Learning, Monetary Policy Rules, and Real Exchange Rate Dynamics

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Abstract

Consider an environment where national central banks set nominal interest rates according to a variant of the Taylor rule and where the exchange rate pricing equation is given by uncovered interestt parity. Then the real exchange rate will be determined not by national price levels, money supplies and output but by expected inflation output gap differentials. Suppose also that market participants are ignorant of the numerical values of the relevant coefficients but attempt to acquire that information using least-squares learning rules. I report evidence that this simple learning environment provides a plausible framework for understanding real dollar–DM exchange rate dynamics from 1976 to 2003. The least-squares learning path for the real exchange rate implied by inflation and output gap data exhibits the real depreciation of the 70s, the great appreciation (1979.4-1985.1) and the subsequent great depreciation (1985.2-1991.1) found in the observed real exchange rate data.

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Introduction

Ever since Meese and Rogoff (1983) reported on the weak to nonexistent relationship between macroeconomic fundamentals and the exchange rate, understanding the macroeconomic determinants of the exchange rate has posed a challenge to exchange rate research. Although progress has been made at understanding long-horizon movements in the exchange rate, the general failure of rational expectations models to explain exchange rate movements in terms of standard macroeconomic fundamentals has come to be known as the *exchange rate disconnect puzzle* [Obstfeld and Rogoff (2000)].[1] One strand of the literature attempts to theoretically model the disconnect [e.g., Devereux and Engel (2002), Kollman (2001), and Duarte and Stockman (2001)].

In this paper, I re-examine the case for connectivity between the real exchange rate and macroeconomic fundamentals. I employ a relatively simple environment where central banks set nominal interest rates according to a variant of the Taylor (1993) rule, uncovered interest parity holds, and market participants acquire knowledge of unknown coefficients by least squares learning rules. The model provides a plausible and useful framework for understanding observed real exchange rate dynamics over the past 27 years. The implied fundamentals driven least-squares learning path for the real dollar-DM exchange rate implied by historical inflation and output gap data displays many of the sizable and lengthy swings exhibited by the data–the depreciation of the late 1970s, the great appreciation of 1979.4–1985.1 and the subsequent great depreciation of 1985.2-1991.1.[2]

This paper is part of a growing literature that recognizes the central role

[1] See Mark (1995), Mark and Sul (2001), Groen (2000,2002), and Rapach and Wohar (2002) who report long-horizon connectedness between the exchange rate and monetary fundamentals. The explanatory power explained in these analyses derive not from the fundamental exchange rate value per se but from the deviation or error between the exchange rate and the implied fundamental value.

[2] I borrow this terminology from Papell (2002) who coined these terms. Engel and Hamilton (1990) refer to these fluctuations as 'long-swings,' and Frankel (1985) referred to dollar strength exhibited in the 80s as the 'dazzling dollar.'

of interest rate reaction functions in exchange rate determination. The fundamentals that govern the exchange rate are not national price levels, money supplies and output levels, as suggested in popular models of exchange rates ranging from disequilibrium Keynesian models of Dornbusch (1976), Mussa (1982) and Obstfeld (1985) to the new open-economy macroeconomics of Obstfeld and Rogoff (1995), but are national inflation rates and output gaps. For example, Engel and West (2002) estimated the rational expectations time path of the real exchange rate implied by reaction function fundamentals and reported that a correlation of 0.4 between the implied real dollar-DM rate and the historical observation from 1979. to 1998 whereas Groen and Matsumoto (2004), calibrate a dynamic general equilibrium to the UK economy where monetary policy operates through interest rate reaction functions.

The exchange rate pricing equation used is uncovered interest parity so expectations of future interest differential movements are a key element in the determination of the current exchange rate. The role of interest rate differentials in the determination of the real exchange rate is not new and differeing strategies for modeling their dynamics have met with varying degrees of success in explaining exchange rate dynamics. Modeling the interest rate as a variant of the Taylor rule introduces a multivariate structure usefully sets up a multivariate structure that produces a richer set of dynamics and more interest rate accurate forecasts than can be obtained from univariate time-series specifications.[3]

The public is assumed to operate in a changing economic environment. While market participants know the general structure of model, they do not know the model's true (and evolving) coefficient values. They work to ac-

[3] e.g., Frankel (1979), Meese and Rogoff (1988), Edison and Pauls (1993), Campbell and Clarida (1987), and Baxter (1994). Mark and Moh (2004) consider nonlinear (threshold) models for real interest rate differentials and find that the implied rational expectations path for the real exchange rate has very little power to explain historical movements in the real exchange rate. For evidence on the importance of a multivariate approach, see Clarida and Taylor (1997) who show that information in the term structure of the forward premium provides significant out-of-sample predictive power for the exchange rate.

quire that information by employing least-squares learning rules much in the same way as an econometrician would take a first pass at the data.[4] The emphasis on a learning environment draws its motivation from at least two sources. First, it is worthwhile to relax the strong informational assumption underlying standard rational expectations analyses of exchange rates that market participants know the very structure that econometricians are struggling to learn. Secondly, an historical explanation of the real exchange rate should acknowledge and attempt to take account of the widespread parameter instability that has plagued research in this area. This instability, which is endemic to empirical exchange rate research, lies at the heart of the Meese-Rogoff (1983) result. Adaptive learning schemes represent a plausible strategy for enabling the public to operate in a changing environment.[5]

One well documented change in the environment that I pay particular attention to is the change in the aggressiveness with which central banks react to changes in inflation that occurred with the appointment of Paul Volker to the Federal Reserve chairmanship in 1979 [Clarida et. al. (1999, 2000)]. In the 1970s, the Fed accommodated increases in inflation which elicited central bank responses that resulted in reductions in the real interest rate. Following the appointment of Paul Volker to the Fed chairmanship, not only the Fed but also foreign central banks have tended to respond much more aggressively to increases in inflation by raising real interest rates. This shift is significant in the exchange rate context because it fundamentally changed the relationship

[4] Lewis (1989a, b) conducts an analysis of Bayesian learning in the foreign exchange market to examine the 1979 changes in the Fed's operating procedures. She focused on shifts in the stochastic process governing monetary aggregates. In the monetary policy literature, Bullard and Mitra (2002) study conditions under which the rational expectations equilibrium is learnable while Orphanides (2003) examines whether the Fed's imperfect knowledge of and attempts to learn the natural rate of unemployment responsible for the inflationary buildup of the 1970s.

[5] The analysis does not suggests an out-of-sample forecasting experiment. Least squares learning, which is estimation with a recursively updated sample, is the standard way that out-of-sample forecasting experiments are conducted. Since dozens of articles have shown this technique to be unable to significantly improve over the random walk forecast, it is unlikely that we will be able to do so here.

between the real exchange rate and national inflation differentials.

The remainder of the paper is as follows. The next section describes the data. Section 2 reports estimates of the central bank's interest rate reaction function under homogeneity restrictions imposed across countries. These estimates provide evidence that the specifying the rule in differential form is not inappropriate and that in differential form, the rule displays the same kind of instability found by Clarida et. al. (2000) for the Fed. Section 3 develops the empirical model. The empirical analysis begins with an examination of the estimated rational expectations real exchange rate. It is useful to begin with such an examination of since the learning agents are attempting to discover the rational expectations equilbrium. The model with learning is presented in subsection 3.2. Section 4 concludes.

1 The Data

The analysis is conducted for the real dollar-DM exchange rate.[6] After first describing the data sources, this section undertakes a coarse examination of the changing relationship between the inflation differential and the real exchange rate.

1.1 Variables and sources.

The data are quarterly observations spanning from 1960.1 to 2003.3. The nominal exchange rate, German short-term nominal interest rates, GDP and potential GDP are from the OECD's Economic Outlook. The imputed DM rate is used from 1998 onward. Goods prices are measured by the real GDP deflator and are from the International Financial Statistics (series code 13499BIRZF). For the US, the Federal funds rate, GDP and potential GDP were obtained from FRED, the St. Louis Fed's data web site.

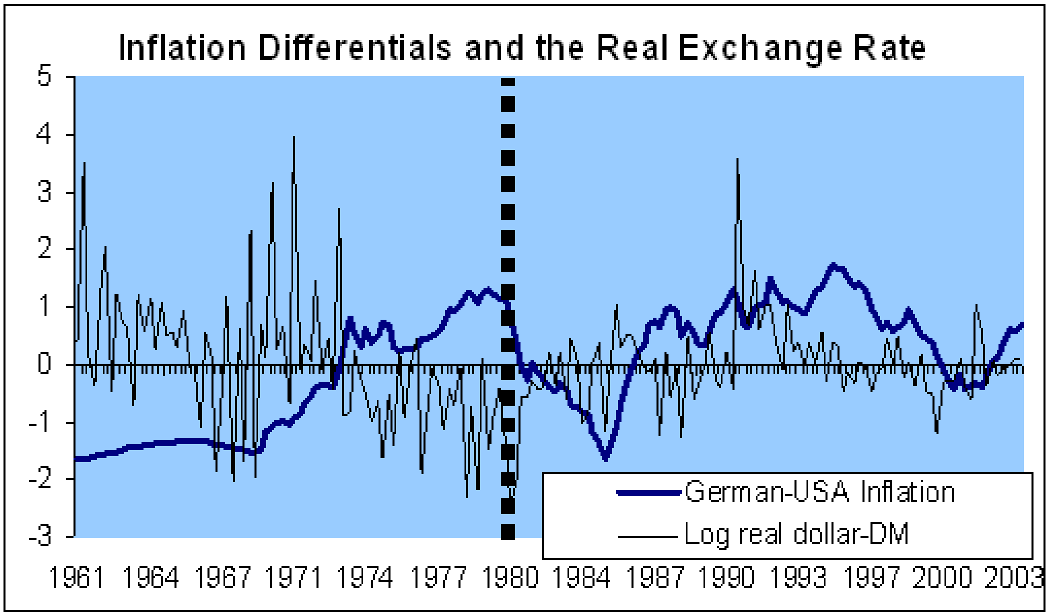
[6] Results for the real dollar-pound, dollar-yen and dollar-Canadian dollar are reported in an appendix.

The output gap is defined to be the percentage deviation of GDP from potential GDP. Quarterly inflation, the output gap and the nominal exchange rate return are stated in percent per annum. The real exchange rate is defined so that an increase signifies a real dollar depreciation.

In addition to potential GDP estimated by the source statistical agency, I construct an alternative measure given by the Hodrick-Prescott (HP) trend. German potential GDP availability from source begins in 1966.1 so these data are spliced together with the deviation from the HP trend to create a series that begins 1960.1.

1.2 Inflation differentials and the real exchange rate

Standardized plots of the quarterly the German-US inflation differential and the log real dollar-DM exchange rate are shown in the top panel of Figure 1. Four broad trends can be seen to characterize the inflation differential. From 1960 through 1979, rising inflation in the US relative to Germany generates a downward trend in the inflation differential. Over the same period, there is a general real depreciation of the dollar. From 1979 to 1992, the trend is reversed and increasing German inflation relative to the US produces an upward drift in relative inflation. A second trend shift appears to occur in 1992 when the German-US inflation differential undergoes a gradual decline through 2003. The trend movements in the log real exchange rate loosely coincides with the German-US inflation differential over the latter two episodes, but the two series move in the opposite directions in the pre 1979 period, suggesting that a change in the relationship between the two series may have occurred at that time. The bottom panel of Figure 1 plots the US-German inflation differential over the pre-1979 sample and the German-US differential over the post 1979 period along with the real dollar. The trends in the real exchange rate and the inflation differentials correspond to one another over the entire sample when the inflation differential is "flipped over" in the first period to allow for a changing relationship.



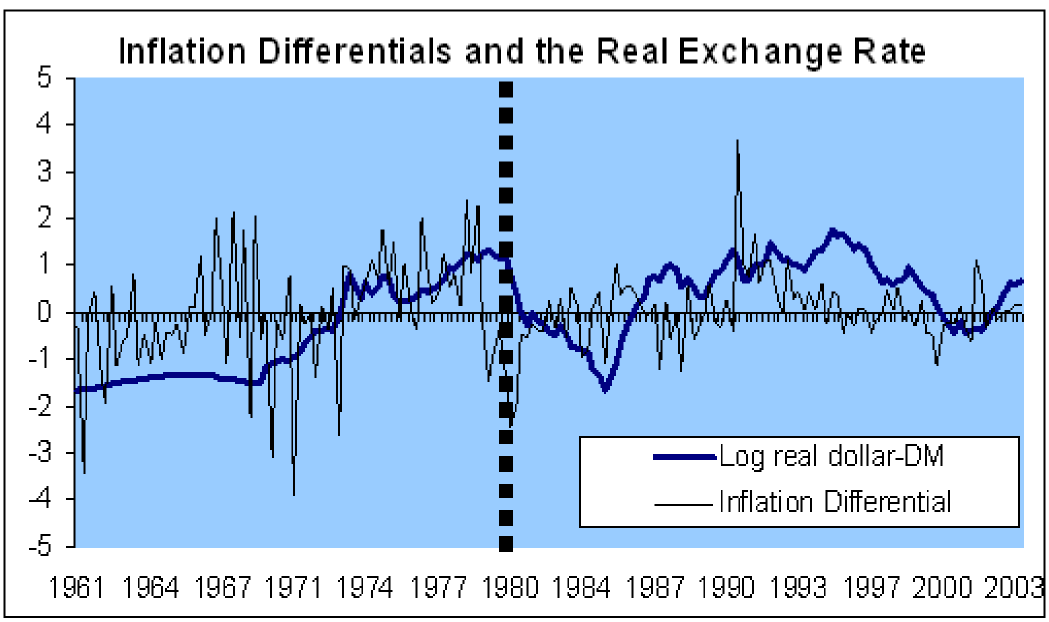


Figure 1: Both figures plot log real dollar-DM rate. Top figure plots German-US inflation differential. Bottom figure plots US-German inflation differential from 1960.2-1979.2 and German-US differential from 1979.3-2003.4.

Thus this informal analysis suggests that the relationship between the trend in the real exchange rate and the inflation differential shifted around 1979. A potentially relevant regime shift is suggested by research in the monetary policy literature which is the change in the conduct of monetary policy that occurred with the appointment of Paul Volker as the Federal Reserve chairman. An examination of this shift in the context of our problem is undertaken in the next section.

2 Interest rate reaction function differentials

This section reports estimation results for the interest rate differential under homogeneity restrictions on the coefficients for expected inflation and the output gap.

Let the US be the home country. German variables are denoted with a 'star' and the German-US differential is denoted with a tilde where $\widetilde{\pi}_t = (\pi_t^* - \pi_t)$, $\widetilde{i}_t = (i_t^* - i_t)$, and $\widetilde{x}_t = (x_t^* - x_t)$ are the German-US differentials in inflation, short-term nominal interest rates and output gaps, respectively.[7] The log nominal dollar price of the deutschemark is denoted by s_t and the log real dollar price of the DM is q_t.

The specification of the interest rate reaction functions follows Clarida et. al. (1998, 2002). In the U.S., the Fed sets its target for the Federal funds rate i_t^T to respond to the deviation of the public's expected inflation from the target inflation rate, $(E_t\pi_{t+1} - \overline{\pi})$ and the output gap x_t according to

$$i_t^T = \overline{i} + \gamma_\pi \left(E_t\pi_{t+1} - \overline{\pi}\right) + \gamma_x x_t, \tag{1}$$

where $\overline{i}$ is the desired nominal Federal funds rate. The central bank and the public employ the same model to forecast future inflation. The actual interest rate is subject to an exogenous interest rate shock and set according

[7] A positive value of x_t indicates that actual GDP lies above potential GDP.

to a partial adjustment mechanism to model the central bank's desire to smooth out changes in the interest rate,

$$i_t = (1-\rho)i_t^T + \rho i_{t-1} + \eta_t, \tag{2}$$

where η_t is an i.i.d. exogenous policy shock.

The foreign central bank acts in an analagous fashion except that it may also react to nominal exchange rate deviations from its 'natural level,' which is taken to be given by purchasing-power parity. Clarida et. al. (1998) found that the feedback from the exchange rate to the German interest rate was statistically significant but quantitatively very small. The foreign target interest rate is

$$i_t^{*T} = \overline{i^*} + \gamma_\pi \left(E_t\pi_{t+1}^* - \overline{\pi^*}\right) + \gamma_x x_t^* + \gamma_s \left(s_t - [p_t - p_t^*]\right). \tag{3}$$

Imposing homogeneity on $(\gamma_\pi, \gamma x)$ gives the empirical specification,

$$\begin{aligned}
\widetilde{i}_t &= (1-\rho)\widetilde{i}_t^T + \rho\widetilde{i}_{t-1} + \widetilde{\eta}_t, && (4)\\
\widetilde{i}_t^T &= \widetilde{\zeta} + \gamma_\pi E_t\widetilde{\pi}_{t+1} + \gamma_x\widetilde{x}_t + \gamma_s q_t, && (5)\\
\zeta &\equiv \left(\overline{i^*} - \overline{i}\right) - \gamma_\pi\left(\overline{\pi^*} - \overline{\pi}\right), && (6)\\
&\quad \widetilde{\eta}_t \overset{iid}{\sim} (0, \sigma^2_{\widetilde{\eta}}). && (7)
\end{aligned}$$

To set up estimation, adding and subtracting $(1-\rho)\gamma_\pi\widetilde{\pi}_{t+1}$ to the right side of (4) and rearranging gives the regression

$$\widetilde{i}_t = \delta + (1-\rho)\left[\gamma_\pi\widetilde{\pi}_{t+1} + \gamma_x\widetilde{x}_t + \gamma_s q_t\right] + \rho\widetilde{i}_{t-1} + \widetilde{\eta}'_t \tag{8}$$

where $\delta = (1-\rho)\zeta$ and $\widetilde{\eta}'_t = \widetilde{\eta}_t - (1-\rho)\gamma_\pi\left[\widetilde{\pi}_{t+1} - E_t\widetilde{\pi}_{t+1}\right]$. Under rational expectations, the composite error term $\widetilde{\eta}'_t$ is uncorrelated with the regressors so that estimation can proceed using generalized method of moments. [8] The

[8] The monetary policy literature places a great deal of emphasis on the magnitude of

Table 1: Relative Reaction Function Estimates: Bundesbank–Fed

Sample	Output gap	γ_π (ase)	γ_x (ase)	γ_q (ase)	ρ (ase)	ζ (ase)	J-statistic (p-value)
60.2-79.2	Source	0.148	-0.126	-0.016	0.858	-0.439	2.571
		(0.482)	(0.221)	(0.015)	(0.063)	(0.267)	(0.860)
79.3-03.4	Source	1.987	0.573	-0.012	0.825	0.258	1.384
		(0.505)	(0.289)	(0.013)	(0.068)	(0.108)	(0.967)
60.2-79.2	HP	-0.127	-0.556	-0.029	0.877	-0.516	2.336
		(0.580)	(0.453)	(0.020)	(0.062)	(0.247)	(0.886)
79.3-03.4	HP	2.048	0.016	0.001	0.795	0.119	1.287
		(0.520)	(0.280)	(0.009)	(0.088)	(0.116)	(0.972)

Notes: Asymptotic standard errors in parentheses.

instrumental variables employed include three lags of the inflation differential, three lags of the output gap differential, three lags of the nominal interest differential, and one lag of the real exchange rate.

The sample is split at 1979.3. Clarida et. al. (2002) report evidence of a significant change in the Fed's response to changes in inflation occurs in 1979, the year that Paul Volker was appointed to the Fed chairmanship. In the pre-Volker sample, increases in inflation were met with a weak response in the nominal interest rate, resulting in a reduction in the real interest rate whereas in the post-1979 period, the Fed responded to increases in inflation with sufficiently aggressive increases in the nominal interest rate to lead to an increase in the real interest rate. Clarida et. al. (1998) report estimates of monetary policy reaction functions for the Bundesbank and several other countries from 1979 to 1993. They find that over this period, the Bundesbank

γ_π. Values less than 1 indicate that an increase in expected inflation elicits a weak response from the central bank that results in a reduction of the real interest rate which stimulates the economy, leading to a further increase in inflation. Values greater than 1 imply that the central bank responds aggressively to an increase in inflation by raising the nominal interest rate sufficiently to raise the real interest rate.

reactions to changes in inflation were similar to those of the post-1979 Fed.[9]

The estimation results are reported in Table 1. The structural shift reported in the literature for the Fed holds reasonably well for the interest differential. The estimated inflation response coefficient is less than 1 over the pre-1979 period and is larger than 1 in the post-1979 period. The estimated output gap coefficient has the wrong sign in the pre-1979 sample but is not statistically significant. The estimated exchange rate response coefficient is not significant. Hansen's test of the overidentifying restrictions does not reject the specification.

A visual account of the fit is provided by Figure 2 which plots the actual interest differential and fitted values using observations from the source estimates of potential GDP. In generating the fitted values, I employ inflation differential forecasts implied by a fourth order bivariate autoregression in the inflation differential and the output gap differential. It can be seen that this simple specification appears to work reasonably well in describing the dynamics of the interest differential. Tractability in the analysis is facilitated by imposing coefficient homogeneity in the interest rate rule across countries. The estimation results suggest that imposing these restrictions is not unreasonable. Since the empirical analysis does not find that γ_s is significant, it will be set to zero in the remainder of the analysis.

3 An empirical model of the real exchange rate

Since the learning public is attempting to discover the rational expectations equilibrium, it is useful begin with an examination of this case. This is done in subsection 3.1. The extension of the model under least-squares learning is undertaken in subsection 3.2.

[9] See also, Gerlach and Schnabel (1999) who estimate monetary policy reaction functions for an average of the EMU countries over a sample spanning from 1990 to 1998.

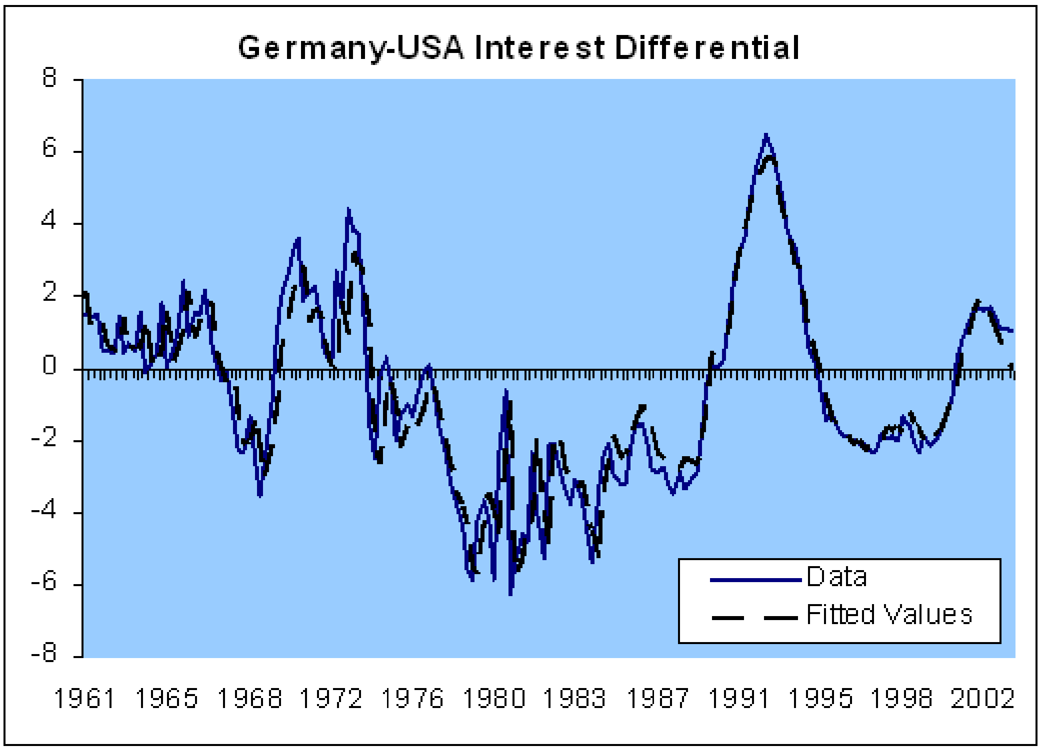


Figure 2: Fitted values employ one-period ahead forecast of inflation differential generated from a fourth-order bi-variate autoregression for the inflation differential and the output gap differential.

3.1 Estimated Rational Expectations Path

The empirical model builds upon uncovered interest parity and the differential in interest differential reaction functions. The approach is relatively unstructured as the dynamics of the inflation differential and the output gap differential are exogenously generated from a bivariate vector autoregression. Market participants view the VAR as the data generating process for inflation and the output gap which they use to construct their forecasts of future inflation. The focus is to examine how well these new macroeconomic fundamentals explain the fluctuations of the real exchange rate rather than to test a particular dynamic general equilibrium model.

The VAR is specified as follows: Let $\widetilde{Y}_t' = (\widetilde{\pi}_t, ..., \widetilde{\pi}_{t-p+1}, \widetilde{x}_t, ..., \widetilde{x}_{t-p+1})$, and $\widetilde{Z}_{vt}' = \left(1, \widetilde{Y}_t'\right)$. The two-equation p-th ordered VAR in regression form is,

$$\widetilde{\pi}_t = b_\pi' \widetilde{Z}_{vt-1}' + \widetilde{v}_{1t}, \tag{9}$$

$$\widetilde{x}_t = b_x' \widetilde{Z}_{vt-1}' + \widetilde{v}_{2t}, \tag{10}$$

which is convenient for estimation. For generating forecasts, it is convenient to rewrite the VAR in companion form,[10],

$$\widetilde{Y}_t = \alpha + A\widetilde{Y}_{t-1} + \widetilde{v}_t$$

Let e_1 be a the row selection vector that is comprised of zeros everywhere except for the (1,1) element which is 1 such that $\widetilde{\pi}_t = e_1 \widetilde{Y}_t$. Since $E_t \widetilde{Y}_{t+1} = \alpha + A\widetilde{Y}_t$, the 1-step ahead expected inflation differential is

$$E_t \widetilde{\pi}_{t+1} = e_1 \left(\alpha + A\widetilde{Y}_t\right). \tag{11}$$

The output gap differential can be recovered from the companion form of the

[10] Any finite ordered VAR(p) can be rewritten in the so-called companion form as a first-ordered VAR.

VAR by defining the selection vector e_2 that is comprised of zeros everywhere except that the (1,p)-th element is 1 so that

$$\widetilde{x}_t = e_2 \widetilde{Y}_t. \tag{12}$$

The nominal exchange rate is priced by uncovered interest parity,

$$s_t = E_t s_{t+1} + \widetilde{i}_t. \tag{13}$$

To price the real exchange rate,add and subtract $E_t \widetilde{\pi}_{t+1}$ from the right hand side of (13) and rearrange to get

$$q_t = E_t q_{t+1} + \widetilde{i}_t - E_t \widetilde{\pi}_{t+1}. \tag{14}$$

Substituting (4),(5), (11), and (12) into (14) gives

$$\begin{aligned} q_t &= E_t q_{t+1} + [\delta + (1-\rho)(\gamma_\pi - 1) e_1 \alpha] \\ &\quad + (1-\rho)(\gamma_x e_2 + (\gamma_\pi - 1) e_1 A) \widetilde{Y}_t + \rho \widetilde{i}_{t-1} + \widetilde{\eta}_t. \end{aligned} \tag{15}$$

Notice how the relationship between the expected real appreciation and the inflation differential is dependent on the response of the central banks to expected inflation. In the pre-1979 period, the two variables trend in opposite directions because with $\gamma_\pi < 1$,a decline in the expected German-US inflation differential leads the public to expect an increase in the German-US real interest differential and a decrease in the real exchange rate. In the post-1979 period with $\gamma_\pi > 1$, a decline in expected inflation differential induces the public to expect an decrease in the German-US interest differential and a real dollar appreciation.

With $\gamma_s = 0$, the real interest parity condition iterates to give the real exchange rate as the undiscounted present value of expected future real interest differentials. A solution will exist if the real interest differential has unconditional mean 0.[11] A rational expectations real exchange rate is the minimum state variable (MSV) solution

$$q_t = \beta_0 + \beta_1{}' Y_t + \beta_2 i_{t-1} + \beta_3 \eta_t, \tag{16}$$

where

$$\beta_2 = \frac{\rho}{1-\rho}, \tag{17}$$

$$\beta_3 = \frac{1}{1-\rho}, \tag{18}$$

$$\beta_1' = \left([\gamma_x e_2 + \gamma_\pi e_1 A] - e_1 A \right) (I - A)^{-1}, \tag{19}$$

$$\beta_0 = -\left(\beta_1' + \beta_2 e_1 \right) \alpha \, (I - A)^{-1}. \tag{20}$$

The restriction on the constant β_0 ensures that the log real exchange rate has zero unconditional mean. This restriction cannot be imposed in the empirical work, however, because the price data are price indices and not actual price levels. Due to the base year price-level embedded into the price indices, the constant term cannot be identified.

3.1.1 Rational Expectations Real Exchange Rate Path

The estimated rational expectations real exchange rate path uses full-sample estimates of the coefficients $(\rho, \gamma_\pi, \gamma_x, \alpha, A)$ using all available data in the sample. The estimates are obtained with a known breakpoint at 1979.3 to allow for the observed change in monetary policy. Market participants are assumed to have known about both regimes and were endowed with knowledge of the coefficient values under each regime. These estimated coefficients give implied values of the exchange rate coefficients in eqs. (17)-(19).[12] We

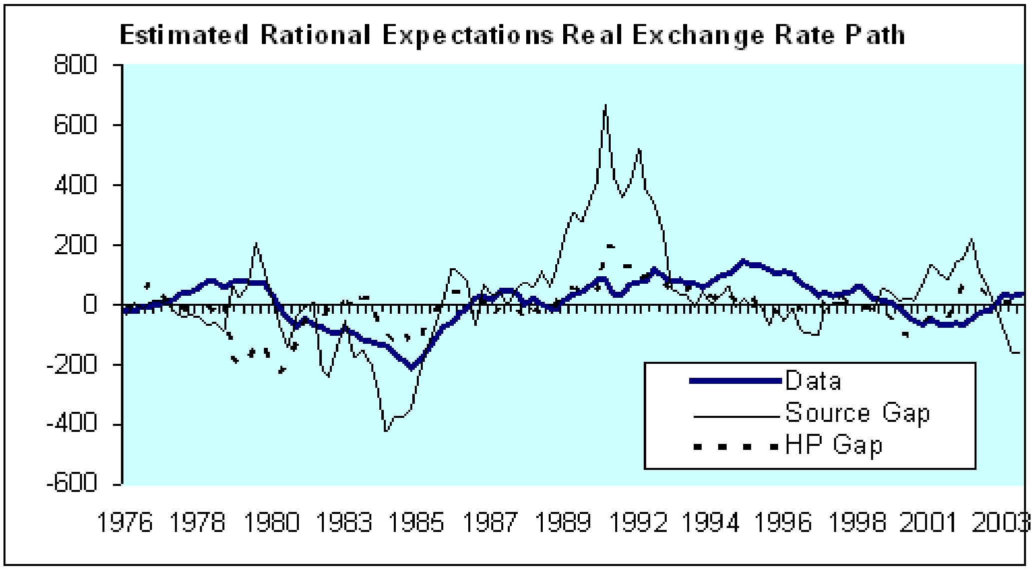
[11] The zero unconditional mean restriction is plausible for the real interest differential and requires $\bar{i} = \bar{\pi}$, from which it follows that $\delta = (1-\rho)(1-\gamma_\pi)\bar{\pi}$

[12] I employ a 4th order VAR for the inflation and output gap differentials, which the BIC rule suggested was appropriate.

note that the implied path is generated entirely by the fundamentals data and does not directly depend on actual real exchange rate observations.

I generate the implied rational expectations real exchange rate beginning in 1976.2 as suggested by Hansen and Hodrick (1982). The argument is that the public may have been expecting a return to the Bretton Woods system and that the flexible exchange rate regime only became fully credible after the IMF's Articles on Exchange Rate arrangements were amended. The estimated time path of the rational expectations real dollar-DM rate are displayed in Figure 3.1.1. The real exchange rate path implied by the HP filtered gap shows only a very loose connection with the real exchange rate data. The path implied with the source constructed output gaps fares better. The implied path misses the real dollar depreciation of the late 1970s but captures the long real appreciation through the mid 1980s and the subsequent depreciation. The implied turning point occurs in 1984.2 whereas in the data it occurs in 1985.1. The implied exchange rate then depreciates from 84.3 to 92.2 whereas in the data, the depreciation more or less continues to 95.3. The implied depreciation is much bigger than that observed in the data. The reason for this is twofold. First, there was a one-time upward spike in relative German inflation in 1991.1 which was combined with a very low (negative) value of the relative German-US output gap. Because the vector autoregression coefficients are estimated over the full sample, information about the spike is contained in these estimates. As we approach the date 1991.1, agents thus are partially able to anticipate the spike. The second reason is that expected high German inflation differential and a low German output gap differential leads people to expect, through the interest rate reaction function, an increase in the German interest differential and a real dollar depreciation.

From 1994.1 to 1997.3, both the implied rational real exchange rate and the data show a gradual real dollar appreciation. The implied exchange rate begins a dollar depreciation in 1997.4 which leads the actual dollar depreca-



tion which begins in 2000.4.

Table 2 quantifies the co-movements between the implied rational real exchange rate path and the data. The table reports the results from regressing the data on the implied real exchange rate, on levels, and changes (in logarithms) at 1, 4, 8, and 16 quarter horizons. The observations are scaled so that exchange returns are expressed in percent per annum. The volatility (measured by the sample standard deviation) of the 1-quarter real exchange rate return in the data is 20.06 percent so the implied return volatility substantially overstates the truth. The estimated slope coefficients, regression R^2s and both the short-and-long horizon return correlations exhibit systematic co-movements both in the level as well as in the changes between the estimated rational expectations fundamental real exchange rate and the actual exchange rate.[13]

[13] These results are qualitatively similar to those reported in Engel and West (2002) who undertake a related analysis. There are several differences in our analyses of the implied rational expectations real real dollar-DM rate. First, Engel and West work with a discounting model $(\gamma_s > 0)$. Second, they equate the actual interest differential to the

Table 2: DM REE Path with Data

		Regression slope	(s.e.)	ρ	R^2
Source output gap	Level	0.214	(0.036)	0.486	0.248
	1-Qtr	0.067	(0.024)	0.256	0.068
Implied Return	4-Qtr	0.111	(0.033)	0.312	0.100
Volatility=76.983	8-Qtr	0.135	(0.036)	0.349	0.124
	16-Qtr	0.168	(0.038)	0.411	0.180
HP output gap	Level	0.313	(0.093)	0.304	0.104
	1-Qtr	0.083	(0.052)	0.151	0.024
Implied Return	4-Qtr	0.099	(0.069)	0.136	0.020
Volatility=36.811	8-Qtr	0.087	(0.079)	0.108	0.012
	16-Qtr	0.011	(0.118)	0.009	0.010

3.2 Learning Dynamics

Unless very strong assumptions about the knowledge and beliefs held by market participants regarding the underlying economic environment, the analysis of the rational expectations real exchange rate path begs the question as to whether the observed real exchange rate path over the past 27 years could have been generated by the model.

Some of these informational assumptions are now relaxed by setting market participants in a learning environment with no model misspecification where they do not know the parameter values in the policy rule or the coefficient values in the VAR that governs actual inflation differentials and the

target interest differential $\left(\tilde{i}_t = \tilde{i}_t^T\right)$, whereas my analysis takes account of central bank's desire to smooth interest rate changes. Third, they impose parameter values for the interest rate reaction functions drawn from estimates reported in the literature whereas mine are estimated from the sample being studied. Fourth, they employ monthly data. They measure goods prices by the CPI and output with industrial production. Also, they construct their output gap as the residual from an output regression on a quadratic trend. Finally, they do not consider the implications of the Volker regime shift and begin their analysis in 1979 under assumption of a single fixed regime.

output gap. In 'real time,' the public knows as much as a would-be econometrician acquires knowledge of the relevant coefficients as would an econometrician by employing least-squares learning rules [Evans and Honkapojian (2001)].

The model generated observations are obtained as follows. Given coefficient values (α_{t-1}, A_{t-1}), for the bivariate VAR on the inflation differential and output gap differential, $(\delta_{t-1}, \rho_{t-1}, \gamma_{\pi,t-1}, \gamma_{x,t-1})$ from the interest rate reaction functions, beliefs concerning the real and nominal interest differential, period t observations are[14]

$$\begin{aligned} i_t &= \left(\delta_{t-1} + \left(1-\rho_{t-1}\right)\gamma_{\pi,t-1}e_1\alpha_{t-1}\right) \\ &\quad + \left(1-\rho_{t-1}\right)\left(\gamma_{x,t-1}e_2 + \gamma_{\pi,t-1}e_1A_{t-1}\right)\widetilde{Y}_t + \rho_{t-1}\widetilde{i}_{t-1} + \widetilde{\eta}_t \qquad (21) \\ r_t &= i_t - e_1\left(\alpha_{t-1} + A_{t-1}\widetilde{Y}_t\right) \qquad (22) \end{aligned}$$

Given coefficient values $\beta'_{t-1} = \left(\beta_{0t-1}, \beta'_{1t-1}, \beta_{2t-1}, \beta_{3t-1}\right)$, agents's perceived law of motion for the real exchange rate draws on the conjectured form of the rational expectations solution,

$$q_t = \beta_{0t-1} + \beta'_{1t-1}\widetilde{Y}_t + \beta_{2t-1}\widetilde{i}_{t-1} + \beta_{3t-1}\widetilde{\eta}_t. \qquad (23)$$

The expected future real exchange rate is then obtained from the perceived law of motion,

$$E_t q_{t+1} = \beta_{0t-1} + \beta'_{1t-1}\left(\alpha_{t-1} + A_{t-1}\widetilde{Y}_t\right) + \beta_{2t-1}\widetilde{i}_t. \qquad (24)$$

The actual law of motion for the real exchange rate is obtained by substitut-

[14]Central banks know the monetary policy reaction functions. That is how they set $\widetilde{i}_t$. The public does not know the coefficient values and must estimate them. The recursive least squares estimates of the policy reaction functions are used in the actual law of motion to account for the possiblity that the policy rule itself has evolved over time. The analysis accounts for this possibility.

ing (22), (23) and (24) into the uncovered interest parity condition,

$$q_t = \tau_{0t} + \tau_{1t}'\widetilde{Y}_t + \tau_{2t}\widetilde{i}_{t-1} + \tau_{3t}\widetilde{\eta}_t \tag{25}$$

where

$$\begin{aligned}
\tau_{0t} &= \beta_{0,t-1} + \left(\beta_{1,t-1} - e_1 + \gamma_{\pi,t-1}\left(\beta_{2,t-1}+1\right)\right)\alpha_{t-1} + \left(1+\beta_{2,t-1}\right)\delta_{t-1} \\
\tau_{1t}' &= \gamma_{x,t-1}e_2 + \beta_{2,t-1}\left(\gamma_{x,t-1}e_2 + \gamma_{\pi,t-1}e_1A_{t-1}\right) \\
&\quad + \left(\left(\gamma_{\pi,t-1}-1\right)e_1 + \beta_{1,t-1}'\right)A_{t-1} \\
\tau_{2t} &= \rho_{t-1}\left(1+\beta_{2,t-1}\right) \\
\tau_{3t} &= 1+\beta_{2,t-1}
\end{aligned}$$

The coefficients are then updated as follows:

1. For the VAR coefficients,

$$R_{v,t} = R_{v,t-1} + g_t\left(\widetilde{Z}_{vt-1}\widetilde{Z}_{vt-1}' - R_{v,t-1}\right) \tag{26}$$

$$(b_{\pi,t}, b_{x,t}) = (b_{\pi,t-1}, b_{x,t-1}) + g_tR_{vt}^{-1}\widetilde{Z}_{vt-1}\left[(\widetilde{\pi}_t, \widetilde{x}_t) - \widetilde{Z}_{vt-1}'(b_{\pi,t-1}, b_{x,t-1})\right] \tag{27}$$

   where g_t is the gain. Standard recursive least-squares estimation is obtained with a decreasing gain set to $g_t = 1/t$. Letting $(\alpha, A) = C(b_\pi', b_x')$ be the mapping from the regression to companion form coefficients, the VAR coefficients are updated according to the rule, $(\alpha_t, A_t) = C(b_{\pi,t}, b_{x,t})$.

2. For the monetary policy rule coefficients, let $\phi' = (\delta, (1-\rho)\gamma_\pi, (1-\rho)\gamma_x, \rho)$ and $\widetilde{Z}_{it}' = \left(1, \widetilde{\pi}_t^e, \widetilde{x}_t, \widetilde{i}_{t-1}\right)$, where $\widetilde{\pi}_t^e = e_1(\alpha_{t-1} + A_{t-1}Y_t)$. The relative reaction function can be compactly restated as $\widetilde{i}_t = \phi'\widetilde{Z}_{it} + \widetilde{\eta}_t$. Now, update according to,

$$
\begin{aligned}
R_{i,t} &= R_{i,t-1} + g_t \left(\widetilde{Z}_{i,t-1} \widetilde{Z}'_{i,t-1} - R_{i,t-1} \right) &\quad (28) \\
\phi_t &= \phi_{t-1} + g_t R^{-1}_{i,t-1} \widetilde{Z}_{i,t-1} \left(\widetilde{i}_t - \phi'_{t-1} \widetilde{Z}_{i,t-1} \right) &\quad (29)
\end{aligned}
$$

3. For the real exchange rate coefficients, let $\widetilde{Z}'_{qt} = \left(1, \widetilde{Y}'_t, \widetilde{i}_{t-1}, \widetilde{\eta}_t\right)$. Then

$$
\begin{aligned}
R_{q,t} &= R_{q,t-1} + g_t \left(\widetilde{Z}_{q,t} \widetilde{Z}'_{q,t} - R_{q,t-1} \right) &\quad (30) \\
\beta_t &= \beta_{t-1} + g_t R^{-1}_{q,t-1} \widetilde{Z}_{q,t} \left(q_t - \beta'_{t-1} \widetilde{Z}_{q,t} \right) &\quad (31)
\end{aligned}
$$

Notice that the implied learning path employs observations only on the monetary policy fundamentals and does not directly employ observations on the real exchange rate.

Several specifications of the gain are considered. The declining gain specification ($g_t = 1/t$) would be appropriate if the public believes that there is a single time invariant structure. If this is true, then under certain regularity conditions, the learning model converges to the rational expectations equilibrium. On the other hand, if the public believed that a regime change occurred at (say) date t', then it might make sense to reset the gain at the time of the known break point ($g_t = 1/(t - t' + 1)$ for $t \geq t'$). A third possibility is that the public understands that they operate in a changing environment but they may or may not know if a regime change has actually occurred. In this case, it might make sense to employ recursive least squares with a constant gain specification as in Orphanides and Williams (2003). Under a constant gain, the least squares coefficients do not converge to constant values. The constant gain makes sense if people expect structural shifts to take place but are not fully aware of the timing of such shifts or whether a shift has in fact occurred at the time of the shift.

The international finance environment has been subject to several innovations over the past three decades. In addition to the 1979 shift in monetary policy, other possible sources of structural instability include German reunification (1990.3), and the breakdown of the European Monetary System following the 1992 crisis. To investigate the potential importance of these events, I consider the following alternative specifications of the gain.

Gain type 0: This is the specification of a constant gain of 0.02. This is the value assumed by Orphanides and Williams (2003), who calibrated the gain to the expectations provided by professional forecasters.

Gain type 1: This is the specification of a decreasing gain $g_t = 1/t$ throughout the entire sample.

Gain type 2: This is a decreasing gain specification which resets in 1979.3 to coincide with a new monetary policy regime.

Gain type 3: This is a decreasing gain specification which resets both in 1979.3 and at German reunification (1990.3).

Gain type 4: This is a decreasing gain specification that resets in 1992.3 to coincide with the European Monetary System crisis.

As in the analysis of the rational expectations path, I begin the sample period for analysis in 1976.2 and use pre-sample observations over the float (beginning in 1973) to estimate initial values of the least-squares coefficients and associated moment matrices.[15] Figure 3.2 plots the alternative implied learning paths along with the data with the output gap defined by the source statistical agencies.[16] The learning paths associated with the alternative gain specifications are qualitatively very similar. Each of the learning

[15]Since the public was grappling with the new and unfamiliar flexible exchange rate regime, it does not make sense to use observations from the Bretton Woods era to obtain starting values since that regime is no longer relevant.

[16]Learning paths with the output gap as the deviation from the HP trend are not presented. While the HP trend may be a reasonable retrospective detrending device, due to the truncation of the HP filter at the endpoints of the sample, it is not appropriate to

paths broadly exhibit the real dollar depreciation in the late 1970s, the great appreciation of the early 1980s and the subsequent great depreciation. The learning path can be seen to exhibit the real dollar depreciation from 1976 through 1981.1 whereas the turning point in the data is 1980.1. The learning path for gain type 3 shows a real dollar appreciation through 1984.3, only two quarters before the turning point in the data as the real dollar continues to gain until 1985.1. From this time, the actual real value of the dollar depreciates until 1988.1, gains for a year then more or less trends downward until1995.2. Within this time, there is a significant gain in the dollar in 1992.1. The implied learning path trends along with the data from 1985 but in contrast to the implied rational expectations path does generate the exaggerated depreciation in 1992.1 when the inflation and output gap data spike. In the learning environment, the 1992.1 spike in the fundamentals is largely unanticipated unlike the rational expectations calculations because those used coefficient estimates estimated on the full sample. Both the actual real exchange rate and the learning paths trend downward through 1998. The learning path then begins a real dollar depreciation that leads the data which begins its own depreciation in 2002.

Table 3 reports the results from regressing the data on the various implied learning paths. The correlation between the levels of the actual real dollar-DM and each of the alternative learning paths ranges from 0.304 (constant gain) to 0.380 (gain type 3). The correlations of quarterly rates of change range from 0.05 to 0.07 whereas the correlations of log changes at the 16 quarter horizon range from 0.51 to 0.56.

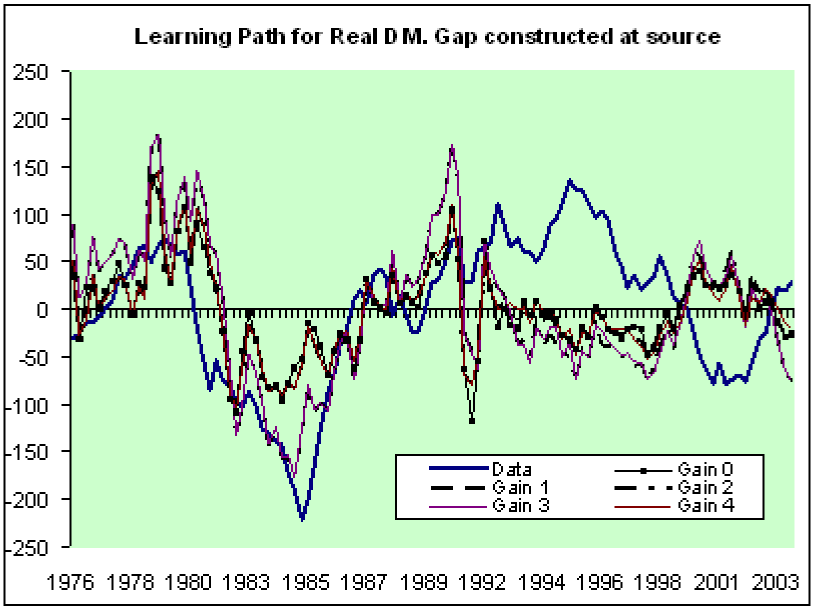
Finally, Figure 3 provides a comparison between the estimated rational expectations path and the (constant gain) learning path. There are several differences between them. First, the volatility of the learning path matches that in the data more closely. Second, the learning path captures the 1976-

---

assume that the retrospectively constructed gap is what the public believed the output gap to have been at the time. To employ the HP trend, one would have to compute the HP gap recursively.

Table 3: DM Learning Path with Data. Source output gap

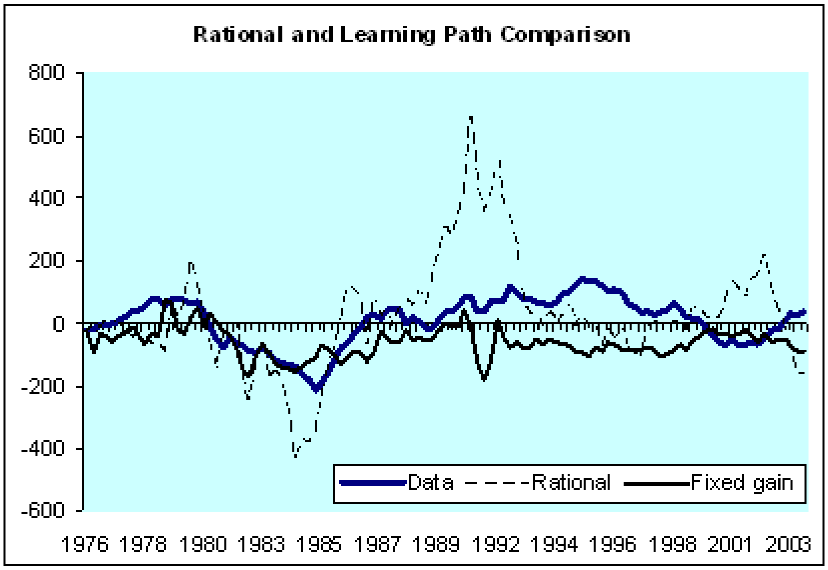
		Gain Specification				
		0	1	2	3	4
Level	slope	0.507	0.516	0.381	0.392	0.543
	(s.e.)	(0.150)	(0.144)	(0.091)	(0.090)	(0.144)
	ρ	0.304	0.321	0.370	0.380	0.337
	R^2	0.104	0.115	0.149	0.156	0.125
1-qtr	slope	0.029	0.023	0.032	0.037	0.025
return	(s.e.)	(0.056)	(0.058)	(0.048)	(0.048)	(0.058)
	ρ_1	0.049	0.039	0.064	0.072	0.040
	R^2	0.003	0.002	0.005	0.006	0.002
4-Qtr	slope	0.118	0.150	0.183	0.188	0.159
return	(s.e.)	(0.099)	(0.097)	(0.075)	(0.074)	(0.097)
	ρ_4	0.115	0.148	0.231	0.238	0.156
	R^2	0.014	0.023	0.055	0.058	0.026
8-qtr	slope	0.331	0.373	0.323	0.326	0.379
return	(s.e.)	(0.131)	(0.126)	(0.086)	(0.086)	(0.125)
	ρ_8	0.242	0.281	0.346	0.350	0.286
	R^2	0.060	0.080	0.122	0.125	0.084
16-qtr	slope	0.756	0.750	0.484	0.500	0.787
return	(s.e.)	(0.130)	(0.123)	(0.076)	(0.076)	(0.122)
	ρ_{16}	0.510	0.528	0.543	0.556	0.549
	R^2	0.273	0.292	0.308	0.323	0.315
Implied	return					
	volatility	34.207	33.248	40.055	39.747	33.178



1981 real dollar depreciation better than the estimated rational expectations path. Third, while both sets of estimates capture the great appreciation and the great depreciation of the 1980s, the estimated rational path predicts too much of a depreciation from 1985 to 1991 whereas the learning path predicts not enough of a depreciation. Both estimates exhibit the turning points in 1991.1 and 1991.3 found in the data. From about 1994 onwards, the qualitative dynamics of the estimated rational path and the learning path are not substantially different.

4 Conclusion

This paper has presented evidence that reconnects the real exchange rate to macroeconomic fundamentals. When interest rates are governed by modern monetary policy rules and agents must adaptively learn about a changing environment including the evoloution of those policy rules, expected inflation and output gap differentials data are able to generate a real exchange rate



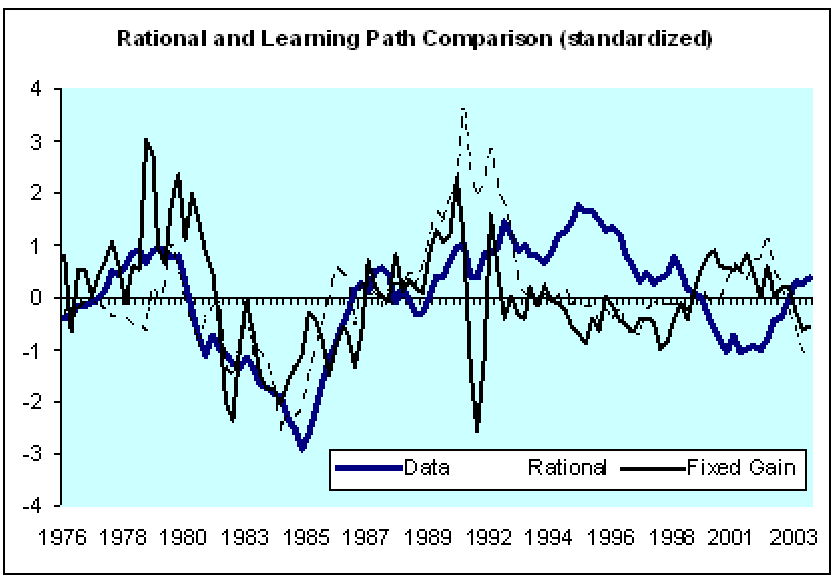


Figure 3: Comparison of estimated rational and learning paths for the real dollar-DM rate. Observations are standardized in the bottom figure.

path that matches major trend shifts in the observed real exchange rate data. In particular, the model provided a fundamentals driven explanation of the great appreciation and the subsequent great depreciation of the 1980s.

The analysis allowed for small departures from full common knowledge rationality. Over the past three decades, foreign exchange market participants have seen themselves operating in and adjusting to a changing environment. Modeling the public as adaptive learners who employed recursive least squares rules suggests that this is a plausible framework for understanding exchange rate dynamics.

The pricing relationship used was simply uncovered interest parity which is entirely standard. One distinguishing feature across various studies that examine the exchange rate–interest differential linkage is how the interest differential is modeled. Drawing on recent research on monetary policy, the framework adopted in the paper modeled the interest differential as a function of expected inflation differentials and output gap differentials. The traditional focus on the monetary fundamentals and the weak links between those fundamentals and the exchange rate has perhaps led to a rush to judgement about the irrelevance of macroeconomic fundamentals in understanding exchange rate dynamics. While alternative approaches based on multiple equilibria (e.g., Flood and Rose (1999)) or micro market structure (Lyons and Evans (2003)) or are worthwhile avenues to pursue, the analysis of this paper suggests that additional work in the macroeconomic context is worthwhile.

References

[1] Baxter, Marianne. "Real exchange rates and real interest differentials: Have we missed the business-cycle relationship?" *Journal of Monetary Economics* 33:1 (February 1994), 5-37.

[2] Bullard, James, and Kaushik Mitra. 2002. "Learning about Monetary Policy Rules," *Journal of Monetary Economics*, 49: 1105-1129.

[3] Campbell, John Y. and Richard Clarida. 1987. "The Dollar and Real Interest Rates," *Carnegie-Rochester Conference Series on Public Policy*, 27, pp. 103-139.

[4] Clarida, Richard, Jordi Gali, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics,* pp 147-180.

[5] Clarida, Richard, Jordi Gali, and Mark Gertler. 1998. "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review,* 42: pp. 1033-1067.

[6] Clarida, Richard and Mark P. Taylor. 1997. "The Term Structure of Forward Exchange Premiums and the Forecastability of Spot Exchange Rates: Correcting the Errors," *Review of Economics and Statistics*, 79(3):pp. 353-61.

[7] Devereux, Michael B. and Charles Engel. 2002. "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect," *Journal of Monetary Economics*, pp. 913-940.

[8] Dornbusch, Rudiger. 1976. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84: pp. 1161–1176.

[9] Duarte, Margaride and Alan C. Stockman. 2001. "Rational Speculation and Exchange Rates," *mimeo*, Federal Reserve Bank of Richmond.

[10] Edison, Hallie and Dianne Pauls. “A Re-assessment of the Relationship between Real Exchange Rates and Real Interest Rates: 1974-1990,” *Journal of Monetary Economics*, 31(2), April 1993, 165-87.

[11] Engel, Charles and Kenneth D. West. 2002. “Taylor Rules and the Deutschemark-Dollar Real Exchange Rate,” *mimeo*, University of Wisconsin.

[12] Engel, Charles and James D. Hamilton. 1990. "Long Swings in the Dollar: Are They in the Data and Do Markets Know it?" *American Economic Review,* 80, pp. 689-713.

[13] Evans, George W. and Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics,* Princeton University Press, Princeton, NJ.

[14] Evans, Martin D.D. and Richard K. Lyons. 2003. "A New Micro Model of Exchange Rate Dynamics," *NBER Working Paper 10379.*

[15] Flood, Robert P. and Andrew K. Rose. 1999. "Understanding Exchange Rate Volatility without the Contrivance of Macroeconomics," *Economic Journal.*

[16] Frankel, Jeffrey. 1979. “On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials,” *American Economic Review*, 69(4): 610-22

[17] Frankel, Jeffrey. 1985. "The Dazzling Dollar," Brookings Papers on Economic Activity 1, 199-217.

[18] Gerlach, Stefan and Gert Schnabel. 1999. "The Taylor Rule and Interest Rates in the EMU Area: A Note," BIS working paper No. 73.

[19] Groen, Jan J.J. 2002. “Cointegration and the Monetary Exchange Rate Model Revisited,” *Oxford Bulletin of Economics and Statistics*, 64, pp. 361-380.

[20] Groen, Jan J.J. 2000. "The Monetary Exchange Rate Model as a Long-Run Phenomenon," *Journal of International Economics*, 52, 2.

[21] Groen, Jan J.J. and Akito Matsumoto. 2003. "Real Exchange Rate Persistence and Systematic Monetary Policy Behavior. *mimeo* Bank of England.

[22] Hansen, Lars P. and Robert J. Hodrick. 1983. "Risk Averse Speculation in the Foreign Exchange Market: An Econometric Analysis of Linear Models, in: J.A. Frenkel, edl., Exchange Rates and International Macroeconomics (University of Chicago Press, Chicago, IL).

[23] Hodrick, Robert J. and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation ," *Journal of Money, Credit, and Banking.* 29: pp. 1–16.

[24] Kollman, Robert. 2001. "Monetary Policy Rules in the Open Economy: Effects on Welfare and the Business Cycle." *mimeo* University of Bonn

[25] Lewis, Karen K. 1989a. "Can Learning Affect Exchange-Rate Behavior? The Case of the Dollar in the Early 1980's," *Journal of Monetary Economics*, 23(1): 79-100

[26] Lewis, Karen K. 1989b. "Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange," *American Economic Review*, 79(4): 621-36

[27] Mark, Nelson C. 1995. "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability," *American Economic Review*, 85,1, pp. 201-218.

[28] Mark, Nelson C. and Donggyu Sul. 2001. "Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post-Bretton Woods Panel," *Journal of International Economics*, 53, pp. 29—52.

[29] Mark, Nelson C. and Young-Kyu Moh. 2004. "What do Real Interest Differentials Tell Us About the Real Exchange Rate? The Role of Non-linearities," *mimeo* Tulane University.

[30] Meese, Richard and Kenneth Rogoff. 1983. "Empirical Exchange Rate Models of the 1970's: Do they Fit Out of Sample?" *Journal of International Economics* 14: pp. 3-24.

[31] Meese,Richard-A and Kenneth Rogoff. 1988. " Was It Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period," *Journal of Finance*, 43(4): 933-48

[32] Mussa, Michael. 1982. "A Model of Exchange Rate Dynamics," *Journal of Political Economy*, 90(1): pp. 74-104.

[33] Obstfeld, Maurice. 1985. "Floating Exchange Rates: Experience and Prospects." *Brookings Papers on Economic Activity* 2: pp. 369–450.

[34] Obstfeld, Maurice and Kenneth Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103: pp.624–660.

[35] Orphanides, Athanasios and John C. Williams. 2003. "The Decline of Activist Stabilization Policy: Natural Rate Misperceptions, Learning, and Expectations," *mimeo,* Federal Reserve Board.

[36] Papell, David. 2002. "The Great Appreciation, the Great Depreciation, and the Purchasing Power Parity Hypothesis," *Journal of International Economics*, May, 51-82.

[37] Rapach, David and Mark Wohar. 2002. "Testing the Monetary Model of Exchange Rate Determination: New Evidence From A Century of data," *Journal of International Economics*, 58, pp. 359-385.

[38] Taylor, John. 1993. "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214.

5 Appendix.

This appendix briefly reports results for the real dollar values of the pound, the yen, and the Canadian dollar. I begin by documenting the data sources for the UK, Japan, and Canada.

5.1 The Data

Most of the data were obtained from OECD Economic Outlook. The beginning of sample availability is summarized in the table

Country	RGDP	Potential GDP	GDP Deflator	Interest	Exchange Rate
UK	1960.1	1979.1	1960.1	1969.1	1960.1
Japan	1960.1	1970.1	1960.1	1969.1	1960.1
Canada	1961.1	1966.1	1961.1	1960.1	1960.1

To obtain a sample that spans from 1960.1 through 2003.4, I made the following adjustments:

1. Interest rates:

   (a) For the UK: Observations from 1960.1 to 1968.4 are the Euro pound rate from the IFS. During this time period, the Euro pound rate is less volatile than the T-bill rate and the deposit rate. It also lies below the T-bill rate and lies above the deposit rate.

   (b) For Japan: Observations from 1960.1 to 1968.4 is the call money rate obtained from the IFS. This interest rate exhibits a close correspondence to the OECD short-term interest rates over the period when both are available.

2. Potential GDP: Observations prior to OECD availability are Hodrick-Prescott trend values.

3. Real GDP and the GDP deflator for Canada 1960.1-1960.4 are obtained from the IFS.

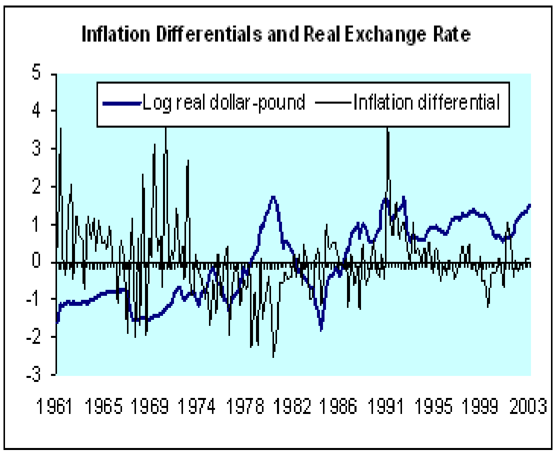
5.2 The real dollar-pound rate.

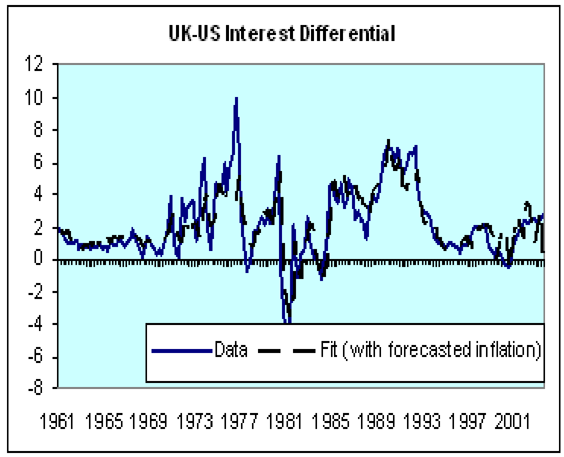
Figures for the real dollar-pound rate are shown in Figure 5.2. Here, it can be seen that the UK-US inflation differential trends downward and the real dollar-pound rate trends upwards from the beginning of the sample to 1980.4 The inflation differential then switches to an upward trend through 1986.along with a real dollar appreciation. Another trend shift appears to be in effect through 1990. The co-movements in the trend real exchange rate and the inflation differential appear to weaken from 1994 onward.

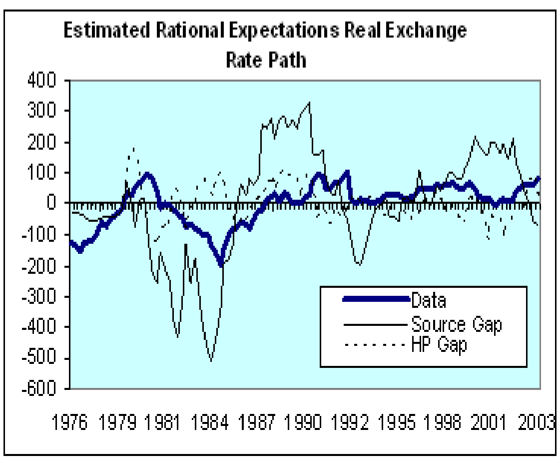
Sample	Output gap	γ_π (ase)	γ_x (ase)	γ_q (ase)	ρ (ase)	$\widetilde{\zeta}$ (ase)	J-Statistic (p-value)
61.2-79.2	Source	0.145	0.115	0.007	0.458	0.560	2.778
		(0.085)	(0.107)	(0.008)	(0.113)	(0.173)	(0.836)
79.3-03.4	Source	1.242	0.569	-0.0373	0.802	1.623	1.023
		(0.613)	(0.272)	(0.022)	(0.078)	(0.835)	(0.985)
61.2-79.2	HP	0.149	0.079	0.005	0.452	0.500	3.321
		(0.084)	(0.138)	(0.008)	(0.112)	(0.151)	(0.768)
79.3-03.4	HP	1.830	0.115	-0.015	0.841	0.256	0.788
		(1.069)	(0.768)	(0.020)	(0.084)	(0.572)	(0.993)

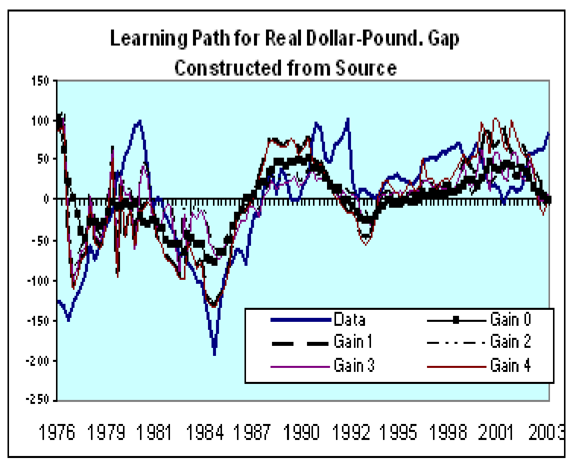
Table 4: Relative Policy Rule Estimates: UK-US

The (relative) policy rule estimates, shown in Table 4, reports estimated coefficients with the predicted signs and suggests that imposing the homogeneity restriction across reaction functions for the Fed and the Bank of England is not unreasonable. The intensity with which both central banks responded to changes in inflationary expectations appears to increase substantially beginning in 1979. The coefficient on the exchange rate in the interest rate reaction function is small in magnitude and not statistically sig-


Inflation Differentials and Real Exchange Rate
Log real dollar-pound
Inflation differential
5
4
3
2
1
0
-1
-2
-3
1961 1965 1969 1974 1978 1982 1986 1991 1995 1999 2003


UK-US Interest Differential
Data
Fit (with forecasted inflation)
12
10
8
6
4
2
0
-2
-4
-6
-8
1961 1965 1969 1973 1977 1981 1985 1989 1993 1997 2001


Estimated Rational Expectations Real Exchange Rate Path
Data
Source Gap
HP Gap
400
300
200
100
0
-100
-200
-300
-400
-500
-600
1976 1979 1981 1984 1987 1990 1992 1995 1998 2001 2003


Learning Path for Real Dollar-Pound. Gap Constructed from Source
Data
Gain 0
Gain 1
Gain 2
Gain 3
Gain 4
150
100
50
0
-50
-100
-150
-200
-250
1976 1979 1981 1984 1987 1990 1992 1995 1998 2001 2003

		Regression slope	(s.e.)	ρ	R^2
Source output gap	Level	0.174	(0.031)	0.468	0.891
	1-Qtr	0.004	(0.031)	0.013	0.009
Implied Return	4-Qtr	0.100	(0.033)	0.284	0.105
Volatility=62.951	8-Qtr	0.131	(0.035)	0.348	0.150
	16-Qtr	0.138	(0.032)	0.400	0.186
HP output gap	Level	-0.122	(0.111)	-0.104	0.861
	1-Qtr	-0.064	(0.045)	-0.133	0.027
Implied Return	4-Qtr	-0.028	(0.059)	-0.046	0.027
Volatility=42.322	8-Qtr	-0.046	(0.079)	-0.058	0.034
	16-Qtr	-0.201	(0.135)	-0.151	0.049

Table 5: Pound REE Path with Data

nificant. Plots of the actual interest differential and the fitted values suggest that the estimated relative interest rate rule has a high degree of explanatory power.

The implied rational expectations path for the real exchange rate does a better job of explaining the actual real exchange rate when the output gap estimated by the source statistical agency is used than the HP filtered output gap. The correlation between the estimated rational path and the actual real exchange rate data is 0.47 in levels and 0.1 in first differences. The correlation at the 4 year horizon, changes in the estimated rational exchange rate and that found in the data rises to 0.4. The volatility of the real exchange rate return found in the data is 20.27 so both (using the source estimated gap and the HP filtered gap) overestimate real exchange rate volatility.

The alternative learning paths exhibit a high degree of co-movement with each other. They also capture many of the swings and turning points of the real exchange rate. The constant gain path is relatively smooth as its implied return volatility (12.6 percent) falls short of that found in the data. The alternative gain paths produce somewhat too much volatility in the range

	gain=0	gain=1	gain=2	gain=3	gain=4
level	0.897	0.642	0.922	0.907	0.622
(s.e.)	(0.159)	(0.091)	(0.158)	(0.140)	(0.086)
ρ	0.470	0.557	0.484	0.522	0.564
R^2	0.891	0.904	0.893	0.899	0.905
1-Qtr	0.230	-0.022	0.037	0.055	0.015
(s.e.)	(0.201)	(0.063)	(0.061)	(0.058)	(0.061)
ρ	0.108	-0.033	0.058	0.089	0.023
R^2	0.020	0.010	0.012	0.017	0.009
4-Qtr	0.476	0.305	0.338	0.309	0.278
(s.e.)	(0.175)	(0.095)	(0.118)	(0.116)	(0.092)
ρ	0.254	0.296	0.267	0.250	0.281
R^2	0.089	0.112	0.096	0.087	0.103
8-Qtr	0.742	0.456	0.642	0.579	0.420
(s.e.)	(0.182)	(0.107)	(0.154)	(0.153)	(0.105)
ρ	0.372	0.387	0.379	0.349	0.367
R^2	0.167	0.179	0.173	0.151	0.163
16-Qtr	0.815	0.595	1.027	0.985	0.579
(s.e.)	(0.182)	(0.106)	(0.182)	(0.182)	(0.106)
ρ	0.417	0.497	0.501	0.483	0.487
R^2	0.199	0.272	0.276	0.259	0.262
Vol	12.561	31.898	32.937	34.097	32.954

Table 6: Pound Learning Path with Data. Source output gap

of 33 percent. The learning path and the data exhibit a high degree of co-movements in levels and in the longer-horizon returns.

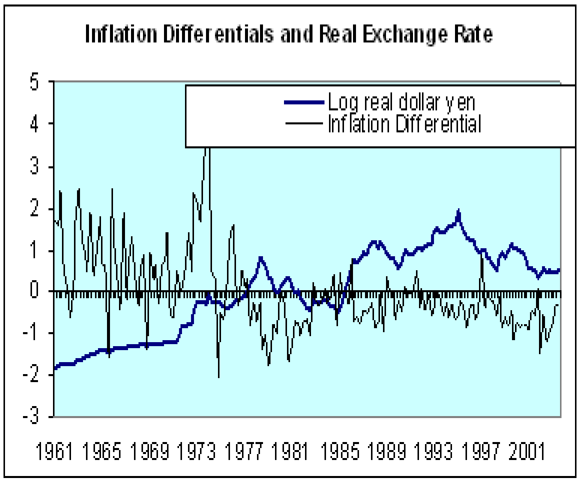
Sample	Output gap	γ_π (ase)	γ_x (ase)	γ_q (ase)	ρ (ase)	$\widetilde{\zeta}$ (ase)	J-statistic (p-value)
61.2-79.2	Source	0.532	0.161	0.000	0.815	0.027	4.088
		(0.650)	(0.298)	(0.014)	(0.078)	(5.598)	(0.664)
79.3-03.4	Source	2.200	0.199	-0.001	0.747	0.088	2.186
		(0.526)	(0.193)	(0.006)	(0.080)	(3.299)	(0.902)
61.2-79.2	HP	0.512	0.658	-0.000	0.676	0.040	6.043
		(0.330)	(0.279)	(0.006)	(0.070)	(4.720)	(0.418)
79.3-03.4	HP	2.100	0.280	-0.001	0.789	0.07785	3.076
		(0.648)	(0.582)	(0.007)	(0.064)	(2.911)	(0.799)

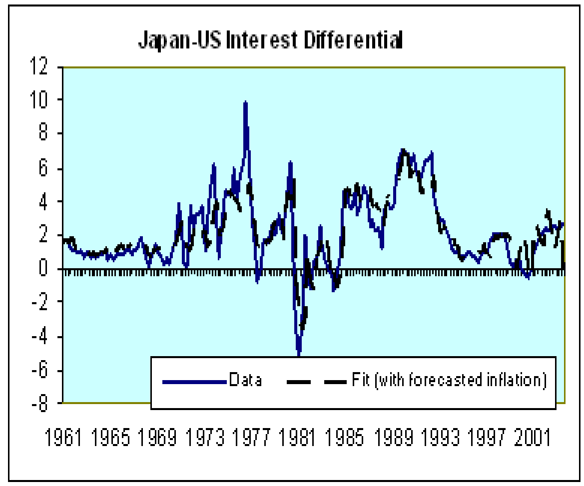
Table 7: Relative Policy Rule Estimates: Japan–USA

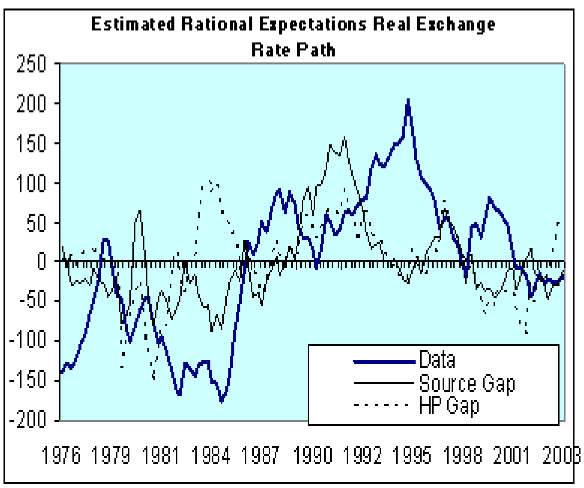
5.3 The Real Dollar-Yen

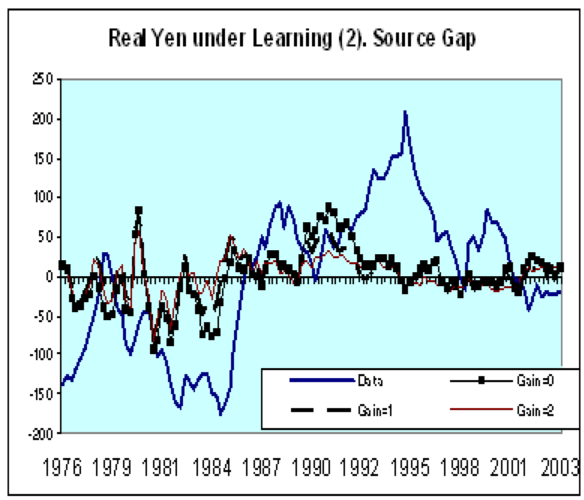
The Japanese-USA inflation differential appears to have three trends. From the beginning of the sample to 1978.4, the real dollar undergoes more or less a steady depreciation while the Japanese-USA inflation differential trends downwards. The inflation differential then drifts upward until 1986.2 where it flattens out. The real dollar gains from 1978.4 to 1985.2, depreciates from that point until 1995.2 when it begins to gain in value through the end of the sample. Here too, the trends in the inflation differential and the real exchange rate go in opposite directions in the pre-1979 data.

The (relative) policy rule estimates, shown in Table 4, reports estimated coefficients with the predicted signs and suggests that imposing the homogeneity restriction across reaction functions for the Fed and the Bank of England is not unreasonable. The intensity with which both central banks responded to changes in inflationary expectations appears to increase substantially beginning in 1979. The coefficient on the exchange rate in the interest rate reaction function is small in magnitude and not statistically significant. The coefficient on the real exchange rate is small in magnitude and not statistically significant. Plots of the actual interest differential and the


Inflation Differentials and Real Exchange Rate
Log real dollar yen
Inflation Differential
5
4
3
2
1
0
-1
-2
-3
1961 1965 1969 1973 1977 1981 1985 1989 1993 1997 2001


Japan-US Interest Differential
12
10
8
6
4
2
0
-2
-4
-6
-8
Data
Fit (with forecasted inflation)
1961 1965 1969 1973 1977 1981 1985 1989 1993 1997 2001


Estimated Rational Expectations Real Exchange Rate Path
250
200
150
100
50
0
-50
-100
-150
-200
Data
Source Gap
HP Gap
1976 1979 1981 1984 1987 1990 1992 1995 1998 2001 2003


Real Yen under Learning (2). Source Gap
250
200
150
100
50
0
-50
-100
-150
-200
Data
Gain=0
Gain=1
Gain=2
1976 1979 1981 1984 1987 1990 1992 1995 1998 2001 2003

		Regression slope	(s.e.)	ρ	R^2
Source output gap	Level	0.688	(0.149)	0.401	0.998
	1-Qtr	0.050	(0.086)	0.056	0.006
Implied Return	4-Qtr	-0.091	(0.115)	-0.076	0.011
Volatility=30.678	8-Qtr	-0.290	(0.126)	-0.222	0.056
	16-Qtr	0.109	(0.143)	0.078	0.017
HP output gap	Level	0.126	(0.168)	0.071	0.998
	1-Qtr	0.017	(0.076)	0.021	0.003
Implied Return	4-Qtr	-0.114	(0.101)	-0.108	0.017
Volatility=31.142	8-Qtr	-0.419	(0.108)	-0.356	0.135
	16-Qtr	-0.393	(0.133)	-0.291	0.097

Table 8: Yen REE Path with Data

fitted values suggest that the estimated relative interest rate rule has a high degree of explanatory power.

The estimated rational expectations path does not exhibit particularly high co-movements with the data. The correlation between the estimated rational expectations real exchange rate and the real exchange rate data (in levels) is 0.4 and is 0.6 in first differences. The correlation between the two series for 4-quarter and 8-quarter returns, however, are negative when the source estimated output gap measures are employed. The real exchange rate return volatility (in the data) is 22.2 percent so the estimated rational expectations path generates too much volatility.

In generating the learning paths, German reunification and the EMS crises seemed to have less relevance to the Japanese market so that results only for gain specifications 0 (constant gain), 1 ($g_t = 1/t$) and 2 (restarting the gain in 1979.3) are shown. The learning paths produce about the right amount of implied return volatility. The constant gain specification appears to produce the highest degree of co-movement with the data but here as well as in the implied rational expectations path, the degree of co-movement

	gain=0	gain=1	gain=2
level	1.107	0.880	0.651
(s.e.)	(0.210)	(0.303)	(0.369)
ρ	0.447	0.266	0.165
R^2	0.998	0.998	0.998
1-Qtr	0.037	0.022	0.022
(s.e.)	(0.099)	(0.104)	(0.114)
ρ	0.036	0.020	0.019
R^2	0.004	0.003	0.003
4-Qtr	-0.101	-0.127	-0.219
(s.e.)	(0.133)	(0.152)	(0.184)
ρ	-0.073	-0.080	-0.115
R^2	0.011	0.012	0.019
8-Qtr	-0.022	0.032	-0.177
(s.e.)	(0.182)	(0.217)	(0.253)
ρ	-0.012	0.015	-0.069
R^2	0.006	0.006	0.011
16-Qtr	0.469	0.520	0.190
(s.e.)	(0.213)	(0.268)	(0.333)
ρ	0.220	0.195	0.059
R^2	0.060	0.050	0.015
Vol	22.557	21.517	19.949

Table 9: Yen Learning Path with Data. Source output gap

Table 10: Relative Policy Rule Estimates: Canada-USA

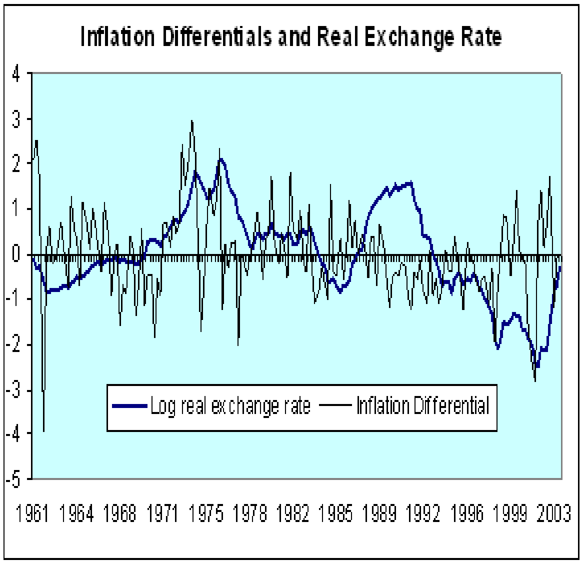
Sample	Output gap	γ_π (ase)	γ_x (ase)	γ_q (ase)	ρ (ase)	$\widetilde{\zeta}$ (ase)	J-statistic (p-value)
61.2-79.2	Source	-0.237	-0.177	0.004	0.689	0.356	5.871
		(0.190)	(0.099)	(0.010)	(0.111)	(0.438)	(0.438)
79.3-03.4	Source	0.516	0.206	0.023	0.785	0.887	0.301
		(0.431)	(0.104)	(0.006)	(0.061)	(0.314)	(0.999)
61.2-79.2	HP	0.757	0.638	0.013	0.872	0.165	1.500
		(0.744)	(0.477)	(0.013)	(0.067)	(0.331)	(0.959)
79.3-03.4	HP	-0.278	-0.804	0.012	0.879	0.293	0.267
		(0.687)	(0.558)	(0.012)	(0.055)	(0.288)	(0.999)

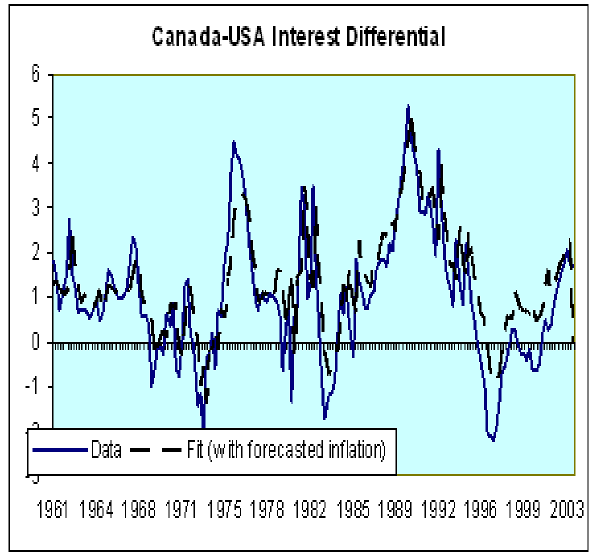
is not particularly high. While the correlation in the levels is 0.45 (and a regression R^2 of 0.998), the 4-quarter and 8-quarter return correlations are negative. At the 16 quarter horizon, the return correlation is 0.22.

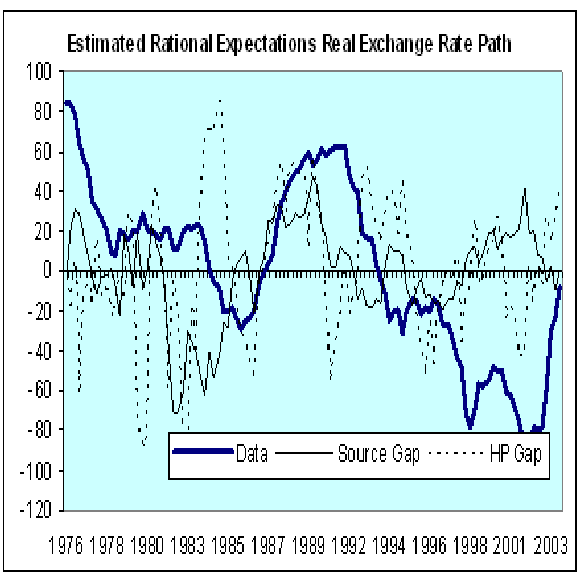
5.4 The Real Dollar-Canadian Dollar Rate

In several respects, the Canadian dollar case (not surprisingly) does not fit the pattern observed for the other three exchange rates. The inflation differential shows a gentle downward trend from 1962 to 1971 then increases substantially during the first oil shock through 1974. The US dollar undergoes a real depreciation from 1962.4 to 1976.2, gains in value through 1986.2, loses value through 1991.4, and gains through 2001.4. The estimated coefficients on the relative policy functions suggest that imposing homogeneity across countries may result in a misspecification.[17] In the pre-1979 period, the Bank of Canada evidently responded more aggressively to changes in inflation (the Bank of Canada's γ_π was larger than the Fed's). In the post-

[17]It turns out that for estimating the least-squares learning path of the real exchange rate that this is not a serious misspecification.


Inflation Differentials and Real Exchange Rate
4
3
2
1
0
-1
-2
-3
-4
-5
Log real exchange rate
Inflation Differential
1961 1964 1968 1971 1975 1978 1982 1985 1989 1992 1996 1999 2003


Canada-USA Interest Differential
6
5
4
3
2
1
0
-1
Data
Fit (with forecasted inflation)
1961 1964 1968 1971 1975 1978 1982 1985 1989 1992 1996 1999 2003


Estimated Rational Expectations Real Exchange Rate Path
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
Data
Source Gap
HP Gap
1976 1978 1980 1983 1985 1987 1989 1992 1994 1996 1998 2001 2003

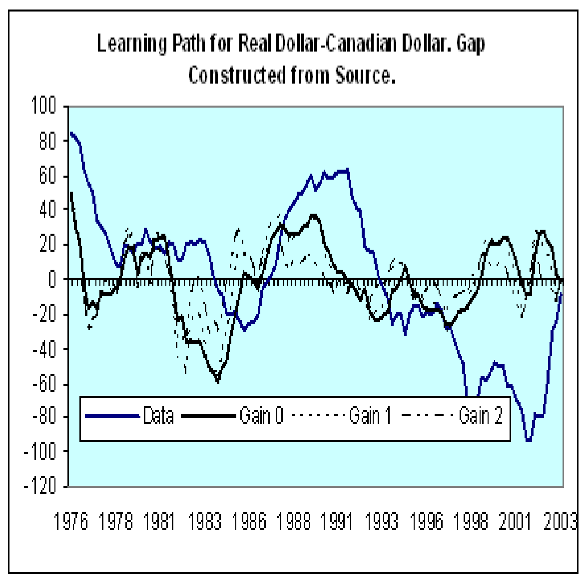

Learning Path for Real Dollar-Canadian Dollar. Gap Constructed from Source.
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
Data
Gain 0
Gain 1
Gain 2
1976 1978 1981 1983 1986 1988 1991 1993 1996 1998 2001 2003

Table 11: CD REE Path with Data

		Regression slope	(s.e.)	ρ	R^2
Source output gap	Level	0.053	(0.172)	0.029	0.871
	1-Qtr	-0.084	(0.063)	-0.126	0.026
Implied Return	4-Qtr	-0.056	(0.090)	-0.061	0.035
Volatility=12.567	8-Qtr	-0.008	(0.117)	-0.007	0.068
	16-Qtr	0.257	(0.119)	0.216	0.162
HP output gap	Level	0.040	(0.111)	0.034	0.871
	1-Qtr	-0.020	(0.027)	-0.073	0.016
Implied Return	4-Qtr	0.016	(0.043)	0.034	0.032
Volatility=29.935	8-Qtr	0.012	(0.054)	0.022	0.068
	16-Qtr	0.038	(0.091)	0.043	0.122

1979 period, the reaction intensity was reversed with the Fed responding more aggressively than the Bank of Canada to changes in inflation. The coefficient on the exchange rate is small and insignificant. While for some purposes, imposing homogeneity of coefficients may be inappropriate, the plots of the fitted values shows that the estimated relative reaction function shows that there is substantial ability for the estimated model to explain the path of the interest differential data.

The return volatility found in the real dollar-Canadian dollar rate is 8.33 percent. The estimated rational expectations path using the source estimates of the output gap generates a bit too much volatility whereas the estimated path using the HP filtered gap estimates generates much too much volatility. Neither set of estimates exhibit very high correlation with the data in levels. At the 16 quarter horizon, there is a 0.22 correlation between changes in the estimated rational expectations real exchange rate and the data.

As in the Japanese case, results for gain specifications 0, 1, and 2 are reported. Here, the results for the implied learning path is somewhat of an improvement over the estimated rational path. The implied return volatilities

Table 12: CD Learning path with data. Source output gap

	gain=0	gain=1	gain=2
level	0.302	0.201	0.149
(s.e.)	(0.179)	(0.176)	(0.271)
ρ	0.158	0.108	0.052
R^2	0.874	0.872	0.871
1-Qtr	0.075	0.051	0.077
(s.e.)	(0.100)	(0.072)	(0.077)
ρ	0.071	0.067	0.095
R^2	0.015	0.015	0.019
4-Qtr	0.158	0.014	-0.044
(s.e.)	(0.100)	(0.081)	(0.102)
ρ	0.150	0.016	-0.041
R^2	0.053	0.031	0.033
8-Qtr	0.230	0.033	-0.212
(s.e.)	(0.112)	(0.110)	(0.163)
ρ	0.199	0.030	-0.127
R^2	0.106	0.069	0.083
16-Qtr	0.357	0.140	-0.324
(s.e.)	(0.114)	(0.121)	(0.212)
ρ	0.307	0.118	-0.155
R^2	0.204	0.132	0.141
Vol	7.961	11.062	10.318

are close to the value of 8.33 found in the data The correlation between the learning real exchange rate and the data is 0.16 in levels, 0.31 at the 16 quarter return horizon, and is positive at the intermediate return horizons considered.