IMF *Staff Papers*
Vol. 45, No. 4 (December 1998)

Virtual Deficits and the Patinkin Effect

ELIANA CARDOSO*

The paper develops a model of inflationary finance that defines the fiscal deficit as a function of the virtual deficit—one that would be observed at zero inflation. It studies the negative relationship between the inflation rate and real government expenditures—the Patinkin effect—a powerful stabilizer during megainflation. The model outperforms other seigniorage models in explaining the persistence of four-digit inflation rates that never explode into an open hyperinflation. It also explains how apparently expansionist fiscal policies end in measured real deficits that are small and compatible with the small amount of seigniorage that can be collected at high inflation rates. [JEL E10, E31, E58, E62]

> *. . . in a situation where—because of coalition considerations—the finance minister does not have the power to force individual ministries to make adequate reductions in their respective budgetary demands and is thus confronted with an overall budget whose planned expenditures far exceed its expected revenues, he may seemingly accept these demands, and then finance the deficit by printing money and letting the resulting inflation enforce the necessary reduction in real government expenditures.*
>
> Don Patinkin (1993, p. 115)

ECONOMISTS THINK of extreme inflation as an unstable process, the instability reinforced by the Tanzi effect—a decline in real tax revenues as inflation rises. But empirical evidence suggests a powerful effect that runs in the other direction through declining real spending levels—the Patinkin effect. This paper introduces the concept of a virtual budget

* Eliana Cardoso is a Lead Specialist in the Poverty Reduction and Economic Management Division of the World Bank's Latin America and the Caribbean Regional Office. When she wrote this paper she was an Advisor in the IMF's Research Department. She thanks Rudi Dornbusch, Stanley Fischer, Robert Flood, Peter Isard, and Paul Masson for helpful comments.

deficit (a deficit that would be observed if inflation were zero), develops a model of inflationary finance, and applies the model to the case of Brazil.

Observed aggregate budget data on nominal, operational, and primary deficits contain very little information about the true fiscal position of the public sector when inflation exceeds 500 percent a year. The Tanzi effect predicts that real tax revenues decline as inflation rises and thus the budget deficit is higher at higher inflation rates. But there is also a reverse Tanzi effect—referred to here as the Patinkin effect. If the Patinkin effect dominates at high inflation rates, real expenditures appear lower than they would be if there were no inflation, and real expenditures tend to increase when inflation disappears. Thus, the fiscal adjustment needed once inflation disappears is usually underestimated. Several factors explain this phenomenon.

- Real interest rates decline with increasing inflation rates and usually rise following stabilization. This rise in real interest rates contributes to the increase in real government expenditures once inflation disappears.
- During periods of high inflation, local governments usually delay payments of salaries and wages. When inflation exceeds 1,000 percent a year, this delay produces a substantial decline in real expenditures. When inflation disappears, delaying payments no longer reduces real expenditures.
- Although governments have learned to lessen gaps in tax collections and to index delayed tax payments to inflation, they still program expenditures with a forecast for inflation that is usually lower than observed inflation. As a consequence, realized real expenditures are much lower than programmed expenditures. When inflation disappears, actual expenditures will be closer to their programmed levels.
- The inflationary revenue of state banks can finance credit subsidies that are not recorded. This revenue disappears when inflation disappears. Furthermore, if inflation conceals banks' weaknesses, and these weaknesses are accentuated by the rise in real interest rates that follows stabilization, the government will have to use fiscal revenues to rescue banks, and recorded real expenditures will increase with stabilization.

Because inflation reduces real expenditures but not real taxes when governments fully index taxes and reduce gaps in tax collections, inflation can be used to accommodate conflicting spending programs of different government levels. Thus, inflation produces operational budget deficits consistent with the amount of real seigniorage that the government needs to finance the deficit.

Section I introduces the concept of a virtual budget deficit and the concept of a Patinkin effect.[1] It also develops a model of inflationary finance. The section discusses multiple equilibria and shows that, even if the virtual budget deficit exceeds the maximum amount of seigniorage that the government can collect, one stable high-inflation equilibrium exists if the Patinkin effect is strong enough. The model outperforms other seigniorage models in explaining the persistence of four-digit inflation rates in countries where inflation never explodes into an open hyperinflation. Furthermore, it explains how expansionist fiscal policies end in measured real deficits compatible with the small amount of seigniorage that can be collected at high inflation rates. The model can also accommodate the traditional analysis of explosive hyperinflations if the Patinkin effect is not strong or if indexing breaks down at extremely high inflation rates. The last part of Section I discusses the shares in seigniorage accruing to the central bank and deposit banks.

Section II applies the model to the case of Brazil, discussing the banking sector's share of seigniorage, interest rate spreads, and nonperforming loans following the Real Plan, instituted in mid-1994. The reduction of the banking sector's share of seigniorage immediately after the stabilization was a consequence of the changes in required reserves. The increase in required reserves and the decline in seigniorage of the banking sector in part explains the increase in interest rate spreads, the high active real interest rates, and the increase in nonperforming loans after stabilization. Section III offers concluding remarks.

I. Budget Deficits and Inflationary Finance

Tanzi (1978) was among the first to explore the impact of inflation on tax revenues. He observed that a rise in inflation could increase or decrease real tax revenues depending on lags in tax collection, built-in elasticity, and indexation. In general, tax collection lags in developing countries, where real tax revenues are assumed to decline as inflation rises, are thought to be long relative to those observed in industrial countries. A rise in inflation would thus increase the budget deficit in developing countries, a process known as the Tanzi effect.

But inflation also affects real expenditures. Bresciani-Turroni (1937, p. 34), one of the first economists to study the relationship between the inflation rate

[1] The concept of the virtual deficit is mentioned by Fischer (1994) in a footnote. He claims that there is a case for calculating a "zero-inflation deficit" (different from the operational deficit) because of the Tanzi effect and because the real interest rate might change if inflation were stabilized.

and the budget deficit, observed that as inflation accelerates, the relationship between the budget deficit and inflation can become negative:

> . . . the German authors, who have maintained that the depreciation of the mark was the cause of the disequilibrium between income and expenditure (in Germany) because, given the imperfect adaptation of income to the monetary depreciation, the yield was diminished, have not considered that in the period now under examination the depreciation of the mark influenced both income and *expenditure* in the same direction. Computed in gold marks, the total expenditure also diminished considerably from July 1919 to February 1920 and more rapidly than the income.

Patinkin (1993) shows how pressure among political coalitions can lead to the use of inflation to erode the real burden of conflicting nominal expenditure demands by different ministries, as in the case of Israel before 1985. Guardia (1992) reports that during high inflation years in Brazil, realized real deficits were always smaller than the programmed real deficits. According to Bacha (1994), programmed real expenditures in Brazil exceeded realized real expenditures because projected inflation was always less than observed inflation, and indexation of expenditures was always avoided.[2]

Government Spending, Deficits, and Inflation

How are spending decisions actually made, in a high-inflation country such as Brazil? In the early and mid-1990s, Brazil's treasury would collect actual federal revenues 10 days at a time, allocate the constitutional shares to state and municipal governments, cover current interest on the public debt, meet the payroll for federal employees, and then allocate the remaining balance to investment and other current expenditures in proportion to congressional appropriations. Then, individual ministries would have discretion over which projects or programs to finance. This system created an arena for bargaining between the national administration and politicians. And bargaining became an important element in securing congressional support for legislation catering to pork and patronage interests of congress members. It also meant that actual real expenditures deviated from programmed real expenditures in significant ways.

Of course, in high-inflation countries some expenditures—such as wages—are indexed. Because indexation is imperfect and linked to past

[2] Bacha (1994) also proposes a deficit finance model in which the budget deficit is represented by a linear inverse function of inflation.

inflation, rising inflation implies declining real wages. Moreover, local governments in Brazil, for instance, used to postpone wage payments when they were short of cash. With double-digit monthly inflation, a 15-day delay in payments implies a significant decline in real expenditures. When the annual inflation rate reaches 4,000 percent a year—as it did in mid-1994—a 15-day delay in payments reduces real expenditures by 15 percent.[3]

With some expenditures indexed and more rigid than others, one would not expect inflation to reduce all expenditures equally in real terms but to affect expenditures that are not subject to strict rules, such as investments by both government agencies and public enterprises. It is this negative relationship between high inflation and real expenditures that can be attributed to the Patinkin effect. When inflation exceeds 1,000 percent a year, observed budget deficits reveal very little about the true fiscal position once inflation is curtailed. If inflation disappears and expenditure commitments remain unchanged, the virtual budget deficit would be much higher than the observed budget deficit at high inflation rates.

One could thus argue that the budget deficit increases with inflation when inflation is low and declines with inflation when inflation is high. At low inflation rates there may be no motivation to index taxes and reduce tax collection gaps, and the Tanzi effect will produce a positive relationship between deficits and inflation. In contrast, when inflation is high, there is a clear incentive to introduce indexation and reduce tax collection gaps. It can also be argued that once arrangements to avoid losses of tax revenues are put in place, they would continue to be used even if inflation were to decline. Thus, in countries with long inflationary histories, we would not observe a positive relationship between inflation and the budget deficit because the Tanzi effect would cease to work. Yet because indexation is perceived as a mechanism that perpetuates inflation, stabilization programs often introduce a clause forbidding indexation, as did the Real Plan in Brazil. In an attempt to eliminate the indexation habit, fines on delayed tax payments were no longer indexed to the price level. This policy could reintroduce the Tanzi effect and the positive association between inflation and the budget deficit at low inflation rates.

At the same time, in a country where political coalitions generate conflicting expenditure demands, inflation can be used to accommodate those demands, and the Patinkin effect becomes operative. Payment delays also start to have a significant impact on real expenditures. In these circumstances, if tax collection continues relatively well, a rise in inflation will

[3] If payment is postponed by 15 days, real outlays are reduced by 3 percent if inflation is 100 percent, by 7 percent if inflation is 500 percent, and by 10 percent if inflation is 1,000 percent.

reduce the budget deficit. Nonetheless, under extremely high inflation rates, any indexation scheme would break down, and the possibility of declining real taxes and increasing deficits would reappear.

Formal Model

The inflationary finance model developed in this subsection is general enough to accommodate different scenarios. That is, at low inflation rates the budget deficit can be assumed to be increasing with inflation or it may be constant. At high inflation rates the model assumes that the Patinkin effect is operative and that it could dominate the Tanzi effect. At even higher inflation rates, indexation could break down and the budget deficit could once again increase with inflation. Thus, a cubic function is a natural candidate to express the share of the budget deficit in GDP as a function of the inflation rate in a form consistent with the stylized findings described previously.

Equation 1 shows the share of the budget deficit in GDP, g, as a function of the inflation rate, π:

$$g = g(0) + a\pi^3 + b\pi^2 + c\pi \qquad a > 0,\ b > 0,\ c > 0, \qquad (1)$$

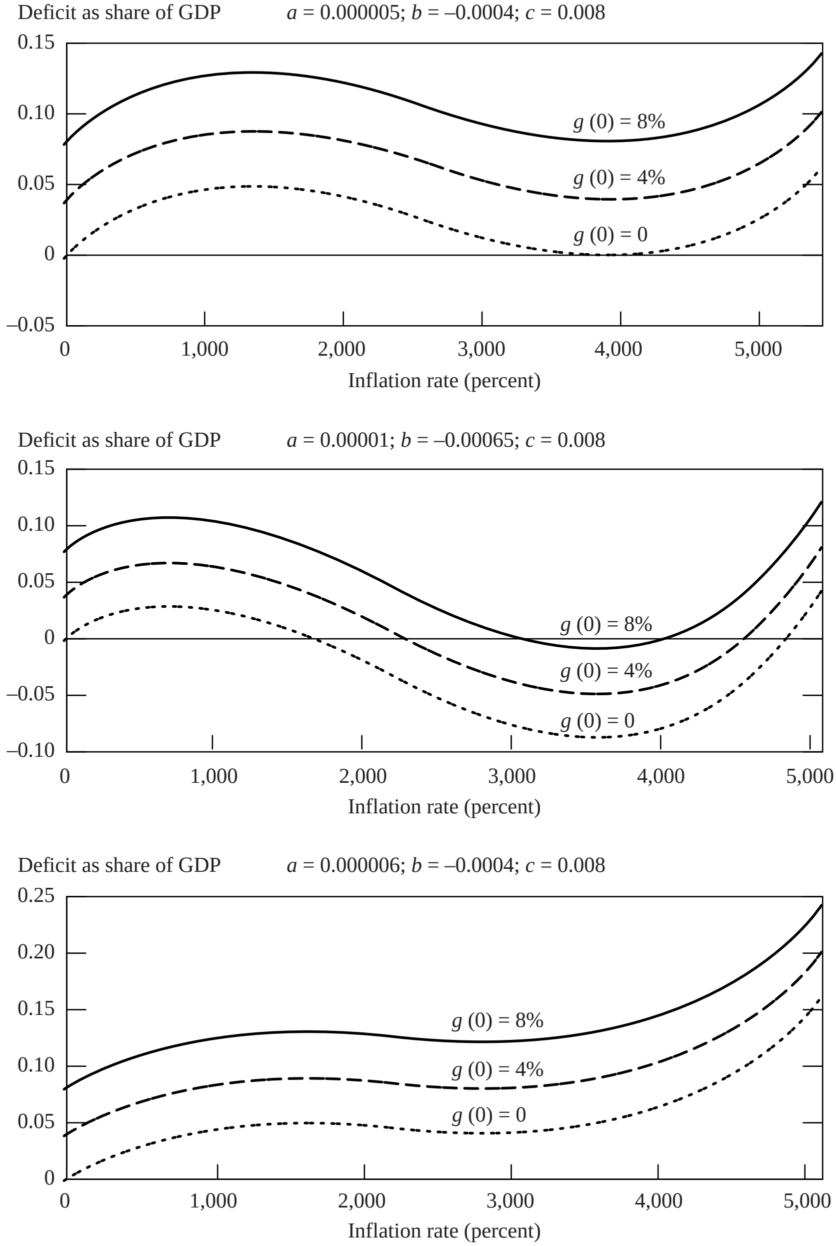
where $g(0)$ is the virtual deficit.

The response of the budget deficit to the inflation rate in equation (1) depends on how strong the Tanzi and the Patinkin effects are at different levels of inflation, that is, it depends on the relative sizes of a, b, and c. Figure 1 shows three different possibilities.

In the first case both effects are relatively modest (a, b, and c are small, that is, the budget deficit does not respond very strongly to inflation). Also, b is big enough relative to a and c to permit the Patinkin effect to dominate the Tanzi effect at annual inflation rates between 1,500 percent and 4,000 percent. The case of a very strong Patinkin effect (a very big b) that would produce a downward-sloping schedule starting at low inflation rates cannot be ruled out but is not considered here.

In the second case, the Patinkin effect is strong enough to generate a budget surplus at annual inflation rates between 2,500 percent and 4,000 percent when the virtual deficit is 4 percent of GDP. This case is of interest if we consider Brazil's experience. Between 1990 and 1994, when inflation averaged close to 2,000 percent a year, the primary surplus was 3.5 percent of GDP. It exceeded 5 percent of GDP in 1994 when inflation reached 2,500 percent. The operational balance was also in surplus in 1993 and 1994. In 1995, when inflation fell to 15 percent, the 1994 operational surplus turned into an operational deficit of approximately 5 percent of GDP. Appendix I discusses in more detail the empirical evidence on the relationship between

Figure 1. *The Response of the Deficit to Inflation*



Note: *a*, *b*, and *c* are parameters of equation 1 and g(0) is the virtual deficit.

inflation, taxes, expenditures, and the different measures of the fiscal deficit in Brazil.

In the third case the Patinkin effect is not strong enough to generate a declining relationship between the budget deficit and inflation. That is, *b* is not big enough relative to *a* and *c* to produce a downward-sloping schedule at any level of inflation.

Tanzi and Patinkin effects are short- and medium-run stories about the monetary authorities, the tax authority, and the spending authorities revising their inflation predictions at different speeds or revising them more slowly than the private sector revises inflation expectations. Eventually, all sectors of government will face their imprecise predictions and try to correct them. Appendix II discusses the government's corrections of its inflation predictions and the implications for a long-run equilibrium.

Equation (2) expresses seigniorage collected by the central bank as a function of the inflation rate:[4]

$$\Delta H/Y = \mu z / v(\pi) \qquad \delta v / \delta\pi > 0, \tag{2}$$

where $\Delta H/Y$ is the ratio of the increase in the monetary base to income, z is the ratio of the monetary base to money, and v is velocity, with a Cagan-type velocity functional form:[5]

$$v = v(0)e^{k\pi}. \tag{3}$$

Since the budget deficit is financed by money creation

$$g\left(g(0),\pi\right) = \mu z/v(\pi), \tag{4}$$

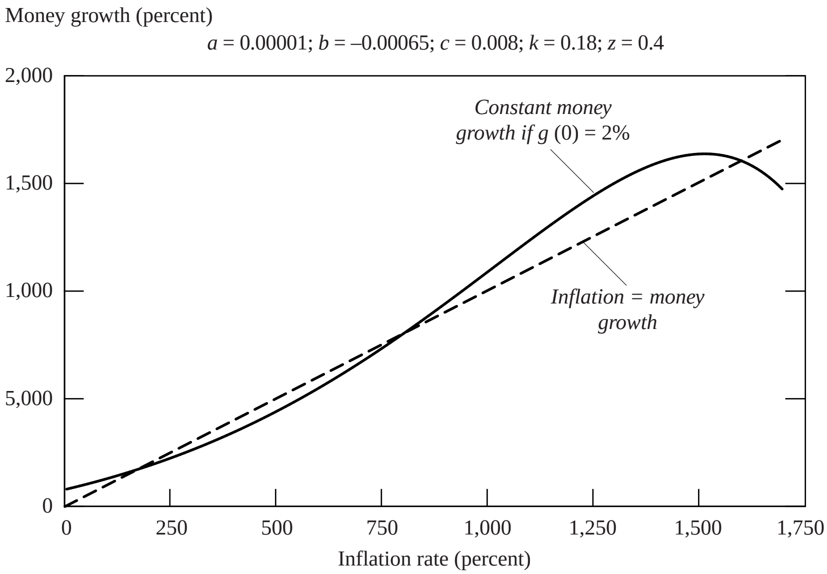
it follows that the required money growth to finance the budget is $\mu = g\,(v/z)$. Money growth increases when the budget deficit, g, exceeds the amount of seigniorage, $z(\mu/v)$, generated by the current rate of money growth:

$$\delta\mu/\delta t = \phi\left(gv/z - \mu\right). \tag{5}$$

[4] To obtain equation (2), define the share in income of seigniorage collected by the central bank as $s \equiv \Delta H/Y$. Given $\Delta H \equiv z\Delta M$, where $z \equiv$ the inverse of the money multiplier (or the ratio of H to M), substitution yields $s \equiv z\Delta M/Y$. Divide and multiply this expression by M, define $\Delta M/M$ (money growth) $\equiv \mu$, and assume that the money supply is equal to the demand for money, $M/Y = 1/v(\pi)$.

[5] In the simulations, the parameters of the Cagan function are consistent with those observed in Brazil, where between 1950 and 1995 the central bank's seigniorage averaged about 2 percent of GDP, but never exceeded 4 percent of GDP, even at four-digit inflation rates. Appendix III contains the empirical evidence on velocity in Brazil between 1950 and 1995.

Figure 2. *Money Growth and Inflation*



Note: *a*, *b*, and *c* are parameters of equation (1). *k* and *z* are defined in equations (2) and (3).

In Figure 2 the constant money growth schedule, $\delta\mu/\delta t = 0$, crosses the vertical axis at the point where money growth generates enough seigniorage to finance the virtual budget deficit, $g(0)$. If inflation is low and the Tanzi effect is strong, the schedule slopes upward but declines with inflation once the Patinkin effect becomes strong enough. The schedule would once again reverse its slope at even higher inflation rates (not represented in Figure 2). Above the curve representing constant money growth, $\delta\mu/\delta t = 0$, money growth exceeds the amount of seigniorage needed to finance the budget deficit and is declining. Below the curve, money growth is not sufficient to finance the budget deficit and is increasing.

In a model with perfect information and no uncertainty, expected inflation is equal to observed inflation. Inflation inertia exists, nevertheless, as a result of formal and informal indexation mechanisms, and inflation moves slowly to catch up with monetary growth:

$$\delta\pi/\delta t = \theta(\mu - \pi). \tag{6}$$

The constant inflation rate, $\delta\pi/\delta t = 0$, is represented in Figure 2 as the 45° line from the origin. Above it the rate of inflation is lower than the rate of money growth, and inflation is rising; below it the rate of inflation exceeds the rate of money growth and inflation is falling. Inflation and money growth are constant and equal at the point where the two schedules cross, that is, where

$$\pi = gv/z. \tag{7}$$

Observe that the necessary condition for a stable equilibrium is that the constant money growth schedule cross the 45° line from above. Figure 2 shows three equilibria for the set of parameters of the budget function shown in the middle panel of Figure 1 and a virtual deficit of 2 percent of GDP. The parameters of the velocity function are those of Brazil, an economy that has been demonetized by a long inflationary history. There is one stable equilibrium at an inflation rate equal to 190 percent, one unstable equilibrium at an inflation rate equal to 790 percent, and another stable equilibrium at an inflation rate equal to 1,600 percent. There is also a fourth unstable equilibrium at an inflation rate in excess of 4,500 percent (not represented in Figure 2).[6]

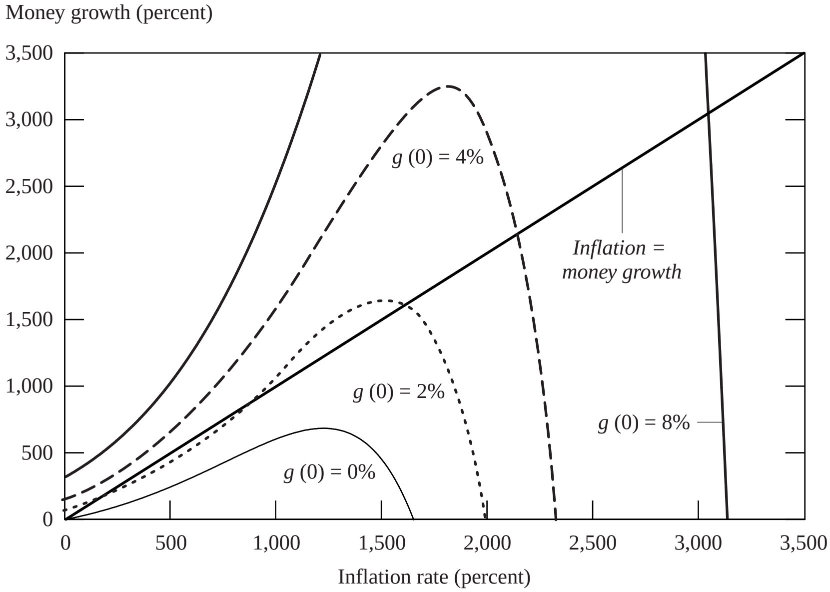
Expansionary Fiscal Policy

An expansionary fiscal policy is defined here as an increase in the virtual budget deficit—that is, a policy that increases $g(0)$, the difference between expenditures and revenues under a zero inflation rate. If the virtual budget deficit is small relative to the amount of seigniorage that can be collected in the economy, multiple equilibria will arise. As fiscal policy becomes more expansionary, the schedule that shows constant money growth shifts upward (Figure 3). If the virtual budget deficit is higher than the amount of seigniorage that can be raised at any inflation rate, and the Patinkin effect is very strong, the schedule showing constant money growth crosses the 45° line once from above. This equilibrium is stable. If the Patinkin effect is not strong, the schedule showing constant money growth does not slope downward and expansionary fiscal policies would lead to hyperinflation.

If the Patinkin effect is strong, there is one stable equilibrium for a large range of budget deficits even if seigniorage collection is small and the revenue-maximizing inflation rate generates an amount of seigniorage that is less than the virtual budget deficit. Convergence to this stable equilibrium is through oscillations. As fiscal policy becomes more expansionary, money growth increases ahead of inflation to generate more seigniorage. Inflation catches up with money growth and then exceeds it.

[6] The fourth unstable equilibrium is also important. If a shock, such as a very big devaluation, drives inflation above the unstable equilibrium, a fiscal contraction or reserve requirement shift that is apparently in the right direction could set off a hyperinflation.

Figure 3. *Money Growth and Inflation Equilibria Under Different Virtual Deficits*



Note: Each curve assumes constant money growth.

Expansionary fiscal policies, which induce an increase in the virtual budget deficit in excess of maximum seigniorage, increase the steady-state inflation rate (Figure 3). In the new steady state, the share of seigniorage in GDP and the share of the realized budget deficit in GDP are smaller than in the initial steady state, as both decline with inflation when the Patinkin effect dominates the Tanzi effect. This model describes the experience of megainflation countries, such as Brazil from the 1980s to the mid-1990s or Israel before 1985, better than other models of seigniorage, in which very expansionary fiscal policies—policies that imply a virtual budget deficit in excess of optimal seigniorage—result in open hyperinflation.[7] In Brazil and in Israel inflation was used to reduce real expenditures and inflation remained at megainflationary levels for long periods without ever exploding into open hyperinflation.

[7] The literature on seigniorage defines optimal seigniorage as that obtained at the revenue maximizing rate of inflation. If the virtual deficit increases above optimal seigniorage, there are cases where no equilibrium exists. For the parameters in this subsection, there is no equilibrium for virtual deficits in excess of 9 percent.

Reserve Requirements and Inflation

The model assumes that the ratio of the monetary base to M1, z, is a constant. This subsection examines this assumption more closely. An increase in the ratio of required reserves to deposits raises z and should in principle increase the central bank's share of total seigniorage, reducing money growth and inflation.[8] If the Patinkin effect is operative, the reduction in inflation would increase the budget deficit, which would be financed by higher seigniorage in the new equilibrium. The higher seigniorage, in return, is a result of the decline in velocity in response to the lower inflation rate. Thus, an increase in the reserves-to-deposit ratio can produce lower inflation even with an unmodified fiscal policy (unchanged $g(0)$).

The combinations of the long-run equilibrium inflation rate and the central bank's seigniorage share, that is, equation (7)—given the virtual budget deficit, $g(0)$, and demand for money, $v(\pi)$—are shown in Figure 4. If the virtual budget deficit equals 2 percent of GDP and the central bank gets two-fifths of the seigniorage revenue ($z = 0.4$), four possible equilibria exist (the fourth, unstable equilibrium at inflation in excess of 4,500 percent is not shown in Figure 4). Among the three equilibria, those corresponding to the low inflation rate and the high inflation rate are stable, while that corresponding to the average inflation rate is unstable. Starting from a stable equilibrium, an increase in the reserves-to-deposit ratio moves the central bank's share in seigniorage upward and reduces inflation.

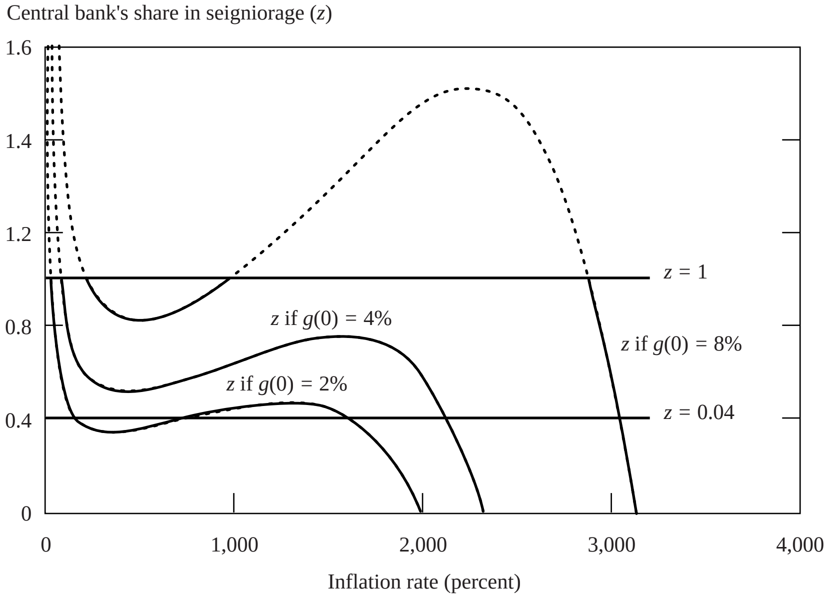
If the virtual deficit is very high—for instance, $g(0) = 9$ percent—the possibility of using the required reserves-to-deposit ratio to reduce inflation practically disappears. Inflation then becomes very inelastic with respect to z because the amount of seigniorage that can be collected at high inflation rates is very low. Thus, increasing the central bank's share of a very small amount of seigniorage (because inflation is very high) will not significantly increase the amount of the budget that can be financed by seigniorage.

If the virtual budget deficit is not so high, but still high relative to optimal seigniorage collection—for instance, $g(0) = 8$ percent of GDP—the schedule becomes discontinuous. But it is still possible to observe different inflation equilibria at very high required reserves-to-deposit ratios. If the economy is stuck at a high inflation equilibrium, a monetary reform that curtails inflation combined with an increase in the reserves-to-deposit ratio can move the economy from a high-inflation equilibrium to a low-inflation

[8] Before calculating the tax on cash balances in Austria, Germany, Greece, Hungary, Poland, and Russia after World War I, Cagan (1956) observes that institutions other than the government typically receive some of the revenue from issuing money.

Figure 4. *The Central Bank's Share in Seigniorage and Inflation*

a = 0.00001; b = –0.00065; c = 0.008; k = 0.18



equilibrium. It is not clear how long this new equilibrium can be sustained if the higher reserves-to-deposit ratio reduces the profits of deposit banks substantially. The increase in required reserves will also have an impact on interest rate spreads that depend on the average costs of funds and the level of reserve requirements. If both spreads and real interest rates increase with stabilization, nonperforming loans may also increase and further contribute to reducing banks' profitability.

II. Reserve Requirements, Interest Rates Spreads, and Nonperforming Loans After Stabilization in Brazil

In mid-1994, Brazil's Real Plan reduced inflation using three types of reforms: a short-lived fiscal adjustment, a monetary reform, and the use of the exchange rate as a nominal anchor. A temporary monetary reform measure linked contracts, prices, wages, and the exchange rate to a single daily escalator and unit of account, the *unidade real de valor*. The adjustment, which began on March 1, 1994, lasted four months. The central bank established a daily parity between the cruzeiro real and the *unidade* *real de valor* based on the current rate of inflation, as reflected in the three most closely watched price indices. Since the cruzeiro real and the *unidade real de valor* depreciated relative to the U.S. dollar at roughly the same rate, most prices and contracts were implicitly set in U.S. dollars. On July 1, 1994, a new currency, the real, was introduced by converting contracts denominated in *unidades real de valor* into reals at a rate of one to one. Brazil's success in bringing down inflation was associated with real exchange rate appreciation. This pattern, similar to that observed in other countries where the exchange rate was used as a nominal anchor runs as follows: there is a real exchange rate appreciation, a rise in real wages, a deterioration in external accounts, an economic boom, and then a slowdown.

Brazil's stabilization was supported by tight monetary policy that was based on an increase in reserve requirements. The increase in required reserves and the decline of inflation led to a substantial decline in the inflationary revenues of deposit banks (Tables 1 and 2). The required reserves-to-deposit ratio increased from an average of 26 percent during January–June 1994 to an average of 64 percent during November 1994–April 1995.[9] With the increase in reserve requirements following implementation of the Real Plan, the share in total seigniorage seized by the central bank (*z*, the inverse of the money multiplier) increased from an average of 60 percent during January–June 1994 to an average of 84 percent in the period January–June 1995. As a consequence, the share in GDP of seigniorage seized by deposit banks fell from 2 percent to close to zero (Figure 5).[10] This decline is consistent with estimates by the *Instituto Brasileiro de Geografia e Estatística* (IBGE, 1997) calculated using a different methodology (Table 2). IBGE calculated banks' inflationary revenue in two steps. First, the difference between the monthly average stock of non-interest-bearing-liabilities and the non-interest-earning assets, *NNL*, was multiplied by the monthly inflation rate of the general price index (*IGP–DI*) to obtain the inflationary revenue. Then monthly revenues were added and the annual sums were divided by GDP. Although IBGE found higher inflationary revenues in deposit banks, the magnitude of the change in the seigniorage revenue of deposit banks between 1993 and 1994 was 2 percent, as it is here.

[9] In the second half of 1994 the required reserves-to-loans ratio increased from 0 percent to 15 percent and the required reserves-to-savings deposits rose from 20 percent to 30 percent, and in May 1995 required reserves-to-time deposits also increased from 20 percent to 30 percent (source: Brazil's central bank).

[10] The share in GDP of deposit banks' seigniorage is: $(1 - z)\Delta M1/GDP = [(1 - R/D)/(1 + C/D)]\Delta M1/GDP = (\Delta M1 - \Delta H)/GDP$, where R/D is the reserves-to-deposit ratio and C/D is the currency deposit ratio. This share was calculated using changes in the average money balances during the year.

Table 1. *Seigniorage in Brazil, 1950–95*

Institution	Average (percentage of GDP)	Standard deviation (percent)
Central bank	2.1	7.5
Deposit banks	1.8	7.5
Total	3.9	1.2

Source: Banco Central.

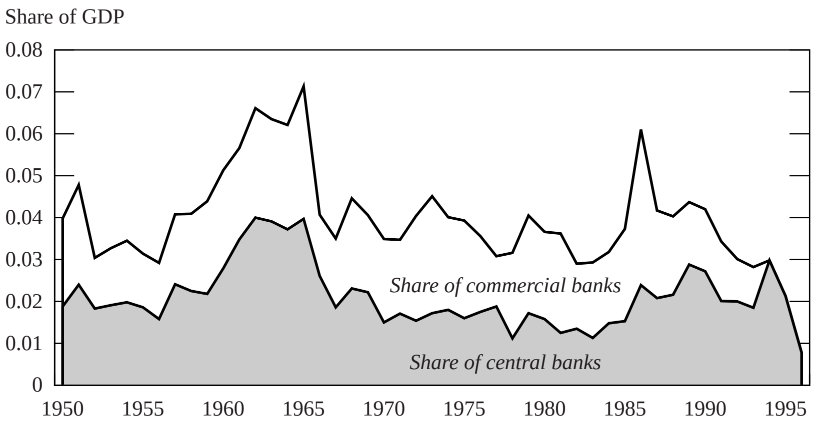
Notes: Total seigniorage is calculated as the increase of the annual average M1 relative to the annual average M1 in the previous year. Seigniorage collected by the central bank is equal to the increase of the annual average monetary base inclusive of all non-interest-bearing required reserves relative to the same variable one year before. The seigniorage collected by deposit banks is equal to the difference between total seigniorage and seigniorage collected by the central bank.

Table 2. *Inflationary Revenue of Private and Public Banks in Brazil, 1990–95*
(Percentage of GDP)

Deposit banks	1990	1991	1992	1993	1994	1995
Private banks	1.4	1.4	1.7	1.6	0.7	–0.0
Public banks	2.6	2.4	2.3	2.7	1.3	0.0
Total	4.0	3.8	4.0	4.2	2.0	0.0

Source: Instituto Brasileiro de Geografia e Estatística (1997).

Figure 5. *Brazilian Banks' Seigniorage Revenue as Percentage of GDP, 1950–96*



Source: Banco Central do Brasil, *Boletim do Banco Central* (monetary base from Table II.2 and non-interest-bearing required reserves from Table II.4)

Notes: Total seigniorage = change in annual average M1. Central bank's seigniorage = change in annual average monetary base + non-interest-bearing required reserves. Deposit bank's seigniorage = difference between total and central bank's seigniorage.

The reduction of the deposit banks' share in total seigniorage is explained by the increase in required reserves. Following the stabilization, the rise in required reserves not only contributed to the increase in seigniorage collection by the central bank, but also explains in part the increase in spreads between passive and active rates. This spread increased from 4 percent a year in January–June 1994 to 86 percent a year in January–June 1995 (Campelo, 1997). Required reserves were gradually reduced, but other factors contributed to keeping the spreads high—such as taxes on financial transactions and the increase in nonperforming loans motivated by the increase in real interest rates. Nonperforming loans doubled from an average of 7.8 percent of total loans during July–September 1994 to an average of 15.6 percent during February–August 1997.

Seigniorage collected by banks did decline with stabilization but seigniorage collected by the central bank did not—at least, not immediately. In 1993, the peak inflation year, seigniorage collected by the central bank was 1.8 percent of GDP. It increased to 3 percent in 1994, the year of the Real Plan, and was 2 percent in 1995—the level of average seigniorage during the high-inflation years.[11] This evidence supports the view that the decline in the inflation rate was achieved through the monetary reform, the fixing of the exchange rate, and tight monetary policy. Stabilization was not achieved through a tightening of fiscal policy, which would have reduced financing of the deficit through seigniorage collected by the central bank. A more balanced policy would not have transferred the revenues from money creation so drastically from deposit banks to the central bank and would have avoided the increase in interest rate spreads and nonperforming loans.

III. Concluding Remarks

The Patinkin effect contributes to the understanding of sustained extreme inflation rates. Using parameters of Brazil's velocity function between 1950 and 1995, and evidence from the relationship between inflation and fiscal deficits in Brazil, the paper simulates an inflationary model in which extremely high inflation rates are stable and do not explode into open hyperinflation.

The paper also argues that in analyzing inflation stabilizations, attention should be paid to the virtual deficit—an estimate of what the deficit would

[11] Because a decline in total seigniorage collection was matched by a decline in seigniorage collection by the commercial banks, leaving seigniorage collected by the central bank unchanged, there was not a wealth effect from the decline in inflation, but only a transfer between the banking sector and the nonbanking sector. In 1996, though, the central bank's seigniorage did decline to 1 percent of GDP.

be if inflation were reduced to zero. The virtual deficit is different from the operational or inflation-adjusted deficit, which deducts the decline in the real value of government debt caused by inflation from the nominal deficit, because it takes into account both the Tanzi and Patinkin effects and because the real interest rate might change if inflation were stabilized.

Following stabilization, fiscal adjustment may have to be more severe than projected, since inflation clouds structural fiscal problems if expenditures are not indexed and the Patinkin effect is strong. Moreover, the high real interest rates that follow stabilization expose banks' weaknesses, which demand fiscal resources for restructuring. If many public banks have accumulated bad loans to local governments, sustainable reform will require an even harsher fiscal effort. By mid-1997, a fiscal adjustment that could sustain recently achieved low inflation had not yet been undertaken in Brazil.

APPENDIX I

The Budget Deficit and Inflation in Brazil

In the model developed in the paper, two important empirical relationships play a role in determining equilibria: the effect of inflation on the budget deficit and the response of velocity to inflation. This Appendix examines the empirical evidence on the relationship between the budget deficit and inflation, and then studies the relationship between velocity and inflation in Brazil during 1949–95.

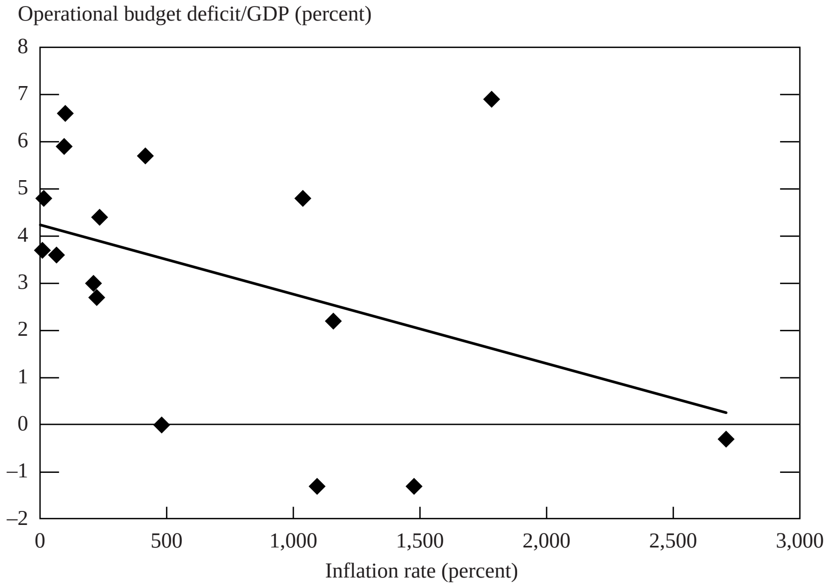
An analysis of Brazil's public finances relies on three concepts of fiscal balance: the public sector borrowing requirement (PSBR), the operational balance, and the primary balance. The PSBR is equal to total revenues less total expenditures of the public sector, which includes all government levels, the central bank, and public enterprises but excludes state and federal banks. Traditional analysis uses the PSBR—which peaked at 83 percent of GDP in 1989 (Table A1)—to assess the impact of the government's actions on aggregate demand and inflation. But the PSBR may not be the appropriate measure of the deficit in countries with high inflation and a high ratio of domestic public debt to GDP (see, for instance, Blejer and Cheasty, 1993). Interest payments rise with the increase in the inflation component of the nominal interest rate on the domestic debt. But these increased payments represent compensation for the erosion of the real value of the debt principal. Payment of the inflation component of the nominal interest rate is thus equivalent to a financing item used to amortize the public debt. It follows that a more meaningful measure of the deficit should exclude the payment of the inflation component of the nominal interest from the PSBR. Exclusion of this component yields the operational balance, which uses the real interest rate in calculating interest payments. Because real interest rates are sensitive to monetary policy and the level of activity in the economy and because interest payments are the result of deficits incurred in previous years, a narrower definition

Table A1. *Brazil: Public Sector Balance, 1983–96*
(Percentage of GDP)

Year	Public sector borrowing requirement	Primary balance (*PB*) (– = deficit)	Real interest payments (*RIP*)	Operational balance (*OB* = *PB* – *RIP*) (– = deficit)
1983	19.9	1.7	4.7	–3.0
1984	23.3	4.2	6.9	–2.7
1985	28.0	2.6	7.0	–4.4
1986	11.3	1.6	5.2	–3.6
1987	32.3	–1.0	4.5	–5.7
1988	53.0	0.9	5.7	–4.8
1989	83.1	–1.0	5.9	–6.9
1990	29.6	4.6	3.3	1.3
1991	27.2	2.8	2.8	0.0
1992	44.2	2.3	4.5	–2.2
1993	58.1	2.6	2.4	0.3
1994	43.8	5.1	3.8	1.3
1995	7.1	0.4	5.1	–4.8
1996	6.1	–0.1	3.8	–3.7

Source: Fundação Getúlio Vargas, 1997.

Figure A1. *Operational Budget Deficit as Share of GDP and Inflation, Brazil, 1981–96*



Source: Banco Central do Brasil.

of fiscal balance, which excludes interest payments from expenditures, could reflect more clearly the discretionary budgetary stance. This measure is the primary balance.[12]

The relationship between the primary budget deficit and inflation in Brazil during 1983–96 was negative; at very high inflation rates, the primary balance was in surplus. The relationship between the operational budget deficit and inflation also seems to be negative (Figure A1). There is no consistent information that would allow the calculation of operational deficits before 1981, ruling out observations for periods of low inflation. Among the 15 observations for the operational budget deficit, only 5 correspond to inflation rates below 200 percent. The 15 observations are too few to permit meaningful empirical results. In a simple ordinary least squares regression, the relationship between the budget deficit and inflation is negative and thus consistent with a strong Patinkin effect.

Even though data for the operational deficit do not exist before 1981, data for some components of expenditures and taxes are available. To show that the existing information is consistent with the hypotheses in Section I of the paper, empirical tests should reject the hypothesis that there is an inverse relationship between taxes and inflation and reject the hypothesis that there is a positive relationship between investment spending and inflation, since investment expenditures are easier to cut than wages and salaries. The relationship between consumption expenditures and inflation is trickier: part of these expenditures (such as wages and salaries) were indexed until recently. Furthermore, there could be a positive relationship between expenditures and inflation with causality running from expenditures to inflation.

Table A2 shows the results of unit root tests for the shares in GDP of income taxes, sales taxes, government consumption expenditures, and investment by public enterprises.[13] The hypothesis that the share in GDP of income taxes, sales taxes, and government's consumption expenditures have a unit root cannot be rejected. The Dickey-Fuller statistic for the share of investment by public enterprises in GDP rejects the unit root hypothesis at the 1 percent and 5 percent levels.

Table A3 shows the results of cointegration tests for income taxes, sales taxes, and inflation. The tests reject any cointegration at the 5 percent significance level. The evidence does not support the existence of a Tanzi effect during the high-inflation years in Brazil.

[12] A difficult issue, which the fiscal figures in Table A1 do not reflect, concerns the quasi-fiscal deficits in federal and state banks, which could be substantial. For instance, the federally owned Banco do Brasil has traditionally subsidized credit to agriculture, and the National Bank of Development (BNDES) subsidizes credit to exporters. In 1996 the treasury recapitalized Banco do Brasil by 7.9 billion reals (more than 1 percent of GDP). This recapitalization has contributed to the increase of total net public debt, estimated to have risen from 30 percent of GDP in 1995 to 35 percent in 1996. The costs of restructuring the banking sector and the impact of these changes on the fiscal budget are not yet clear. And with the end of high inflation, bad loans from state banks to state governments have also emerged as a serious problem.

[13] Data for taxes and government consumption expenditures are from Brazil's national accounts for the years between 1965 and 1994, except where noted. Income taxes are collected by the central government. The sales taxes are value-added taxes collected by state governments (ICMS) and by the central government (IPI). Data for investment by public enterprises between 1980 and 1996 are from the Treasury Department of the Finance Ministry.

Table A2. *Brazil: Unit Root Tests on Income Taxes, Sales Taxes, Government Consumption Expenditures, and Public Enterprises' Investment*

	Income taxes/ GDP	Sales taxes/ GDP	Government consumption/ GDP	Public enterprises' investment/ GDP
Augmented Dickey-Fuller Test (equation includes intercept, change in lagged variable, and trend)	–2.2365	–1.6858	–1.2540	–3.9658
1 percent critical value	–4.3226	–4.3226	–4.3082	–4.7315
5 percent critical value	–3.5796	–3.5796	–3.5731	–3.7611
Phillips-Perron Test (equation includes intercept)	–2.8305	–2.4995	0.2289	–0.8000
1 percent critical value	–3.6752	–3.6752	–3.6661	–3.9228
5 percent critical value	–2.9665	–2.9665	–2.9627	–3.0659

Notes: Sales taxes and government expenditure taxes data from 1965–95, and public enterprises investment data from 1980–96.

Table A3. *Brazil: Johansen Cointegration Test Statistics for Taxes and Inflation, 1965–94*

	Eigenvalue	Likelihood ratio	5 percent critical value	1 percent critical value
Hypothesis: There is no cointegration between income taxes and inflation	0.241	11.25	15.41	20.04
Hypothesis: There is no cointegration between sales taxes and inflation	0.1104	6.026	15.41	20.04

Notes: The log likelihood is 43.33. The lag interval is 1 to 1.

On the consumption expenditure side, the Johansen cointegration test indicates one cointegrating equation at the 5 percent significance level, and the relationship between consumption expenditures and inflation is positive (Table A4). But considering wage indexation and the reversed causality between expenditures and inflation, this result, even if it does not support the Patinkin effect, is not surprising. A Granger causality test gives mixed results: with a one-year lag, the test cannot reject the hypothesis that government spending does not cause inflation. But with a one-year lag and a two-year lag, the test rejects the hypothesis that government spending does not cause inflation. With a two-year lag, the test indicates that there is a 22 percent probability that inflation does not cause government spending (Table A5).

There are fewer observations for investment by public enterprises than for the other variables, and thus the tests are weaker. Still they reject the hypothesis of a unit root for the share of investment by public enterprises in GDP. The ordinary least-squares regressions reported in Table A6 show that an increase in inflation reduces this share. The coefficient is significant and robust to different specifications.

The empirical findings are broadly in line with the hypothesis of a negative relationship between real budget deficits and high inflation rates in Brazil. Such

Table A4. *Brazil: Johansen Cointegration Test for Government Consumption Expenditures and Inflation, 1965–95*

Cointegration Test

	Eigenvalue	Likelihood ratio	5 percent critical value	1 percent critical value
Hypothesis: There is no cointegration	0.686		19.96	24.60
Hypothesis: At most one cointegration equation exists	0.044	1.30	9.24	12.97

Normalized Cointegrated Coefficients
(One cointegrating equation)

Government consumption expenditures	Inflation	Constant
1.00	–0.0046 (0.0003)	–0.100

Notes: Standard errors are in parentheses. The log likelihood is 58.82. The lag interval is 1 to 1.

Table A5. *Brazil: Granger Causality Tests for Government Consumption Expenditures and Inflation, 1965–95*

Null hypothesis	Lags	Observations	*F*-statistic	Probability
Inflation does not cause government consumption expenditures	1 (one year)	30	2.75	0.110
Government consumption expenditures do not cause inflation	1 (one year)	30	0.44	0.510
Inflation does not cause government consumption expenditures	2 (two years)	29	1.59	0.220
Government consumption expenditures do not cause inflation	2 (two years)	29	7.81	0.002

a relationship stems from the interaction of two forces. First, Brazil's tax system has been continuously adjusted to protect real tax collections; collection lags are small, and until mid-1994 late payments and fines were indexed. As a consequence, the ratio of tax revenues to GDP varied little despite enormous oscillations in inflation. The share of central government revenues in GDP remained around 15 percent of GDP between 1986 and 1993, when inflation peaked. In contrast, not all expenditures were indexed and realized real expenditures were less than programmed real expenditures. Furthermore, the share of investment by public enterprises shows a significant negative relationship with inflation.

Table A6. *Regression Analysis*
(Dependent variable: ratio of public enterprises' investment to GDP, 1980–96)

Constant	3.39	0.74
	(5.69)	(2.51)
Inflation	–0.07	–0.03
	(–1.89)	(–4.18)
Public enterprises' investment (–1)	—	–0.60
	—	(–4.401)
Dummy for period after 1985	—	0.80
	—	(7.74)
Adjusted R^2	0.14	0.87

Note: *t*-statistics in parentheses corrected by Newey-West standard errors and covariance.

APPENDIX II

Government Predicted Inflation and the Long Run

Equation (1) leaves out inflation predictions by the government. If we define government-predicted inflation as λ, equation (1) can be written as

$$g = g(0) + a(\pi^3 - \lambda^3) + b(\pi^2 - \lambda^2) + c(\pi - \lambda). \quad (8)$$

The government revises its predicted inflation slowly:

$$\delta\lambda/\delta t = \xi(\pi - \lambda). \quad (9)$$

In the long-run steady state, $\pi = \lambda$ and $g = g(0)$. In that case, the constant money growth schedule, $\delta\mu/\delta t = 0$, is $g(0) = \mu z/v(\pi)$ and is represented by the upward-sloping schedule labeled as long-run constant money growth in Figure A2. If the virtual budget deficit is 2 percent of GDP and the simulation uses the same parameters used in the formal model Section I, there are two long-run equilibria corresponding to the intersections of the long-run constant money growth schedule and the 45º line in Figure A3. These two equilibria correspond to the two equilibria found in seigniorage models such as those discussed in Mundell (1971) or Bruno and Fischer (1986), for instance. In models where the inflation rate does not jump, the high-inflation long-run equilibrium is unstable. In the model in this paper, as the government's predicted inflation rate approaches the actual inflation rate, it becomes impossible to reconcile planned expenditures and revenues, and the economy moves into hyperinflation.

Using the same parameters used in the other simulations in the paper, Figure A3 illustrates a situation in which the virtual budget deficit is 8 percent of GDP. As long as the government's predicted inflation rate is below the actual inflation rate and the Patinkin effect is strong, there is a high-inflation medium-run equilibrium. But there is no long-run equilibrium if the government's predicted inflation matches the observed inflation rate, as shown in Figure A3, where the long-run constant money growth does not intercept the 45º line.

Figure A2. *Constant Money Growth in the Medium and Long Run*

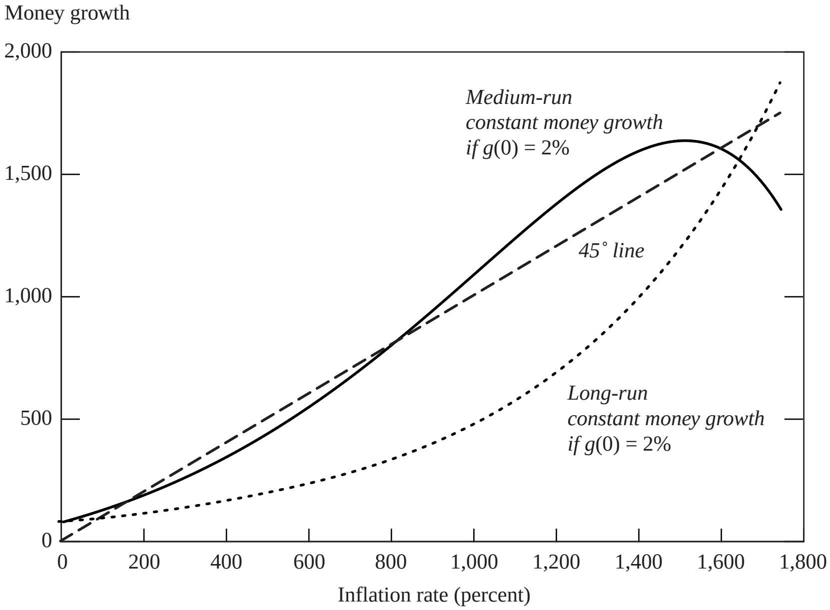
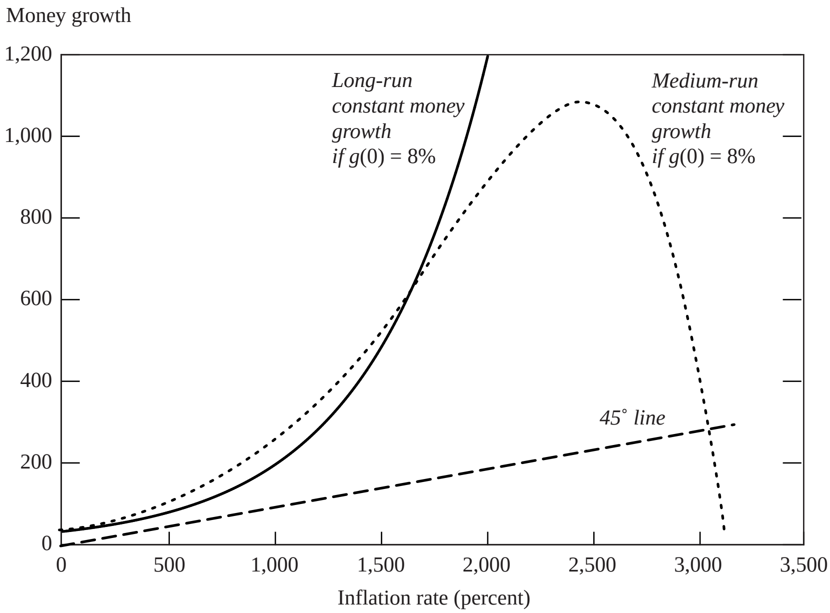


Figure A3. *Constant Money Growth in the Medium and Long Run*



APPENDIX III

Velocity and Seigniorage

Velocity—defined as the ratio of income to Ml—increased in Brazil from 4.5 in 1950 to 32 in 1993, when annual inflation reached 2,700 percent. At the same time, the ratio of income to high-powered money peaked at 52. Basic descriptive statistics for velocity and inflation in Brazil between 1950 and 1995 appear in Table A7. Figure A4 shows the very strong positive relationship between the growth rate of velocity and the growth rate of inflation during that period.

Unit root tests on velocity and inflation cannot reject the hypothesis that velocity and inflation have unit roots (Table A8). Velocity and inflation are endogenous variables in models in which the demand for money depends on expected inflation and inflation depends on money growth. These models predict that velocity and inflation are cointegrated. Cointegration tests strongly reject the hypothesis that there is no cointegration of velocity and inflation—that is, that velocity and inflation do not have an equilibrium condition that keeps their proportion constant in the long run. (Table A9) The logarithm of velocity and inflation are also cointegrated, and the normalized cointegrating vector shows a coefficient of –0.18, a value consistent with the theoretical prediction of a positive relationship between the logarithm of velocity and inflation, as in Cagan's function for the demand for money (Table A10).

Seigniorage

The share in GDP of seigniorage collected by the central bank in Brazil has averaged 2 percent of GDP during the past 47 years. It remained unchanged in 1994–95 after the Real Plan succeeded in sharply reducing inflation. Seigniorage collected by deposit banks declined.

Total seigniorage, the revenue from money creation collected by the central bank and deposit banks, is defined as

$$TS_t \equiv M_t - M_{t-1}, \tag{10}$$

where TS is seigniorage and M is currency and non-interest-bearing demand deposits. The portion of seigniorage that accrues to the central bank corresponds to the change in high-powered money (currency and reserves), and the portion of seigniorage that accrues to deposit banks is the change in non-interest-bearing demand deposits minus reserves.

The ratio of total seigniorage to income is denoted by ts. In the steady state,

$$ts = \frac{1}{v}\left[\frac{(1+x)(1+\pi)-1}{(1+x)(1+\pi)}\right], \tag{11}$$

where v is velocity and x is the growth rate of real income.[14] If the growth rate of real income were zero in the steady state, then, in the steady state, with constant velocity, total seigniorage would be equal to the inflation tax, $\tau = (1/v)[\pi/(1 + \pi)]$.

[14] To obtain equation (10), divide equation (9) by current income, Y_t, then divide and multiply by lagged income. In steady state $v_t = v_{t-1} = v$, and $(1 + \mu) = (1 + \pi)(1 + x)$, where μ is the growth rate of money, π is the inflation rate, and x is the growth rate of real income.

Figure A4. *Growth Rate of Velocity and Growth Rate of Inflation in Brazil, 1950–95*

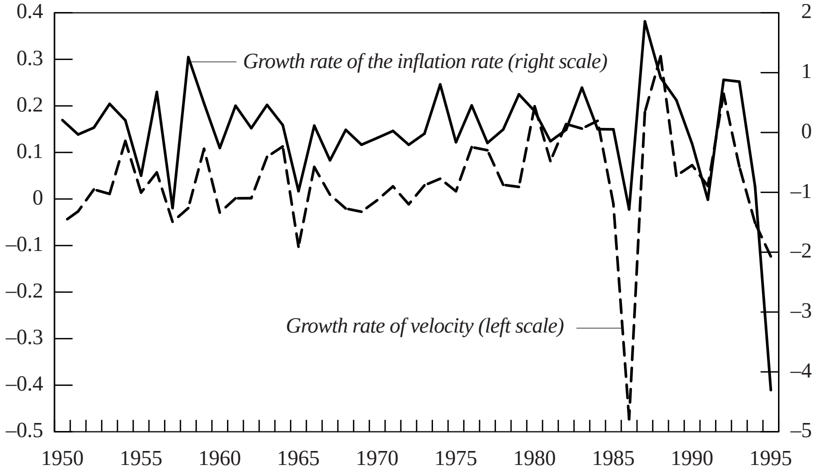


Table A7. *Brazil: Descriptive Statistics of Velocity and Inflation, 1949–95*
(Number of observations = 47)

Measure	Velocity levels (Income/M1)	Inflation rate (Average percent per year)
Mean	11.0	259
Median	7.0	39
Minimum	4.5	7
Maximum	32.0	2,700

Sources: Central bank and author's calculations.

Table A8. *Brazil: Unit Root Tests on Velocity and Inflation, 1949–95*
(Number of observations = 47)

	Velocity	Inflation	Critical values
Augmented Dickey-Fuller test (equation includes intercept and change in lagged variable)	–0.358	–3.332	–3.581[a] –2.927[b]
Phillips-Perron Test (equation includes intercept)	0.373	–2.721	–3.578[a] –2.926[b]
(equation includes intercept and trend)	–1.649	–3.319	–4.168[a] –3.509[b]

[a] 1 percent critical value.
[b] 5 percent critical value.

Table A9. *Brazil: Johansen Cointegration Test Statistics for Velocity and Inflation, 1949–95*

Cointegration Test				
	Eigenvalue	Likelihood ratio	5 percent critical value	1 percent critical value
Hypothesis: There is no cointegration	0.4977	33.03	19.96	24.60
Hypothesis: There is at most one cointegrating equation	0.0444	2.04	9.24	12.97

Normalized Cointegrated Coefficients
(One cointegrating equation)

Velocity	Inflation	Constant
1.0000	–2.279 (0.354)	–6.4946 (1.116)

Notes: Standard errors are in parentheses. The log likelihood is –157.5117. The lag interval is 1 to 1.

Table A10. *Brazil: Johansen Cointegration Test for Log (Velocity) and Inflation, 1949–95*

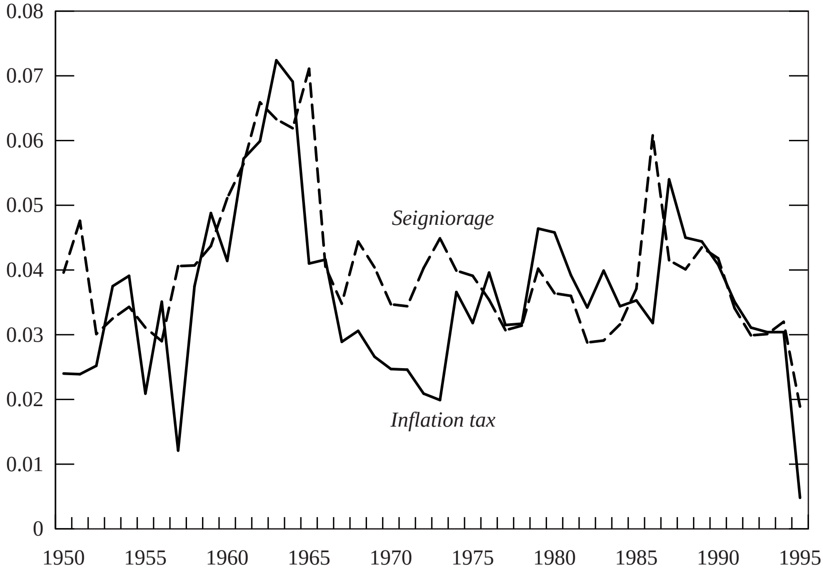
Cointegration Test				
	Eigenvalue	Likelihood ratio	5 percent critical value	1 percent critical value
Hypothesis: There is no cointegration	0.3909	26.94	19.96	24.60
Hypothesis: There is at most one cointegrating equation	0.0978	4.63	9.24	12.97

Normalized Cointegrated Coefficients
(One cointegrating equation)

Log (Velocity)	Inflation	Constant
1.0000	–0.1813 (0.0370)	–1.842 (0.1319)

Notes: Standard errors are in parentheses. The log likelihood is –39.63252. The lag interval is 1 to 1.

Figure A5. *Seigniorage and Inflation Tax of Brazilian Banks as Share of GDP, 1950–95*



In the short run, money growth is different from the inflation rate because velocity moves and the growth rate of real income is different from zero. Thus, as expected, a comparison of short-run total seigniorage to GDP ratios and short-run inflation tax to GDP ratios between 1950 and 1995 shows that the two ratios were never identical because the growth rate of money and the inflation rate differed. But during 1950–95 both short-run seigniorage and the inflation tax for the entire banking system, including the central bank, never exceeded 8 percent of GDP in any year (Figure A5). Total seigniorage collected by the central bank and deposit banks together averaged 4 percent of GDP.

REFERENCES

Bacha, Edmar, 1994, "O Fisco e a Inflação: Uma Interpretação do Caso Brasileiro," *Revista de Economia Política*, Vol. 14 (1), pp. 5–17.

Blejer, Mario I., and Adrienne Cheasty, 1993, "How to Measure the Fiscal Deficit: Analytical and Methodological Issues" (Washington: International Monetary Fund).

Bresciani-Turroni, Costantino, 1937, *The Economics of Inflation*, translated by Millicent E. Sayers (London: Allen and Unwin).

Bruno, Michael, and Stanley Fischer, 1986, "Israel's Inflationary Process: Shocks and Accommodation," in *The Israeli Economy: Maturing Through Crises*, ed. by Yoram Ben-Porath (Cambridge, Massachusetts: Harvard University Press).

Cagan, Phillip, 1956, “The Monetary Dynamics of Hyperinflation,” in *Studies in the Quantity Theory of Money*, ed. by Milton Friedman (Chicago: University of Chicago Press).

Campelo, Jr., Aloísio, 1997, “Por Que os Juros Não Caem,” *Conjuntura Econômica*, Vol. 51 (June), pp. 35–6.

Fischer, Stanley, 1994, “Modern Central Banking,” in *The Future of Central Banking: The Tercentenary Symposium of the Bank of England*, ed. by Forrest Capie and others (New York: Cambridge University Press).

Fundação Getúlio Vargas, 1997, *Conjuntura Econômica* (Rio de Janeiro: Fundação Getúlio Vargas, May).

Guardia, E.R., 1992, “Orçamento Público e Política Fiscal: Aspectos Institucionais e a Experiência Recente” (unpublished M.A. dissertation; São Paulo: Universidade de Campinas).

Instituto Brasileiro de Geografia e Estatística, 1997, *Sistema Financeiro, Uma Análise a Partir das Contas Nacionais*, 1990–1995 (Rio de Janeiro: IBGE).

Mundell, Robert A., 1971, “Deficit Finance and Growth,” Chapter 4 in *Monetary Theory: Inflation, Interest, and Growth in the World Economy* (Pacific Palisades, California: Goodyear Publishing Company).

Patinkin, Don, 1993, “Israel’s Stabilization Program of 1985, Or Some Simple Truths of Monetary Theory,” *Journal of Economic Perspectives*, Vol. 7 (Spring), pp. 103–28.

Tanzi, Vito, 1978, “Inflation, Real Tax Revenue, and the Case for Inflationary Finance: Theory with an Application to Argentina,” *Staff Papers*, International Monetary Fund, Vol. 25 (September), pp. 417–51.