Optimal Devaluations
PRELIMINARY AND INCOMPLETE

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Abstract

We analyze optimal policy in a simple small open economy model with price setting frictions. In particular, we study the optimal response of the nominal exchange rate following a terms of trade shock. We depart from the New Keynesian literature in that we explicitly model internationally traded commodities as intermediate inputs in the production of local nal goods and assume that the small open economy takes this price as given. This modi - cation is not only in line with the long standing tradition of small open economy models, but also changes the optimal movements in the exchange rate. In contrast with the recent Small Open Economy New Keynesian literature, our model is able to reproduce the co-movement between the nominal exchange rate and the price of exports, as it has been documented in the commodity currencies literature. While we show there are preferences for which price stability is optimal even without exible scal instruments, our model suggests that more attention should be given to the coordination between monetary and scal policy (taxes) in small open economies that are heavily dependent on exports of commodities. The model we propose is a useful framework to study fear of oating.

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1. INTRODUCTION

The purpose of this paper is to study the optimal response of monetary and exchange rate policy to a change in the price of a commodity a small open economy actively trades in international markets. This question is very important for many economies in the world. Indeed, commodity prices are very volatile and, in many cases, exports of commodities are a sizable fraction of foreign trade. In Figure 1, we plot monthly data on prices for a set of commodities for the last 10 years. The prices are expressed in constant dollars as a fraction of the price in January 2000. In Table 1 we report the principal commodity exports for a selection of small open economies and their shares in total good exports. We also report the aggregate share of good exports over total exports and of total exports over gross domestic product.[1]

Concern regarding shocks to commodity prices runs very high in the political agenda of these countries. For small open economies (say Chile) a drop in the exportable commodity price (cooper) is seen as recessionary; the same happens following an increase in the price of the importable commodity (oil).[2] It is precisely to hedge against this uncertainty that in recent years, countries in which the government either owns or taxes the firms that produce a particular commodity, like Norway (oil) and Chile (cooper), passed legislation forcing the Treasury to save in foreign assets in periods when the commodity prices are "high", in order to be able to spend more in times in which the prices are "low". While it is clear that volatility of international commodity prices can give rise to fiscal policies like the one just described, it is less clear what are its implications, if any, regarding monetary and exchange rate policy. In small open economies (SOE), movements in the nominal exchange rate are important shock absorbers. In a world with fully flexible prices, this should not be important. But in the presence of nominal rigidities, as emphasized in the "New open economy macroeconomics" literature, shocks to the terms of trade could lead to inefficient real effects. That literature, however, has largely ignored the effects of commodity price shocks. This is the main theme of our paper.

The one we address is a central question for policy design in small open economies. For example, both New Zealand and Chile have explicitly adopted an inflation targeting policy. This means that the Central Bank defines an inflation rate on the consumer price index as its main policy objective. Therefore, the Central Bank abstains from foreign exchange interventions and the nominal exchange rate is fully market determined. It turns out that the resulting volatility of the nominal exchange rate is very high and that it moves negatively with the international

[1] Total imports of commodities can also be large, but they are not so concentrated in a few goods. That is why we do not report a table similar to Table 1 for imports.

[2] Chile imported over 90% of the oil consumed during the last 10 years.

price of the exportable in small open economies that follow inflation targeting.[3] Figure 2 depicts the nominal exchange rate and the dollar price of the main exportable commodity for Chile and Norway as deviations from trend. The shocks are very large. In Table 2, we report several moments for these variables. The table makes clear that the volatility of these shocks are large, as are their correlations.[4] The current literature that studies optimal monetary policy with price frictions in small open economies has largely ignored commodities and is unable to reproduce these facts.

It is precisely because of this volatility that the law allows Central Banks to deviate from the pure inflation targeting policy under "special circumstances." The Central Bank of Chile did so in April 2008 and announced a program for buying international reserves (for an amount close to 40 percent of the existing stock) after the nominal exchange rate went from over 750 pesos per dollar in March 2003 to below 450 in March 2008. The program was suspended with only 70 percent of the announced purchases completed in September 2008, once the exchange rate jumped back to around 650 pesos. A new program to buy reserves was announced in January 2011, with a total amount over 40 percent of the existing stock. The exchange rate last January was around 475 pesos per dollar. The justification used by the board of the Central Bank of Chile was that "The international economy presents an unusual state, characterized by high commodity prices, low interest rates, slow recovery of the developed economies, and depreciation of the US dollar."[5]

Is this an optimal policy in a small open economy facing large shocks to commodity prices? The model we analyze in this paper builds from the existing literature and provides a step forward in providing an answer to that question.

Following the seminal work of Obstfeld and Rogoff (1995, 1996), there has been growing interest in studying optimal policy in open economies with frictions in the setting of prices or wages. A branch of the literature, like Obstfeld and Rogoff (2000), Engel (2001), and many others, focuses on the two-country case.[6] This literature emphasizes the relationship between the strategic interactions in two-country models and optimal exchange rate policy, and in most cases it focuses on the flexible versus fixed exchange rate regimes debate. Gali and Monacelli (2005) specifically consider the case of the small open economies and several other papers followed, like

[3] To the extent that these countries succeed in stabilizing in ation, the nominal exchange rate volatility translates into real exchange rate volatility.

[4] See Chen and Rogo (2003) and other papers in the commodity currencies literature.

[5] The statement can be found in http://www.asipla.cl/2011/01/04/banco-central-interviene-el-mercado-cambiario-con-compra-de-us12-000-millones/. The translation to English has been made by the authors.

[6] An incomplete list also includes Corseti and Pesenti (2001) and (2005), Devereux and Engel (2003), Benigno and Benigno (2003), Duarte and Obstfeld (2004), Ferrero (2005) and Adao, Correia and Teles (2005).

Faia and Monacelli (2008) and de Paoli (2009).

The main innovation of our paper, relative to the New Keynesian literature just cited, is to explicitly model commodities as intermediate goods in production, using a model similar in spirit to the one used by Burstein, Da Neves, and Rebelo (2003) and Burstein, Eichenbaum, and Rebelo (2007).[7] Following the tradition on small open economy models, the international price of these commodities is exogenous to the economy we consider. In previous papers, only domestic inputs - typically labor - enter into the production function of domestic final goods. The final goods are produced by local monopolists and are traded internationally. In our model, domestic inputs *and* traded commodities enter the production function of the final goods. Then, as in the previous models, the final goods are produced by local monopolists and can be traded internationally.

This is the obvious modification to make, given the motivation of the paper: To study optimal monetary and exchange rate policy in the presence of shocks to commodity prices. But it is also important, as we will clearly demonstrate in the paper, for two other reasons. First, in the standard model, an increase in the price of importables is, contrary to the concerns mentioned above, expansionary.[8] The reason is that a reduction in the international relative price of local *final* goods implies, via a substitution effect in preferences, an increase in world—and local—demand for the local composite good which in turns increases local production. On the contrary, in our model, when the increase is on the price of the *intermediate* importable—relative to the exportable intermediate—the units of labor required to import one unit of the importable intermediate increases and it is therefore contractionary. Second, in the model without traded commodities, the shock to the terms of trade does not change local costs, so it does not interact in an interesting way with the domestic price frictions.[9] To the extent that the emphasis is not to discuss the optimal policy response following terms of trade shocks—as it is the case with the previous literature—this may be inessential. Given the emphasis of this paper, this is a key distinction.

On the methodological front, we also depart from the literature in that we consider distorting fiscal instruments, as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), and

[7] In our model, commodities are intermediate inputs that are traded internationally in perfectly competitive markets. This assumption, very common in the small open economy literature in the 70's and 80's has been dropped in the New Keynesian small open economy literature.

[8] Our model follows the literature in that nal goods are produced by monopolists, so the price of exportables is endogenous, even in the small open economy case. Thus, a negative shock to the terms of trade means an increase in the price of importables.

[9] This is also a consequence of the Dixit-Stiglitz formulation. As demands faced by monopolist of nal goods have constant elasticities, changes in demand a ect quantities sold, but not prices, since domestic marginal costs are not a ected by changes in the price of foreign nal goods.

Correia, Nicolini and Teles (2008). This approach has the advantage of making explicit all the existing distortions in the economy. The analysis thus provides a minimal set of monetary and fiscal instruments required to achieve the second best allocation. One can then use the model to evaluate the welfare cost of imposing restrictions on the available instruments. Indeed, it has become standard in the literature to assume that while monetary and exchange rate policy are flexible, in the sense that they can be made time and state dependent, fiscal policy is not. The model of the paper can easily be used to evaluate optimal policy with restrictions on the set of instruments.

We study a representative agent economy with final goods produced by monopolistically competitive firms—so firms have power to set prices—and tradable intermediate inputs—so we can analyze the optimal policy response following terms of trade shocks. Final goods are produced using domestic labor[10] and two intermediate inputs (one importable and one exportable). The exportable intermediate good is produced by perfectly competitive firms that take the international price as given and produce using domestic labor and a non-tradeable input in fixed supply, which we label "land".[11] The price of the importable intermediate input is also given to the country. We follow the literature and assume a Calvo-type price rigidity, in which only a randomly selected group of final goods firms are allowed to change prices in any given period. We also follow the tradition of the recent New Keynesian literature and assume a cashless economy where currency only plays the role of a numeraire.

The fiscal policy instruments that we consider are consumption, labor income, and dividend taxes, as well as import and export tariffs. We also allow the government to issue state contingent bonds. We abstract from the question of the best intermediate target for monetary policy and also from the question of implementability. We characterize sequences of nominal interest rates and nominal exchange rates, $\{R_t, S_t\}_{t=0}^{\infty}$, that are consistent with the optimal allocation, but we abstract form the bigger question of how to implement that allocation. It is well known that while exchange rate rules implement a unique allocation, interest rate rules lead to global indeterminacy. As it is standard in Ramsey analysis, we abstract from time inconsistency and assume full commitment. Thus, whichever role the exchange rate can have in fostering good—or bad!—reputation will be absent in this analysis.

We first show, in Section 2, how the introduction of commodities implies that domestic costs interact with commodity prices and changes the transmission mechanism of nominal exchange

[10] We interpret labor very broadly, including all services that are non-tradable and that are essential to production.

[11] This input should be intrepreted more broadly. It could represent oil or copper reserves in teh case of exhaustible resources.

rate movements. We also show that the model can theoretical be consistent with the evidence on Table 2 in countries that follow inflation targeting. Movements in the exchange rate become key to stabilize costs, and therefore prices.

In Section 3, we solve for the Ramsey allocation. We show that if taxes can be flexible, price stability is optimal, as in Gali and Monacelli (2005). Thus, their policy implication survives in a different model, that can potentially replicate the moments on Table 2, and where the transmission mechanism of exchange rate movements is very different. The reason is that in these models with price frictions, price stability implies production efficiency, as it becomes clear in the discussion that follows. Production efficiency is a feature of the optimal allocation in many environments. We also show that for preferences that are widely used in the New Keynesian literature (Gali and Monacelli (2005), Farhi, Gopinath and Itskhoki (2011) among many others), the optimal taxes are constant over time and state[12]. Thus, in this case, the model justifies a policy that stabilizes prices even if the nominal exchange rate is subject to very large fluctuations and taxes cannot be made flexible. These preferences exhibit constant elasticities for labor and aggregate consumption. In addition, aggregate consumption must be a Cobb-Douglas function of domestic and foreign consumption. Thus, under the assumptions of the model, a case for "fear of floating" can only be made based on preferences only.[13] In Section 4, we show that the model can reproduce the behavior of the nominal exchange rate in Chile and Norway (as depicted in Figure 2 and Table 2), characterize the optimal monetary and exchange policy, and provide welfare computations under alternative assumptions regarding preferences. WORK IN PROGRESS.

2. THE MODEL

The model is composed of a small open economy, which we call home, and the rest of the world. There are two final goods that can be internationally traded, one of them produced at home and the other produced in the rest of the world. The home economy faces a downward sloping demand for the final good it produces but is unable to affect any other international price—hence the term semi-small open economy. There is also international trade in two intermediate inputs (commodities) that are used in the production of intermediate goods. Home is inhabited by households, the government, competitive firms that produce the final good, competitive firms that

[12] Similar results have been found for closed economies - see Zhou (1992).

[13] It should be noted, however, that we only consider the case of domestic producer price frictions. Allowing for local currency price frictions or adding wage frictions would change the implications of this model. In the jargon of the New Keynesian literature, the divine coincidence falls apart in those cases. We leave the analysis of these cases for future research.

produce one of the tradeable commodities, and a continuum of firms that produce differentiated intermediate goods.

Time is denoted by $t = 0, 1, ...\infty$. We let μ_t denote the state of the economy at period t and $\mu^t = (\mu_0, \mu_1, ..., \mu_t)$ the history of states up to time t. We denote by $\pi(\mu^t)$ the probability of μ^t conditional on μ_0 and, for simplicity, we assume that μ_t belongs to a finite set of events for all t.

Households

There is a representative household that has preferences over contingent sequences of two final consumption goods, C_t^h and C_t^f, and labor N_t. Utility function is weakly separable between the final consumption goods and labor, and represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where $0 < \beta < 1$ is a discount factor, $C_t = H\left(C_t^h, C_t^f\right)$ is a function homogeneous of degree one and increasing in each argument, and $U(C, N)$ is increasing in the first argument, decreasing in the second, and concave.

Financial markets are complete. We let $B_{t,t+1}$ and $B_{t,t+1}^*$ denote one-period discount bonds denominated in domestic and foreign currency respectively. These are bonds issued at period t that pay one unit of the corresponding currency at period $t+1$ on a particular state of the world and zero otherwise.

The household's budget constraint is given by

$$\begin{aligned} &(1+\tau_t^c)\left(P_t^h C_t^h + P_t^f C_t^f\right) + E_t\left[Q_{t,t+1}B_{t,t+1} + S_t Q_{t,t+1}^* B_{t,t+1}^*\right] \leq \\ &W_t(1-\tau_t^n) N_t + B_{t-1,t} + S_t B_{t-1,t}^*, \end{aligned} \tag{2}$$

where S_t is the nominal exchange rate between domestic and foreign currency, W_t is the nominal wage rate, τ_t^c is a tax on final goods, τ_t^n is a labor income tax, $Q_{t,t+1}$ is the domestic currency price of the one period contingent domestic bond normalized by the conditional probability of the state $\pi\left(\mu_{t+1}|\mu_t\right)$, and $Q_{t,t+1}^*$ is the analogous foreign currency price of the foreign bond. We assume that dividends are fully taxed.

Using the budget constraint at periods t and $t+1$ and rearranging gives the no-arbitrage condition between domestic and foreign bonds

$$Q_{t,t+1} = Q^*_{t,t+1} \frac{S_t}{S_{t+1}}. \tag{3}$$

It is convenient to work with the present value budget constraint. To that end, for any integer $k > 0$, we let $Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+2} ... Q_{t+k-1,t+k}$ be the currency price of one unit of domestic currency at a particular history μ^{t+1} in terms of domestic currency at time t, and an analogous definition holds for $Q^*_{t,t+k}$. Iterating forward on (2) and imposing the no-Ponzi condition $\lim_{t\to\infty} E_0 \left[Q_{0,t} B_t + S_t Q^*_{0,t} B^*_t\right] \geq 0$ gives

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[(1+\tau^c_t) \left(P^h_t C^h_t + P^f_t C^f_t \right) - W_t (1-\tau^n_t) N_t \right] \leq 0. \tag{4}$$

Here we have assumed that initial financial wealth is zero, or $B_{-1,0} = B^*_{-1,0} = 0$.

The household maximizes (1) subject to (4). The optimality conditions are given by

$$\frac{H_{C^h}\left(C^h_t, C^f_t\right)}{H_{C^f}\left(C^h_t, C^f_t\right)} = \frac{P^h_t}{P^f_t} \tag{5}$$

$$\frac{U_C(C_t, N_t) H_{C^h}\left(C^h_t, C^f_t\right)}{-U_N(C_t, N_t)} = \frac{P^h_t (1+\tau^c_t)}{W_t (1-\tau^n_t)} \tag{6}$$

$$\frac{U_C(C_t, N_t) H_{C^h}\left(C^h_t, C^f_t\right)}{P^h_t (1+\tau^c_t)} = \beta Q_{t,t+1} \frac{U_C(C_{t+1}, N_{t+1}) H_{C^h}\left(C^h_{t+1}, C^f_{t+1}\right)}{P^h_{t+1}\left(1+\tau^c_{t+1}\right)} \tag{7}$$

Government

The government sets monetary and fiscal policy and raises taxes to pay for exogenous consumption of the home final good, G^h_t.[14] Monetary policy consists of rules for the nominal interest rate R_t and the exchange rate S_t. Fiscal policy consists of labor taxes τ^n_t, consumption taxes τ^c_t, export taxes τ^h_t on final goods, and dividend taxes τ^d_t. There are two sources of pure rents in the model: the dividends of intermediate good firms and the profits of commodity producers—equivalently, one can think of the latter as the rents on a fixed factor of production, like land.

[14] It is straightforward to also let the government consume foreign goods. For simplicity, we ignore this possibility.

Throughout the paper, we assume that all rents are fully taxed so that $\tau_t^d = 1$ for all μ^t.[15]

Final good firms

Perfectly competitive firms produce the domestic final good Y_t^h by combining a continuum of non-tradeable intermediate goods indexed by $i \in (0,1)$ using the technology

$$Y_t^h = \left[\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution between pair of intermediate goods. Given prices $P_i^h(\mu^t)$ for each $i \in (0,1)$ and the final good price $P^h(\mu^t)$, the firm's problem implies the cost minimization condition

$$y_{it} = Y_t^h \left(\frac{P_{it}^h}{P_t^h}\right)^{-\theta} \tag{8}$$

for all $i \in (0,1)$. Integrating this condition over all varieties and using the production function gives a price index relating the final good price and the prices of the individual varieties,

$$P_t^h = \left(\int_0^1 P_{it}^{h1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{9}$$

Commodities sector

There are two tradeable commodities, denoted by x and z, that are used as inputs in the production of intermediate goods. The home economy, however, is only able to produce the x commodity. The z commodity must be imported. We denote by P_t^x and P_t^z the local currency prices of commodities x and z respectively.

Commodity x is produced according to the technology

$$X_t = A_t (n_t^x)^\rho, \tag{10}$$

where n_t^x is labor, A_t is the level of productivity, and $0 < \rho \leq 1$. Implicit in this technology is the assumption of a fixed factor of production (when $\rho < 1$) which we broadly interpret as land.

[15] If pure rents are not fully taxed, the Ramsey government will use the in ation tax to partially tax those rents (Schmitt-Grohe and Uribe, 2004) . We deliberately abstract from this role of monetary policy because we believe that scal policy is the natural instrument to tax rents.

Profit maximization then requires

$$\rho P_t^x A_t \left(n_t^x\right)^{\rho-1} = W_t. \tag{11}$$

Because the two commodities can be freely traded, the law of one price holds:

$$\begin{aligned} P_t^x &= S_t P_t^{x*} \\ P_t^z &= S_t P_t^{z*}. \end{aligned} \tag{12}$$

Here, P_t^{x*} and P_t^{z*} denote the foreign currency prices of the x and z commodities.

Intermediate good firms

Each intermediate good $i \in (0,1)$ is produced by a monopolistic competitive firm who uses labor and the two tradeable commodities with the technology

$$y_{it} = \frac{Z_t x_{it}^{\eta_1} z_{it}^{\eta_2} \left(n_{it}^y\right)^{\eta_3}}{\eta_1^{\eta_1} \eta_2^{\eta_2} \eta_3^{\eta_3}}$$

where x_{it} and z_{it} is the demand for commodities, n_{it}^y is labor, Z_t denotes the level of productivity, $\eta_j \geq 0$ for $j = 1, 2, 3$, and $\sum_{j=1}^{3} \eta_j = 1$.[16]

The associated nominal marginal cost function is common across intermediate good firms and given by

$$MC_t = \frac{\left(P_t^x\right)^{\eta_1} \left(P_t^z\right)^{\eta_2} W_t^{\eta_3}}{Z_t}.$$

Using (11) and (12), the nominal marginal cost can be written as $MC_t = S_t MC_t^*$, where MC_t^*, the marginal cost measured in foreign currency, is given by

$$MC^*\left(\mu^t\right) = \frac{\left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3}}{Z_t}. \tag{13}$$

That is, the marginal cost in foreign currency depends on the exogenous international commodity prices, on technological factors, and on the equilibrium allocation of labor in the commodities sector.

[16] Our results generalize to any constant returns to scale technology.

In addition, cost minimization implies that final goods firms choose the same ratio of inputs,

$$
\begin{aligned}
\frac{x_{it}}{n_{it}^{y}} &= \frac{\eta_1}{\eta_3}\rho A_t \left(n_t^x\right)^{\rho-1} \\
\frac{z_{it}}{n_{it}^{y}} &= \frac{\eta_2}{\eta_3}\frac{P_t^{x*}}{P_t^{z*}}\rho A_t \left(n_t^x\right)^{\rho-1} \quad \text{for all } i \in (0,1)\,.
\end{aligned} \tag{14}
$$

where we used equation (11).

Introducing (14) into the production function gives

$$
y_{it} = n_{it}^{y}\frac{Z_t}{\eta_3}\left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{1-\eta_3}\left(P_t^{x*}\right)^{\eta_2}\left(P_t^{z*}\right)^{-\eta_2}. \tag{15}
$$

Each monopolist i faces the downward sloping demand curve (8). We follow the standard tradition in the New Keynesian literature and impose Calvo price rigidity. Namely, in each period, intermediate good firms are able to reoptimize nominal prices with a constant probability $0 < \alpha < 1$. Those that get the chance to set a new price will set it according to

$$
p_t^h = \frac{\theta}{\theta-1}E_t\sum_{j=0}^{\infty}\eta_{t,j}\frac{\left(P_{t+j}^x\right)^{\eta_1}\left(P_{t+j}^z\right)^{\eta_2}W_{t+j}^{\eta_3}}{Z_{t+j}}, \tag{16}
$$

where

$$
\eta_{t,j} = \frac{(\alpha\beta)^j\frac{\left(1-\tau_{t+j}^d\right)u_C(t+j)}{\left(1+\tau_{t+j}^C\right)}\left(P_{t+j}^h\right)^{\theta-1}Y_{t+j}}{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\frac{\left(1-\tau_{t+j}^d\right)u_C(t+j)}{\left(1+\tau_{t+j}^C\right)}\left(P_{t+j}^h\right)^{\theta-1}Y_{t+j}}. \tag{17}
$$

The price level in (18) can be written as

$$
P_t^h = \left[(1-\alpha)\,p_t^{h1-\theta} + \alpha P_{t-1}^{h1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{18}
$$

Implications of price stability.—

A monetary policy that successfully stabilizes the domestic price of the final good must stabilize the marginal cost. Indeed, note that if

$$
\frac{\left(P_{t+j}^x\right)^{\eta_1}\left(P_{t+j}^z\right)^{\eta_2}W_{t+j}^{\eta_3}}{Z_{t+j}} = MC \text{ for all } t
$$

then

$$\begin{aligned} p_t^h &= \frac{\theta}{\theta-1} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{\left(P_{t+j}^x\right)^{\eta_1} \left(P_{t+j}^z\right)^{\eta_2} W_{t+j}^{\eta_3}}{Z_{t+j}} \\ &= MC \frac{\theta}{\theta-1} E_t \sum_{j=0}^{\infty} \eta_{t,j} = MC \frac{\theta}{\theta-1} \text{ for all } t \end{aligned}$$

But

$$MC = S_t \frac{\left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3}}{Z_t}$$

so stabilizing marginal costs implies

$$S_t = \frac{1}{MC} \frac{Z_t}{\left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3}}$$

Thus, the volatility of the nominal exchange rate depends on the volatility of the exogenous shocks $(P_t^{x*}, P_t^{z*}, A_t, Z_t)$ and on the allocation of labor across sectors. In addition, the correlation between S_t and P_t^{x*} will be negative, as in Table 2. Therefore a small open economy that follows inflation targeting will experience fluctuations on the exchange rate that depend on movements on commodity prices and productivity shocks, as well as on the properties of the input-output matrix (the parameters $\rho, \eta^1, \eta^2, \eta^3$).[17]

Foreign sector and feasibility

We assume an isoelastic foreign demand for the home final good of the form

$$C_t^{h*} = \left(K_t^*\right)^{\gamma} \left(P_t^{h*}\right)^{-\gamma} \tag{19}$$

where $\gamma > 1$, P_t^{h*} is the foreign currency price of the home final good and K_t^* is a stochastic process.

The government imposes a tax $\left(1+\tau_t^h\right)$ on final goods exported to the rest of the world and a tariff $(1+\tau_t^m)$ to final good imports. The law of one price on domestic and foreign final goods

[17] In a recent paper Farhi, Gopinath and Itskhoki (2011) argue that a devaluation can be repliated with changes in scal instruments in a way that is very general: no information is reuired regarding details of the economy. As this model makes clear, this is no loger the case once we have foreign inputs in the production process: taxes on inputs will be required and information regarding the input-output matrix will be required.

then requires

$$
\begin{aligned}
P_t^h\left(1+\tau_t^h\right) &= S_t P_t^{h*} \\
P_t^f &= S_t P_t^{f*}\left(1+\tau_t^m\right)
\end{aligned}
\tag{20}
$$

where P_t^{f*} is the foreign currency price of the foreign final good.

Net exports measured in foreign currency are given by

$$
m_t^* = P_t^{h*}C_t^{h*} - P_t^{f*}C_t^f + P_t^{x*}\left[X_t - \int_0^1 x_{it}di\right] - P_t^{z*}\int_0^1 z_{it}di \tag{21}
$$

Thus, the net foreign assets of the country, denoted by $B^*\left(\mu^t\right)$, evolve according to

$$
B_{t-1,t}^* + m_t^* = E_t B_{t,t+1}^* Q_{t,t+1}^*.
$$

Solving this equation from period 0 forward, gives the economy foreign sector feasibility constraint measured in foreign currency at time 0

$$
E_0 \sum_{t=0}^{\infty} Q_{0,t}^* m_t^* = -B_{-1,0}^*. \tag{22}
$$

In addition, market clearing in domestic final goods requires

$$
Y_t^h = C_t^h + C_t^{h*} + G_t^h, \tag{23}
$$

and labor market feasibility is given by

$$
N_t = \int_0^1 n_{it}^y di + n_t^x. \tag{24}
$$

3. THE RAMSEY PROBLEM

We assume that the government is able to commit to a particular policy chosen at the initial period and never deviates from it.

To characterize the optimal policy, the Ramsey taxation literature finds necessary and sufficient conditions that an allocation has to satisfy to be implantable as an equilibrium (Lucas and Stokey, 1983; Chari and Kehoe, 1999). In our model, however, these sufficient conditions cannot be characterized in terms of the allocation alone. The constraints imposed by the price setting restrictions on the equilibrium allocation make the equilibrium set a difficult object to analyze. We thus follow a different approach and define a *relaxed* set of allocations that contains the set of equilibrium allocations for any degree of price stickiness α. The relaxed set is defined in terms of necessary conditions that any equilibrium allocation must satisfy.

Proposition 1: *Given domestic currency prices P_{it}^h, any equilibrium allocation of the economy with commodities satisfies*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_C\left(C_t, N_t\right) \left(H_{C^h}\left(C_t^h, C_t^f\right) C_t^h + H_{C^f}\left(C_t^h, C_t^f\right) C_t^f \right) + U_N\left(C_t, N_t\right) N_t \right] = 0, \tag{25}$$

$$\frac{Z_t}{\eta_3} \left[\rho A_t \left(n_t^x\right)^{\rho-1} \right]^{1-\eta_3} \left(P_t^{x*}\right)^{\eta_2} \left(P_t^{z*}\right)^{-\eta_2} \left(N_t - n_t^x\right) = D_t \left[C_t^h + C_t^{h*} + G_t^h \right], \text{ and} \tag{26}$$

$$m_t^* = K_t^* \left(C_t^{h*}\right)^{\frac{\gamma-1}{\gamma}} - P_t^{f*} C_t^f + P_t^{x*} A_t \left(n_t^x\right)^{\rho} - \left(\frac{1-\eta_3}{\eta_3} \right) P_t^{x*} \rho A_t \left(n_t^x\right)^{\rho-1} \left[N_t - n_t^x \right] \tag{27}$$

where

$$D_t = \int_0^1 \left(P_{it}^h / P_t^h \right)^{-\theta} di \tag{28}$$

is an index of price dispersion across domestic final good firms. This index satisfies $D_t \geq 1$ with equality if and only if $P_{it}^h = P_t^h$ for all $i \in (0,1)$.

Proof: in the Appendix 1.

Condition (25) summarizes the household's optimization problem, (26) is market clearing in the market for home final goods, and (27) is net exports.

Our strategy is to find the allocation that maximizes the household's utility among all allocation satisfying the conditions in Proposition 1. We call this the *relaxed optimal allocation*. In particular, we define the relaxed set of allocations as the set of allocations $\left\{ C_t^h, C_t^f, C_t^{h*}, N_t, n_t^x, m_t^* \right\}$ such that there are domestic currency prices P_t^h and P_{it}^h for $i \in (0,1)$, and foreign currency prices P_t^{h*} such that (22), (25), (26), (27), and (28) hold, and such that the prices P_t^h and P_{it}^h satisfy (9).

The relaxed set of allocations imposes less restrictions on the allocation than the equilibrium set. In particular, the relaxed set allows for firm specific prices P_{it}^h, disregards the constraint imposed by the price setting restriction (**16**) and ignores the no-arbitrage condition (3). It then follows that any equilibrium allocation delivers utility no greater than that attained under the allocation that maximizes utility among allocations in the relaxed set. We next show, however, that given sufficiently flexible fiscal instruments, there is a government policy such that the optimal allocation in the relaxed set. Therefore, the relaxed optimal allocation is the best allocation among all equilibrium allocations.

We start by noting that the relaxed optimal allocation requires setting $D_t = 1$ for all t. That is, the price of all intermediate good firms must be the same and equal to $P_{it}^h = P_t^h$ for all $i \in (0, 1)$. This is so because $D_t = 1$ is the value that attains production efficiency. To see this, note that the term D_t only appears in equation (26). Given a level of output of home final goods (the left side of equation (26)), consumption of home final goods is maximized when $D_t = 1$. In other words, the price frictions imply that, in equilibrium, otherwise identical firms may be setting different prices. If this is the case, the equilibrium does not exhibit production efficiency and the allocation lies inside the production possibility frontier. As it turns out production efficiency is a property of the second best, as it has been pointed by Diamond and Mirrlees (1973).

But $D_t = 1$ can only occur if monetary policy is able to implement constant intermediate good prices. That is, monetary policy must be such that firms that are able to reoptimize prices will choose to set the same constant price in every period. For the rest of this section we consider the relaxed Ramsey problem under constant prices.

It is convenient to define the distorted utility function

$$V\left(C^h, C^f, N; \lambda\right) \equiv U(C, N) + \lambda\left(U_C(C, N)\left(H_{C^h}\left(C^h, C^f\right)C^h + H_{C^f}\left(C^h, C^f\right)C^f\right) + U_N(C, N)N\right)$$

where λ is the Lagrange multiplier on the implementability constraint (25). The distorted utility function includes the contribution of constraint (25) to utility.

The Lagrangian of the relaxed Ramsey problem is

$$\begin{aligned}
&\max E_0 \sum_{t=0}^{\infty} \beta^t V\left(C_t^h, C_t^f, N_t; \lambda\right) \\
&+E_0 \sum_{t=0}^{\infty} \varphi_t \left[\frac{Z_t}{\eta_3}\left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{1-\eta_3} \left(P_t^{x*}\right)^{\eta_2} \left(P_t^{z*}\right)^{-\eta_2} \left(N_t - n_t^x\right) - C_t^h - C_t^{h*} - G_t^h\right] \\
&+\zeta E_0 \sum_{t=0}^{\infty} Q_{0,t}^* \left[K_t^* \left(C_t^{h*}\right)^{\frac{\gamma-1}{\gamma}} - P_t^{f*} C_t^f + P_t^{x*} A_t \left(n_t^x\right)^{\rho} - \left(\frac{1-\eta_3}{\eta_3}\right) P_t^{x*} \rho A_t \left(n_t^x\right)^{\rho-1} \left[N_t - n_t^x\right]\right]
\end{aligned}$$

where φ_t is the Lagrange multiplier on (26) and ζ is the (constant) multiplier on the foreign sector feasibility constraint (22).

After some algebra, we can write the necessary conditions for an optimum as

$$\beta^t V_{C^h}\left(C_t^h, C_t^f, N_t; \lambda\right) = \varphi_t \tag{29}$$

$$\beta^t V_{C^f}\left(C_t^h, C_t^f, N_t; \lambda\right) = \zeta Q_{0,t}^* P_t^{f*} \tag{30}$$

$$-\beta^t V_N\left(C_t^h, C_t^f, N_t; \lambda\right) = \zeta Q_{0,t}^* P_t^{x*} \rho A_t \left(n_t^x\right)^{\rho-1} \tag{31}$$

$$\varphi_t = \zeta Q_{0,t}^* \frac{\left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3} \left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2}}{Z_t} \tag{32}$$

$$\varphi_t = \frac{\gamma-1}{\gamma} \zeta Q_{0,t}^* K_t^* \left(C_t^{h*}\right)^{\frac{-1}{\gamma}} \tag{33}$$

Note that the condition with respect to labor resembles the condition with respect to the foreign consumption aggregate. By dividing both equations, we obtain the following relationship

$$-\frac{V_N\left(C_t^h, C_t^f, N_t; \lambda\right)}{V_{C^f}\left(C_t^h, C_t^f, N_t; \lambda\right)} = \frac{P_t^{x*}}{P_t^{f*}} \rho A_t \left(n_t^x\right)^{\rho-1}$$

so the marginal rate of substitution between labor and the foreign consumption aggregate (using the Ramsey planner preferences) is equalized to the price of the commodity relative to that of the foreign final good adjusted by the local productivity of labor in the production of the commodity. Thus, the presence of commodities implies that labor becomes effectively a traded good and terms of trade shocks directly affect local costs, a key determinant of domestic pricing decisions.

Given that the aggregator H is constant returns to scale, by Diamond and Mirrlees homogenous taxation result, the margin between domestic and foreign consumption will not be distorted. In addition, as the elasticity of demand for the final domestic good is constant, the optimal mark up will be constant. Therefore, the taxes τ_t^h, τ_t^m are constant, satisfying

$$\frac{\theta}{\theta-1} = \left(1+\tau_t^m\right)$$

$$\left(1+\tau_t^h\right)\frac{\theta}{\theta-1} = \left(\frac{\gamma}{\gamma-1}\right)$$

The first equation implies that the optimal tariff on the final foreign goods,τ_t^m ,is equal to the local mark-up domestic producers impose on domestic final goods. In this way, the relative price domestic consumers face is equal to the marginal rate of transformation. The second equation implies that the tax τ_t^h corrects the local mark up chosen by the domestic monopolists so as foreign consumers face the optimal mark-up.

At this level of generality, little can be said regarding consumption and labor income taxes. Time and state varying labor income taxes will move so as the Ramsey allocation satisfies the intraperiod equilibrium condition

$$\frac{U_C\left(C_t, N_t\right) H_{C^h}\left(C_t^h, C_t^f\right)}{-U_N\left(C_t, N_t\right)} = \frac{P_t^h\left(1+\tau_t^c\right)}{W_t\left(1-\tau_t^n\right)}$$

while consumption taxes will move over time so the Ramsey allocation satisfies the intertemporal equilibrium condition

$$\frac{U_C\left(C_t, N_t\right) H_{C^h}\left(C_t^h, C_t^f\right)}{P_t^h\left(1+\tau_t^c\right)} = \beta Q_{t,t+1}^* \frac{S_t}{S_{t+1}} \frac{U_C\left(C_{t+1}, N_{t+1}\right) H_{C^h}\left(C_{t+1}^h, C_{t+1}^f\right)}{P_{t+1}^h\left(1+\tau_{t+1}^c\right)}$$

since the nominal exchange rate moves so as to stabilize local marginal costs. Notice that, as long as the nominal exchange rate is very volatile, the growth rate of consumption taxes will also be very volatile.

However, price stability is a feature of the second best, so the nominal exchange rate must move so as to stabilize domestic marginal costs, as discussed above, according to

$$S_t = \frac{1}{MC} \frac{Z_t}{\left(P_t^{x*}\right)^{1-\eta_2}\left(P_t^{z*}\right)^{\eta_2}\left[\rho A_t\left(n_t^x\right)^{\rho-1}\right]^{\eta_3}}$$

For example, in the particular case of $\eta_3 = 0$, and ignoring productivity shocks $\left(A_t = A, Z_t = Z\right)$, then

$$\ln S_t = k - \left(1-\eta_2\right)\ln P_t^{x*} - \eta_2 \ln P_t^{z*}$$

so

$$V(\ln S_t) = \left(1-\eta_2\right)^2 V(\ln P_t^{x*}) + \eta_2^2 V(\ln P_t^{z*}) + 2\left(1-\eta_2\right)\eta_2 Cov(\ln P_t^{z*}, \ln P_t^{x*})$$

$$Cov(\ln S_t, \ln P_t^{x*}) = -\left(1-\eta_2\right) V(\ln P_t^{x*}) - \eta_2 Cov(\ln P_t^{z*}, \ln P_t^{x*})$$

as long as $Cov(\ln P_t^{z*}, \ln P_t^{x*}) > 0$ as it is the case with commodities, the covariance (and therefore the correlation) between the nominal exchange rate and the price of the exportable commodity will be negative.

As we mention before, the result requires flexible tax instruments. In the next proposition, we show that for some preferences, optimal tax rates are constant across states and periods.

Proposition 2. *Suppose the utility function is of the form*

$$U(C,N,m) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\psi}}{1+\psi} + u(m), \quad \sigma, \psi > 0$$

and the consumption aggregator $C = H\left(C^h, C^f\right)$ is Cobb-Douglas. Then, the optimal policy sets a constant intratemporal wedge across dates and states, and does not distort the intertemporal wedge at all. That is, we don't need the consumption taxes to match the no-arbitrage condition.[18]

Proof: See Appendix 2.

Thus, as long as preferences can be well approximated by the ones specified in Proposition 2, price stability is optimal and no case can be made for "fear of floating". In the next Section, we numerically solve the model to evaluate how well it can reproduce the moments in Table 2. We are in the process of characterizing optimal policy under alternative preferences and computing welfare losses of stabilizing prices when preferences depart from the ones described in Proposition 2 and taxes are not flexible.

4.NUMERICAL SOLUTIONS

In this section we provide a preliminary quantitative exploration of our model. We show that there is a reasonable parametrization of the model that is able to generate, under the optimal policy, correlations and volatilities of nominal exchange rate similar to those observed in the data. We note, however, that the purpose of this section is to illustrate that our model is able to rationalize these observations rather than to provide an exhaustive calibration of the model.

We consider the following preferences

$$U(C_t^h, C_t^f, N_t) = \omega \log C_t^h + (1-\omega)\log C_t^f - \kappa N_t^{1+\psi} / (1+\psi).$$

As we showed in Proposition 2, the optimal policy with these preferences imply that labor and consumption taxes are state and time independent.

We assume that government consumption G_t^h and the level of the foreign demand K_t^* are constant. There are four exogenous shocks in the model $\{P_t^{x*}, P_t^{z*}, Z_t, A_t\}$. We assume that the logarithm of each component follows a first order autoregressive process. Note, however, that in the Ramsey allocation the shocks P_t^{z*} and Z_t come bundled as $(P_t^{z*})^{\eta_2}/Z_t$. The relevant state variable is thus $\tilde{P}_t^z = (P_t^{z*})^{\eta_2}/Z_t$. As is well known, if $\log P_t^{z*}$ and $\log A_t$ both follow a first

[18] The absense of intertemporal distortions holds in a more general speci cation of the utility function, $C^{1-\sigma}/(1-\sigma) + v(N) + u(N)$ for any function N. The proof is identical to that of proposition 1.

order autogression, then $\log \tilde{P}_t^{z*}$ is distributed as an ARMA(2,1) process. Thus, the state of the economy at time t is summarized by the vector $\mu_t = \left[\log P_t^{x*}, \log \tilde{P}_t^{z*}, \log A_t\right]$, where

$$\log P_t^{x*} = a^x + b^x \log P_{t-1}^{x*} + \varepsilon_t^x, \tag{34}$$

$$\log \tilde{P}_t^{z*} = a^z + b_1^z \log \tilde{P}_{t-1}^{z*} + b_2^z \log \tilde{P}_{t-2}^{z*} + \varepsilon_t^z + c^z \varepsilon_{t-1}^z, \tag{35}$$

$$\log A_t = a^A + b^A \log A_{t-1} + \varepsilon_t^A. \tag{36}$$

We allow the shocks ε_t^x, ε_t^z and ε_t^A to be contemporaneusly correlated.

Under the optimal policy, the first order conditions from the Ramsey problem implies that the optimal allocation is a time invariant function of μ_t and of the (constant) Lagrange multipliers λ and ζ. We compute the model numerically as follows. We choose a grid of points for each exogenous shock. Given an initial condition $B_0 = B_0^* = 0$ and a guess for the multipliers λ and ζ, we numerically solve the system of equations (29)–(33) at each grid point. We evaluate the solution at other points using linear interpolation. We next check whether the present value constraints (22) and (25) are satisfied at equality given the proposed multipliers (λ, ζ)—we evaluate these constraints using Monte Carlo simulations by drawing many histories of shocks from the processes (34), (35), and (36) and evaluating the constraint using sample averages. We ajust (λ, ζ) until both constraints are satisfied.

For the parametrization of the model, we choose a small contribution of labor in the commodity sector ($\rho = 0.1$), consistent with the observation that commodities are not too labor intensive. Regarding the intermediate goods sector, we choose a small contribution of the home commodity ($\eta_1 = 0.01$), a moderate contribution of the foreign commodity ($\eta_2 = 0.29$), and a large contribution of labor ($\eta_3 = 0.7$). We set the stochastic process for the home commidity by running a quarterly autogression on oil prices. The process for both productivity shocks are standard, and the process for the foreign commodity is the same as that for the home commidity. We next recover the implied ARMA(2,1) process for the combined shock. We also impose that the correlation between the shocks ε_t^z and ε_t^x is 0.1. The other shocks are uncorrelated. Table 3 displays all the parameters used in the simulation.

Table 4 reports the implied volatility of the logarithm of the exchange rate and the correlation of the (log) exchange rate with the (log) home commodity price. We compute these statistics by running 2000 simulations of length 1000 and computing sample averages. The proposed calibration is able to roughly reproduce the moments displayed in Table 2, with a standard deviation of about 6.4 percent, and a correlation of -49 percent. The top panel of Figure 3 displays the associated histograms across simulations of these statistics. The lower panel shows two typical realized histories of length 200 of exchange rates and home commodity prices as (log) percentage deviations from their sample means.

In summary, we find that there is a reasonable parametrization that is able to reproduce the observed volatility and correlation of exchange rate with commodity prices. This is just a preliminary computation as we are ultimately interested in comparing welfare across different policy scenarios. For that purpose, we need to solve the model by constraining some fiscal instrument, allowing for preferences different from those in Proposition 2, and so forth.

5.CONCLUSIONS

To be done.

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APPENDIX 1

Proof of Proposition 1: Condition (25) summarizes the household's behavior and follows from introducing (5), (6), and (7) into (4) evaluated at equality. Integrating (8), (14), and (15) over $i \in (0,1)$ and rearranging gives

$$\int_0^1 x_{it} di = \frac{\eta_1}{Z_t} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3} \left(P_t^{x*}\right)^{-\eta_2} \left(P_t^{z*}\right)^{\eta_2} D_t Y_t^h \tag{A1}$$

$$\int_0^1 z_{it} di = \frac{\eta_2}{Z_t} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3} \left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2-1} D_t Y_t^h \tag{A2}$$

$$\int_0^1 n_{it}^y di = \frac{\eta_3}{Z_t} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3-1} \left(P_t^{x*}\right)^{-\eta_2} \left(P_t^{z*}\right)^{\eta_2} D_t Y_t^h \tag{A3}$$

where D_t is an index of price dispersion defined as

$$D_t = \int_0^1 \left(P_{it}^h / P_t^h\right)^{-\theta} di.$$

Introducing (A3) into the labor market feasibility condition (24) gives

$$N_t = n_t^x + \frac{\eta_3}{Z_t} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3-1} \left(P_t^{x*}\right)^{-\eta_2} \left(P_t^{z*}\right)^{\eta_2} D_t Y_t^h. \tag{A4}$$

Next, using (A1) and (A2) we can write

$$P_t^{x*} \int_0^1 x_{it} di + P_t^{z*} \int_0^1 z_{it} di = \frac{1-\eta_3}{Z_t} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{\eta_3} \left(P_t^{x*}\right)^{1-\eta_2} \left(P_t^{z*}\right)^{\eta_2} D_t Y_t^h.$$

Using the (A4) into the previous equation then gives

$$P_t^{x*} \int_0^1 x_{it} di + P_t^{z*} \int_0^1 z_{it} di = \left(\frac{1-\eta_3}{\eta_3}\right) P_t^{x*} \rho A_t \left(n_t^x\right)^{\rho-1} \left[N_t - n_t^x\right]$$

Inserting this expression, (10), and (19) into (21) we obtain

$$NX_t^* = K_t^* \left(C_t^{h*}\right)^{\frac{\gamma-1}{\gamma}} - P_t^{f*} C_t^f + P_t^{x*} A_t \left(n_t^x\right)^{\rho} - \left(\frac{1-\eta_3}{\eta_3}\right) P_t^{x*} \rho A_t \left(n_t^x\right)^{\rho-1} \left[N_t - n_t^x\right].$$

Likewise, introducing (A4) into the feasibility condition (26) gives

$$D_t \left(C_t^h + G_t^h + C_t^{h*}\right) = \frac{Z_t}{\eta_3} \left[\rho A_t \left(n_t^x\right)^{\rho-1}\right]^{1-\eta_3} \left(P_t^{x*}\right)^{\eta_2} \left(P_t^{z*}\right)^{-\eta_2} \left(N_t - n_t^x\right)$$

It remains to prove that $D_t \geq 1$, with equality if and only if $P_{it}^h = P_t^h$ for almost all i except for those in a set of measure zero. Let $w_{it} = \left(P^h_{it}\right)^{1-\theta}$. It then follows that $\left(P^h_{it}\right)^{-\theta} = w_{it}^{\theta/(\theta-1)}$, which is a strictly convex function of w_{it}. Therefore, Jensen's inequality implies

$$\int_0^1 \left(P^h_{it}\right)^{-\theta} di = \int_0^1 w_{it}^{\theta/(\theta-1)} di \geq \left(\int_0^1 w_{it} di\right)^{\frac{\theta}{\theta-1}} = \left(P^h_t\right)^{\theta}$$

with strict equality if an only if $P^h_{it} = P^h_t$ almost everywhere.$\square$

APPENDIX 2

In this appendix, we show specific preferences for which optimal taxes are constant. In this appendix we consider the case $\rho = 1$ and $\eta^3 = 0$. The result holds for the general case, but the Ramsey conditions are more cumbersome.

$$\sum_{t=0}^{\infty} \sum_{\mu^t} \beta^t U\left(C\left(\mu^t\right), N\left(\mu^t\right), m(\mu^t)\right) \pi\left(\mu^t\right),$$

The composite good $C(s^t)$ is

$$C(\mu^t) = H\left(C^h(\mu^t), C^f(\mu^t)\right), \tag{37}$$

where the function $H(\bullet)$ is homogeneous of degree one and increasing in each argument. This function implies an aggregate price index

$$P(\mu^t) = h\left(P^h(\mu^t), P^f(\mu^t)\right), \tag{38}$$

where $h(\bullet)$ is homogeneous of degree one and increasing in each argument.

The choice of the aggregates $C\left(\mu^t\right)$, $N\left(\mu^t\right)$, and $m\left(\mu^t\right)$ satisfy the conditions

$$\frac{\beta^t \pi(\mu^t) U_C(\mu^t)}{P(\mu^t)\left(1 + \tau^c\left(\mu^t\right)\right)} = \frac{Q(\mu^t|\mu^0) U_C(\mu^0)}{P(\mu^0)\left(1 + \tau^c\left(\mu^0\right)\right)}, \tag{39}$$

$$-\frac{U_N(\mu^t)}{U_C(\mu^t)} = \frac{W(\mu^t)}{P(\mu^t)} \frac{\left(1 - \tau^n(\mu^t)\right)}{\left(1 + \tau^c\left(\mu^t\right)\right)}, \tag{40}$$

$$\frac{U_m(\mu^t)}{U_C(\mu^t)} = \frac{R\left(\mu^t\right) - 1}{R\left(\mu^t\right)}. \tag{41}$$

Ramsey Problem: Define the pseudo-utility function

$$V\left(C, N, M\right) = U\left(C, N, m\right) + \lambda\left[U_C C + U_N N + U_m m\right]$$

where λ is the Lagrange multiplier on the implementability constraint. We also attach multipliers $\varphi\left(\mu^t\right)$ to the constraint $U_m \geq 0$ and a multiplier ξ to the foreign sector feasibility constraint The first order necessary conditions for $C(\mu^t)$, $N(\mu^t)$, and $m(\mu^t)$ are

$$\beta^t \pi\left(\mu^t\right)\left[V_C\left(\mu^t;\lambda\right)+\varphi\left(\mu^t\right)U_{mC}\left(\mu^t\right)\right]=\xi Q^*\left(\mu^t|\mu^0\right)h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right] \tag{42}$$

$$-\beta^t \pi\left(\mu^t\right)\left[V_N\left(\mu^t;\lambda\right)+\varphi\left(\mu^t\right)U_{mN}\left(\mu^t\right)\right]=\xi Q^*\left(\mu^t|\mu^0\right)P^{x*}\left(\mu^t\right)A^x\left(\mu^t\right) \tag{43}$$

$$V_m\left(\mu^t;\lambda\right)+\varphi\left(\mu^t\right)U_{mm}\left(\mu^t\right)=0 \tag{44}$$

We now provide the decentralization of fiscal policy. Consider the equilibrium equations

$$P^h\left(\mu^{t-1}\right)=\frac{\theta}{\theta-1}S\left(\mu^t\right)MC^*\left(\mu^t\right), \tag{45}$$

$$P\left(\mu^t\right)=S\left(\mu^t\right)h\left(\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right), \tag{46}$$

$$\frac{W\left(\mu^t\right)}{P\left(\mu^t\right)}=\frac{P^{x*}\left(\mu^t\right)A^x\left(\mu^t\right)}{h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]} \tag{47}$$

Using the household's condition (39) at periods t and $t+1$ gives

$$\frac{\beta\pi\left(\mu^{t+1}|\mu^t\right)U_C\left(\mu^{t+1}\right)}{P\left(\mu^{t+1}\right)}\frac{P\left(\mu^t\right)}{U_C\left(\mu^t\right)}\frac{1+\tau^c\left(\mu^t\right)}{1+\tau^c\left(\mu^{t+1}\right)}=Q\left(\mu^{t+1}|\mu^t\right)$$

Using the parity condition(**??**) to get rid of $Q\left(\mu^{t+1}|\mu^t\right)$ and then (46) gives

$$Q^*\left(\mu^{t+1}|\mu^t\right)=\frac{1+\tau^c\left(\mu^t\right)}{1+\tau^c\left(\mu^{t+1}\right)}\beta\pi\left(\mu^{t+1}|\mu^t\right)\frac{U_C\left(\mu^{t+1}\right)}{U_C\left(\mu^t\right)}\frac{h\left(\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right)}{h\left(\frac{\theta}{\theta-1}MC^*\left(\mu^{t+1}\right),P^{f*}\left(\mu^{t+1}\right)\right)} \tag{48}$$

Using 47 into (40) gives

$$-\frac{U_N(\mu^t)}{U_C(\mu^t)}=\frac{P^{x*}\left(\mu^t\right)A^x\left(\mu^t\right)}{h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]}\frac{\left(1-\tau^n(\mu^t)\right)}{\left(1+\tau^c\left(\mu^t\right)\right)} \tag{49}$$

Rewrite now the Ramsey first order conditions (42) and (43) as

$$Q^*\left(\mu^{t+1}|\mu^t\right)=\beta\pi\left(\mu^{t+1}|\mu^t\right)\frac{V_C\left(\mu^{t+1};\lambda\right)}{V_C\left(\mu^t;\lambda\right)}\frac{h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]}{h\left[MC^*\left(\mu^{t+1}\right),P^{f*}\left(\mu^{t+1}\right)\right]} \tag{50}$$

$$-\frac{V_N\left(\mu^t;\lambda\right)}{V_C\left(\mu^t;\lambda\right)}=\frac{P^{x*}\left(\mu^t\right)A^x\left(\mu^t\right)}{h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]} \tag{51}$$

Then, equations (48) and 50 gives

$$\frac{1+\tau^c\left(\mu^{t+1}\right)}{1+\tau^c\left(\mu^t\right)}=\frac{V_C\left(\mu^t;\lambda\right)}{V_C\left(\mu^{t+1};\lambda\right)}\frac{U_C\left(\mu^{t+1}\right)}{U_C\left(\mu^t\right)}\frac{h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]}{h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^{t+1}\right),P^{f*}\left(\mu^{t+1}\right)\right]}\frac{h\left[MC^*\left(\mu^{t+1}\right),P^{f*}\left(\mu^{t+1}\right)\right]}{h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]} \tag{52}$$

and equations (49) and (51) gives

$$\frac{1-\tau^n(\mu^t)}{1+\tau^c\left(\mu^t\right)}=\frac{U_N(\mu^t)}{U_C(\mu^t)}\frac{V_C\left(\mu^t;\lambda\right)}{V_N\left(\mu^t;\lambda\right)}\frac{h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]}{h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]} \tag{53}$$

Conditions (52) and (53) gives the optimal intertemporal and intratemporal wedges derived from the optimal policy.

If preferences are as in Proposition 2

$$U\left(C,N,m\right)=\frac{C^{1-\sigma}}{1-\sigma}-\frac{N^{1+\psi}}{1+\psi}+u\left(m\right),\quad \sigma,\psi>0 \tag{54}$$

we have that

$$-\frac{V_N\left(\mu^t;\lambda\right)}{V_C\left(\mu^t;\lambda\right)}=\frac{N\left(\mu^t\right)^{\psi}\left[1+\lambda\left(1+\psi\right)\right]}{C\left(\mu^t\right)^{-\sigma}\left[1+\lambda\left(1-\sigma\right)\right]}\text{ and }-\frac{U_N\left(\mu^t\right)}{U_C\left(\mu^t\right)}=\frac{N\left(\mu^t\right)^{\psi}}{C\left(\mu^t\right)^{-\sigma}}$$

and

$$\frac{V_C\left(\mu^{t+1};\lambda\right)}{V_C\left(\mu^t;\lambda\right)}=\frac{U_C\left(\mu^{t+1}\right)}{U_C\left(\mu^t\right)}=\left(\frac{C\left(\mu^t\right)}{C\left(\mu^{t+1}\right)}\right)^{\sigma}$$

Moreover, if $H\left(C^h,C^f\right)$ is Cobb-Douglas, by duality the cost function $h\left(\cdot\right)$ is also Cobb-Douglas, therefore

$$h\left[\frac{\theta}{\theta-1}MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]=\left(\frac{\theta}{\theta-1}\right)^{\zeta}h\left[MC^*\left(\mu^t\right),P^{f*}\left(\mu^t\right)\right]$$

where ζ is the exponent on the first argument of $h\left(\cdot\right)$.

Using these two conditions into (53) gives

$$\frac{1-\tau^n(\mu^t)}{1+\tau^c\left(\mu^t\right)}=\frac{1+\lambda\left(1-\sigma\right)}{1+\lambda\left(1+\psi\right)}\left(\frac{\theta}{\theta-1}\right)^{\zeta}$$

which proves that the intratemporal wedge is constant.

Likewise, using these conditions into (52) gives

$$\frac{1+\tau^c\left(\mu^{t+1}\right)}{1+\tau^c\left(\mu^t\right)}=1$$

so that the optimal allocation does not distort the intertemporal margin. In particular, a fiscal policy that decentralizes the allocation has

$$\begin{aligned} 1-\tau^{n}(\mu^{t}) &= \frac{1+\lambda\,(1-\sigma)}{1+\lambda\,(1+\psi)}\left(\frac{\theta}{\theta-1}\right)^{\zeta} \\ \tau^{c}\left(\mu^{t}\right) &= 0 \end{aligned}$$

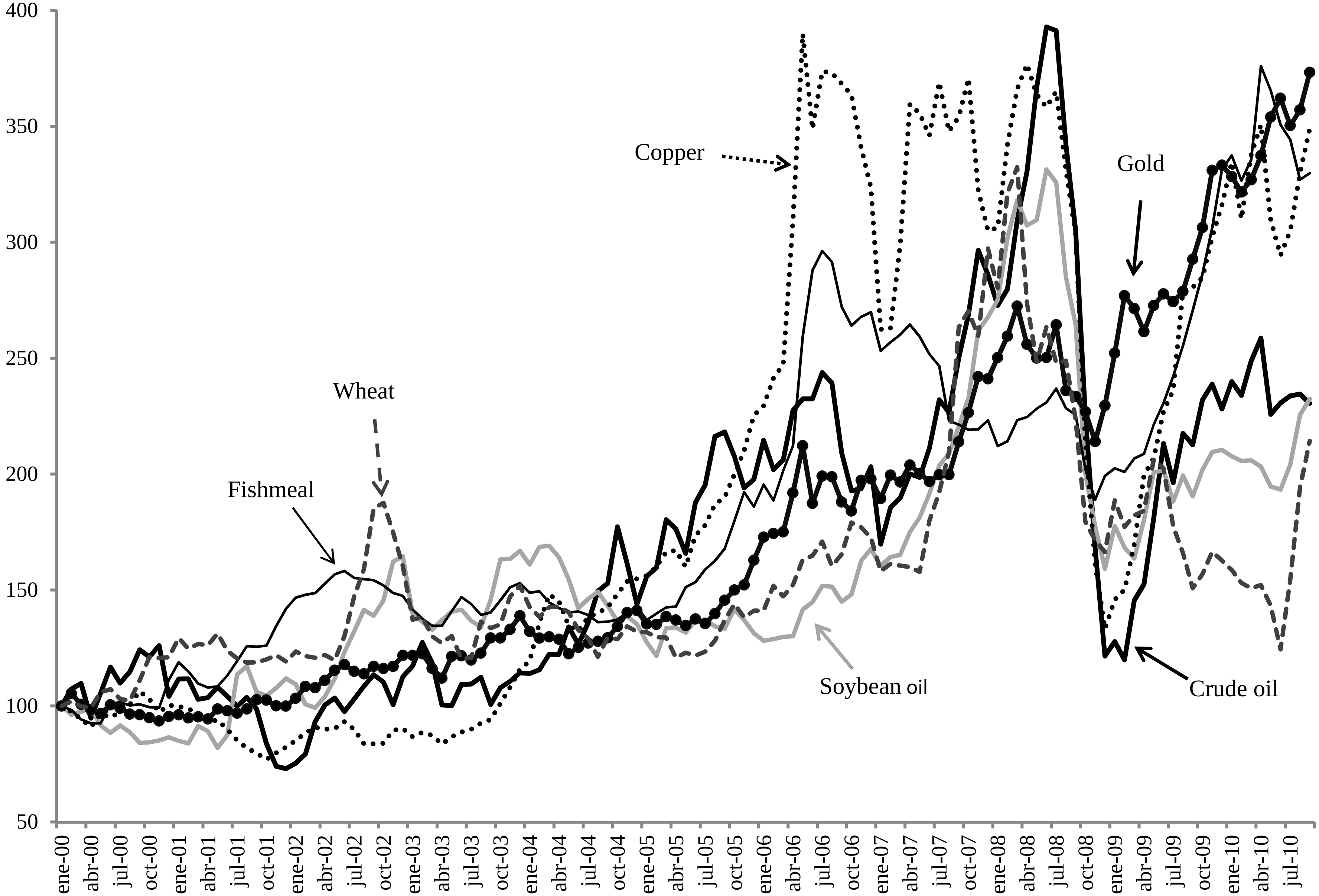
for all μ^t. This finishes the proof.

TABLE 1. Principal commodity exports in selected countries

Panel A	Principal commodity exports (monthly averages since Jan 2000)			Share in good exports (%)			
	C1	*C2*	*C3*	*C1*	*C2*	*C3*	*Total*
Argentina	Soybean and products	Petroleum and products	Wheat	23	9	4	36
Australia	Coal	Iron ore	Gold	14	9	5	28
Brazil	Soybean and products	Petroleum and products	Iron oxides	9	8	7	24
Chile	Copper	Marine products		45	7	-	52
Iceland	Marine products	Aluminium		53	25	-	78
New Zealand	Diary produce	Meat and edible offal	Wood and products	19	13	7	39
Norway	Petroleum and products	Marine products		57	5	-	62
Peru	Copper	Gold	Marine products	20	19	8	47

Panel B	Aggregate shares (%)		
	Goods/Total Exports	*Total Exports/GDP*	*Goods/GDP*
Argentina	87	22	6.7%
Australia	78	20	4.4%
Brazil	87	13	2.7%
Chile	83	39	16.8%
Iceland	65	37	18.7%
New Zealand	74	30	8.6%
Norway	76	44	20.7%
Peru	87	22	9.0%

Sources: National statistics agencies. Columns labeled *C1-C3* report the most important commodities and their shares in total exports of goods. Column labeled *Total* reports the share of the three principal commodities on total good exports. Commodity exports data are monthly and the last observation varies by country: Argentina, Jan2000 - Jun2010; Australia, Jan2000 - Oct2010; Brazil, Jan2000 - Oct2010; Chile, Jan2000 - Nov2010; Iceland, Jan2000 - Oct2010; New Zealand, Jan2000 - Oct2010; Norway, Jan2000 - Oct2010; and Peru, Jan2000 - Sep2010.


Copper
Gold
Wheat
Fishmeal
Soybean oil
Crude oil
400
350
300
250
200
150
100
50
ene-00
abr-00
jul-00
oct-00
ene-01
abr-01
jul-01
oct-01
ene-02
abr-02
jul-02
oct-02
ene-03
abr-03
jul-03
oct-03
ene-04
abr-04
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oct-04
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ene-06
abr-06
jul-06
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ene-07
abr-07
jul-07
oct-07
ene-08
abr-08
jul-08
oct-08
ene-09
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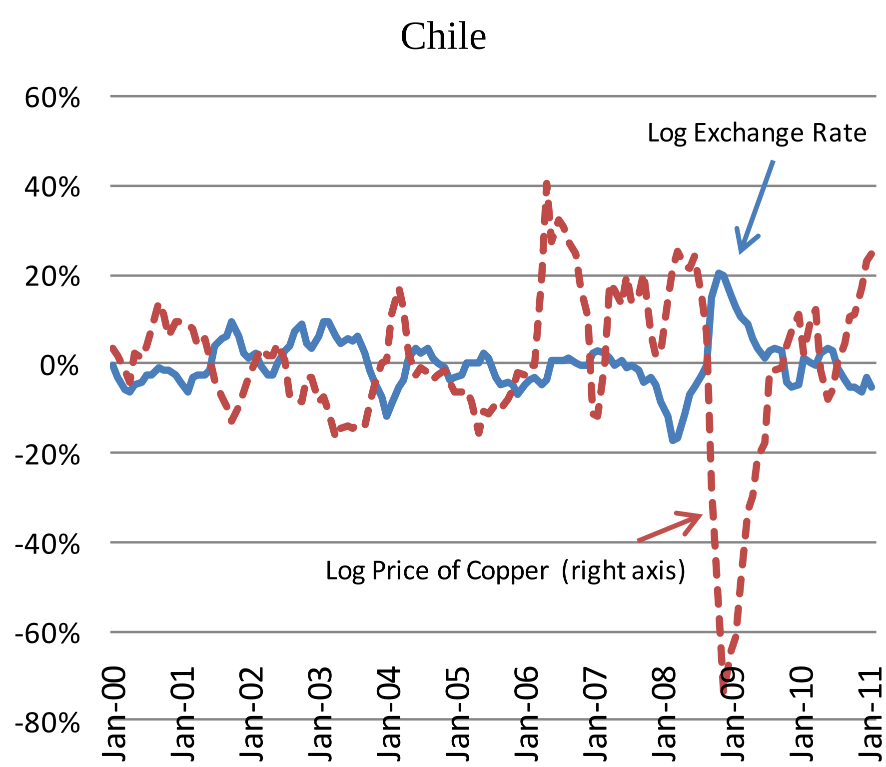
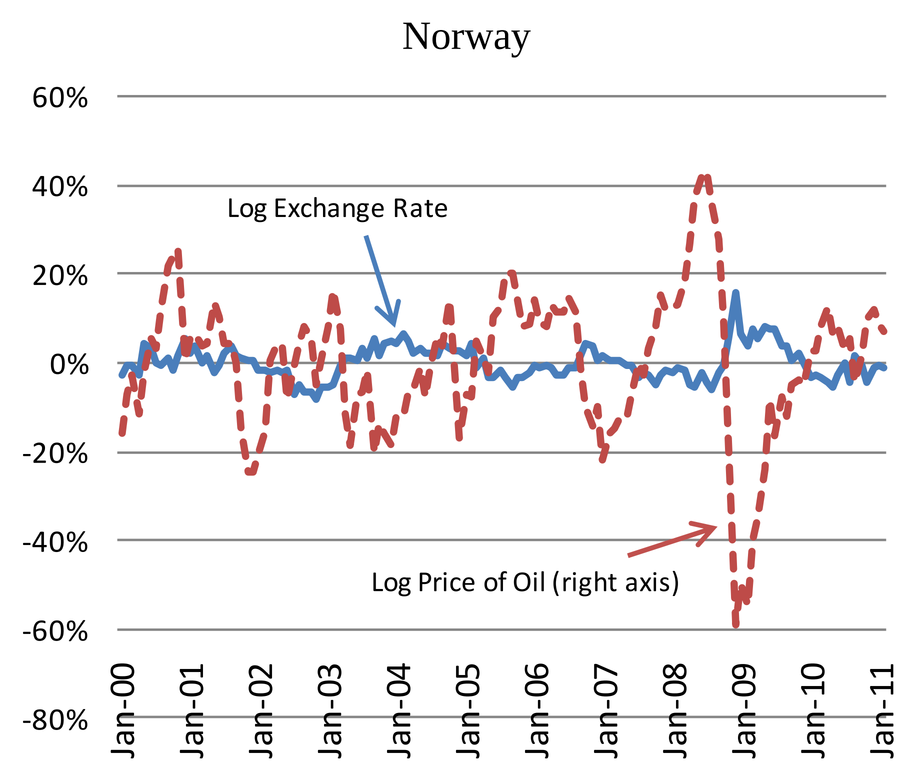
oct-09
ene-10
abr-10
jul-10

HP-Filtered Exchange Rate and Commodity Price
Data shown as percentage deviation from trend





Note: Series are first logged and then HP-filtered with a smoothing parameter of 14400

Summary Statistics Exchange Rate and Commodity Price

Data shown as percentage deviation from trend

	In US dollars		In Euros	
	Std. Deviation	Correlation	Std. Deviation	Correlation
Chile				
Exchange Rate	0.0506	-0.4729	0.0580	-0.5132
Price of Copper	0.1241		0.1266	
Norway				
Exchange Rate	0.0559	-0.5438	0.0316	-0.4332
Price of Oil	0.1459		0.1374	

Note: Data is first logged and then HP-filtered with a smoothing parameter of 14400

Table 3. Parameters

Symbol	*Description*	*Value*
ω	Preferences	0.6
κ	Preferences	20
ψ	Preferences	1
β	Discount factor	0.987
ρ	Technology commodity	0.1
η_1	Technology intermediate	0.01
η_2	Technology intermediate	0.29
η_3	Technology intermediate	0.70
G^h	Government consumption	0.30
K^*	Foreign demand	1
γ	Foreign demand elasticity	2
P_t^{f*}	Foreign nal good price	1
a^x	Parameter home commodity price	0.16
b^x	Parameter home commodity price	0.96
σ^x	Sd deviation shock home commodity price	0.15
$\rho\left(\varepsilon_t^x, \varepsilon_t^z\right)$	Correlation shocks home commodity vs bundle shock	0.1

Table 4. Volatility and Correlation

Standard deviation log exchange rate	0.064
Correlation log exchange rate with log home commodity price	-0.49

Figure 3. Quantitative exploration of the model