Carry Trades, Monetary Policy and Speculative Dynamics*

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Abstract

We ask when currency carry trades are associated with destabilizing dynamics in the foreign exchange market, and investigate the role of monetary policy rules in setting off such dynamics. In a model where the exchange rate has a long-term fundamental anchor, we find that carry trades can be stabilizing or destabilizing at shorter horizons, depending on the propensity of capital inflows to overheat the recipient economy. In the destabilizing case, we solve for a unique equilibrium that exhibits the classic pattern of the carry trade recipient currency appreciating for extended periods, punctuated by sharp falls.

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1 Introduction

Low interest rates maintained by advanced economy central banks in the aftermath of the 2008 financial crisis have ignited a fierce debate about capital flows to emerging economies. The accusation is that such capital flows are driven by speculative carry trades that unduly destabilize exchange rates by seeking to exploit the interest rate differences between advanced and emerging economies. Beyond the debate in the policy blogosphere and in the financial press[1], there is much that remains to be understood about the mechanisms underpinning the link between monetary policy and capital inflows.

One set of questions turns on whether speculative trading is stabilizing or destabilizing. Milton Friedman (1953) famously argued for the stabilizing effect of speculation when arbitrageurs, anticipating an eventual reversion of prices to some fundamental value, take positions that hasten the adjustment. Against this, many commentators have emphasized the role of monetary policy rules that increase the potential for carry trades to be associated with destabilizing speculation that take exchange rate away from some perceived fundamental value. In an ECB policy paper on the global spillovers in monetary policy, Moutot and Vitale (2009) argue that the spread of inflation-targeting and other monetary policy rules that raises policy rates in reaction to overheating domestic economic conditions may actually exacerbate capital inflows and undermine the effectiveness of higher interest rates in dampening demand.

For emerging economies, the interaction between domestic monetary policy and capital inflows can pose particularly difficult challenges in cooling an an overheating economy. An IMF working paper (Kamil (2008)) describing the experience of Colombia lays out the policy dilemma in the following terms.

> Foreign investors, realizing that the central bank would eventually focus on taming inflation (and eventually let the exchange rate appreciate), took unprecedented amounts of leveraged bets against the central bank in the derivatives market—thereby limiting the effectiveness of intervention. Paradoxically, then, the [Colombian central bank's] perceived strong commitment to inflation actually undermined its ability to influence the exchange rate. [Kamil (2008, pp.6-7)]

[1] See, for instance, the full page feature in the Financial Times entitled "Carried Away", April 30th, 2010.

Although the above quote is for Colombia, the same words could be applied to many other economies, especially in the aftermath of the global financial crisis where we have the conjunction of low U.S. dollar interest rates but booming emerging economies with surging capital inflows. Of the large economies, the case of Brazil has attracted particular attention due to its combination of high nominal interest rates and evidence of an overheating economy.[2]

In this paper, we ask when carry trades are liable to trigger destabilizing speculation in the foreign exchange market, and ask what role monetary policy plays in the speculative dynamics. We pose this question in a model of exchange rates that are anchored in the long-run to some known economic fundamentals, but where short-term speculative activity may push the exchange rate away from this known long-run fundamental value. We show that the model is general enough to accommodate both the stabilizing mode of speculative activity in the manner of Friedman (1953), but is also capable of generating destabilizing dynamics where speculators' actions to engage in the carry trade take on the attributes of *strategic complements* - that is, the action by other speculators to engage in the carry trade increases the attractiveness of engaging in the carry trade oneself.

Crucially, whether speculative activity is stabilizing or de-stabilizing depends on the monetary policy rule used by the capital recipient economy's central bank. The more sensitive is the recipient economy to overheating due to capital inflows, the more sensitive is the central bank's monetary policy rule to such inflows. But then, greater capital inflows fuel further increases in the interest rate of the recipient country, increasing the attractiveness of the carry trade, and fuelling the capital inflows. We show that when the conditions are right, the interaction between capital inflows and monetary policy can create a vicious circle that encourages destabilizing carry trade inflows.

Perhaps the poster child for the perverse interaction between monetary policy and carry trade inflows is Iceland in the run-up to the 2008 financial crisis. Even before the carry trade took on significant proportions, Iceland was already an overheating economy. The central bank had embraced inflation targeting in 2001, and kept official rates at high levels to try and curb an investment and consumption boom. The high interest rate differential fuelled capital inflows via the banking sector and through the

[2] Brazil recently raised its benchmark selic rate to 11.25% from 10.75%. See the WSJ article on Jan 20, 2011 "Brazil Raises Key Interest Rate" at: http://online.wsj.com/article/SB10001424052748704590704576092471442713938.html

issuance of eurobonds - the so-called "glacier bonds" (see Jonsson, p. 70) that were used to fund investment projects in Iceland. The outstanding notional amount of glacier bonds culminated above 30% of Icelandic GDP. Unsurprisingly, such large inflows led to a sharp appreciation in the Krona combined with overheating in the domestic economy. In particular, for reasons discussed below, the exchange rate appreciation did not have important demand-switching effects that could have offset the expansionary impact of foreign capital inflows. Thus, the carry trade exerted a positive feedback on official rates. As Iceland raised interest rates in response to the overheating economy, the higher interest rate differential attracted capital inflows that fuelled the investment boom that exacerbated the overheating economy. The inflation-targeting central bank raised interest rates further in response, giving a further twist to the vicious circle of an appreciating exchange rate and further capital inflows.

In the case of Iceland, several specific factors explain why this feedback loop between capital inflows and interest rates has been particularly strong. First, capital inflows fuelled a housing boom, and housing prices entered the CPI in Iceland, creating an almost mechanical link between carry trades and official rates (see Jonsson p.69). Also, exchange rate appreciation had limited impact in cooling down the economy due to the nature of domestic output. For example, a large fraction of Icelandic exports is fish, subject to binding regulatory constraints on catch volume, and thus not sensitive to price fluctuations. Also, most productive investments were long gestation projects related to (U.S. dollar denominated) commodities, by nature quite inelastic to the krona/yen or krona/euro exchange rates. The Chief Economist of the Central Bank of Iceland discusses the role of these factors and others in the apparent ineffectiveness of the exchange rate channel at cooling down the economy in Sighvatsson (2007).

The role of the housing and commodity sectors were also important in New Zealand, another important destination economy for carry trade inflows before the 2008 crisis. Our model will actually be posed in terms of the pre-crisis carry trade scenario where the funding currency is Japanese yen, and the destination currency is the New Zealand dollar. In our model, investors consume in yen but contemplate making deposits in New Zealand so as to benefit from the interest rate differential. The interest rate on New Zealand dollar is set with the primary goal of stabilizing inflation. Capital inflows boost domestic output and raise inflationary pressures. In addition, exchange rate fluctuations affect the price level of the traded fraction of consumption, but NZ dollar appreciation has limited demand composition effects that could cool down the economy.

With this basic set-up, we build our theoretical framework in two steps. First, we consider a simple deterministic version of our model and ascertain the conditions under which the actions across speculators to engage in the carry trade are strategic substitutes or strategic complements. We show that both cases are possible, and point to the crucial role played by the monetary policy reaction function of the recipient country in determining the answer. When carry trades are strategic complements, self-fulfilling departures from the steady state can be supported in equilibrium, much as in the vicious circle described for Iceland above.

Our fully developed theoretical framework develops the basic model by introducing small exogenous noise which leads to the selection of a unique, dominance-solvable equilibrium in the game played by speculative traders. Our equilibrium selection technique draws on the tools developed by Frankel and Pauzner (2000) and Burdzy, Frankel and Pauzner (2001) to solve dynamic coordination games. We show that our model is capable of generating the typical time path of exchange rates associated with carry trade recipient currencies where extended periods of slow appreciations of the high rate currency are stochastically punctuated by endogenous crashes. Currency traders refer to such extreme patterns as “going up by the stairs and coming down in the elevator” (see Breedon, 2001)[3].

In the unique equilibrium, the exchange rate exhibits history-dependence and depends on a highly nonlinear fashion on exogenous shocks. There are two forces that drive such speculative dynamics. First, cumulative foreign investment boosts output and thus official rates. Second, inflation expectations contribute to support these dynamics. When the size of the carry trade is large, it also implies that the carry trade has a small upside risk and a large downside risk because the dollar is overvalued and the economy is near full capacity. This asymmetric risk profile goes against coordination on the continuation of the carry trade. But this skewed risk profile is partially compensated by the fact that imported inflation risk is accordingly skewed in the opposite direction, which helps to foster coordination on speculative dynamics. In the case of small exogenous shocks, closed-form solutions generate insights into the impact of the primitive parameters of the model on the nature of the price paths. Slower-moving carry traders, and more active monetary policy lead to more "bubbly" exchange rate paths, with more history-dependence, and more rare and dramatic episodes of the New Zealand dollar "going down in the elevator."

[3] Gagnon and Chaboud (2007), Gyntelberg and Remolona (2007) and Brunnermeier, Nagel and Pedersen (2009) document the skewed nature of carry trade returns.

Related Literature

This paper relates to several strands of literature. Although our model does not address asset pricing issues directly, the equilibrium price paths in our paper that reproduce the pattern of "going up by the stairs and coming down in the elevator" are suggestive of skewed distributions of outcomes and the possible explanatory role of the peso problem in the foreign exchange markets. In a series of papers, Burnside, Eichenbaum, Kleshchelski and Rebelo (2006, 2007, 2008) have explored the extent to which conventional asset pricing models can explain the returns to carry trade positions, and point to the importance of rare jumps in the stochastic discount factor itself - a form of peso problem. Burnside (2010) is a recent further exploration of the limits of traditional risk factors to account for returns on foreign exchange markets, and how broadly applicable such risk factors are to traditional asset markets. Brunnermeier, Nagel and Pedersen (2009) study the link between carry trades and skewed distributions of exchange rates for funding and destination currencies, while Farhi and Gabaix (2009) argue that the possibility of rare disasters can account for the excess returns associated with carry trades.

Our paper has points of contact with the strand of the literature where coordination motives enter in speculative decisions by economic agents, such as the "collective moral hazard" models of Farhi and Tirole (2010) or Schneider and Tornell (2004). In these models, the government bails out speculators if their aggregate losses are sufficiently large. This creates a coordination motive among speculators. Excessively risky positions become attractive if a sufficiently large number of speculators crowd into them, because it activates the put induced by the bailout. This creates room for multiple equilibria. Speculators in our setup coordinate to induce a high interest rate on their NZ dollar holdings. Our main contribution to this literature is to develop a model in which, instead of multiple Pareto-ranked steady-states, similar externalities generate a unique equilibrium path with endogenous triggers of "bubble-and-crash"-like patterns. In particular, our closed-form equilibria are useful to understand how the probability and severity of such "bubbles and crashes" vary with the strength of positive externalities among traders. Given the important debate on the role of collective moral hazard in the recent financial crisis, we believe that our approach is a useful first step towards more applied and quantitative models of coordination-based financial instability.

Third, our model of sequential trading at random discrete dates relates to the literature on asset pricing with search frictions pioneered by Duffie, Garleanu, and Pedersen (2005). This literature has recently studied liquidity crises by developing models of out-of-steady-state price dynamics (see, e.g., Lagos, Rocheteau, and Weill, 2010). Our approach is complementary. We make a number of simplifying assumptions that renders the determination of asset prices more straightforward than in these papers. The gain from these more stylized aspects of our setup is that we fully characterize a stochastic steady-state with interesting nonlinear price dynamics.

Finally, the tools developed by Frankel and Pauzner (2000) and Burdzy, Frankel and Pauzner (2001) are reminiscent of the solution to static coordination games with private information - "global games" - introduced by Carlsson and van Damme (1993), and popularized by Morris and Shin (1998). The global-game framework relies on simultaneous actions at each date, and on the presence of very accurate private information. The approach developed by Burdzy, Frankel, and Pauzner seems better suited to the foreign exchange market for two reasons. First, we assume sequential trades, which arguably describes decentralized over-the-counter markets such as the FX market better than simultaneous submissions of market orders. Second, equilibrium uniqueness in our setup does not rely on the presence of private information. This again seems appropriate in the case of FX markets in which it is unlikely that material private information be available to inside traders.

2 Model

Time is continuous and is indexed by $t \in [0, +\infty)$. There are two currencies, Japanese yen and New Zealand dollar (simply, "yen" and "NZ dollar" henceforth). Yen serves as the *numéraire*. There are two types of agents - first, a continuum of unit mass of carry traders (or traders, or speculators henceforth), and second, a liquidity provider in the foreign exchange market. Carry traders are risk-neutral and discount the future at the rate $\delta > 0$. Carry traders consume in yen but may trade yen or dollar-denominated assets. All yen-denominated assets are in perfectly elastic supply at the expected rate of return δ. Carry traders face limits on the size of their trading positions, both for long and short positions. For simplicity, we will normalize the position limits and assume that a carry trader can invest a total of at most one yen, and that short positions are not permitted.

Each carry trader has a chance to rebalance its portfolio at discrete dates with constant arrival rate $\lambda > 0$. We assume that the Poisson processes associated with these rebalancing dates are independent across carry traders.

In between two such trading dates, a trader consumes instantaneous asset returns if they are positive, or refinances them if negative. This way, trading positions - the yen amount invested in each asset - are kept constant between trading dates.

Let x_t denote the aggregate yen amount invested in NZ dollar assets by the carry traders at date t. From the law of large numbers, a fraction λdt of the carry traders have an opportunity to rebalance their portfolios at each time interval dt, and so $(x_t)_{t\geq 0}$ follows the law of motion given by

$$\dot{x}_t = \lambda\left(b_t - x_t\right), \tag{1}$$

where $b_t \in [0,1]$ is the average yen amount invested in dollar-denominated assets by traders who have a chance to readjust their holdings at date t.

We assume that there is a "fundamental" value of the NZ dollar to which the exchange rate - that is, the yen amount required to purchase one NZ dollar - will ultimately revert to, but we will allow this fundamental anchor to be possibly quite weak in the interim. Formally, we model the fundamental anchor in the following way. At some random date, the trading in the foreign exchange market comes to a halt, at which time the exchange rate reverts to its "fundamental" value v where $v > 0$. This "fundamental value" v is known from the outset. All NZ dollar positions are liquidated at this terminal date, and no further trading takes place. Figuratively, we dub this random terminal date the "day of reckoning".

The "day or reckoning" is a fictional device which cannot be taken literally. However, in the model, it serves the useful purpose of allowing the fundamentals to exert an influence on the decisions of individual traders who trade *in anticipation* of its possible arrival. In terms of the model, our interest is in the dynamics of the economic system *before* the arrival of the day of reckoning.

We assume that the day of reckoning arrives according to a Poisson process with arrival rate $\rho \geq 0$. The arrival rate ρ is our way of parametrizing the strength of the fundamental anchor on the exchange rate. The smaller is ρ, the looser is the fundamental anchor and the longer the exchange rate may deviate from its fundamental value v. In the remainder, we study the evolution of the economy before this "day of reckoning" occurs.

The yen return earned by a carry trader on its dollar investments before the "day of reckoning" is driven by two processes - the evolution of the exchange rate and the instantaneous dollar rate of return on dollar denominated assets. We now describe these two processes.

Determination of Exchange Rate

At each rebalancing date, carry traders meet a competitive representative bank that is willing to trade currencies. At each date, the bank observes the net yen order flow $\dot{x}_t dt$ associated with traders' rebalancing decisions, and then quotes a competitive exchange rate. The bank is risk neutral and consumes in NZ dollars. It discounts the future at the rate γ, which is also the fixed instantaneous rate of return on its NZ dollar balances. It earns an instantaneous return of $z(y)$ NZ dollars when it holds a balance of y yen, where

$$z\left(y\right)=1-e^{-my},\ m>0.$$

Concavity of z implies that the bank has a preference for a balanced portfolio in the two currencies, which we may motivate by an aversion to inventory risk.[4] The particular functional form for z allows us to obtain a simple exchange rate process resulting from the carry traders' decisions. For all $t\geq 0$, let p_t denote the exchange rate - the yen amount required to purchase one NZ dollar at date t. We have

Lemma 1 *The NZ dollar exchange rate at date t is a function of x_t only and satisfies*

$$p_t=\frac{e^{mx_t}}{m\gamma}. \tag{2}$$

Proof. See Appendix A.■

For notational simplicity we normalize $m\gamma=1$ in the remainder.

Determination of Dollar Return

Carry traders earn the nominal deposit rate on NZ dollars, which we assume is identical to the official rate set by the New Zealand monetary authority - the Reserve Bank of New Zealand (RBNZ). The publicly known objective of the RBNZ is to maintain the inflation rate at a constant level i, and it uses adjustments in the official rate to control inflation. We denote by r_t the instantaneous official rate. The New Zealand price index π_t and domestic output y_t are assumed to follow the following static reduced form relationships.

[4] Bacchetta and van Wincoop (2009) and Hau and Rey (2006) use the analogous device of imperfect liquidity supply by currency traders with exogenous private valuations of the currencies to determine the exchange rate.

First, there is a Phillips curve relationship between inflation and output, given by

$$\frac{\dot{\pi}_t}{\pi_t} = \kappa y_t - \mu \frac{\dot{p}_t}{p_t} + c, \tag{3}$$

where y_t is the date t output, c, κ, μ are constants with $\kappa, \mu > 0$. Thus, inflation is positively related to output, and inversely related with NZ dollar appreciation. Output follows the IS curve relationship given by:

$$y_t = \nu x_t - \xi (r_t - i) + k, \tag{4}$$

where k, ν, ξ are constants with $\nu, \xi > 0$. New Zealand output is positively related to the net foreign capital position x_t, so that foreign capital inflows are expansionary.[5] However, output is negatively related to the real interest rate $r_t - i$.

The timing of actions within each time interval $(t, t + dt)$ is the following. First, carry traders who have the opportunity to rebalance their portfolio do so. This determines the aggregate net order $\dot{x}_t dt$ and the exchange rate fluctuation $\frac{\dot{p}_t}{p_t}$. Observing this, the RBNZ announces a nominal interest rate r_t, and output is produced. Using (4) to eliminate y_t from (3), we see that the RBNZ can ensure that

$$\frac{\dot{\pi}_t}{\pi_t} = i$$

with an interest rule of the form

$$r_t = \delta - \eta - \alpha \frac{\dot{p}_t}{p_t} + \beta x_t, \tag{5}$$

where

$$\alpha = \frac{\mu}{\kappa \xi}, \; \beta = \frac{\nu}{\xi}.$$

Thus α and β are large when capital inflows have a large impact on output (ν large), exchange rate fluctuations have an important impact on inflation (μ large), the official rate has little impact on output (ξ small), and taming inflation requires large output contractions (κ small). In the remainder, we will track how the coefficients α and β figure in the speculative dynamics. We will focus on the case $\alpha < 1$ because, as argued in Section 4.2, we believe it is empirically plausible.

[5] Although we measure net foreign capital in terms of x_t, it would be straightforward to use instead the cumulative amount of NZ dollars that carry traders bring into New Zealand $x_0 + \int_0^t \frac{\dot{x}_s}{p_s} ds = x_0 + \frac{1}{m}\left(e^{-m x_0} - e^{-m x_t}\right)$.

This modelling of the interplay of carry trades and monetary policy has two crucial ingredients:

1. $\nu > 0$. That is, purchases of NZ dollar assets by carry traders are expansionary. Notice that x_t is related to both the foreign capital stock and the exchange rate level. That $\nu > 0$ captures that the expansionary effect of foreign inflows is more important than the demand-switching impact of an expensive currency. For the reasons laid out in the introduction, this is plausible in the economies targeted by carry traders.

2. Carry traders move first within each period, and the central bank sets monetary policy to reach its inflation target. In this sense the central bank is the Stackelberg follower, and cannot commit ex ante to an explicit policy rule.

Our modelling of monetary policy yields an interest rate rule (5) that is linear in spot variables. This simplifies the exposition, but we should say here that more complexity could be accommodated in the analysis without materially affecting our main conclusions. Any monetary policy rule where the official rate (and hence the deposit rate) increases w.r.t to rational forecasts of future foreign capital stocks and decreases when the central bank expects future exchange rate appreciation would yield similar results.

3 Stabilizing Versus Destabilizing Carry Trades

Having characterized the simple responses of the liquidity supplier and the central bank to carry traders' actions in (2) and (5), we can boil down the problem into a simple game between carry traders. For a given initial position x_0, the process of rebalancing decisions $(b_t)_{t\geq 0}$ fully characterizes the paths for all variables in the economy until the arrival of the day of reckoning. The instantaneous rebalancing decisions $(b_t)_{t\geq 0}$ are the result of maximization decisions of the carry traders who seek the highest expected return to their portfolios.

We proceed to examine the strategic interaction between the traders, and in particular what determines whether the carry traders' decisions are strategic substitutes or complements - in other words, whether the incentive to engage in the carry trade is increasing or decreasing in the incidence of the carry trade in the wider population. To gain some intuition, suppose that a carry trader has a chance to rebalance its portfolio at some date t, and believes that future aggregate rebalancing positions until the day of reckoning are given by the deterministic process $(b_{t+u})_{u\geq 0}$. Let R_t denote the net unit return that this carry trader expects to earn by switching from yen to NZ dollar assets. We can write this net return as follows.

Lemma 2

$$R_t = \int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\frac{\dot{p}_{t+u}}{p_{t+u}} + r_{t+u} - \delta + \rho \frac{v - p_{t+u}}{p_{t+u}} \right] du. \tag{6}$$

Proof. See Appendix A.■

Expression (6) can be interpreted as follows. The term $e^{-(\lambda+\rho+\delta)u}$ inside the integral sign means that the trader (with discount rate δ) will hold the portfolio chosen at date t until one of two events happen: either the next rebalancing date arrives (with intensity λ), or the day of reckoning arrives (with intensity ρ). Until the first of these two dates, the trader earns an instantaneous return that has three components. The first component $\frac{\dot{p}_{t+u}}{p_{t+u}}$ is the instantaneous exchange rate appreciation of the NZ dollar. The second component $r_{t+u} - \delta$ is the interest rate differential between the NZ dollar and yen. The third component $\rho \frac{v-p_{t+u}}{p_{t+u}}$ is the capital gain/loss if the day of reckoning arrives before the next rebalancing date.

Plugging (2) and (5) into (6), the expected return R_t can be written as

$$\int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[m(1-\alpha)\dot{x}_{t+u} + \beta x_{t+u} + \rho \left(v e^{-m x_{t+u}} - 1 \right) - \eta \right] du,$$

which, by virtue of (1), further simplifies into

$$\int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\begin{array}{c} \lambda m(1-\alpha)\left(b_{t+u} - x_{t+u}\right) + \beta x_{t+u} \\ +\rho\left(v e^{-m x_{t+u}} - 1\right) - \eta \end{array} \right] du. \tag{7}$$

This expression offers insights into the nature of the strategic interaction between carry traders, and in particular whether traders' decisions to engage in the carry trade are strategic substitutes or complements. If the date$-t$ speculator expects that the incidence of carry trades will be high in the future (i.e. that the paths $(x_{t+u})_{u\geq 0}$ and $(b_{t+u})_{u\geq 0}$ take high values), does the trader have an incentive to switch to NZ dollar assets at date t? If the answer is yes, then the carry trades are mutually reinforcing and traders' actions are strategic complements. Under these circumstances, we may expect deviations of the exchange rate from its fundamental value sustained by self-fulfilling beliefs of NZ dollar appreciation or depreciation. If the answer is no, then the situation resembles strategic substitutability, and the exchange rate path will be self-stabilizing in that individually rational actions by the traders hasten the return of the exchange rate toward its fundamental value.

An inspection of the expression in (7) sheds some light on how parameter values determine the answer to this question. We can see first that if β is large - that is, if the official NZ dollar rate is sensitive to carry traders' holdings at a given date x_t - then the attractiveness to engage in the carry trade is increasing in the future path of the incidence of carry trades as given by $(x_{t+u})_{u\geq 0}$. A small ρ also implies that the fundamental anchor that ties the exchange rate is too weak to offset this potentially destabilizing impact of β. Under these conditions, the circumstances are ripe for the formation of self-fulfilling deviations of the exchange rate paths from the fundamental value.

The impact of $\lambda m(1-\alpha)$ on traders interactions is more subtle. This coefficient determines how future changes in carry traders' positions $b_{t+u}-x_{t+u}$ affect the decisions of current carry traders. We will see in Section 4 that large and protracted deviations of the exchange rate from its fundamental value are more frequent when the coefficient $\lambda m(1-\alpha)$ is smaller. The broad intuition is that a small $\lambda m(1-\alpha)$ implies that future traders will create limited losses for the current ones if they decide to bet in the opposite direction from them. This makes current traders more prone to trade in the same direction as their predecessors without the fear of being caught in the opposite trades of their successors. Notice that this coefficient is small in particular if inflation is very sensitive to exchange rate fluctuations (α close to 1). In this case, if one expects for example a future depreciation of the NZ dollar $\dot{p}_{t+u} = -\lambda m p_{t+u}$, then expectations of high future imported inflation and thus a high official rate mitigates the negative impact of depreciation on total return.

The next results formalize these intuitions on the role of β and ρ. In order to state the conditions on the parameters more precisely, we first characterize steady-states with constant exchange rate and interest rate. By steady-state, we mean steady-state until the day of reckoning.

Lemma 3

i) Let $x^* \in (0,1)$. The path $b_t = x_t = x^*$ for all t is a steady-state with constant exchange rate $p^* = e^{mx^*}$ if and only if $f(x^*) = 0$, where

$$f(x) = \beta x - \eta + \rho\left(ve^{-mx} - 1\right).$$

ii) Suppose

$$v \in (1, e^m) \text{ and } \beta > \eta. \tag{8}$$

Then if ρ is sufficiently large or sufficiently small, there exists a unique steady-state with constant exchange rate.

Proof. i) Substitute b_{t+u} and x_{t+u} with x^* in (7).

ii) Suppose is ρ sufficiently large that

$$\beta + \rho v e^{-m} < \eta + \rho < \rho v. \tag{9}$$

This implies that $f(0) > 0$ and $f(1) < 0$. Since in addition

$$f'(x) = \beta - \rho m v e^{-mx}$$

is strictly increasing, f' must be either negative, or negative then positive. (If it was positive, one would have $f(0) < f(1)$.) In any case this implies that f has a unique zero $x^* \in (0,1)$. Consider now ρ sufficiently small that

$$\beta + \rho v e^{-m} > \eta + \rho > \rho v. \tag{10}$$

Condition (10) implies $f(0) < 0$ and $f(1) > 0$. Since f' is increasing, this implies again that f has a unique zero x^*. ■

When such a steady-state exists, it satisfies a version of *uncovered interest parity* that takes account of the adjustment of the exchange rate toward the fundamental value associated with the "day of reckoning". To see this, note that in a steady-state characterized by x^* we have

$$\delta + \beta x^* - \eta + \rho \left(\frac{v - p^*}{p^*} \right) = \delta$$

A high x^* means that p^* is high, or that the exchange rate is perceived to be currently overvalued given its fundamental value v. Accordingly, investors require a high instantaneous return on NZ dollar $\delta - \eta + \beta x^*$ as compensation for the depreciation of the NZ dollar at the day of reckoning from p^* to v.

The parameter ρ plays a key role in determining the nature of such steady states with constant exchange rate. First, in line with the above discussion, if ρ is large enough (and with some additional conditions), the unique constant steady state is stable in the following sense. Starting from any initial conditions, the unique rationalizable outcome in this economy is for the exchange rate to converge to this steady state at the fastest rate that is consistent with the aggregate adjustment of portfolios. In this sense, when ρ is large, speculation is *stabilizing*. This case of stabilizing speculation is presented separately as Proposition 10 in Appendix B, and is proved there. Conversely, when the fundamental anchor is loose in the sense that ρ is small, the conditions are ripe for deviations of the exchange rate from its fundamental value driven by strategic effects across speculative traders. The next proposition formalizes this.

Proposition 4 *Suppose that (8) and (10) hold. There exist steady-states other than the constant one. If $x_0 \geq x^*$ then $b_t = 1$ for all t is a steady-state, while if $x_0 \leq x^*$, then $b_t = 0$ for all t is a steady-state.*

In words, when ρ is small enough (so that the fundamental anchor is loose) there are multiple rationalizable outcomes in the trading game depending on the initial aggregate holding $x_0 \in (0, 1)$. If $x_0 \geq x^*$, then all speculators holding only NZ dollars is a rationalizable path, while if $x_0 \leq x^*$ then all speculators holding yen only is a rationalizable path. Under such action profiles, the exchange rate path is determined by the mutually reinforcing nature of the traders' actions that push the exchange rate away from the steady state value x^*. In this sense, Proposition 4 describes the case where speculation is destabilizing.

Our focus from here will be on this case of *destabilizing* speculation, when ρ is small. We first prove Proposition 4 and then consider how the multiplicity will give way to a unique equilibrium by the addition of fundamental shocks.

Proof of Proposition 4. The expected return (7) can be rewritten as

$$\int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\lambda m(1-\alpha)\dot{x}_{t+u} + f\left(x_{t+u}\right)\right] du. \tag{11}$$

As mentioned in the proof of Lemma 3, condition (10) implies that $f(x) \leq 0$ over $[0, x^*]$ and $f(x) \geq 0$ over $[x^*, 1]$. From (11), this readily implies that for $x_0 \geq x^*$ (respectively $x_0 \leq x^*$) $\dot{x}_t = \lambda\left(1 - x_t\right)$ is a steady-state (respectively $\dot{x}_t = -\lambda x_t$ is a steady-state).■

When speculation is destabilizing, the steady-state x^* is unstable in the sense that, starting from the steady state exchange rate p^*, the self-interested actions of speculators will push the exchange rate to its minimal or maximal value. If all traders believe that all subsequent traders will enter the carry trade, then the action to enter (or stay in) the NZ dollar carry trade is justified by the fact that i) the NZ dollar will appreciate and ii) the NZ dollar interest rate will stay sufficiently high that the rate differential more than compensates for the (low) probability of the arrival of the day of reckoning. A symmetric argument establishes the possibility of destabilizing downward trajectory for the NZ dollar exchange rate.

However, we now show that adding some residual uncertainty to this model enables us to obtain a unique dominance-solvable outcome for the case when ρ is small.

4 Uniqueness of Equilibrium with Shocks

The multiplicity of equilibria when ρ is small is not robust to the addition of noise in carry trade returns. Adding vanishingly small shocks on instantaneous returns, we obtain a unique dominance-solvable equilibrium for parameter values that would lead to multiple predictable outcomes absent such shocks. The technique we draw on is from the work of Burdzy, Frankel and Pauzner (2001) and Frankel and Pauzner (2000), who showed that in binary action coordination games with strategic complementarities, the addition of small stochastic shocks to the fundamentals of the payoffs generates a unique, dominance solvable outcome in games where players adjust their actions with Poisson arrival rates.

Formally, we assume in this section that the date t instantaneous return on the carry trade has an exogenous stochastic component. The instantaneous return is of the form:

$$\frac{\dot{p}_t}{p_t} + r_t - \delta + \rho \frac{v - p_t}{p_t} + w_t,$$

where

$$w_t = \sigma W_t,$$

with W_t a standard Wiener process, and $\sigma > 0$. We interpret these shocks as unexpected evolutions of the New Zealand economy, or possible "policy shocks" that emanate from the actions of the monetary authority. Assuming persistent shocks greatly simplifies the analysis. We will discuss more realistic shocks in section 4.1. The next proposition outlines cases where despite an arbitrarily small ρ - even possibly $\rho = 0$, the economy has a unique rationalizable outcome.

Proposition 5 *Suppose*

$$\beta > m\left(\lambda\left(1-\alpha\right) + \rho v\right). \tag{12}$$

Then there exists a unique rationalizable outcome. It is characterized by a decreasing Lispchitz function l such that

$$b_t = \mathbf{1}_{\{w_t > l(x_t)\}}.$$

In other words, traders enter the carry trade when (w_t, x_t) is on the right of the frontier defined by $\{w = l\left(x\right)\}$ in the plane (w, x), and exit the carry trade otherwise.

Proof. The proof as the same broad structure as the proof of Theorem 1 in Frankel and Pauzner (2000). Some parts are more complex, however, because expected returns depend only on future values of x_t in their model, while it depends on future x_t and $\dot{x}_t$ in ours. Define $l_0(x_t)$ such that if $w_t = l_0(x_t)$, then a trader rebalancing at date t is indifferent between entering the carry trade or not if she believes that for all $u \geq 0$, $b_{t+u} = 0$. We have:

Lemma 6 *l_0 is Lipschitz decreasing.*

Proof. See Appendix A.■

Define now $l_1(x_t)$ such that if $w_t = l_1(x_t)$, then a trader rebalancing at date t is indifferent between entering the carry trade or not, if she believes that for all $u \geq 0$, $b_{t+u} = \mathbf{1}_{\{w_{t+u} > l_0(x_{t+u})\}}$. That is, she expects that traders will enter the carry trade when (w_{t+u}, x_{t+u}) is on the right of the frontier $\{w = l_0(x)\}$ in the plane (w, x), and exit it otherwise. Note that l_1 is well defined since Theorem 1 in Burdzy et al. (1998) shows that the stochastic differential equation

$$\begin{aligned} \dot{x}_t &= -\lambda x_t \text{ for } w_t < l_0(x_t) \\ \dot{x}_t &= \lambda(1 - x_t) \text{ for } w_t > l_0(x_t) \end{aligned}$$

is such that for almost every path of w_t, there is a unique Lipschitz path x_t that starts from a given x_0. We have:

Lemma 7

l_1 is Lipschitz decreasing. The Lipschitz constant of l_1 is smaller than that of l_0.

Proof. See Appendix A.■

By iterating this process, we can obtain the boundary l_∞ for the region where a trader exiting the carry trade can be eliminated by iterated dominance. l_∞ is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The boundary l_∞ defines an equilibrium strategy since, if all traders hold yen to the left and hold dollar to the right, the indifference point between dollar and yen for the trader also lies on l_∞. We now show that this equilibrium is actually the unique rationalizable outcome.

Consider a translation to the left of the graph of l_∞ in (w, x) so that the whole of the curve lies in a region where holding yen is dominant. Call this translation l'_0. To the left of l'_0, holding yen is dominant. Then construct l'_1 as the rightmost translation of l'_0 such that a trader must choose yen to the left of $l_1^{'}$ if she believes that other traders will play according to $l_0^{'}$. By iterating this process, we obtain a sequence of translations to the right of $l_0^{'}$. Denote by l'_∞ the limit of the sequence. Refer to Figure 1.

[Figure 1 here]

The boundary l'_∞ does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of $l_0^{'}$. However, we know that if all others were to play according to the boundary l'_∞, then there is at least one point A on Z'_∞ where the trader is indifferent between holding yen and holding dollar. If there were no such point as A, this would imply that l'_∞ is not the rightmost translation, as required in the definition.

We claim that l'_∞ and l_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between l'_∞ and l_∞. Then, choose point B on l_∞ such that A and B have the same height - i.e. have the same second component. But then, since the shape of the boundaries of l'_∞ and l_∞ and the values of x are identical, the paths starting from A must have the same distribution as the paths starting from B up to the constant difference in the initial values of w. This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B. If she were indifferent at A, she would strictly prefer to hold dollar at B, and if she is indifferent at B, she would strictly prefer to hold yen at A. But we constructed A and B so that traders are indifferent in both A and B. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap", and we must have $l'_\infty = l_\infty$.■

Proposition 5 can be illustrated in Figure 2. The curve l divides the (w, x)-space into two regions. Proposition 5 states that in the unique equilibrium, any trader decides to enter the carry trade to the right of the l curve, and exit the carry trade to the left of the l curve. Thus, p_t will tend to rise in the right hand region, and tend to fall in the left hand region, as indicated by the arrows in Figure 2.

[Figure 2 here]

The dynamics of the flow of funds implied by the unique equilibrium is given by:

$$\dot{x}_t = -\lambda x_t + \lambda 1_{\{w_t > l(x_t)\}}. \tag{13}$$

where $1_{\{.\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations,* and are studied in Bass and Burdzy (1999) and

Burdzy et al. (1998). From Theorem 1 in Burdzy et al. (1998), for a given initial x_0, and for almost every sample path of w, there exists a unique Lipschitz solution $(x_t)_{t\geq 0}$ to the differential equation (13) defining the price dynamics for l Lipschitz decreasing.

Some suggestive features of the price dynamics can be seen from Figure 2. Starting on the frontier l, a positive shock will pull the system on the right of it. Unless the path of w is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that the dollar appreciates for a while so that x_t becomes close to 1. In this case, the exchange rate obeys

$$\frac{\dot{p}}{p} = \lambda m\,(1-x) \simeq 0.$$

If cumulative negative shocks eventually lead the system back to the left of the frontier, the rate of depreciation is

$$\frac{\dot{p}}{p} \simeq -\lambda m$$

In other words, when p is high and the currency crosses the boundary from above, there is a sharp depreciation that was preceded by a slow appreciation. Such dynamics are suggestive of the price paths of high-yielding currencies in carry trades that "go up by the stairs and come down in the elevator". Proposition 5 demonstrates the impact of adding some uncertainty to the carry return. The multiplicity of equilibria reported in the previous section resulted from the feature that, if the fundamentals were fixed and known, then one cannot rule out all other players trading in one direction, provided that the fundamentals were consistent with such a strategy. However, the introduction of shocks changes the picture radically. Trades are far less nimble than the shifts in the carry. Thus, choosing to enter the carry trade *versus* exiting the carry trade entails a substantial degree of commitment over time to fix one's trading strategy. Suppose that the (w, x) pair is close to a dominance region, but just outside it. If w is fixed, it may be possible to construct an equilibrium for both actions, but when w moves around stochastically, it may wander into the dominance region between now and the next opportunity that the trader gets to trade. This gives the trader some reason to hedge her bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on. Essentially, when adding shocks, the unstable situations described in Proposition 4 can be solved the same way as the stable situations described in Proposition 10 when condition (12) holds.

That l is decreasing implies that price paths exhibit hysteresis. If the dynamic system (w_t, x_t) is in the area where buying is dominant $(w_t > l(x_t))$, then the buy pressure takes the system away from l, making the continuation of a bullish market even more likely, all else equal. Thus, although uniquely defined by the paths of shocks, exchange rate paths exhibit strong history-dependence and nonlinearities. The following numerical simulation illustrates this.

[Figure 3 here]

This is a simple spreadsheet simulation with 5,000 draws that is not meant to be quantitatively meaningful, but only to offer some intuition. Compare points A and B. At both points, the path crosses l after negative shocks on w. Even though they correspond to fairly similar values of w, the two crossings have different consequences. In the first one, there is no krach because w bounces back sufficiently quickly that the frontier is crossed the other way. Thus the carry trade resumes after a brief scare. Reaching B has a more significant impact because w spends more time at low levels around B, and thus beliefs shift more significantly around it. Then, the carry trade resumes around point C only, which is considerably higher than the level at which carry traders were willing to step in again after the negative shock in A: the bifurcation in B has created a new higher threshold for the carry trade to be acceptable. Finally, point D exhibits a major breakdown of the carry trade. Beliefs about others' beliefs have become so pessimistic at this point that even the fairly high levels reached by w towards the end of the path are not sufficient to spark carry trades again.

4.1 The Limiting Case of Small Shocks

As the volatility of shocks σ tends to zero, it is possible to characterize the shape of the frontier l and the behaviour of the exchange rate more precisely, even with closed-form solutions if in addition $\rho = 0$. In order to stress the dependence of the frontier $l(x)$ on the parameter σ, we denote it $l(\sigma, x)$ in this section. Suppose the economy is in the state $(l(\sigma, x_t), x_t)$ at date t. That is, it is on the frontier. For some arbitrarily small $\varepsilon > 0$, introduce the two stopping times

$$\begin{aligned} T_1 &= \inf_{u \geq 0} \{x_{t+u} \notin (\varepsilon, 1-\varepsilon)\}, \\ T_0 &= \sup_{0 \leq u < T_1} \{x_{t+u} \neq l(\sigma, x_{t+u})\}. \end{aligned}$$

In words, T_1 is the first date at which x_t gets close to 0 or 1, and T_0 is the last date at which x_t crosses the frontier before T_1. If T_0 is small in distribution, it means that the economy is prone to bifurcations. That is, it never stays around the frontier for long. Upon hitting it, it quickly heads towards the lowest or the highest exchange rate. The next proposition shows that this is actually the case when shocks are small. This, in turn, yields a simple explicit determination of the frontier.

Proposition 8 *Assume that $\rho = 0$, (12) holds, and*

$$\beta\delta > 2m\lambda\,(\lambda + \delta)\,(1 - \alpha). \tag{14}$$

Then for all $x \in (0, 1)$

$$\lim_{\sigma \to 0} l(\sigma, x) = \eta - \frac{1}{2\lambda + \delta}\left[(\beta\delta - 2m\lambda\,(\lambda + \delta)\,(1 - \alpha))\,x + \lambda\,(\beta + m\,(\lambda + \delta)\,(1 - \alpha))\right]. \tag{15}$$

If $w_t = l(\sigma, x_t)$, then as $\sigma \to 0$, T_0 converges to 0 in distribution, and the probability that $\dot{x}_t > 0$ (respectively $\dot{x}_t < 0$) over $[T_0, T_1]$ converges to $1 - x_t$ (x_t respectively).

Proof. See Appendix A.■

An explicit determination of the bifurcation boundary l is useful because, at least in the case in which (14) holds, it generates insight into the impact of the primitive parameters of the model on exchange rate dynamics. If the function l has a smaller slope in absolute value, then its graph is closer to a vertical line in the plane (w, x). All else equal, the frontier should obviously be crossed more often when it is more vertical. Conversely, the larger the slope of l (in absolute value), the more likely are protracted bifurcations towards 0 or 1. From expression (15), as shocks become small and (14) holds, such long stays of x_t around 0 or 1 are more likely to occur when all else equal, λ becomes smaller and α and β larger. Thus, with more illiquid investments and a very active stance of monetary policy, long bifurcations are more likely. The intuition is exactly the one that we got from expression (7). A high λ makes carry trades at different dates look like strategic substitutes, while large α and β make them resemble strategic complements. When the former is small and the latter are large, the coordination motives behind the carry trade are quite important, and the fundamental shocks have to be quite large in order to stop traders from bifurcating one way or the other. In the opposite case, fundamental shocks play a more direct role in traders' decisions and a less important role in the formation of their beliefs about future actions. Thus shock lead to more frequent reversals of the carry trade.

Proposition 9 *Assume that* $\rho = 0$, *(12) holds, and*

$$\beta\delta < 2m\lambda(\lambda + \delta)(1 - \alpha). \tag{16}$$

Then there exists $l^* \in (0, 1)$ *such that for all* $x \in (0, 1)$

$$\lim_{\sigma \to 0} l(\sigma, x) = l^*$$

Proof. See Appendix A.■

When (16) holds and shocks become small, then the system no longer displays history dependence. In this limiting case, traders' decisions depend only on the current value of w, and no longer on the value of x. Thus the correlation between policy shocks and exchange rate fluctuations should be stronger in this limiting case than in the one outlined in Proposition 8 in which carry traders change their mind only rarely. Clearly the opposite of UIP should hold since traders enter the carry trade when the interest rate differential is sufficiently large.

Finally, the limiting case of small shocks also has another interesting feature. As shown in Burdzy et al. (1998), if the volatility of shocks is sufficiently small, then equilibrium uniqueness also holds when shocks have a drift that depends on x_t and t. This allows for instance for mean-reversion.

4.2 Orders of magnitude

Even though our stylized approach is not aimed at a quantitative exercise, it is worthwhile checking that the orders of magnitude under which the parameters are conducive to speculative dynamics - in particular, under which condition (12) is satisfied, are plausible. The Taylor rule coefficients commonly discussed for the United States assign a coefficient of 1.5 to the realized inflation rate. Monetary policy is likely to be at least as active in emerging economies in which the central bank has recently started to build inflation-targeting credibility. Assuming a degree of openness of 40%, a rough computation leads to a value of the coefficient α of around $0.4 \times 1.5 = 0.6$. Bacchetta and van Wincoop (2009) claim an average two-year rebalancing frequency to be plausible in FX markets in general, and assume it in order to quantitatively explain the forward discount bias. Accordingly, we believe that a value for λ of around 50% is reasonable. If the carry trade can move the exchange rate by, say 30% ($m = 30\%$) and $\rho = 0$ (very loose fundamental anchor), then (12) holds as soon as the coefficient β is larger than 6%. Given the amplitude of the fluctuations in official rates rates recently observed, it seems plausible that the cumulative funds from the carry trade, that reached more than 30% of the Icelandic GDP for instance, can have a maximal impact of more than 600 basis points on the official rate. In sum, these admittedly rough orders of magnitude suggest at least that (12) is not implausible. It also shows that the assumption of slow-moving carry traders (λ relatively small) is important for this condition to hold. This assumption seems particularly relevant for the New Zealand or Iceland carry trades that involved retail investors. The glacier bonds denominated in Icelandic krona or the uridashi bonds used by Japanese investors to invest in New Zealand had a typical maturity of 1 to 5 years, and were principally purchased by unhedged retail investors with no access to a liquid secondary market. For such an investor, a two-year average commitment to the carry trade seems plausible.

5 Concluding Remarks

The implications of our model are twofold. First, as illustrated in Figure 3, we predict speculative dynamics that are characterized by history-dependence and a non-linear relationship between observed shocks and the exchange rate. To be sure, related types of "bubbly" dynamics can be derived from models of "rational" bubbles such as in Blanchard and Watson (1982), or Froot and Obstfeld (1991). Such models, however, typically feature a continuum of equilibria. Our unique equilibrium delivers sharper predictions on the probability and distribution of large price fluctuations given history.

Second, insights from Proposition 8 can be tested. More precisely, our predictions on the relationship between the presence of an important population of slow-moving carry traders, the stance of monetary policy, and the amount of negative shocks that are necessary to create a large depreciation of the high rate currency given history are novel and testable in principle. One can study for instance, the links between cumulative shocks to monetary policy and the probability of a sharp New Zealand dollar depreciation through the lens of our theory. In sum, we offer microfoundations for a new model of exchange rate regime switching that can be tested in principle.

Our approach is positive in essence. Starting from a reduced-form modelling of monetary policy, we focussed on developing a detailed model of "bubbly" exchange rate dynamics with a unique equilibrium. Still, we believe that it delivers some normative insights into the policies that the central bank can implement in order to stabilize the exchange rate and the economy. Obviously, credible threats of capital control (increasing ρ) can restore a stable exchange rate, as suggested in Proposition 10. More interestingly, if the RBNZ could ex ante commit to a less active monetary policy (smaller α and β), this would make the bifurcation frontier more vertical, and therefore lead to less dramatic bifurcations of the exchange rate and less extreme reversals. In our stylized model in which inflation is always under control, this would also generate a more stable equilibrium path for the output. A full normative analysis of optimal monetary policy along these lines would require a more full-fledged macroeconomic model. But we believe that this paper, in which the sensitivity of exchange rate dynamics to policy parameters is uniquely determined and easy to characterize, offers a useful first building block towards this aim.

The stripped-down nature of our model has enabled us to extract analytical solutions from a potentially very complex problem. However, this simplicity has costs, too. Although we have incorporated a monetary policy rule into the model, there is the prior question of where such a rule comes from. Indeed, the fact that monetary policy plays the role of the backdrop to the strategic interactions among speculative traders raises interesting questions of how the existing literature on optimal monetary policy (such as Clarida, Gali and Gertler (2000) and Gali and Monacelli (2005)) might be amended if it were to include departures from uncovered interest parity. The current literature on optimal monetary policy are fully-fledged general equilibrium models, but have traditionally assumed that uncovered interest parity (UIP) holds. The consequences of the failure of UIP in such models would be of great interest and be worthy of further investigation.

Appendix A

5.1 Proof of Lemma 1

Let y_t denote the cumulated number of yens sold by carry traders to the bank between 0 and t, and d_t denote the cumulated number of NZ dollars paid by the bank in exchange. We have

$$y_t = x_t,\ d_t = -\int_0^t \frac{\dot{x}_s}{p_s} ds.$$

That the bank is competitive implies that she quotes at each date the exchange rate at which she is indifferent between making the trade or not, or such that

$$\frac{d}{dt}\left(\gamma d_t + z(y_t)\right) = \gamma \dot{d}_t + z'(x_t)\dot{x}_t = 0,$$

and $\dot{d}_t = -\frac{\dot{x}_t}{p_t}$implies in turn $\gamma p_t = \frac{1}{z'(x_t)}$.■

5.2 Proof of Lemma 2

After investing one yen in dollars at date t, the trader is locked into the position until the first of two dates: the next rebalancing date that arrives with intensity λ and the intervention date that arrives with intensity ρ. The expected return earned on the dollar investment over this period is thus

$$\int_0^{+\infty} e^{-(\lambda+\rho)u} \left[(\lambda+\rho)\int_0^u e^{-\delta s}\left(\frac{\dot{p}_{t+s}}{p_{t+s}} + r_{t+s}\right) ds + \rho e^{-\delta u}\frac{v - p_{t+u}}{p_{t+u}}\right] du. \tag{17}$$

The term $\frac{\dot{p}_{t+s}}{p_{t+s}} + r_{t+s}$ is the return earned per unit of time before the end of the period. It is comprised of the exchange rate fluctuation $\frac{\dot{p}_{t+s}}{p_{t+s}}$ and of the interest on dollar r_{t+s}. (Recall that these returns are not compounded since by assumption the trader keeps a constant yen position.) The second term is the capital gain/loss in case the intervention occurs before the rebalancing date. If she keeps her yen instead, her expected return over the period is

$$\int_0^{+\infty} \delta e^{-(\lambda+\rho+\delta)u} du. \tag{18}$$

Integrating the first term in (17) by parts and then substracting (18) yields 6). ■

Proof of Lemma 6

Consider (w_t, x_t) and (w'_t, x'_t) such that

$$0 < x_t < x'_t < 1, w_t = l_0(x_t), w'_t = l_0(x'_t).$$

Both a trader who starts at (w_t, x_t) and one who starts at (w'_t, x'_t) expect zero return from the carry trade if all subsequent traders exit it. The difference in their expected returns, Δ, is thus also equal to zero. Thus,

$$\Delta = \int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\varphi(x'_{t+u}) - \varphi\left(x_{t+u}\right) + w'_t - w_t\right] du = 0,$$

where

$$\varphi(x) = \left(\beta - \lambda m \left(1-\alpha\right)\right) x + \rho v e^{-mx}.$$

Thus

$$\frac{l_0(x_t) - l_0(x'_t)}{\lambda+\rho+\delta} = \int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\varphi(x'_t e^{-\lambda u}) - \varphi\left(x_t e^{-\lambda u}\right)\right] du,$$

and l_0 is Lipschitz decreasing since φ is Lipschitz, increasing from (12). ■

5.3 Proof of Lemma 7

Let us first show that l_1 is decreasing. The idea is to reason Brownian path by Brownian path. More precisely, consider (w_t, x_t) and (w'_t, x'_t) such that

$$0 < x_t < x'_t < 1, w_t = l_1(x_t), w'_t = l_1(x'_t).$$

Suppose $l_1(x_t) < l_1(x'_t)$. Consider two paths $\left(w'_{t+u}\right)_{u\geq 0}$ and $\left(w_{t+u}\right)_{u\geq 0}$ such that for all $u \geq 0$

$$w'_{t+u} - w_{t+u} = w'_t - w_t.$$

The difference in expected returns *conditional on these particular Brownian paths* is:

$$\Omega = \int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\begin{array}{c} \lambda m \left(1-\alpha\right)\left(b'_{t+u} - b_{t+u}\right) \\ +\varphi(x'_{t+u}) - \varphi\left(x_{t+u}\right) + l_1\left(x'_t\right) - l_1\left(x_t\right) \end{array}\right] du.$$

From Lemma 2 in Burdzy et al. (1998), $(w_t, x_t) < (w'_t, x'_t)$ implies that for all $u \geq 0$

$$x'_{t+u} \geq x_{t+u}.$$

But then, since it is also the case that $w'_{t+u} \geq w_{t+u}$, and l_0 is decreasing, it must be that whenever (w_{t+u}, x_{t+u}) is on the right of the frontier, so is (w'_{t+u}, x'_{t+u}). This implies that for all u,

$$b'_{t+u} \geq b_{t+u} \ .$$

Since φ is increasing, $\Omega > 0$ along every such pair of paths, which contradicts that the expected returns (now with expectations over almost all possible Brownian paths) are equal.

We now show that l_1 is Lipschitz with a constant smaller than

$$K = \sup_{x \neq y} \frac{l_0(y) - l_0(x)}{x - y}.$$

Suppose by contradiction that there exists two points on l_1 such that $x' > x$ and

$$\frac{l_1(x) - l_1(x')}{x' - x} > K. \tag{19}$$

Consider again two paths with starting points

$$x_t = x, w_t = l_1(x) \text{ and } x'_t = x', w'_t = l_1(x'),$$

and such that $w'_{t+u} - w_{t+u} = w'_t - w_t$. Let us show that for all $u \geq 0$,

$$b_{t+u} \geq b'_{t+u}. \tag{20}$$

Let us define the sequence $(T_n)_{n \geq 0}$ as follows. First,

$$T_0 = 0,$$

Then for all $k \geq 0$,

$$\begin{aligned} T_{2k+1} &= \inf_{u \geq T_{2k}} \left\{ w'_{t+u} = l_0\left(x'_{t+u}\right)\right\}, \\ T_{2k+2} &= \inf_{u \geq T_{2k+1}} \left\{ w_{t+u} = l_0\left(x_{t+u}\right)\right\}, \end{aligned}$$

with the convention that if one of these sets is empty for some $n \geq 1$, then $T_m = +\infty$ for all $m \geq n$. We prove by iterations that (20) holds over $[T_{2k}, T_{2k+2}]$. Let us first show it over $[T_0, T_2]$. If $T_1 = +\infty$ then $b'_{t+u} = 0$ and (20) is true. Otherwise, since $b'_{t+u} = 0 \leq b_{t+u}$ for $0 \leq u \leq T_1$ then it must be that

$$x'_{t+T_1} - x_{t+T_1} \leq (x'_t - x_t)e^{-\lambda T_1}. \tag{21}$$

because for all $u \in [0, T_1]$

$$\begin{aligned} \dot{x}'_{t+u} - \dot{x}_{t+u} &= \lambda \left(b'_{t+u} - b_{t+u}\right) - \lambda \left(x'_{t+u} - x_{t+u}\right) \\ &\leq -\lambda \left(x'_{t+u} - x_{t+u}\right). \end{aligned}$$

(21) implies

$$x'_{t+T_1} - x_{t+T_1} < x'_t - x_t < \frac{l_1 (x_t) - l_1 (x'_t)}{K} = \frac{w_{t+T_1} - w'_{t+T_1}}{K}.$$

This implies that (w_{t+T_1}, x_{t+T_1}) is strictly on the right of the frontier l_0. If $T_2 = +\infty$ then $b_{t+T_1+u} = 1$ and the result is established. Otherwise $b_{t+u} = 1 \geq b'_{t+u}$ for $u \in [T_1, T_2]$ and thus

$$\begin{aligned} x'_{t+T_1+T_2} - x_{t+T_1+T_2} &\leq (x'_{t+T_1} - x_{t+T_1}) e^{-\lambda T_2} \\ &< x'_t - x_t < \frac{l_1 (x_t) - l_1 (x'_t)}{K} = \frac{w_{t+T_1+T_2} - w'_{t+T_1+T_2}}{K}, \end{aligned}$$

which implies that $\left(w'_{t+T_1+T_2}, x'_{t+T_1+T_2}\right)$ is strictly on the left of the frontier l_0. But then we can apply over $[T_2, T_4]$ what we just did over $[T_0, T_2]$, with $\left(w'_{t+T_1+T_2}, x'_{t+T_1+T_2}\right)$ and $(w_{t+T_1+T_2}, x_{t+T_1+T_2})$ in lieu of (w_t, x_t) and (w'_t, x'_t), and so on until one T_n is infinite. If all the T_n are finite, then it is straightforward to see that the above reasoning can be used for a proof by recursion that $b_t \geq b'_t$ over intervals $[T_{2k}, T_{2k+2}]$.

Now, if (19) and therefore (20) were true starting from x_t and x'_t, then the differential expected return (computed over all future paths):

$$\Lambda = E_t \int_0^{+\infty} e^{-(\lambda+\rho+\delta)u} \left[\begin{array}{c} \lambda m (1-\alpha) \left(b'_{t+u} - b_{t+u}\right) + \\ \varphi(x'_{t+u}) - \varphi(x_{t+u}) + l_1 (x'_t) - l_1 (x_t) \end{array} \right] du.$$

would be strictly negative, which would contradict the definition of l_1. First, (20) implies that the first term in the integrand is negative. Second, that

$$x'_{t+u} - x_{t+u} \leq \left(x'_t - x_t\right) e^{-\lambda u}. \tag{22}$$

over all future parallel paths, together with the fact that l_1 is strictly steeper than l_0 between x_t and x'_t, implies that the integral over the second term in the integrand is strictly negative: We know from the proof of Lemma 6 that it was equal to 0 when we had l_0 instead of l_1, and inequality (22) was an equality. With $l_1 (x'_t) - l_1 (x_t) < l_0 (x'_t) - l_0 (x_t)$, it must be strictly negative ■

Proof of Proposition 8

Let

$$\psi(x) = \eta - \frac{1}{2\lambda+\delta}\left[\left(\beta\delta - 2m\lambda(\lambda+\delta)(1-\alpha)\right)x + \lambda\left(\beta + m(\lambda+\delta)(1-\alpha)\right)\right].$$

Suppose that a trader can rebalance at a date t at which $w_t = \psi(x_t)$. Suppose that the trader believes that subsequent traders will play according to the frontier ψ. Since $\psi'(x_t) \neq 0$, we know from Theorem 2 in Burdzy et al. (1998) that, as $\sigma \to 0$, the economy bifurcates quickly - in the sense that T_0 tends to 0 in distribution, upwards with prob. $1 - x_t$ or downwards with prob. x_t. Thus, in order to prove the proposition we only need to show that ψ is actually the frontier in the limit when $\sigma \to 0$. If the economy bifurcates upwards, then

$$\dot{x}_{t+u} = \lambda(1-x_t)e^{-\lambda u},\ x_{t+u} = 1-(1-x_t)e^{-\lambda u},$$

while if it bifurcates downwards we have

$$\dot{x}_{t+u} = -\lambda x_{t+u},\ x_{t+u} = x_t e^{-\lambda u}.$$

Thus, as $\sigma \to 0$, the agent's expected return tends to

$$\begin{aligned}&(1-x_t)\int_0^{+\infty} e^{-(\lambda+\delta)u}\left[\lambda m(1-\alpha)(1-x_t)e^{-\lambda u} + \beta\left(1-(1-x_t)e^{-\lambda u}\right)\right]du \\ &+x_t\int_0^{+\infty} e^{-(\lambda+\delta)u}\left[-\lambda m(1-\alpha)x_t e^{-\lambda u} + \beta x_t e^{-\lambda u}\right]du + \frac{\psi(x_t)-\eta}{\lambda+\delta},\end{aligned}$$

and simple algebra shows that this is equal to

$$\frac{\left(\beta\delta - 2m\lambda(\lambda+\delta)(1-\alpha)\right)x_t + \lambda\beta + m\lambda(\lambda+\delta)(1-\alpha)}{(2\lambda+\delta)(\lambda+\delta)} + \frac{\psi(x_t)-\eta}{\lambda+\delta}. \quad (23)$$

It is easy to see that this is equal to 0. Thus it must be that

$$\lim_{\sigma\to 0} l(\sigma,x) = \psi(x)$$

since the expected return stays bounded away from zero as $\sigma \to 0$ if $w_t \neq \lim_{\sigma\to 0} l(\sigma, x_t)$. ■

Proof of Proposition 9

Suppose that there exists $y \in (0,1)$ such that

$$\frac{\partial \lim_{\sigma \to 0} l(\sigma, y)}{\partial x} \neq 0.$$

Again, we know from Theorem 2 in Burdzy et al. (1998) that in this case, if σ becomes small, then starting from $(l(\sigma, y), y)$ the economy quickly bifurcates. From (23), as $\sigma \to 0$, the expected return from entering the carry trade at date t if $(w_t, x_t) = (l(\sigma, y), y)$ tends to

$$\frac{(\beta\delta - 2m\lambda (\lambda + \delta)(1 - \alpha)) x_t + \lambda\beta + m\lambda (\lambda + \delta)(1 - \alpha)}{(2\lambda + \delta)(\lambda + \delta)} + \frac{\lim_{\sigma \to 0} l(\sigma, x_t) - \eta}{\lambda + \delta}.$$

Inequality (16) then implies that the frontier would become strictly increasing as $\sigma \to 0$, which contradicts Proposition 5. Thus it must be that there exists no such y, and that $l(\sigma, x)$ has a limit that does not depend on x when $\sigma \to 0$.■

Appendix B

In this appendix, we consider the case of *stabilizing* speculation and prove the following result for the case without Brownian shocks.

Proposition 10 *Suppose ρ is sufficiently large that*

$$\beta + \rho v e^{-m} < \eta + \rho < \rho v. \tag{24}$$

and $\frac{\lambda}{\rho}$ sufficiently large that

$$\beta > m\left(\rho v - (\lambda + \rho + \delta)(1 - \alpha)\right). \tag{25}$$

Then, there exists a unique dominance solvable outcome. This outcome is characterized by the unique $x^* \in (0,1)$ such that $f(x^*) = 0$. Trading strategies are such that for all $t \geq 0$

$$b_t = x^* \times \mathbf{1}_{\{x_t \leq x^*\}} + (1 - x^*)\, \mathbf{1}_{\{x_t < x^*\}}.$$

In words, traders enter the carry trade whenever $p_t < p^*$, exit the carry trade when $p_t > p^*$, and invest x^* yens in dollars otherwise.

Proof. Inequality (24) implies that $f(0) > 0$ and $f(1) < 0$. Since in addition

$$f'(x) = \beta - \rho m v e^{-mx}$$

is strictly increasing, f' must be either negative, or negative then positive. (If it was positive, one would have $f(0) < f(1)$.) In any case this implies that f has a unique zero $x^* \in (0,1)$, is strictly positive over $(0, x^*)$, and strictly negative over $(x^*, 1)$.

We show that all paths but the one that converges to x^* as quickly as possible can be ruled out by rational traders. Consider a given rationalizable process $(x_t)_{t \geq 0}$. Let

$$\overline{x} = \sup\left\{y : \text{Prob}\left(\exists t \geq 0 \ s.t.\ x_t \geq y\right) > 0\right\}.$$

In other words, $\overline{x}$ is the upper bound of all values that have some probability of being reached in the future. We first establish the following:

Lemma 11 *Suppose $\overline{x} > x^*$. Under condition (25), there exists $\eta > 0$, such that for all $t \geq 0$, if*

$$x_t \geq \overline{x} - \eta$$

then the net expected return from entering the carry trade at date t must be strictly negative.

Proof of Lemma 11. It is sufficient to show that there exists η sufficiently small such that if $\overline{x} > x^*$, then for any arbitrary sample path $(x_t)_{t\geq 0}$, the net expected return from the carry trade conditional on this sample path is strictly negative. For such a given sample path, the traders who have a rebalancing date at t face an expected return

$$R_t = \frac{f(x_t)}{\lambda+\rho+\delta} + \int_0^{+\infty} e^{-(\lambda+\rho)u} \left[\begin{array}{c} (\lambda+\rho+\delta)\, m\,(1-\alpha)\,(x_{t+u}-x_t) \\ +f(x_{t+u}) - f(x_t) \end{array} \right] du. \tag{26}$$

Since f is continuous and negative over $[x^*, 1)$, there exists $\eta_1 > 0$ such that

$$x_t \geq \overline{x} - \eta_1 \rightarrow \frac{f(x_t)}{\lambda+\rho+\delta} < \frac{2}{3}\frac{f(\overline{x})}{\lambda+\rho+\delta}.$$

Condition (25) implies that

$$g(x) = (\lambda+\rho+\delta)\, m\,(1-\alpha)\, x + f(x)$$

is strictly increasing, Lipschitz. Thus, there exists η_2 sufficiently small such that the integral on the right-hand side of (26) can be made smaller than $\frac{-f(\overline{x})}{3(\lambda+\rho+\delta)}$ for $x_t \geq \overline{x} - \eta_2$ since by definition of $\overline{x}$:

$$x_t \geq \overline{x} - \eta_2 \rightarrow \forall s, x_{t+u} - x_t \leq \eta_2$$

For $\eta = \inf\{\eta_1, \eta_2\}$, we have

$$x_t \geq \overline{x} - \eta \ \rightarrow \ R_t \leq \frac{f(\overline{x})}{3\,(\lambda+\rho+\delta)} < 0.$$

Since the net expected return is strictly negative given any sample path, it is also strictly negative with expectations taken over all future paths.□

Lemma 11 implies that there exists $\eta > 0$ such that as soon as the process $(x_t)_{t\geq 0}$ is above $\overline{x} - \eta$, traders strictly prefer to exit the carry trade. Thus it must be that $\overline{x} = x_0$ otherwise this would contradict the definition of $\overline{x}$. Lemma 11 also implies that the process $(x_t)_{t\geq 0}$ is strictly decreasing in the band $[x_0, x_0 - \eta]$. Since the result holds for any initial path value, it must be that all rationalizable processes starting above x^* are strictly decreasing until x_t reaches x^*.

Step 2. One can show in a symmetric way that

$$x_t < x^* \rightarrow \forall u > 0, x_{t+u} > x_t.$$

Step 3. Steps 1 and 2 imply that the path $(x_t)_{t\geq 0}$ decreases strictly until it reaches x^* when $x_0 > x^*$, increases strictly until it reaches x^* when

$x_0 < x^*$, and then stays constant once x^* is reached. It is clear from (26) that this implies that R_t is strictly negative (respectively positive) when x_t is strictly larger (respectively smaller) than x^*. Thus $b_t = 0$ when $x_t > x^*$ and $b_t = 1$ when $x_t < x^*$ is the only rationalizable outcome.$\blacksquare$

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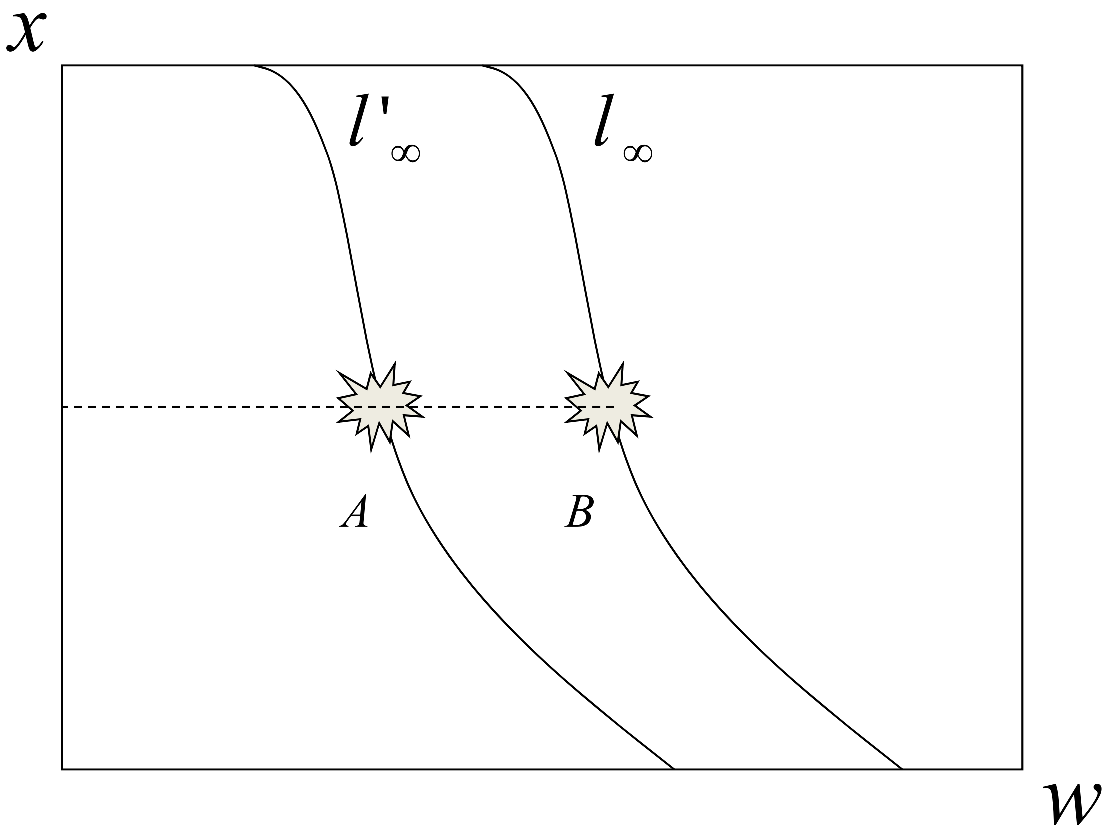

$$l'_{\infty} = l_{\infty}$$

Figure 1. If a trader is in *A* and thinks that other traders enter the carry trade if and only if they are to the right of l'_{∞}, then future price trajectories will just be horizontal translations of the trajectories realized when a trader is in *B* and thinks that other traders enter the carry trade if and only if they are to the right of l_{∞}. Thus a trader can be indifferent between both situations only if *A* and *B* correspond to the same *w* and thus $l_{\infty} = l'_{\infty}$.

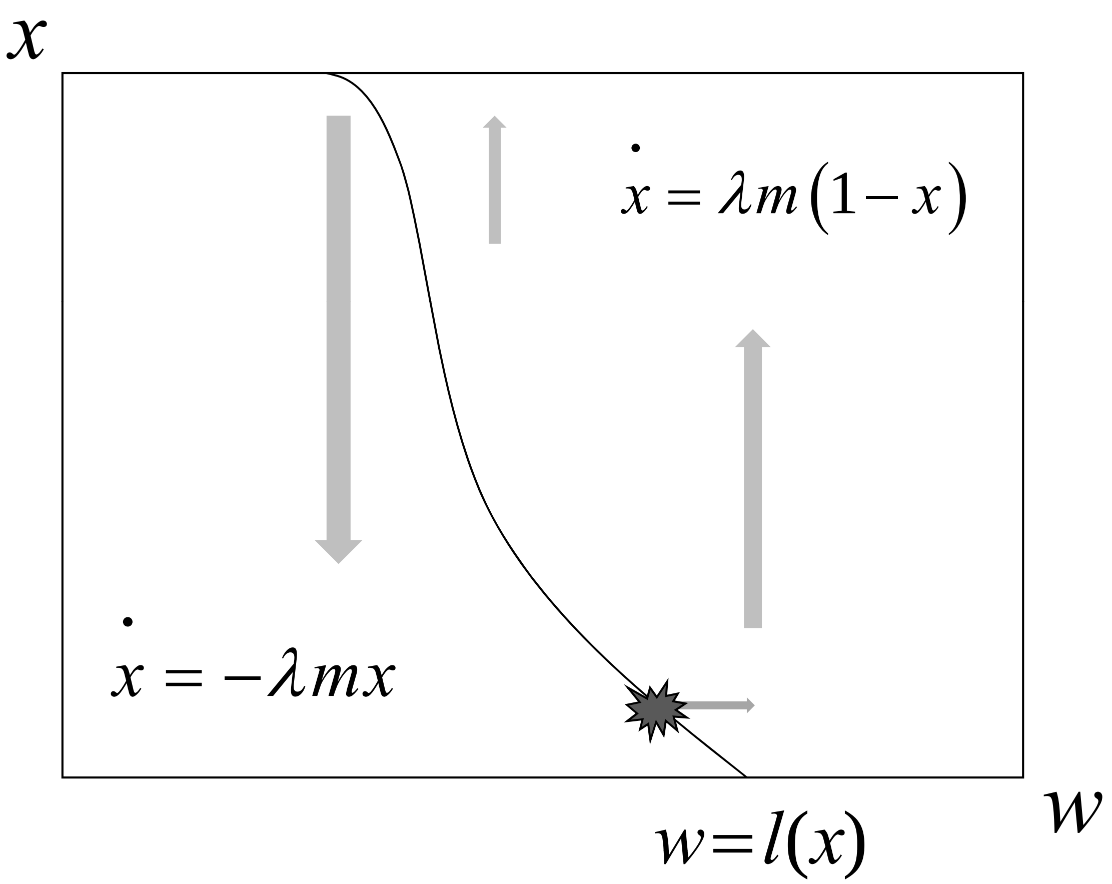

Figure 2. Unique outcome with shocks.

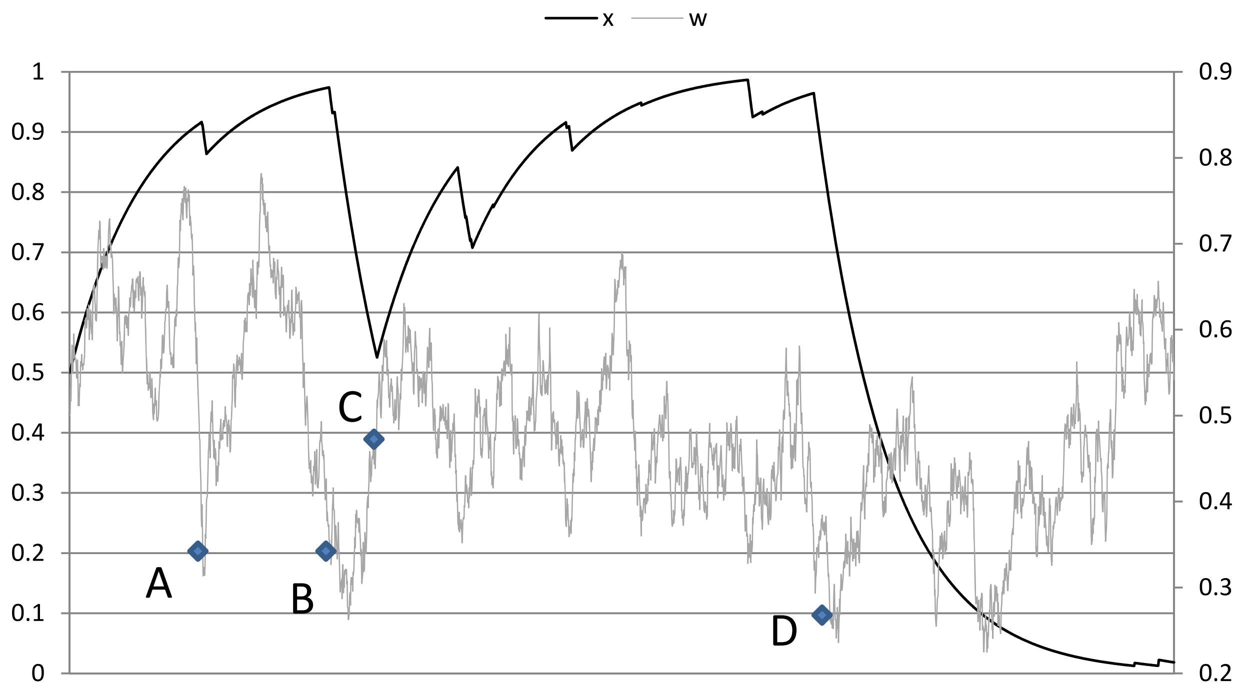

Figure 3. Sample paths of *x* and *w.* The frontier is *w=0.65-0.3x* and the system starts on the frontier at (0.5,0.5). The scale of *x* is on the left, *w* on the right.