Timing Fiscal Retrenchment in the Wake of Deep Recessions

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Abstract

Government debt levels in industrialized economies are rising rapidly at a time when monetary policy remains constrained by the zero lower bound (ZLB) on policy rates. This confronts policymakers with a formidable challenge. On the one hand, reducing government spending under these circumstances directly curtails demand, potentially undermining economic recovery. On the other hand, delaying consolidation may lead to rising sovereign risk premia that spill over to private sector funding conditions and cannot be offset by monetary policy. In this paper, we use a new Keynesian model to study how the timing of fiscal retrenchment affects economic outcomes at the ZLB. The model incorporates a risk premium that rises endogenously with the deterioration of the fiscal outlook. We find that if public debt is low, delaying government spending cuts leads to output gains relative to earlier fiscal retrenchment. However, if public debt is already high, there is a strong case for immediate policy action, because high debt levels—via their impact on the risk premium—enlarge the region of parameters for which the equilibrium is indeterminate and thus magnify the risk that expectations become unanchored.

Keywords: Fiscal consolidation, Monetary policy, Zero lower bound, Risk premium

JEL-Codes: E62, E52, E32

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"Those countries with serious fiscal challenges need to accelerate the pace of consolidation. We welcome the recent announcements by some countries to reduce their deficits in 2010..." (G20 Communiqué, June 2010)

"The right thing, overwhelmingly, is to do things that will reduce spending and/or raise revenue after the economy has recovered - specifically, wait until after the economy is strong enough that monetary policy can offset the contractionary effects of fiscal austerity." (Paul Krugman, June 2010)

1 Introduction

The global financial crisis has sent public debt in the industrialized world on a sharply higher trajectory. As a result, the sustainability of public finances has moved into the focus of market concerns, and fiscal consolidation has become a policy priority. Yet the appropriate timing of consolidation remains subject to intense controversy. On the one hand, consolidation efforts may prematurely withdraw support from a still-fragile economic recovery, at a time when monetary policy remains constrained by the zero lower bound (ZLB) on nominal interest rates. On the other hand, delaying consolidation measures for too long may lead to sharply rising risk premia, which in turn weigh down on private demand.

In this paper, we take up the question of when to start fiscal consolidation in the wake of deep recessions. We assume that the recession is triggered by a very large negative shock, such that monetary policy becomes constrained—the central bank cannot lower the policy rate as much as it would like, given the ZLB. During such episodes, government spending can affect output more strongly than in normal times, as shown by Christiano et al. (2009) and Woodford (2010), among others. The reason lies in the interaction of fiscal and monetary policy. In normal times, the expansionary effect of government spending triggers a rise in inflation and a monetary tightening, which crowds out private demand. Hence, output multipliers are typically moderate. In contrast, when the ZLB on policy rates is binding the central bank will not respond to the positive fiscal impulse; any increase in inflation due to higher government spending will simply translate into lower real interest rates, raising private demand. For the same reason, fiscal retrenchment can be particularly harmful to economic activity during deep recessions.

However, there is an important countervailing consideration. As government revenue declines during the recession and the public debt stock rises, steps to reduce the deficit may be necessary to prevent or at least contain a rise in the risk premium that investors demand for holding the country's sovereign debt. To the extent that fiscal consolidation succeeds in lowering this risk premium, it may thus have a stimulating effect on economic activity. A key reason is the close link between public and private financing costs. As emphasized in both International Monetary Fund (2010) and European Central Bank (2010), government bond yields typically set a floor for, or at least have a strong influence on, domestic corporate bond yields. Consequently, a reduction in sovereign risk premia can lower funding costs in the wider economy and stimulate private consumption and investment. This effect can be particularly desirable in deep recessions, when monetary policy is constrained by the ZLB and hence unable to offset adverse spillovers from government risk premia to private sector funding costs.

Against this backdrop, the present paper studies formally how the timing and intensity of fiscal retrenchment measures affect current economic activity. Our analysis is based on a new Keynesian model, which we specify so as to capture key features that have shaped the fiscal policy debate in major industrialized countries since the beginning of the global financial crisis. Specifically, we posit that the risk premium on government debt rises in the expected level of government debt (drawing on the risk premium specification by García-Cicco et al. (2009)), and that higher sovereign risk premia can spill over to private sector interest rates.

We perform quantitative experiments to analyze the effect on economic activity of alternative fiscal plans that differ with respect to the timing of expenditure cuts after the initial recessionary impulse. We do so by expanding on earlier work of ours, notably by integrating sovereign risk premia into the analysis of Corsetti et al. (2010). In this earlier paper, we showed that credible plans for future spending cuts can enhance current fiscal stimulus during a deep recession, provided that the fiscal retrenchment does not start too early into the economic recovery. The mechanism relies on the effect of fiscal policy decisions on the path of real interest rates: firms facing nominal rigidities have an incentive, all else equal, to set lower prices ahead of the period in which the government is expected to reduce spending. This, in turn, creates disinflationary pressures which induce the central bank to lower policy rates. If cuts are projected to occur sufficiently far in the future, when the economy is already on the recovery path and the ZLB no longer binding, the anticipated fiscal retrenchment immediately lowers long-term interest rates and thus stimulates activity during the recessionary period.[1] Note that, for this mechanism to work, it is crucial that the central bank can offset the disinflationary effect of spending cuts once they occur. If instead the cuts take place too early, i.e. while monetary policy is still constrained by the ZLB, fiscal retrenchment is counterproductive, as its disinflationary impact translates into higher real interest rates.

The present paper revisits this mechanism, with a view toward assessing how the inclusion of sovereign risk premia alters the macroeconomic and budget dynamics relative to our previous results. Overall, we find that high risk premia raise the stakes for the timing of fiscal consolidation. Perhaps unsurprisingly, the case for early consolidation can become stronger if

[1] Eggertsson (2001) also stresses that anticipated spending cuts may have an expansionary effect at the zero lower bound.

initial debt levels, and hence risk premia, are high. In extreme cases, the drop in real interest rates caused by a well-timed fiscal retrenchment may even be strong enough to generate non-Keynesian effects. More strikingly, however, we find that in such high-debt scenarios even moderate delays to fiscal retrenchment can significantly increase the risk of indeterminacy—a situation in which the government's fiscal plans do not pin down a unique macroeconomic equilibrium, potentially giving rise to highly adverse dynamics.

To shed additional light on these findings, in the second part of the paper we derive analytical results using a simplified version of our model. We show that the effect of the government's fiscal retrenchment depends on the interplay of (a) the output elasticity of the government budget, (b) the initial level of sovereign indebtedness, (c) the sensitivity of the sovereign risk premium with respect to the level of debt, and (d) the extent to which sovereign risk spills over to the private sector. Specifically, cutting government spending in a deep recession can actually support output and stabilize the economy if the government budget is not very responsive to economic activity and initial debt is high. A specific result of our analysis, however, relates to the effect of risk premia on the stability of equilibria. In particular, the risk of indeterminacy greatly increases in the debt-elasticity of interest rates.

The text is structured as follows. Section 2 presents empirical evidence with respect to the risk premium dynamics stressed in this paper. Section 3 describes the model economy. Section 4 reports results from analyzing different consolidation paths. The endogenous response of the risk premium complicates the analysis of the adjustment path considerably, as the equilibrium may become indeterminate. In order to build intuition for the effects at work, Section 5 uses a simplified version of the model that can be solved analytically. Section 6 concludes.

2 Evidence on government debt and risk premia

Our model-based analysis below accounts for the possibility that higher public debt leads to increasing sovereign risk premia which eventually spill over into private sector financing conditions. In the following we briefly discuss some evidence in support of these channels.

Recent developments in sovereign bond markets confirm the notion that a large current and/or projected future stock of public debt is a key indicator of sovereign risk. Figure 1 bears out this relationship. It plots credit default swap spreads of industrialized economies against the level of projected end-2010 gross public debt.[2] For the countries shown, CDS spreads are systematically higher, the higher the level of projected gross public debt. In fact, the risk premia appear to rise disproportionately as the debt level goes up. A similar relationship exists between CDS spreads and net public debt;[3] and between CDS spreads and debt held

[2] Credit default swaps (CDS) are insurance contracts that cover the repayment risk on an underlying bond. The CDS spread indicates the annual insurance premium to be paid by the buyer. Accordingly, a higher perceived default probability on the underlying bond implies, ceteris paribus, a higher CDS spread.

[3] In theory, net debt should represent a more accurate picture of a sovereign's financial position. In practice,

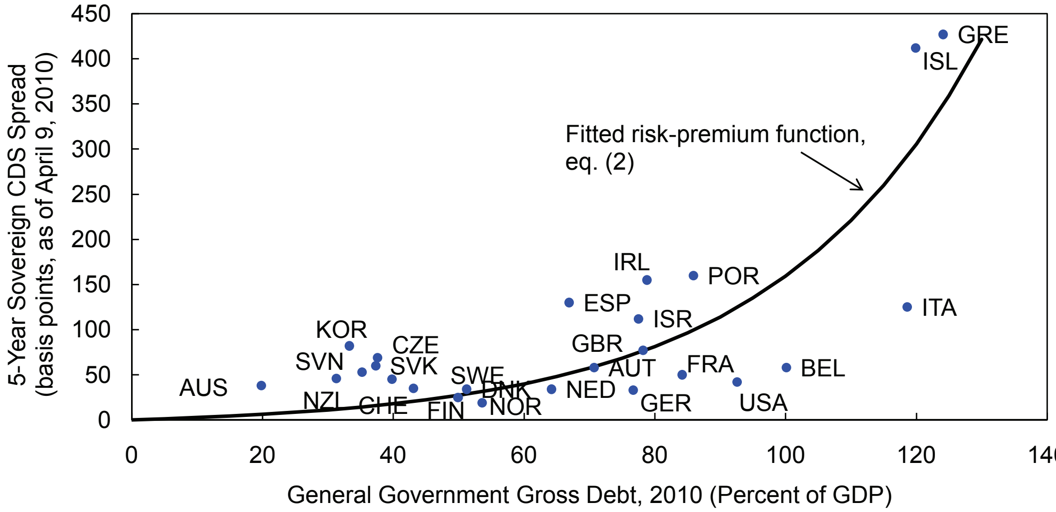

Figure 1: Sovereign Risk Premia vs. General Government Gross Debt 2010. Source: IMF, Global Financial Stability Report, April 2010. Note: Excludes Japan. The black solid line is the risk-premium curve (2).

by nonresidents, whether expressed in relation to GDP or in relation to annual government revenue, compare Figure 2. In addition, the slope of the risk premium curve plotted in Figure 1 is not even particularly extreme. Within a mere four weeks from the cutoff point for the IMF's April 2010 Global Financial Stability Report, from which the data of Figure 1 are taken, CDS spreads had more than doubled in the case of the most vulnerable sovereign debtors; see Figure 3.

The finding of a systematic relationship between fiscal variables and sovereign risk premia is by no means new or specific to the recent turmoil in sovereign bond markets. Instead, similar relationships have been documented in a number of papers, including Reinhart and Sack (2000), Ardagna et al. (2007), Baldacci et al. (2008), Haugh et al. (2009), or Baldacci and Kumar (2010). These studies generally use regression techniques to determine the marginal impact of particular fiscal variables, notably the levels of debt and deficits, on long-term government bond rates. A common finding is that countries with higher debt and deficits face higher financing costs, consistent with the notion of increased default risk. Ardagna et al. (2007) explicitly focus on possible nonlinearities in the relationship and find that bond rates rise disproportionately for very high levels of debt, as suggested by the previous figures. Needless to say, actual financing costs appear to be affected by a range of additional factors as well, including the quality of the country's fiscal institutions, the extent to which a country relies on foreign bond investors, or the level of world interest rates.[4] The latter point also hints at the important fact that risk premia can vary significantly over time, as evidenced in

however, net debt data for different countries are not always comparable due to differences in definition. Moreover, if some of the government's financial assets cannot readily be liquidated in times of funding stress, gross liabilities remain a relevant concept from the viewpoint of financial vulnerability.

[4] For instance, it has been argued that risk premia on Japan's public debt have remained very low despite high (gross) debt levels, because of the economy's large stock of private domestic savings.

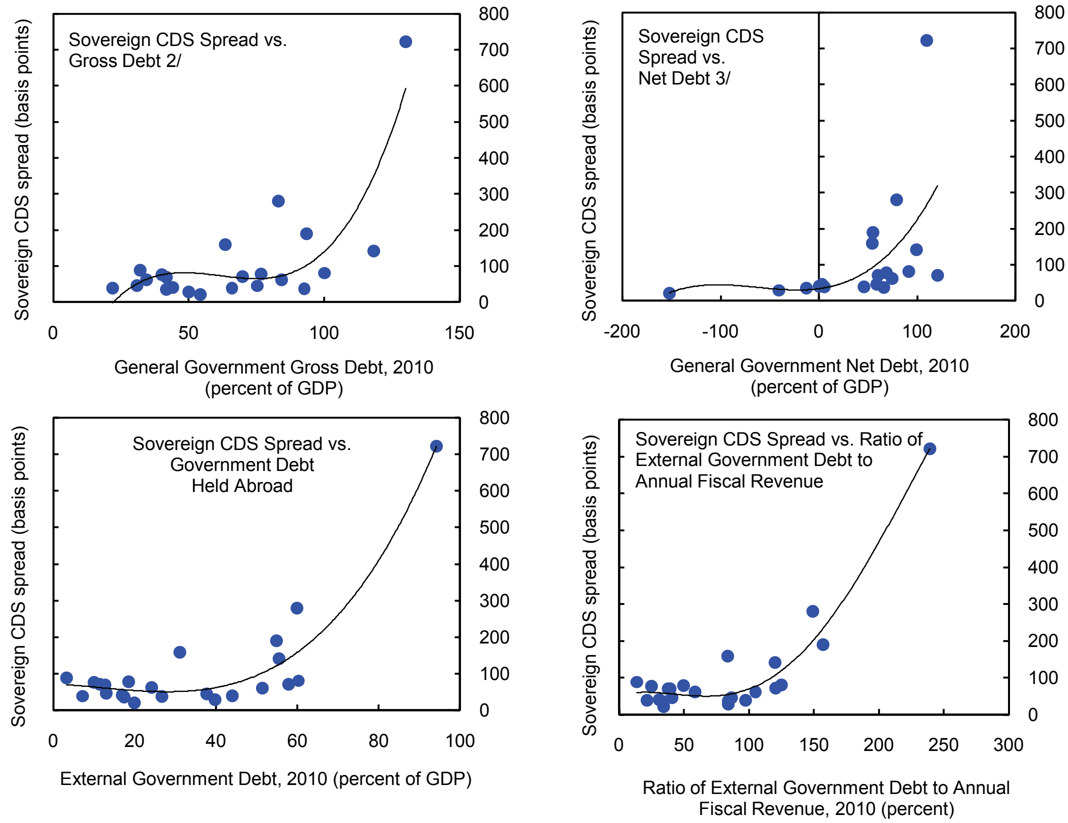

Figure 2: Relationship between Sovereign Risk Premia and Public Debt. Sources: Bloomberg; and IMF, Global Financial Stability Report, October 2010, Table 1. Notes: 1/ Risk premia shown are the CDS spreads (5-year contracts) on April 30, 2010 for the following sovereigns: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Republic of Korea, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, United Kingdom, and United States. Solid lines indicate third- or fourth-order polynominal regression lines. 2/ Excludes Japan. 3/ Net debt data unavailable for Czech Republic, Republic of Korea, Slovak Republic, and Slovenia.

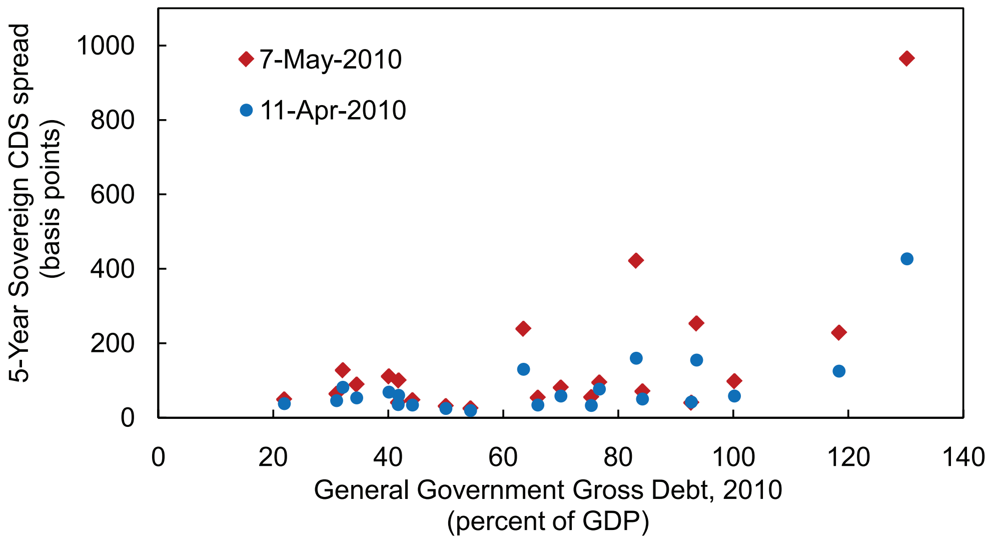

Figure 3: Relationship between Sovereign Risk Premia and Public Debt. Same as Figure 1 but comparing two different dates for CDS prices, i.e., April 11, 2010 and May 7, 2010.

Figure 3. Nonetheless, high current and/or projected debt is consistently found to be a key determinant of government financing costs.

Long-term government bond yields, in turn, are an important benchmark for broader financing conditions in a given country. A prominent reflection of this is the notion of a "sovereign ceiling." In a strict interpretation, it implies that no debtor in a given country can have a better credit quality than the government, given the latter's capacity to extract private sector resources through taxation. In reality, some authors, including Durbin and Ng (2005), have documented exceptions to this rule, notably for firms with substantial export earnings or close links to a foreign firm. Even then, however, sovereign bond yields and corporate bond yields comove significantly as sovereign risk spills over to corporate risk; compare, for example, the literature review in Cavallo and Valenzuela (2007). This observation is in line with the recent experience. As Figure 4 shows, risk premia on European banks rose nearly one-for-one with the risk premia of the corresponding sovereigns. As such, weak fiscal positions can affect risk premia for the economy as a whole.

Despite their relevance, risk premia on sovereign and/or private debt are routinely abstracted from in standard business cycle models. One key reason is the difficulty of modeling the underlying sovereign default risk. In this paper, we sidestep the question of the specific microfoundations determining sovereign risk premia, and rely instead on an operational approach informed by the data and the studies summarized above. Specifically, we posit that the risk premium on sovereign debt (and, through spillovers, the risk premium on private debt) is a function of current and future government debt levels; see equation (2) below. For the functional form we choose parameters so as to match the relationship depicted in 1. While explicit microfoundations would obviously be desirable, we argue that our simple approach

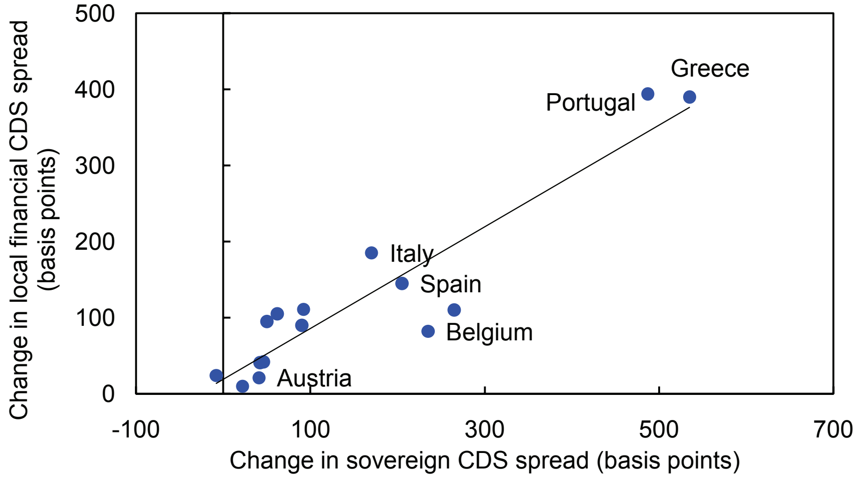

Figure 4: Comovement of Sovereign and Local Financial CDS Spreads, October 2009-June 2010. Source: IMF, Global Financial Stability Report, April 2010, Figure 4.

captures the essence of how sovereign risk premia interact with policy decisions on the timing of fiscal retrenchment, which are the subject of our analysis.

3 The Model

The model economy is the textbook New Keynesian model, as, for example, in Woodford (2003), except that we add a link between government debt levels and private sector spreads. Private agents face a retail interest rate. This retail interest rate is priced off the interest rate on government debt. The interest rate an sovereign debt includes a sovereign risk premium over and above the interbank rate that the central bank steers. As a result, private sector interest rates may be affected by government risk premia. This is meant to capture, in a stylized manner, the spill-over from sovereign risk to the private sector, emphasized in the sovereign debt ceiling literature, see, e.g., Durbin and Ng (2005).

We do not provide an explicit model of the underlying channels, but focus on the macroeconomic effects of a rising government risk premium.[5] In the following exposition of the model, capital letters denote nominal variables, small letters real variables.

[5] Private assets may become more risky as the government's credit rating falls for several reasons. The banking system, for example, may be exposed to higher sovereign default risk and reduces lending, which, in turn, raises private sector interest rates. In addition, the prospect of government default may raise the scope for events that negatively impact on the private sector, see European Central Bank (2010) for further discussion.

3.1 Households

The representative household chooses consumption c_t and employment n_t to maximize

$$E_t \sum_{i=0}^{\infty} (e^d_{t+i}\beta^i) \left[\frac{c_{t+i}^{1-\sigma}}{1-\sigma} - \kappa_n \frac{n_{t+i}^{1+\omega}}{1+\omega} \right], \ \sigma, \kappa_n, \omega > 0.$$

c_t is a CES bundle of differentiated goods c_{jt}, $j \in [0,1]$, with prices P_{jt}, and associated demand functions

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} c_t,$$

where $\epsilon > 0$ is the elasticity of demand and P_t is the aggregate price level. e_t is a unit-mean shock to the time-discount factor $\beta \in (0,1)$. The period budget constraint in nominal terms is

$$\int_0^1 p_{jt} c_{jt} dj + T_t + B_t + D_t = (1-\tau_t) n_t W_t + B_{t-1} R^{\text{gov}}_{t-1} + D_{t-1} R^{\text{pr}}_{t-1} + \Psi_t. \tag{1}$$

T_t are lump-sum taxes. Nominal wages, W_t, are flexible and determined in a competitive labor market. τ_t is the labor tax rate. Ψ_t are dividends paid by firms. There are two types of bonds in the economy, government bonds and private-sector bonds. Government bonds pay gross nominal interest rate R^{gov}_t in $t+1$. B_t marks the amount outstanding of these. Government bonds are directly subject to sovereign risk premia. We parameterize the risk premium on government debt relative to safe overnight money, similar to García-Cicco et al. (2009), relating it to the expected level of government debt as follows

$$\log(R^{\text{gov}}_t) = \log(R_t) + E_t \left(\exp \left(\left[b^{\text{avg}}_{t+1}/4y - \psi_1 \right] / \psi_2 \right) + \psi_3 \right) /100, \tag{2}$$

where ψ_1 and ψ_2 parameterize the risk premium.[6] Parameter ψ_3 is used to ensure consistency in the sense that at zero debt levels there is no risk premium. The formulation ensures that the risk premium of government debt is always non-negative. Rather than assuming that the risk premium depends on the debt level at one particular time in the future or the debt level today, the level of debt that governs the risk premium is a weighted average of current and future expected debt:

$$b^{\text{avg}}_t = (1-\zeta) b_t + \zeta E_t \left\{ b^{\text{avg}}_{t+1} \right\}.$$

The larger $\zeta \in [0,1)$, the more weight have future debt levels in the determination of the risk premium. In the calibration below, we will set ζ such that the half-life reflects the average debt maturity.

Another set of bonds is private (inside) debt that is in zero net supply and pays interest R^{pr}_t. The rate of interest on private debt is priced off the yield on short-term government bonds.

[6] García-Cicco et al. (2009) model an emerging market context, and assume that the domestic real rate is the world real rate plus a risk premium term that depends on the aggregate level of external debt.

In particular, it is equal to a weighted average of the risk-less interbank rate that the central bank targets, R_t, and the interest rate paid on government bonds, R_t^{gov}:

$$\log(R_t^{\text{pr}}) = \alpha \log(R_t^{\text{gov}}) + (1-\alpha)\log(R_t), \tag{3}$$

where $\alpha \in [0,1]$ gives the relative weight. If $\alpha = 0$, private bond markets are immune to government risk premia. Higher government risk premia in that case affect real allocations in the economy only to the extent that the debt is financed through higher distortionary taxes or if the path of debt is explosive from which we abstract here. If, more realistically, α is closer to unity, there is an increasing amount of spill-overs from government debt to private bond markets. These spill-overs directly affect the consumption savings decision and thus distort the economy unless the central bank neutralizes them.

3.2 Firms

There is a unit mass of firms in the economy, indexed by $j \in [0,1]$. These produce differentiated goods according to

$$y_{jt} = z n_{jt},$$

where $z > 0$. Firms are subject to Calvo-staggered price setting. In each period, a random fraction of firms, $1-\theta$, $\theta \in [0,1]$, can update their price. These firms' optimizing problem is

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} \beta_{t,t+i}\theta^i \left[\frac{P_{jt} y_{jt+i}(P_{jt})}{P_{t+j}} - \frac{w_{t+i} y_{jt+i}(P_{jt})}{z}\right]$$

subject to demand function $y_{jt} = (P_{j,t}/P_t)^{-\epsilon} y_t$. The latter reflects that real Government spending, g_t, is isomorphic to consumption.[7] Above $\beta_{t,t+i}$ is the stochastic discount factor between t and $t+i$. The aggregate price level is given by

$$P_t = \left(\int_0^1 P_{jt}^{(1-\epsilon)}\right)^{1/(1-\epsilon)}.$$

The first-order condition results in the standard New Keynesian Phillips curve with marginal costs given by w_t/z.

3.3 The government

Nominal government debt evolves according to

$$B_t = B_{t-1} R_{t-1}^{\text{avg,gov}} + G_t - T_t - \tau_t n_t W_t. \tag{4}$$

[7] While not spelled out above, our simulations further assume that firms, that do not reoptimize, update their prices by the steady-state inflation rate.

The government collects taxes, $T_t + \tau_t n_t W_t$, and engages in government spending, G_t. It pays the coupon on existing debt, $B_{t-1} R_{t-1}^{\text{avg,gov}}$, and issues new debt to satisfy its period budget constraint. We note here, that the government pays a weighted average of current and past short-term bond yields so as to reflect the maturity structure of government debt. In particular, we assume that

$$R_t^{\text{avg,gov}} = \rho^{\text{avg,gov}} R_{t-1}^{\text{avg,gov}} + (1 - \rho^{\text{avg,gov}}) R_t^{\text{gov}},$$

The half-life of this process is equal to $\log(0.5) - \log(\rho^{\text{avg,gov}})$ quarters. We later set this equal to the average maturity of government debt.

Most of the paper focuses on lump-sum taxes. Lump-sum taxes will be set according to

$$\frac{t_t - g}{y_t} = \frac{t - g}{y} + \phi_{T,y} \log(y_t/y) + \phi_{T,b} \left[b_{t-1} - \overline{b} \right], \tag{5}$$

where g marks steady-state government spending. This formulation is in line with estimated deficit rules and incorporates a debt-stabilization motive, see, e.g., Bohn (1998). $\phi_{T,y}$ is a key parameter in the current paper, as it measures the semi-elasticity of tax revenue with respect to output.

The central bank operates by influencing the overnight rate in the interbank market, R_t. It conducts open-market operations with a view to influence the private sector interest rate, R_t^{pr}, that is the relevant rate for consumption and savings decisions. The central bank's target level for this rate is denoted by $R_t^{pr,*}$. The central bank's target level follows a Taylor-type rule

$$\log(R_t^{pr,*}/R^{pr}) = \phi_\Pi \log(\Pi_t/\overline{\Pi}) + \phi_y \log(y_t/y_t^{\text{flex}});\ \phi_\Pi > 1, \phi_y \geq 0. \tag{6}$$

Above, $R^{\text{pr}} = \overline{\Pi}/\beta$ denotes the steady-state private-sector interest rate, and $\overline{\Pi}$ denotes the target for the (gross) inflation rate, $\Pi_t = P_t/P_{t-1}$. When government debt is low and stable, this rate moves in close sync with the overnight interest rate, R_t. Rising sovereign risk premia, however, drive a state-dependent wedge between the two rates. Also, the central bank can cushion any rise in risk premia by reducing the interbank rate. When the target rate, $R_t^{pr,*}$, is low, however, or if sovereign risk premia are high, the ZLB on interest rates may become binding. More precisely, implementing the policy, the central bank is constrained by the ZLB on its policy instrument: $R_t \geq 1$.[8] This is not to deny the possibility that central banks can affect economic conditions even when the short-term nominal interest rate is at the lower bound, as indeed several central banks have attempted through various unconventional operations since 2008. However, the significant uncertainty about the effectiveness and risks

[8] The target level for the private interest rate (6) implies the following target rule for the central bank's instrument:

$$\log(R_t^*) = \log(R) + \frac{1}{1-\alpha} \left[\phi_\Pi \log(\Pi_t/\overline{\Pi}) - \alpha \log(R_t^{\text{gov}}/R^{\text{gov}}) \right].$$

The central bank sets $R_t = \max\{R_t^*, 1\}$.

of such operations may itself be regarded as a policy constraint.

3.4 Market clearing

Total demand is given

$$y_t = c_t + g_t.$$

Total supply of output is given by

$$y_t = zn_t/s_t,$$

where s_t measures price dispersion, as in Schmitt-Grohé and Uribe (2004):

$$s_t = (1-\theta)(p_t^*)^{-\epsilon} + \theta(\Pi/\Pi_t)^{-\epsilon}s_{t-1},$$

where p_t^* is the relative price chosen by optimizing firms in period t, and Π is the steady-state inflation rate.

4 The timing of fiscal retrenchment

In this section, we use model simulations to explore how the timing of fiscal retrenchment affects an economy that recovers from a deep recession in the course of which the ZLB became binding. Relative to our earlier work on the same subject (Corsetti et al. 2010), here we explicitly allow for a response of the sovereign risk premium to the consolidation effort.

We envisage a scenario in which, in the absence of a fiscal reaction, the economy is assumed to remain at the ZLB for several quarters, caught in a deep and protracted downturn; a similar scenario is considered, for example, in Eggertsson and Woodford (2003). We then investigate the effect of a program of government spending cuts relative to this benchmark scenario. The fiscal program is announced in the initial period, but may prescribe that the spending cuts be implemented with a certain delay. Our goal is to contrast the macroeconomic transmission of early fiscal measures with measures that will take effect later in time.

In our model economy, there are two main channels through which fiscal consolidation affects the economy, depending strongly on the timing of its implementation. On the one hand, when the policy rate is at the ZLB, any reduction in the sovereign and private risk premium directly translates into a reduction of the real interest rate faced by private agents. To the extent that early consolidation measures reduce the debt burden, and thereby the risk premium, they also stimulate economic activity. On the other hand, a reduction in government spending lowers demand and thus exerts a disinflationary effect. If this cannot be accommodated by monetary policy, because the economy is at the ZLB, for example , disinflation translates directly into an increase of real interest rates, with adverse effects on economic activity and, through the fall in tax revenue, on the government budget and risk premia.[9] If fiscal correction is instead implemented when the central bank can accommodate it by lowering rates, its transmission is less contractionary and its anticipation while the economy is at the ZLB can actually raise demand. The relative strength of either channel is therefore likely to differ sharply with the timing of the spending cuts, as shown by our quantitative explorations below.

4.1 Calibration

A time period in the model is one quarter. A specific feature of our model economy is that the risk premium depends on the level of sovereign debt. In order to operationalize this idea, we postulate the nonlinear functional form (2). This is governed by three parameters. In order to pin these down, we use the empirical, cross-country relationship between the level of gross public debt and interest rate spreads (as measured by credit default swaps, CDS) displayed in Figure 1. The parameters which minimize the sum of squared deviations are $\psi_1 = 1.28$, $\psi_2 = 0.32$, and $\psi_3 = -0.02$. While Figure 1 suggests that, across countries, there is an obvious relationship between debt levels and the risk premium, we reiterate a caveat already mentioned in Section 2. Despite the apparent cross-sectional relationship between CDS spreads and debt levels shown by Figures 1 and 2, there is considerable variation in this relationship both over time and across countries. In particular, for any given level of outstanding debt these rates can vary substantially with the institutional environment in which each country operates and (expectations of) the fiscal stance in the more distant future, for which current and expected debt levels are not necessarily sufficient statistics. Clearly, our way of modeling is only a first cut at these aspects. Next, parameter α in the model indexes the degree to which sovereign risk premia spill over to private sector interest rates. In our baseline scenario, we set $\alpha = 1$, which corresponds to a one-for-one spillover. Empirically, we motivate this assumption with the evidence displayed by Figure 4, which shows a strong co-movement between sovereign CDS spreads and senior financial CDS spreads during the most recent bond market turmoil. The remaining parameters are chosen such that the model captures key features of the US economy. Estimates provided by the OECD for the output responsiveness of the government budget suggest the parametrization $\phi_{T,y} = 0.34$.[10] The baseline abstracts from distortionary taxation. To determine the Frisch elasticity of labor supply, we set $\omega = 1/1.9$ in line with the arguments provided by Hall (2009). We set the degree of price stickiness to $\theta = 0.9$, a

[9] Conversely, abstracting from risk premia, an increase in government spending raises inflation and reduces real rates at the ZLB. Government spending multipliers are thus likely to be large, see Christiano et al. (2009).

[10] See Girouard and André (2005); to obtain these estimates, the OECD follows a disaggregated approach, distinguishing four sources of tax revenues: personal income tax, social security contributions, corporate income tax and indirect taxes; in addition the estimates take into account unemployment-related transfers. For all five categories, the output elasticity is decomposed into i) the tax-base elasticity of a particular revenue/expenditure type and ii) the output elasticity of the tax/expenditure base in question. These components are quantified on the basis of different estimation strategies and combined to compute the output semi-elasticity of the budget, which, in our model, is captured by the parameter $\phi_{T,y}$.

value which results in an empirically realistic slope of the Phillips curve.[11] We normalize steady-state output to unity (by appropriately choosing z), and set the own-price elasticity of demand to a customary value of $\epsilon = 11$. We set the value of κ_n so as to target steady-state employment of $n = 1/3$. For our baseline scenario, we assume log utility, or $\sigma = 1$. As regards $\overline{b}$, in our simulations the long-run target level for government debt is equal to 60 percent of annual GDP. This value is chosen so as to be in line with the average ratio of gross financial liabilities to GDP for the consolidated US general government sector since 1984 (based on OECD data). The long-run target level of government spending (consumption and investment) relative to the size of GDP is $\overline{g} = 0.19$, based on the US average between 1984 and 2006. The inflation target, $\overline{\Pi}$, is set to an annualized rate of 2 percent. The steady-state level for the central bank's target interest rate, R, is 4.6 percent annualized. These values are broadly in line with US averages over the last 20 years and imply a value for the time-discount factor of $\beta = 0.9937$. The coefficients in the Taylor rule are $\phi_\Pi = 1.5$ and $\phi_y = 0$.

As the average maturity of privately held marketable interest-bearing public (federal) US debt in the 12 months through June 2010 was 52 months, we set $\rho^{\text{avg, gov}} = 0.5^{1/(52/3)} = 0.96$. Parameter ζ, which indexes the maturity structure of debt determining the risk premium, is set equal to the same value. Note that this also matches, if only roughly, the horizon of the CDS contracts shown in Figure 1. Finally, in this paper we are interested in examining the short-run transmission of fiscal retrenchment on the economy, when sovereign risk premia depend on debt dynamics. Parameter $\phi_{T,b}$, which determines how fast debt will be stabilized over time in the absence of spending cuts, is set to a positive, but small value, $\phi_{T,b} = 0.01$. This implies that debt stabilization will phase in, but only slowly over time. Moreover, unless stated otherwise, the simulations below assume that there is no response of taxes to the debt level for the first 21 periods. This more than covers the entire period of the ZLB episode.

4.2 Baseline: a deep recession and no spending cuts

To set the stage for our analysis, we begin by studying the impact of a large recessionary shock in the absence of any spending cuts. For four different initial debt levels, Figure 5 shows the evolution of the economy after a sudden but persistent increase in the consumers' time-discount factor. While the steady-state debt level is assumed to be the same in each of these economies, namely 60 percent, we consider different values for the initial level of debt relative to GDP: 60, 90, 100, and 110 percent, respectively. The 90 percent debt-to-GDP-ratio is in line with the level of gross US government debt in 2010, the higher values are representative of the fiscal situation in some other OECD countries. The depth and persistence of the demand shock is calibrated to match the following criteria in the economy

[11] Our parametrization implies a slope of the Phillips curve of $\kappa = 0.0117$. Gali and Gertler (1999) report estimates for the slope of the Philips curve, given by $(1 - \beta\theta)(1 - \theta)/\theta$ in the range between 0.007 and 0.047. More recently, Altig et al. (2010) report an estimate of 0.014.

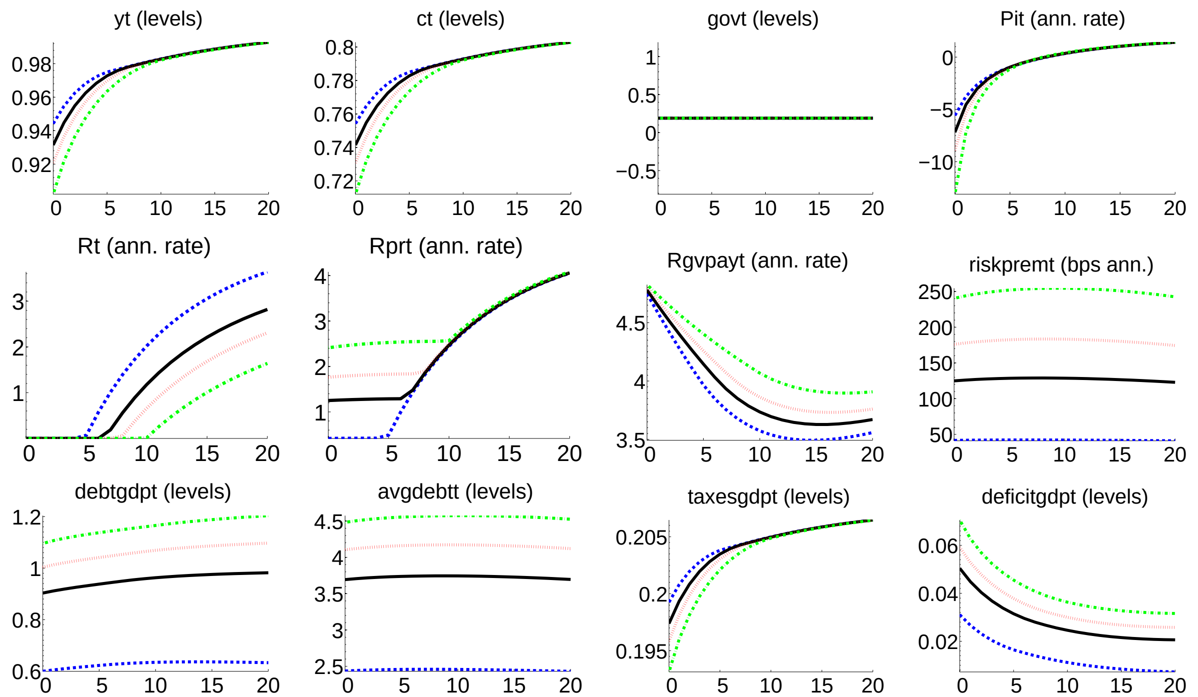

Figure 5: A deep recession without discretionary spending measures. Shown are the responses in the economy for the same shock, but different initial debt levels. The blue dashed line shows the responses with an initial debt-to-GDP ratio of 60 percent; black solid line: 90 percent initial debt-to-GDP ratio; red dotted: 100 percent initial debt-to-GDP ratio; green dashed dotted line: 110 percent initial debt-to-GDP ratio.

with an initial debt level of 90%: first, the initial drop of output is about 7 percent, which conforms with the gap between the CBO measure of potential output and actual GDP in 2010H1 in the US; second, the lower bound remains binding for six additional quarters, which, at the time of writing, roughly reflects the markets' expectation of the persistence of the ZLB situation as embedded in the fed funds futures path. More precisely, for the shock process we parameterize $e^d_{t+1}/e_t = \rho_d e^d_t / e^d_{t-1} + \epsilon^d_t$. The implied values for the persistence parameter and the initial innovation are $\rho_d = 0.91$ and $\epsilon^d_0 = 0.0118$, respectively. Note that the latter amounts to assuming a 1.18 percent shock to the time-discount factor.

Results from our first set of simulations are reported in Figure 5. This figure reports impulse responses in economies with an initial debt level of 60, 90, 100, and 110 percent, denoted by the blue dashed line, the black solid line, the red dotted line, and the green dashed-dotted line respectively. As apparent from the figure, the sovereign risk premium drives a wedge between the central bank and government rates of interest, and thereby also between private sector interest rates and the central bank rate. Not only does a higher initial level of debt make the recession deeper, it also prolongs the time span over which the ZLB constraint remains binding, due to the fact that the risk premium remains persistently high. The difference is as large as four quarters when comparing the highest-debt economy with the lowest-debt economy in the figure. By the same token, Figure 5 also shows a notable increase in the government budget deficit and in government debt as the ZLB situation unfolds. Nonetheless, the risk premium is barely affected in the low-debt economy, whereas the premium rises for some time if the initial debt-to-GDP ratio is higher. In our interpretation, the large additional output losses and deficit effects that ensue at higher debt levels clearly underscore the importance of fiscal prudence in good times.

4.3 The effects of spending cuts with different implementation dates

Starting from the baseline scenario displayed in Figure 5, we now consider a program of fiscal tightening measures. We posit a consolidation scenario in which the government cuts spending by 1 percent of steady-state GDP for four consecutive quarters. The program is announced at time 0, but may be structured in such a way that the actual cuts are enacted at a later date. While, of course, real-world consolidation packages are likely to involve longer-lasting cuts and a time-varying intensity, the sharp consolidation measures we focus on serve to highlight most clearly the relevant transmission mechanism for a sizeable fiscal correction aimed at producing tangible immediate effects on debt dynamics.[12] As before, we distinguish the effects of these spending cuts in economies that converge to the same equilibrium in the long run, but start at different initial levels of debt, namely 60, 90, 100 and 110 percent of GDP.

[12] Spending packages that entail more protracted cuts can be thought of as a combination of different packages with the characteristics discussed in the text.

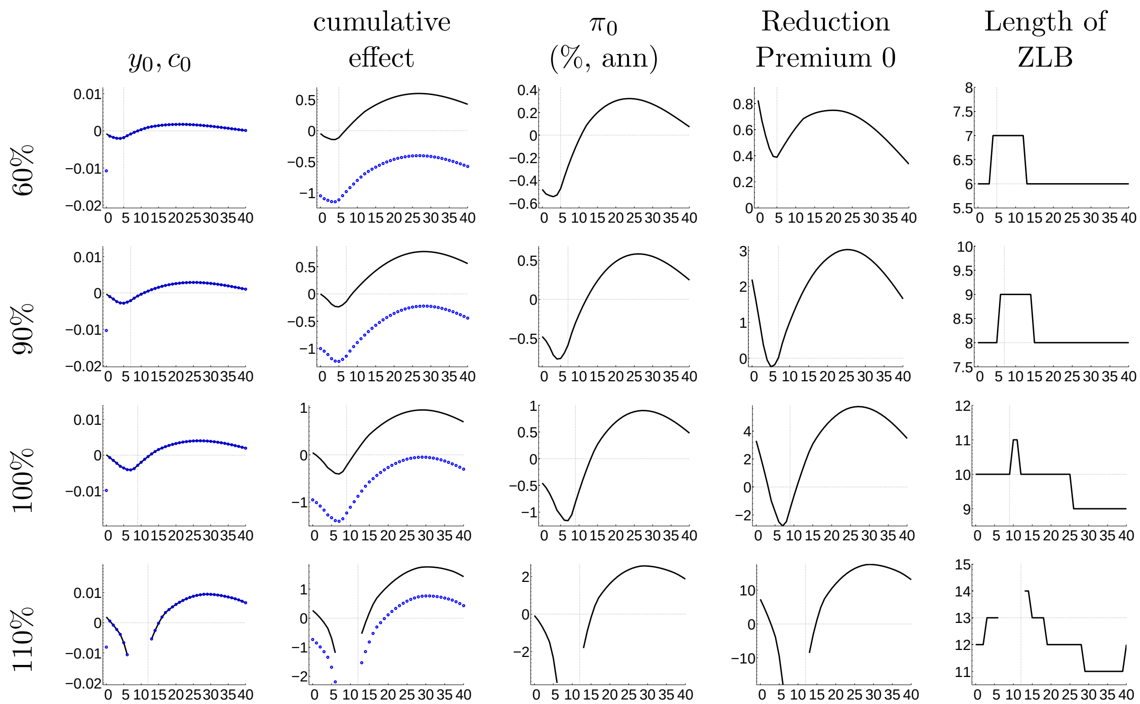

Figure 6: Effect of government spending cuts relative to the baseline scenario (no cuts). A cut in government spending of 1 percent of steady-state GDP lasts for four consecutive quarters. It is enacted in either period 0, period 1, ..., or period 40. The different rows show different levels of initial debt (steady-state debt-level is 60 percent in each case). The left column shows the period-0 response of consumption (black solid) and output (blue dots). The second column entitled "cumulative effect" plots the cumulative increase in output relative to the baseline over the first 100 quarters divided by the cumulative change in government spending (both discounted at the time discount factor); black solid lines show the cumulative effect on consumption, blue dots the effect on output. The third column shows the period-0 effect on inflation (annualized and in percentage points). The fourth column shows the reduction in the period-0 risk premium due to the spending reversal (bps, ann.). The final column shows the length of the ZLB episode. For the figures, the event "exit from the ZLB" is triggered when interest rates rise above 50 bps (ann.). In each of the panels, the vertical line indicates the last quarter at the ZLB in the baseline without any fiscal retrenchment. A horizontal dotted line for the multiplier and inflation graphs marks no change relative to the baseline. In the ZLB graphs, the vertical line marks the ZLB exit in the baseline. The algorithm did not solve for configurations in which the lines are broken.

Figure 6 displays a number of statistics pertaining to the different experiments that we perform for each of our economies. In each graph, the horizontal axis indicates in which period spending cuts begin. In interpreting the simulations, it is important to bear in mind that, independently of the timing of their implementation, the fiscal consolidation programs all have the same length and depths, and all become known in period 0. For instance, '0' on the horizontal axis means that the four-quarter contraction in government spending starts at the beginning of the simulation horizon; '1' means that the cuts are programmed to start with a delay of one quarter and so on.

The panels in the left column of the figure show the period-0 response of consumption (black solid) and output (blue dots) under the consolidation programs relative to the case of no fiscal adjustment (shown in Figure 5). The second column entitled "cumulative effect" plots the cumulative increase in output or consumption, again relative to the baseline with no fiscal reaction, over the first 100 quarters (discounted at the time discount factor) divided by the cumulative change in government spending (also discounted at the time discount factor) over the same period; a black line shows the cumulative effect on consumption, blue dots the effect on output. A positive value means that the cumulative spending cuts increase c or y cumulatively over the relevant time horizon. The third column shows the period-0 effect of the announcement of the consolidation program on inflation (annualized and in percentage points). The fourth column shows the drop in the period-0 risk premium (in basis points, annualized). The final column, in turn, shows the number of quarters during which the economy persists at the ZLB, depending on when the spending cuts are implemented. For these graphs, the economy is assumed to "exit from the ZLB" when policy rates rise above 50 bps (annualized). In each of the graphs a vertical line shows the last period during which the lower bound would be binding in the baseline economy without a fiscal adjustment.

Turning to our findings, note first that spending cuts that are enacted immediately (*immediate implementation*) tend to reduce both period-0 consumption and output (left column). In addition, they tend to prolong the time during which the economy remains stuck at the ZLB (right column). The cumulative effects of spending cuts on output and consumption are negative as well in this case (second column). An exception is the case where initial debt is high (last row): here we find a (small) increase in consumption (but not in output) upon impact, as well as a positive cumulative effect for consumption. To understand these results note that immediate implementation lowers the risk premium in all scenarios (column four). However, the effect of lower risk premia is dominated in all but the highest-debt scenario by the disinflationary impact of the spending cuts (column three), which, all else equal, raises real interest rates and thereby weakens private demand.

In contrast, if implementation is delayed considerably (*late implementation*), the impact responses of consumption and output are positive—as is the cumulative effect on consumption. To understand this result, recall that, in the absence of fiscal retrenchment, the economy exits the ZLB after 6 quarters. Delaying correction further implies that, once spending cuts exert their disinflationary effects, monetary policy is again able to offset them by lowering policy rates, in both nominal and real terms. At time zero, the anticipation of this decline in short-term real interest rates translates into a drop in the long-term rate—the rate which is relevant to private spending decisions. This mechanism then sets in motion a virtuous cycle, by which a rise in demand leads to higher inflation and thus to lower real interest rates for the time period at the ZLB, which further serves to increase demand. It is by this mechanism that delaying fiscal consolidation beyond the period in which the economy exits from the ZLB proves to be beneficial, in terms of containing the upfront fall in consumption and output when the recessionary shock hits.[13] The effect of the anticipated future retrenchment on demand and economic activity in the short run more than compensates for the adverse effect of somewhat higher risk premia in the case of delayed consolidation.[14]

More interesting is the case of an intermediate timing in the implementation of spending cuts (*intermediate implementation*). Indeed, this is the case for which we find the least beneficial effects of fiscal consolidation, in all debt scenarios and across all statistics of interest. Not only are the impact and cumulative effects on consumption and output negative in this case, but the decline in the risk premium is also more muted than in the case of immediate or late implementation. The key issue here is that the anticipation of near-term spending cuts, i.e., at a time when the economy has not fully emerged from recession, tends to exacerbate the disinflationary effects of the initial recessionary shock over that horizon. This translates into even higher long-term real rates and therefore lower demand on impact. Because of the adverse cyclical effects, public finances actually deteriorate, and the positive budget effect of the spending cuts can be dominated by the cyclical drop in tax revenue. In fact, in all but the lowest-debt scenario, the consolidation effort via spending cuts turns out to be self-defeating, resulting in rising risk premia.

It is informative to compare the effects of changing the timing of the fiscal program across the different initial levels of debt. In one dimension, our figure suggests minor differences across scenarios. Specifically, late implementation appears to be the best option in terms of cumulative output effects, whether or not the initial debt is low.[15] However, there is also a

[13] A key role is played by staggered forward-looking price setting, which implies that the disinflationary effect of spending cuts makes itself felt before these cuts are actually implemented, as discussed in Corsetti et al. 2010.

[14] Note that the central bank can offset changes in the sovereign risk premium after the ZLB ceases to bind. As a result, the effect on real interest rates that comes with early spending cuts is confined to the ZLB period, which is relatively short in all the cases considered.

[15] To understand this finding note that none of the early-timed consolidations are truly non-Keynesian, i.e. output always falls at the time of the spending cut. This implies that the initial inflation response in the early-timed consolidations is negative throughout the ZLB period, which drives the real rate of interest upward. At the same time, the reduction in output means less tax revenue. This reduces the effectiveness of the spending cuts in lowering the debt level. Thus, while the early spending cuts reduce the debt level and the risk premium, this effect is not sufficiently strong to make up for the negative demand-side effects

striking difference. In the high-debt scenario, there is a considerable range of implementation periods for which numerically we obtain no solution at all. For this range, no statistics are reported in the last row of Figure 6. Picking up on this finding, the next section highlights that an economy with high debt can be prone to belief-driven equilibria. To understand why, consider the possibility that during the ZLB period agents expect some drop in output. A drop in output means less tax revenue, higher deficits, higher debt and thus, ultimately, a higher risk premium. At the lower bound, a rise in the risk premium cannot be offset by monetary action: this is possible only in the distant future, after the economy exits from the ZLB. But a rising risk premium would immediately push back in time the exit from the ZLB, thereby delaying the time before a monetary reaction is possible. As a result, expectations of negative output developments can become self-fulfilling in high-debt economies, with a high and rising risk premium weighing heavily on output and thus confirming agents' beliefs. We explore this issue in greater detail in the next section.

Before doing so, however, we conclude this section with an observation. In our stylized model, the beneficial effects from delayed consolidation are large when spending cuts start at least four years after the beginning of the recession. Hence, for some countries, the actual policy choice may be restricted to a more narrow set of near-term consolidation options. Although our model does not allow us to formally elaborate on this, it may be reasonable to expect that such commitment problems might be particularly acute for economies that face high stocks of public debt, especially where these coincide with a track record of weak fiscal institutions. In this case, intermediate horizons for delayed consolidation, and thus the risk of indeterminacy highlighted by our model, are likely to have practical relevance. Over these horizons, attempts to delay fiscal retrenchment may be frustrated by the emergence of self-fulfilling equilibria, which may eventually lead governments to resort to immediate action.

5 Government spending and the risk premium at the ZLB

In this section, we reconsider our main results analytically, using a simplified version of our framework. Specifically, we study a linear(ized) version of the model around the deterministic steady state, but take into account the non-linearity induced by the ZLB on interest rates by means of a Markov-structure, as in Christiano et al. (2009) and Woodford (2010). Also different from the analysis in the previous section, we posit that the consumption preference shock (alternatively, the financial premium shock) which leads the economy into the lower bound, persists for a future period with probability $\mu \in [0, 1)$. Hence, the end of the ZLB episode is no longer endogenous with respect to fiscal policy. Moreover, we also assume that the risk premium is a function of the expected deficit rather than of the stock of debt, and all taxes are lump sum. We thus disregard debt dynamics, implicitly assuming that debt will

on the real rate of interest.

be stabilized eventually.

Importantly, however, the model economy shares a key feature with the one analyzed so far: for any given inflation target, a higher (steady-state) risk premium (in this section corresponding to a higher steady-state deficit), a) raises the likelihood that the economy will be pushed into the ZLB and b) increases the depth of the ensuing recession—strengthening the case for fiscal prudence in good times.

In the following, we revisit analytically our results suggesting that an endogenous risk premium increases the risk of belief-driven equilibria. We first establish how this risk is altered if government spending is output elastic. Specifically, we show that pro-cyclical spending (so that spending is cut during a recession) enlarges the range of parameters for which the equilibrium is unique and stable (while the opposite is true if the risk premium does not respond to deficit).[16] Next we reconsider the determinants of the sign and magnitude of the spending multiplier when the economy is at the ZLB.

5.1 The simplified model economy

Under our simplifying assumption, there is no intrinsic persistence: once the shock ceases to persist, the economy leaves the lower bound and reverts to its non-stochastic steady state. Denoting deviations from steady-state with a tilde, and log deviations with a hat, the linearized IS equation in our model can be expressed as

$$\tilde{y}_t - \tilde{g}_t = E_t\tilde{y}_{t+1} - E_t\tilde{g}_{t+1} - \varrho\left[\widehat{R}_t + \alpha\widetilde{\psi}_t + \widehat{\Delta}_t - E_t\widehat{\Pi}_{t+1}\right]. \quad (7)$$

$\widehat{\Delta_t}$ is the shock to the household's inter-temporal discount factor. Π_t marks the gross inflation rate and ψ_t marks the risk premium, or the "sovereign spread." As regards parameters, $\varrho = \frac{1-g}{\sigma}$, where g is the steady-state level of government spending and $1/\sigma$ is the intertemporal elasticity of substitution in consumption. $\alpha \in [0, 1]$ indicates to what extent to which sovereign risk spills over to the private sector.

The risk premium is posited to increase with the expected deficit,

$$\widetilde{\psi}_t = \xi E_t(\tilde{g}_{t+1} - \tilde{t}_{t+1}).$$

with $\xi \geq 0$. Linearizing equation (5) and disregarding the debt-feedback term while at the ZLB, the government revenue at the ZLB depends on output according to $\tilde{t}_t = \chi\tilde{y}_t$, whereas the elasticity of taxes with respect to output is $\chi = (t-g)/y + \phi_{T,y}$. Combining this expression

[16] Here we focus on local determinacy once the economy has reached the lower bound. Another strand of the literature examines global determinacy in the New Keynesian model and is concerned with preventing the economy to fall into a liquidity-trap in the first place. Benhabib et al. (2002) for example propose switching to non-Ricardian fiscal policy. These mechanisms will rule out liquidity traps by making the low-inflation steady state fiscally unsustainable. Mertens and Ravn (2010) study the efficacy of fiscal policy in belief-driven equilibria.

with that for the risk premium, we can rewrite the risk premium as a function of government spending and output only:

$$\widetilde{\psi}_t = \xi E_t(\tilde{g}_{t+1} - \chi \tilde{y}_{t+1}). \tag{8}$$

Monetary policy takes the same form as in (6), respecting the lower bound:

$$\widehat{R_t} = \max\{\phi_\pi \widehat{\Pi}_t - \alpha \widetilde{\psi}_t, -\log(R)\}. \tag{9}$$

where $\phi_\Pi > 1$, so that determinacy is ensured when away from the lower bound; and R marks the steady-state value of the central bank's policy rate.
The Phillips curve is given by

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{g}_t, \tag{10}$$

where $\beta \in (0,1)$ is the time-discount factor, $\kappa_y := \kappa[\omega + 1/\varrho]$ and $\kappa_g = \kappa/\varrho$, where $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ is the slope of the Phillips curve.

5.2 An in-depth analysis of determinacy issues

At the ZLB, an important risk faced by policymakers is that expectations become unanchored. We therefore begin our analysis by assessing how a risk premium affects the determinacy properties of the model. Proposition 1 gives the restrictions on parameters for ensuring that the equilibrium is (locally) determinate. Scrutinizing these conditions, it emerges that accounting for the sovereign risk premium reduces the determinacy region.

Proposition 1 *In the economy summarized by equations* (7)*,* (8) (9)*,* (10)*, let* $\widehat{\Delta}_t$ *take on a positive value* $\widehat{\Delta} > 0$ *in period zero, and remain such with probability* μ *in each subsequent period, until it reverts to zero, after which* $\widehat{\Delta}_t = 0$ *forever. Furthermore, let the value of* $\widehat{\Delta}$ *be large enough that the lower bound is binding initially. With an a-cyclical* $\tilde{g}_t$*, this economy has a unique bounded equilibrium if and only if*

$$a)\quad a < 1/(\beta\mu), \quad \text{and} \quad b)\quad (1-\beta\mu)(1-a) > \mu\varrho\kappa_y,$$

where $a := \mu + \mu\alpha\xi\chi\varrho$ *and* $\kappa_y := \kappa[\omega + 1/\varrho]$.

Proof. See Appendix A. ■

If the risk premium does not impact on the economy, $\xi = 0$, as in Christiano et al. (2009) and Woodford (2010), condition a) is always satisfied. So there will be a unique bounded equilibrium if and only if condition b) holds, that is if $(1-\beta\mu)(1-\mu) > \mu\varrho\kappa_y$. The previous literature has shown that the set of "fundamental" parameters for which this condition holds is larger, (i) the less persistent the lower bound situation (in our parameterization, the smaller

μ), (ii) the lower the interest-sensitivity of demand (the smaller ϱ) and (iii) the flatter the Phillips curve (the smaller κ_y).

Relative to these findings, our analysis shows that the range of parameters for which the equilibrium is determinate actually shrinks in the presence of an endogenous risk premium. Namely, condition a) is violated if either the risk premium is sufficiently responsive to the deficit or if the tax revenue is sufficiently responsive to output (χ is large enough). Note that the same parameters are also key determinants for whether condition b) is satisfied.

In our first proposition, we rule out the possibility that government spending systematically responds to output. We now show that allowing for this possibility substantially alters the determinacy properties of equilibria. Specifically, we find that at the ZLB determinacy may well require spending to be pro-cyclical, i.e. recessions must be matched with contemporaneous spending cuts, improving the fiscal outlook of the country.

Specifically, we assume that government spending takes on a value of $\tilde{g}_t = \varphi\tilde{y}_t$, with $\varphi < 1$ as long as the economy remains at the zero bound, while immediately reverting to its steady-state level once the economy leaves the lower bound. For this case, the following proposition summarizes the conditions for the existence of a unique bounded equilibrium.

Proposition 2 *In the economy specified in Proposition 1, let $\tilde{g}_t$ take on a value of $\tilde{g}_t = \varphi\tilde{y}_t$, when the economy is at the ZLB, 0 otherwise. Suppose further that $\varphi \neq 1$. Define $a^* := \mu + \mu\alpha\xi\chi^*\varrho^*$; $\kappa_y^* = \kappa_y - \varphi\kappa_g$; $\chi^* := \chi - \varphi$, and $\varrho^* = \varrho/(1-\varphi)$. There exists a unique bounded equilibrium if and only if:*

1. *with $a^* > 0$*

$$a)\quad a^* < 1/(\beta\mu), \quad \text{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\varrho^*\kappa_y^*,$$

$$[\text{ and if } \varphi > 1], \quad c)\quad (1+\beta\mu)(1+a^*) > -\mu\varrho^*\kappa_y^*$$

2. *with $a^* < 0$:*

$$a)\quad (1+\beta\mu)(1+a^*) > -\mu\varrho^*\kappa_y^* \quad \text{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\varrho^*\kappa_y^*.$$

Proof. See Appendix A. ■

This establishes that the determinacy region depends on how government spending responds systematically to output while the economy persists at the lower bound.

Corollary 3 *Under the conditions of Proposition 2, the following special cases obtain:*

1. *With no endogenous risk premium ($\alpha\xi = 0$), the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ($\varphi < 0$), rather than*

acyclical. In addition, the range of fundamental parameters implying determinacy of the equilibrium is smaller, the larger φ.

2. *With an endogenous risk premium $\alpha\xi > 0$, instead, the range of parameters for which the equilibrium is determinate often is larger if spending is pro-cyclical, i.e., if spending is cut during a deep recession. More precisely:*

   (a) *If the conditions of Part 1) of Proposition 2 hold and $\varphi \in (0,1)$ and $\chi < 1 - \frac{\kappa\omega}{(1-\beta\mu)\alpha\xi}$, then the range of fundamental parameters for which the equilibrium is determinate is at least as large as in the absence of an endogenous response in spending, and can be larger. Note that this case is more likely the less elastic the tax revenue to the state of the economy (the smaller χ), and the more responsive the country's risk premium to the deficit (the larger ξ).*

   (b) *It may occur that the equilibrium is indeterminate if government spending does not respond to output, but becomes determinate with a mild procyclical response that satisfies $\frac{1+\alpha\xi\varrho\chi}{1+\alpha\xi\varrho} < \varphi < 1$ (this is is only case under which the conditions of Case 2) of Proposition 2 can hold). Note that this inequality is more likely satisfied the steeper the risk premium and the less elastic the response of taxes.*

Proof. See Appendix A. ■

Corollary 3 establishes that countercyclical spending policies ($\varphi < 0$) enlarge the range of fundamental parameters for which the equilibrium is determinate in the absence of a response in the risk premium. When the risk premium is endogenous, instead, the opposite may be true: determinacy is more likely if fiscal retrenchment is undertaken during deep recessions, when the economy is at the ZLB. By the conditions discussed in the corollary, to rule out indeterminacy a pro-cyclical spending reaction is required the most, when the risk premium is sufficiently steep.

For the baseline parameterization of the model, we illustrate our results graphically in Figure 7 for different values of the slope of the risk premium and of the response coefficient of government spending. Across the graphs in the figure we also vary the output elasticity of taxes. Grey areas indicate determinacy regions, while white areas indicate equilibrium indeterminacy. Observe that, in our numerical examples, as the risk premium becomes sufficiently responsive, there may be no countercyclical policy which ensures determinacy.

5.3 The spending multiplier at the lower bound

Our model also sheds light on a widely discussed issue that regards the size of the multiplier at the ZLB. Focusing the analysis on parameter values for which the equilibrium is unique, we now turn to a formal characterization of the spending multiplier on output. In doing so, we

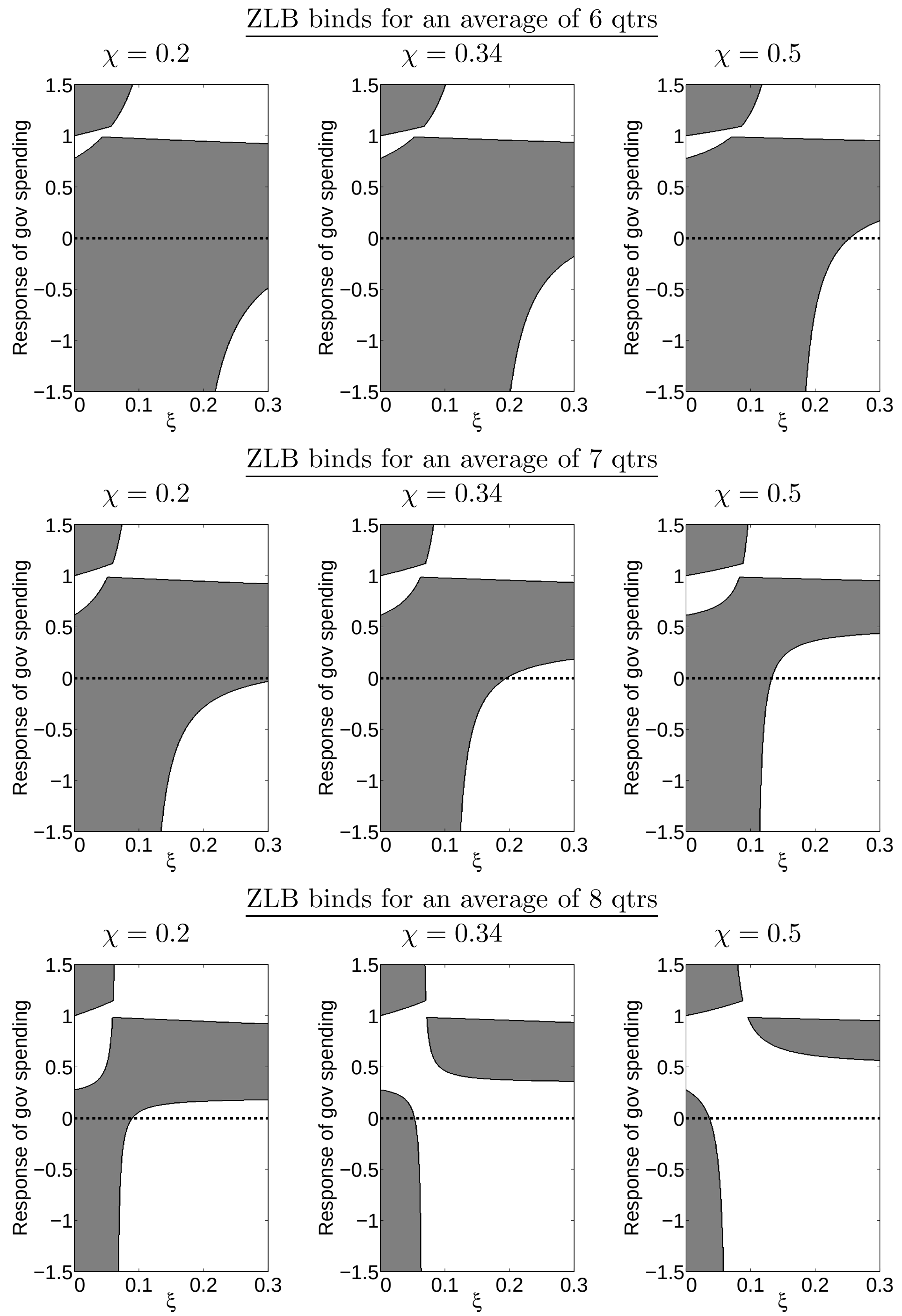

Figure 7: Determinacy regions with endogenous response of government spending to state of the economy in recession. Grey areas mark parameterizations that imply determinacy. White areas mark parameterizations for which the equilibrium is indeterminate. y-axis: response of government spending to output, φ ($\tilde{g}_t = \varphi \tilde{y}_t$). In each panel, a horizontal dashed line marks the case in which government spending does not respond to output in the ZLB phase, $\varphi = 0$. x-axis: response of risk premium with respect to deficit, ξ. From top to bottom: ZLB is expected to bind for 6, 7, or 8 quarters, $\mu = 5/6, 6/7, 7/8$. From left to right: tax elasticity $\chi = 0.2, 0.35$, and 0.5.

follow Woodford (2010) and Christiano et al. (2009) and assume that government spending takes on a value of g_l above its steady-state value only when the economy is at the ZLB, otherwise it is set to its steady-state level. Proposition 4 summarizes how the multiplier and the depth of the recession depend on the interplay of the slope of the risk premium and the output multiplier of taxes.

Proposition 4 *Under the conditions spelled by Proposition 1 for a unique bounded equilibrium to exist, let government spending take on a value of g_l whenever the lower bound is binding, and zero otherwise. As before, define $a = \mu + \mu\alpha\xi\chi\varrho$, and $b = \mu + \mu\varrho\alpha\xi$. Then, while the economy is at the ZLB, output is given by*

$$y_l = \vartheta_r(\log(R) - \Delta) + \vartheta_g \, g_l,$$

where

$$\vartheta_r = \frac{\varrho(1-\beta\mu)}{(1-\beta\mu)(1-a) - \mu\varrho\kappa_y} > 0 \tag{11}$$

and

$$\vartheta_g = \frac{(1-\beta\mu)(1-b) - \mu\varrho\kappa_g}{(1-\beta\mu)(1-a) - \mu\varrho\kappa_y}. \tag{12}$$

Proof. See Appendix A. ■

Proposition 4 implies – somewhat straightforward given the structure of the model – that for a given target level of inflation, the higher the steady-state risk premium is, the deeper is the recession when the shock hitting the economy is severe enough to drive it into the ZLB. The reason is that the risk premium reduces the average level of the central bank rate for any given inflation rate. Specifically, the steady state of the model economy gives $\log(R_t^{\text{pr}}) = \log(R_t) + \alpha\psi$, where $\psi = \exp((b/4y - \psi_1)/\psi_2) + \psi_3)/100$ is the steady-state risk premium. To the extent that higher steady-state deficits (or debt, in the more general version of the model) increase the sovereign risk premium, and this spills over to the private sector ($\alpha > 0$), a wedge emerges between the private sector interest rate and the central bank's interest rate, $\log(R) = \log(R^{\text{pr}}) - \alpha\psi = \Pi/\beta - \alpha\psi$. So $\log(R)$ will be the smaller the bigger the risk premium: as there is less leeway for the central bank to counteract recessionary shocks in the first place, the lower bound will become binding more easily (for smaller values of Δ).

Let ϑ_g denote the government spending multiplier at the ZLB. The following corollary states the conditions under which such multiplier is positive. The corollary also states conditions under which the multiplier is larger than one, and so large that government spending becomes self-financing.

Corollary 5 *Under the parameter restrictions of Proposition 1:*

1. *The government spending multiplier,* ϑ_g, *is positive if and only if*

$$(1-\mu) - \frac{\mu \varrho \kappa_g}{1-\beta\mu} > \mu\alpha\xi\varrho. \tag{13}$$

   Note that, conversely, the spending multiplier will be negative if the risk premium sufficiently affects the economy, i.e., if $\alpha\xi$ *is large enough.*

2. *If* $\alpha\xi = 0$*, provided that the conditions for determinacy in Proposition 1 are satisfied, the government spending multiplier is strictly larger than one. This case corresponds to the analysis by Christiano et al. (2009) and Woodford (2010).*

3. *If* $\alpha\xi > 0$*, the government spending multiplier is unambiguously larger than one if* $\chi > 1 - \frac{\kappa\omega}{\alpha\xi(1-\beta\mu)}$*, that is, if the tax revenue rises sufficiently fast with output. In addition, government spending at the lower bound is self-financing if* $\vartheta_g > 1/\chi$.

Proof. See Appendix A. ■

Corollary 5 establishes that, abstracting from the sovereign risk premium, the government spending multiplier is unambiguously positive as long as the equilibrium is determinate. In other words, when the economy is at the ZLB, government spending is an effective stabilization tool. This however implies that retrenchment while at the lower bound, or indeed even shortly after the end of the ZLB situation, is bound to reduce output disproportionately, see Corsetti et al. (2010). Conversely, in economies where sovereign risk spills over to the private sector (with $\alpha\xi > 0$), Corollary 5 also unveils that the sign and magnitude of the multiplier will generally depend on the interplay of the risk premium with the tax elasticity. In fact, the cut-off between positive and negative values for the multiplier coincides with the point in which the positive effects on demand of government spending just balance the negative risk premium effects from a deterioration of the budget deficit (for a sufficiently steep risk premium).

5.4 Future austerity and current economic activity

So far, we have discussed fiscal retrenchment while the economy is still at the ZLB. We now consider a retrenchment that is designed to take effect only once the economy has left the ZLB. As discussed in Corsetti et al. (2010), fiscal consolidation some time in the future does reduce demand and inflation at that time, but may have positive effects on impact. In fact, if spending consolidation is implemented when the central bank is no longer constrained by the ZLB, in reaction to it policy rate will fall in both nominal and real terms (recall, $\phi_\pi > 1$). As discussed below, with staggered price setting, this fall will actually occur before the spending cuts take effect. In anticipation of a path of lower interest rates, long-term interest rates will contract as of today, thereby crowding in consumption and output even if the economy continues to be at the ZLB.

However, by the very nature of this transmission mechanism, the exact timing of consolidation is crucial. As already mentioned, when firms anticipate the future drop in demand, because of nominal rigidities they start to reduce prices before government demand actually falls. In other words, inflation falls in anticipation of the retrenchment. If much of this happens too close to the period in which the economy has left the ZLB, its effect on the real rate of interest can be perverse, that is, real rates may rise while the economy still is at the ZLB—hampering the recovery. These considerations are summarized in the following proposition and corollary.[17]

Proposition 6 *In the economy specified in Proposition 1, let $\tilde{g}_t$ take on a value of $\tilde{g}_t = 0$ whenever the ZLB is binding. Once the ZLB ceases to bind, $\tilde{g}_t = g_a < 0$, in the first-period, and subsequently with probability $\nu \in [0, 1)$. Otherwise $\tilde{g}_t = 0$ forever. Assuming that the conditions for determinacy are satisfied, output while at the ZLB is given by*

$$
\begin{aligned}
y_l \;=\; \tfrac{1}{d}\Big[\;& \varrho(1-\beta\mu)[\log(R)-\Delta] + (1-\mu)(1-\beta\mu)(1+\varrho\alpha\xi\chi)(y_a - g_a) + \varrho(1-\mu)\pi_a \\
& - (1-\mu)(1-\beta\mu)\varrho\alpha\xi(1-\chi)g_a \;\Big],
\end{aligned}
\tag{14}
$$

where $d = (1-\beta\mu)[1-a] - \varrho\mu\kappa_y$, $a := \mu + \mu\alpha\xi\chi\varrho$ as in Proposition 1, and y_a and π_a denote, respectively, output and inflation in the austerity period, equal to

$$
y_a = \frac{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_g}{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_y} g_a, \tag{15}
$$

and

$$
\pi_a = \frac{(1-\nu)(\kappa_y - \kappa_g)}{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_y} g_a. \tag{16}
$$

Corollary 7 shows that the effect of future austerity on output while at the ZLB is positive provided that not too much austerity is expected to be concentrated very early after the ZLB ceased to bind (so if ν is large enough).

Corollary 7 *Under the conditions in Proposition 6,*

1. *if $\alpha\xi = 0$, future austerity enhances current economic activity unless too much of it is expected to occur too close to the exit from the ZLB. More precisely, there exists a value of $\nu \in [0, 1)$ such that $y_l > 0$ if $g_a < 0$. This is the case for any $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.*

[17] The scenario that is described in Proposition 6 strictly speaking mixes two different elements of a retrenchment, namely the timing of it and its persistence. We have computed an alternative scenario which disentangles these two elements. The scenario assumed that some time after the economy had left the ZLB there is a period of retrenchment with persistence ν. The timing of the start of the retrenchment was random, however, and retrenchment would start with probability $1-\gamma$, $\gamma \in [0, 1)$ in any given period after the ZLB had ceased to bind. The results were, qualitatively similar to those displayed here: Future retrenchment had a positive effect on output while at the ZLB if retrenchment did not start too early after the end of the ZLB phase (or too late). The formulae in that scenario were unwieldy, however, so we rather report the simpler case given by Proposition 6.

2. *if* $\alpha\xi > 0$ *and, furthermore,* $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$*, the effect of future austerity on current economic activity* (y_l) *is more positive than in the absence of the risk premium, regardless of the size of the tax multiplier.*

3. *Provided that the effect of future austerity on* y_l *is positive, the magnitude of such effect will be larger, the more sensitive the economy is to the risk premium (the larger is* $\alpha\xi$*)(as long as determinacy obtains).*

4. *In addition, there are parameterizations for which the effect of future austerity is positive if* $\alpha\xi > 0$*, while it is negative if* $\alpha\xi = 0$*. At the same time, ill-timed austerity for which* $y_l < 0$ *may be particularly detrimental if* $\alpha\xi > 0$ *and* $\chi > 0$*.*

Proof. See Appendix A. ■

The corollary lends further support to our earlier results that a delayed implementation of spending cuts - with the appropriate timing - can have a particularly strong positive effect on output while the economy is at the ZLB, especially if the risk premium is steep enough. For this result, we again stress the caveats highlighted at the end of Section 4.

6 Conclusion

Given the sharp deterioration of public finances in the wake of the global financial crisis, most industrialized countries are facing a period of significant fiscal retrenchment, including sizeable spending cuts. Starting from this observation we analyze how the economic impact of such spending cuts depends on the precise timing of their implementation. The question has particular relevance in the wake of deep recessions, when monetary policy is constrained by the zero lower bound (ZLB) on nominal interest rates.

Under these circumstances, delaying fiscal retrenchment until monetary policy is no longer constrained may be beneficial for economic activity, because the disinflationary effect of the fiscal retrenchment could then be offset by accommodative monetary policy. In anticipation of such fiscal-monetary interaction, real long-term interest rates would fall prior to the actual spending cuts, supporting demand during the initial recession period. This mechanism is discussed in detail by Corsetti et al. (2010).

In the present paper, we enrich the analysis by adding an important feature that had been absent from our earlier analysis but is likely to play a key role in policymakers' decisions about the right timing of fiscal retrenchment, i.e., sovereign risk premia. Consistent with the empirical evidence, we assume that sovereign risk premia rise as a function of the public debt stock; and that an increase in the sovereign risk premium spills over into private sector funding costs as well. Through this channel, early fiscal consolidation can unfold a positive effect on aggregate demand and output to the extent that it reduces the real interest rate faced by private borrowers. The contribution of this paper is to evaluate quantitatively, in a new Keynesian model, how this effect alters the decision on the appropriate timing of consolidation measures.

Our analysis shows that timing is indeed crucial for the overall macroeconomic response to fiscal retrenchment. We find, in particular, that the initial level of debt interacts in a critical manner with the timing of the spending cuts. If initial debt is low, delaying the implementation of spending cuts for a relatively long time is preferable in terms of their cumulative output effect. By contrast, an intermediate timing of implementation that envisages spending cuts while the economy is still emerging from the recession turns out to be most harmful irrespectively of the level of debt. The reason is that the anticipation of spending cuts in the near future, when the economy has not yet fully emerged from recession, tends to exacerbate the disinflationary and thus recessionary effect of the initial negative shock.

The most interesting result, however, relates to the effect of risk premia on the determinacy of equilibrium in the economy. Specifically, at high levels of initial debt, even short delays in the implementation of retrenchment are found to raise issues of equilibrium indeterminacy—causing expectations can become unanchored so the path of the economy is no longer uniquely pinned down by economy fundamentals. We explore the issue analytically in a simplified version of our model. Using this framework, we show that the presence of endogenous risk premia reduces the parameter space for which the equilibrium is uniquely determined. However, we also find that under some configurations indeterminacy can be avoided if fiscal retrenchment is initiated very early on. As such, our analysis provides a rationale for policy formulation conditional on the initial health of public finances.

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A Proofs

A.1 Proof of Proposition 1

The economy, stripped from exogenous variables, is given by

$$E_t z_{t+1} = A z_t,$$

where $z_t = [\tilde{y}_t; \widehat{\pi}_t]$ and

$$A = \frac{1}{a\mu\beta} \begin{bmatrix} \mu\beta + \varrho\mu\kappa_y & -\varrho\mu \\ -a\kappa_y & a \end{bmatrix},$$

where $a = (\mu + \mu\varrho\alpha\xi\chi)$. The Blanchard-Kahn conditions for determinacy require that matrix A have two roots outside the unit circle. Woodford (2003) gives the following necessary and sufficient conditions for determinacy:
either (Case I): (i) $det(A) > 1$, (ii) $det(A) - tr(A) > -1$, and (iii) $det(A) + tr(A) > -1$,
or (Case II): (i) $det(A) - tr(A) < -1$ and (ii) $det(A) + tr(A) < -1$.

In the current case, $det(A) = \frac{1}{a\mu\beta}$ and $tr(A) = \frac{1}{a\mu\beta}[\mu\beta + \varrho\mu\kappa_y + a]$. Since both $det(A) > 0$ and $tr(A) > 0$ Case II cannot be satisfied. Checking Case I, condition (iii) holds since both terms are positive. Condition (i) is equivalent to condition a) in the proposition. Condition (ii) of Case I is equivalent to condition b) in the proposition. □

A.2 Proof of Proposition 2

In this case

$$A = \frac{1}{a^*\mu\beta} \begin{bmatrix} \mu\beta + \varrho^*\mu\kappa_y^* & -\varrho^*\mu \\ -a^*\kappa_y^* & a^* \end{bmatrix},$$

where a^*, ϱ^* and κ_y^* are defined in the proposition.

1. Note that under the restriction that $a^* > 0$, $det(A) > 0$. Therefore it cannot be the case that $det(A) - tr(A) < -1$ and $det(A) + tr(A) < -1$. This means that determinacy can only obtain under the conditions of Case I. In addition, if $\varphi < 1$, then $tr(A) > 0$, so $det(A) + tr(A) > -1$. Condition c) is therefore obsolete if $\varphi < 1$.

2. For $a^* < 0$, $det(A) < 0$, so Case I cannot hold. The conditions given in the proposition are those pertaining to Case II. □

A.3 Proof of Corollary 3

1. If $\alpha\xi = 0$, $a^* = \mu > 0$. As a result condition Case 1 of Proposition 2 gives the relevant condition. First note that condition a) will be satisfied always. Condition c) holds for $\varphi < 1$. What remains to be checked therefore is whether condition b) holds for $\varphi < 0$ whenever it holds for $\varphi = 0$, and holds for some fundamental parameters for which it would not hold otherwise. That is true if

$$(1-\beta\mu)(1-a^*) - \mu\varrho^*\kappa_y^* > (1-\beta\mu)(1-a) - \mu\varrho\kappa_y,$$

or equivalently (for $\alpha\xi = 0$),

$$\mu\varrho^*\kappa_y^* - \mu\varrho\kappa_y < 0.$$

Substituting, the condition reads

$$\mu\kappa\left[\frac{\varrho\omega}{1-\varphi} + 1\right] - \mu\kappa\left[\varrho\omega + 1\right] < 0.$$

This reduces to $\varphi/(1-\varphi) < 0$ which is true for $\varphi < 0$. So the range of fundamental parameters for which determinacy obtains is bigger with a countercyclical government spending response in this case than in the absence of any response. What remains to be shown is that a stronger response further increases the range of fundamental parameters for which determinacy obtains. To see this, observe that the left-hand side of condition b) in Case 1 of Proposition 2 is independent of φ. The right-hand side is given by

$$\mu\varrho^*\kappa_y^* = \mu\kappa[1 + \omega\varrho/(1-\varphi)].$$

The right-hand side is strictly increasing in φ. As a result, the set of parameters for which the condition will bind will be the larger the more negative φ is.

2. (a) The range of fundamental parameters for which determinacy holds is bigger if $a^* < a$, and if

$$(1-\beta\mu)(a-a^*) > \mu\varrho^*\kappa_y^* - \mu\varrho\kappa_y.$$

   $a^* < a$ boils down to $\frac{\chi-\varphi}{1-\varphi} < \chi$, which is true for $\varphi < 1$. The second condition reduces to

$$(1-\beta\mu)\alpha\xi(1-\chi)\frac{\varphi}{1-\varphi} > \kappa\omega\frac{\varphi}{1-\varphi}.$$

   For $\varphi \in (0,1)$ this yields $\chi < 1 - \frac{\kappa\omega}{(1-\beta\mu)\alpha\xi}$, the condition in the corollary.

   (b) $a^* < 0$ means $\frac{1+\alpha\xi\varrho\chi}{1+\alpha\xi\varrho} < \varphi < 1$, so this is the only case in which Part 2 of Proposition 2 can be satisfied. □

A.4 Proof of Proposition 4

The assumed Markov structure means that output, inflation and government spending (in deviation from steady state) will take on the same values while the lower bound binds, y_l, π_l and g_l, respectively, and values of zero thereafter. The IS curve thus implies

$$y_l - g_l = \mu(y_l - g_l) - \varrho[-\log(R) + \Delta + \mu\alpha\xi(g_l - \chi y_l) - \mu\pi_l].$$

And the Phillips curve implies

$$\pi_l = \mu\beta\pi_l + \kappa_y y_l - \kappa_g g_l.$$

Solving these equations for y_l and π_l gives for y_l:

$$y_l = \vartheta_r[\log(R) - \Delta] \; + \vartheta_g g_l,$$

where ϑ_r and ϑ_g take on the values given in the proposition. In addition, $\vartheta_r > 0$: the numerator is positive, and the denominator is positive, too, by the condition b) for determinacy in Proposition 1. □

A.5 Proof of Corollary 5

1. Under the restrictions for determinacy of Proposition 1, the denominator of ϑ_g is unambiguously positive. $\vartheta_g > 0$ thus requires $(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > 0$ which solves to the expression in equation (13).

2. The conditions for determinacy require that $(1-\beta\mu)(1-a) - \mu\varrho\kappa_y > 0$, so the denominator of ϑ_g is positive. The same condition can be also used to prove that the numerator of ϑ_g is positive. Extending the above inequality yields:

$$(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > -(1-\beta\mu)(b-a) - \mu\varrho(\kappa_g - \kappa_y).$$

   Note that $\kappa_g < \kappa_y$. In addition, note that $b = a$ if $\alpha\xi = 0$. This proves that $(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > 0$ if $\alpha\xi = 0$ and under the conditions of Proposition 1.

3. For $\alpha\xi > 0$, $\vartheta_g > 1$ is equivalent, after substituting for κ_g and κ_y, to $\chi > 1 - \frac{\kappa\phi}{\alpha\xi(1-\beta\mu)}$. The deficit is given by $g - \chi y_l$. Spending will thus be self-financing if $1 - \chi\vartheta_g < 0$. □

A.6 Proof of Proposition 6

For the austerity phase the IS equation is given by

$$(y_a - g_a)(1-\nu) = -\varrho\left[\phi\pi_a - \nu\pi_a\right].$$

The Phillips curve is given by

$$\pi_a = \beta\nu\pi_a + \kappa_y y_a - \kappa_g g_a.$$

These two equations solve to expressions (15) and (16). While at the lower bound, the IS equation is given by

$$y_l(1-\mu) = (1-\mu)(y_a - g_a) - \varrho\left[-\log(R) + \Delta + \alpha\xi[(1-\mu)g_a - \chi(1-\mu)y_a - \chi\mu y_l] - \mu\pi_l - (1-\mu)\pi_a\right].$$

The Phillips curve is given by

$$\pi_l = \beta\mu\pi_l + \beta(1-\mu)\pi_a + \kappa y_l.$$

Solving the latter two equations leads to the equation for output, y_l, (14).

A.7 Proof of Corollary 7

1. For the case $\xi\alpha = 0$, we have that $y_l = 1/d[(1-mu)(1-\beta\mu)(y_a - g_a) + \varrho(1-\mu)\pi_a]$. Note that $d = (1-\mu)(1-\beta\mu) - \varrho\mu\kappa_y > 0$ since Proposition 6 assumes determinacy of the equilibrium (cp. condition b) in Proposition 1).
   $y_l > 0$ thus requires

   $$(1-\mu)(1-\beta\mu)(y_a - g_a) + \varrho(1-\mu)\pi_a > 0. \tag{17}$$

   Substitute for y_a and π_a using equations (15) and (16). Further note that the denominator in the expressions for π_a and y_a is positive (we have assumed determinacy in Proposition 6, so $\phi_\pi > 1$, and therefore especially $\phi_\pi - \nu > 0$). Furthermore, observe that $\kappa_y - \kappa_g > 0$, and that $g_a < 0$. Using these observations, inequality (17) resolves to $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

2. Let y_l^w denote the size of output at the lower bound with a response of the risk-premium ($\alpha\xi > 0$). Denote with a superscript ^{o} the terms in the absence of a response of the risk-premium. For example, let y_l^o denote the size of output in the absence of a response of the risk premium ($\alpha\xi = 0$).
   Note, first, that y_a, g_a, π_a are independent of the risk-premium.
   Note, second, that $d^w = (1-\beta\mu)(1-\mu-\mu\alpha\xi\chi\varrho) - \varrho\mu\kappa_y < d^o$.
   Note, third, that $d^w > 0$ by the assumption of determinacy. Thus $[\frac{d^o}{d^w} - 1] > 0$.
   Condition $y_l^w > y_l^o$, after substituting for y_a and π_a, and after dividing by $g_a < 0$, is

equivalent to

$$\begin{aligned}&\left[\tfrac{d^o}{d^w}-1\right]\varrho(1-\mu)(\kappa_g-\kappa_y)\left[(1-\beta\mu)(\phi_\pi-\nu)-1+\nu\right]\\&+\tfrac{d^o}{d^w}\varrho(\phi_\pi-\nu)(\kappa_g-\kappa_y)\varrho\alpha\xi\chi(1-\mu)(1-\beta\mu)\\&+(1-\mu)(1-\beta\mu)\left[(1-\nu)(1-\beta\nu)+\varrho(\phi_\pi-\nu)\kappa_y\right]\varrho\alpha\xi(\chi-1)<0.\end{aligned}$$

The second row is non-positive. The third row is strictly negative (since $\chi \in [0,1)$). $\kappa_g - \kappa_y < 0$, so the first row will be strictly negative if $(1-\beta\mu)(\phi_\pi - \nu) - 1 + \nu > 0$, which is equivalent to $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

3. $\frac{\partial y_l}{\partial(\alpha\xi)} = -\frac{1}{d^2} n \frac{\partial d}{\partial(\alpha\xi)} + \frac{1}{d}\frac{\partial n}{\partial(\alpha\xi)}$, where n denotes the denominator in the expression for y_l. d is positive in the case of determinacy. n is positive for those cases in which y_l reacts positively to future austerity.
$\frac{\partial d}{\partial(\alpha\xi)} = -(1-\beta\mu)\mu\varrho\chi < 0$.
$\frac{\partial n}{\partial(\alpha\xi)} = -(1-\mu)(1-\beta\mu)\varrho(1-\chi)g_a + (1-\mu)(1-\beta\mu)\varrho\chi(y_a - g_a) > 0$, since $g_a < 0$ and $(y_a - g_a) > 0$. Putting these elements together proves the claim. □