19. Price Indices Using an Artificial Data Set

A. Introduction

19.1 In order to give the reader some idea of how much the various index numbers might differ using a real data set, all of the major indices defined in the previous chapters are computed using an artificial data set consisting of prices and quantities for eight commodities over five periods (see Section B).[1] The period can be thought of as between one and five years. The trends in the data are generally more pronounced than one would see in the course of a year. The eight commodities can be thought of as the net deliveries to the final demand sector of all industries in the economy. The first six commodities are outputs and correspond to the usual private consumption plus government consumption plus investment plus export deliveries to final demand, whereas the last two commodities are imports (and hence are indexed with a negative sign).

19.2 In Section C, the same final-demand data set is used in order to compute the midyear indices that were described in Chapter 17. Recall that these indices have an important practical advantage over superlative indices because they can be computed using current data on prices and lagged data on quantities (or equivalently, using lagged data on expenditures).

19.3 In Section D, the additive percentage change decompositions for the Fisher ideal price index that were discussed in Section C.8 of Chapter 16 are illustrated using the final-demand data set on eight commodities.

19.4 In Section E.1, price and quantity data for three industrial sectors of the economy are presented. This industrial data set is consistent with the final-demand data set listed in Section B.1 below. Sections E.2 through E.4 construct value-added deflators for these three industries. Only the Laspeyres, Paasche, Fisher, and Törnqvist formulas are considered in Section E and subsequent sections since these are the formulas that are likely to be used in practice.

19.5 In Section F, the industry data are used in order to construct national output price indices, national intermediate input price deflators, and national value-added deflators. The construction of a national value-added deflator by aggregating the national output and intermediate input price indices is undertaken in Section F.4. This two-stage national value-added deflator is then compared with its single-stage counterpart and also with the final-demand deflator constructed in Section B.

B. Price Indices for Final-Demand Components

B.1 Final-demand data set

19.6 The price and quantity data for net deliveries to final demand are listed in Tables 19.1 and 19.2 below. For convenience, the period t nominal expenditures, $p^t \cdot q^t \equiv \sum_{i=1}^{8} p_i^t q_i^t$, have been listed along with the corresponding period t expenditure shares, $s_i^t \equiv p_i^t q_i^t / p^t \cdot q^t$, in Table 19.3. Typically, the statistical agency will not have quantity data available; only price and expenditure data will be collected. However, given the information in Table 19.3, the period t net expenditure shares s_n^t may be multiplied by period t total net expenditures $p^t \cdot q^t$ in order to obtain final-demand expenditures by commodity. Then these commodity expenditures may be divided by the corresponding prices in Ta-

[1]Lowe and Young indices are not calculated for this data set; however, they are available in Chapter 19 of the *Consumer Price Index Manual* (International Labour Organization and others, 2004) to allow comparisons with the other major indices.

ble 19.1 in order to obtain the implicit quantities listed in Table 19.2.[2]

19.7 The trends that are built into the tables can be explained as follows. Think of the first four commodities as the final-demand consumption of various classes of *goods* in some economy, while the next two commodities are the consumption of two classes of *services*. Think of the first good as *agricultural consumption and exports*. The final-demand quantity for this good mildly fluctuates around 30 units of output, while its price fluctuates more violently around 1. However, as the rest of the economy grows, the share of agricultural output declines to about one-half of its initial share. The second good is *energy consumption* in final demand. The quantity of this good trends up gently during the five periods with some fluctuations. However, note that the price of energy fluctuates wildly from period to period.[3] The third good is *traditional manufactures*. There are rather high inflation rates for this commodity for periods 2 and 3, which diminish to a very low inflation rate by the end of our sample period.[4] The final-demand consumption of traditional manufactured goods is more or less static in our data set. The fourth commodity is *high-technology manufactured goods*; for example, computers, video cameras, compact discs, etc. The demand for these high-tech commodities grows tenfold over our sample period, while the final period price is only one-fifth of the first-period price. The fifth commodity is *traditional services*. The price trends for this commodity are similar to traditional manufactures, except that the period-to-period inflation rates are a bit higher. However, the demand for traditional services is growing much more strongly than for traditional manufactures. Our sixth commodity is *high-technology services*; for example, telecommunications, wireless phones, Internet services, stock market trading, etc. For this final commodity, the price is trending downward very strongly to end up at 20 percent of the starting level, while demand increases fivefold. The final two commodities are *energy imports* and *imports of high-technology manufactured goods*. Since imports are intermediate inputs to the economy as a whole, the quantities for these last two commodities are indexed with minus signs. The prices and quantities for the two imported commodities are more or less proportional to the corresponding final consumption demand prices and quantities. The movements of prices and quantities in this artificial data set are more pronounced than the year-to-year movements that would be encountered in a typical country. However, they do illustrate the problem that is facing compilers of the producer price index: namely, *year-to-year price and quantity movements are far from being proportional across commodities, so the choice of index number formula will matter*.

19.8 Every price statistician is familiar with the *Laspeyres index*, P_L, defined by equation (15.5) in the main text of Chapter 15, and the *Paasche index*, P_P, defined by equation (15.6). These indices are listed in Table 19.4 along with the two unweighted indices that were considered in Chapters 15 and 16: the *Carli index* defined by equation (16.45) and the *Jevons index* defined by equation (16.47). The indices in Table 19.4 compare the prices in period t with the prices in period 1; that is, they are *fixed-base indices*. Thus, the period t entry for the Carli index, P_C, is simply the arithmetic mean of the eight price relatives, $\sum_{i=1}^{8}(1/8)\left(p_i^t/p_i^1\right)$, while the period t entry for the Jevons index, P_J, is the geometric mean of the eight price relatives, $\prod_{i=1}^{8}\left(p_i^t/p_i^1\right)^{1/8}$.

19.9 Note that by period 5, the spread between the fixed-base Laspeyres and Paasche price indices is fairly large: P_L is equal to 1.6343 while P_P is 1.2865, *a spread of about 27 percent*. Since these indices have exactly the same *theoretical* justification, it can be seen that the choice of index number formula matters a great deal. There is also a substantial spread between the two unweighted indices by period 5: the fixed-base Carli index is equal to 0.9125, while the fixed-base Jevons index is 0.6373, *a spread of about 43 percent*. However,

[2]Typically, the prices will be price relatives or averages of price relatives, but if the base period is equal to period 1, then these relative prices will all be unity in period 1.

[3]This is an example of the price-bouncing phenomenon noted by Szulc (1983). Note that the fluctuations in the price of energy that have been built into our data set are not that unrealistic: in the recent past, the price of a barrel of crude oil has fluctuated from US$10 to US$37. Note that agricultural prices also bounce but not as violently.

[4]This corresponds roughly to the experience of most industrialized countries over a period starting in 1973 and ending in the mid 1990s. Thus, roughly five years of price movement are compressed into one of the periods.

Table 19.1. Prices for Eight Commodities

Period t	Final Demand of Goods: Agriculture exports p_1^t	Energy p_2^t	Traditional manufacturing p_3^t	High-tech manufacturing p_4^t	Services: Traditional services p_5^t	High-tech services p_6^t	Imports: Energy imports p_7^t	High-tech imports p_8^t
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.3	2.0	1.3	0.7	1.4	0.8	2.1	0.7
3	1.0	1.0	1.5	0.5	1.7	0.6	1.0	0.5
4	0.7	0.5	1.6	0.3	1.9	0.4	0.6	0.3
5	1.0	1.0	1.7	0.2	2.0	0.2	1.0	0.2

Table 19.2. Quantities for Eight Commodities

Period t	Final Demand of Goods: Agriculture exports q_1^t	Energy q_2^t	Traditional manufacturing q_3^t	High-tech manufacturing q_4^t	Services: Traditional services q_5^t	High-tech services q_6^t	Imports: Energy imports q_7^t	High-tech imports q_8^t
1	30	10	40	10	45	5	–28	–7
2	28	8	39	13	47	6	–20	–9
3	30	11	38	30	50	8	–29	–21
4	32	14	39	60	56	13	–35	–42
5	29	12	40	100	65	25	–30	–70

Table 19.3. Net Expenditures and Net Expenditure Shares for Eight Commodities

Period t	$p^t \cdot q^t$	Final Demand of Goods: Agriculture exports s_1^t	Energy s_2^t	Traditional manufacturing s_3^t	High-tech manufacturing s_4^t	Services: Traditional services s_5^t	High-tech services s_6^t	Imports: Energy imports s_7^t	High-tech imports s_8^t
1	105.0	0.2857	0.0952	0.3810	0.0952	0.4286	0.0476	–0.2667	–0.0667
2	134.5	0.2706	0.1190	0.3770	0.0677	0.4892	0.0357	–0.3123	–0.0468
3	163.3	0.1837	0.0674	0.3491	0.0919	0.5205	0.0294	–0.1776	–0.0643
4	187.8	0.1193	0.0373	0.3323	0.0958	0.5666	0.0277	–0.1118	–0.0671
5	220.0	0.1318	0.0545	0.3091	0.0909	0.5909	0.0227	–0.1364	–0.0636

Table 19.4. Fixed-Base Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.2875	1.1853
3	1.4571	1.3957	0.9750	0.8868
4	1.5390	1.3708	0.7875	0.6240
5	1.6343	1.2865	0.9125	0.6373

Table 19.5. Chained Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.2875	1.1853
3	1.3743	1.4834	1.0126	0.8868
4	1.4374	1.5349	0.7406	0.6240
5	1.4963	1.5720	0.8372	0.6373

more troublesome than this spread is the fact that *the unweighted indices are far below both the Paasche and Laspeyres indices* by period 5.[5] Thus, when there are divergent trends in both prices and quantities, it will usually be the case that unweighted price indices will give very different answers than their weighted counterparts. Since none of the index number theories considered in previous chapters supported the use of unweighted indices, the use of unweighted formulas is not recommended for aggregation at the higher level, that is, when data on weights are available. However, in Chapter 20, aggregation at the lower level is considered for weights that are unavailable, and the use of unweighted index number formulas will be revisited. Finally, note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[6]

19.10 It is of interest to recalculate the four indices listed in Table 19.4 using the *chain principle* rather than the *fixed-base principle*. Our expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chained indices are listed in Table 19.5.

19.11 It can be seen comparing Tables 19.4 and 19.5 that chaining eliminated about three-fourths of the spread between the fixed-base Paasche and Laspeyres indices for period 5. However, even the chained Paasche and Laspeyres indices differ by about 8 percent in period 3, so the choice of index number formula still matters. In Table 19.4, the fixed-base Laspeyres exceeds the fixed-base

[5]The reason for this is that when using weighted indices, the imports of high-technology goods are offset by the final-demand expenditures on high-technology goods to a large extent; that is, commodities 6 and 8 have the same dramatic downward price trends, but their quantity trends are opposite in sign and cancel each other out to a large extent. However, when calculating the unweighted indices, this cancellation does not occur, and the downward trends in the prices of commodities 6 and 8 get a much higher implicit weight in the unweighted indices.

[6]This is the Theorem of the Arithmetic and Geometric Mean; see Hardy, Littlewood, and Polyá (1934) and Chapter 20.

Table 19.6. Asymmetrically Weighted Fixed-Base Indices

Period t	P_{PAL}	P_{GP}	P_L	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1520	1.1852	1.1552	1.1811	1.2009	1.1906
3	1.5133	1.4676	1.4571	1.4018	1.3957	1.3212
4	1.6628	1.5661	1.5390	1.4111	1.3708	1.2017
5	1.7673	1.6374	1.6343	1.4573	1.2865	1.0711

Paasche, while in Table 19.5, the positions are reversed for the respective chained indices. Such differences for fixed-base Laspeyres and Paasche were shown in Appendix 15.1 of Chapter 15 to depend on the sign of the correlation between relative price changes and average quantity changes.[7] Note that chaining did not affect the Jevons index. This is an advantage of the index, but the lack of weighting is a fatal flaw. The truth would be expected to lie between the Paasche and Laspeyres indices and in Table 19.5. However, the unweighted Jevons index is far below this acceptable range. Note that chaining did not affect the Carli index in a systematic way for our particular data set: in period 3, the chained Carli index is above the corresponding fixed-base Carli, but in periods 4 and 5, the chained Carli index is below the fixed-base Carli.

19.12 A systematic comparison of all of the *asymmetrically weighted price indices* is now undertaken. The *fixed-base indices* are listed in Table 19.6. The fixed-base *Laspeyres* and *Paasche indices*, P_L and P_P, are the same as those indices listed in Table 19.4. The *Palgrave index*, P_{PAL}, is defined by equation (16.55). The indices denoted by P_{GL} and P_{GP} are the *geometric Laspeyres* and *geometric Paasche indices*,[8] which are special cases of the fixed-weight geometric indices defined by Konüs and Byushgens (1926); see equations (16.75) and (16.76). For the *geometric Laspeyres index*, P_{GL}, let the weights α_i be the *base-period expenditure shares*, s_i^1. This index should be considered an alternative to the fixed-base Laspeyres index, since each of these indices makes use of the same information set. For the *geometric Paasche index*, P_{GP}, let the weights α_i be the *current-period expenditure shares*, s_i^t. Finally, the index P_{HL} is the *harmonic Laspeyres index* that was defined by equation (16.59).

19.13 By looking at the period 5 entries in Table 19.6, it can be seen that the spread between all of these fixed-base asymmetrically weighted indices has grown to be even larger than our earlier spread of 27 percent between the fixed-base Paasche and Laspeyres indices. In Table 19.6, the period 5 Palgrave index is about 1.65 times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, *this illustrates the point that due to the nonproportional growth of prices and quantities in most economies today, the choice of index number formula is very important*.

19.14 If there were no negative quantities in the final-demand vectors, then it is possible to explain why certain elements of the indices in Table 19.6

[7]Forsyth and Fowler (1981, p. 234) show how the relative positions of fixed and chained Laspeyres depend on the sign of their respective correlation coefficients. With the former, it is the correlation between price changes and quantities for periods 0 and t; with the latter, it is that between periods $t-1$ and t. The latter are more likely to take account of substitution effects leading to differences between the two.

[8]Vartia (1978, p. 272) used the terms *logarithmic Laspeyres* and *logarithmic Paasche*, respectively.

Table 19.7. Asymmetrically Weighted Indices Using the Chain Principle

Period t	P_{PAL}	P_{GP}	P_L	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1520	1.1852	1.1552	1.1811	1.2009	1.1906
3	1.3444	1.4050	1.3743	1.4569	1.4834	1.6083
4	1.4229	1.4730	1.4374	1.5057	1.5349	1.6342
5	1.4942	1.5292	1.4963	1.5510	1.5720	1.6599

are bigger than others. If all weights are positive, it can be shown that a *weighted arithmetic mean* of N numbers is equal to or greater than the corresponding *weighted geometric mean* of the same N numbers, which in turn is equal to or greater than the corresponding *weighted harmonic mean* of the same N numbers.[9] It can be seen that the three indices P_{PAL}, P_{GP}, and P_P all use the current-period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1), but P_{PAL} is a weighted *arithmetic* mean of these price relatives, P_{GP} is a weighted *geometric* mean of these price relatives, and P_P is a weighted *harmonic* mean of these price relatives. Thus, if there are no negative components in final demand, we have the following, according to Schlömilch's inequality:[10]

(19.1) $P_{PAL} \geq P_{GP} \geq P_P$.

However, due to the existence of imports in each period (which leads to negative quantities for these components of the final-demand vector), the inequalities in equation (19.1) are not necessarily true. Viewing Table 19.6, it can be seen that the inequalities in equation (19.1) hold for periods 3, 4, and 5 but not for period 2. It can also be verified that the three indices P_L, P_{GL}, and P_{HL} all use the base-period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1), but P_L is a weighted *arithmetic* mean of these price relatives, P_{GL} is a weighted *geometric* mean of these price relatives, and P_{HL} is a weighted *harmonic* mean of these price relatives. If all of these shares were nonnegative, then we have the following, according to Schlömilch's inequality:[11]

(19.2) $P_L \geq P_{GL} \geq P_{HL}$.

However, due to the existence of imports in each period, the inequalities in equation (19.2) are not necessarily true. Viewing Table 19.6, it can be seen that the inequalities in equation (19.2) hold for periods 3, 4, and 5 but not for period 2.

19.15 Now continue with the systematic comparison of all of the *asymmetrically weighted price indices*. These indices that use the *chain principle* are listed in Table 19.7. Viewing Table 19.7, it can be seen that the use of the chain principle dramatically reduced the spread between all of the asymmetrically weighted indices compared with their fixed-base counterparts in Table 19.6. For period 5, the spread between the smallest and largest asymmetrically weighted fixed-base index was 65 percent, but for the period 5 chained indices, this spread was reduced to 11 percent.

19.16 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory; that is, as was seen in Chapter 17, a superlative index is exact for a representation of the producer's production function or the corresponding unit revenue function that can provide a second-order approximation to arbitrary technologies that satisfy certain regularity conditions. In Chapters

[9]This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood, and Polyá (1934, chapter 11).

[10]These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[11]These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

Table 19.8. Symmetrically Weighted Fixed-Base Indices

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1831	1.1827	1.1814	1.1778	1.1781	1.1788
3	1.4343	1.4339	1.4327	1.4261	1.4264	1.4248
4	1.4866	1.4840	1.4820	1.4525	1.4549	1.4438
5	1.5447	1.5320	1.5193	1.4500	1.4604	1.4188

Table 19.9. Symmetrically Weighted Indices Using the Chain Principle

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1831	1.1827	1.1814	1.1778	1.1781	1.1788
3	1.4307	1.4257	1.4298	1.4278	1.4288	1.4290
4	1.4893	1.4844	1.4889	1.4853	1.4861	1.4862
5	1.5400	1.5344	1.5387	1.5337	1.5342	1.5338

Table 19.10. Fixed-Base Superlative Single-Stage and Two-Stage Indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1778	1.1830	1.1831	1.1837	1.1814	1.1835	1.1827	1.1829
3	1.4261	1.4259	1.4343	1.4351	1.4327	1.4341	1.4339	1.4325
4	1.4525	1.4713	1.4866	1.4974	1.4820	1.4990	1.4840	1.4798
5	1.4500	1.4366	1.5447	1.5440	1.5193	1.5208	1.5320	1.5191

15–17, four primary superlative indices were considered:

- The *Fisher ideal price index, P_F*, defined by equation (15.12);
- The *Walsh price index, P_W*, defined by equation (15.19) (this price index also corresponds to the quantity index Q^1 defined by equation [17.26]);[12]
- The *Törnqvist-Theil price index, P_T*, defined by equation (15.81); and
- The *implicit Walsh price index, P_{IW}*, that corresponds to the Walsh quantity index Q_W defined by equation (16.34).

These four symmetrically weighted superlative price indices are listed in Table 19.8 using the fixed-base principle. Also listed in this table are

[12]Since square roots of negative quantities are not feasible, the sign conventions are changed when calculating this index: change the negative quantities into positive quantities and change the corresponding positive prices into negative prices.

two symmetrically weighted (but not superlative) price indices:[13]

- The Marshall-Edgeworth price index, P_{ME}, defined by equation (15.18) and
- The Drobisch price index, P_{DR}, defined in Paragraph 15.19.

19.17 Note that the Drobisch index P_{DR} is always equal to or greater than the corresponding Fisher index P_F. This follows from the facts that the Fisher index is the geometric mean of the Paasche and Laspeyres indices while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices; an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed-base asymmetrically weighted indices, Table 19.6, with the symmetrically weighted indices, Table 19.8, *it can be seen that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices*. The spread was 1.7673/1.0711 = 1.65 for the asymmetrically weighted indices but only 1.5447/1.4188 = 1.09 for the symmetrically weighted indices. If the analysis is restricted to the superlative indices listed for period 5 in Table 19.8, then this spread is further reduced to 1.5447/1.4500 = 1.065; that is, the spread between the fixed-base superlative indices is only 6.5 percent compared with the fixed-base spread between the Paasche and Laspeyres indices of 27 percent (1.6343/1.2865 = 1.27). The spread between the superlative indices can be expected to be further reduced by using the chain principle.

19.18 The symmetrically weighted indices are recomputed using the chain principle. The results may be found in Table 19.9. A quick glance at Table 19.9 shows that *the combined effect of using both the chain principle as well as symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles*. The spread between all of the symmetrically weighted indices in period 5 is only 1.5400/1.5337 = 1.004 or 0.4 percent, which is the same as the spread between the four superlative indices in period 5.[14]

19.19 The results listed in Table 19.9 reinforce the numerical results tabled in R.J. Hill (2000) and Diewert (1978, p. 894): *the most commonly used chained superlative indices will generally give approximately the same numerical results*.[15] This is in spite of the erratic nature of the fluctuations in the data in Tables 19.1 to 19.3. In particular, the chained Fisher, Törnqvist, and Walsh indices will generally approximate each other very closely.

19.20 Attention is now turned to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see Section C of Chapter 17 for a discussion of the issues and a listing of the formulas used. In our artificial data set, the first four commodities are aggregated into a *goods aggregate*, the next two commodities into a *services aggregate,* and the last two commodities into an imports aggregate. In the second stage of aggregation, these three price and quantity components will be aggregated into a net final-demand price index.

19.21 The results are reported in Table 19.10 for our two-stage aggregation procedure using period 1 as the *fixed base* for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indexes, P_W and P_{IW}. Viewing Table 19.10, it can be seen that the fixed-base single-stage superlative indices generally approximate their fixed-base two-stage counterparts fairly closely. The divergence between the single-stage Fisher index P_F and its two-stage counterpart P_{F2S} in period 5 is 1.4500/1.4366 = 1.008 or 0.8 percent. The other divergences are even less.

[13]Diewert (1978, p. 897) showed that the Drobisch-Sidgwick-Bowley price index approximates any superlative index to the second order around an equal price and quantity point; that is, P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall-Edgeworth index P_{ME} is also pseudo-superlative.

[14]On average over the last four periods, the chain Fisher and the chain Törnqvist indices differed by 0.0046 percentage points.

[15]More precisely, the superlative quadratic mean of order r price indices P^r defined by equation (17.28) and the implicit quadratic mean of order r price indices $P^{r}*$ defined by equation (17.25) will generally closely approximate each other provided that r is in the interval $0 \leq r \leq 2$. Note that when one or more of the quantities being aggregated is negative (as in the present situation), the sign conventions are changed when calculating Q^r or $P^{r}*$: change the negative sign on import quantities to positive and make the import prices negative.

Table 19.11. Chained Superlative Single-Stage and Two-Stage Indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1778	1.1830	1.1831	1.1837	1.1814	1.1835	1.1827	1.1829
3	1.4278	1.4448	1.4307	1.4309	1.4298	1.4378	1.4257	1.4282
4	1.4853	1.5059	1.4893	1.4907	1.4889	1.4991	1.4844	1.4871
5	1.5337	1.5556	1.5400	1.5419	1.5387	1.5499	1.5344	1.5372

19.22 Using *chained indices*, the results are reported in Table 19.11 for our two-stage aggregation procedure. Again, the single-stage approach and its two-stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indexes, P_W and P_{IW}. Viewing Table 19.11, it can be seen that the chained single-stage superlative indices generally approximate their fixed-base two-stage counterparts quite closely. The divergence between the chained single-stage Fisher index P_F and its two-stage counterpart P_{F2S} in period 5 is 1.5556/1.5337 = 1.014 or 1.4 percent. The other divergences are all less than this. Given the large dispersion in period-to-period price movements, these two-stage aggregation errors are not large. However, the important point that emerges from Table 19.11 is that *the use of the chain principle has reduced the spread across all eight single-stage and two-stage superlative indices* compared with their fixed-base counterparts in Table 19.10. The maximum spread for the period 5 chained index values is 1.4 percent, while the maximum spread for the period 5 fixed-base index values is 7.5 percent.

C. Midyear Indices

19.23 The next formulas to illustrate using our artificial data set are the arithmetic- and geometric-type midyear indices defined in Section E of Chapter 17. Recall that these indices are due to Schultz (1998) and Okamoto (2001). Basically, midyear indices are fixed-basket indices, where the basket of quantities being priced is midway between the base period and the current period. If the current period t less the base period 1 is an even integer, then the quantity vector $q^{(t-1)/2}$ is used as the midyear basket. If the current period t less the base period 1 is an odd integer, then the midyear basket is an average of the two midyear quantity vectors, $q^{t/2}$ and $q^{(t/2)+1}$. If the arithmetic average of these two midyear baskets is taken, the sequence of *fixed-base arithmetic-type midyear indices*, $P_{OSA}{}^t$, is obtained, defined by equation (17.50) in Chapter 17. If the geometric average of these two midyear baskets is taken, the sequence of *fixed-base geometric-type midyear indices* is obtained, $P_{OSG}{}^t$, defined by equation (17.51) in Chapter 17.[16] Recall also that going from period 1 to period 2, the period 2 *midyear arithmetic-type index number* $P_{OSA}{}^2$ is equal to $P_{ME}(p^1,p^2,q^1,q^2)$, the Marshall- (1887) Edgeworth (1925) price index for period 2. In addition, the period 2 *midyear geometric-type index number* $P_{OSG}{}^2$ is equal to $P_W(p^1,p^2,q^1,q^2)$, the Walsh (1901) price index for period 2.[17]

19.24 The two sequences of *fixed-base midyear price indices*, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, along with the corresponding *fixed-base Fisher, Törnqvist, and Walsh price indices*, $P_F{}^t$, $P_T{}^t$, and $P_W{}^t$, respectively, are listed in Table 19.12. Note that for odd t, the arithmetic- and geometric-type midyear indices, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, coincide. This is as it should be because when t is odd, both indices are set equal to

[16]Since the quantity vectors have two negative components (and thus, one cannot take square roots of these negative components), the sign conventions need to be changed when evaluating these geometric-type midyear indices; make all quantities positive but change the prices of the import components from positive to negative. Thus, when calculating a geometric-type midyear index where it is necessary to take the geometric average of two midyear quantity vectors, the same sign conventions are used as when calculating Walsh price indices where the same problem occurred.

[17]As usual, when calculating this Walsh price index, switch the signs of the negative import quantities to positive signs and make the corresponding import prices negative.

Table 19.12. Fixed-Base Arithmetic- and Geometric-Type Midyear Indices

Period t	P_{OSA}	P_{OSG}	P_F	P_T	P_W
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1788	1.1814	1.1778	1.1831	1.1814
3	1.4286	1.4286	1.4261	1.4343	1.4327
4	1.4747	1.4783	1.4525	1.4866	1.4820
5	1.5385	1.5385	1.4500	1.5447	1.5193

Table 19.13. Chained Arithmetic- and Geometric-Type Midyear Indices

Period t	P_{OSA}	P_{OSG}	P_F	P_T	P_W
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1788	1.1814	1.1778	1.1831	1.1814
3	1.4286	1.4286	1.4278	1.4307	1.4298
4	1.5230	1.5263	1.4853	1.4893	1.4889
5	1.5388	1.5388	1.5337	1.5400	1.5387

the Schultz midyear index, since there is a single unique midyear basket in this case. The two sequences of midyear indices differ only for even t, since in the even case, there are two midyear baskets and a decision must be made on arithmetic or geometric averaging of these baskets. Note also that the Walsh index for period 2 is equal to the corresponding geometric-type midyear index, since this is true by construction. Finally, note that with the exception of the Fisher fixed-base index, P_F, the fixed-base indices listed in Table 19.12 approximate each other surprisingly well, given the tremendous variability that was built into the underlying data set. The relatively low results for the fixed-base Fisher index may arise from the relatively low results for the fixed-base Paasche index in Table 19.4 and its high spread. When chained-base Laspeyres and Paasche indices were calculated in Table 19.5, the spread was much less, with the Paasche index being pulled up *above* the Laspeyres index to a figure quite close to the Törnqvist and Walsh indices. This seems to suggest that the relatively low Paasche fixed-base index result in Table 19.4, and thus, the fixed-base Fisher index in Table 19.12, was biased downward.

19.25 The chained counterparts to the indices listed in Table 19.12 are now considered. Recall that the chained sequence of arithmetic- and geometric-type midyear indices was defined by equations (17.54) and (17.55), respectively, in Chapter 17. The two sequences of *chained midyear price indices*, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, along with the corresponding *chained Fisher, Törnqvist, and Walsh price indices*, $P_F{}^t$, $P_T{}^t$, and $P_W{}^t$, respectively, are listed in Table 19.13. Note that for odd t, the chained arithmetic- and geometric-type midyear indices, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, coincide. This is as it should be because when t is odd, both indices are set equal to chained Schultz midyear indices. What is striking in looking at Table 19.13 is how close the chained midyear indices are to their chained superlative counterparts for odd periods. For year 5, the maximum spread among the five indices is the spread between the chained Fisher and Törnqvist indices, which was only 1.5400/1.5337 = 1.004 or 0.4 percent. The explanation for this rather remarkable result is that for odd periods, the underlying price and quantity data have fairly smooth trends; and, under these circumstances, the midyear indices would be expected to approximate the superlative Walsh index rather closely, as was

Table 19.14. An Additive Percentage Change Decomposition of the Fisher Index

Period t	P_F–1	$v_{F1}\Delta p_1$	$v_{F2}\Delta p_2$	$v_{F3}\Delta p_3$	$v_{F4}\Delta p_4$	$v_{F5}\Delta p_5$	$v_{F6}\Delta p_6$	$v_{F7}\Delta p_7$	$v_{F8}\Delta p_8$
2	0.1778	0.0791	0.0816	0.1079	–0.0316	0.1678	–0.0101	–0.2389	0.0220
3	0.2122	–0.0648	–0.0716	0.0571	–0.0331	0.1084	–0.0105	0.2037	0.0231
4	0.0403	–0.0541	–0.0363	0.0224	–0.0519	0.0616	–0.0121	0.0744	0.0363
5	0.0326	0.0459	0.0326	0.0198	–0.0396	0.0302	–0.0187	–0.0653	0.0277

indicated in Chapter 17. However, for periods 2 and 4, the underlying data bounce considerably, so the trends in the data switch abruptly. Therefore, under these conditions, it is expected that the midyear indices could deviate from their superlative counterparts. This expectation is borne out by looking at the entries for period 4 in Table 19.12, where the two midyear indices are about 2 to 3 percent higher than their chained superlative counterparts.

19.26 The conclusion that emerges from Tables 19.12 and 19.13 is that midyear indices approximate their superlative counterparts surprisingly well but not perfectly. Given the large amount of variability in the underlying price and quantity data, it appears that the *midyear indices could be used to give very good advanced estimates of superlative indices*, which cannot necessarily be evaluated on a timely basis.

D. Additive Percentage Change Decompositions for the Fisher Index

19.27 The final formulas that are illustrated using the artificial final expenditures data set are the *additive percentage change decompositions* for the Fisher ideal index that were discussed in Section C.8 of Chapter 16. The *chain links* for the Fisher price index will first be decomposed using the Diewert (2002a) decomposition formulas shown in equations (16.41) through (16.43). The results of the decomposition are listed in Table 19.14. Thus, $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t - 1$ to t, and *the decomposition factor* $v_{Fi}\Delta p_i = v_{Fi}\,(p_i^t - p_i^{t-1})$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for i = 1,2,…,8. Viewing Table 19.14, it can be seen that the price index going from period 1 to 2 grew 17.78 percent, and the major contributors to this change were the increases in the price of commodity 1, finally demanded agricultural products (7.91 percentage points); commodity 2, finally demanded energy (8.16 percentage points); commodity 3, finally demanded traditional manufactures (10.79 percentage points); commodity 5, traditional services (16.78 percentage points); and commodity 7, energy imports (–23.89 percentage points). The sum of the last eight entries for period 2 in Table 19.14 is equal to 0.1778, the percentage increase in the Fisher price index going from period 1 to 2. Note that although the price of energy imports *increased* dramatically in period 2, the contribution to the overall price change is *negative* due to the fact that the quantity of energy imports is indexed with a negative sign. Similarly, although the price of high-technology imports *decreased* dramatically in period 2, the contribution to the overall price change is *positive* due to the fact that the quantity of high-technology imports is indexed with a negative sign.[18] Care must be taken, therefore, in interpreting the last two columns of Table 19.14, because there are negative quantities for some components of the aggregate.[19] It can be seen that a big price change in a particular component i combined with a big expenditure share in the

[18]Since the expenditure share of high-technology imports is small, the large decrease in price does not translate into a large change in the overall Fisher price index for final-demand expenditures.

[19]The counterintuitive numbers in the last two columns of Table 19.14 help to explain why the deflator for final-demand expenditures (or the GDP deflator as it is commonly known) is not a satisfactory indicator of inflationary pressures in the economy because a large *increase* in the relative price of imported goods leads to a *decrease* in the index.

Table 19.15. Van Ijzeren's Decomposition of the Fisher Price Index

Period t	$P_F - 1$	$v_{F1}*\Delta p_1$	$v_{F2}*\Delta p_2$	$v_{F3}*\Delta p_3$	$v_{F4}*\Delta p_4$	$v_{F5}*\Delta p_5$	$v_{F6}*\Delta p_6$	$v_{F7}*\Delta p_7$	$v_{F8}*\Delta p_8$
2	0.1778	0.0804	0.0834	0.1094	–0.0317	0.1697	–0.0101	–0.2454	0.0220
3	0.2122	–0.0652	–0.0712	0.0577	–0.0322	0.1091	–0.0105	0.2021	0.0225
4	0.0403	–0.0540	–0.0361	0.0224	–0.0515	0.0615	–0.0121	0.0741	0.0360
5	0.0326	0.0458	0.0326	0.0197	–0.0393	0.0300	–0.0186	–0.0652	0.0275

two periods under consideration will lead to a big decomposition factor, v_{Fi}.

19.28 Our final set of computations illustrates the *additive percentage change decomposition* for the Fisher ideal index that is due to Van Ijzeren (1987, p. 6), mentioned in Section C.8 of Chapter 16.[20] The *price* counterpart to the *additive decomposition* for a *quantity* index, shown in equation (16.35), is:

$$(19.3)\ P_F(p^0,p^1,q^0,q^1) = \frac{\sum_{i=1}^{8} q_{Fi}^* p_i^t}{\sum_{i=1}^{8} q_{Fi}^* p_i^0},$$

where the reference quantities need to be defined somehow. Van Ijzeren (1987, p. 6) showed that the following reference weights provided an *exact additive representation for the Fisher ideal price index*:

$$(19.4)\ q_{Fi}^* \equiv \left(\frac{1}{2}\right) q_i^0 + \left[\left(\frac{1}{2}\right) q_i^t \Big/ Q_F\left(p^0,p^1,q^0,q^1\right)\right];$$

$i = 1,2,\ldots,8,$

where Q_F is the overall Fisher quantity index. Thus, using the Van Ijzeren quantity weights $q_{Fi}*$, the *following Van Ijzeren additive percentage change decomposition for the Fisher price index* is obtained:

$$(19.5)\ P_F(p^0,p^1,q^0,q^1) - 1 = \frac{\sum_{i=1}^{8} q_{Fi}^* p_i^1}{\sum_{i=1}^{8} q_{Fi}^* p_i^0} - 1$$

$$= \sum_{i=1}^{8} v_{Fi}^* \left(p_i^1 - p_i^0\right),$$

where the *Van Ijzeren weight* for commodity i, $v_{Fi}*$, is defined as

$$(19.6)\ v_{Fi}* \equiv \frac{q_{Fi}^*}{\sum_{i=1}^{8} q_{Fi}^* p_i^0};\ i = 1,\ldots,8.$$

The *chain links* for the Fisher price index will again be decomposed using equations (19.2) to (19.4) listed above. The results of the decomposition are listed in Table 19.15. Thus, $P_F - 1$ is the *percentage change in the Fisher ideal chain link* going from period $t - 1$ to t and the *Van Ijzeren decomposition factor* $v_{Fi}*\Delta p_i$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1,2,\ldots,8$.

19.29 Comparing the entries in Tables 19.14 and 19.15, it can be seen that the differences between the Diewert and Van Ijzeren decompositions of the Fisher price index are *very small*. This is somewhat surprising given the very different nature of the two decompositions.[21] As was mentioned in Section C.8 of Chapter 16, the Van Ijzeren decomposition of the chain Fisher *quantity* index is used

[20]It was also independently derived by Dikhanov (1997) and used by Ehemann, Katz, and Moulton (2002).

[21]The terms in Diewert's decomposition can be given economic interpretations, whereas the terms in the other decomposition are more difficult to interpret from an economic perspective. However, Reinsdorf, Diewert, and Ehemann (2002) show that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and the two quantity vectors are equal.

by the Bureau of Economic Analysis in the United States.[22]

E. Industry Price Indices

E.1 Industry data set

19.30 A highly simplified economy consisting of three industrial sectors is considered. The three sectors are the *agricultural sector* (or primary sector), the *manufacturing sector* (or secondary sector), and the *services sector* (or tertiary sector). It is assumed that all transactions go through the services sector. This might appear to be a bit unusual initially. However, recall that transportation services reside in the services sector. Hence, imported goods are delivered as intermediate inputs to the agricultural and manufacturing sectors using service transportation inputs, or they are delivered directly to the final-demand sector—again using service sector transportation, storage, retailing, and wholesaling services. Similarly, the agricultural sector produces unprocessed food that is delivered by the services sector to the manufacturing sector for further processing and packaging. That manufactured food output is then again delivered by the services sector to the final-demand sector.[23]

19.31 Three outputs and intermediate inputs are distinguished for the agricultural sector. The first commodity is agricultural output delivered to the services sector. This is the only output of this sector. There are two intermediate inputs used in the agricultural sector: commodity 2 is deliveries of nonenergy materials (fertilizer, etc.) to agriculture from the services sector, and commodity 3 is deliveries of energy from the services sector to agriculture. These prices and quantities are denoted by p_n^{At} and q_n^{At} for $n = 1, 2, 3$ and $t = 1,...,5$. Note that q_1^{At} is positive (because commodity 1 is an output) and q_2^{At} and q_3^{At} are negative (since commodities 2 and 3 in the agriculture sector are intermediate inputs). The data for the agriculture sector for five periods are listed in Table 19.16 (on the next page).

[22]See Ehemann, Katz, and Moulton (2002).

[23]Our treatment of industrial transactions is an extension of Kohli's (1978) approach to modeling the treatment of imports as flowing first through the production sector of the economy rather than being directly delivered to final demand or other industrial sectors.

19.32 Two outputs and three intermediate inputs are distinguished for the manufacturing sector, five commodities in all.

- Commodity 1 is processed agricultural output delivered to the services sector;
- Commodity 2 is traditional manufactures delivered to the services sector;
- Commodity 3 is deliveries of transported agricultural intermediate inputs delivered from the services sector;
- Commodity 4 is deliveries of energy from services to manufacturing; and
- Commodity 5 is inputs of business services.

These prices and quantities are denoted by p_n^{Mt} and q_n^{Mt} for $n = 1,...,5$ and $t = 1,...,5$. Note that q_1^{Mt} and q_2^{Mt} are positive (because these commodities are outputs) and q_3^{Mt}, q_4^{Mt}, and q_5^{Mt} are negative (since commodities 3, 4, and 5 in the manufacturing sector are intermediate inputs). The data for the manufacturing sector for five periods are listed in Table 19.17 (on the next page).

19.33 Eleven service sector outputs and five service sector intermediate inputs, or 16 commodities in all, are distinguished. The 11 *outputs* are listed as follows:

- Commodity 1 is food deliveries to final demand;
- Commodity 2 is energy deliveries to final demand;
- Commodity 3 is traditional manufacturing deliveries to final demand;
- Commodity 4 is deliveries of high-technology manufactured goods to final demand;
- Commodity 5 is delivery of personal services to final demand;
- Commodity 6 is deliveries of high-technology services to final demand;
- Commodity 7 is deliveries of materials to agriculture;
- Commodity 8 is deliveries of energy to agriculture;
- Commodity 9 is delivery of materials to manufacturing;
- Commodity 10 is deliveries of energy to manufacturing; and
- Commodity 11 is deliveries of business services to manufacturing.

Table 19.16. Price and Quantity Data for the Agriculture Sector

Period	p_1^A	p_2^A	p_3^A	q_1^A	q_2^A	q_3^A
1	1.0	1.0	1.0	20.0	–3.0	–6.0
2	1.5	1.4	2.2	16.0	–2.0	–4.0
3	1.1	1.6	1.1	20.0	–3.0	–5.0
4	0.6	1.4	0.7	23.0	–3.0	–6.0
5	1.0	1.7	1.1	19.0	–3.0	–5.0

Table 19.17. Price and Quantity Data for the Manufacturing Sector

Period	p_1^M	p_2^M	p_3^M	p_4^M	p_5^M	q_1^M	q_2^M	q_3^M	q_4^M	q_5^M
1	1.0	1.0	1.0	1.0	1.0	26.0	36.0	–22.0	–6.0	–8.0
2	1.3	1.2	1.4	2.0	1.2	23.0	35.0	–19.0	–5.0	–9.0
3	1.1	1.4	1.1	1.1	1.6	26.0	34.0	–22.0	–5.0	–10.0
4	0.8	1.5	0.7	0.8	1.8	27.0	35.0	–23.0	–5.0	–11.0
5	1.0	1.6	1.0	1.1	1.9	25.0	36.0	–21.0	–5.0	–11.0

Table 19.18. Price Data for the Services Sector

t	p_1^S	p_2^S	p_3^S	p_4^S	p_5^S	p_6^S	p_7^S	p_8^S	p_9^S	p_{10}^S	p_{11}^S	p_{12}^S	p_{13}^S	p_{14}^S	p_{15}^S	p_{16}^S
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.3	2.0	1.3	0.7	1.4	0.8	1.4	2.2	1.4	2.0	1.2	2.1	0.7	1.5	1.3	1.2
3	1.0	1.0	1.5	0.5	1.7	0.6	1.6	1.1	1.1	1.1	1.6	1.0	0.5	1.1	1.1	1.4
4	0.7	0.5	1.6	0.3	1.9	0.4	1.4	0.7	0.7	0.8	1.8	0.6	0.3	0.6	0.8	1.5
5	1.0	1.0	1.7	0.2	2.0	0.2	1.7	1.1	1.0	1.1	1.9	1.0	0.2	1.0	1.0	1.6

The five *intermediate inputs* into the services sector are listed as follows:

- Commodity 12 is imports of energy into the economy;
- Commodity 13 is imports of high-technology manufactures into the economy;
- Commodity 14 is deliveries of agricultural output to services;
- Commodity 15 is deliveries of processed food from manufacturing to services; and
- Commodity 16 is deliveries of traditional manufacturing to services.

These prices and quantities are denoted by p_n^{St} and q_n^{St} for $n = 1,...,16$ and $t = 1,...,5$. Note that q_1^{St} to q_{11}^{St} are positive (because these commodities are outputs) and q_{12}^{St} to q_{16}^{St} are negative (since these commodities in the services sector are intermediate inputs). The service sector price and quantity data for the 16 commodities are listed in Tables 19.18 and 19.19, respectively.

Table 19.19. Quantity Data for the Services Sector

t	q_1^S	q_2^S	q_3^S	q_4^S	q_5^S	q_6^S	q_7^S	q_8^S	q_9^S	q_{10}^S	q_{11}^S	q_{12}^S	q_{13}^S	q_{14}^S	q_{15}^S	q_{16}^S
1	30	10	40	10	45	5	3	6	22	6	8	–28	–7	–20	–26	–36
2	28	8	39	13	47	6	2	4	19	5	9	–20	–9	–16	–23	–35
3	30	11	38	30	50	8	3	5	22	5	10	–29	–21	–20	–26	–34
4	32	14	39	60	56	13	3	6	23	5	11	–35	–42	–23	–27	–35
5	29	12	40	100	65	25	3	5	21	5	11	–30	–70	–19	–25	–36

Table 19.20. Agriculture Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1455	1.2400	1.1918	1.2000
3	0.9636	0.9750	0.9693	0.9679
4	0.3273	0.3857	0.3553	0.3472
5	0.7545	0.7636	0.7591	0.7478

Table 19.21. Agriculture Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Price Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1455	1.2400	1.1918	1.2000
3	0.9238	0.9803	0.9516	0.9579
4	0.3395	0.3808	0.3596	0.3584
5	0.7104	0.8646	0.7837	0.7758

19.34 The sectoral data above satisfy the conventions of national income accounting in that every value transaction (which is of the form $p_n^{et}q_n^{et}$, where e denotes a sector and n denotes a commodity) in each sector has a *matching transaction* in another sector for each period and each sector. It should be noted that no attempt has been made to balance the supply and demand for each commodity across sectors; put another way, no attempt has been made to produce *balanced input output tables in real terms,* commodity by commodity across sectors. In order to produce such constant dollar input output tables, it is necessary to make assumptions about margins in each sector; a primary commodity is, for example, transformed as it progresses from the agriculture sector to the various downstream sectors. However, these margins are not constant from period to period, which makes it difficult to interpret constant dollar input output tables. Moreover, as goods are transformed through the manufacturing process, they often lose their initial identities, which again makes it difficult to interpret a constant dollar input output table. The approach used in this chapter avoids all of these problems by focusing on transactions between each pair of sectors in the industrial classification. For each pair of sectors, these intersector transactions can be further classified using a commodity classification, which is what has been done in the data set above, but there is no attempt to have a uniform commodity classification across all sectors.

19.35 In the next three subsections, value-added deflators for each of the three industrial sectors are calculated. Only fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist indices will be computed, since these are the ones most likely to be used in practice.

E.2 Value-added deflators for the agriculture sector

19.36 The data listed in Table 19.16 for the agriculture sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.20 (on preceding page).

19.37 From Table 19.20 it can be seen that all four value-added deflators are close to each other for the odd periods; but for the even periods (when agricultural and energy prices *bounce* or are quite different from their longer term normal values), the Paasche and Laspeyres indices differ considerably. However, for all periods, the two superlative indices are quite close to each other.

19.38 The data listed in Table 19.16 for the agriculture sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist price value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.21 (on preceding page).

19.39 It can be seen, comparing Tables 19.20 and 19.21, that the chained indices show considerably *more* variation than their fixed-base counterparts. Here is an example of a sector where chaining does not reduce the spread between the Paasche and Laspeyres value-added deflators. The reason why chaining does not reduce the spread is that agriculture is an example of a sector where price bouncing is much more important than divergent trends in relative prices. The commodities that have divergent prices are high-technology goods and services, and the agriculture sector does not use or produce these commodities. Even though chaining did not reduce the spread between the Paasche and Laspeyres indices for the agriculture sector, it can be seen that the chained Fisher and Törnqvist price indices are still close to each other, although they are somewhat higher than their fixed-base counterparts for the later periods.

E.3 Value-added deflators for the manufacturing sector

19.40 The data listed in Table 19.17 for the manufacturing sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.22 .

19.41 From Table 19.22, it can be seen that the divergence between the fixed-base Laspeyres and Paasche value-added deflators for the manufacturing sector grows steadily from period 3 when it is 3.6 percent to period 5 when it is 4.4 percent. However, the divergence between the two superlative value-added deflators is quite small for all periods.

19.42 The data listed in Table 19.17 for the manufacturing sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.23.

19.43 Comparing Tables 19.22 and 19.23, it can be seen that chaining did *not* reduce the spread between the Paasche and Laspeyres value-added deflators for the manufacturing sector; the spread between these two chained indices in period 5 is 7.0 percent, whereas it was only 4.4 percent for the corresponding fixed-base indices. The explanation for this result is the same as it was for agriculture: (traditional) manufacturing is an example of a sector where the bouncing behavior of energy prices is much more important than divergent trends in relative prices. The commodities that have divergent prices are high-technology goods and services, and the traditional manufacturing sector does not use or produce these commodities. Comparing Tables 19.22 and 19.23, it can also be seen that chaining did *not* reduce the spread between the Fisher and Törnqvist value-added deflators for the manufacturing sector. Again, bouncing energy prices explain this result. However, the chained Fisher and Törnqvist price indices are still quite close to each other.

E.4 Value-added deflators for the services sector

19.44 The data listed in Tables 19.18 and 19.19 for the services sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.24.

Table 19.22. Manufacturing Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	0.9462	0.9800	0.9629	0.9599
3	1.3615	1.3261	1.3437	1.3425
4	1.5462	1.4870	1.5163	1.5265
5	1.5308	1.4667	1.4984	1.4951

Table 19.23. Manufacturing Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	0.9462	0.9800	0.9629	0.9599
3	1.2937	1.3711	1.3318	1.3430
4	1.4591	1.5476	1.5027	1.5217
5	1.4335	1.5345	1.4832	1.5013

Table 19.24. Services Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2368	1.2675	1.2521	1.2561
3	1.5735	1.4768	1.5244	1.5344
4	1.7324	1.4820	1.6023	1.6555
5	1.8162	1.2971	1.5348	1.6547

Table 19.25. Services Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2368	1.2675	1.2521	1.2561
3	1.4763	1.6056	1.5396	1.5324
4	1.6104	1.7331	1.6706	1.6662
5	1.6364	1.7410	1.6879	1.6870

19.45 From Table 19.24, it can be seen that the divergence between the fixed-base Laspeyres and Paasche value-added deflators for the services sector grows steadily from period 2 when it is 2.5 percent to period 5 when it is 40.0 percent. However, the divergence between the two superlative value-added deflators is much smaller but does grow over time to reach 7.8 percent in period 5.

19.46 The data listed in Tables 19.18 and 19.19 for the services sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.25.

19.47 Comparing Tables 19.24 and 19.25, it can be seen that chaining has substantially reduced the spread between the Paasche and Laspeyres value-added deflators for the services sector. In period 5, the divergence between the chained Paasche and Laspeyres is only 6.4 percent, compared with the 40 percent divergence between their fixed-base counterparts. Similarly, chaining has reduced the spread between the two superlative indices; in period 5, the chained Fisher and Törnqvist value-added deflators differ only by 0.05 percent, compared with the 7.8 percent divergence between their fixed-base counterparts. Chaining reduces divergences between the four indices for the services sector because several outputs and intermediate inputs for this sector have strongly divergent trends in their prices. This divergent prices effect overwhelms the effects of bouncing agricultural and energy prices.

F. National Producer Price Indices

F.1 The national output price index

19.48 In order to construct a national output price index, all that is required is to collect the outputs from each of the three industrial sectors and apply normal index number theory to these value flows. There is 1 output in the agriculture sector, 2 outputs in the manufacturing sector, and 11 outputs in the services sector, or 14 outputs in all. The price and quantity data pertaining to these 14 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist output price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.26.

19.49 Since there are divergent trends in the relative prices of outputs in the economy, it should come as no surprise that the Paasche and Laspeyres output price indices grow farther apart over time, reaching a difference of 25.7 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 7.2 percent in period 5. The expectation is that chaining will reduce these divergences.

19.50 The price and quantity data pertaining to the 14 sectoral outputs in the economy are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist output price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.27.

19.51 Comparing Tables 19.26 and 19.27, it can be seen that chaining has indeed reduced the differences between the various national output price

Table 19.26. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Output Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3551	1.3295	1.3422	1.3424
3	1.2753	1.2226	1.2487	1.2575
4	1.1622	1.0305	1.0944	1.1203
5	1.3487	1.0697	1.2011	1.2880

Table 19.27. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Output Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3551	1.3295	1.3422	1.3424
3	1.3033	1.2477	1.2752	1.2751
4	1.1806	1.1119	1.1457	1.1456
5	1.3404	1.2221	1.2799	1.2813

indices. The period 5 difference between the Paasche and Laspeyres price indices is only 9.7 percent, compared with a difference of 25.7 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist price indices is only 0.1 percent, compared with a difference of 7.2 percent for their fixed-base counterparts.

F.2 The national intermediate input price index

19.52 In order to construct a national intermediate input price index, it is necessary only to collect the intermediate inputs from each of the three industrial sectors and apply normal index number theory to these value flows.[24] There are 2 intermediate inputs in the agriculture sector, 3 intermediate inputs in the manufacturing sector, and 5 intermediate inputs in the services sector, or 10 intermediate inputs in all. The price and quantity data pertaining to these 10 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist intermediate input price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.28.

19.53 Since there are divergent trends in the relative prices of intermediate inputs in the economy, it should come as no surprise that the Paasche and Laspeyres intermediate input price indices grow farther apart over time, reaching a difference of 28.6 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 6.7 percent in period 5. The expectation is that chaining will reduce these divergences.

19.54 The price and quantity data pertaining to the 10 sectoral intermediate inputs in the economy

[24]In this section, the negative quantities are changed into positive quantities.

Table 19.28. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Intermediate Input Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4846	1.4310	1.4575	1.4582
3	1.1574	1.1069	1.1319	1.1397
4	0.9179	0.8086	0.8615	0.8817
5	1.1636	0.9049	1.0261	1.0997

Table 19.29. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Intermediate Input Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4846	1.4310	1.4575	1.4582
3	1.2040	1.1168	1.1596	1.1597
4	0.9485	0.8627	0.9046	0.9052
5	1.1759	1.0296	1.1003	1.1030

are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist intermediate input price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.29.

19.55 Comparing Tables 19.28 and 19.29, it can be seen that chaining has reduced the differences between the Paasche and Laspeyres intermediate input price indices. The period 5 difference between the chained Paasche and Laspeyres price indices is 12.4 percent, compared to a difference of 28.6 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist price indices is only 0.2 percent, compared to a difference of 6.7 percent for their fixed-base counterparts.

F.3 The national value-added deflator

19.56 In order to construct a national value-added deflator, all that is needed is to collect all of the outputs and intermediate inputs from each of the three industrial sectors and apply normal index number theory to these value flows. There are 2 intermediate inputs and 1 output in the agriculture sector, 2 outputs and 3 intermediate inputs in the manufacturing sector, and 11 outputs and 5 intermediate inputs in the services sector, or 24 commodities in all. The price and quantity data pertaining to these 24 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.30.

19.57 Since there are divergent trends in the relative prices of outputs and intermediate inputs in the economy, it should come as no surprise that the fixed-base Paasche and Laspeyres value-added deflators grow farther apart over time, reaching a difference of 27.0 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 6.5 percent in period 5. As usual, our expectation is that chaining will reduce these divergences.

Table 19.30. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period *t*	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1778	1.1831
3	1.4571	1.3957	1.4261	1.4343
4	1.5390	1.3708	1.4525	1.4866
5	1.6343	1.2865	1.4500	1.5447

Table 19.31. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period *t*	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1778	1.1831
3	1.3743	1.4834	1.4278	1.4307
4	1.4374	1.5349	1.4853	1.4893
5	1.4963	1.5720	1.5337	1.5400

19.58 The price and quantity data pertaining to the 24 sectoral outputs and intermediate inputs in the economy are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist national value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.31.

19.59 Comparing Tables 19.30 and 19.31, it can be seen that chaining has reduced the differences between the Paasche and Laspeyres deflators. The period 5 difference between the chained Paasche and Laspeyres deflators is 5.1 percent, compared to a difference of 27.0 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist deflators is only 0.4 percent, compared to a difference of 6.5 percent for their fixed-base counterparts.

19.60 At the beginning of this chapter, the Laspeyres, Paasche, Fisher, and Törnqvist *final-demand deflators* were calculated using a fixed base in Tables 19.4 and 19.8 and using the chain principle in Tables 19.5 and 19.9. If these final-demand deflators are compared with their *national value-added deflator* counterparts listed in Tables 19.30 and 19.31, the reader will find that *these two types of deflator give exactly the same answer*. It was assumed that *all transactions are classified on a bilateral sectoral basis*; that is, all transactions between each pair of sectors in the economy are tracked. Under these conditions, if any of the commonly used index number formulas are used, then it can be shown that the final-demand deflator will be *exactly equal* to the national value-added deflator.[25]

[25]The index number formula used must be consistent with either Hicks' (1946, pp. 312–13) or Leontief's (1936) aggregation theorems. That is, if all prices vary in strict proportion across the two periods under consideration, then the price index is equal to this common factor of proportionality (Hicks); or if all quantities vary in strict proportion across the two periods under consideration, then the quantity index that corresponds to the price index is equal to this common factor of proportionality (Leontief). See Allen and Diewert (1981, p. 433) for additional material on these aggregation theorems.

Table 19.32. Two-Stage Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1815	1.1830
3	1.4571	1.3957	1.4259	1.4379
4	1.5390	1.3708	1.4510	1.5018
5	1.6343	1.2865	1.4485	1.5653

Table 19.33. Two-Stage Chained National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1815	1.1830
3	1.3743	1.4834	1.4281	1.4277
4	1.4374	1.5349	1.4853	1.4861
5	1.4963	1.5720	1.5342	1.5368

F.4 National two-stage aggregation

19.61 The national output price index and the national intermediate input price index have been constructed. It is natural to use the two-stage aggregation explained in Section D of Chapter 17 to aggregate these two indices into a national value-added deflator. This result can then be compared with the national value-added deflator that was obtained in the previous section (which was a single-stage aggregation procedure). This comparison is undertaken in this section.

19.62 Using the computations made in the previous section and the theory outlined in Section D of Chapter 17, *two-stage fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively, were constructed. The resulting two-stage national value-added deflators are listed in Table 19.32.

19.63 Comparing the two-stage value-added deflators listed in Table 19.32 with the corresponding single-stage deflators listed in Table 19.30, it can be seen that the Paasche and Laspeyres estimates *are exactly the same*, but there are some small differences between the single-stage and two-stage Fisher and Törnqvist value-added deflators. For period 5, the difference in the two fixed-base Fisher deflators is only 0.1 percent, and the difference in the two fixed-base Törnqvist deflators is 1.3 percent.

19.64 Using the computations made in the previous section and the theory outlined in Section D of Chapter 17, *two-stage chained* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively, were constructed. The resulting two-stage national value-added deflators are listed in Table 19.33.

19.65 Comparing the two-stage chained value-added deflators listed in Table 19.33 with the corresponding chained single-stage deflators listed in Table 19.31, it can be seen that the Paasche and Laspeyres estimates *are exactly the same*, but there are some small differences between the single-stage and two-stage Fisher and Törnqvist value-added deflators. For period 5, the difference in the chained Fisher deflators is only 0.03 percent, and the difference in the two chained Törnqvist deflators is 0.2 percent. Thus, chaining has led to a closer correspondence between the single-stage and two-stage national value-added deflators.