Macroeconomic Interdependence and the International Role of the Dollar

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Abstract

The U.S. dollar holds a dominant place in the invoicing of international trade. Not only are most U.S. exports and imports invoiced in dollars, the currency plays a significant role for trade flows that do not involve the United States. Likewise euros are used on trade transactions with the European periphery and between those periphery countries. We analyze how this second dimension of the international role of a currency, in which it serves as a vehicle currency for international trade, impacts global interdependence and monetary policy. Using a simple center-periphery model, we show that the prevalence of a vehicle currency magnifies the exposure of periphery countries to the center’s monetary policy, even when direct trade flows between these countries and the center are limited. Our results indicate that the invoicing of intra-periphery trade in a vehicle currency can generate inefficient and costly fluctuations in relative prices.

Keywords: exchange rate, pass-through, center-periphery, invoicing, center, periphery, monetary policy.

JEL codes: F41, F42

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1 Introduction

The prominent role of the U.S. dollar is a major feature of the global economy. In addition of its central position in international reserve holdings and financial markets, the dollar is extensively used in the currency of invoicing in international trade flows. Its role as an invoicing currency in international trade encompasses two dimensions exposted in Goldberg and Tille (2005). The first relates to trade flows to and from the United States, which are overwhelmingly invoiced in dollars. The second dimension is the sizable use of the dollar in trade flows for which the United States is neither the origin nor the destination country, known as the vehicle currency role of the dollar. The euro also plays both of these roles, although mainly with countries that are directly peripheral to the euro area.

This paper analyzes how these two dimensions of the international role of a vehicle currency affect the international transmission of shocks and policy. The literature recognizes the pattern of international trade invoicing to be a central component of international interdependence, as pricing decisions made by producers drive the extent to which exchange rate fluctuations are transmitted to import prices (Corsetti and Pesenti 2005a, Engel and Devereux 2003, Obstfeld and Rogoff 2002). When exporters set their prices in their own currency, exchange rate movements are fully passed-through to the prices paid by consumers, leading to expenditure-switching in consumption between goods produced in various countries. By contrast, if exporters set their prices in the currency of their customers, exchange rate pass-through to consumer prices does not occur and consumers do not experience the relative price movements that would otherwise induce expenditure switching. The design of optimal monetary policy is substantially affected by the (in)ability of exchange rate movements to impact consumer prices.

While most contributions to this literature on optimal monetary policy design assume the degree of exchange rate pass-through is the same for all trade partners, i.e. symmetric, several contributions have explored an asymmetric pass-through. Evidence of such asymmetry abounds between the U.S. and the Euro area countries with their respective trade partners as documented by Campa and Goldberg (2005), Faruque (2006), and Ihrig et al (2006). Corsetti and Pesenti (2005a,b) and Devereux, Shi, and Xu (2006) analyze two-country models where there is full exchange rate pass-through for exports originating in the home country (the U.S.) and no pass-through for trade flows originating in the foreign country (the rest of the world). By focusing on a two-country environment, the existing results on optimal monetary policy apply to direct macroeconomic interdependence between these partners, which is the first dimension of the international role of a currency. Optimal monetary policy under asymmetric pricing has been demonstrated
to differ noticeably from under symmetric pricing.

Our theoretical analysis goes beyond the existing contributions on macroeconomic interdependence by focusing on the importance of the second dimension of the international role of a currency, namely its use in trade flows that do not directly involve the country whose currency is used for invoicing purposes. We tackle this issue by introducing a general equilibrium three-country model with a center country, such as the United States, and two periphery countries. The model is enriched by allowing for home bias in consumption between the center and the periphery: the consumption basket of agents in the center country is tilted toward goods produced in the center, while the baskets of agents in either periphery country are tilted towards goods produced in the collective periphery. Under the extreme case of complete home bias, the center and the periphery are fully disconnected in terms of direct trade flows.

The main results of our analysis are that utilizing a vehicle currency in intra-periphery trade has significant effects that extend beyond its role in center-periphery trade. First, consumption in the periphery is more sensitive to monetary policy in the center, both compared to cases of symmetric pass-through and compared to cases where the international role of the dollar is limited to invoicing transactions between the center and periphery. Second, spillovers from the center occur even when the center and the periphery do not engage in direct trades with each other. Third, vehicle currency use in intra-periphery trade can lead to sizable welfare losses for periphery countries by generating inefficient fluctuations in relative prices. We find that the gains from cooperation are largest for the countries with the most volatile shocks.

Our emphasis on the international role of the dollar in intra-periphery trade is consistent with the insights of Cook and Devereux (2006). They consider a partial equilibrium model where the center is taken as exogenous, and apply it to the East Asian crisis of 1997-1998. Their results point to the role of the dollar in intra-Asia trade as a central feature in accounting for the magnitude and persistence of the crisis.

The paper is organized as follows. Section 2 presents empirical evidence on the international role of the dollar and euro in international trade transactions. Section 3 presents a simple center-periphery model. Section 4 explores the design of optimal monetary policy in a stochastic setup, with a numerical illustration of the main results. Section 6 concludes.
2 Evidence on vehicle currency use in international trade

Our focus on vehicle currency use in international trade is highly relevant given the documented international role of the dollar and the emerging role of the euro. The extensive international roles of the dollar and euro are demonstrated in Tables 1 and 2 which present data on invoicing from Goldberg and Tille (2005) and ECB publications, and on international trade transactions.

To illustrate the first dimension of the international role of the dollar, focus on columns (1), (2), (4) and (5) of Table 1. Columns (1) and (4) show the share of country exports and imports, respectively, that are invoiced in U.S. dollars. Columns (2) and (5) show the share of the country’s trade that is bilateral with the United States. Looking across countries, the use of the dollar in invoicing goes well beyond the role of the United States as a direct trade counterparty. Columns (3) and (6) show the share of “dollar bloc” countries in bilateral trade transactions. The vehicle currency role of the dollar is especially striking for Asian countries: more than 80 percent of the exports of Korea, Malaysia and Thailand are invoiced in dollars, while the United States accounts for at most one-fifth of these countries exports. Figure 1 illustrates the prominent role of the dollar by contrasting its use as an invoicing currency (vertical axis) against the role of the United States as a trading partner (horizontal axis) for various countries. The figure clearly shows that the use of the dollar goes well beyond trade flows that involve the United States. The second dimension of the role of the dollar is therefore strong, with the dollar being used to a sizable extent in the invoicing of trade flows that do not involve the United States. Cook and Devereux (2006) similarly emphasize the role of the dollar in the invoicing of trade between Asian countries.

Table 2 on the international role of the euro shows strong differences across regions in currency use. Asian economies seldom use euros for invoicing export or import transactions. Country proximity to the euro area plays a substantial role in explaining the use of euros in international trade transactions, as does whether a country has pending goals of joining the euro area [Goldberg and Tille (2006), Goldberg (2007), ECB (2006), Kamps (2006)]. For these countries, trade with the center and other periphery countries are largely conducted in euros. This point is illustrated in Figure 2, which show accession country the use of euros in invoicing export or import transactions plotted against the share of that country’s trade going to the euro area and other euro-bloc countries. The placement of the points along the diagonal shows that these periphery countries are mainly using the euro on trade with the center and trade with the rest of the periphery.
3 A simple center-periphery model

3.1 Geographical structure and timing

For establishing the role of vehicle currency use in macroeconomic interdependence, we use a three-country variant of the workhorse 'new open economy macroeconomics’ model introduced by Obstfeld and Rogoff (1995). As we build on a workhorse setup in the literature, our exposition focuses on our novel elements and the corresponding intuitive interpretations. A detailed exposition of the technical steps is found in an Appendix available on request.

The world is comprised of three countries: A, B and C. Country A represents a "center" country, while countries B and C are "periphery" countries. In terms of country size, the center country A and the overall periphery represent half the world, of which the overall size is normalized to unity, and the two periphery countries B and C are of equal size, each accounting for a quarter of the world. There is a continuum of differentiated brands available for consumption, indexed along a unit interval. Firms in country A produce brands on the $0 - 0.5$ interval, firms in country B produce brands on the $0.5 - 0.75$ interval, and firms in country C produce brands on the $0.75 - 1$ interval.

Each country is inhabited by a representative consumer who purchases all brands available in the world economy. In terms of notation, consumption levels are indexed with a subscript for the country where consumption takes place, and a superscript for the country where the good is produced. Specifically, $C_i^j(z)$ is the consumption in country $i$ of the brand $z$ produced in country $j$. Individual brands are aggregated into indexes, as detailed below, and $C_i^j$ is the consumption in country $i$ of the index of all brands produced in country $j$. The indexes themselves are aggregated further into the overall consumption, with $C_i$ being the overall consumption index in country $i$.

The prices of the various goods are indexes along similar lines. $P_i^j(z)$ is the price paid by the consumers in country $i$ for each unit of brand $z$ produced in country $j$. The prices of the various brands produced in a given country are aggregated into a country-of-origin price index, with $P_i^j$ being the price index charged in country $i$ for the brands produced in country $j$. These indexes are in turn aggregated in the overall consumer price index $P_i$. Prices are expressed in the currency of the country where the goods are consumed, namely $i$.

We consider a one-period stochastic model, with some decisions taken before shocks are realized and other taken after. The firms that produce goods set their prices at the beginning of the period. The various shocks then occur, and the monetary authorities react to them, leading to movements in
exchange rate and, possibly, import prices. Consumption and production take place. The ex-post output is demand-driven, with firms meeting the demand they face at their preset prices. While prices are set before the realization of shocks, this is done through forward-looking optimization with firms ex ante knowing the distribution of shocks and the rules followed by the monetary authorities. While considering this type of static model can appear restrictive, the functional forms we consider imply a dynamic version of the model boils down to a succession of one-period models (Corsetti and Pesenti 2005a).\footnote{Our assumption of a unit elasticity of substitution between goods produced in different countries, along with a log utility of consumption, always ensures full-risk sharing.}

### 3.2 Consumption allocation

While all goods are traded, we allow for home bias in consumption between the center and periphery goods. Specifically, the representative consumer in country $A$ allocates her overall consumption across the various brands to maximize the following index:

$$C_A = (\alpha)^{-\alpha} \left(\frac{1 - \alpha}{2}\right)^{-(1-\alpha)} (C_A^A)^{\alpha} (C_A^B C_A^C)^{1-\alpha}$$

(1)

The elasticity of substitution between goods produced in different countries is set at one. The sub-index by country of origin are given by:

$$C_A^A = \left[\frac{\lambda - 1}{\lambda} \int_0^{0.5} (C_A^A(z))^{\frac{\lambda - 1}{\lambda}} dz\right]^\frac{1}{\lambda - 1}$$

$$C_A^B = \left[\frac{\lambda - 1}{\lambda} \int_{0.5}^{0.75} (C_A^B(z))^{\frac{\lambda - 1}{\lambda}} dz\right]^\frac{1}{\lambda - 1}$$

$$C_A^C = \left[\frac{\lambda - 1}{\lambda} \int_{0.75}^{1} (C_A^C(z))^{\frac{\lambda - 1}{\lambda}} dz\right]^\frac{1}{\lambda - 1}$$

$\lambda > 1$ is the elasticity of substitution between brands produced in the same country. Similarly, the representative consumer in country $B$ and $C$ allocates her consumption across the various brands to maximize:

$$C_i = (1 - \alpha)^{-(1-\alpha)} \left(\frac{\alpha}{2}\right)^{-\alpha} (C_i^A)^{1-\alpha} (C_i^B C_i^C)^{\frac{\alpha}{2}} \quad i = B, C$$

(2)

The coefficient $\alpha \in [0.5, 1]$ in (1)-(2) reflects the degree of home bias, in terms of periphery vs. center goods, and allows us to vary the degree of integration between the center and the periphery. One extreme corresponds
to a fully integrated world with no home bias ($\alpha = 0.5$). The other extreme corresponds to a disconnected world with no trade between the center and the periphery ($\alpha = 1$). The home bias is defined solely in terms of center vs. periphery, and there is no corresponding bias between goods produced in the periphery.

The allocation of consumption is computed following the usual steps and reflects the relative prices. For instance, the allocation of purchases by the consumer in country $A$ is:

$$C^A_A(z) = 2\alpha \left[ \frac{P^A_A(z)}{P^A_A} \right]^{-\lambda} \left[ \frac{P^A_A}{P^A_A} \right]^{-1} C_A$$

$$C^j_A(z) = 2(1 - \alpha) \left[ \frac{P^j_A(z)}{P^j_A} \right]^{-\lambda} \left[ \frac{P^j_A}{P^A_A} \right]^{-1} C_A \quad j = B, C$$

The price indexes represent the minimal expenditure required to purchase one unit of the corresponding index. In particular, the consumer price index is:

$$P_A = (P^A_A)^{\alpha} (P^B_A P^C_A)^{1-\alpha}$$

The allocation of consumption in country $B$ and $C$ is computed along similar lines, with the consumer price index in country $i = B, C$ being:

$$P_i = (P^A_i)^{1-\alpha} (P^B_i P^C_i)^{\alpha/2}$$

### 3.3 Money and effort

The consumer in country $i$ maximizes a simple utility over consumption, real balances and hours worked:

$$U_i = E \left[ \ln (C_i) + \chi \ln \left( \frac{M_i}{P_i} \right) - \kappa H_i \right] \quad i = A, B, C$$

where $E$ denotes the expectation operator, from the point of view of the beginning of the period. $C_i$ is the aggregate consumption index, $M_i/P_i$ denotes the real money balances and $H_i$ denotes the hours worked by the consumer. $\chi$ and $\kappa$ are scaling parameters. The simple functional form in (5) allows us to derive our results with the minimal amount of technical complexity. The budget constraint faced by the consumer in country $i$ is:

$$P_i C_i + M_i = \Pi_i + W_i H_i - T_i$$

where $\Pi_i$ denotes the profits of the firms in country $i$, which are owned by the local consumer, $W_i$ is the wage rate, and $T_i$ is a lump-sum tax paid to
the government of country $i$.\footnote{Without loss of generality we assume that initial cash holdings are zero.} The first-order conditions with respect to real balances and hours worked lead to the money demand and labor supply:

$$M_i = \chi P_i C_i \quad \quad W_i = \kappa P_i C_i = \frac{\kappa}{\chi} M_i \quad \quad \quad \quad (7)$$

### 3.4 Structure of pricing

As firms set their prices before the realization of shocks and the associated response by monetary policy, the currency in which prices are set plays a central role. Prices for sales to foreign countries can be set in different currencies, implying different sensitivity of the import prices paid by consumers to exchange rate movements, the so-called exchange rate pass-through. Our paper focuses on how alternatives pattern of trade invoicing alters the transmission of monetary policy and its optimal design. Throughout the paper we take the pattern of invoicing to be set exogenously. While a growing literature has focused on the determinants of invoicing (Bacchetta and van Wincoop (2005), Devereux, Engel and Storegaard (2004), Goldberg and Tille (2005)) the models considered go beyond our simple setup. For instance, Goldberg and Tille (2005) point to the key role of decreasing returns to scale in generating a concern by firms for demand volatility. In the present paper we instead consider a constant return to scale technology to keep the technical complexity to a minimum. Encompassing endogenous invoicing choice in our analysis would require a richer model, a step that we leave for future research.

Firms set the price for domestic sales in the domestic currency, but prices for sales abroad can be set in different currencies. A firm located in country $j$ sets a price $\tilde{P}^j_j(z)$ in its own currency for domestic sales. Its exports are invoiced in a basket of the three available currencies, with the weight of each being in the $[0, 1]$ interval. The weights are denoted by $\gamma$ with a subscript indicating the country of destination, as well as superscripts indicating the country of production and the currency of invoicing. Specifically $\gamma_{i;\text{cur } k}^j$ is the share of currency $k$ in the invoicing of exports from country $j$ to country $i$. These exogenous invoicing weights are the same for all firms in the exporting country.

The pricing choice for the firm producing brand $z$ in country $j$ and exporting to country $i$ consists of choosing a price $\tilde{P}^j_i(z)$ such that the price paid by the importing consumer in her own currency, $i$, is:

$$P^j_i(z) = \tilde{P}^j_i(z) \sum_{k=A,B,C} \left( \frac{S_k}{S_i} \right) \gamma_{i;\text{cur } k}^j = \tilde{P}^j_i(z) \left( S_i \right)^{-1} (S_B)^{\gamma_{i;\text{cur } B}^j} (S_C)^{\gamma_{i;\text{cur } C}^j} \quad \quad (8)$$
where $S_i$ is the exchange rate between currency $A$ and currency $i$. It is expressed as the amount of currency $A$ per unit of currency $i$, so an increase corresponds to a bilateral depreciation of currency $A$. The exchange rate between currency $i$ and currency $k$, in terms of the amount of currency $i$ per unit of currency $k$, is then given by $S_k/S_i$. The case of producer currency pricing (PCP) corresponds to $\gamma_{i,\text{cur}}^j = 1$, while the case of local currency pricing (LCP) corresponds to $\gamma_{i,\text{cur}}^i = 1$. Pricing in a vehicle currency (VCP) corresponds to $\gamma_{i,\text{cur}}^j = \gamma_{i,\text{cur}}^i = 0$.

For brevity, we focus on five corner cases of invoicing, as illustrated by Figure 3. For each case Figure 3 depicts the trade flows between the various countries along with the currency used in the invoicing of the specific flows (for instance a label $C$ on the arrow from country $C$ to country $A$ indicates that exports from $C$ to $A$ are invoiced in the currency of country $C$). Case 1 and Case 2 are fully symmetric invoicing behavior as applied to all trade flows. In Case 1, referred to as PCP-SYM, producer currency pricing applies to all trade flows, symmetrically across countries. This scenario is characterized by complete exchange rate pass-through to all import prices. In Case 2, referred to as LCP-SYM, local currency pricing applies to all trade flows. In this scenario there is no exchange rate pass-through and all import prices are fully insulated from exchange rate movements.

The next three cases constrain all trade transactions involving the center country $A$ to be invoiced in the center’s currency. In terms of notations, all of these cases start with DOL-, but differ along the dimension of invoicing the trade flows that pass between the two periphery countries, $B$ and $C$. DOL- means that there is complete pass-through of exchange rate movements to the consumer prices for goods sold by country $A$ to the periphery, but no pass-through to consumer prices for imports from the periphery in country $A$. In the DOL-PCP case, intra-periphery trade flows are invoiced in producer currency, and there is complete bilateral exchange rate pass-through within the periphery. In the DOL-LCP case, intra-periphery trade flows are invoiced in the currencies of the respective consumers and there is no bilateral exchange rate pass-through. The last case, DOL-DOL, captures the international role of currency $A$ along the second dimension discussed in the introduction. In that case all trade flows worldwide, including intra-periphery flows, are invoiced in currency $A$. In particular, this implies that exchange rate fluctuations between currency $A$ and either of the periphery currencies affect the price of intra-periphery imports relative to the local goods in the periphery.
3.5 Technology and output

Firms use a simple technology with constant returns to scale over labor hours worked in production of good \( z \), \( H_i(z) \):

\[
Y_i(z) = K_i H_i(z) \quad i = A, B, C \tag{9}
\]

The country-wide productivity terms \( K \)'s are subject to random shocks, and firms set their prices before the realization of these shocks. The demands faced by the various firms are computed by aggregating the allocation of consumption derived above across the various agents. Using the pricing structure detailed above, the output of a firm producing brand \( z \) in country \( A \) is equated to demand by consumers in \( A, B, \) and \( C \):

\[
Y_A(z) = \alpha \left[ \tilde{P}_A(z) \right]^{-\lambda} (P_A^A)^{\lambda-1} P_A C_A
\]

\[
+ \frac{1 - \alpha}{2} \left[ \tilde{P}_B(z) (S_B)^{\gamma^A_{\text{cur}, B}} (S_C)^{\gamma^A_{\text{cur}, C}} \right]^{-\lambda} (P_B^A)^{\lambda-1} P_B C_B
\]

\[
+ \frac{1 - \alpha}{2} \left[ \tilde{P}_C(z) (S_B)^{\gamma^A_{\text{cur}, B}} (S_C)^{\gamma^A_{\text{cur}, C}} \right]^{-\lambda} (P_C^A)^{\lambda-1} P_C C_C \tag{10}
\]

The demands faced by firms in country \( B \) and \( C \) are computed similarly. In equilibrium all firms in a given country are identical. We can then drop the \( z \) index and write (10) in terms of per-capita output:

\[
Y_A = \alpha \frac{P_A C_A}{P_A^A} + \frac{1 - \alpha}{2} \left[ \frac{P_B C_B}{P_B^A} + \frac{P_C C_C}{P_C^A} \right] \tag{11}
\]

4 Solution of the model

4.1 Exchange rates

We abstract from government spending and assume that the seigniorage income from monetary creation is repaid to the domestic households as lump sum income. As in a strict monetary approach to exchange rate determination, exchange rates reflect the ratios of the monetary stances adjusted for money demand shocks, regardless of the structure of invoicing:

\[
S_B = \frac{M_A}{M_B} \quad ; \quad S_C = \frac{M_A}{M_C} \tag{12}
\]

Equation (12) show that exchange rates are fully determined by the relative monetary stances, a feature that is common to the various contributions in the literature. As a result, the volatility of exchange rate fluctuations that the model generates is well below the one observed in the data. This shortcoming does not necessarily alter our results however. We could include shocks
to the money demand to which the central bank cannot react, interpreting these fluctuations as financial market shocks. The exchange rate is then also affected by these financial shocks and can display a much larger volatility. Still, the optimal monetary response to productivity shocks is not affected and all our results go through, as long as productivity and financial shocks are uncorrelated.³

4.2 The flexible price allocation

A useful benchmark is given by the situation where goods prices are fully flexible. If firms can adjust their prices following the realization of shocks and the response by monetary authorities, they set them as a constant markup over marginal cost, which is the wage adjusted for productivity. Using the labor supply (7) the price set by a firm in country $j$ for sales to country $i$ is expressed as follows, in terms of country $j$ currency:

$$P^j_i = \frac{\lambda \kappa}{\lambda - \frac{1}{\lambda}} \frac{1}{M_j K_j}$$

(13) shows that the law of one price holds, as a given good sells for the same price in any country. This price reflects the ratio between the monetary stance in country $i$ and productivity.

The ability of firms to reset prices implies that money demand shocks have no real effects and productivity shocks in a country affect output in that country one-for-one, with no impact on hours worked. Consumptions are driven by weighted averages of productivity shocks, with the weights corresponding to the shares of the various goods in the consumption baskets (1)-(2). Without loss of generality, we assume that productivity shocks are log-normal, with mean zero. Abstracting from the direct impact of real balances on utility, the welfare (5) is the same in all three countries and reflects structural parameters:

$$U_{i, \text{flexible prices}} = E \left[ \ln \left( C_i \right) - \kappa H_i \right] = \Phi$$

(14)

where $\Phi = \ln \left( \frac{\lambda - 1}{\lambda \kappa} \right) - \frac{\lambda - 1}{\lambda}$.

4.3 Optimal price setting

When prices have to be set in advance, a firm in country $j$ sets its prices in order to maximize the expected discounted value of its profits. As all firms are domestically owned, the discount factor is the marginal utility of income

³If the monetary authorities can observe the financial shock, the optimal response is the to fully offset them and they do not enter the model.
in country $j$. Using the pass-through structure (8), the labor supply (7) and the solution for the exchange rate (12), the home country price set by a firm in country $j$ for sales to country $i$ is written as:

$$
\tilde{P}^j_i = \frac{\lambda \kappa}{\lambda - 1} E \left( \frac{1}{K_j} (M_A)^{\gamma^i_{j, \text{cur} A}} (M_B)^{\gamma^i_{j, \text{cur} B}} (M_C)^{\gamma^i_{j, \text{cur} C}} \right) \tag{15}
$$

$$
\gamma^i_{j, \text{cur} j} = 1 \quad \gamma^i_{j, \text{cur} k \neq j} = 0
$$

The optimal preset price (15) is similar to the optimal flexible price (13), with important differences. Prices are again set as a markup over marginal cost, given by a ratio between monetary stances and productivity in the country where the goods are made. (15) shows that the markup is over the expected marginal cost, as opposed to its realized value in (13). In addition, the marginal cost in (15) reflects a weighted average of the monetary stances in all countries, reflecting their role in the invoicing of trade, while only the domestic monetary stance matters in (13). The later point of course does not apply to domestic sales which are fully invoiced in the domestic currency, as shown in the second row of (15).

### 4.4 The prominent role of the center

Our first step is to compute the impact of monetary policies of each country on consumption. We compute the consumption levels from the money demand (7), the consumer price indexes (3)-(4), the pass-through structure (8) and the solution for the exchange rate (12). The resulting consumption in country $i$ takes the following form:

$$
C_i = \Theta_i (M_A)^{\xi_A} (M_B)^{\xi_B} (M_C)^{\xi_C} \tag{16}
$$

where the $\xi^i$s are coefficients that reflect the pattern of invoicing. The term $\Theta_i$ reflects the variables that are predetermined at time of the shocks, and is not affected ex-post by the actual realization of shocks and monetary stances. It is important to bear in mind that $\Theta_i$ is affected by monetary policy in ex-ante terms, as it includes the preset prices $\tilde{P}^j_i$, which are affected by the exact rule followed by the monetary authorities. (16) shows the sensitivity of consumption to ex-post realizations of the monetary stances and the shocks.

Several points emerge. First, the impact of monetary policy in country $i$ on consumption in country $j$ (the $\xi^i$s) reflects the extent to which consumer
prices in country $i$ are invoiced in currency $j$.\footnote{For instance, we can show that the coefficients for consumption in country $B$ are:}

This implies that the impact of monetary policy in country $A$ on consumption in the periphery reflects the use of currency $A$ in invoicing of international trade. Second, productivity shocks have no direct ex-post impact on consumption. While they affect the marginal costs of firms, the firms are prevented from adjusting their prices in response, hence the productivity shocks do not affect real balances and consumption. This does not mean that they are irrelevant, but that their impact operates through the levels at which prices $\bar{P}_j$ are set, which enters the $\Theta$ terms.

A worldwide measure of consumption can be computed as a weighted average of (16) with the weights reflecting the size of the various countries:

$$C_W = (C_A)^{0.5} (C_B C_C)^{0.25}$$

Aggregate measures of $\Theta_W$ and $M_W$ are constructed along similar lines. In the symmetric invoicing cases PCP-SYM and LCP-SYM, the impact of the monetary stance in each country simply reflects its size: $C_W = \Theta_W M_W$. By contrast, the prominence of currency $A$ in trade invoicing in the DOL- cases leads the monetary stance in the center to have a disproportionate impact on world consumption. In the DOL-PCP and DOL-LCP cases, we write:

$$C_W = \Theta_W M_W \left[ \frac{M_A}{(M_B)^{0.5} (M_C)^{0.5}} \right]^{1-\alpha/2}$$

A monetary expansion in country $A$ boosts worldwide consumption by a factor that exceeds the size of the center country in the world economy. Conversely, the monetary stances in periphery countries have a relatively small impact. The asymmetry is more pronounced in the DOL-DOL case:

$$C_W = \Theta_W M_W \left[ \frac{M_A}{(M_B)^{0.5} (M_C)^{0.5}} \right]^{2-\alpha/4}$$

The prominent impact of monetary policy in the center is illustrated in Figure 4 which shows the the impact of a 1 percent increase in $M_A$ on world-wide consumption, $C_W$, in percent, depending on the degree of integration, $\alpha$. Under PCP-SYM and LCP-SYM the increase in consumption reflects
the size of country $A$ regardless of the degree of integration. By contrast, worldwide consumption increases by more in all DOL-cases, especially when intra-periphery trade flows are invoiced in currency $A$ (DOL-DOL). In the later case, monetary policy in the center disproportionately boosts worldwide consumption even when there are no trade flows between the center and the pariphery ($\alpha = 1$).

5 Optimal monetary policy

5.1 Welfare and the impact of monetary policy

The goal of monetary policy is to maximize some combination of the welfare of the representative agents in the various countries, given by (5). We take the standard approach of ignoring the small direct impact of real balances on welfare and focusing on consumption and hours:

$$U_i = E \ln (C_i) - \kappa EH_i$$

(17)

Under our specification, expected hours worked boil down to a simple function of the structural parameters of the economy, regardless of the structure of invoicing, a well-known feature of such models (Corsetti and Pesenti 2005a): $\kappa EH_i = (\lambda - 1) / \lambda$. The welfare (17) can then be assessed by focusing on the consumption component. The welfare of agent in country $i$ is given by taking the expected value of the log of (16), and explicitly writing the preset prices in $\Theta_i$ by using the optimal pricing rule (15). The key element is that the preset prices are affected by the expected monetary stances, as shown by (15).

The first step towards setting the optimal monetary stance is to compute the marginal impact of monetary policy in a given state of nature $s$ on the expected log of consumption. The resulting derivatives can be expressed in terms of log-linear approximations around a steady state where productivity and velocity are constant. Expressing log deviations by San-Serif variables, the marginal impact of monetary stance in country $A$ in state $s$ on the expected log consumption in country $A$ is written as:

$$\frac{\partial E \ln (C_A)}{\pi_s \partial M_{A,s}} = -\alpha (m_{A,s} - k_{A,s})$$

$$-\frac{1 - \alpha}{2} \gamma_A B, \operatorname{cur} A \left[ \gamma_A B, \operatorname{cur} A m_{A,s} + \gamma_A B, \operatorname{cur} B m_{B,s} + \gamma_A B, \operatorname{cur} C m_{C,s} - k_{B,s} \right]$$

$$-\frac{1 - \alpha}{2} \gamma_A C, \operatorname{cur} A \left[ \gamma_A C, \operatorname{cur} A m_{A,s} + \gamma_A C, \operatorname{cur} B m_{B,s} + \gamma_A C, \operatorname{cur} C m_{C,s} - k_{C,s} \right]$$
where $\pi_s$ is the probability of state $s$ being realized. Similar expressions can be derived for the marginal impact of the monetary stance in any country on the expected log consumption in any country.

The optimal monetary policy is computed by setting some combination of these marginal impacts to zero, with different objectives translating into different combinations as detailed below. This gives a log linear relation between the monetary stance and the various shocks, that we refer to as a policy rule. Our analysis focuses on the design of optimal rule and we abstract from the issue of discretionary policy. As the expected shocks are zero ($E k_t = 0$), the expected log-deviation of the monetary stance is also zero ($Em_t = 0$).\(^5\)

Using the forward looking prices (15), the welfare in the various countries can be written in terms of the variances of the monetary stances and shocks, as well as the invoicing structure. For instance, the welfare in country $A$ is:

$$
\hat{U}_A = \frac{1}{2} \text{Var}[m_A - k_A] - \frac{1}{2} \text{Var}[\gamma_A^B \text{cur} A m_A + \gamma_A^B \text{cur} B m_B + \gamma_A^C \text{cur} C m_C - k_B]
$$

$$
= \frac{1}{2} \text{Var}[\gamma_A^C \text{cur} A m_A + \gamma_A^C \text{cur} B m_B + \gamma_A^C \text{cur} C m_C - k_C]
$$

(18)

where \text{Var} denotes the variance. $\hat{U}_A$ is the difference of the welfare from its value under flexible prices (14), with $\hat{U}_A = 0$ indicating that the welfare under preset prices corresponds to the level under flexible prices. The final step in assessing the welfare consists of substituting the monetary stances by using the policy rules.

5.2 Optimal monetary policy in a decentralized setting

5.2.1 Monetary rules

We first consider a decentralized Nash equilibrium where each monetary authority focuses on maximizing the welfare of its own residents only, and ignores any impact on the welfare of residents in other countries. The policy stances in state $s$ are then set to satisfy the following first-order conditions:

$$
\frac{\partial E \ln (C_A)}{\pi_s \partial M_{A,s}} = \frac{\partial E \ln (C_B)}{\pi_s \partial M_{B,s}} = \frac{\partial E \ln (C_C)}{\pi_s \partial M_{C,s}} = 0
$$

This gives a linear system of three equations in three unknowns, $m_{i,s}$ for $i = A, B, C$ and three exogenous productivity shocks.\(^6\) For convenience, we define

\(^5\)This result also follows in a specification that permits exchange rate volatility as arising from volatile money demand

\(^6\)If we had money demand shocks that led to exchange rate volatility, the optimal policy would call for a full offset regardless of the invoicing structure: when the monetary
the following periphery-wide measure of shocks:

\[ k_{P,s} = \frac{(k_{B,s} + k_{C,s})}{2} \]

Consider the optimal response to productivity shocks. Under PCP-SYM the optimal policy fully focused on domestic shocks, as described in Obstfeld and Rogoff (2002):

\[ m_{i,s} = k_{i,s} \quad i = A, B, C \quad (19) \]

Under LCP-SYM the optimal policy react to a weighted average of shocks, with the weights reflecting home bias:

\[
\begin{align*}
    m_{A,s} &= \alpha k_{A,s} + (1 - \alpha) k_{P,s} \\
    m_{B,s} &= m_{C,s} = (1 - \alpha) k_{A,s} + \alpha k_{P,s}
\end{align*}
\]

If the center and the periphery are fully integrated (\( \alpha = 1 \)), monetary policy in each country reacts to the worldwide average of shocks.

Under any of the DOL- cases, the optimal policy in country \( A \) reacts to a weighted average of shocks, exactly as in the LCP-SYM case. Turning to the periphery countries, the optimal policy is focused on domestic shocks under both the DOL-PCP and the DOL-DOL cases:

\[ m_{i,s} = k_{i,s} \quad i = B, C \quad (21) \]

Under DOL-LCP, monetary policy in the periphery reacts only to the average periphery shock:

\[ m_{B,s} = m_{C,s} = k_{P,s} \quad (22) \]

Notice that if \( k_{B,s} = k_{C,s} \) the DOL- setups are the same, and monetary policy in the periphery follows the shock in the periphery. This corresponds to a two-country center-periphery version of the model.

The optimal policy is asymmetric in the DOL- cases, as the monetary stances in the periphery are never affected by shocks in the center, while the center’s policy reacts to periphery shocks, an aspect that can be found in Corsetti and Pesenti (2005a,b) and Devereux, Shi, and Xu (2006). Intuitively, the preset component of exports from country \( A \) in a DOL- case is always insulated from monetary policy in the periphery. From (15) we write:

\[ \tilde{P}_B^A = \tilde{P}_C^A \times E(M_A/K_A) \]

authorities can observe them, as they can simply offset the shocks one-for-one. If the money demand shocks occur once policy has already been set, they would have no implication for the conduct of policy as long as they are uncorrelated with the observed productivity shocks, which we assume to be the case. If the two types of shocks were correlated, the monetary authorities would react to productivity shocks not only for their own sake, but also due to the fact that they provide information on money demand shocks.
where $\propto$ denotes a proportionality. The monetary authorities in country $B$ then have no leeway on these prices, and focus on shocks on the periphery as this affects the preset prices of domestic goods and imports from the other periphery country.

By contrast, in the center country $A$ the monetary stance affects import prices:

$$\hat{P}_A^B \propto E(M_A/K_B) \quad \hat{P}_A^C \propto E(M_A/K_C)$$

Policy then reacts to a combination of shocks in the periphery, to lower import prices, and shocks in the center, to lower the price of domestic goods.

### 5.2.2 Exchange rate volatility

The optimal monetary policy in the various cases can be concisely illustrated through the volatility of exchange rate. Under PCP-SYM the exchange rates move one-for-one with the bilateral productivity shocks:

$$\begin{align*}
Var(s_i)^{\text{Nash,PCP-SYM}} &= Var[k_A - k_i] \quad i = B, C \\
Var(s_B - s_C)^{\text{Nash,PCP-SYM}} &= Var[k_C - k_B]
\end{align*}$$

where the superscripts indicates the case of decentralized policy setting, and the subscripts denote the invoicing structure. Under LCP-SYM exchange rates are much less volatile:

$$\begin{align*}
Var(s_i)^{\text{Nash,LCP-SYM}} &= (2\alpha - 1)^2[k_A - k_i] \quad i = B, C \\
Var(s_B - s_C)^{\text{Nash,LCP-SYM}} &= 0
\end{align*}$$

The reduced volatility stems from the fact that the optimal monetary policy calls for large exchange rate movements when they lead to efficient movements in relative prices, as in the PCP-SYM case. If prices are insulated, as in the LCP-SYM case, exchange rate movements are not useful.

A similar pattern is observed in the DOL-LCP case:

$$\begin{align*}
Var(s_i)^{\text{Nash,DOL-LCP}} &= \alpha^2 Var[k_A - k_P] \quad i = B, C \\
Var(s_B - s_C)^{\text{Nash,DOL-LCP}} &= 0
\end{align*}$$

While exchange rate movements are smaller than under PCP-SYM, the fluctuations of bilateral exchange rates between the center and either periphery country are larger than under LCP-SYM (as $\alpha < 1$) because such fluctuations lead to an efficient realignment of import prices in the periphery.

In both the DOL-PCP and DOL-DOL the volatility of bilateral exchange rates between the center and a periphery country is affected by the intra-
periphery productivity differential \((k_B - k_C)\):

\[
\begin{align*}
Var(s_B)^{Nash}_{DOL-PCP/DOL} &= Var\left[\alpha (k_A - k_P) - \frac{1}{2}(k_B - k_C)\right] \\
Var(s_C)^{Nash}_{DOL-PCP/DOL} &= Var\left[\alpha (k_A - k_P) + \frac{1}{2}(k_B - k_C)\right] \\
Var(s_B - s_C)^{Nash}_{DOL-PCP/DOL} &= Var[k_C - k_B]
\end{align*}
\]

This pattern reflects that fact that the periphery monetary policy is fully inward-looking in this case. As a result, the monetary stance in the periphery country with the most volatile shocks fluctuates by more, leading to larger movements in the exchange rate vis-à-vis the center. Assuming that the shocks in the center and the periphery are not correlated, we write:

\[
Var(s_B)^{Nash}_{DOL-PCP/DOL} - Var(s_C)^{Nash}_{DOL-PCP/DOL} = \alpha (Var[k_B] - Var[k_C])
\]

### 5.2.3 Welfare under productivity shocks

Turning to the role of productivity shocks, in the PCP-SYM case the welfare in all countries is equal to the flexible price allocation:

\[
\left(\hat{U}_i\right)^{Nash}_{PCP-SYM} = 0 \quad i = A, B, C
\]

This is the standard result that when exchange rate are fully transmitted to import prices, inward-looking monetary policies lead to efficient movements in international relative prices and monetary policy can fully bring the economy around the obstacle of nominal rigidities.

In all other cases of invoicing selection, monetary policy cannot bring the economy to the flexible price allocation. The welfare levels are driven by the volatility of the productivity differentials between the center and the periphery, \(k_A - k_P\), and between the two periphery countries, \(k_B - k_C\). Under LCP-SYM, exchange rate movements cannot lead to efficient movements in international relative prices and welfare is lowered, especially in the periphery countries where imports account for a larger share of the consumption basket:

\[
\begin{align*}
\left(\hat{U}_A\right)^{Nash}_{LCP-SYM} &= -\frac{\alpha (1 - \alpha)}{2} Var[k_A - k_P] - \frac{1 - \alpha}{8} Var[k_B - k_C] \\
\left(\hat{U}_i\right)^{Nash}_{LCP-SYM} &= -\frac{\alpha (1 - \alpha)}{2} Var[k_A - k_P] - \frac{\alpha}{8} Var[k_B - k_C] \quad i = B, C
\end{align*}
\]

Under any DOL- case consumer prices in country \(A\) are fully insulated from exchange rate movements and \(A\)'s welfare is identical to the LCP-SYM case (24). By contrast, the periphery counties are adversely affected by the
volatility of the center-periphery productivity differential. Even though the price of center’s goods sold in the periphery varies with the exchange rate, the fluctuations of the exchange rate do not lead to efficient price movement because the center’s monetary policy is not geared solely towards domestic shocks (Devereux, Shi, and Xu 2006). Under DOL-PCP the intra-periphery differential entails no cost for them as the optimal policy leads to efficient movements in the relative prices between them:

\[
\left( \hat{U}_i \right)_{\text{Nash DOL-PCP}}^{\text{Nash DOL-PCP}} = -\frac{(1 - \alpha)^3}{2} \text{Var} [k_A - k_P] \quad i = B, C
\]  

(25)

By contrast, the intra-periphery productivity differential entails a cost under DOL-LCP as monetary policy cannot induce efficient movements in international prices:

\[
\left( \hat{U}_i \right)_{\text{Nash DOL-LCP}}^{\text{Nash DOL-LCP}} = -\frac{(1 - \alpha)^3}{2} \text{Var} [k_A - k_P] - \frac{\alpha}{8} \text{Var} [k_B - k_C] \quad i = B, C
\]  

(26)

Under DOL-DOL the volatility of the center-periphery productivity differential entails a larger cost. Intuitively, shocks in the center affect monetary policy in the center. This not only leads to movements in the exchange rates between the center and the periphery countries that are not fully efficient, but also generates inefficient fluctuations in the relative prices between the two periphery countries:

\[
\left( \hat{U}_B \right)_{\text{Nash DOL-DOL}}^{\text{Nash DOL-DOL}} = -\left[ \frac{(1 - \alpha)^3}{2} + \frac{\alpha^3}{4} \right] \text{Var} [k_A - k_P]
\]

(27)

\[
\left( \hat{U}_C \right)_{\text{Nash DOL-DOL}}^{\text{Nash DOL-DOL}} = -\left[ \frac{(1 - \alpha)^3}{2} + \frac{\alpha^3}{4} \right] \text{Var} [k_A - k_P] - \frac{\alpha}{16} \text{Var} [k_B - k_C] - \frac{\alpha^2}{4} \text{Covar} [k_A - k_P] [k_B - k_C]
\]  

(28)

(27)-(28) show that the welfare is not necessarily equalized across the two periphery countries in the DOL-DOL case, while it was it all the other cases. Assuming that the shocks in the center and the periphery are not correlated, the welfare is higher in the periphery country with the most volatile shocks:

\[
\left( \hat{U}_B \right)_{\text{Nash DOL-DOL}}^{\text{Nash DOL-DOL}} - \left( \hat{U}_C \right)_{\text{Nash DOL-DOL}}^{\text{Nash DOL-DOL}} = \frac{\alpha^2}{4} (\text{Var} [k_B] - \text{Var} [k_C])
\]
This result is counterintuitive, as we would expect a country with more volatile shocks to be worse off. However this is not the case because monetary policies in the periphery countries are more efficient at offsetting domestic shocks than foreign ones. Consider the case where productivity is most volatile in country $B$. From (15) the price charged for domestic sales by country $B$ firms is:

$$
\hat{\tilde{P}}_B^B \propto E(M_B/K_B)
$$

where $\propto$ denotes a proportionality. This price is fully stabilized by the inward-looking policy chosen in the DOL-DOL case (21). By contrast, the price charged by firms of country $B$ for sales in country $C$ is:

$$
\hat{\tilde{P}}_C^B \propto E(M_A/K_B)
$$

which is not affected by the monetary stance in either periphery country. As a result, a periphery country can offset the impact of its own shocks on domestic prices, but the other periphery country bears the full weight of these shocks. By contrast, the impact would be lessened in the DOL-PCP and DOL-LCP as prices would be at least partially stabilized by monetary policy in the periphery.

Our analysis highlights the contrast between the center-periphery dimension highlighted by other researchers and the intra-periphery dimensions that we have added in this modeling exercise. For instance, the welfare impact of similar policies can be different depending on the invoicing structure. Consider for simplicity the case where shocks in the periphery countries are perfectly correlated ($\text{Var}[k_B - k_C] = 0$). In this situation the monetary policy rules are the same in all three DOL- cases. The welfare for periphery countries is however smaller in the DOL-DOL case than in either the DOL-PCP or DOL-LCP cases, because fluctuations in the bilateral exchange rate between the center and either periphery country now entail inefficient movements in the prices of trade between the two periphery countries.

Another illustration of the contrast stems from the comparison of welfare in the center and the periphery, considering again that $\text{Var}[k_B - k_C] = 0$ for simplicity. From (25)-(26) and (24), periphery countries have a higher welfare level than the center under DOL-PCP and DOL-LCP. Intuitively, fluctuations in the center-periphery exchange leads to movements in import prices in the periphery that are partially efficient, while import prices in the center are fully set. This benefit along the center-periphery dimension is associated with a cost along the intra-periphery dimension under the DOL-DOL case, as the exchange rate movements then lead to inefficient fluctuations in the relative price of periphery goods. From (27) and (24) the welfare is lower in the periphery than in the center when center-periphery trade flows are limited ($\alpha < 0.71$), as the situation is then dominated by the intra-periphery cost.
5.3 Optimal monetary policy in a cooperative setting

5.3.1 Monetary rules

We now turn to the gains from cooperation in the conduct of monetary policy. Cooperation limited to the periphery countries presents a first case. In this situation, the monetary authorities in the periphery set their rules by taking into account the impact on all periphery consumers. By contrast the monetary authorities in the center focus only on their domestic welfare. This limited cooperation leads to the exact same policy rules than the decentralized setup considered above and entails no welfare gain.7

Consider instead a global cooperation setup in which monetary authorities in any country choose their rule to maximize the weighted average of the welfare of various consumers:

\[ 0 = \frac{\partial E \ln (C_A) + \frac{1}{2} \partial E [\ln (C_B) + \ln (C_C)]}{\partial M_{i,s}} \quad i = A, B, C \]

As in the decentralized setup, this gives a linear system of three equations in three unknowns, \( m_{i,s} \) for \( i = A, B, C \) and six exogenous productivity and money demand shocks. We focus on the response to productivity shocks, as policy either fully offsets money demand shocks or ignore them, both in the decentralized and cooperative cases.

Cooperation entails no gains in the PCP-SYM case, as the decentralized policy already fully offsets the inefficiency due to nominal rigidities. A similar result holds for the LCP-SYM case, is because the monetary stance in a given country does not affect the preset level of prices abroad. Cooperation therefore leads to no gain in the symmetric cases, as shown by Corsetti and Pesenti (2005a).8

We therefore focus on the DOL- cases which represent important scenarios when there are strong center countries with vehicle currencies. Our first result is that monetary policy in the periphery countries is the same as in the decentralized outcome (21)-(22). This reflects two aspects. First, prices in the center are only affected by the monetary policy rule in the center, as can be seen from (15), setting \( j = A \) and \( \gamma^{i,\text{cur}}_A = 1 \). The policy rule in the periphery is then not affected by taking welfare in the center into account, as it does not affect it. Second, while prices in the periphery are affected by monetary policy in both periphery countries in general, the impact is similar to a two-country situation with symmetric invoicing. The same logic applies as in the PCP-SYM and LCP-SYM cases and the policy rules are the same in a cooperative as in a decentralized situation.

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7 Cooperation could possibly be beneficial if a given trade flow is invoiced in a basket of different currencies, a case that we leave to future research.

8 Cooperation can still be beneficial in the case of partial exchange rate pass-through.
By contrast, cooperation matters for the optimal policy rule in the center, reflecting the fact that the center’s monetary stance has a substantial impact on the periphery countries that is not taken into account in a decentralized setting. Under both DOL-PCP and DOL-LCP we get:

\[ m_{A,s} = \frac{1}{2 - \alpha} k_{A,s} + \left( 1 - \frac{1}{2 - \alpha} \right) k_{P,s} \]  

(29)

Comparing (29) to the policy under the decentralized setting (20) shows that the monetary authorities in the center are more inward looking under the cooperative setting, with their own shocks receiving a larger weight in the policy rule:

\[ \frac{1}{2 - \alpha} > \alpha \]

Intuitively, a monetary expansion in the center following a positive shock there leads to a depreciation of the center’s currency against both periphery currencies. As there is full exchange pass-through from the center to the periphery, this lowers import prices in the periphery. This constitutes an efficient response to the productivity gain which is ignored by the center monetary authorities in the decentralized setting.

Under the DOL-DOL case the cooperative monetary policy of the center is:

\[ m_{A,s} = \frac{2}{4 - \alpha} k_{A,s} + \left( 1 - \frac{2}{4 - \alpha} \right) k_{P,s} \]  

(30)

Comparing (29)-(30) shows that under a cooperative setting monetary policy in the center reacts less to its own shocks in the DOL-DOL case:

\[ \frac{2}{4 - \alpha} < \frac{1}{2 - \alpha} \]

Intuitively, movements in the exchange rate between the center and the periphery countries now affect the price paid by consumer in country B for country C goods, and conversely. A monetary expansion in the center following a productivity gain there then makes intra-periphery imports cheaper, which is an inefficient response as productivity in the periphery has not changed. Contrasting the cooperative policy in the DOL-DOL case (30) with the policy under the decentralized setting (20) shows that the cooperative calls for a larger reaction to center shocks only when the center and periphery are closely integrated, as the benefit of exchange rate fluctuations along the center-periphery dimension then dominates the costs along the intra-periphery dimension:

\[ \frac{2}{4 - \alpha} > \alpha \Leftrightarrow \alpha < 0.59 \]
5.3.2 Welfare

Across all DOL- cases the welfare of periphery countries is higher under the cooperative setting, at the expense of the center country. For brevity, we contrast the welfare under the cooperative setting with that under the decentralized policy setting. In both the DOL-PCP and DOL-LCP cases we write:

\[
\left( \hat{U}_A \right)_{\text{Gain}}^{\text{DOL-PCP/LCP}} = -\frac{(1 - \alpha)^4}{2} \left( \frac{1}{2 - \alpha} \right)^2 \text{Var} \left[ k_A - k_P \right] < 0 \tag{31}
\]

\[
\left( \hat{U}_i \right)_{\text{Gain}}^{\text{DOL-PCP/LCP}} = \frac{(1 - \alpha)^3}{2} \left[ 1 - \left( \frac{1}{2 - \alpha} \right)^2 \right] \text{Var} \left[ k_A - k_P \right] > 0 \tag{32}
\]

\[ i = B, C \]

where the gain superscript denotes the welfare difference between the cooperative and decentralized settings, with a positive value indicating that the country is better off when monetary policy is conducted under cooperation.

Two main points emerge from (31)-(32). First, only the volatility of the center-periphery productivity differential matters. Intuitively, a cooperative setting only affects the optimal rule in the center country. As intra-periphery trade flows are invoiced in one of the periphery currencies, the intra-periphery dimension is not affected by monetary policy in the center, hence does not enter (31)-(32). Second, the gain from cooperation is equalized across the two periphery countries, even if one of these countries faces more volatile shocks.

Turning to the DOL-DOL case we write:

\[
\left( \hat{U}_A \right)_{\text{Gain}}^{\text{DOL-DOL}} = -\left[ \frac{\alpha}{2} \left( \frac{2 - \alpha}{4 - \alpha} \right)^2 + \frac{1 - \alpha}{2} \left( \frac{2}{4 - \alpha} \right)^2 \right] \text{Var} \left[ k_A - k_P \right] \tag{33}
\]

\[
\left( \hat{U}_B \right)_{\text{Gain}}^{\text{DOL-DOL}} = \left[ \frac{1 - \alpha}{2} \left( 1 - \alpha \right)^2 - \left( \frac{2 - \alpha}{4 - \alpha} \right)^2 \right] \text{Var} \left[ k_A - k_P \right] \tag{34}
\]

\[
- \left[ \frac{\alpha}{2} \frac{1}{2 - \alpha} - \frac{\alpha^2}{4} \right] \text{Covar} \left[ k_A - k_P \right] \left[ k_B - k_C \right] \]

\[
\left( \hat{U}_C \right)_{\text{Gain}}^{\text{DOL-DOL}} = \left[ \frac{1 - \alpha}{2} \left( 1 - \alpha \right)^2 - \left( \frac{2 - \alpha}{4 - \alpha} \right)^2 \right] \text{Var} \left[ k_A - k_P \right] \tag{35}
\]

\[
+ \left[ \frac{\alpha}{2} \frac{1}{2 - \alpha} - \frac{\alpha^2}{4} \right] \text{Covar} \left[ k_A - k_P \right] \left[ k_B - k_C \right] \]

The bracket terms that multiply \( \text{Var} \left[ k_A - k_P \right] \) in (33)-(35) are positive for any value of \( \alpha \), implying that cooperation boosts the welfare of the periphery countries, at the expense of the center.
(34)-(35) show that in general the welfare is not equalized between the two periphery countries. The bracket in the second terms in (34)-(35) can be positive or negative, depending on \( \alpha \), implying that the relative gain between the two periphery countries is driven by the relative volatility of their own shocks as well as the degree of integration between the center and the periphery. If shocks in the center and the periphery are uncorrelated, (34)-(35) imply:

\[
\left( \hat{U}_B \right)_{\text{DOL-DOL}}^{\text{Gain}} - \left( \hat{U}_C \right)_{\text{DOL-DOL}}^{\text{Gain}} = \left[ \frac{\alpha}{2} - \frac{\alpha^2}{4} \right] (\text{Var} [k_B] - \text{Var} [k_C])
\]

The term in bracket is positive only when the center and the periphery are closely integrated (\( \alpha < 0.59 \)). When the integration is less than this threshold, the gain from cooperation is larger for the periphery country with the least volatile productivity shocks. Recall that the prices of goods shipped between the two periphery countries are:

\[
\tilde{P}_B^B \propto E(M_A/K_B) \quad \tilde{P}_C^C \propto E(M_A/K_C)
\]

where \( \propto \) denotes a proportionality. Focus on a case where the center and periphery are loosely integrated (\( \alpha \) is large), and consider that shocks are more volatile in country \( B \). Under a decentralized policy, the center monetary stance responds little to shocks in either periphery country. This is inefficient as the marginal cost then fluctuates, leading to higher preset prices. The problem is more acute for country \( C \) has the import price it pays is driven by the volatility of \( M_A/K_B \) which is high. By contrast the issue is less important for country \( B \) as \( M_A/K_C \) fluctuates by less. When policy is undertaken under a cooperative setting, the center monetary authorities take their impact on the intra-periphery dimension into account and their stance reacts more to periphery shocks. Consequently, the volatility of \( M_A/K_B \) and \( M_A/K_C \) is reduced, thereby lowering prices in the periphery and improving welfare. The gain is moderate for country \( B \), as \( M_A/K_C \) was not too volatile to start with, but more substantial for country \( C \) which bore the cost of the high volatility in \( M_A/K_B \). As a result, the gain from cooperation is larger for the country with the least volatile shocks.

This aspect is related to our finding that in the DOL-DOL case the welfare under a decentralized policy setting is highest in the periphery country with the most volatile shocks, because its own policy can offset its own shocks while the other periphery country cannot use their policy to offset the shocks. This implies that the periphery country with the most volatile shocks can "take care of itself" by reacting to its volatile shocks, while the other periphery country cannot and needs monetary policy in the center to address the issue.
5.4 A numerical illustration

Our results are illustrated by means of a simple example, focusing on the impact of the degree of center-periphery integration $\alpha$ which is the central parameter in our analysis. We first focus on the role of productivity differentials between the center and the periphery by assuming that productivity shocks in country $B$ and country $C$ are perfectly correlated ($\text{Var}[k_B - k_C] = 0$), but are independent from shocks in the center. We set the standard deviation of productivity shocks at 5% for all countries, leading to a standard deviation for the center-periphery productivity differential of 7% ($\text{Var}[k_A - k_P] = (0.07)^2$).

The welfare levels under a decentralized policy are presented in Figure 5, recalling that they are always zero under PCP-SYM. The welfare in the center country $A$ is the same under LCP-SYM and any DOL- case (dotted line). This also corresponds to the welfare in either periphery country in the LCP-SYM case, as the situation is symmetric for all countries. The first dimension of the international role of the center currency is beneficial to the periphery, as it allows for partially efficient movement in relative prices along the center-periphery dimension. The welfare in either periphery country (dashed line) is then higher than in the center in either the DOL-PCP or the DOL-LCP case. The second dimension of the international role is however costly for the periphery as it leads to inefficient movements in relative prices along the intra-periphery dimension. This has a substantial impact on welfare in either periphery country (thick line), especially when the center and the periphery are not tightly connected as the benefit along the center-periphery dimension is then secondary.

Figure 6 shows the gains and losses from following a cooperative monetary policy, relative to the decentralized policy. Recall that cooperation leads to different rules only in the DOL- cases. Panel A shows that cooperation is costly to the center country $A$, especially under the DOL-DOL case when the center and the periphery are not closely connected. This situation is symmetric for the periphery countries (panel B), with large gains in the DOL-DOL case when monetary policy in the center takes its impact on the intra-periphery prices into account.

Figure 6 shows that the relation between integration and the gains from cooperation depends on the exact structure of invoicing. In the DOL-PCP and DOL-LCP cases, the externality of the center monetary policy is limited to the center-periphery dimension, and cooperation matters most when the center and the periphery are closely connected. Cooperation then calls for more exchange rate volatility (a stronger reaction of the center’s policy to its own shocks) as this leads to efficient movements in import prices in the periphery.

By contrast, another externality emerges along the intra-periphery dimen-
sion in the DOL-DOL case. Cooperation along this dimension calls for less exchange rate volatility as it leads to inefficient movements in intra-periphery import prices. Overall the policy under cooperation balances the two externalities. If the center and the periphery are closely connected, the two aspects cancel each other out and the gain from cooperation is limited (thick line). When the connection is looser, the intra-periphery aspect dominates and cooperation leads to substantial gains.

We next assess the impact of asymmetric shocks in the periphery. We still assume that shock in the periphery countries are perfectly correlated, but we take them to be twice as volatile in country $B$ as in country $C$. Specifically, we set the standard deviation of shocks in country $B$ and $C$ at 6.6% and 3.3% respectively. With the standard deviation of shocks in the center kept at 5%, this ensures that the standard deviation of the center-periphery productivity differential remains at 7%, and the standard deviation of the intra-periphery productivity differential, $k_B - k_C$, is equal to 3.3%.

The welfare under a decentralized monetary policy is shown in Figure 7. Panel A presents the LCP-SYM and DOL-PCP and DOL-LCP cases. The pattern in the LCP-SYM case is similar as in Figure 5, with welfare being somewhat lower in both periphery countries. The first dimension of the international role of the center currency is again beneficial to both periphery countries, especially when there is full pass-through at the intra-periphery level (DOL-PCP). The DOL-DOL case is presented in Panel B. The welfare in the center country $A$ (dotted line) is similar to Figure 5. Similarly, welfare in either periphery country is low when the integration between the center and the periphery is limited. The main difference from Figure 5 is the presence of a substantial gap between the two periphery countries, with welfare being lower in the country where shocks are the least volatile (country $C$) as bears the burden of highly volatile shocks in the other periphery country.

The gains from cooperation are presented in Figure 8. As our parametrization keeps the standard deviation for the center-periphery productivity differential unchanged, the gains in the DOL-PCP and DOL-LCP cases are the same as in Figure 6, and so is the loss for country $A$ in the DOL-DOL case. Figure 8 then focuses on the gains for the periphery countries in the DOL-DOL case. While the pattern is similar to Figure 6, cooperation is especially beneficial to the country with the least volatile shocks, as long as the center and the periphery are not tightly connected.9

---

9The average gain between the two periphery countries correspond to the one in Figure 6.
6 Conclusion

In this paper, we analyze the impact of the international role of the dollar on macroeconomic interdependence using a simple open economy model consisting of a center, such as the United States or the euro area, and two periphery countries. We distinguish between two dimensions of the international role of a currency, namely its use in transactions involving the center as issuer and its use in transactions between periphery countries. The latter aspect has received little attention in the literature, despite its empirical relevance. We first show that monetary policy in the center country has a substantial impact on the periphery countries when intra-periphery trade is invoiced in the center currency. Specifically, a monetary expansion in the center depreciates its currency and makes imported goods cheaper in both periphery countries, boosting trade flows between these two countries. This channel operates independently of trade flows between the center and the periphery.

We then show that the second dimension of the international role of the center currency can have sizable implications for the design of monetary policy. In a decentralized setup monetary policy in the center country reacts to productivity shocks without taking account of the fact that the ensuing movements in exchange rates affect the relative price of intra-periphery imports. In the center-periphery model of the paper the policy rule is then suboptimal from a global perspective and can lead to sizable welfare losses for the periphery countries. Consequently a cooperative monetary policy would lead to a substantial improvement in welfare. Our analysis also shows that the gains from cooperation are largest for the periphery country with the least volatile shocks.

Our setup can be extended in several directions. An interesting avenue of research will be to endogeneize the choice of the invoicing currency, and assess how the degree of exchange rate pass-through interacts with the design of monetary policy.
References


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<tr>
<th>Country</th>
<th>Year of Invoicing Observation and Trade Shares</th>
<th>Share invoiced in Dollars</th>
<th>Share sold to the US</th>
<th>Share sold to &quot;Dollar Bloc&quot; Countries</th>
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Notes:

- **Invoicing data and trade data refer only to “extra euro-area” trade.**

- **Dollar bloc countries are:** Afghanistan, Antigua & Barbuda, Azerbaijan, the Bahamas, Bahrain, Bangladesh, Barbados, Belize, Bermuda, China, Djibouti, Dominica, Ecuador, El Salvador, Ethiopia, Grenada, Guinea, Guyana, Hong Kong, India, Iraq, Jordan, Kazakhstan, Kenya, Kuwait, Lebanon, Malaysia, Maldives, Mongolia, Mozambique, Netherlands Antilles, Oman, Pakistan, Panama, Qatar, St Vincent & the Grenadines, Sao Tome & Principe, Saudi Arabia, St Kitts & Nevis, St Lucia, Sudan, Suriname, Syrian Arab Republic, Trinidad & Tobago, Ukraine, United Arab Emirates, Vietnam, Yemen, and Zimbabwe. In the case that trade data to one of these countries is not available, reported trade shares do not include trade with that country in the numerator.
## Table 2: Dimensions of the International Role of the Euro

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<sup>a</sup>Invoicing data and “euro bloc” trade data refer only to “extra euro-area” trade.

<sup>b</sup>Euro bloc countries are: Albania, Benin, Bosnia & Herzegovina, Bulgaria, Burkina Faso, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Republic of Congo, Cote D’Ivoire, Croatia, Cyprus, Czech Republic, Denmark, Equatorial Guinea, Estonia, Gabon, Guinea-Bissau, Hungary, Latvia, Lithuania, Macedonia, Mali, Malta, Niger, Poland, Reunion (Thailand trade statistics only), Romania, San Marino, Senegal, Serbia, Slovakia, Slovenia, Togo, Turkey. In the case that trade data to one of these countries is not available, reported trade shares do not include trade with that country in the numerator.

NA: Not Applicable
Figure 1: Use of the U.S. Dollar in International Export Transactions

Exports to the US, Share of Total Exports (percent)
US Dollar Invoicing Share in Country Exports (percent)

Source: DOTS and various national sources
Euro-area country data, with the exception of Italy data, refer to extra-euro area trade and invoicing.
Figure 2: Use of Euro in Periphery Transactions

Country's Trade to the Euro Area and Euro Bloc as a Share of Total Country Trade, %

Euro Invoicing Share in Country Trade
Figure 3: Five cases of invoicing

The letters denote the currency used for the invoicing of the corresponding trade flows.

PCP-SYM

```
Country A
B <---- A ----> C
|      |      |
|      |      |
|      |      |

Country B
```

LCP-SYM

```
Country A
A <---- B ----> C
|      |      |
|      |      |
|      |      |

Country B
```

DOL-PCP

```
Country A
A <---- A ----> A
|      |      |
|      |      |
|      |      |

Country B
```

DOL-LCP

```
Country A
A <---- A ----> A
|      |      |
|      |      |
|      |      |

Country B
```

DOL-DOL

```
Country A
A <---- A ----> A
|      |      |
|      |      |
|      |      |

Country B
```

Country C
Figure 4: World consumption change following a unit expansion of monetary policy in country A

Figure 5: Welfare under decentralized monetary policy
Shocks are equally volatile in country B and C
Figure 6: Gain from cooperation
Welfare under cooperative policy - welfare under decentralized policy
Shocks are equally volatile on country B and C

Panel A: Country A

Panel B: Country B and C
Figure 7: Welfare under decentralized monetary policy

Shocks are twice as volatile in country B as in country C

Panel A: LCP-SYM and DOL-PCP / LCP

Panel B: DOL-DOL
Figure 8: Gain from cooperation, DOL-DOL case
Shocks are twice as volatile in country B as in country C