Does a Currency Union Need a Capital Market Union?

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Abstract

We study financial linkages and risk sharing in four types of currency unions: segmented markets; a banking union; a capital market union; and complete financial markets. We analyze how these economies respond to various shocks. Broadly speaking, we find that a banking union is efficient at sharing demand shocks (deleveraging, fiscal consolidation), while a capital market union is necessary to share supply shocks (productivity/quality shocks, markup shocks). We present theoretical results where either type of union can exactly replicate the complete market allocation, as well as calibration of welfare gains.
Failures of risk sharing lie at the heart of many economic crises, including the one that recently threatened the viability of the eurozone. Martin and Philippon (2014) decompose the Eurozone crisis into three components: private leverage cycles, sovereign risks, and sudden stops/banking crises. In some countries (e.g., Greece) the fiscal component dominates, while in some others (e.g., Spain) the sudden stop component looms larger. The comparison with the United States is particularly illuminating when we think about risk sharing. The private leverage cycles are large and heterogenous across U.S. states just as much as across E.U. countries, and their immediate consequences are also similar. In Europe however, unlike in the U.S., private leverage cycles are amplified by sudden stops and widening spreads in funding costs across countries. As the costs of fund diverged between countries within the Eurozone, the weaker countries sunk deeper into recession. These are clear signs of inefficient risk sharing.

The creation of a banking union is a deliberate answer to these issues. Focusing on banks is a natural step for Europe since banks intermediate most of its financial flows. The funding cost of banks has a direct impact on the funding conditions of households and firms. The main purpose of the banking union is to guarantee that funding conditions remain the same across regions within the Eurozone, and in particular that they are not directly affected by domestic sovereign risk. There is broad agreement that some form of banking union is necessary to ensure the stability of the currency union. There is disagreement, however, on how far it needs to go, and in particular on the necessity of a full-fledged common deposit insurance scheme.

This suggests two questions that we aim to answer in this paper. First, how large are the gains from building a banking union? Second, should Europe stop there, or are there large gains from building a capital market union in addition to a banking union?

We study a currency union with nominal (wage) rigidities under four degrees of financial integration: (i) segmented markets as observed during the eurozone crisis; (ii) a banking union where funding costs are equalized across regions; (iii) a capital market union with optimal cross border equity holdings; and (iv) a complete market economy. We also consider a planner’s economy that internalizes aggregate demand externalities.

We then ask how these four model economies respond to two types of shocks: demand shocks (triggered by public or private deleveraging), and supply shocks (TFP, terms of trade, etc.). We find that a banking union is enough to deal with leveraging/deleveraging shocks, both public and private. However, a capital market union is necessary to attain (or approximate) the complete market outcome.
Table 1: Summary of Results

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<th>Supply Shocks</th>
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<td>Segmented Markets (SM)</td>
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when there are productivity shocks.

The starting point of our analysis is Martin and Philippon (2014), who provide a framework and an identification strategy to study country dynamics within the Eurozone. Our contribution is to extend their analysis to study spillovers across countries, in particular by modeling aggregate demand spillovers and monetary policy, and to analyze the desirability of capital market integration within a currency union. Martin and Philippon (2014) disentangle the shocks that hit the Eurozone and quantify the relative importance of fiscal policy, private leverage and financial spreads. They find that credit spreads play an important role in exacerbating the Eurozone crisis. They run counterfactuals where spreads are capped and they find that this would have significantly reduced the recession in the Eurozone periphery. We think of a banking union as an institution that guarantees that (risk-adjusted) bank funding costs remain the same in all regions irrespective of the shocks that hit these regions. In particular, bank funding costs are not directly affected by the health of their sovereigns. Since banks are the main source of private funds for households and firms, we model the banking union as a currency union were the private sector costs of funds are equalized across regions. We ask whether such a banking union is sufficient to replicate a complete market economy, and we show that it depends on the shocks. For deleveraging shocks, public or private, we find that the banking union is enough. Note that deleveraging creates an aggregate drag on the economy in any case, but borrowing and lending across regions allows an efficient sharing of the burden of adjustment created by the deleveraging.

Table 1 summarizes our results.

Finally, we study two extensions of the (relatively) simple model. First, we study the implications of allowing borrowers to default. This amounts to a transfer from savers to borrowers. In the class of model that we consider, default can be ex-post efficient. In an open economy setting, an interesting questions arises: who bears the cost of default? We show that this depends on the ultimate ownership of claims and, in particular, of bank equity. The second extension is to introduce physical capital, which is technically challenging in this class of model.
**Related Literature**  Our paper is related to several lines of research in economic integration, open economy macroeconomics and macroeconomics models in the New Keynesian tradition, as well as studies of the causes and consequences of the Eurozone crisis.

Martin and Philippon (2014) and Lane (2012) provide detailed accounts of the principal drivers of the Eurozone crisis; the specific role of the boom/bust cycle in capital flows is analyzed by Lane (2013) and Gourinchas and Obstfeld (2012).

There is a large literature on risk sharing in currency unions. Bayoumi and Masson (1995) discuss the issue of risk sharing and fiscal transfer before the creation of the Euro, and Asdrubali et al. (1996) provide evidence for the US. The Eurozone crisis spurred interest in this topic. Hepp and von Hagen (2013) provide evidence from Germany. Allard and Brooks (2013) summarizes the existing evidence. Farhi and Werning (2017) analyze risk sharing in a currency union in a model with nominal rigidities. They show that fixed exchange rates increase the value of risk sharing and that complete markets do not lead to constrained efficient risk sharing. Blanchard et al. (2014) address the issue of cross-border fiscal multipliers. Bolton and Jeanne (2011) analyze the transmission of sovereign debt crises through the banking systems of financially integrated economies.

If we agree on the necessity of a banking union, the open question then becomes: What other features are necessary for a successful currency union? Considering the U.S. experience, two such features immediately come to mind: fiscal union and capital market union. A fiscal union improves risk sharing via net fiscal transfers across regions. A capital market union improves risk sharing via financial markets, i.e. equity and fixed income flows apart from cross-border bank flows.

In a two-country, two-good endowment economy with fully flexible prices, Cole and Obstfeld (1991) show that the welfare gain from allowing trading of assets across borders is small, since adjustments in the terms of trade provide insurance against country specific fluctuations. Heathcote and Perri (2002) find that in a similar setup but with endogenous production a framework in which countries are in asset market autarky matches cross-country correlations much better than the complete markets model. The analysis in Keloh and Perri (2002) goes a step further in that it endogenizes the incompleteness of markets by introducing enforcement constraints that require each country to prefer the allocation it receives by honoring it’s liabilities than living in autarky from any given time onward. The model goes some way towards explaining cross-country correlations observed in data, but at the cost of generating counterfactually low cross-border investment.

In contrast to studies which preserve the basic structure of the real business cycle model (the IRBC
literature), studies in the new open economy (NOE) literature, following Obstfeld and Rogoff (1995), introduce nominal rigidities in the style of New Keynesian business cycle models as well as market incompleteness into the open economy framework. Schmitt-Grohe and Uribe (2013) emphasize the role of downward wage rigidity in the Eurozone recession. An important shortcoming of models in this tradition is that they are unable to pin down a unique, endogenously determined steady state, and are inherently non-stationary. Ghironi (2006) provides a succinct discussion of the evolution of this literature and attempts to deal with these issues. More recent studies, for example Gali and Monacelli (2008), circumvent the issue entirely by assuming complete asset markets. This is the approach followed, for example, in Blanchard et al. (2014) who model the Eurozone as a two-country (core and periphery) version of Gali and Monacelli (2008).

A common thread in both IRBC and NOE research is that the composition of financing flows is not discussed in detail beyond distinguishing between complete markets and non-contingent bond economies, as explained in Devereux and Sutherland (2011) and Coeurdacier and Rey (2012). The authors provide a simple approximation method for portfolio choice problems in general equilibrium models that are solved using first order approximations around a non-stochastic steady state. An exception to this theme are papers which specifically address one of the enduring puzzles in open economy macroeconomics, the home equity bias puzzle. Coeurdacier and Gourinchas (2011) solve jointly for the optimal equity and bond portfolio in an environment with multiple shocks. In Heathcote and Perri (2013), home bias arises because endogenous international relative price fluctuations make domestic assets a good hedge against labor income risk. Coeurdacier et al. (2010) emphasize trade in stocks and bonds: domestic equity hedges labor income risk while terms of trade shocks are hedged using domestic and foreign bonds.

Following Bernanke and Gertler (1989), many macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke et al. (1999), or Cooley et al. (2004)). In all these models, the availability of credit limits corporate investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Curdia and Woodford (2009) analyze the implication for monetary policy of imperfect intermediation between borrowers and lenders. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, as do Mian and Sufi (2010), that this makes a difference for the model’s cross-sectional implications. In particular, a striking feature
of the data is the strong correlation between household leverage and employment across regions and over time, as documented in Midrigan and Philippon (2010) for the US.\textsuperscript{1} Our framework builds on the tradition of Campbell and Mankiw (1989), that feature impatient and patient consumers. This type of model has been used by Gali et al. (2007) to analyze the impact of fiscal policy on consumption and by Eggertsson and Krugman (2012) to analyze macroeconomic dynamics during the Great Recession.

Fornaro (2014) and Benigno and Romei (2014) study the effect of deleveraging shocks in open economies with nominal rigidities. Fornaro (2014) compares the consequences of a tightening of the exogenous borrowing limit in Bewley economies with and without nominal rigidities and fixed exchange rates. Benigno and Romei (2014) consider a two-country model where one country is a debtor and the other a creditor in the steady state, and analyze the effect of a tightening in the borrowing limit. In contrast to these studies, we analyze a model where each country is populated by both borrowers and savers. This is an important distinction because we know from the extensive research on the US experience that deleveraging in a closed economy (or in an open economy without a sudden stop) can in itself create a recession.

\section{Model}

We consider a currency union composed of several regions, each of which is populated by a (potentially different) measure of infinitely lived households. Each region produces a tradable domestic good and households consume both the domestic and the foreign goods. We will consider two main specifications. One with two regions, home and foreign, as in standard models of international trade, and another with a continuum of countries, each one small relative to the union, as in Gali and Monacelli (2008). Some results are easier to establish in one setup than in the other.

Following Mankiw (2000) and Eggertsson and Krugman (2012), we assume that within each region, households are heterogeneous in their degree of time preference. Specifically, in each region there is a fraction $\chi$ of impatient households, and $1 - \chi$ of patient ones. Patient households (indexed by $i = s$ for savers) have a higher discount factor than borrowers (indexed by $i = b$ for borrowers): $\beta_s > \beta_b$. We denote the regions home and foreign, and indicate foreign variables and parameters with superscript $\ast$. The economies differ only with respect to the menu of traded assets, which affect only savers'\textsuperscript{1}

\textsuperscript{1}For instance, Mian and Sufi (2010) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balance sheets and were able to draw on existing credit lines (Ivashina and Scharfstein (2008)).
borrowing constraints, so we first describe their common structure.\footnote{For ease of exposition the equations presented below are valid when the two regions’ populations are of equal measure; we relegate the general model to the appendix.}

\section{Preferences and technology}

Households of each type and in each region derive utility from consumption and labor:

\[
E_t \sum_{t=0}^{\infty} \beta^t [\log C_{i,t} - \nu (N_{i,t})], \text{ for } i = b, s
\]

where \(C_{i,t}\) is a composite good that aggregates goods produced by the home (\(C_h\)) and foreign (\(C_f\)) countries

\[
\log C_{i,t} = (1 - \alpha) \log (C_{h,i,t}) + \alpha \log (C_{f,i,t}),
\]

and \(\alpha < \frac{1}{2}\) is a measure of the openness to trade of the economy; equivalently, \(1 - \alpha\) measures home bias in consumption. With these preferences, the home consumption-based price index (CPI) is

\[
P_t = (P_{h,t})^{1-\alpha} (P_{f,t})^\alpha,
\]

where \(P_{h,t}\) and \(P_{f,t}\) are the time \(t\) producer price indices (PPI) in the home and foreign countries, respectively. Assuming the law of one price, the foreign price index is \(P_{f,t}^* = (P_{f,t}^*)^{1-\alpha} (P_{h,t}^*)^\alpha\).

The home and foreign goods are in turn compositions of intermediate goods produced in each of the countries, which are aggregated into the final consumption goods using the following constant elasticity of substitution technologies:

\[
C_h = \left[ \int_0^1 c(j)^{1/\epsilon} \, dj \right]^{1/\epsilon}
\]

The PPIs in each region are therefore:

\[
P_{h,t} = \left[ \int_0^1 p_t(j)^{1-\epsilon} \, dj \right]^{1/\epsilon}
\]

where \(p_t(j)\) are prices of intermediate goods. The production of intermediate goods is linear in labor \(N_t\), so the marginal cost of production for each intermediate producer is simply the wage rate \(W_t\).
1.2 Wage setting

We assume that wages are sticky and we ration the labor market uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we loose some tractability.\(^3\) Wage dynamics are determined by a Phillips curve with slope \(\kappa\):

\[
W_t = W_{t-1} (1 + \kappa (N_t - N_{ss})),
\]

where \(N_{ss}\) is steady-state employment. The monopolistically competitive intermediate goods producers set their prices flexibly every period. It follows that:

\[
p_t(j) = P_{h,t} = \mu W_t, \ \forall j, t
\]

where \(\mu \equiv \epsilon / (\epsilon - 1)\) is a markup over the marginal cost \(W_t\). Since intermediate goods producers charge a markup over marginal cost, they earn profits

\[
\Pi_t = (P_{h,t} - W_t) N_t = (\mu - 1) W_t N_t.
\]

1.3 Borrowers’ budget constraint

The budget constraint of impatient households (borrowers) in each country is given by

\[
\frac{B_{t+1}}{R_t} + W_t N_t - T^b_t = P_t C_{b,t} + B_t
\]

Where \(B_t\) is the face value of debt issued in period \(t - 1\) by borrowers, \(R_t\) is the nominal interest rate between \(t\) and \(t + 1\) and \(T^b_t\) are lump sum taxes. Borrowing is denominated in units of the currency of

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\(^3\)In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix this by assuming a low elasticity of labor supply, which essentially boils down to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state \(\nu' (n^*_i) = (1 - \tau_j) \frac{w_j}{n^*_i}\). We can then ration the labor market relative to their natural rate: \(n_{i,j,t} = \frac{n^*_i (\tau)}{\sum_i n^*_i (\tau)} n_{j,t}\) where \(n^*_i (\tau)\) is the natural rate for household \(i\) in country. This ensures consistency and convergence to the correct long run equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.
the monetary union and is subject to an exogenous limit $B_{t+1}$:

$$B_{t+1} \leq \bar{B}_{t+1}$$

We will assume throughout that borrowers are impatient enough that they always borrow up to the constraint, so $B_{t+1} = \bar{B}_{t+1}$.

### 1.4 Monetary and fiscal policy

Monetary policy follows a Taylor rule. The nominal interest rate in the currency union $\bar{R}$ is set by the central bank according to

$$\bar{R}_t = R_{ss} \left( \left( \frac{Y_t}{Y_{ss}} \right) \left( \frac{Y^*_t}{Y^*_{ss}} \right) \phi_Y \left( \left( \frac{\pi_t}{\pi_{ss}} \right) \left( \frac{\pi^*_t}{\pi^*_{ss}} \right) \right) \phi_\pi \right),$$

where $R_{ss}$, $Y_{ss}$ and $\pi_{ss}$ are the steady state interest rate, output and inflation, respectively. The government budget constraint is:

$$\frac{B^g_{t+1}}{R_t} = P_{h,t} G_t - T_t + B^g_t,$$  \hspace{1cm} (1)

The rate on government debt is $R_t$ and tax receipts are $T_t = \chi T^g_t + (1 - \chi) T^*_t$.

### 1.5 Savers’ budget constraint in each of the economies

**Segmented Markets (SMU) and Banking Union (BU)** Savers save at the rate $R^s_t$. The savers’ budget constraint is

$$S_t + W_t N_t - T^*_t + \frac{\Pi_t}{1 - \chi} = P_t C_{s,t} + \frac{S_{t+1}}{R^s_t},$$

where $\Pi_t$ are per-capita profits from intermediate good producers. Only savers in each country have claims to these profits, so $\frac{\Pi_t}{1 - \chi}$ are profits per saver. Under BU, the interest rate at home is always equal to the interest rate in the union: $R_t = \bar{R}_t$ for all $t$. Under SMU, on the other hand, we can have $R_t \neq \bar{R}_t$.

**Capital Market Union (CMU)** In a capital market union savers can trade a non-contingent bond, a home stock and a foreign stock. The savers’ budget constraints in the home region is

$$S_t + W_t N_t - T^*_t + \phi_t \left( V_t + \frac{\Pi_t}{1 - \chi} \right) + (1 - \phi_t) \left( V^*_t + \frac{\Pi^*_t}{1 - \chi} \right) = \varphi_{t+1} V_t + (1 - \varphi^*_{t+1}) V^*_t + P_t C_{s,t} + \frac{S_{t+1}}{R_t}.$$
Similarly, abroad we have

\[ S_t^* + W_t^* N_t^* - T_t^* + \varphi_t^* \left( V_t^* + \frac{\Pi_t^*}{1 - \chi} \right) + (1 - \varphi_t) \left( V_t + \frac{\Pi_t}{1 - \chi} \right) = \varphi_{t+1} V_{t+1}^* + (1 - \varphi_{t+1}) V_t + P_t^* C_{s,t}^* + \frac{S_{t+1}^*}{R_t}, \]

where \( \varphi_t \) are the home savers’ aggregate holdings of the home stock and \( \varphi_t^* \) are the foreign savers’ holdings of the foreign stock. Moreover, \( V_t \) and \( V_t^* \) are the prices of the home and foreign stock, respectively, that represent claims to the aggregate profit streams in the countries.

**Complete Markets** In the complete markets economy, savers have access to a full set of state contingent securities. We denote purchases at time \( t \) of securities paying off one unit of currency at time \( t+1 \) contingent on the realization of state \( s_{t+1} \) following history \( s^t \) by \( D_{t+1} (s_{t+1}, s^t) \); this security has a time \( t \) price \( Q_t (s_{t+1}, s^t) \):

\[ S_t + W_t N_t - T_t + \frac{\Pi_t}{1 - \chi} + \int_{s_{t+1}} Q_t (s_{t+1}, s^t) D_{t+1} (s_{t+1}, s^t) = D_t (s_{t+1}, s^t) + P_tC_{s,t} + \frac{S_{t+1}}{R_t} \]

### 1.6 Equilibrium conditions

Let us summarize the key equations of the model. Demand functions for the home and foreign consumption bundles by savers and borrowers are given by

\[
P_{h,t}C_{h,i,t} = (1 - \alpha) P_tC_{i,t} \tag{2}
\]

\[
P_{f,t}C_{f,i,t} = \alpha P_tC_{i,t}.
\]

Since borrowers are always at their borrowing limit, their consumption is determined by their budget constraint

\[
P_tC_{b,t} = \frac{\bar{B}_{t+1}}{R_t} + W_t N_t - T_t - \bar{B}_t \tag{3}
\]

Savers are unconstrained and their consumption is determined by their Euler equation and budget constraint (which differs across economies, as presented in section 1.5):

\[
\frac{1}{P_tC_t} = \beta R_t E_t \left[ \frac{1}{P_{t+1}C_{t+1}} \right] \tag{4}
\]
Market clearing in goods is given by

\[ N_t = \chi C_{h,b,t} + (1 - \chi) C_{h,s,t} + \chi^* C^*_{h,b,t} + (1 - \chi^*) C^*_{h,s,t} + G_t, \]

where \( G_t \) is spent on home goods only. Substituting in for demand functions and expressing in nominal terms, nominal output is

\[ P_{h,t} N_t = (1 - \alpha) (\chi P_t C_{b,t} + (1 - \chi) P_t C_{s,t}) + \alpha^* (\chi^* P^* t C^*_{b,t} + (1 - \chi^*) P^* t C^*_{s,t}) + P_{h,t} G_t. \]

Finally, market clearing for bonds requires

\[ (1 - \chi) S_{t+1} + (1 - \chi^*) S^*_{t+1} = \chi B_{t+1} + \chi^* B^*_{t+1} + B^d_t + B^d^* t, \]

and (if available) for Arrow-Debreu securities \( D_t (s_{t+1}, s^t) = D^*_t (s_{t+1}, s^t) \) for all \( s_{t+1} \).

2 Deleveraging Shocks under SMU and BU

2.1 Small Open Economies in a BU

In this section we study demand shocks under BU, coming from private borrowing or fiscal policy. We first derive analytical results for the impact of these shocks on a small open economy. We then study the case of two regions. Finally, we also study the effect of a deleveraging shock large enough to make the zero lower bound on the nominal interest rate binding.\(^4\)

Under BU, the funding cost is the same in all regions. Let us first define the \( k \)-period discount rate from the savers’ perspective as \( R_{t,k} \equiv R_t \times \ldots \times R_{t+k-1} \), with the convention \( R_{t,0} = 1 \). We also define \( \tilde{Y}_t \equiv P_{h,t} N_t - T_t \) as private disposable income and \( F_t \) as nominal exports (in the case of two regions this is simply \( F_t = \alpha^* P^* t C^*_t \)).

The first step is to write the current account equilibrium in market values. We then have the following Lemma:

\(^4\)To solve the model when the ZLB occasionally binds we use Guerrieri and Iacovello’s OccBin toolbox; see Guerrieri and Iacovello (2014) for details.
Lemma 1. The inter-temporal current account condition for country $j$ is

$$\alpha \left( (1 - \chi) S_t - \chi B_t + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}^*}{R_{t,k}} \right) = (1 - \chi) S_t - \chi B_t - B_t^{\beta} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}}. \quad (7)$$

Proof. See Appendix. \qed

On the left we have the net present value of all future imports, which is a share $\alpha$ of private wealth, which itself equals financial wealth plus the value of disposable income. On the right we have net foreign assets plus the value of exports ($F_t$). The key point here is that the inter-temporal current-account condition pins down the NPV of disposable income, as a function of current assets and foreign demand. With unit demand elasticity (log-preferences) nominal exports are exogenous to the small open economy.

The next step is to consider the program of the savers. With log-preferences, we can write the savers’ problem as

$$\max \mathbb{E}_t \sum_{t \geq 0} \beta^t \log (P_t C_{s,t})
\quad P_t C_{s,t} + \frac{S_{t+1}}{R_t} = S_t + \tilde{Y}_t^*$$

The inter-temporal budget constraint of savers is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \frac{P_{t+k} C_{s,t+k}}{R_{t,k}} = S_t + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}^*}{R_{t,k}}, \quad (8)$$

where $\tilde{Y}_t^* = W_t N_t - T_{s,t}^* + \frac{\Pi_{t}}{1-\chi}$ is the disposable income of savers. Savers have a claim on corporate equity and might face different taxes than borrowers who earn $\tilde{Y}_t^b = W_t N_t - T_{b,t}^*$. To derive our first result, we need to make a connection between the disposable income of savers $\tilde{Y}_t^s$ that enters Equation (8), and the average disposable income $\tilde{Y}_t = (1 - \chi) \tilde{Y}_t^s + \chi \tilde{Y}_t^b$ that enters Equation (7). If taxes are arbitrary, there is of course very little that we can say. We therefore restrict our attention to a class of fiscal policies where the following condition holds.

Condition 1. The present value of savers’s income is a simple function (linear, affine, etc.) of
that of average disposable income

\[ \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}} \sim \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}} \]

Condition 1 imposes restrictions on fiscal policy, but it holds in many natural settings. The simplest example is uniform flat taxation of all income at rate \( \tau_t \), i.e., \( T^b_t = \tau_t W_t \), and \( T^s_t = \tau_t \left( W_t + \frac{\Pi_t}{1-\chi} \right) \). Therefore all taxes, income and profits are proportional to \( W_t \). In particular \( \tilde{Y}_t = \mu (1 - \tau_t) W_t \), and therefore \( \tilde{Y}^s_t = \frac{1}{\mu} \left( 1 + \frac{\mu - 1}{1-\chi} \right) \tilde{Y}_t \). All disposable incomes are directly proportional, period-by-period, which is much stronger than Condition 1. Note the important fact that markups are constant: we will return to this issue in the next section.

If we combine Lemma 1 and Condition 1, we obtain the following result.

**Lemma 2.** Under Condition 1 and log-preferences, nominal spending by savers \((P_t C_{s,t})\) does not react to private credit shocks \((\bar{B}_{t+1})\) or to fiscal policy (neither \(G_t\) nor \(T_t\)). Spending only reacts to interest rate and foreign demand shocks.

**Proof.** Lemma 1 shows that the net present value of disposable income is a function of exactly four variables:

\[ \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}} \equiv \Omega_t \left( S_t, B_t, B^g_t, \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}} \right) \]

where the first three variables (saving, household debt, public debt) are predetermined at time \( t \) and the last one (exports in euros) is exogenous under log preferences. Therefore, equation (8) is in fact

\[ \mathbb{E}_t \sum_{k=0}^{\infty} \frac{P_{t+k} C_{s,t+k}}{R_{t,k}} \sim S_t + \Omega_t \]

So current spending of savers only depends on \( \Omega_t \) and the path of interest rates. In particular, for given \( \Omega_t \) and interest rates, it cannot depend on contemporaneous or future private credit and fiscal policy.

Lemma 2 clarifies the behavior of savers. Their nominal spending reacts neither to credit shocks nor to fiscal shocks. This is surprising because changes in borrowing constraints or in fiscal policy create fluctuations in GDP and in disposable income \( \tilde{Y}_t \). The key result is that the macroeconomic
value of these inter-temporal disturbances cancels out in a particular way. Suppose $B_{t+1}$ goes up. This increases demand by impatient households, creates a boom, and increases the disposable income of savers. However, when the debt is repaid, with interest, it lowers GDP in such a way that the net present value of disposable income does not change. Changes in $B^h_{t+1}, G_{t+1}, T_{t+1}$ have no effect on the permanent nominal income of patient agents. As a result, patients agents keep their nominal spending constant. These results hinge on our use of the preferences of Cole and Obstfeld (1991). In a small open economy, foreign demand is exogenous, so the proposition completely characterizes the equilibrium behavior of savers. Finally, it is worth emphasizing that our results refer to expenditures, not real consumption. Even when expenditures remain constant, real consumption moves with inflation; in realistic settings, inflation responses are relatively small. We can now state our first main result.

**Proposition 1.** For a small open economy subject to private and public leveraging and deleveraging shocks, the Banking Union achieves the Complete Markets allocation.

**Proof.** Under BU, the interest rate is the same in all regions and is independent of idiosyncratic shocks to the SOE. Given interest rates, savers’ spending $P_tC_{s,t}$ is constant. On the other hand, the complete market outcome is characterized by the Backus-Smith condition, which, with log preferences, takes the form

$$\frac{C^*_{s,t}}{C_{s,t}} \sim \frac{P_t}{P_t^*}$$

Since shocks to an SOE do not affect foreign prices or quantities, it follows that the complete market condition is also that $P_tC_{s,t}$ remains constant. Hence, in response to deleveraging shocks, the BU replicates the complete market economy.

Proposition 1 shows that a banking union is enough to deal with any cross-sectional distribution of debt deleveraging and fiscal shocks. Martin and Philippon (2014) show that segmented markets, on the other hand, can be very inefficient. They find that spreads go up during episodes of private deleveraging, mostly because of stress in the banking sector. This leads savers (or firms under Q-theory) to cut spending precisely when the economy is in recession, exacerbating the downturn.

### 2.2 Two Countries and ZLB

Our next task is to study the case of shocks hitting a large economy. Proposition 1 is exactly correct in a SOE (i.e., a small periphery country). With two economies, foreign demand depends (partly)
on domestic demand and therefore on domestic deleveraging. In addition, the central bank reacts by changing the risk free rate.

In spite of these differences we find that the result of Proposition 1 remains (approximately) correct. The intuition is as follows. First, we know that savers do not react in a SOE. With two countries, foreign demand is endogenous, but this effect is small because it depends on two consecutive cross-border spillovers: the pass-through of domestic demand onto foreign income, and then from foreign income back to foreign demand for home goods. The spillover is quantitatively small.

The second important difference is the Taylor rule. Of course, the reaction of the monetary authority has a direct impact on the dynamics of the economy. But the key point is that this impact is the same under BU and under CM. Why? Because savers face the same interest rate in both countries.

The impulse response to a domestic deleveraging shock in each of the two-region economies is depicted in Figure 1. The responses of all variables except the $S_t$ and $S_{t+1}$ are virtually the same in each of the economies. The ranking of the magnitude of equilibrium adjustment in domestic savings $S_t$ is intuitive: in the BU economy greater adjustment is required than in the complete markets economy.

The aggregate (currency union-wide) response to a deleveraging shock obviously depends on how monetary policy reacts. Our results show that, conditional on whatever the central bank does, the BU and complete market economies behave in virtually identical ways after deleveraging shock. One might wonder, however, if this result could be over-turned if the central bank is constrained by the zero lower bound. We find that this is not the case: our result also holds at the ZLB. Figure 2 depicts impulse responses to a deleveraging shock large enough to make the ZLB bind. Naturally, when the ZLB binds the central bank is unable to lower the interest rate enough to stabilize aggregate employment in the currency union.

We conclude that a successful banking union – or any union that guarantees that rates of returns are equalized across regions – is enough to deal with demand shocks stemming from fiscal policy or from private credit cycles.

### 2.3 Welfare Costs of Market Segmentation

The goal of this section is to present a simple model to quantify the welfare losses from market segmentation in a currency union, and hence the welfare gains from implementing a banking union. Under segmented markets, the private costs of funds are not equalized across regions. Martin and Philippon (2014) and Gourinchas et al. (2016) quantify the extent of the dispersion in funding costs
Figure 1: Private Deleveraging in 2-Country Model

Note: Impulse responses to permanent -5% shock to $\bar{B}_t$. 
Figure 2: Private Deleveraging in 2-Country Model with ZLB

Note: Impulse responses to permanent -5% shock to $\bar{B}_t$. 
Table 2: Consumption volatilities under segmented markets and a banking union

<table>
<thead>
<tr>
<th></th>
<th>Segment markets</th>
<th>Banking union</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savers’ consumption volatility</td>
<td>8.2%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Borrowers’ consumption volatility</td>
<td>9.3%</td>
<td>8.6%</td>
</tr>
<tr>
<td>Aggregate consumption volatility</td>
<td>7.2%</td>
<td>2.8%</td>
</tr>
</tbody>
</table>

during the Eurozone crisis. The simplest interpretation is that domestic banks intermediate savings and investment, so that the private cost of fund is pinned down by the banking system. Formally, in log-deviations from steady state, we have

\[ r_t = r_t^b \]

where \( r_t^b \) is the banks’ funding cost. We can then consider a small island subject to two shocks, a spread shock \( r_t^s \) and a private leverage shock \( \bar{B}_t \). The deleveraging shock can be modeled as (at annual frequency):

\[
\log \bar{B}_{i,t} - \log \bar{B}_{i,t-1} = -0.01 \times (\log \bar{B}_{i,t-1} - \log \bar{B}) + 0.85 \times (\log \bar{B}_{i,t-1} - \log \bar{B}_{i,t-2}) + 0.04 \epsilon_{i,t}^b
\]

where \( \epsilon \) is normalized (0,1) and \( \bar{B} \) is the steady-state leverage. The spread follows a persistent process

\[ r_t^b = 0.9 r_t^{bs} + 0.003 \epsilon_t^r \]

and the correlation between the two shocks is

\[
corr (\epsilon_{i,t}^b, \epsilon_t^r) = -0.3
\]

So when spreads go up, debt goes down. Spread differences between countries increase consumption volatility and lower welfare. The banking union lowers consumption volatility by equalizing interest rates between countries.

Table 2 shows the consumption volatilities under segmented markets and a banking union. Moving from segmented markets to a banking union leads to a 86% reduction in the volatility of (log) consumption of savers and to a 7.4% reduction for borrowers. Under segmented markets, the borrowers’ consumption volatility is 14% higher than that of savers. Due to logarithmic preferences the welfare benefits of these changes are still relatively small. However, we could increase this welfare gain by raising savers’ risk aversion possibly through the use of recursive preferences (Epstein and Zin (1989)).
3 Quality Shocks

3.1 Theory

In this section we look at technology shocks in the form of “quality” shocks to the goods sold by domestic firms. Formally, we model these shocks as an unanticipated change to $\alpha^*$, the preference of foreigners for domestic goods. In response to these shocks, domestic firms become more profitable, while foreign firms become less profitable. The banking union will not be able to share this kind of risk. However, the capital market union can in principle share these types of shocks. This result is highlighted in the following proposition:

**Proposition 2.** It is possible to replicate the complete market allocation in a capital market union subject to (home or foreign) quality shocks using static equity positions and no-cross country borrowing.

**Proof.** Given symmetric countries the complete market condition is $P_tC_t = P_t^*C_t^*$. Imposing symmetric and constant stock positions as well as constant labor taxes and borrowing limits (these are relaxed later), the savers’ budget constraints are

$$\tilde{B} + W_t N_t - T + \varphi \frac{\Pi_t}{1 - \chi} + (1 - \varphi) \frac{\Pi_t^*}{1 - \chi} = P_t C_{s,t} + \frac{\tilde{B}}{R_t}$$

and

$$\tilde{B} + W_t^* N_t^* - T + \varphi \frac{\Pi_t^*}{1 - \chi} + (1 - \varphi) \frac{\Pi_t^*}{1 - \chi} = P_t^* C_{s,t} + \frac{\tilde{B}}{R_t}$$

Quality shocks affect firm profits and labor income. Subtracting the borrowing constraints, imposing the complete market condition and using the production function yields

$$(W_t N_t - W_t^* N_t^*) \left(1 + \frac{(2 \varphi - 1)(\mu - 1)}{1 - \chi}\right) = 0$$

Away from the indeterminacy case where labor income in both countries is always the same, we can solve

$$\varphi = \frac{1}{2} - \frac{1}{2} \frac{1 - \chi}{\mu - 1}$$

With these stock positions the complete market condition holds for arbitrary labor income realizations. The complete market condition also ensures that the Euler equations for the stocks and the bond hold. Therefore the above stock positions and no-cross country borrowing constitute an equilibrium. □
To efficiently share quality shocks, savers should underweight home stocks. In practice various frictions might lead savers to do the opposite and overweight home stocks. This type of capital market union with partially segmented equity markets is able to share some but not all of the risks related with quality shocks.

Figure 3 shows the outcome to a quality shock in a banking union, a partial capital market union (with equal weights on home and foreign stocks), and complete markets (equivalently, a CMU with optimal weights). In a complete market savers’ spendings in the two countries are equalized. Proposition 2 shows that, if stock positions are chosen correctly, the capital market outcome coincides with the complete market case. However, with equal weights on home and foreign stocks, savers’ spending in the home country increases relative to that in the foreign country. However, this increase is milder than in a banking union.

Simultaneous Quality and Demand Shocks

Proposition 1 shows that a banking union is able to share demand shocks using dynamic borrowing. Proposition 2 argues that a capital market union can share quality shocks using static equity positions. These propositions can be combined to yield the following result

**Proposition 3.** When each country is small, it is possible to replicate the complete market allocation in a capital market union subject to both (idiosyncratic) deleveraging and (arbitrary) quality shocks using static equity positions and dynamic cross-country borrowing.

Proof: See Appendix.

3.2 Welfare Benefits of a Capital Market Union

In this section we estimate the welfare gains of moving from a banking union to a capital market union. We consider an economy subject to deleveraging and quality shocks. The quality shocks are chosen to match empirical estimates of consumption and export growth volatilities. The results are given in Table 3. Moving from a banking union to a capital market union with optimally chosen stock positions results in a 42% reduction in the consumption volatility of savers and to a 26% reduction for borrowers.
Notes: Impulse responses to 10% shock to $\alpha^*$. CMU 0.5 has exogenous equal weights on home and foreign stocks. Complete markets is equivalent to a CMU with optimal weights, as explained in Proposition 2. BU is CMU with zero weight on foreign stocks.
Table 3: Consumption volatilities under a banking union and a capital market union, deleveraging and quality shocks

<table>
<thead>
<tr>
<th></th>
<th>Banking union</th>
<th>Capital market union</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savers’ consumption volatility</td>
<td>8.3%</td>
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</tr>
<tr>
<td>Borrowers’ consumption volatility</td>
<td>10.3%</td>
<td>7.6%</td>
</tr>
<tr>
<td>Aggregate consumption volatility</td>
<td>8.6%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

4 Extensions

We study two extensions of the model. The first is to introduce physical capital. The second is to consider default on private debts.

4.1 Technology Shocks in a model with Capital

[TO BE COMPLETED]

4.2 Allowing Borrowers to Default

In this section we study deleveraging with default by some borrowers. Allowing borrowers to default can have a significant impact on deleveraging dynamics. A default is a transfer from savers to borrowers. If borrowers can default, the required spending cut is smaller, and this limits the macroeconomic impact of deleveraging. This is the direct effect of default. The indirect effects work through the banking system, as non performing loans reduce banks’ solvency and limit their ability to lend. In that sense, default can lead to a credit crunch. We consider each one in turn.

Default and Deleveraging

Consider a debt deleveraging shock at home, which reduces the gross debt of impatient agents. Without default, the budget constraint of an impatient agent is given by equation (3). We start from a steady state with constant $\bar{B}_t = B_0$ and we tighten the limit to $\bar{B}_1 < B_0$. We let $\eta$ be the fraction of deleveraging achieved through default. The budget constraint at time 0 then becomes

$$P_0C_{0,0} = W_0N_0 - T_0 + \frac{\bar{B}_1}{K_0} - (1 - \eta)\bar{B}_0$$

We assume no further default from time 1 onward.

The next question is: Who bears the cost of default? In a closed economy setup credit losses must be borne by domestic savers. In an open economy it depends on who owns the debt. Let $\omega$ be the
fraction of home debt owned by home savers. After the default, we have

\[ S_0 = E_{-1} [S_0] - \omega \eta \bar{B}_0 \]

and

\[ S_0^* = E_{-1} [S^*_0] - (1 - \omega) \eta \bar{B}_0 \]

The debt can be directly held, or it can be intermediated by banks. In the later case, the crucial issue becomes the equity ownership of banks. The most direct interpretation of these equations is that household debt is directly held by other households. An equivalent setup is to imagine that domestic banks intermediate domestic household debt. Domestic banks make loans to households and finance themselves with debt and equity. Foreign households own a share \((1 - \omega)\) of domestic bank equity. When borrowers default, bank equity takes a hit, and foreign household end up absorbing \((1 - \omega) \eta \bar{B}_0\) just as if they held the debt directly.

**Lemma 3. Equivalence of cross-border bond funds and foreign bank equity ownership.**

The two following economies are equivalent in their sharing of credit risk: (1) foreign households hold a fraction \(1 - \omega\) of bonds backed by domestic loans; or (2) domestic households borrow from domestic banks, but foreign households own a fraction \(1 - \omega\) of domestic bank equity.

The lemma makes clear that, as far as sharing default risk is concerned, different types of capital market integration are equivalent. One option is to package loans into bonds and sell them abroad. Another option is to let banks retain the loans and encourage cross-border ownership of bank equity.

We now present impulse responses to deleveraging and default shocks. Figure 4 presents impulse responses to a 5% deleveraging shock when part of deleveraging is achieved through default, for different values of \(\eta\) (keeping \(\omega = 0.5\)). For the case of deleveraging achieved completely through default (\(\eta = 1\)), the deleveraging shock has no effect. The effects of the deleveraging shocks are larger the smaller the fraction of deleveraging achieved through default.

For a large enough deleveraging shock, such that the zero lower bound on the nominal interest binds, deleveraging through default is especially attractive from the point of view of employment stabilization: in the most extreme case in which deleveraging occurs only through default (\(\eta = 1\)) and the incidence of default by domestic borrowers falls only on domestic savers (\(\omega = 1\)), employment is unchanged in both countries and consequently the central bank does not change its policy rate. In the polar opposite case, \(\eta = 0\), full employment stabilization requires an unfeasibly large fall in the
Figure 4: Deleveraging with Default

Notes: Impulse responses to permanent -5% shock to $B_t$ with default. $\eta = 0$ means no default, $\eta = 1$ means complete default. Cross-border bank equity is set at $\omega = 0.5$. 

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nominal interest rate. This comparison is illustrated in Figure 5 (for $\omega = 0.5$).
Figure 5: Deleveraging with Default at ZLB

Notes: Impulse responses to permanent -5% shock to $B_t$ with default. $\eta = 0$ means no default, $\eta = 1$ means complete default. Cross-border bank equity is set at $\omega = 0.5$. 

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Notes: Impulse responses to permanent -5% shock to $B_t$ with default. $\eta = 0$ means no default, $\eta = 1$ means complete default. Cross-border bank equity is set at $\omega = 0.5$. 

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**Default and Banks’ Funding Costs** In a model à la Gertler and Kiyotaki (2013), the funding cost of banks depends on current and expected credit losses. Gourinchas et al. (2016) find that private credit losses are well described by

\[ d^p_t = -\bar{d}_y y_t + \bar{d}_b b_{t-1} + \zeta^{def}_t, \]

where \( y_t \) is (disposable) income. Non performing loans increase when private debt is high or when income goes down. This simple log-linear relation capture more than 80% of the time variation of non performing loans in Greece. The exogenous shocks \( \zeta^{def}_t \) is a fundamental source of risk and well approximated by an AR(1) process. In the simplest model, these effects lead to a log-linear equation for banks’ funding cost

\[ r^b_t = \bar{r}_t + \xi^p E_t \left[ d^p_{t+1} \right] + \epsilon_t \]

where \( r_t \) is the policy rate, \( E_t \left[ d^p_{t+1} \right] \) are expected credit losses (non-performing loans to domestic firms and households). Finally, we have already discussed the negative correlation between banks funding costs and lending.

This extension connects defaults, bank fundings costs, and lending. In that sense, it connects SU, BU and CMU.

[TO BE COMPLETED]

## 5 Conclusion

We define a banking union as an institution that ensures that private funding costs are equalized across regions in a currency union. We define a capital market union as the optimal cross-border ownership of corporate equity.

We find that a banking union can efficiently share demand shocks. Remarkably, with unit demand elasticities, the banking union exactly replicates the complete market outcome. In other cases, it is quantitatively close. We find that a capital market union can efficiently share productivity and terms-of-trade shocks.

We estimate that the gains of moving from SM to BU are large if we calibrate the model with recent data. The gains of moving from BU to CMU are smaller but still significant. [TO BE COMPLETED].
APPENDIX

A Equilibrium Conditions

A.1 Home

\[ P_t C_{b,t} = \frac{\bar{B}_{t+1}}{R_t} + W_t N_t - T_t - \bar{B}_t \]

\[ \frac{1}{P_t C_{s,t}} = \beta R_t E_t \left[ \frac{1}{P_{t+1} C_{s,t+1}} \right] \]

\[ P_{h,t} N_t = (1 - \alpha) \left( \chi P_t C_{b,t} + (1 - \chi) P_t C_{s,t} \right) + \frac{N^{*}_{ss}}{N^{*}_{ss}} \alpha^{*} \left( \chi^{*} P_t^{*} C_{b,t}^{*} + (1 - \chi^{*}) P_t^{*} C_{s,t}^{*} \right) + T_t \]

\[ \Pi_t = (P_{h,t} - W_t) N_t \]

\[ P_{h,t} = \mu W_t \]

\[ W_t = W_{t-1} (1 + \kappa (N_t - N^{*})) \]

\[ S_t + Y_t - T_t + \frac{(1 - \phi^{*}) \Pi_t + \phi \Pi^{*} \frac{N^{*}_{ss}}{N^{*}_{ss}}}{1 - \chi} = P_t C_{s,t} + \frac{S_{t+1}}{R_t} \]

A.2 Foreign

\[ P_t^{*} C_{b,t}^{*} = \frac{\tilde{B}_{t+1}^{*}}{R_t} + W_t^{*} N_t^{*} - T_t^{*} - \tilde{B}_{t}^{*} \]

\[ \frac{1}{P_t^{*} C_{s,t}^{*}} = \beta R_t E_t \left[ \frac{1}{P_{t+1}^{*} C_{s,t+1}^{*}} \right] \]

\[ P_{h,t}^{*} N_t^{*} = (1 - \alpha^{*}) \left( \chi^{*} P_t^{*} C_{b,t}^{*} + (1 - \chi^{*}) P_t^{*} C_{s,t}^{*} \right) + \frac{N^{*}_{ss}}{N^{*}_{ss}} \alpha^{*} \left( \chi P_t C_{b,t} + (1 - \chi) P_t C_{s,t} \right) + T_t^{*} \]
\[ \Pi^*_t = (P^*_{h,t} - W^*_t) N^*_t \]

\[ P^*_{h,t} = \mu^* W^*_t \]

\[ W^*_t = W^*_{t-1} (1 + \kappa^* (N^*_t - N^*_{ss})) \]

\[ S^*_t + W^*_t N^*_t - T^*_t + \frac{(1 - \phi) \Pi^*_t + \phi \Pi^* \frac{N^*_{ss}}{N^*_t}}{1 - \chi^*} = P_t C_{s,t} + \frac{S^*_{t+1}}{R_t} \]

### A.3 Union-wide

\[ R_t = R_{ss} \left( \left( \frac{Y_t}{Y_{ss}} \right)^{N_{ss}^* / N_{ss}^*} \left( \frac{Y_t}{Y_{ss}} \right)^{N_{ss}^* / N_{ss}^*} \right) \frac{\phi_Y}{\phi_Y} \left( \left( \frac{\pi_t}{\pi_{ss}} \right)^{N_{ss}^* / N_{ss}^*} \left( \frac{\pi_t}{\pi_{ss}} \right)^{N_{ss}^* / N_{ss}^*} \right) \]

and

\[ N_{ss} (1 - \chi) S_{t+1} + N_{ss}^* (1 - \chi^*) S_{t+1}^* = N_{ss} \chi B_{t+1} + N_{ss}^* \chi^* B_{t+1}^* \]

### B Proof of Lemma 1

Define the k-period ahead discount rate for \( k \geq 1 \) from the savers' perspective

\[ R_{j,t,k} \equiv (1 + r_{j,t}) \cdots (1 + r_{s,j,t+k-1}) \]

and the convention \( R_{j,t,0} = 1 \).

Let us start from market clearing for the home good:

\[ P_{h,t} N_t = (1 - \alpha) (\chi P_t C_{b,t} + (1 - \chi) P_t C_{s,t}) + \alpha^* (\chi^* P^*_t C^*_{b,t} + (1 - \chi^*) P^*_t C^*_{s,t}) + P_{h,t} G_t \]

Using the budget constraints of the agents and of the government we get

\[ \alpha_j \tilde{Y}_{j,t} = (1 - \alpha_j) \chi_j \left( \frac{B_{j,t+1}^h}{1 + r_{j,t}} - B_{j,t}^h \right) - (1 - \alpha_j) (1 - \chi_j) \left( \frac{S_{j,t+1}}{1 + r_{j,t}} - S_{j,t} \right) + F_{j,t} + \frac{B_{j,t+1}^g}{1 + r_{j,t}} - B_{j,t}^g \]
Summing and rearranging the terms, we get

\[
\alpha_j \left( \tilde{Y}_{j,t} + \frac{\tilde{Y}_{j,t+1}}{R_{j,t,1}} \right) = (1 - \alpha_j) \chi_j \left( \frac{1}{R_{j,t,1}} \frac{B_{j,t+2}^h}{1 + r_{j,t+1}} - B_{j,t}^h \right) - (1 - \alpha_j) (1 - \chi_j) \left( -S_{j,t} + \frac{1}{R_{j,t,1}} \frac{S_{j,t+2}}{1 + r_{j,t+1}} \right) + F_{j,t} + \frac{F_{j,t+1}}{R_{j,t,1}} \]

\[
+ \frac{1}{R_{j,t,1}} \frac{B_{j,t+2}^g}{1 + r_{j,t+1}} - B_{j,t}^g \]

to write:

\[
\alpha_j \left( \tilde{Y}_{j,t} + \frac{\tilde{Y}_{j,t+1} + \tilde{Y}_{j,t+2}}{R_{j,t,1}} \right) = - (1 - \alpha_j) \chi_j \left( B_{j,t}^h - \frac{1}{R_{j,t,2}} \frac{B_{j,t+3}^h}{1 + r_{j,t+2}} \right) + (1 - \alpha_j) (1 - \chi_j) \left( S_{j,t} - \frac{S_{j,t+3}}{R_{j,t,3}} \right) + F_{j,t} + \frac{F_{j,t+1}}{R_{j,t,1}} + \frac{F_{j,t+2}}{R_{j,t,2}} \]

\[
- B_{j,t}^g + \frac{1}{R_{j,t,2}} \frac{B_{j,t+3}^g}{1 + r_{j,t+2}} \]

Therefore for a generic horizon \( K \)

\[
\sum_{k=0}^{K} \alpha_j \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k-1}} = (1 - \alpha_j) \left( (1 - \chi_j) S_{j,t} - \chi_j B_{j,t}^h \right) - B_{j,t}^g + \sum_{k=0}^{K} \frac{F_{j,t+k}}{R_{j,t,k}} \\
- (1 - \chi_j) (1 - \alpha_j) \frac{S_{j,t+K+1}}{R_{j,t,K+1}} + \frac{1}{R_{j,t,K}} \left( \frac{(1 - \alpha_j) \chi_j B_{j,t+K+1}^h}{1 + r_{j,t+K}} + \frac{B_{j,t+K+1}^g}{1 + r_{j,t+K}} \right) \]

We take the limit and we impose a No-Ponzi condition

\[
\lim_{K \to \infty} E_t \left[ \frac{S_{j,t+K+1}}{R_{j,t,K+1}} \right] = 0 \]

\[
\lim_{K \to \infty} E_t \left[ \frac{1}{R_{j,t,K}} \frac{B_{j,t+K+1}^h}{1 + r_{j,t+K}} \right] = 0 \]

\[
\lim_{K \to \infty} E_t \left[ \frac{1}{R_{j,t,K}} \frac{B_{j,t+K+1}^g}{1 + r_{j,t+K}} \right] = 0 \]

The inter-temporal current account condition is

\[
\alpha_j E_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}} = E_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}} - (1 - \alpha_j) (\chi_j B_{j,t}^h - (1 - \chi_j) S_{j,t}) - B_{j,t}^g \]
C  Proof of Proposition 3

Step 1: Extending Propositions 1-2  To begin, we need to extend some of the earlier results. First we extend proposition 2 to include multiple symmetric countries. Given that there are $I$ symmetric countries, the complete market condition is $C_{t,s,i}P_{t,i} = C_{t,s,j}P_{t,j}$, $i, j = 1, ..., I$. With symmetric stock positions and uniform tax rate on labor and capital income, the budget in each country is

$$P_{t,i}C_{s,t,i} = B(1 - \frac{1}{R_t}) + W_{t,i}N_{t,i}(1 - \tau) + \frac{\varphi(\mu - 1)W_{t,i}N_{t,i}(1 - \tau)}{1 - \chi} + \sum_{j \neq i} (1 - \varphi) (\mu - 1)W_{t,j}N_{t,j}(1 - \tau)$$

Deducting the conditions for two countries $i$ and $j \neq i$ we obtain

$$P_{t,i}C_{s,t,i} - P_{t,j}C_{s,t,j} = (W_{t,i}N_{t,i} - W_{t,j}N_{t,j})(1 - \tau) + \frac{\varphi(\mu - 1)W_{t,i}N_{t,i} - (\mu - 1)W_{t,j}N_{t,j}(1 - \tau)}{1 - \chi} - (\mu - 1)\frac{W_{t,i}N_{t,i} - W_{t,j}N_{t,j} 1 - \varphi}{1 - \chi} (1 - \tau) = (1 - \tau) (W_{t,i}N_{t,i} - W_{t,j}N_{t,j}) \left(1 + \varphi \frac{\mu - 1}{1 - \chi} - \frac{1 - \varphi \mu - 1}{T - 1 - 1 - \chi}\right)$$

Imposing the complete market condition and ignoring the indeterminacy case, we need

$$1 + \varphi \frac{\mu - 1}{1 - \chi} - \frac{1 - \varphi \mu - 1}{T - 1 - 1 - \chi} = 0$$

From this one can solve

$$\varphi = \frac{1}{I} - \frac{I - 1 1 - \chi}{\mu - 1}$$

Savers in each country should invest $\frac{1}{T - \lambda} + \frac{1 - \chi}{\mu - 1}$ in home stocks and $\frac{1}{I} + \frac{1 - \chi}{\mu - 1}$ in the stocks of each foreign country. Savers should overweight foreign stocks. Next we need to extend proposition 1 to include static equity positions. Using manipulations similar to those in the proof of Lemma 1, we can write

$$W_{t,i}N_{t,i}(\mu - (1 - \alpha)(1 + \varphi(\mu - 1))) = F_{t,i} + \alpha G_{t,i} + (1 - \chi)(1 - \alpha) \left(\frac{B_{t+1,i}}{R_t} - B_{t,i}\right)$$

$$- \chi(1 - \alpha) \left(\frac{S_{t+1,i}}{R_t} - S_{t,i}\right) + (1 - \alpha)(1 - \chi) \Gamma_{t,i} + (1 - \alpha) \left(\frac{B_{t+1,i}^g}{R_t} - B_{t,i}^g\right)$$
Here $\Gamma_t$ is the savers’ income from foreign stocks. For simplicity assume labor income as well as home and foreign capital gains are all taxed at the same rate $\tau$ and that the government does not take new debt. From this we obtain

$$W_{t,i}N_{t,i}(\mu - (1 - \alpha)(1 + \varphi(\mu - 1)) - \alpha\tau - \alpha\tau\varphi(\frac{\mu - 1}{1 - \chi}) = F_{t,i} + (1 - \chi)(1 - \alpha)\left(\frac{B_{t+1,i}}{R_t} - B_{t,i}\right)$$

$$\chi(1 - \alpha)\left(\frac{S_{t+1,i}}{R_t} - S_{t,i}\right) + ((1 - \alpha)(1 - \chi) + \alpha\tau)\Gamma_{t,i} + (1 - \alpha)\left(\frac{B_{t+1,i}}{R_t} - B_{t,i}^g\right)$$

From this it is possible to write labor income as a function of exogenous variables. Then one can write the value of savers’ spending as a function of exogenous variables $S_t, B_t, B_t^g, E_t \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t+k}}$ and $E_t \sum_{k=0}^{\infty} \frac{\Gamma_{t+k}}{R_{t+k}}$ as in the proof of Lemma 1. This generalizes proposition 1.

**Step 2: The Main Argument**  Given symmetric borrowing patterns the above stock positions perfectly share shocks affecting labor income such as quality shocks. These shocks need not be idiosyncratic. Idiosyncratic deleveraging shocks do not distort symmetry. This is because the savers hold a constant amount in non-contingent savings. If borrowers pay back debt, the savers can substitute this by lending more to foreign countries.

**D Calibration**

**D.1 Symmetric calibration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\chi$</td>
<td>Fraction of impatient</td>
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<tr>
<td>$\beta$</td>
<td>Discount factor of savers</td>
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<td>$\alpha$</td>
<td>Openness to trade</td>
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<td>$\kappa$</td>
<td>Slope wage Phillips curve</td>
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<td>$\epsilon$</td>
<td>Elasticity domestic intermediates</td>
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<td>$\varphi$</td>
<td>Portfolio share CMU economy</td>
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<tr>
<td>$\phi_Y$</td>
<td>Taylor rule - output gap</td>
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<tr>
<td>$\phi_\pi$</td>
<td>Taylor rule - inflation</td>
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</tbody>
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References


