Recent Developments in Stress Testing Market Risk

Gerald Krenn
Financial Markets Analysis and Surveillance Division

Hosted by the International Monetary Fund
Washington, DC – May 2-3, 2006

The views expressed in this paper are those of the author(s) only, and the presence of them, or of links to them, on the IMF website does not imply that the IMF, its Executive Board, or its management endorses or shares the views expressed in the paper.
Recent Developments in Stress Testing Market Risk

Presentation at the IMF Expert Forum on Advanced Techniques on Stress Testing
May 2, 2006

Gerald Krenn

Financial Markets Analysis and Surveillance Division

Gerald.Krenn@oenb.at

www.oenb.at
Agenda

I. Traditional stress tests for market risk

II. Maximum Loss as a risk measure uncovering harmful scenarios

III. Integration of market and credit risk stress testing
I: Traditional Stress Tests

Ingredients for stress testing

- Portfolio: In our case the trading book (subject to market risk)
- Scenarios: possible market states \( r \)
  \( r = (r_1, \ldots, r_n) \) vector of risk factor values
  \( r_i \) are: interest rates, exchange rates, equity indices etc.
- Portfolio valuation function \( P \) as a function of \( r \): \( P = P(r) \)
- Current state of the market: \( r_{CM} \)
- Hence, current portfolio value: \( P(r_{CM}) \)

Performing stress tests

1. Select scenarios \( r_{stress1}, r_{stress2}, \ldots \) (according to some criterion)
2. Calculate portfolio values \( P(r_{stress1}), P(r_{stress2}), \ldots \)
3. Derive some measure of riskiness of the scenarios
I: Traditional Stress Tests

How to select scenarios

- Standard scenarios
- Historical scenarios
- Subjective worst case scenarios
I: Dangers of Traditional Stress Tests

- A stress scenario for one portfolio might be a lucky strike for another portfolio
- Stress tests with standard and historical scenarios may nourish a false illusion of safety
- Subjective worst case scenarios are often too implausible to trigger management action

But: Stress Tests can be the basis of informed risk decisions ...
... if the scenarios are plausible
... if we are confident there are no worse scenarios
II: Maximum Loss

- Good overview on Maximum Loss in doctoral thesis by Studer (1997)
- Can be interpreted as a risk measure that avoids dangers of traditional stress tests
- Choose a trust region $TR$: A set of scenarios above a certain minimal plausibility threshold

$$\text{MaxLoss}_{TR}(P) := \sup_{r \in TR} \{P(r_{CM}) - P(r)\}$$

- Maximum Loss defined as:

- “Above the plausibility threshold no loss worse than MaxLoss can happen”

Choice of trust region

- By means of the multivariate risk factor distribution
- Trust region shall have some predefined probability ($p$) and contain only scenarios with “highest density”
- In case risk factors have an elliptic distribution (e.g. multivariate normal, Student-t):
  Trust region is an ellipsoid of scenarios with Mahalanobis distance to $r_{CM}$ below some threshold $k_p$:

$$TR = \left\{ r : (r - r_{CM})' \Sigma^{-1} (r - r_{CM}) \leq k_p \right\}$$

($\Sigma$ is the co-variance-matrix)
II: Trust Region: Area of Highest Density
II: Within Trust Region: Find Scenario with Smallest Portfolio Value (= Maximum Loss)
II: Benefits of Maximum Loss

- Maximum Loss not only quantifies risks but also identifies a worst case scenario (among all scenarios in the trust region)
- Searching for worst case scenarios yields more harmful and more plausible scenarios than other ways to identify stress scenarios
- Sample portfolio consisting of options on different international stock indices
  - Stress scenarios are identified in different ways
    - Worst case according to the recommendations of the DPG (Derivatives Policy Group)
    - Recurrence of Black Friday in October 1987
    - Worst case scenario implied by Maximum Loss

<table>
<thead>
<tr>
<th></th>
<th>Relative Loss</th>
<th>Plausibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst DPG</td>
<td>- 183%</td>
<td>once in 10 yrs</td>
</tr>
<tr>
<td>Black Friday</td>
<td>- 154%</td>
<td>once in 19 yrs</td>
</tr>
<tr>
<td>Worst Case (ML)</td>
<td>- 279%</td>
<td>once in 8 yrs</td>
</tr>
</tbody>
</table>
II: Benefits of Maximum Loss

Identifying key risk factors of the worst case scenario = Locating the vulnerable spots of a portfolio

Example: Again option portfolio

<table>
<thead>
<tr>
<th>Report</th>
<th>Risk Factors</th>
<th>Rel. Changes</th>
<th>Loss</th>
<th>Explanatory Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Report 1</td>
<td>FTSE100</td>
<td>-13%</td>
<td>206</td>
<td>74%</td>
</tr>
<tr>
<td>Report 2</td>
<td>FTSE100 DJI</td>
<td>-13% -8%</td>
<td>264</td>
<td>94%</td>
</tr>
<tr>
<td>Report 3</td>
<td>FTSE100 DJI NIK225</td>
<td>-13% -8% -5%</td>
<td>271</td>
<td>97%</td>
</tr>
</tbody>
</table>

Explanatory Power = \( \frac{\text{Loss} (r_{\text{report}})}{\text{Loss} (r_{\text{worst case}})} \)
II: The Problem of Dimensional Dependence

- \( n \) ... number of risk factors on which the portfolio depends
- Let’s consider an elliptic risk factor distribution; trust regions are then ellipsoids
- The trust region shall have probability \( p \)
- When \( k \) (the “radius” of the ellipsoid) is fixed, \( p \) depends on \( k \) (and on \( n \)):
  - e.g. for multivariate normal distribution:

\[
p(k,n) = 1 - F_{\chi^2_n}(k^2) = 1 - \frac{1}{2^{n/2} \Gamma(n/2)} \int_0^{k^2} s^{n-1} e^{-s} ds
\]

- To get trust regions with some predefined probability \( p \), \( k \) has to increase as \( n \) increases
- If we add an “empty risk factor” (i.e. a factor on which the portfolio value does not depend), the radius \( k \) has to increase in order to hold \( p \) fixed
- We therefore are searching for MaxLoss within a larger trust region when we add an empty risk factor
- Also MaxLoss is likely to be larger once having added an empty risk factor
III: Systemic Risk Monitor (SRM) – Basic Structure

Distribution of Risk Factor Changes

Scenarios

Market Risk Losses

Interbank Network Model

Non Interbank Credit Risk Losses

Default Statistics of Banking System
Decomposition Fundamental, Contagious Defaults
III: Risk Factors in SRM

• SRM analysis market risk and credit risk simultaneously
• As risk factors we have market risk factors as well as credit risk drivers
• Time horizon in SRM is 3 months
  – Implies that length of risk factor time series will be limited; e.g. with quarterly data starting in 1980 we get about 100 data points
  – For numerical stability (estimation of covariance matrix for the grouped t-copula) number of observations should clearly exceed number of risk factors
• Therefore parsimonious selection of risk factors (trade-off with accuracy of valuation)
  – Interest rates:
    • 5 currencies EUR, USD, CHF, JPY, GBP
    • Maturities: 3 months, 1 year, 5 years, 10 years
  – 2 equity indices (national, international)
  – 4 exchange rates: EUR vis-à-vis USD, CHF, JPY, GBP
• Credit risk drivers should be able to explain PDs in different industrial sectors
  \[
  \mu_{i,t} = \frac{e^{X_t \beta_i}}{1 + e^{X_t \beta_i}} + \xi_{i,t}
  \]
  – 8 credit risk factors were selected (e.g. GDP, Consumer Price Index, Unemployment rate, international stock index)
III: Stress Testing in SRM

- 26 market risk factors + 8 credit risk factors = 34 risk factors
- These factors we wish to model statistically
  - Allows for a Monte Carlo-simulation for analyzing the actual situation (sampling from the unconditional distribution)
  - Allows for a Monte Carlo-simulation for **stress testing** (sampling from the conditional distribution)
- For stress testing, a set of risk factors is set to some predefined values
- Remaining factors are sampled from the conditional distribution
III: Statistical Modeling of Risk Factors

- Multivariate distribution of risk factors is estimated in a 2-step procedure:
  - Step 1: Modeling of marginal distribution of each risk factor by models which are optimized with respect to their out-of-sample density forecast
  - Step 2: Modeling of dependencies between individual risk factors by a grouped t-copula
- Our goal is to have enough flexibility in order to capture
  - Marginal distributions of the various risk factors
  - Patterns of dependence between risk factors
- Market risk factors and Credit risk factors are treated in a common statistical model
III: Selecting Marginal Distributions

Goal: Find a statistical model for risk factor changes over the horizon of one quarter

Aspects to be considered when selecting a model

• Maybe modeling risk factors at higher frequencies (basic periods: daily, weekly,…)
  and aggregating these models to a quarter can exploit information contained in
  higher frequency data
• Maybe there are GARCH effects even for quarterly data
• Different alternatives for the distribution of residuals:
  – Normal
  – Student t
  – Extreme value
III: Marginal Distributions: Tests and Results

- The resulting 36 models were applied to 19 (sufficiently long) time series
- 2 statistical tests were applied to each model:
  - Test 1: According to de Raaij und Raunig (2002)
  - Test 2: Kolmogorov-Smirnov-Test for N(0,1)
- Table shows number of accepted time series per model (according to test 1 and 2)

<table>
<thead>
<tr>
<th>Basic period</th>
<th>No GARCH</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>t</td>
</tr>
<tr>
<td></td>
<td>T1  T2</td>
<td>T1  T2</td>
</tr>
<tr>
<td>1d</td>
<td>0    0</td>
<td>0    0</td>
</tr>
<tr>
<td>5d</td>
<td>0    0</td>
<td>0    1</td>
</tr>
<tr>
<td>10d</td>
<td>0    0</td>
<td>0    2</td>
</tr>
<tr>
<td>20d</td>
<td>0    2</td>
<td>1    6</td>
</tr>
<tr>
<td>30d</td>
<td>0    5</td>
<td>8    16</td>
</tr>
<tr>
<td>60d</td>
<td>9    16</td>
<td>9    13</td>
</tr>
</tbody>
</table>

Results

- Aggregation does not yield good results: Quarterly data already contain all the information
- GARCH models are slightly superior to constant volatility
- Extreme value distribution for residuals appears best
III: Marginal Distributions: Final Model Selection

- No aggregation of higher frequency data, i.e. use quarterly data directly

- GARCH
  - Testing procedure favors consideration of GARCH effects
  - Makes sense for analysis of current situation
  - Should be used with care for stress tests

- Distribution of Residuals
  - Extreme value distribution performs best in the test procedures
  - Simulations show that extreme value distribution leads to too extreme risk factor movements
  - SRM now uses t-distribution as marginals
    - Also leads to extreme risk factor movements in some cases
    - Hence restriction: degrees of freedom > 4,1
III: Match between Historical and Generated Data

- Many time series show more extreme movements than a normal distribution (small degree of freedom)

- Sample of 10,000 scenarios (no GARCH) is compared with the historical input data
  - Standard deviations match pretty well
  - Kolmogorov-Smirnov test for the null hypothesis, that historical data and generated sample have the same distribution
  - Is rejected only in two cases at alpha = 0.05

<table>
<thead>
<tr>
<th>Risk factor</th>
<th># degrees of freedom</th>
<th>Standard deviation</th>
<th>p-Value KS-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hist. Data</td>
<td>Generated Sample</td>
<td></td>
</tr>
<tr>
<td>Usd 03M</td>
<td>4.1</td>
<td>0.3%</td>
<td>0.297</td>
</tr>
<tr>
<td>Usd 01Y</td>
<td>4.1</td>
<td>1.2%</td>
<td>0.981</td>
</tr>
<tr>
<td>Usd 05Y</td>
<td>4.7</td>
<td>4.0%</td>
<td>0.668</td>
</tr>
<tr>
<td>Usd 10Y</td>
<td>4.8</td>
<td>6.7%</td>
<td>0.382</td>
</tr>
<tr>
<td>Chf 03M</td>
<td>4.1</td>
<td>0.2%</td>
<td>0.846</td>
</tr>
<tr>
<td>Chf 01Y</td>
<td>4.6</td>
<td>0.8%</td>
<td>0.926</td>
</tr>
<tr>
<td>Chf 05Y</td>
<td>9.0</td>
<td>1.9%</td>
<td>0.922</td>
</tr>
<tr>
<td>Chf 10Y</td>
<td>9.6</td>
<td>3.2%</td>
<td>0.821</td>
</tr>
<tr>
<td>Jpy 03M</td>
<td>4.1</td>
<td>0.3%</td>
<td>0.000</td>
</tr>
<tr>
<td>Jpy 01Y</td>
<td>4.1</td>
<td>0.8%</td>
<td>0.044</td>
</tr>
<tr>
<td>Jpy 05Y</td>
<td>4.1</td>
<td>2.6%</td>
<td>0.677</td>
</tr>
<tr>
<td>Jpy 10Y</td>
<td>4.1</td>
<td>4.6%</td>
<td>0.964</td>
</tr>
<tr>
<td>Gbp 03M</td>
<td>4.1</td>
<td>0.3%</td>
<td>0.710</td>
</tr>
<tr>
<td>Gbp 01Y</td>
<td>4.1</td>
<td>1.0%</td>
<td>0.853</td>
</tr>
<tr>
<td>Gbp 05Y</td>
<td>4.1</td>
<td>3.0%</td>
<td>0.825</td>
</tr>
<tr>
<td>Gbp 10Y</td>
<td>4.1</td>
<td>5.3%</td>
<td>0.919</td>
</tr>
</tbody>
</table>
III: Match between Historical and Generated Data

2 selected time series
- Austrian traded index (top)
- JPY 3 months (bottom)

Left: Historical data in a normal probability plot

Middle: Generated sample (size=10,000; no GARCH) in a normal probability plot

Right: QQ-Plot for assessing the match between historical and generated data
  - ATX fits well
  - JPY 3M: historical extremes are not captured (a result of the restriction dof > 4,1)
III: Modeling Dependencies: Grouped t-Copula

- **Copula** models dependencies between risk factors
  - Copula is the part of the multivariate distribution which is not contained in the marginal distributions
- Concept of **tail-dependence** for assessing dependencies
  - The coefficient of tail-dependence \( \lambda \) between two variables is defined as:
    \[
    \lambda := \lim_{v \to 1^-} \mathbb{P}(X_1 > G_1^{-1}(v) \mid X_2 > G_2^{-1}(v)) > 0;
    \]
  - Is roughly speaking the probability that one variable is very large (small) given the other variable is very large (small)
  - In case \( \lambda > 0 \), “one variable can pull up (down) the other variable”
- For the multivariate normal distribution we have \( \lambda = 0 \) (no tail-dependence)
  - Real data show tail-dependence
- An alternative is given by the t-copula
  - There is tail-dependence between risk factors (\( \lambda > 0 \))
  - Scenarios can be generated easily in a Monte Carlo-simulation
  - Drawback: between all risk factors there is the same tail-dependence
III: Modeling Dependencies: Grouped t-Copula

- As an alternative to the t-copula the grouped t-copula was introduced by Daul et al. (2003)
  - Risk factors are arranged into groups
  - Within each group risk factors have the same tail-dependence
  - Each group is characterized by a parameter (degrees of freedom)
- Grouped t-copula was adopted for SRM
  - Is suited equally well for MC-simulations as the plain t-copula
  - In SRM risk factors were arranged into 4 groups (in parentheses: estimated degrees of freedom)
    - Credit risk factors (20)
    - FX (14)
    - Equity (5)
    - Interest rates (11)
III: Simulation

- In SRM we need the multivariate distribution in order to generate scenarios for the Monte Carlo-simulation
- For the grouped t-copula efficient algorithms exist for sampling from the
  - un-conditional distribution
  - conditional distribution (a set of risk factors is set to predefined values)
- E.g. algorithm for un-conditional distribution (Daul et al. (2003))

1. Generate a random vector $Z \sim N(0, \rho)$, where $\rho$ is a linear correlation matrix, and generate an independent random variable $U \sim U(0,1)$

2. Denote by $G_{\nu}$ the distribution function of $\chi^2_\nu$ and let $R_k := G_{\nu_k}^{-1}(U)$ for $k = 1, \ldots, m$.

3. Then the vector

$$Y = \left( \frac{Z_1}{\sqrt{R_1/\nu_1}}, \ldots, \frac{Z_{\nu_1}}{\sqrt{R_1/\nu_1}}, \frac{Z_{\nu_1+1}}{\sqrt{R_2/\nu_2}}, \ldots, \frac{Z_{\nu_1+\nu_2}}{\sqrt{R_2/\nu_2}}, \ldots, \frac{Z_n}{\sqrt{R_m/\nu_m}} \right)$$

has by equation (A.18) a grouped t-copula with Student marginals.

4. Denote by $t_{\nu}$ the distribution function of the one-dimensional Student distribution. Then

$$X := \left( H_{1}^{-1}(t_{\nu_1}(Y_1)), \ldots, H_{\nu_1}^{-1}(t_{\nu_1}(Y_{\nu_1})), H_{\nu_1+1}^{-1}(t_{\nu_2}(Y_{\nu_1+1})), \ldots, H_{\nu_m}^{-1}(t_{\nu_m}(Y_{\nu_m})) \right),$$

have grouped t-copula and marginal distribution functions $H_{\nu}$, $\nu = 1, \ldots, n$. 
Literature

http://www.bis.org/bcbs/events/rtf05breuer.pdf


