Risk Premia in Crude Oil Futures Prices

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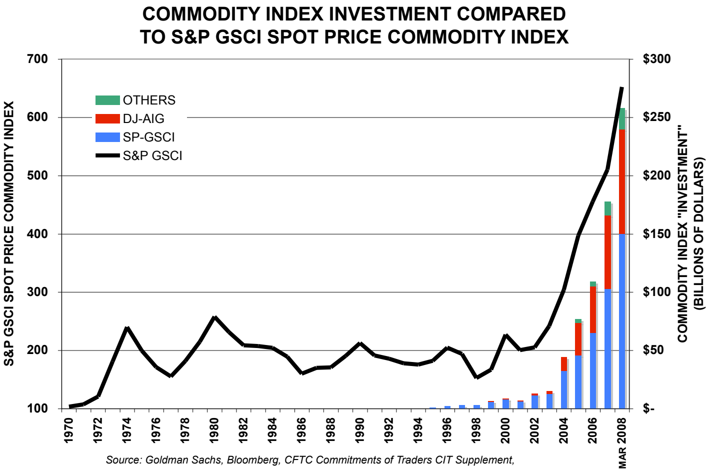
One estimate of total commodity-index assets



Source: Michael Masters, Testimony before U.S. Senate, 2008.

Literature on Commodities Markets Financialization

Policy discussion

- Masters (2008), Kennedy (2012)

Academic literature

- **Survey:** Irwin and Sanders (2011) and Fattouh, Kilian, and Mahadeva (2012)
- **Correlation:** Tang and Xiong (2011), Buyuksahin and Robe (2010, 2011), Fattouh, Kilian, and Mahadeva (2012)
- **Structural VAR:** Kilian and Murphy (2011), Lombardi and van Robays (2011) and Juvenal and Petrella (2011)
- **Prediction regression:** Brunetti, Buyuksahin, and Harris (2011), Irwin and Sanders (2011a, b, 2012), Stoll and Whaley (2010), Alquist and Gervais (2011), Buyuksahin and Harris (2011), Singleton (2011), Hamilton and Wu (2012)

Effect of financialization on the futures price

Masters' argument: increased volume of buying may drive up the futures price

Possible mechanism: Sellers willing to take other side if compensated in the form of higher return

Our paper: explores whether this could operate through changes in the risk premium

Keynes' theory of normal backwardation

Keynes (1930)

- Producers hedge by selling futures contracts, and pay a premium
- Arbitrageurs are forced to take the other side, exposed to non-diversifiable risk, and compensated

Empirical support

- Carter, Rausser, and Schmitz (1983), Chang (1985), Bessembinder (1992), De Roon, Nijman, and Veld (2000), and Acharya, Lochstoer, and Ramadorai (2010)

Impact of Financial Investors?

- Buy commodities futures for portfolio diversification
- Exert a similar effect in the opposite direction
- Shift the receipt of the risk premium from the long side to the short side of the contract

Similarity between effects of index investing and quantitative easing

Hamilton and Wu (JMCB 2012) relate price of risk to supplies of Treasury debt in QE context

This paper: We investigate the relation between the *price of risk* and *volume of contracts* in the index investing environment.

Contribution

Methodology:

- Build the interaction between commercial hedgers or financial investors and arbitrageurs into an affine factor framework
- Model the dynamics of risk premia with no-arbitrage conditions
- Develop a new algorithm for estimation using unbalanced data

Significant changes in oil future risk premia since 2005

- Risk premia to the long position smaller or even negative
- Risk premia more volatile

Implications:

- Financial investors become more important determining risk premia
- They become the natural counterparties of commercial hedgers

Seasonal variation of risk premia over the month

Outline

Arbitrageur's problem

F_{nt} =price at t of contract of maturity n
z_{nt} =notional holdings in contracts of maturity n
z_{nt}/F_{nt} = number of barrels purchased with contract n
Arbitrageur's cash flow for $t+1$

$$W_{t+1} = \sum_{n=1}^{N} z_{nt} \frac{F_{n-1,t+1} - F_{nt}}{F_{nt}}.$$

Arbitrageur's optimization problem

$$\max_{\{z_{1t},\ldots,z_{nt}\}} E_t(W_{t+1}) - (\gamma/2)\text{Var}_t(W_{t+1}).$$

Assumptions

Log price linear in $(m \times 1)$ factors x_t

$$f_{nt} = \log F_{nt} = \alpha_n + \beta_n' x_t.$$

Factor dynamics

$$x_{t+1} = c + \rho x_t + \Sigma u_{t+1} \quad u_{t+1} \sim \text{ i.i.d. } N(0, I_m)$$

Equilibrium

Arbitrageur's FOC

$$\alpha_{n-1} + \beta'_{n-1}(c + \rho x_t) - \alpha_n - \beta'_n x_t + (1/2)\beta'_{n-1}\Sigma\Sigma'\beta_{n-1} = \beta'_{n-1}\lambda_t$$

$$\text{where } \lambda_t = \gamma\Sigma\Sigma' \left(\sum_{\ell=1}^{N} z_{\ell t}\beta_{\ell-1} \right).$$

If counterparty demands $(z_{\ell t})$ are affine functions of x_t, then in equilibrium risk prices will take affine form

$$\lambda_t = \lambda + \Lambda x_t.$$

Factor loading iterations, analogous to ATSM recursion

$$\beta'_n = \beta'_{n-1}\rho - \beta'_{n-1}\Lambda$$

$$\alpha_n = \alpha_{n-1} + \beta'_{n-1}c + (1/2)\beta'_{n-1}\Sigma\Sigma'\beta_{n-1} - \beta'_{n-1}\lambda.$$

Outline

Data Structure

Definition of four "weeks"

$j_t = 1:$ last business day of the previous month

$j_t = 2:$ 5th business day

$j_t = 3:$ 10th business day

$j_t = 4:$ expiration day (third business before the 25th calendar day)

Unbalanced panel: the nearest three contracts

$$y_t = \begin{cases} (f_{3t}, f_{7t}, f_{11,t})' & \text{if } j_t = 1 \\ (f_{2t}, f_{6t}, f_{10,t})' & \text{if } j_t = 2 \\ (f_{1t}, f_{5t}, f_{9t})' & \text{if } j_t = 3 \\ (f_{0t}, f_{4t}, f_{8t})' & \text{if } j_t = 4 \end{cases} .$$

Level and slope

Definition

$$y_{1t} = H_1 y_t$$

$$H_1 = \begin{bmatrix} 0 & (1/2) & (1/2) \\ 0 & -1 & 1 \end{bmatrix}.$$

Model implies

$$f_{nt} = \alpha_n + \beta_n' x_t.$$

Priced exactly

$$y_{1t} = A_{1,j_t} + B_{1,j_t} x_t$$

with

$$x_{t+1} = c + \rho x_t + \Sigma u_{t+1}$$

$$\Rightarrow y_{1t} | y_{t-1}, y_{t-2}, ..., y_0 \sim N(\phi_{j_t} + \Phi_{j_t} y_{1,t-1}, \Omega_{j_t})$$

where $\phi_{j_t}, \Phi_{j_t}, \Omega_{j_t}$ depend on structural parameters
$\theta : (c, \rho, \Sigma, c^Q, \rho^Q, \lambda, \Lambda, \alpha_0, \beta_0)$

Near Contract

Definition

$$y_{2t} = H_2 y_t$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Priced with measurement error

$$y_{2t} = A_{2,j_t} + B_{2,j_t} x_t + \sigma_{e,j_t} u_{e,t}$$

with

$$y_{1t} = A_{1,j_t} + B_{1,j_t} x_t$$

$$\Rightarrow y_{2t} | y_{1t}, y_{t-1}, y_{t-2}, ..., y_0 \sim N(\gamma_{j_t} + \Gamma_{j_t} y_{1t}, \sigma^2_{e,j_t})$$

where $\gamma_{j_t}, \Gamma_{j_t}$ depend on structural parameters θ

Step 1: OLS for unrestricted VAR

Unrestricted VAR

$$y_{1t}|y_{t-1}, y_{t-2}, ..., y_0 \sim N(\phi_{j_t} + \Phi_{j_t} y_{1,t-1}, \Omega_{j_t})$$

$$y_{2t}|y_{1t}, y_{t-1}, y_{t-2}, ..., y_0 \sim N(\gamma_{j_t} + \Gamma_{j_t} y_{1t}, \sigma^2_{e,j_t})$$

where $\phi_{j_t}, \Phi_{j_t}, \Omega_{j_t}, \gamma_{j_t}, \Gamma_{j_t}, \sigma_{e,j_t}$ are unrestricted

Step 1: OLS for unrestricted VAR

Log likelihood function

$$\begin{aligned}
\mathcal{L} &= \sum_{t=1}^{T} [\log g(y_{1t}; \phi_{j_t} + \Phi_{j_t} y_{1,t-1}, \Omega_{j_t}) \\
&\quad + \log g(y_{2t}; \gamma_{j_t} + \Gamma_{j_t} y_{1t}, \sigma^2_{e,j_t})] \\
&= \sum_{j=1}^{4} \sum_{t=1}^{T} \delta(j_t = j) \log g(y_{1t}; \phi_j + \Phi_j y_{1,t-1}, \Omega_j) \\
&\quad + \sum_{j=1}^{4} \sum_{t=1}^{T} \delta(j_t = j) \log g(y_{2t}; \gamma_j + \Gamma_j y_{1t}, \sigma^2_{ej})
\end{aligned}$$

Reduced form parameters

$$\pi : (\phi_1, \Phi_1, \Omega_1, \gamma_1, \Gamma_1, ..., \phi_4, \Phi_4, \Omega_4, \gamma_4, \Gamma_4)$$

MLE ($\hat{\pi}$) can be obtained by OLS, with each week of month as separate sample

Step 2: MCSE for structural parameters

Hamilton and Wu (J Econometrics 2012)

- Idea: choose structural parameters θ that would imply reduced-form coefficients $\pi(\theta)$ as close as possible to the unrestricted estimates $\hat{\pi}$.
- Asymptotically equivalent to full MLE.
- Computational advantages
- Interpretive advantages

Step 2: MCSE for structural parameters

Model implies

$$\pi = g(\theta)$$

Minimum-chi-square estimation

$$min T[\hat{\pi} - g(\theta)]' \hat{R} [\hat{\pi} - g(\theta)]$$

where $\hat{R}$ is the information matrix of $\hat{\pi}$

Minimized value is asymptotically χ^2 with degrees of freedom given by number of parameters in π minus number in θ

Outline

Introduction

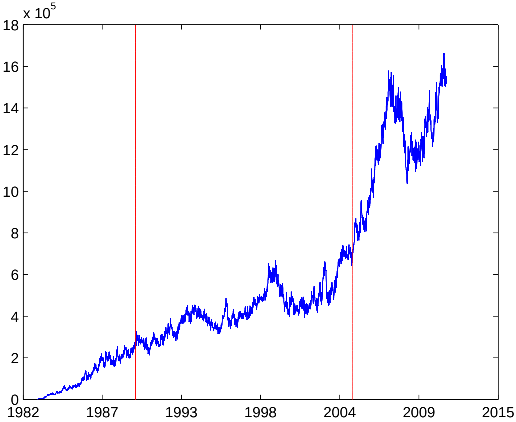
Model

Estimation

Empirical Results

Conclusion

Data



Sample: January 1990 - December 2004, January 2005 - June 2011
Liklihood ratio test of structural break: $p = 2.2 \times 10^{-16}$

Risk Price

$$\lambda_t = \lambda + \Lambda x_t$$

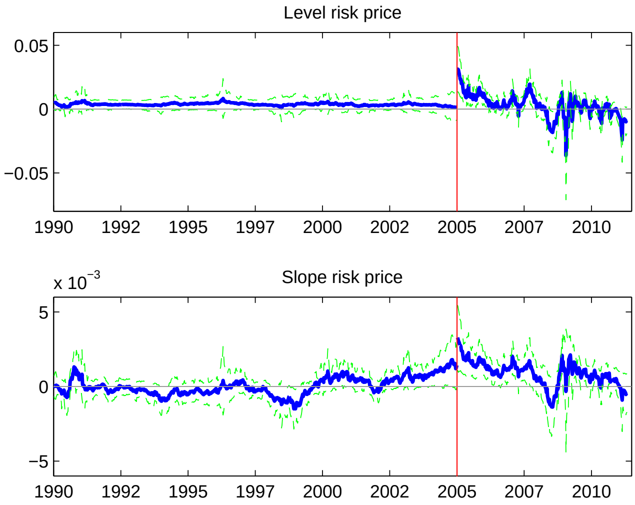
1990-2004

- First element of $\lambda + \Lambda \bar{x}$ is 0.0037 (0.0018).
  $\Rightarrow$ Positive compensation for long position.

2005-2011

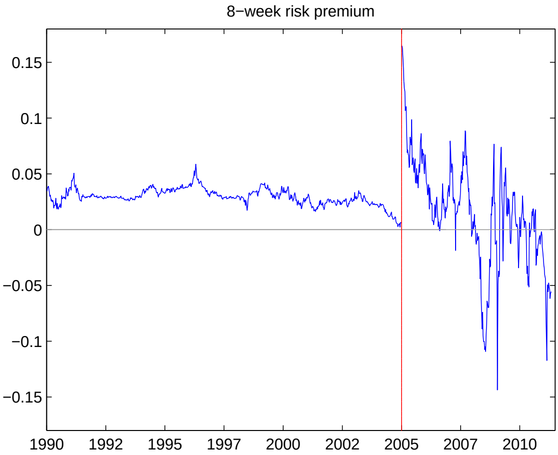
- Large negative value for Λ_{12}
  $\Rightarrow$ When the spread gets sufficiently high, a long position in the 1- and 2-month contracts would on average lose money.
- First element of $\lambda + \Lambda \bar{x}$ is smaller, and no longer significant
  $\Rightarrow$ The average reward for taking long positions in the second subsample is not as evident in the first subsample

Risk Price


Level risk price
0.05
0
−0.05
1990
1992
1995
1997
2000
2002
2005
2007
2010
Slope risk price
x 10^−3
5
0
−5
1990
1992
1995
1997
2000
2002
2005
2007
2010

Risk Premium

$$rp_t = \tilde{f}_{nt} - f_{nt}$$



Implications

Positive $\bar{\lambda}_t(1)$ from 1990-2004

- Arbitrageurs: take long positions, accept positive expected earnings
- Commercial producers: hedge by short positions, pay for insurance

Index fund buyers explain why a long position no longer has a positive return.

- Serve as counterparty for commercial hedgers
- Don't demand risk compensation

Positive return to a spreading position from 2005-2011

- Arbitragers buy long-term futures from oil producers, and sell short-term futures to index-fund investors

Conclusion

Methodology

- Affine factor model for studying the interaction between hedgers and arbitrageurs in oil futures market
- Estimation with unbalanced panel
- Diagnostic tools

Empirical findings

- Prior to 2005, positive compensation for a long position, with low variation of risk premium
  $\Rightarrow$ the premium comes from hedging demand by commercial producers
- Since 2005, lower and often negative compensation for a long position, with higher volatility
  $\Rightarrow$ Increased participation by financial investors change the nature of risk premia