Linkages Across Sovereign Debt Markets

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Motivation

- Theory of sovereign default studies countries in isolation
- Sovereign debt crises happen in bunches
Percentage of Countries in Default

Source: Reinhart and Rogoff 2011
European Debt Crisis

- Default fears: Greece, Ireland, Italy, Portugal, Spain
  - Interest rate spreads co-move
- Lending banks at the center of crisis
  - Banks’ exposure to GIIPS seen as a major concern
  - In German banks, loans to GIIPS are 130% of their capital
Spreads

Percent

Greece
Ireland
Italy
Portugal
Spain

2006 2007 2008 2009 2010 2011 2012 2013 2014
0 5 10 15 20 25
This Paper

- Dynamic multicountry model of sovereign debt linkages
  - Countries borrow, default, and renegotiate with common lenders

- Countries default together because
  - Renegotiating together lowers debt recovery
  - Rolling over debt is more expensive
This Paper

- Dynamic multicountry model of sovereign debt linkages
  - Countries borrow, default, and renegotiate with common lenders
- Countries default together because
  - Renegotiating together lowers debt recovery
  - Rolling over debt is more expensive
- Predictions consistent with historical cross-country data
  - Default probabilities higher when others default
  - Renegotiation probabilities higher when others renegotiate
  - Recovery rates lower when others renegotiate
- Model accounts for 50% of spread correlation across GIIPS
Sovereign default and renegotiation in single countries:
Eaton & Gersovitz (1981), Arellano (2008), Yue (2010), Benjamin & Wright (2011)

Here multiple countries linked through debt markets

Risk premia in sovereign bonds market: Borri and Verdelhan (2010), Presno and Pouzo (2012), Gilchrist, Yue, and Zakrajsek (2012)

Here risk premium endogenous to countries’ choices

Contagion in capital flows through common lender:
Calvo and Mendoza (2000), Kyle and Xiong (2001)

Here contagion through default choices

Default and contagion: Lizarazo (2010), Park (2012)

Here strategic interactions among countries and renegotiation
Simple Model

- Two periods no-uncertainty
- Two borrowing countries and continuum of lenders
- Countries are strategic big players; lenders are competitive
- Countries differ in initial debt $b = \{b_1, b_2\}$
- Countries borrow, default, and renegotiate
  - Default entails costs: output and autarky
Borrowing Countries: Consumption

Period 1: Countries decide whether to repay or default

- If repay \((d_i = 0)\), borrow
  \[ c_i = y - b_i + q_i(b, d, b') b'_i \]
  where \(b'_i\) is the bond price.

- If default \((d_i = 1)\), output loss and not borrow
  \[ c_i = y^d \]
Borrowing Countries: Consumption

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  bond price

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Period 2: Pay debt or recovery

- Non-defaulters pay debt: $c'_i = y' - b'_i$
- Defaulters renegotiate
  \[ c'_i = y' - \phi_i(d, b') \]
  recovery
Borrowing Countries: Consumption

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Period 2: Pay debt or recovery

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- Defaulters renegotiate
  \[ c'_i = y' - \phi_i(d, b') \]
  where \(\phi_i(d, b')\) is the recovery

Countries are linked through bond price and recovery
Borrowing Countries: Default Decision

- Country $i$ defaults if default value higher than repaying value

$$u(y - b_i + q_i(b, d_{-i})\bar{b}) + \beta u(y' - \bar{b}) < u(y^d) + \beta u(y' - \phi_i(d_{-i}))$$

- If repay borrow to limit: $\bar{b} = y' - y^d$

- States and choices of other country affects $\phi_i'$ and $q_i$
  - Low $\phi_i$ and low $q_i$ increase default incentives

- Default is more likely when $b_i$ is high

- Default cutoff $\hat{b}_i(b_{-i}, d_{-i})$

$$d_i = 1 \text{ if } b_i \geq \hat{b}_i(b_{-i}, d_{-i})$$
Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining

- One country defaults and renegotiates

\[
\max_{\phi_i} \left[ u(y' - \phi_i) - u(y^d) \right]^\theta \left[ u^L(y_L + \bar{b} + \phi_i) - u^L(y_L + \bar{b}) \right]^{1-\theta}
\]

- Lenders' outside option lower when both countries renegotiate
- Joint renegotiation ⇒ recovery lower ⇒ defaults more, ⇒ \(\hat{b}_i\) tighter
Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining

- One country defaults and renegotiates

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\max_{\phi_i} \left[ u(y' - \phi_i) - u(y^d) \right]^\theta \left[ u^L(y_L + b + \phi_i) - u^L(y_L + b) \right]^{1-\theta}
\]

- Two countries default and renegotiate

\[
\max_{\phi_1, \phi_2} \left[ u(y' - \phi_1) - u(y^d) \right]^\theta \left[ u(y' - \phi_2) - u(y^d) \right]^\theta \left[ u^L(y_L + \phi_1 + \phi_2) - u^L(y_L) \right]^{1-\theta}
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\]

- Lenders’ outside option lower when both countries renegotiate

- Joint renegotiation \rightarrow \text{recovery lower} \rightarrow \text{defaults more,}
  \Rightarrow \hat{b}_i \text{ tighter}
Bond Price Functions

- Prices solve lenders’ demand system
- Two countries repay

\[ q_i = \delta \frac{u'_L(y_L + 2\bar{b})}{u'_L(y_L + b_i - q_i\bar{b} + b_{-i} - q_{-i}\bar{b})} \]

- Price \( q_i \) increases with repayment of large \( b_{-i} \)
- Foreign defaults

\[ q_i = \delta \frac{u'_L(y_L + 2\bar{b})}{u'_L(y_L + b_i - q_i\bar{b})} \]

- Foreign repays small \( b_{-i} \) ⇒ \( \hat{b}_i \) tighter
Best Responses

\[
\hat{b}(b_{-i}, d_{-i} = 1) \quad \hat{b}(b_{-i}, d_{-i} = 0)
\]

**Dependency zone**

- \(d_i = 0\) if \(d_{-i} = 0\)
- \(d_i = 1\) if \(d_{-i} = 1\)

In the dependency zone, foreign default leads to home default.
In dependency zone, foreign default leads to home default
Equilibrium

Multiple equilibrium

\[ \hat{b}(b_i, d_i = 1) \]

\[ \hat{b}(b_i, d_i = 0) \]

\[ d_i = 0 \]
\[ d_{-i} = 1 \]

\[ d_i = 1 \]
\[ d_{-i} = 1 \]

\[ d_i = 0, d_{-i} = 0 \]
\[ d_i = 1, d_{-i} = 1 \]
Equilibrium: Zones

\begin{align*}
\hat{b}(b_{-i}, d_{-i} = 1) & \quad \hat{b}(b_{-i}, d_{-i} = 0)
\end{align*}

Independent Repay

Self-Fulfilling

Dependent Default

Dependent Repay

Independent Default

\[ b_{-i}, b_i \]
Main Predictions Simple Model

- Default more when others default
- Renegotiate more when others renegotiate
- Recovery lower with joint renegotiation
## Cross Country Data

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>Renegotiation</th>
<th>Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction in Default_it</td>
<td>1.36***</td>
<td>-0.88***</td>
<td>0.92***</td>
</tr>
<tr>
<td>Fraction Renegotiating_it</td>
<td>-2.13*</td>
<td>4.60**</td>
<td>-7.39***</td>
</tr>
<tr>
<td>Debt/GDP_it</td>
<td>0.11**</td>
<td>-0.03*</td>
<td>-0.21***</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Adjusted $R^2$</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>0.28</td>
<td>2682</td>
</tr>
<tr>
<td>Renegotiation</td>
<td>0.06</td>
<td>552</td>
</tr>
<tr>
<td>Recovery</td>
<td>0.34</td>
<td>139</td>
</tr>
</tbody>
</table>


- Theory predictions are consistent with historical data
Quantitative Model

- Infinite horizon and stochastic income
- Time varying debt and default choices
- Spreads compensates for expected default loss
- Same strategic interactions for default and renegotiation
- Markov equilibrium:
  - Optimal choices for default, renegotiation, and debt are Nash
  - Bond price and recovery functions are consistent with default and renegotiation decisions
- Equilibrium selection: Outcome that maximizes country values
Quantitative analysis

- Use data from Greece, Italy, Spain, and historical debt recoveries to parameterize model

- Model can account for half of the correlation in spreads and debt exposures across countries

- Strategic interactions and joint renegotiations are most important forces
## Model and Data

**Calibrated moments (%)**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean risk free rate</td>
<td>4.0</td>
<td>4.2</td>
</tr>
<tr>
<td>Mean spread</td>
<td>1.4</td>
<td>1.6</td>
</tr>
<tr>
<td>Volatility risk free rate</td>
<td>1.4</td>
<td>1.6</td>
</tr>
<tr>
<td>Volatility spread</td>
<td>2.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Volatility exposure</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Mean recovery</td>
<td>60</td>
<td>66</td>
</tr>
<tr>
<td>$\Delta$ in recovery</td>
<td>-16</td>
<td>-13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with multiple rene.</td>
</tr>
</tbody>
</table>

**Other moments**

<table>
<thead>
<tr>
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<th>Data</th>
<th>Model</th>
</tr>
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<tbody>
<tr>
<td>Correlation of spread</td>
<td>0.97</td>
<td>0.43</td>
</tr>
<tr>
<td>Correlation of exposure</td>
<td>0.56</td>
<td>0.30</td>
</tr>
</tbody>
</table>
### Debt Linkages

<table>
<thead>
<tr>
<th>Home</th>
<th>Overall Mean</th>
<th>Foreign Good Credit</th>
<th>Foreign Bad Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default prob.</td>
<td>4.5</td>
<td>2.9</td>
<td>37.3</td>
</tr>
<tr>
<td>Rene. prob.</td>
<td>98</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>Recovery</td>
<td>66</td>
<td>71</td>
<td>90</td>
</tr>
<tr>
<td>Spread</td>
<td>1.6</td>
<td>1.6</td>
<td>1.9</td>
</tr>
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- Default more when others default or do not renegotiate
- Renegotiate more when others repay or renegotiate
- Recovery is reduced when others repay or renegotiate
What Drives Results?

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<tr>
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<th>Benchmark</th>
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<td>Small</td>
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<td>0.52</td>
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<td>0.51</td>
</tr>
<tr>
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<td>0.34</td>
<td>0.45</td>
<td>0.32</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>1.6</td>
<td>0.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Dependent events</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>25</td>
<td>35</td>
<td>31</td>
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Strategic interactions are most important force
Conclusion

- Developed a model of sovereign debt linkages
- Defaults and renegotiations are correlated
  - Strategic interactions of countries trading with common lenders
Parameters

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad g(c_L) = \frac{c_L^{1-\alpha}}{1-\alpha} \]

Uncorrelated AR(1) shock process from Greek GDP \( \rho = 0.88 \), \( \eta = 0.03 \)
Borrowers’ risk aversion \( \sigma = 2 \)

<table>
<thead>
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<th>Value</th>
<th>Target</th>
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<tr>
<td>Lender’s income ( y_L = 1.4 )</td>
<td>German yield: mean and vol.</td>
</tr>
<tr>
<td>Lender’s risk aversion ( \alpha = 0.65 )</td>
<td>Greek spread: mean and vol.</td>
</tr>
<tr>
<td>Lender’s discount factor ( \delta = 0.96 )</td>
<td>Recovery rate mean and cond.</td>
</tr>
<tr>
<td>Output cost after default ( \lambda = 0.016 )</td>
<td>Exposure vol.</td>
</tr>
<tr>
<td>Borrowers’ discount factor ( \beta = 0.82 )</td>
<td></td>
</tr>
<tr>
<td>Borrower’s bargaining power ( \theta = 0.38 )</td>
<td></td>
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</table>
Consumption and expected value for arbitrary strategy \((d, b')\)

\[
w_i(s, d, b') = u(c_i(s, d, b')) + \beta \sum \pi(y', y) v_i(s'(d, b'))
\]

- \(v_i\): future value given by the Markov allocations
- \(w_i\): payoff for arbitrary strategies \((d, b')\)
- Future state \(s' = (b', h', y')\) induced by strategies \((d, b')\)
Each period has two sequential stages:

- Default-renegotiation stage
  Countries decide on $d_i$: repay/default or renegotiate/not

- Borrowing stage: given states $(s, d)$
  Non-defaulting countries decide on borrowing $b'$
  Cournot competition

*We consider Markov equilibrium*
Borrowers Countries: Borrowing Stage

- Borrowing determined by Cournot competition
  - Borrowing best response of country $i$
    \[ x_i^b(b'_{-i}, s, d) = \{ b'_i : \max_{b'_i} w_i(s, d, b') \} \]
  - Optimal borrowing $(B_1(s, d), B_2(s, d))$ is Nash
    \[ B_i(s, d) = x_i^b(B_{-i}(s, d), s, d) \quad \text{for all } i \]
Optimal default and renegotiation is Nash

Default-renegotiation best response of country $i$

$$x_i^d(d_{-i}, s) = \{ d_i : \max_{d_i} w_i(s, d, B(s, d)) \}$$

Optimal default-renegotiation strategy $(D_1(s), D_2(s))$

$$D_i(s) = x_i^d(D_{-i}(s), s)$$

Markov equilibrium given price and recovery functions

$$v_i(s) = w_i(s, D(s), B(s, D(s)))$$
Lenders

- Competitive with preferences: $E \sum_{t=0}^{\infty} \delta^t g(c_{Lt})$

- More patient than borrowers $\delta > \beta$, dislike volatility $g''(.) < 0$

- Dividends depend on credit phases and countries’ choices

$$c_L = y_L + \sum_{i=1}^{2} [1 - D_i(s)] [(1 - h_i) (b_i - Q_i(s) b_i') + h_i \Phi_i(s)]$$

net repayment from country $i$

- $Q_i(s)$ is prices of bonds; $\Phi_i(s)$ is the recovery
Lenders FOC

▶ Lenders’ kernel

\[ m(s', s) = \frac{\delta \pi(y', y) g'_L(s')}{g'(c_L(s))} \]

▶ Price of bonds

\[ Q(s) = E \ m(s', s) \left[ (1 - D(s')) + D(s') \zeta(s') \right] \]

▶ Risk adjusted present value of recovery rate

\[ \zeta(s) = E \left[ m(s', s)(1 - D(s')) \frac{\Phi(s')}{b'} + D(s')\zeta(s') \right] \]
Bond price schedule

Price tight with foreign default, high borrowing, and not renegotiate
Recovery schedule

- Foreign default
- Foreign not rene.
- Foreign rene.
- Foreign repay: optimal borrowing

▶ Recovery low in joint renegotiations