## BACKGROUND PAPER TO BOPETG ISSUES \# 26

## INDEXATION OF DEBT INSTRUMENTS

An Illustrative Example Prompted By Revised BOPTEG Issues Paper \# 26

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This paper explores the characteristics of the four alternative methodologies for the accrual of interest on indexed debt instruments. The analysis within the paper is based upon a set of arithmetic examples. The examples are constructed on assumptions which are essentially arbitrary, but which are nevertheless considered capable of delivering robust conclusions.

## The Basic Model

A five year zero coupon bond is issued for $\$ 1000$ with its redemption value indexed to a commodity price index. The expected yield to maturity (YTM) at issue is $8 \%$ which represents a risk premium of 2 pp over the opportunity cost of funds invested in a conventional zero coupon bond. The bond is traded in the secondary market where its quoted value reflects both the expected level of the indicator index at maturity and the current opportunity cost of funds. Other things being equal, an increase in the expected maturity value of the commodity index will be associated with an increase in the current market value of the debt instrument, while an increase in the opportunity cost of funds would be associated with a fall in the current market value of the debt instrument. It is further assumed that the market interest rate is effectively independent of the commodity index, i.e., the return on holding the commodity is determined by fundamentals in the supply and demand for the commodities included in the index and by expectations about those fundamentals.

Consider first the case where the opportunity cost of funds - the market interest rate - is unchanged at $6 \%$ over the life of the bond. What data are then needed to construct accrued interest flows and associated stock reconciliations under the various alternatives?

## Alternative 1: Interest based on the actual path of the commodity index

In its pure form, in which data are prepared after redemption, this approach only requires the value of the indicator series at the time of redemption and the actual market values of the debt instrument at each reference date. In practice the market value series is sufficient because the value of the indicator series will be embodied within the value of the debt instrument at redemption. Before the redemption date, some estimation methods will have to be used that may require the actual path of the indicator series.

## Alternative 2: Interest based on expected YTM at the time of issue

This approach again requires the market value of the debt instrument plus the expected redemption value of the instrument at the time of issue.

## Alternative 3: Interest based on expected current YTM in each period

This approach requires the actual and expected path for the value of the debt instrument. Notwithstanding the assumption of no change in the opportunity cost of funds, these values can diverge because actual values are affected by unforeseen changes in the indicator series.

## Alternative 4: Separating the embedded derivative

For the debtor approach, interest flows are equivalent to those delivered by applying the unchanged market interest rate (using the information for similar instruments that are not indexed). Under this approach, the market value of the debt instrument will need to be decomposed into an element representing the value of the implied $8 \%$ bond $(6 \%+2 \mathrm{pp}$ risk premium), and the residual representing the value of the imputed derivative.

## The Data

The starting point for the example is the actual path of the indicator series which is assumed exogenous, and the expected path of this series at the time of issue.

Table 1:

|  | 0 | 1 | 2 | 3 | 4 | 5 | Yield to <br> redemption |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Actual indicator | 1000 | 1070 | 1130 | 1290 | 1480 | 1403 | $7.0 \%$ |
| Expected values <br> at time of issue | 1000 | 1080 | 1166 | 1260 | 1360 | 1469 | $8.0 \%$ |
| Period Changes <br> in expected <br> values at issue |  | 80 | 86 | 94 | 100 | 109 |  |
| Amortised values <br> based on actual <br> redemption value | 1000 | 1070 | 1145 | 1225 | 1311 | 1403 |  |
| Period Changes <br> in amortised <br> value |  | 70 | 75 | 80 | 86 | 92 |  |

At the end of period 1 , it can be seen that the commodity index has underperformed its expected path at the time of issue. It is assumed that this will cause the expected redemption value to be revised and thereby lead to the current market value of the debt instrument also falling below its expected value at the time of issue. For the purpose of this illustration, the expected YTM of the commodity index at the end of period $t$ is taken to be the weighted average of the previous expected YTM and the current growth in the index, with weights of 0.7 and 0.3 respectively. Thus in period 1 the expected YTM of the index is $7.7 \%$ giving an expected redemption value of 1440 . With market interest rates and the required risk premium constant this gives a market value for the debt instrument of 1058 to deliver a YTM to new investors of $8 \%$. Following this assumed relationship through to later periods delivers the following:

Table 2:

|  | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Expected <br> redemption <br> values | 1469 | 1440 | 1387 | 1538 | 1641 | 1403 |
| Expected <br> remaining YTM | $8.00 \%$ | $7.70 \%$ | $7.07 \%$ | $9.20 \%$ | $10.86 \%$ |  |
| Market value of <br> debt instrument | 1000 | 1058 | 1101 | 1319 | 1519 | 1403 |

In this example, the market value of the debt instrument underperforms initial expectations in years 1 and 2 but then substantially over-performs in years 3 and 4 before collapsing in year 5 in response to an unanticipated downturn in the commodity index. The calculations above are shown only for illustrative purposes, in practice market values of securities at a point in time are available from markets and should be used.

Returning to the 1993 SNA and the four alternatives discussed in the IMF paper for computing interest we see the following:

## 1993 SNA:

Interest accruing in each accounting period is determined using the movement in the index during that period. Estimates of interest are not revised. It can be seen that total interest payments equal the difference between the issue and redemption price of the bond and that revaluations net to zero.

Table 3:

| Year | Opening <br> Balance | Interest | Revaluation | Closing <br> Balance |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1000 | 70 | -12 | 1058 |
| 2 | 1058 | 60 | -17 | 1101 |
| 3 | 1101 | 160 | 58 | 1319 |
| 4 | 1319 | 190 | 10 | 1519 |
| 5 | 1519 | -77 | -39 | 1403 |
| Total |  | 403 | 0 |  |

## Alternative 1 method (a):

Interest accruing in each accounting period is determined using the movement in the index during that period and revised when the actual redemption value is known. Note that the only difference with the 1993 SNA is that the estimates of interest based on the movements in the relevant index are revised once when actual redemption value is known.

Table 4:

| Year | Opening Balance | Interest |  | Revaluation |  | Closing Balance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Initial | Final at end of year 5 | Initial | Final at end of year 5 |  |
| 1 | 1000 | 70 | 70 | -12 | -12 | 1058 |
| 2 | 1058 | 60 | 75 | -17 | -32 | 1101 |
| 3 | 1101 | 160 | 80 | 58 | 138 | 1319 |
| 4 | 1319 | 190 | 86 | 10 | 114 | 1519 |
| 5 | 1519 | -77 | 92 | -39 | -208 | 1403 |
| Total |  | 403 | 403 | 0 | 0 |  |

## Alternative 1 method (b):

Interest accruals are determined using the most recent observation of the index and revised continuously.

Table 5:

| Year | Opening <br> Balance | Interest (estimates made in the year) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
| 1 | 1000 | 70 | 63 | 89 | 103 | 70 |
| 2 | 1058 |  | 67 | 96 | 114 | 75 |
| 3 | 1101 |  |  | 105 | 125 | 80 |
| 4 | 1319 |  |  |  | 138 | 86 |
| 5 | 1519 |  |  |  |  | 92 |
| Total |  |  |  |  |  | 403 |

In this example, the interest accruals for the entire period, from the beginning through the current period, are derived using the most recent observation of the index. This involves continuous revision of past data until the debt instrument is redeemed. The closing balances are given by the market values of the instrument at the end of the period (they will be the same as in Table 4). Each period, the difference between the closing balance and the opening balance plus interest accrued gives holding gains/losses.

## Alternative 2

Interest is defined, for the life of the instrument, as the difference between expected redemption value at the time of issue and the issue price. Thus, interest accruals are calculated using the expected YTM at issue.

Table 6:

| Year | Opening <br> Balance | Interest | Revaluation | Closing <br> Balance |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1000 | 80 | -22 | 1058 |
| 2 | 1058 | 86 | -43 | 1101 |
| 3 | 1101 | 94 | 124 | 1319 |
| 4 | 1319 | 100 | 100 | 1519 |
| 5 | 1519 | 109 | -225 | 1403 |
| Total |  | 469 | -66 |  |

Total interest flows are higher than under the 1993 SNA approach and Alternative 1, but are offset by a net negative revaluation (holding loss) reflecting the difference between the actual and expected redemption value at the time of issue. Estimates of interest are not revised.

## Alternative 3

Interest is defined, for the remaining life of the instrument, as the difference between the expected redemption value at the current time and the current market price.

Table 7:

| Year | Opening <br> Balance | Interest | Revaluation | Closing <br> Balance |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1000 | 80 | -22 | 1058 |
| 2 | 1058 | 85 | -42 | 1101 |
| 3 | 1101 | 88 | 130 | 1319 |
| 4 | 1319 | 106 | 94 | 1519 |
| 5 | 1519 | 122 | -238 | 1403 |
| Total |  | 481 | -78 |  |

It should be remembered here that market interest rates are unchanged. The approach nevertheless delivers a different result from alternatives 1 and 2 because these latter compute interest based on an assumption of the instrument being held from issue to redemption. Alternative 3 uses an approach based on the expected return in the secondary market.

## Alternative 4

Interest accruals and values of embedded derivative are separated. For the debtor approach, this means that interest is determined using the expected YTM at issue. The value of the embedded derivative reflects any deviation of the interest accruals from actual movements in the relevant index.

Table 8:
Standardised Bond

| Year | Opening <br> Balance | Interest | Revaluation | Closing <br> Balance |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1000 | 80 |  | 1080 |
| 2 | 1080 | 86 | 0 | 1166 |
| 3 | 1166 | 94 | 0 | 1260 |
| 4 | 1260 | 100 | 0 | 1360 |
| 5 | 1360 | 109 | 0 | 1469 |
| Total |  | 469 | 0 |  |

Derivative

| Year | Opening <br> Balance | Interest | Revaluation | Closing <br> Balance |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | -22 | -22 |
| 2 | -22 | 0 | -43 | -65 |
| 3 | -65 | 0 | 124 | 59 |
| 4 | 59 | 0 | 100 | 159 |
| 5 | 159 | 0 | -225 | -66 |
| Total |  | 0 | -66 |  |

The relationship between Alternative 4 and Alternative 2 can be seen easily from these tables. Summing the corresponding cells of the standard bond and derivative in table 8 restores the Alternative 2 presentation in table 6.

In this particular example in the paper, it is assumed that only the market expectation about future path of the index changes (general market interest rate and credit risk remain unchanged). Therefore, it is considered that the change in the market value of the combined instrument due to revaluations reflects the effect of the index, which passes through embedded derivative. If there were changes in interest rates as well, then one would assume that the change in market values due to the change in general interest rate would be attributed to the bond rather than the derivative. When expectation on the future path of the index as well as market interest rate and credit risk change, it could become difficult to disentangle the effect of the index on the value of the combined instrument from the effect of these other factors. An alternative approach would be to consider that all revaluations pass through the embedded derivative.

The same example could be used to illustrate the effect of a change in the Market interest rate. For example, if the market rate were to rise from $6 \%$ to $6.5 \%$ at the end of year 1 with no change to the risk premium, then, other things being equal, the market value of the debt instrument would be 1039 , i.e. giving an expected yield of $8.5 \%$ to an unchanged expected redemption value of 1440 . The interest flows under alternative 1 and 2 would be unaffected by this change, although the subsequent revaluations would change. Under Alternative 3, a larger negative revaluation (holding loss) would be recorded in period 1 while in years 2 and following the recorded interest stream would change. Under Alternative 4, the impact of the change in the interest rate would be reflected in the market value of the standard bond with equivalent revaluation changes. The relationship between Alternative 4 and Alternative 2 would continue to hold.

## Exploring the four alternatives

The recorded accrued interest measures for the four core alternatives described in the STA paper are not too dissimilar in this example because the actual redemption value of the commodity index is assumed to be fairly close to its expected value when the debt instrument was issued. But the story could be a very different one.

For example, the STA paper notes (and the above table confirms) that estimation approach A under Alternative 1 could give rise to negative interest in some periods. In fact, Alternative 1 is capable of delivering negative interest in every period - this would automatically occur if the commodity index turned out to be lower at the end of period 5 than when the debt instrument was issued at the end of period 0 . This might be thought to be an extreme example but is nevertheless one which the favoured method should accommodate.

Suppose that expectations at the time of issue were the same as in the earlier example, but that the actual value of the commodity index starts drifting down straight away and is down to 950 by year 5 . Alternative 1 would then deliver negative interest flows of approximately 10 per year while Alternative 2 would deliver the same positive interest stream as in the original example as expectations at the point of issue would be fixed.

But we would also expect some sizeable changes in the market value of the debt instrument. Using the same arbitrary expectations model as before, the expected YTM of the commodity
index at the end of year 1 would be $5.3 \%$ (weighted average of $8 \%$ and $-1 \%$ ) giving an expected value at redemption of 1217. Secondary market investors in the debt instrument will still expect a yield of $8 \%$, because the opportunity cost of funds and the risk premium have not change, so the market value of the debt instrument in year 1 will be 894.

Applying the same assumptions as before, a series for the expected value of the commodity index at the time of redemption of the debt instrument can be calculated for each period and the associated market value of the debt instrument worked out. This gives:

Table 9:

| Year | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Expected <br> value of <br> commodity <br> index at <br> time of <br> redemption | 1469 | 1217 | 1084 | 1011 | 971 | 950 |
| Market <br> value of <br> Debt <br> Instrument | 1000 | 894 | 860 | 867 | 899 | 950 |

Based on these data, the computed interest flows under the four alternatives would be as follows:

Table 10:

| Year | 1993 SNA | Alt. 1 (after revision) | Alt. 2 | Alt. 3 | Alt. 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Based on the actual movements | -10 | 80 | 80 | 80 |
| 2 |  | -10 | 86 | 72 | 86 |
| 3 |  | -10 | 94 | 69 | 94 |
| 4 |  | -10 | 100 | 69 | 100 |
| 5 |  | -10 | 109 | 72 | 109 |
| Total |  | -50 | 469 | 362 | 469 |

The precise numbers here are not important - the data are illustrative and rely on an arbitrary assumption about how expectations of future performance are updated. But the broader story they tell about the characteristics of interest under the various alternatives appears robust and so the examples are useful.

The first question concerns the interpretation of an accruals methodology capable of delivering negative interest. While the fact that the investor has sustained a loss in this case is clear, the nature of the loss is presented differently under the four approaches.
Alternatives 2 and 3 both impute positive interest flows offset by holding losses (downward revaluations) of the debt instrument. Alternative 4 similarly imputes a positive interest stream to a synthetic bond plus an accumulated liability position in derivatives. While Alternative 1 presents the whole of the loss as a reverse flow of interest with no net holding gain or loss.

Whilst this example has been deliberately chosen to represent an extreme situation, it should be clear that Alternativel does not portray interest as equivalent to the service provided by the provision of capital. The investor knows when acquiring the debt instrument that it is capital uncertain so it makes no sense to presume that the whole of the change in value through its life represents the effect of reinvested interest (or involuntary disinvestments in this particular case). This argument is particularly forceful where measured interest is negative, but is a more general concern - had the commodity index risen from 1000 to 1050 over the life of the instrument, the recording of a $1 \%$ per annum interest stream in an environment where the risk free opportunity cost is $6 \%$ can make little sense. The reality must be that the investor has earned interest at the market rate but has sustained a holding loss. Such an interpretation is, however, at odds with the SNA text which regards the difference between the issue and redemption value of the instrument as interest.

The market rate of return is the key. While the redemption value of the debt instrument is linked to a commodity (or some other) index, the debt instrument is traded in a market where investors require a market return independent of the performance of commodity markets. So, in the last example, if the market expectation at the time of issue had been that the commodity index would fall to 950 by year 5 , then the debt instrument would have needed to be issued at a lower price (\$647) in order to deliver an expected return in line with the market rate and risk premium.

With this in mind, Alternative 2 may better characterise the standard debtor (historic cost) approach to interest measurement. It computes interest based on the expected rather than actual return on holding the instrument from issue to redemption. As such, interest is consistent with the market return at the time of issue.

Similarly, Alternative 4 can be characterised as presenting a standard zero coupon bond under the debtor approach with a separately valued embedded derivative. This approach may be considered consistent with IFRS guidance on the disclosure and valuation of instruments with embedded derivatives - namely to present the underlying and the derivative separately at market or fair value.

However, it should be noted that interest measurement under either Alternative 2 or 4 is invariant to actual movements in the commodity index. This results from the principle of determining interest flows at the point of issue and so is a feature of the debtor approach. The CYTM approach to interest measurement in Alternative 3 uses all available data from the commodity market to update expectations and so provides an interest measure reflecting both the opportunity cost of funds and developments in the commodity index.

## Conclusion

Of the three Alternative s (1,2 and 4) on which views are sought, I consider the choice to lie between Alternative 2 and Alternative 4. These two Alternatives are effectively equivalent so my preference between them is a weak one.

Adoption of either of these Alternatives over Alternative 1, will require a change to the current SNA guidance, that, when principal is indexed, the difference between the eventual redemption price and the issue price is treated as interest accruing over the life of the asset. In the light of these examples, interest under the debtor approach is measured as the difference between the issue price and the expected redemption value at the time of issue, with any difference between the actual and expected value recorded as a holding gain or loss.

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