The views expressed in this paper are those of the author(s) only, and the presence of them, or of links to them, on the IMF website does not imply that the IMF, its Executive Board, or its management endorses or shares the views expressed in the paper.
The Marginal Product of Capital

Francesco Caselli and James Feyrer
Is capital efficiently allocated across countries?

Yes, if MPK equalized

No, if MPK differs

This paper’s answer: yes
First pass: one sector model

Constant returns and competitive markets

Capital Income in country $i = MPK^i \times K^i$

Then

$$MPK^i = \frac{\alpha^i Y^i}{K^i}$$

where $\alpha^i$ is measured capital share in income (country specific!)

No functional form assumptions
First two estimates of $MPK$

Using total capital share

$$MPKN^i = \alpha^i_w \frac{Y^i}{K^i}$$

Using reproducible capital share

$$MPKL^i = \alpha^i_k \frac{Y^i}{K^i}$$
Second pass: 2-sector model

International cost of borrowing is $R^*$

Price of output is $P_y^i$

Price of equipment is $P_k^i$

Equipment-investment decision

$$\frac{P_y^i(t) \cdot MPK_y^i(t) + P_k^i(t + 1)(1 - \delta)}{P_k^i(t)} = R^*$$

Abstracting from capital gains

$$\frac{P_y^i \cdot MPK_y^i}{P_k^i} = R^* - (1 - \delta)$$

This rate-of-return equalization condition easily extends to $J$-sectors
Estimates 3 and 4

Total capital share with price correction

\[ PMPKN^i = \frac{\alpha^i_w P^i_y Y^i}{P^i_k K^i} = \frac{P^i_y}{P^i_k} MPKN^i, \]

Reproducible capital share with price correction

\[ PMPKL^i = \frac{\alpha^i_k P^i_y Y^i}{P^i_k K^i} = \frac{P^i_y}{P^i_k} MPKL^i. \]
Data

One cross-section: 1996

$Y$: real per worker, from PWT6.1

$K$: PIM, $\delta = 0.6$, $I$ from PWT6.1

$P_y, P_k$: dollar price of a bundle of final/capital goods. PWT6.1

$\alpha^i_w$: from Bernanke and Gürkaynak (2001), via Gollin (2002). 1-labor share. Various corrections to NA figures to deal with non-corporate sector
Data (cont.): Reproducible capital share

World Bank (2006): estimates of total wealth and its components (based on present value of estimated rents)
Figure 1: The Marginal Product of Capital
Table 1: Average Return to Capital in Poor and Rich Countries

<table>
<thead>
<tr>
<th></th>
<th>Rich Countries</th>
<th>Poor Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPKN</td>
<td>11.4 (2.7)</td>
<td>27.2 (9.0)</td>
</tr>
<tr>
<td>MPKL</td>
<td>7.5 (1.7)</td>
<td>11.9 (6.9)</td>
</tr>
<tr>
<td>PMPKN</td>
<td>12.6 (2.5)</td>
<td>15.7 (5.5)</td>
</tr>
<tr>
<td>PMPKL</td>
<td>8.4 (1.9)</td>
<td>6.9 (3.7)</td>
</tr>
</tbody>
</table>

Deadweight loss calculation

Counterfactual world distribution of $k$ and $y$ if existing $K$ redistributed to equalize MPKs

Answer: very small changes, and trivial world GDP gain
Conclusions

Lucas was right: credit frictions in international markets do not explain why poor countries do not receive capital.

Capital does not flow to poor countries because of low TFP, low human capital, and low $P_y/P_k$. 
Implication for aid policy

Large poor-rich “physical” $MPK$ differentials usually seen as good reason to increase aid flows

But with small poor-rich “financial” $MPK$ differentials increased aid flows likely to be offset by increased private flows in opposite direction
Deadweight loss calculation

Counterfactual world GDP if existing $K$ redistributed to equalize MPKs

(Abstract from changes in aggregate $K$)

(Not a normative exercise!)
Have to assume Cobb-Douglas

\[ Y_{ij} = Z_{ij}^{\beta_{ij}} K_{ij}^{\alpha_i} (X_{ij} L_{ij})^{1-\alpha_i-\beta_{ij}} \]

Domestic and international no arbitrage

\[ \frac{P_{ij}}{P_k} \alpha_i Z_{ij}^{\beta_{ij}} (K_{ij}^*)^{\alpha_i-1} (X_{ij} L_{ij})^{1-\alpha_i-\beta_{ij}} = PMPK^* \]

Dividing last two

\[ K_{ij}^* = \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{1}{1-\alpha_i}} K_{ij} \]

(Hence \( Z_{ij} \), and \( L_{ij} \) indeed unchanged). Aggregating across sectors

\[ K_i^* = \sum_j K_{ij}^* = \sum_j \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{1}{1-\alpha_i}} K_{ij} = \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{1}{1-\alpha_i}} K_i \]

Aggregating across countries

\[ \sum K_i^* = \sum K_i = \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{1}{1-\alpha_i}} K_i. \]

Solve for \( PMPK^* \) and then compute \( K_i^* \) and \( Y_i^* \)
Figure 2:
Table 2: Average Changes in Equilibrium Capital Stocks under $MPK$ Equalization

<table>
<thead>
<tr>
<th></th>
<th>Unweighted</th>
<th>Weighted by Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rich Countries</td>
<td>Poor Countries</td>
</tr>
<tr>
<td>$MPKN$</td>
<td>-12.9%</td>
<td>274.5%</td>
</tr>
<tr>
<td>$MPKL$</td>
<td>-6.2%</td>
<td>86.6%</td>
</tr>
<tr>
<td>$PMPKN$</td>
<td>0.1%</td>
<td>71.8%</td>
</tr>
<tr>
<td>$PMPKL$</td>
<td>0.6%</td>
<td>-10.6%</td>
</tr>
</tbody>
</table>

Notes: see Table 1
\[ Y_{ij}^* = Z_{ij}^{\beta_{ij}} (K_{ij}^*)^{\alpha_i} (X_{ij} L_{ij})^{1-\alpha_i-\beta_{ij}} = \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{\alpha_i}{1-\alpha_i}} Y_{ij} \]

\[ Y_i^* = \left( \frac{PMPK^*}{PMPK_i} \right)^{\frac{\alpha_i}{1-\alpha_i}} Y_i \]
Figure 3:

MPKN

PMPKN

MPKL

PMPKL
Table 3: Average Changes in Equilibrium Output per Worker under $MPK$ Equalization

<table>
<thead>
<tr>
<th></th>
<th>Unweighted</th>
<th>Weighted by Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rich Countries</td>
<td>Poor Countries</td>
</tr>
<tr>
<td>$MPKN$</td>
<td>-3.0%</td>
<td>76.7%</td>
</tr>
<tr>
<td>$MPKL$</td>
<td>-0.7%</td>
<td>16.8%</td>
</tr>
<tr>
<td>$PMPKN$</td>
<td>1.1%</td>
<td>24.7%</td>
</tr>
<tr>
<td>$PMPKL$</td>
<td>0.7%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Notes: see Table 1
Dead Weight Losses

\[ \frac{\sum (Y^* - Y)}{\sum Y}. \]

Table 4: World Output Gain from MPK Equalization

<table>
<thead>
<tr>
<th></th>
<th>No Price Adjustment</th>
<th>With Price Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Natural-Capital Adjustment</td>
<td>2.9%</td>
<td>1.4%</td>
</tr>
<tr>
<td>With Natural-Capital Adjustment</td>
<td>0.6%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Authors’ calculations.
Table 5: Counterfactual $MPK$ under $MPK$ Equalization

<table>
<thead>
<tr>
<th></th>
<th>No Price Adjustment</th>
<th>With Price Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Natural-Capital Adjustment</td>
<td>12.7%</td>
<td>12.8%</td>
</tr>
<tr>
<td>With Natural-Capital Adjustment</td>
<td>8.0%</td>
<td>8.6%</td>
</tr>
</tbody>
</table>

Authors’ calculations.
\( k_i^* = \Pi_i \times \Lambda_i, \)

where

\[
\Pi_i = \left( \frac{\alpha_i}{PMPK^* P_{k,i}} \right)^{\frac{1}{1-\alpha_i}},
\]

and

\[
\Lambda_i = \left[ z_i^{\beta_i} (X_i)^{1-\alpha_i-\beta_i} \right]^{\frac{1}{1-\alpha_i}},
\]

\[
\text{var} \left[ \log(k^*) \right] = \text{var} \left[ \log(\Pi) \right] + \text{var} \left[ \log(\Lambda) \right] + 2 \times \text{cov} \left[ \log(\Pi), \log(\Lambda) \right]
\]

Splitting covariance, contributions are 54% and 46%
Theories of $P_y/P_k$

Taxes on capital purchases (e.g. Chari et al.).

Relative productivity of investment sector (e.g. Hsieh and Klenow).
Time Series Results
Figure 4: The Dead Weight Loss of $MPK$ Differentials

Notes: see Figure 1