# Exchange Rate Disconnect in General Equilibrium

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# Motivation

- Exchange Rate Disconnect (ERD) is one of the most pervasive and challenging puzzles in macroeconomics
  - — exchange rates are present in all international macro models
  - — yet, we do not have a satisfactory theory of exchange rates

- Broader ERD combines five exchange-rate-related puzzles:
  1. Meese-Rogoff (1983) puzzle
     *NER follows a volatile RW, uncorrelated with macro fundamentals*
  2. PPP puzzle (Rogoff 1996)
     *RER is as volatile and persistent as NER, and the two are nearly indistinguishable at most horizons (also related Mussa puzzle)*
  3. LOP/Terms-of-Trade puzzle (Engel 1999, Atkeson-Burstein 2008)
     *LOP violations for tradables account for nearly all RER dynamics*
     *ToT is three times less volatile than RER*
  4. Backus-Smith (1993) puzzle
     *Consumption is high when prices are high (RER appreciated)*
     *Consumption is five times less volatile than RER*
  5. Forward-premium puzzle (Fama 1984)
     *High interest rates predict nominal appreciations (UIP violations)*

# Our Approach

- The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles
- We provide a unifying theory of exchange rates, capturing simultaneously all stylized facts about their properties

# Our Approach

- The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles
- We provide a unifying theory of exchange rates, capturing simultaneously all stylized facts about their properties
- A theory of exchange rate (disconnect) must specify:
  1. The exogenous shock process driving the exchange rate
     - — little empirical guidance here; we adopt a general approach and select from a variety of candidate shocks
     - — we prove that only the financial shock (in the exchange rate market) satisfies the necessary condition for ER disconnect
  2. The transmission mechanism muting the response of the macro variables to exchange rate movements relies on:
     - — strategic complementarities in price setting resulting in PTM
     - — low elasticity of substitution between home and foreign goods
     - — home bias in consumption

     all admitting tight empirical discipline

$\rightarrow$ incomplete markets are important, but not nominal stickiness

# MODELING FRAMEWORK

# Model setup

- Two countries: home (Europe) and foreign (US, denoted w/$*$)
- Nominal wages $W_t$ in euros and $W_t^*$ in dollars, the numeraires
- $\mathcal{E}_t$ is the nominal exchange rate (price of one dollar in euros)
- Baseline model:
  - representative households
  - representative firms
  - one internationally-traded foreign-bond
- We allow for all possible shocks/CKM-style wedges:

$$\mathbf{\Omega}_t = (w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t, \eta_t, \xi_t, \psi_t)$$

and foreign counterparts

# Equilibrium conditions

1. Households:
   - (i) labor supply and asset demand ▸ show
   - (ii) expenditure on home and foreign good ▸ show
     - — $\gamma$ expenditure share on foreign goods
     - — $\theta$ elasticity of substitution between home and foreign goods
2. Firms:
   - (i) production and profits ▸ show
   - (ii) price setting ▸ show
     - — $\alpha$ strategic complementarity elasticity in price setting
3. Government: balanced budget ▸ show
4. Foreign: symmetric ▸ show
5. GE: market clearing and country budget constraint ▸ show

# DISCONNECT IN THE LIMIT

# Disconnect in the Autarky Limit

- Consider an economy in autarky = complete ER disconnect
  - (i) NER is not determined and can take any value
  - (ii) this has no effect on domestic quantities, prices or interest rates
  - (iii) as price levels are determined independently from NER, RER moves one-to-one with NER

- **Definition**: Exchange rate disconnect in the autarky limit

$$\lim_{\gamma\to 0} \frac{\mathrm{d}\mathbf{Z}_{t+j}}{\mathrm{d}\varepsilon_t} = \mathbf{0} \quad \forall j \qquad \text{and} \qquad \lim_{\gamma\to 0} \frac{\mathrm{d}\mathcal{E}_t}{\mathrm{d}\varepsilon_t} \neq 0,$$

- **Proposition 1**: *The model cannot exhibit exchange rate disconnect in the limit with zero weight on:*
  - (i) *LOP deviation shocks:* $\eta_t$
  - (ii) *Foreign-good demand shocks:* $\xi_t$
  - (iii) *Financial (international asset demand) shocks:* $\psi_t$

- A pessimistic result for IRBC and NOEM models

# Admissible Shocks

- Intuition: two international conditions
  - risk sharing: $\mathbb{E}_t \left\{ R^*_{t+1} \left[ \Theta^*_{t+1} - \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} e^{\psi_t} \right] \right\} = 0$
  - budget constraint: $B^*_{t+1} - R^*_t B^*_t = NX^*(\mathcal{Q}_t; \eta_t, \xi_t)$
- In the limit, shocks to these conditions have a vanishingly small effect, while other shocks still have a direct effect

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- **Proposition 2**: *In the limit, $\psi_t$ is the only shock that simultaneously and robustly produces:*
  (i) *positively correlated ToT and RER (Obstfeld-Rogoff moment)*
  (ii) *negatively correlated relative consumption growth and real exchange rate depreciations (Backus-Smith correlation)*
  (iii) *deviations from the UIP (negative Fama coefficient).*

$\Rightarrow$ $\psi_t$ is the prime candidate shock for a **quantitative** model of ER disconnect

# BASELINE MODEL

OF EXCHANGE RATE DISCONNECT

# Ingredients

1. Financial exchange rate shock $\psi_t$ only: ▸ microfoundations

$$\psi_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$$

   — persistent ($\rho \lesssim 1$, e.g. $\rho = 0.97$) w/small innovations ($\sigma_\varepsilon \gtrsim 0$):

$$\psi_t = \rho \psi_{t-1} + \varepsilon_t, \qquad \beta\rho < 1$$

   — important limiting case: $\beta\rho \to 1$

2. Transmission mechanism
   (i) Strategic complementarities: $\alpha = 0.4$ (AIK 2015)
   (ii) Elasticity of substitution: $\theta = 1.5$ (FLOR 2014)
   (iii) Home bias: $\gamma = 0.07 = \frac{1}{2} \frac{\text{Imp+Exp}}{\text{GDP}} \frac{\text{GDP}}{\text{Prod-n}}$ (for US, EU, Japan)

- Other parameters:

$$\beta = 0.99, \quad \sigma = 2, \quad \nu = 1, \quad \phi = 0.5, \quad \zeta = 1 - \phi$$

# Roadmap

1. Equilibrium exchange rate dynamics
2. Real and nominal exchange rates
3. Exchange rate and prices
4. Exchange rate and quantities
5. Exchange rate and interest rates

# Exchange Rate Dynamics

1. The international risk sharing condition:

$$\mathbb{E}_t \left\{ e^{\psi_t} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \Theta_{t+1} - \Theta^*_{t+1} \right\} = 0 \quad \Rightarrow \quad \psi_t = -d_1 \cdot \mathbb{E}_t \Delta e_{t+1},$$

   with $d_1 > 1$ and $\lim_{\gamma \to 0} d_1 = 1$

2. Intertemporal budget constraint:

$$\beta b^*_{t+1} - b^*_t = nx_t, \quad nx_t = \gamma d_2 \cdot e_t, \quad d_2 > 0$$

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## Proposition

*When $\psi_t \sim$AR(1), the equilibrium exchange rate follows ARIMA:*

$$\Delta e_t = \rho \Delta e_{t-1} + \frac{\beta / d_1}{1 - \beta\rho} \left( \varepsilon_t - \frac{1}{\beta} \varepsilon_{t-1} \right).$$

*This process becomes arbitrary close to a random walk as $\beta\rho \to 1$.*

*This is the unique equilibrium solution, non-fundamental solutions do not exist.*

properties

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$$(1 - \rho L) \Delta e_t = \frac{\beta / d_1}{1 - \beta\rho} \left( 1 - \frac{1}{\beta} L \right) \varepsilon_t.$$

*This process becomes arbitrary close to a random walk as $\beta\rho \to 1$.*

*This is the unique equilibrium solution, non-fundamental solutions do not exist.*

▸ properties

# RER and the PPP Puzzle

## Proposition

*RER and NER are tied together by the following relationship:*

$$q_t = \frac{1}{1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} e_t.$$

- Intuition:

$$p_t = w_t + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma} q_t$$
$$p_t^* = w_t^* - \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma} q_t$$

- $(q_t - e_t) \xrightarrow[\gamma \to 0]{} 0$
- Relative volatility: $\frac{\mathrm{std}(\Delta q_t)}{\mathrm{std}(\Delta e_t)} = \frac{1}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} = 0.75$
- Heterogenous firms and/or LCP sticky prices can further increase volatility of RER

## Exchange Rates and Prices

- Three closely related variables:

$$\mathcal{Q}_t = \frac{P_t^* \mathcal{E}_t}{P_t} \qquad \mathcal{Q}_t^P = \frac{P_{Ft}^* \mathcal{E}_t}{P_{Ht}} \qquad \mathcal{S}_t = \frac{P_{Ft}}{P_{Ht}^* \mathcal{E}_t}$$

- Two relationships:

$$q_t = (1-\gamma) q_t^P - \gamma s_t$$
$$s_t = q_t^P - 2\alpha q_t$$

- In the data: $q_t^P \approx q_t$, $\mathrm{std}(\Delta q_t) \gg \mathrm{std}(\Delta s_t)$, $\mathrm{corr}(\Delta s_t, \Delta q_t) > 0$

- **Proposition**:
$$q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t \quad \text{and} \quad s_t = \frac{1-2\boldsymbol{\alpha}(1-\gamma)}{1-2\gamma} q_t$$
  - conventional models with $\alpha = 0$ cannot do the trick
  - $\alpha$ needs to be positive, but not too large

# Exchange Rates and Prices

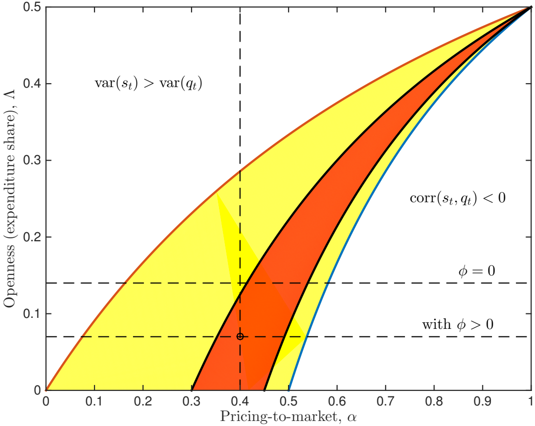



Figure : Terms of trade and Real exchange rate

# Exchange Rate and Quantities

- Static relationship between consumption and RER: ▸ show
  (i) labor supply
  (ii) labor demand
  (iii) goods market clearing

- Proposition: Static expenditure switching implies:

$$c_t - c_t^* = -\frac{2\boldsymbol{\theta(1-\alpha)}\frac{1-\gamma}{1-2\gamma} + \nu + \frac{2\gamma}{1-2\gamma}\frac{\nu+\phi}{1-\phi}}{(\sigma\nu+1)(1-\phi) + \frac{2\gamma}{1-2\gamma}\sigma\nu}\frac{2\gamma}{1-2\gamma}q_t$$

# Exchange Rate and Quantities

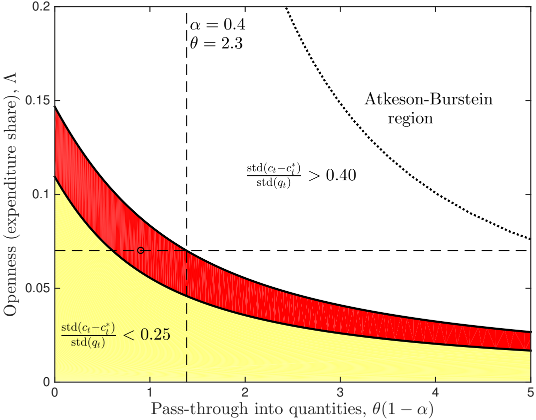



Figure : Exchange rate disconnect: relative consumption volatility

# Exchange Rate and Interest rates

- Two interest rate conditions:

$$\psi_t = (i_t - i_t^*) - \mathbb{E}_t \Delta e_{t+1} \quad \text{and} \quad i_t - i_t^* = \frac{d_1 - 1}{d_1} \psi_t$$

## Proposition

*Fama-regression coefficient:*

$$\mathbb{E}\{\Delta e_{t+1} | i_{t+1} - i_{t+1}^*\} = \beta_F (i_{t+1} - i_{t+1}^*), \qquad \beta_F \equiv -\frac{1}{d_1 - 1} < 0.$$

*In the limit $\beta\rho \to 1$:*

(i) *Fama-regression $R^2 \to 0$*

(ii) $\mathrm{var}(i_t - i_t^*)/\mathrm{var}(\Delta e_{t+1}) \to 0$

(iii) $\rho(\Delta e_t) \to 0$, *while* $\rho(i_t - i_t^*) \to 1$

(iv) *the Sharpe ratio of the carry trade: $SR_C \to 0$*

* *carry trade return:* $r_{t+1}^C = x_t \cdot (i_t - i_i^* - \Delta e_{t+1})$ *with* $x_t = i_t - i_i^* - \mathbb{E}_t \Delta e_{t+1}$

# ER Disconnect: Summary

| | | Data | Baseline | Robustness | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | $\theta = 2.5$ | $\alpha = 0$ | $\gamma = .15$ | $\rho = 0.9$ | $\sigma = 1$ |
| 1. | $\rho(\Delta e)$ | 0.00 | $-0.02$ (0.09) | | | | $-0.05$ | |
| 2. | $\rho(q)$ | 0.94 | $0.93^{\star}$ (0.04) | | | | 0.87 | |
| | $HL(q)$ | 12.0 | $9.9^{\star}$ (6.4) | | | | 4.9 | |
| | $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.88 | 0.75 | | | 0.54 | | |
| 3. | $\sigma(\Delta s)/\sigma(\Delta q)$ | 0.34 | 0.30 | | 1.16 | 0.46 | | |
| | $\sigma(\Delta q^P)/\sigma(\Delta q)$ | 0.95 | 1.10 | | 1.16 | 1.26 | | |
| 4. | $\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$ | $-0.25$ | $-0.31$ | $-0.42$ | $-0.42$ | $-0.81$ | | $-0.48$ |
| 5. | Fama $\beta_F$ | $\lesssim 0$ | $-8.1^{\star}$ (4.7) | | | | | |
| | Fama $R^2$ | 0.02 | 0.04 (0.02) | | | | 0.07 | |
| | $\sigma(i - i^*)/\sigma(\Delta e)$ | 0.10 | 0.03 (0.01) | | | | | |
| | Carry $SR$ | 0.20 | 0.21 (0.04) | | | | 0.29 | |

Note: Baseline parameters: $\gamma = 0.07$, $\alpha = 0.4$, $\theta = 1.5$, $\rho = 0.97$, $\sigma = 2$, $\nu = 1$, $\phi = 0.5$, $\mu = 0$, $\beta = 0.99$. Results are robust to changing $\nu$, $\phi$, $\mu$ and $\beta$. $^{\star}$Asymptotic values: $\rho(q) = 1$, $HL(q) = \infty$, $\beta_F = -4.6$.

# EXTENSIONS

# Extensions

1. Monetary model with sticky prices
   - — different transmission mechanism
   - — similar quantitative conclusions for $\psi_t$ shock
   - — gives half of the Mussa puzzle

2. Multiple shocks:
   - — productivity, monetary, foreign good and asset demand

3. Limits-to-arbitrage model of the financial sector
   - — multiple foreign assets, non-zero NFA, and valuation effects

4. Heterogeneous firms (following AIK 2015)
   - — small local firms ($\alpha=0$, $\phi^*=0$) vs large exporters ($\alpha=0.5$, $\phi^*=0.3$)
   - $\rightarrow$ further mutes the transmission mechanism (Switzerland model)
   - — mix of LCP and PCP $\longrightarrow$ International Price System

# Monetary model

- Standard New Keynesian Open Economy model
- Baseline: LCP sticky prices with strategic complements ($\alpha$)
- Taylor rule: $i_t = \rho_i i_{t-1} + (1 - \rho_i)\delta_\pi \pi_t + \varepsilon_t^m$
- New transmission: $i_t$ does not respond directly to the $\psi_t$ shock, but instead through inflation it generates
- Results:
  1. monetary shock alone results in numerous ER puzzles
  2. financial shock $\psi_t$ has quantitative similar properties, with two exceptions:
     + RER becomes more volatile and more persistent and NER becomes closer to a random walk
     − RER is negatively correlated with ToT

# Multishock model

| | Data | One shock $\psi$ | Multiple shocks $\xi + a$ | $\xi + a + m$ |
|---|---|---|---|---|
| $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.88 | 0.75 | 0.79 | 1.01 |
| $\rho(\Delta q, \Delta e)$ | 0.98 | 1 | 0.96 | 1.00 |
| $\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$ | 0.25 | 0.31 | 0.30 | 0.23 |
| $\rho(\Delta c - \Delta c^*, \Delta q)$ | $-0.28$ | $-1$ | $-0.22$ | $-0.17$ |
| $\sigma(\Delta nx)/\sigma(\Delta q)$ | 0.12 | 0.25 | 0.30 | 0.28 |
| $\rho(\Delta nx, \Delta q)$ | $\gtrsim 0$ | 1 | $-0.01$ | 0.01 |
| Fama $\beta_F$ | $\lesssim 0$ | $-8.1$ | $-0.6$ | 0.2 |
| Fama $R^2$ | 0.02 | 0.04 | 0.01 | 0.00 |
| Carry $SR$ | 0.20 | 0.21 | 0.16 | 0.19 |
| **Decomposition of $\mathrm{var}(\Delta q_t)$** | | | | |
| $\psi$-shock | | 100% | 53% | 61% |
| $\xi$-shock | | — | 39% | 31% |
| $a$-shock | | — | 8% | 5% |
| $m$-shock | | — | — | 2% |

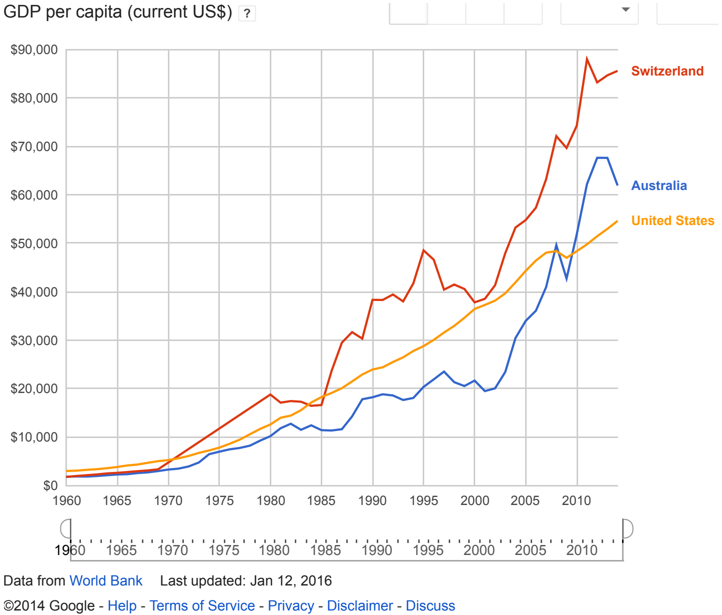
▸ show horserace

# Conclusion

- Exchange rates have been very puzzling for macroeconomists
- We propose a unifying theory of exchange rates, in which:
  1. Nominal exchange rate follows a near random walk and correlates little with other macro variables
  2. RER tracks closely NER, with very long half-lives
  3. ToT respond weakly to RER due to LOP deviations
  4. Consumption is higher when RER is appreciated, yet the relationship between the two is weak
  5. High interest rates predict nominal appreciations, yet with a very low $R^2$, and the Sharpe ratios on the carry trades are low
- An empirically successful theory of ER must rely on:
  1. a financial shock in the exchange rate market
  2. a transmission mechanism that mutes the response to ER

# APPENDIX

# Motivation


GDP per capita (current US$) ?
$90,000
$80,000
$70,000
$60,000
$50,000
$40,000
$30,000
$20,000
$10,000
$0
1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010
Switzerland
Australia
United States
1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010
Data from World Bank Last updated: Jan 12, 2016
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# Horserace: Single-shock models

back to slides

| Moment | Data | $\psi_t$ shock Baseline | $\psi_t$ shock Monetary | $a_t$ shock IRBC | $m_t$ shock NOEM |
|---|---|---|---|---|---|
| **1-2. PPP Puzzle and Meese-Rogoff:** | | | | | |
| $\rho(q)$ | 0.94 | 0.93 (0.04) | 0.92 (0.05) | 0.92 (0.04) | 0.65 (0.07) |
| $\rho(\Delta e)$ | 0.00 | −0.02 (0.09) | −0.04 (0.09) | 0.55 (0.15) | −0.15 (0.08) |
| $\frac{\sigma(\Delta q)}{\sigma(\Delta e)}$ | 0.88 | 0.75 | 1.00 | 38.7 | 0.94 |
| **3. Terms of trade:** | | | | | |
| $\frac{\sigma(\Delta s)}{\sigma(\Delta q)}$ | 0.34 | 0.30 | −0.80 | 1.16 | −0.91 |
| **4. Backus-Smith:** | | | | | |
| $\frac{\sigma(\Delta c - \Delta c^*)}{\sigma(\Delta q)}$ | −0.25 | −0.31 | −0.19 | 0.64 | 0.50 |
| **5. Forward premium puzzle:** | | | | | |
| Fama $\beta$ | $\lesssim 0$ | −8.1 (4.7) | −2.0 (1.7) | 1.06 (0.07) | 1.1 (0.3) |
| Fama $R^2$ | 0.02 | 0.04 (0.02) | 0.02 (0.02) | 0.75 (0.07) | 0.10 (0.04) |
| $\frac{\sigma(i - i^*)}{\sigma(\Delta e)}$ | 0.10 | 0.03 (0.01) | 0.08 (0.03) | 0.83 (0.09) | 0.29 (0.03) |
| Carry $SR$ | 0.20 | 0.21 (0.04) | 0.21 (0.04) | 0 | 0 |

# New Mechanisms

1. Exchange rate dynamics:
   - $\longrightarrow$ near random-walk behavior emerging from the intertemporal budget constraint under incomplete markets
   - $\longrightarrow$ small but persistent expected appreciations require a large unexpected devaluation on impact
2. PPP puzzle
   - $\longrightarrow$ no wedge between nominal and real exchange rates, unlike IRBC and NOEM models
3. Violation of the Backus-Smith condition:
   - $\longrightarrow$ we demote the dynamic risk-sharing condition from determining consumption allocation
   - $\longrightarrow$ instead static market clearing determination of consumption
4. Violation of UIP and Forward premium puzzle:
   - $\longrightarrow$ small persistent interest rate movements support consumption allocation, disconnected from volatile exchange rate
   - $\longrightarrow$ negative Fama coefficient, yet small Sharpe ratio on carry trade

# Households

back to slides

- Representative home household solves:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1+1/\nu} L_t^{1+1/\nu} \right)$$

$$\text{s.t.} \quad P_t C_t + \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^* \mathcal{E}_t}{e^{\psi_t} R_t^*} \leq B_t + B_t^* \mathcal{E}_t + W_t L_t + \Pi_t + T_t$$

- Household optimality (labor supply and demand for bonds):

$$e^{\kappa_t} C_t^{\sigma} L_t^{1/\nu} = \frac{W_t}{P_t},$$

$$R_t \mathbb{E}_t \left\{ \Theta_{t+1} \right\} = 1,$$

$$e^{\psi_t} R_t^* \mathbb{E}_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \Theta_{t+1} \right\} = 1,$$

where the home nominal SDF is given by:

$$\Theta_{t+1} \equiv \beta e^{\Delta \chi_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

# Demand

back to slides

- Consumption expenditure on home and foreign goods:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft}$$

arises from a homothetic consumption aggregator:

$$C_{Ht} = (1-\gamma) e^{-\gamma \xi_t} h\left(\tfrac{P_{Ht}}{P_t}\right) C_t,$$

$$C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\tfrac{P_{Ft}}{P_t}\right) C_t$$

- The foreign share and the elasticity of substitution:

$$\gamma_t \equiv \frac{P_{Ht} C_{Ht}}{P_t C_t}\Big|_{\substack{P_{Ht}=P_{Ft}=P_t \\ \xi_t=0}} = \gamma$$

$$\theta_t \equiv -\frac{\partial \log h(x_t)}{\partial \log x_t}\Big|_{x_t=1} = \theta$$

# Production and profits

back to slides

- Production function with intermediates:

$$Y_t = e^{a_t} L_t^{1-\phi} X_t^{\phi}$$

$$MC_t = e^{-a_t} \left(\frac{W_t}{1-\phi}\right)^{1-\phi} \left(\frac{P_t}{\phi}\right)^{\phi}$$

- Profits:

$$\Pi_t = (P_{Ht} - MC_t) Y_{Ht} + (P^*_{Ht} \mathcal{E}_t - MC_t) Y^*_{Ht},$$

where $\quad Y_t = Y_{Ht} + Y^*_{Ht}$

- Labor and intermediate goods demand:

$$W_t L_t = (1-\phi) MC_t Y_t$$

$$P_t X_t = \phi MC_t Y_t$$

and fraction $\gamma_t$ of $P_t X_t$ is allocated to foreign intermediates

## Prices

back to slides

- We postulate the following price setting rule:

$$P_{Ht} = e^{\mu_t} MC_t^{1-\alpha} P_t^{\alpha}$$

$$P_{Ht}^* = e^{\mu_t + \eta_t} \left( MC_t / \mathcal{E}_t \right)^{1-\alpha} P_t^{*\alpha}$$

- LOP violations:

$$\mathcal{Q}_{Ht} \equiv \frac{P_{Ht}^* \mathcal{E}_t}{P_{Ht}} = e^{\eta_t} \mathcal{Q}_t^{\alpha}$$

where the real exchange rate is given by:

$$\mathcal{Q}_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}$$

# Government

◂ back to slides

- Government runs a balanced budget, using lump-sum taxes to finance expenditure:

$$P_t G_t = P_t e^{g_t},$$

where fraction $\gamma_t$ of $P_t G_t$ is allocated to foreign goods

- The transfers to the households are given by:

$$T_t = \left(e^{-\psi_t} - 1\right) \frac{B^*_{t+1} \mathcal{E}_t}{R^*_t} - P_t e^{g_t}$$

# Foreign

back to slides

- Foreign households and firms are symmetric, subject to:

$$\{\chi_t^*, \kappa_t^*, \xi_t^*, a_t^*, \mu_t^*, \eta_t^*, g_t^*\}$$

- Foreign households only differ in that they do not have access to the home bond, which is not internationally traded.

  As a result, their only Euler equation is for foreign bonds:

$$R_t^* \mathbb{E}_t \left\{ \Theta_{t+1}^* \right\} = 1, \qquad \Theta_{t+1}^* \equiv \beta e^{\Delta \chi_{t+1}^*} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}$$

# Equilibrium

back to slides

1. Labor market clearing
2. Goods market clearing, e.g.:

$$Y_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h\left(\frac{P_{Ht}^*}{P_t^*}\right)[C_t^* + X_t^* + G_t^*]$$

3. Bond market clearing:

$$B_t = 0 \quad \text{and} \quad B_t^* + B_t^{*F} = 0$$

4. Country budget constraint:

$$\frac{B_{t+1}^* \mathcal{E}_t}{R_t^*} - B_t^* \mathcal{E}_t = NX_t, \quad NX_t = P_{Ht}^* \mathcal{E}_t Y_{Ht}^* - P_{Ft} Y_{Ft},$$

and we define the terms of trade:

$$\mathcal{S}_t \equiv \frac{P_{Ft}}{P_{Ht}^* \mathcal{E}_t}$$

# Microfoundations for $\psi_t$ shock

back to slides

Risk premium shock: $\psi_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$

1. International asset demand shocks (in the utility function)
   — e.g., Dekle, Jeong and Kiyotaki (2014)
2. Noise trader shocks and limits to arbitrage
   — e.g., Jeanne and Rose (2002)
   - noise traders can be liquidity/safety traders
   - arbitrageurs with downward sloping demand
   - multiple equilibria $\longrightarrow$ Mussa puzzle
3. Heterogenous beliefs or expectation shocks
   — e.g., Bacchetta and van Wincoop (2006)
   - huge volumes of currency trades (also order flows)
   - $\psi_t$ are disagreement or expectation shocks
4. Financial frictions (e.g., Gabaix and Maggiori 2015)
5. Risk appetite (e.g., Brunnermeier, Nagel and Pedersen 2009)

# Properties of the Exchange Rate

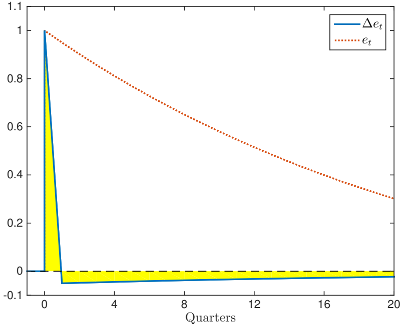
back to slides

- Near-random-walk behavior (as $\beta\rho \to 1$)

$\mathrm{corr}(\Delta e_{t+1}, \Delta e_t) \to 0$ $\qquad \dfrac{\mathrm{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\mathrm{var}(\Delta_k e_{t+k})} \to 1$ $\qquad \dfrac{\mathrm{std}(\Delta e_t)}{\mathrm{std}(\psi_t)} \to \infty$

(a) $\rho = 0.96$ and $\beta = 0.99$

(b) $\rho = 0.99$ and $\beta = 0.995$



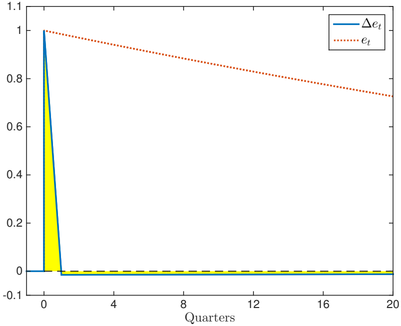




Figure : Impulse response of the exchange rate $\Delta e_t$ to $\psi_t$

# Properties of the Exchange Rate

back to slides

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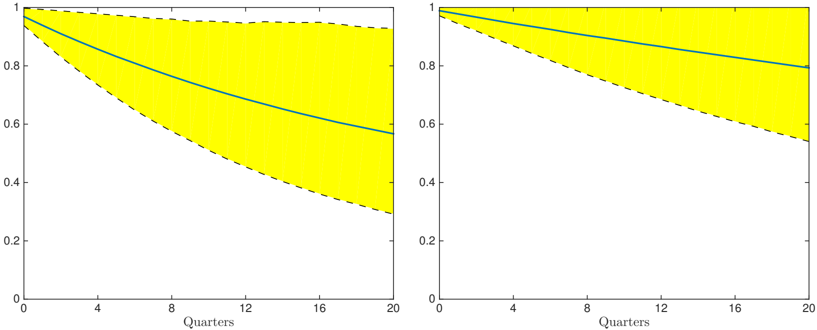


Figure : Contribution of the unexpected component (in small sample)

# RER Persistence

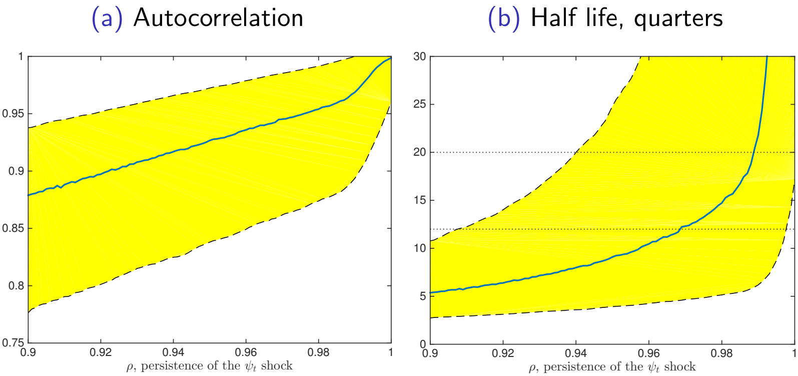


Figure : Persistence of the real exchange rate $q_t$ in small samples

# Exchange Rate and Quantities

back to slides

- Labor Supply:

$$\sigma \tilde{c}_t + \frac{1}{\nu}\tilde{\ell}_t = -\frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t$$

— recall that: $p_t = w_t + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t$

- Labor Demand:

$$\tilde{\ell}_t = \tilde{y}_t + \frac{\phi}{1-\phi}\frac{\gamma}{1-2\gamma}q_t.$$

- Goods market clearing:

$$\tilde{y}_t = \frac{\zeta}{\zeta + \frac{2\gamma}{1-2\gamma}}\tilde{c}_t + \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - (1-\zeta)}{\zeta + \frac{2\gamma}{1-2\gamma}}\frac{\gamma}{1-2\gamma}q_t$$

# Exchange Rate and Interest Rate

(a) Fama $\beta_F$ (b) Fama $R^2$ (c) Sharpe ratio

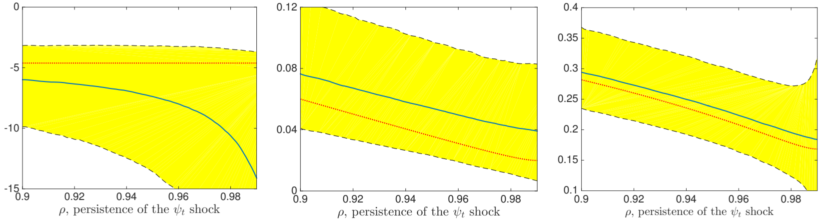


Figure : Deviations from UIP (in small samples)

# Mechanism

1. An international asset demand shock $\varepsilon_t > 0$ results in an immediate sharp ER depreciation to balance the asset market
2. Exchange rate then gradually appreciates (as the $\psi_t$ shock wears out) to ensure the intertemporal budget constraint
3. Nominal and real devaluations happen together, and the real wage declines
4. Devaluation is associated with a dampened deterioration of the terms of trade and the resulting expenditure switching towards home goods
5. Consumption falls to ensure equilibrium in labor and goods markets
6. Consumption fall is supported by an increase in the interest rate, which balances out the fall in demand for domestic assets