard distributions. Hansen

[10]The initial income variable ly_0 is computed as the five-year average of real income per capita in PPP terms for the previous five-year period, allowing the identification of ly_0 under fixed effects.

(1996, 1999) suggests a bootstrap method to simulate the asymptotic distribution of the following likelihood ratio test of H_0:

$$LR_0 = \left(S_0 - S_1\right)/\hat{\sigma}^2 \tag{5}$$

where S_0, and S_1 are the residual sum of squares under H_0: $\gamma_1 = \gamma_2$, and H_1: $\gamma_1 \neq \gamma_2$, respectively; and $\hat{\sigma}^2$ the residual variance under H_1. In other words, S_0 and S_1 are the residual sum of squares for equation (1) without and with threshold effects, respectively. The asymptotic distribution of LR_0 is nonstandard and strictly dominates the χ^2 distribution. The distribution of LR_0 depends in general on the moments of the sample; thus critical values cannot be tabulated. Hansen (1999) shows how to bootstrap the distribution of LR_0.

An interesting question is whether an inflation threshold, for example, of 10 percent is significantly different from a threshold of 8 percent or 15 percent. In other words, can the concept of confidence intervals be generalized to threshold estimates? Chan and Tsay (1998) show that in the case of a continuous threshold model studied here, the asymptotic distribution of all parameters, including the threshold level, have a *normal* distribution.[11] More precisely, define $\Phi = (\mu_i\ \mu_t\ \gamma_1\ \gamma_2\ \theta', \pi^*)$ as the set of all parameters, including the threshold level. Chan and Tsay (1998) show that the NLLS estimate $\hat{\Phi}$ of Φ (described above) is asymptotically normally distributed:[12]

$$\hat{\Phi} \sim N\left(\Phi, U^{-1}VU^{-1}\right) \tag{6}$$

where $U = E(H_{it}H_{it}')$, $V = E(e_{it}^2 H_{it}H_{it}')$, $H_{it} = (-\tilde{X}_{it}, \gamma_1(1 - d_{it}\pi^*) + \gamma_2 d_{it}\pi^*)$, $\tilde{X}_{it}$ is the vector of all right-hand-side variables in equation (1), and NT is the total number of observations. An estimate of U and V are given by
$\hat{U} = \sum_{i=1}^{N}\sum_{t=1}^{T} \hat{H}_{it}\hat{H}_{it}'/(NT)$ and $\hat{V} = \sum_{i=1}^{N}\sum_{t=1}^{T} \hat{e}_{it}^2\hat{H}_{it}\hat{H}_{it}'/(NT)$ with $\hat{H}_{it} = (-\tilde{X}_{it}, \hat{\gamma}_1(1 - d_{it}\pi^*) + \hat{\gamma}_2 d_{it}\pi^*)$.

III. Estimation and Inference Results

Test for Existence of Threshold Effects

The first step is to test for the existence of a threshold effect in the relationship between real GDP growth and inflation using the likelihood ratio, LR_0, discussed above. This involves estimating equation (1) and computing the residual sum of squares (RSS) for threshold levels of inflation ranging from $\underline{\pi}$ to $\overline{\pi}$. The threshold estimate is the one that minimizes the sequence of RSSs. The test for the existence of threshold effects has been conducted using the full sample and two subsamples (industrial and developing countries). The results are summarized in Table 1.

[11]Hansen (2000) derives the asymtotic distribution for the discontinuous threshold model.
[12]For an application of this method, see Cox, Hansen, and Jimenez (1999).

Table 1. Test Results of Threshold Effects

Sample	Search Range for Thresholds	Threshold Estimate (%)	LR_0	Critical Values	Significance Levels
All Countries	{1, 2, 3,…, 100}	11	10.59	7.47	0.001
Industrial Countries	{1, 2, 3,…, 30}	1	8.80	6.63	0.005
Developing Countries	{1, 2, 3,…, 100}	11	10.89	6.21	0.000

Note: The second column gives the range over which the search for the threshold effect is conducted, the third column gives the threshold estimate in percent, the column LR_0 gives the observed value of the likelihood ratio, the fifth column gives the critical values, and the last column gives the corresponding significance level, both computed using the bootstrap distributions (corresponding to the three samples) of LR_0. For a more detailed discussion on the computation of the bootstrap distribution of LR_0, see Hansen (1999).

The second column gives the range over which the search for the threshold effect is conducted. For the full sample, $\underline{\pi}$ =1 percent, $\overline{\pi}$ =100 percent, and the increment is 1 percent, which yields 100 panel regressions of equation (1).[13] The minimization of the vector of 100 RSSs occurs at the inflation level of 11 percent (see Figure 3). Repeating the same procedure for the subsamples yields a threshold estimate of 11 percent for developing countries and 1 percent for industrial countries. Note that the threshold level for industrial countries is much lower than that for developing countries.[14] The column LR_0 in Table 1 gives the observed value of the likelihood ratio. The significance levels have been computed using the bootstrap distributions (corresponding to the three samples) of LR_0.[15] The null hypothesis of no threshold effects can be rejected at least at the 1 percent significance level for all three samples. Thus the data strongly support the existence of threshold effects.

Estimation Results

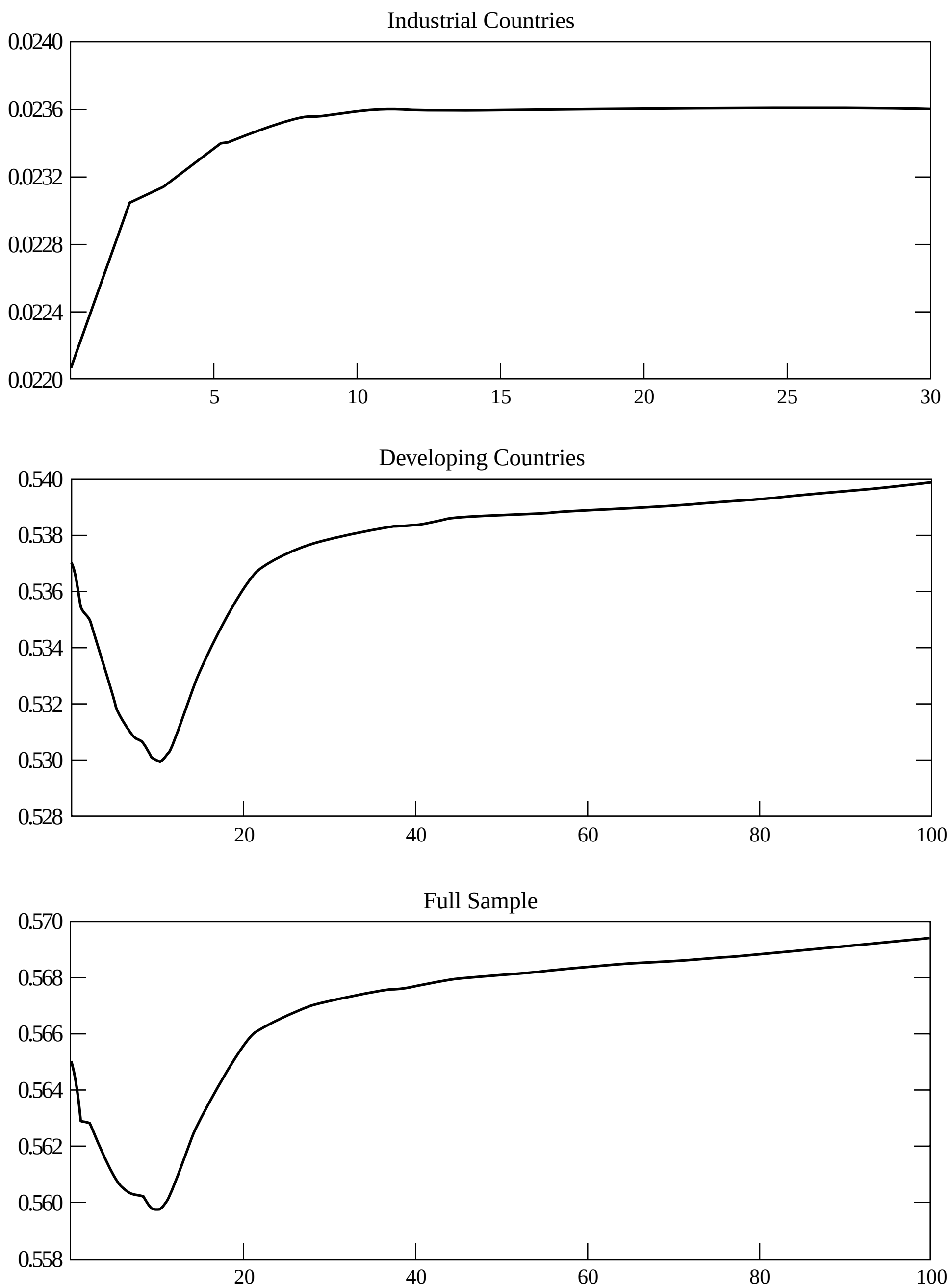
Table 2 provides the estimation results of equation (1) for the three samples. Fixed effects and time dummies have been included (but not reported) to control for cross-country heterogeneity and time effects. For the full sample, for which the threshold estimate is 11 percent, all coefficients have the right sign and are statistically significant at the 1 percent level. Recall that the existence of a threshold effect cannot be inferred simply from a classical test of equality between γ_1 and γ_2 as the distribution of the *t*-statistic for this variable is highly nonstandard under the null hypothesis of no threshold effect. This is why the null hypothesis has been tested using the boot-

[13]For industrial countries the upper bound has been set to 30 percent.

[14]Since the threshold estimate occurs at the lower bound of the search range for developed countries (corner solution), the question is whether the minimum is at 1 percent or less than 1 percent. This question cannot be answered with five-year-averaged observations as there are only 12 observations with an inflation rate below 1 percent for industrial countries. However, this question will be re-examined in the next section with yearly data which provide more observations with low inflation.

[15]For a more detailed discussion on the computation of the bootstrap distribution of LR_0, see Hansen (1999).

Figure 3. Residual Sum of Squares as a Function of the Threshold Level (Five-Year Average)



Note: Figure 3 shows the residual sum of squares (RSS) from equation (1) as a function of the threshold level of inflation for the three samples. The minimum of the RSS sequence determines the threshold estimate, which occurs at 1 percent for industrial countries, and 11 percent for developing countries and the full sample.

Table 2. NLLS With Fixed Effects (Five-Year Average)

Dependent Variable: $d\log(gdp)$

Independent Variables	All	Industrial	Developing
$(1-d^{\pi^*})[\log(\pi)-\log(\pi^*)]$	0.00049	0.05991	0.00109
	(–0.66)	(2.53)a	(1.33)
$d^{\pi^*}[\log(\pi)-\log(\pi^*)]$	–0.00895	–0.00643	–0.00895
	(–4.70)a	(–4.23)a	(–4.42)a
ly_0	–0.02506	–0.03634	–0.02551
	(–13.20)a	(–15.58)a	(–11.08)a
igdp	0.15090	0.10640	0.15910
	(5.01)a	(3.47)a	(4.96)a
$d\log(pop)$	0.04947	–0.01557	0.05095
	(2.33)a	(–0.23)	(2.33)a
σ_{tot}	–0.00020	–0.00031	–0.00019
	(–2.56)a	(–1.17)	(–2.33)a
Threshold (%)	11	1	11
	(64.42)a	(9.10)a	(58.59)a
NxT	905	165	740
R^2	0.43	0.80	0.39

Note: The panel has 8 observations (T), that is five-year averages over 1960–98, for 140 countries (N). The variables are inflation, π; the log of initial income, ly_0; gross domestic investment over GDP, *igdp*; the growth rate of population, $d\log(pop)$; and the standard deviation of terms of trade, $\sigma(tot)$. The dummy variable d^{π^*} takes one for inflation rates greater than the threshold estimate (π^*) and zero otherwise. The *t*-statistics, given in parentheses, are computed from White heteroskedasticity-consistent standard errors. The letters "a", "b", "c", indicate statistical significance at 1, 5, and 10 percent, respectively. The growth rate of a variable *x* is approximated by the first difference of the log of *x*, $d\log(x)$. The estimated time dummies and country-specific effects are not reported.

strap distribution of the likelihood ratio $LR_0(\pi)$. However, the distribution of the *t*-values of all explanatory variables retain their usual distribution under the alternative hypothesis of a threshold effect. Furthermore, Chan and Tsay (1998) show that the asymptotic distribution of all coefficients, including the threshold, is multivariate *normal* with a variance-covariance matrix given by equation (6).

In the previous sub-section, we established the existence of a threshold for all three samples; the next important question is how precise are these estimates? This requires the computation of the confidence region around the threshold estimate. While the existence of threshold effects in the relationship between inflation and growth is well accepted, the precise level of the inflation threshold is still subject to debate. Indeed, as discussed earlier, based on existing studies, the range could be between 2.5 percent and 40 percent. If the confidence region shows that the threshold estimate is not significantly different from a large number of other potential threshold levels, that would imply that there is substantial uncertainty about the threshold level. Interestingly, the confidence intervals here are very tight, which implies that the thresholds are precisely estimated. Indeed, the 95 percent confidence intervals for the whole sample, industrial countries, and developing countries, are [10.66, 11.34], [0.89, 1.11], and [10.62, 11.38], respectively.

Two basic conclusions can be drawn from this set of statistical tests. First, the threshold is around an inflation rate of 1 percent for industrial economies and 11 percent for developing countries. Second, these threshold estimates are very precise. One needs to ask why the threshold level for developing countries is higher than the threshold level for industrial countries. There are at least two possible conjectures that we can make. First, the long history of inflation in many developing countries led them to adopt widespread indexation systems to negate, at least partially, the adverse effects of inflation. Once in place, these indexation mechanisms make it possible for governments in these countries to run higher rates of inflation without experiencing adverse growth effects (because relative prices do not change that much). Second, to the extent that inflation is viewed as a tax on financial intermediation, governments, faced with a target level of expenditure will, in the absence of conventional taxes, levy the inflation tax. Accordingly, the differential threshold levels for the effects of inflation on growth for industrial and developing countries could reflect the higher level of conventional taxation in the former than in the latter. Thus, while relatively small increases in inflation in industrial countries adversely affect investment (by raising the effective cost of capital goods), productivity, and growth, in developing countries, with relatively low levels of conventional taxes, a larger inflation tax is required to have the same growth-inhibiting effects.[16]

While inflation below its threshold level has no significant effect on growth, inflation rates above the threshold level have a significant negative effect on growth for the whole sample. Dividing the sample into industrial and developing countries yields some interesting insights. First, both groups show a positive relationship between growth and inflation below their respective threshold levels (although it is statistically significant only for industrial countries for which the threshold level is at 1 percent), and a significant and a more powerful negative relationship for inflation rates above the threshold. As expected, investment as a share of GDP and population growth have a positive and significant effect on growth (except for industrial countries for which population growth is statistically insignificant). On average, an increase in the investment–GDP ratio of 5 percentage points will boost real GDP growth by 0.80 percentage points for developing countries and by 0.53 percentage points for industrial countries. In the empirical growth literature, the log of the initial GDP per capita (ly_0) has been generally included in growth regressions to test conditional convergence. Conditional convergence holds if the coefficient on ly_0 is negative.[17] Thus, convergence occurs for all samples. The rate of convergence among industrial countries is faster than for developing countries, corroborating the results of previous studies, which find that conditional convergence is stronger among industrial countries.[18]

The first three panels of Table 3 illustrate the regression results reported in Table 2 for the full sample, for industrial countries, and for developing countries,

[16]Roubini and Sala-i-Martin (1995) and Cukierman, Edwards, and Tabellini (1992) have developed models that yield results along these lines. We are grateful to Paul Cashin and John McDermott for bringing this possible explanation to our attention.

[17]A negative coefficient on ly_0 implies that countries with initially low income per capita tend to grow faster than countries with higher income per capita.

[18]See, for example, Mankiw, Romer, and Weil (1992).

Table 3. Numerical Illustration of the Effects of Inflation on Growth

(In percent)

π_0= 3 percent

	Five-Year-Average			Yearly Data		
	All	Industrial	Developing	All	Industrial	Developing
Threshold π	11%	1%	11%	9%	3%	12%
4	0.01	–0.18	0.03	0.02	–0.27	0.01
5	0.02	–0.33	0.06	0.03	–0.47	0.02
6	0.03	–0.45	0.08	0.04	–0.64	0.03
9	0.05	–0.71	0.12	0.06	–1.01	0.05
11	0.06	–0.84	0.14	–0.18	–1.20	0.06
15	–0.21	–1.03	–0.14	–0.54	–1.49	–0.24
20	–0.47	–1.22	–0.39	–0.88	–1.75	–0.63
25	–0.67	–1.36	–0.59	–1.15	–1.96	–0.93
30	–0.83	–1.48	–0.76	–1.36	–2.13	–1.18
40	–1.09	–1.66	–1.01	–1.70	–2.39	–1.56
60	–1.45	–1.93	–1.38	–2.18	–2.77	–2.11

Note: This table shows the effect on growth of gradually increasing inflation from an initial inflation rate (π_0) of 3 percent to 60 percent, using estimates of the fixed-effects model with yearly and five-year-average data. For example, increasing inflation from 3 percent to 25 percent entails a loss in growth of 1.17 percent using the full sample estimates with yearly data. Shaded areas indicate a crossing of a threshold.

respectively. The three panels show the effect on growth of gradually increasing inflation for a hypothetical economy with an initial inflation rate of 3 percent.[19] The maximum growth that a developing country, with an initial inflation rate of 3 percent, can gain through further inflation is 0.14 percentage points (by moving from an annual inflation rate of 3 percent to 11 percent). This magnitude very likely overestimates the positive effect of inflation as investment over GDP (*igdp*) was held constant while moving inflation from 3 to 11 percent. However, Fischer (1993) has shown that inflation also has a negative and significant indirect effect on growth through its effect on investment. This indirect effect is not taken into account here. From our results, the positive effect rapidly changes into a negative one as inflation increases above the threshold. For example, an increase in inflation from 3 to 40 percent will reduce growth by 1.01 percentage points in developing countries and by 1.66 percentage points in industrial countries. The effect of inflation on growth for any pair of inflation rates in the first column is simply equal to the difference between their growth effects. For example, reducing a developing country's annual inflation rate from 60 percent to 15 percent will increase its GDP growth by 1.24 percentage points. The log transformation implies that the effect on growth will be identical for an economy that moves from a 3 percent inflation rate to 6 percent and an economy that increases its inflation rate from 4 percent to 8 percent. This is because, in both cases, the inflation rate is doubled. Of course, this property holds only for inflation changes that do not induce a crossing of the threshold.

[19]The shaded areas in Table 3 indicate inflation rates that are above the threshold level of inflation.

IV. Robustness

Sensitivity to Fixed Effects

Since panel estimation can be quite sensitive to the use of fixed effects, equation (1) has also been estimated without fixed effects. Tables 2 and 4 show similar results. In particular, the estimates of threshold levels are identical. However, omitting fixed effects weakens the negative effect of inflation on growth for developing countries above the threshold level of inflation, and lowers the rate of convergence among countries.

Sensitivity to High-Inflation Observations

Bruno and Easterly (1998) and Easterly (1996) have argued that the negative relationship between inflation and growth holds only for high-inflationary economies. They show that excluding observations with annual inflation rates of 40 percent or more weakens the negative relationship between inflation and

Table 4. NLLS Without Fixed Effects (Five-Year Average)

Dependent Variable: $d\log(gdp)$

Independent Variables	All	Industrial	Developing
$(1-d^{\pi^*})[\log(\pi)-\log(\pi^*)]$	0.00061	0.05667	0.00074
	(0.98)	(2.85)a	(1.11)
$d^{\pi^*}[\log(\pi)-\log(\pi^*)]$	–0.00574	–0.00737	–0.00586
	(–2.71)a	(–5.00)a	(–2.65)a
ly_0	–0.00262	–0.02582	–0.00228
	(–7.52)a	(–67.76)a	(–4.47)a
igdp	0.11580	0.09150	0.10810
	(7.78)a	(10.93)a	(6.10)a
$d\log(pop)$	0.03873	0.27128	0.03234
	(3.06)a	(8.15)a	(2.08)b
σ_{tot}	–0.00021	–0.00104	–0.00019
	(–2.61)a	(–4.58)a	(–2.24)b
Threshold estimate (%)	11	1	11
	(25.42)a	(7.40)a	(23.97)a
NxT	905	165	740
R^2	0.21	0.70	0.19

Note: The panel has 8 observations (T), that is five-year averages over 1960–98, for 140 countries (N). The variables are inflation, π; the log of initial income, ly_0; gross domestic investment over GDP, *igdp*; the growth rate of population, $d\log(pop)$; and the standard deviation of terms of trade, $\sigma(tot)$. The dummy variable d^{π^*} takes one for inflation rates greater than the threshold estimate (π^*) and zero otherwise. The *t*-statistics, given in parentheses, are computed from White heteroskedasticity-consistent standard errors. The letters "a", "b", "c", indicate statistical significance at 1, 5, and 10 percent, respectively. The growth rate of a variable *x* is approximated by the first difference of the log of *x*, $d\log(x)$. The estimated time dummies and country-specific effects are not reported.

Table 5. NLLS With Fixed Effects (Five-Year Average)

Excluding Observations With Inflation Greater Than 40 Percent Dependent Variable: *d*log(*gdp*)

Independent Variables	All	Industrial	Developing
$(1-d^{\pi^*})[\log(\pi)-\log(\pi^*)]$	0 .00101	0.06227	0.00166
	(1.34)	(2.77)a	(1.85)c
$d^{\pi^*}[\log(\pi)-\log(\pi^*)]$	–0.01983	–0.00709	–0.02067
	(–9.04)a	(–4.10)a	(–9.02)a
ly_0	–0.02286	–0.03461	–0.02299
	(–9.00)a	(–13.91)a	(–6.89)a
igdp	0.15100	0.09860	0.16100
	(4.71)a	(2.83)a	(4.77)a
*d*log*(pop)*	0.06509	–0.00132	0.06595
	(3.06)a	(–0.02)	(3.01)a
σ_{tot}	–0.00027	–0.00040	–0.00026
	(–2.43)a	(–0.98	(–2.25)b)
Threshold estimate (%)	12	1	12
	(130.21)a	(9.94)a	(123.11)a
NxT	838	160	678
R^2	0.43	0.80	0.39

Note: The panel has 8 observations (T), that is five-year averages over 1960–98, for 140 countries (N). The variables are inflation, π; the log of initial income, ly_0; gross domestic investment over GDP, *igdp*; the growth rate of population, *d*log*(pop)*; and the standard deviation of terms of trade, $\sigma(tot)$. The dummy variable d^{π^*} takes one for inflation rates greater than the threshold estimate (π^*) and zero otherwise. The *t*-statistics, given in parentheses, are computed from White heteroskedasticity-consistent standard errors. The letters "a", "b", "c", indicate statistical significance at 1, 5, and 10 percent, respectively. The growth rate of a variable *x* is approximated by the first difference of the log of *x*, *d*log(*x*). The estimated time dummies and country-specific effects are not reported.

growth. Their methodology differs from ours in that theirs is not based on regression analysis but on mean comparisons before, during, and after inflation crises (defined as inflation episodes above 40 percent). To test their hypothesis within our framework, equation (1) was re-estimated with five-year-averaged data excluding observations with inflation rates higher than 40 percent. The results are presented in Table 5.

The results turn out to be very close to the estimates with the full sample (given in Table 2). In fact, the threshold estimates without high inflation observations for developing countries are almost identical to the estimates obtained with all the data.[20]

Sensitivity to the Location of the Threshold

Figure 4 shows the sensitivity of the effect of inflation on growth when the threshold level varies from 1 percent to 50 percent. The three panels (corresponding to the three samples) depict the effect of inflation on growth for

[20]In Table 5, the elimination of observations with inflation rates above 40 percent restricts the grid over which the search for threshold effects can be conducted.

economies with an inflation rate below the threshold level (solid line) and for economies with inflation rates above the threshold level (dotted line). These effects are given by the coefficients γ_1 and γ_2 in equation (1). The vertical line indicates the threshold estimate. The following points emerge from Figure 4:

(i) the high- and low-inflation effects are most sensitive to the location of the threshold over the 1 to 20 percent range;

(ii) the positive effect of inflation on growth is only present for inflation rates lower than 5 percent for industrial countries and 18 percent for developing countries;

(iii) for developing countries, the inflation effect on growth, which is negative over the whole range, strengthens as the threshold increases, which implies a worsening of the negative effect of inflation on growth as inflation increases; and

(iv) for industrial countries, the inflation effect, while remaining negative over the entire range, first weakens (in absolute value) as the inflation threshold increases, reaches a minimum around a threshold of 15 percent, and strengthens thereafter.

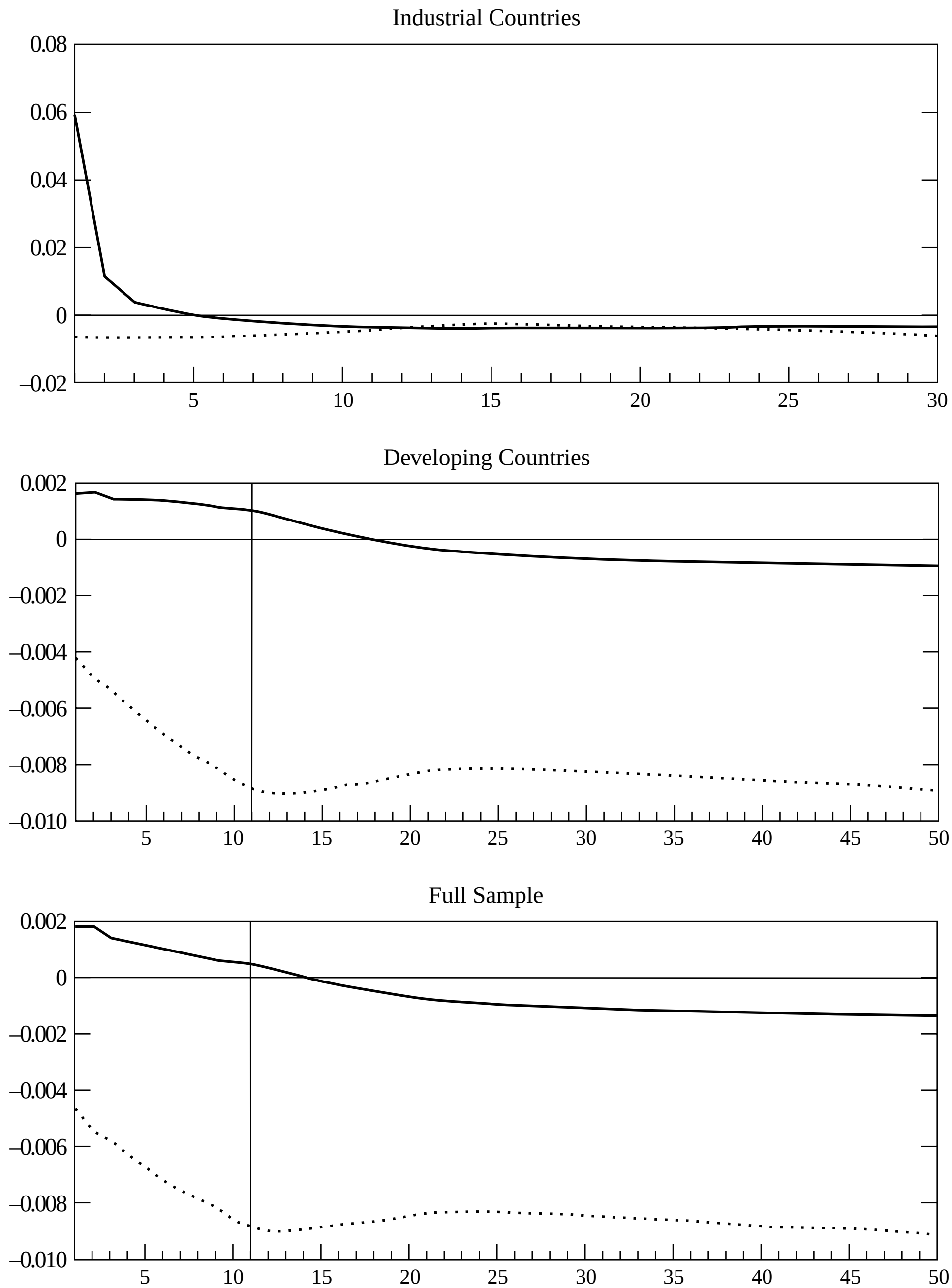
Sensitivity to Data Frequency

The estimation and inference in the previous section were based on five-year averages of the data. This procedure has become common practice in empirical growth literature and aims at filtering out business cycle fluctuations and allowing the focus to be on the medium- and long-term trends in the data. Estimation of equation (1) has also been carried out with annual data in order to examine two issues. First, it is interesting to analyze how data frequency changes the location and the magnitude of the threshold effect and the parameter estimates of equation (1). Second, while noisier, annual data provide more degrees of freedom, especially at the tails of the distribution for inflation. In particular, the inflation threshold for industrial countries was estimated at 1 percent, which was the lower bound of the grid search for threshold effects. The question raised earlier was whether the threshold was at 1 percent or at less than 1 percent. With the five-year averages, there were not enough observations with inflation at less than 1 percent, whereas annual data provide enough low-inflation observations to answer the question.

Table 6 gives the threshold estimate and parameter estimates of equation (1).[21] A comparison of Tables 2 and 6 reveals some interesting points. First, the threshold estimates are somewhat different but very close. The threshold estimates with yearly data are slightly higher for both industrial and developing countries (3 percent versus 1 percent for industrial countries, and 12 percent versus 11 percent for developing countries). Second, the high-inflation effect (that is, γ_2) is more powerful for yearly data. This is illustrated in the last three columns of Table 3. As expected, the fit is poorer with yearly data, but the threshold levels of inflation are precisely estimated.

[21]There is a small difference in the specification of equation (1) reported in Tables 2 and 6. In Table 2, equation (1) has the five-year standard deviation of terms of trade as an explanatory variable, whereas in Table 6, it is replaced by the growth rate of terms of trade since the standard deviation cannot be computed for yearly data. If both variables are included in equation (1), when estimated with five-year-averaged data, both become insignificant.

Figure 4. Sensitivity of the Effect of Inflation on Growth to the Threshold Level (Five-Year Average)



Note: The low (solid line) and high (dotted line) effects are respectively given by the coefficients γ_1 and γ_2 in equation (1). The threshold varies from 1 to 50 percent, except for industrial countries where the range is from 1 to 30 percent. The vertical line indicates the estimate of the inflation threshold.

Table 6. NLLS With Fixed Effects (Yearly Data)

Dependent Variable: *d*log(*gdp*)

Independent Variables	All	Industrial	Developing
$(1–d^{\pi^*})[\log(\pi)–\log(\pi^*)]$	0 .00054	0.00143	0.00043
	(–1.99)b	(3.22)a	(1.59)
$d^{\pi^*}[\log(\pi)–\log(\pi^*)]$	–0.01180	–0.00923	–0.01347
	(–6.00)a	(–5.17)a	(–5.56)a
Igdp	0.07820	0.02690	0.07860
	(4.11)a	(0.92)	(3.83)a
*d*log*(pop)*	–0.01557	–0.02750	–0.01701
	(–0.16)	(–.20)	(–0.17)
*d*log*(tot)*	–4.72E–05	–0.01583	–1.77E–05
	(–0.00)	(–1.03)	(–0.00)
Threshold estimate (%)	9	3	12
	(81.37)a	(25.24)a	(87.68)a
NxT	4264	950	3414
R^2	0.14	0.50	0.12

Note: The panel has potentially 39 observations (T), covering 1960–98, for 140 countries (N). The variables are inflation, π; gross domestic investment over GDP, *igdp*; the growth rate of population, *d*log*(pop)*; and the growth rate of terms of trade, *d*log*(tot)*. The dummy variable d^{π^*} takes one for inflation rates greater than the threshold estimate (π*) and zero otherwise. The *t*-statistics, given in parentheses, are computed from White heteroskedasticity-consistent standard errors. The letters "a", "b", "c", indicate statistical significance at 1, 5, and 10 percent, respectively. The growth rate of a variable *x* is approximated by the first difference of the log of *x*, *d*log*(x)*. The estimated country-specific effects are not reported.

The 95 percent confidence intervals for the whole sample, industrial countries, and developing countries, are [8.78, 9.22], [2.76, 3.24], and [11.80, 12.20], respectively. Considering the few number of observations with very low inflation rates for the five-year-averaged data, the 3 percent threshold estimate (versus 1 percent with smoothed data) for industrial countries may well be more reliable.

Sensitivity to Additional Explanatory Variables

As explained in Section I, only variables that were found to be robust in the empirical growth literature were included in the regression equation linking inflation to growth. The use of fixed effects also helps capture cross-country differences in GDP growth. Since endogenous growth theory has emphasized the role of human capital in the growth process of a country, equation (1) has been augmented by including a human capital variable. Following the empirical growth literature, human capital is proxied by enrollment rates in the primary, secondary, and tertiary schools.[22] All three variables came out statistically insignificant. Furthermore, their inclusion does not significantly change the results. In fact, the

[22]The data on enrollment were taken from the World Bank's Global Development Network Growth Database maintained by William Easterly and Hairong Yu.

threshold values remain the same. The reason may be that the three proxies (primary, secondary, and tertiary enrollment) are highly correlated with the initial income variable (ly_0). A regression of the former on the latter yields an R^2 of 0.98, 0.92, and 0.98, respectively. In other words, the initial income variable appears to be picking up most of the cross-country variation in school enrollment.

Financial development is another important variable that was emphasized by King and Levine (1993). Following this approach, we used three different proxies for financial depth. The first measures the size of the formal financial intermediary sector relative to economic activity (the ratio of liquid liabilities of the financial system, measured by M3 when it is available and M2 otherwise, to GDP); the second proxy measures the proportion of credit allocated to the private sector (the ratio of claims on the nonfinancial private sector to total domestic credit);[23] and the third is simply the second normalized by GDP instead of total domestic credit. Adding these variables did not change the estimated threshold values at all.

V. Conclusions

This paper re-examines the issue of the existence of threshold effects in the relationship between inflation and growth using new econometric techniques that provide appropriate procedures for estimation and inference. The data cover 140 developing and industrialized countries for the period 1960–98.[24] Estimates were obtained for panels with five-year averaged data as well as yearly data.

The empirical results strongly suggest the existence of a threshold beyond which inflation exerts a negative effect on growth. The threshold is lower for industrial than for developing countries (the estimates are 1–3 percent and 11–12 percent for industrial and developing countries, respectively, depending on the estimation method). The thresholds are statistically significant at 1 percent or less. The confidence intervals are very tight, which implies that the threshold estimates are very precise.

The negative and significant relationship between inflation and growth for inflation rates above the threshold level is robust with respect to the estimation method, perturbations in the location of the threshold level, the inclusion or exclusion of high-inflation observations, data frequency, and alternative specifications. Interestingly, using yearly data yields threshold levels that are close to the estimates from the five-year-averaged data (12 percent for developing countries and 3 percent for industrial countries) and a stronger negative relationship between inflation and growth.[25] Thus, as in Bruno and Easterly (1998), the relationship between inflation and growth is stronger at high frequencies. At the same time, our results suggest a strong and negative effect of inflation on growth even when data have been averaged over five years.

While the results are informative, some caveats are important to bear in mind when interpreting these results. First, the estimated relationship between inflation

[23]Data for these variables were taken from the IMF's *International Financial Statistics* (IFS). Claims on the nonfinancial private sector is IFS line 32d and domestic credit is IFS lines 32a to 32f excluding 32e.

[24]As mentioned earlier, the period is shorter for a large number of developing countries.

[25]The confidence regions for the threshold estimates with yearly data are wider, reflecting the noisier nature of yearly data versus the five-year-averaged data.

and growth does not provide the precise channels through which inflation affects growth—beyond the fact that, because investment and employment are controlled for, the effect is primarily through productivity.[26] This also implies that the total negative effect of inflation may be understated. Second, inflation is not an exogenous variable in the growth-inflation regression, and the coefficient estimates may be biased. The seriousness of this problem will depend, to a large extent, on whether the causality runs mainly from inflation to growth, in which case the endogeneity problem may not be serious, or the other way around, in which case a bias may be present. As argued by Fischer (1993), the causality is more likely to run predominantly from inflation to growth, in which case the problem of simultaneity bias may not be very important. However, this assumption needs to be explicitly tested.[27] Finally, inflation may have adverse effects on the economy beyond that on growth. These effects have not been considered here and warrant serious study.

In conclusion, policymakers around the world during the last decade or so have recognized that lowering inflation is conducive to improved growth performance. The goal has become one of bringing inflation down to single digits, or close to single digits, and keeping it there. The results in this paper provide strong empirical support for this view.

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[26]Fischer (1993) examines the effects of inflation on investment, employment, and total factor productivity.

[27]The estimation method used here has not been extended to standard econometric methods of handling simultaneity.

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