The Dark Corners of the Labor Market

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EUI/CEPR/IMF conference on Secular Stagnation, Growth and Real Interest Rates

Florence, June 2015
The main lesson of the crisis is that we were much closer to those dark corners than we thought—and the corners were even darker than we had thought too. – Olivier Blanchard (2014), in “Where Danger Lurks”.
Single vs multiple steady states

Single steady state

Multiple steady states

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Dark Corners

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Models with multiple steady-state rates of unemployment

This paper

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- Focus on labor market stocks and flows for the United States.
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- Estimate reduced-form model and compute implied steady states.
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- Preview of findings: at least 3 steady states with different unemployment rates:
  - I_A: 5% (stable)
  - I_B: 10% (unstable)
  - I_C: >10% (stable)
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2. Search and matching models

- Basic Diamond-Mortensen-Pissarides (DMP) model + extension with skill losses à la Pissarides (1992)
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- Preview of findings:
  - basic DMP model fails to explain data by a wide margin.
  - extension with quantitatively moderate skill losses generates multiple steady states.
Part 1: reduced-form model
Reduced-form model

\[
    u_t = \left(1 - \rho_{f,t}\right)u_{t-1} + \rho_{x,t} \left(1 - \rho_{f,t}\right) (1 - u_{t-1})
\]

\[
    \rho_{x,t} = \rho_x(S_t) \\
    \rho_{f,t} = \rho_f(S_t)
\]

- \( u_t \): unemployment rate
- \( \rho_{f,t} \): job finding rate
- \( \rho_{x,t} \): job loss rate
- \( S_t \): aggregate state

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Steady states

Example 1: \[ \rho_x(S_t) = \bar{\rho}_x, \quad \rho_f(S_t) = \gamma_0 + \gamma_1 u_{t-1} \]

\[ \Rightarrow \]

\[ \bar{u} = (1 - \gamma_0 - \gamma_1 \bar{u}) \bar{u} + \bar{\rho}_x (1 - \gamma_0 - \gamma_1 \bar{u}) (1 - \bar{u}) \]

quadratic equation, two solutions for \( \bar{u} \).

Example 2: \[ \rho_x(S_t) = \bar{\rho}_x, \quad \rho_f(S_t) = \gamma_0 \]

\[ \Rightarrow \]

\[ \bar{u} = (1 - \gamma_0) \bar{u} + \bar{\rho}_x (1 - \gamma_0) (1 - \bar{u}) \]

linear equation, one solution for \( \bar{u} \).
Estimating steady states

- Estimate forecasting equations:

\[
E_t \rho_{r,t+k} \equiv E \left[ \rho_r (S_{t+k}) | S_t \right]
\]

for \( r = x, f \) and for some forecast horizon \( k \geq 1 \).
Estimating steady states

- Estimate forecasting equations:

\[ \mathbb{E}_t \rho_{r,t+k} \equiv \mathbb{E} [\rho_r(S_{t+k}) | S_t] \]

for \( r = x, f \) and for some forecast horizon \( k \geq 1 \).

- Combine estimated equations with transition to compute implied steady state(s).
Estimating steady states

- Estimate *forecasting* equations:

\[ \mathbb{E}_t \rho_{r,t+k} \equiv \mathbb{E} [ \rho_r (S_{t+k}) | S_t ] \]

for \( r = x, f \) and for some forecast horizon \( k \geq 1 \).

- Combine estimated equations with transition to compute implied steady state(s).

- Issue: \( S_t \) may not be entirely observable.
Estimating steady states

- Estimate \textit{forecasting} equations:

\[\mathbb{E}_t \rho_{r,t+k} \equiv \mathbb{E} [\rho_r (S_{t+k}) | S_t]\]

for \( r = x, f \) and for some forecast horizon \( k \geq 1 \).

- Combine estimated equations with transition to compute implied steady state(s).

- Issue: \( S_t \) may not be entirely observable.
  - Exploit that information in \( S_t \) is implicitly revealed by observed outcomes.
Estimating steady states

- Partition $S_t = \{ s_{1,t}, s_{2,t} \}$, where $s_{1,t}$ contains lags of $\rho_{x,t}$, $\rho_{f,t}$ and $u_t$. Assume $s_{2,t}$ contains two additional state variables (may be unobserved).
Partition $\mathcal{S}_t = \{s_{1,t}, s_{2,t}\}$, where $s_{1,t}$ contains lags of $\rho_{x,t}$, $\rho_{f,t}$ and $u_t$. Assume $s_{2,t}$ contains two additional state variables (may be unobserved).

$s_{2,t}$ uniquely pinned down by observed outcomes $\rho_{f,t} = \rho_f \left( \{s_{1,t}, s_{2,t}\} \right)$ and $\rho_{x,t} = \rho_x \left( \{s_{1,t}, s_{2,t}\} \right)$. Forecast becomes:

$$\mathbb{E}_t \rho_{r,t+k} \equiv \mathbb{E} \left[ \rho_r (\mathcal{S}_{t+k}) \mid \mathcal{S}_t \right],$$

$$= \mathbb{E} \left[ \rho_r (\mathcal{S}_{t+k}) \mid s_{1,t}, \rho_{f,t}, \rho_{x,t} \right].$$
Model specifications

Compare three specifications for job finding rate:

(I) $E_t \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \epsilon_{t+k}$

(II) $E_t \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \gamma_3 u_t + \epsilon_{t+k}$

(III) $E_t \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \gamma_3 u_t + \gamma_3 u_t^2 + \epsilon_{t+k}$

Specify AR(1) process for job loss rate $\rho_{x,t}$. 
Data


- CPS data on unemployment rate and flow rate from U to E (gross-flows).
  - similar results with duration-based flow data

- Construct job loss rate to be consistent with transition identity.

- IV estimator to account for noise in observations, using lagged values as instruments.
Data

A. Job finding rate ($\rho_f$)

B. Job loss rate ($\rho_x$)

C. Unemployment rate ($u$)
Data

\[ u_t^* = \frac{\rho_x, t (1 - \rho_f, t)}{\rho_x, t (1 - \rho_f, t) + \rho_f, t} \]
Model diagnostics

\[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \epsilon_{t+k} \]

Model (II): \[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \beta_3 u_t + \epsilon_{t+k} \]

Model (III): \[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \beta_3 u_t + \beta_4 u_t^2 + \epsilon_{t+k} \]
Two year ahead forecasts

Realized job finding rate versus forecasts (1 year moving averages)

- **realization**
- **forecast model (I)**
- **forecast model (II)**
- **forecast model (III)**
Steady state curve for $\rho^*_x, t = \rho_x$. Shaded area’s denote 90 percent (bootstrapped) confidence bands.
Phase diagram
Phase diagram

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Phase diagram

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Part 2: search and matching models
Model: general setup

- Random search and matching model in tradition of Diamond, Mortensen and Pissarides.
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- Economy populated by measure of risk-neutral households who transition between employment and unemployment and who own firms.
Random search and matching model in tradition of Diamond, Mortensen and Pissarides.

Economy populated by measure of risk-neutral households who transition between employment and unemployment and who own firms.

Unemployment creates a loss of human capital (Pissarides (1992)).
Model: general setup

- Random search and matching model in tradition of Diamond, Mortensen and Pissarides.
- Economy populated by measure of risk-neutral households who transition between employment and unemployment and who own firms.
- Unemployment creates a loss of human capital (Pissarides (1992)).
- Only source of aggregate uncertainty is exogenously varying probability of job loss $\rho_{x,t}$. 
Rate of job loss is revealed and job losses take place.

Job losers and previously unemployed workers find a job with an endogenous probability $\rho_{f,t}$. Vacancies ($v_t \geq 0$) are posted at a cost $\kappa > 0$ per unit and filled with an endogenous probability $q_t$.

Production and consumption take place. Employed workers produce $\bar{A}$ units of goods and receive a wage. Unemployed workers receive $b < \bar{A}$ units of goods.
Model: skill losses

- Job losers who immediately find a new job retain their productivity.

- Job losers who become unemployed need to be re-trained upon re-employment, at a cost $\chi \geq 0$ to the employer. Basic DMP model is obtained by setting $\chi = 0$.

- The fraction of job searchers with reduced skills, $p_t$, is given by:

$$p_t = \frac{u_{t-1}}{u_{t-1} + \rho_{x,t} (1 - u_{t-1})}.$$
Vacancy posting (free-entry) condition

\[
\frac{\kappa}{q_t} + p_t \chi = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^t s_{t,t+k} (\bar{A} - w_{t+j}) + \xi_t \right\}
\]

where

- \( s_{t,t+k} \equiv \prod_{j=1}^{k} (1 - \rho_{x,t+j}) \) is the probability that the match survives until period \( t + k \)
- \( \xi_t \) is the Lagrange multiplier on the constraint \( \nu_t \geq 0 \).
Matching function:

\[ m_t = s_t^{\alpha} v_t^{1-\alpha}, \]

where \( s_t \equiv u_{t-1} + \rho x_{t} (1 - u_{t-1}) \) is the number of searchers \( \Rightarrow \rho_{f,t} = \frac{m_t}{s_t} \)

and \( q_t = \frac{m_t}{v_t} = \rho_{f,t}^{\frac{\alpha}{\alpha-1}} \).

Assume firms have all bargaining power \( \Rightarrow w_t = \bar{w} = b \) (rigid real wage). Could be relaxed.
Model summary

\[ u_t = \left(1 - \rho_{f,t}\right) u_{t-1} + \rho_{x,t} \left(1 - \rho_{f,t}\right) (1 - u_{t-1}) \quad (1) \]

\[ p_t = \frac{u_{t-1}}{u_{t-1} + \rho_{x,t} (1 - u_{t-1})} \quad (2) \]

\[ \rho_{x,t} = (1 - \lambda_x) \bar{\rho}_x + \lambda_x \rho_{x,t-1} + \epsilon_{x,t} \quad (3) \]

\[ \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left(\chi p_{t+1} - \xi_{t+1} + \kappa \rho_{f,t+1}^{\frac{\alpha}{1-\alpha}}\right) = \chi p_t - \xi_t + \kappa \rho_{f,t}^{\frac{\alpha}{1-\alpha}} - \bar{A} + \bar{w} \quad (4) \]

An equilibrium is characterized by laws of motion for \( u_t, \rho_{f,t}, \rho_{f,t}, p_t \) and \( \xi_t \) that satisfy the above four equations, and the complementary slackness condition \( \rho_{f,t} \xi_t = 0 \). The state of the aggregate economy can be summarized as

\[ S_t = \left\{ \rho_{x,t}, u_{t-1} \right\} . \]
Phase diagram: no skill losses
Phase diagram: skill losses
Parameter values

- Model period: 1 month
- Steady-state targets:

<table>
<thead>
<tr>
<th>target</th>
<th>no skill losses</th>
<th>skill losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{u}^A$</td>
<td>0.055</td>
<td>0.055</td>
</tr>
<tr>
<td>$\bar{u}^B$</td>
<td>–</td>
<td>0.095</td>
</tr>
</tbody>
</table>
## Parameter values

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>no skill losses</th>
<th>skill losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>$1.04^{\frac{1}{12}}$</td>
<td>$1.04^{\frac{1}{12}}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>matching function elast.</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>vacancy cost</td>
<td>0.989</td>
<td>0.989</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>worker productivity</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{\rho}_x$</td>
<td>s.s. job loss rate</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>persistence job loss rate shocks</td>
<td>0.896</td>
<td>0.896</td>
</tr>
<tr>
<td>$\bar{\sigma}_x$</td>
<td>s.t. deviation job loss shocks</td>
<td>$7.91e^{-4}$</td>
<td>$7.91e^{-4}$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>re-training cost</td>
<td>0</td>
<td>0.688</td>
</tr>
<tr>
<td>$b$</td>
<td>flow from unemployment</td>
<td>0.997</td>
<td>0.985</td>
</tr>
</tbody>
</table>
Propagating deterministic simulation

unemployment rate \( u_t \)

DMP model + skill losses

basic DMP model

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Simulation

job loss rate ($\rho_{x,t}$)

0.016 0.018 0.02 0.022 0.024 0.026 0.028 0.03

unemployment rate ($u_t$)

data
DMP model with skill losses
basic DMP model
constant job finding rate
job finding rate ($\rho_{f,t}$)

- Data
- DMP model with skill losses
- Basic DMP model
- Constant job finding rate

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Conclusion

- Multiple-steady state model provides superior description of data.
- Threshold at around 10% unemployment.
- Possibly large and non-linear policy implications.
Appendix: firm decision problem

Large firms with constant returns-to-scale technologies decide on number of vacancies ($v_t$), hires ($h_t$) and employment ($n_t$). Decision problem:

$$V(n_{t-1}, S_t) = \max_{h_t, n_t, v_t} \left( (\bar{A} - w_t) n_t - \left( (\chi - d_t) p_t + \frac{\kappa}{q_t} \right) h_t \right)$$

$$+ \beta E_t V(n_t, S_{t+1}) ,$$

subject to

$$n_t = \left( 1 - \rho_{x,t} \right) n_{t-1} + h_t ,$$

$$h_t = q_t v_t ,$$

$$h_t \geq 0 ,$$

where $w_t$ is the wage and $d_t$ is a possible wage deduction for newly hired workers with reduced skills.