9. PPI Calculation in Practice

A. Introduction

9.1 This chapter provides a general description of the ways PPIs are calculated in practice. The methods used in different countries are not exactly the same, but they have much in common. Both compilers and users of PPIs are interested in knowing how most statistical offices actually set about calculating their PPIs.

9.2 As a result of the greater insights into the properties and behavior of price indices that have been achieved in recent years, it is now recognized that some traditional methods may not necessarily be optimal from a conceptual and theoretical viewpoint. Concerns have also been voiced in a number of countries about possible biases that may be affecting PPIs. These issues and concerns need to be addressed in the *Manual*. Of course, the methods used to compile PPIs are inevitably constrained by the resources available, not merely for collecting and processing prices but also the revenue data needed for weighting purposes. In some countries, the methods used may be severely constrained by a lack of resources.

9.3 The calculation of PPIs usually proceeds in two stages. First, price indices are estimated for the *elementary aggregates,* and then these *elementary price indices* are averaged to obtain *higher-level indices* using the relative values of the revenue weights for elementary aggregates as weights. Section B starts by explaining how the elementary aggregates are constructed and which economic and statistical criteria need to be taken into consideration in defining the aggregates. The index number formulas most commonly used to calculate the elementary indices are then presented and their properties and behavior illustrated using numerical examples. The pros and cons of the various formulas are considered together with some alternative formulas that might be used. The problems created by disappearing and new products are also explained, as are the different ways of imputing for missing prices.

9.4 Section C of the chapter is concerned with the calculation of higher-level indices. The focus is on the ongoing production of a monthly price index in which the elementary price indices are averaged, or aggregated, to obtain higher-level indices. Price updating of weights, chain linking, and reweighting are discussed, with examples provided. The problems associated with introduction of new elementary price indices and new higher-level indices into the PPI are also covered. The section explains how it is possible to decompose the change in the overall index into its component parts. Finally, the possibility of using some alternative and rather more complex index formulas is considered.

9.5 Section D concludes with data editing procedures, since these are an integral part of the process of compiling PPIs. It is essential to ensure that the right data are entered into the various formulas. There may be errors resulting from the inclusion of incorrect data or from entering correct data inappropriately and errors resulting from the exclusion of correct data that are mistakenly believed to be wrong. The section examines data editing procedures that try to minimize both types of errors.

B. Calculation of Price Indices for Elementary Aggregates

9.6 PPIs typically are calculated in two steps. In the first step, the elementary price indices for the elementary aggregates are calculated. In the second step, higher-level indices are calculated by averaging the elementary price indices. The elementary aggregates and their price indices are the basic building blocks of the PPI.

B.1 Composition of elementary aggregates

9.7 Elementary aggregates are constructed by grouping individual goods and individual services into relatively homogeneous products and transactions. They may be formed for products in various regions of the country or for the country as a whole. Likewise, elementary aggregates may be formed for different types of establishments or for various subgroups of products. The actual formation of elementary aggregates thus depends on the circumstances and the availability of information, and they may therefore be defined differently in different countries. However, some key points should be observed:

- Elementary aggregates should consist of groups of goods or services that are as similar as possible, and preferably fairly homogeneous.
- They should also consist of products that may be expected to have similar price movements. The objective should be to try to minimize the dispersion of price movements within the aggregate.
- The elementary aggregates should be appropriate to serve as strata for sampling purposes in light of the sampling regime planned for the data collection.

9.8 Each elementary aggregate, whether relating to the whole country, an individual region, or a group of establishments, will typically contain a very large number of individual goods, services, or products. In practice, only a small number can be selected for pricing. When selecting the products, the following considerations need to be taken into account:

(i) The transactions selected should be ones with price movements believed to be representative of all the products within the elementary aggregate.

(ii) The number of transactions within each elementary aggregate for which prices are collected should be large enough for the estimated price index to be statistically reliable. The minimum number required will vary between elementary aggregates, depending on the nature of the products and their price behavior.

(iii) The object is to try to track the price of the same product over time for as long as possible, or for as long as the product continues to be representative. The products selected should therefore be ones that are expected to remain on the market for some time so that like can be compared with like.

B.1.1 Aggregation structure

9.9 The aggregation structure for a PPI is discussed in Chapter 4, Section C.4, and in Figure 4.1. Using a classification of business products such as PRODCOM, CPC, or CPA, the entire set of produced goods and services covered by the overall PPI can be divided into broad *sections, divisions,* and *groups,* then further refined into smaller *classes* and *subclasses.* Each elementary aggregate is assigned a product code. This enables statistical offices to aggregate elementary indices at the lowest level to higher product classes, groups, divisions, etc. In addition, each elementary aggregate is assigned an industry (activity) code from a standard industrial classification such as ISIC or NACE and thus can be aggregated by industry from the four-digit to the three-digit and higher levels. The overall PPI should be the same whether aggregated by industry or product as long as each elementary aggregate has the same weight in the industry and product aggregations.

9.10 The methods used to calculate the elementary indices from the individual price observations are discussed below. Working from the elementary price indices, all indices above the elementary aggregate level are higher-level indices that can be calculated from the elementary price indices using the elementary revenue aggregates as weights. The aggregation structure is consistent so that the weight at each level above the elementary aggregate is always equal to the sum of its components. The price index at each higher level of aggregation can be calculated on the basis of the weights and price indices for its components—that is, the lower-level or elementary indices. The individual elementary price indices are not necessarily sufficiently reliable to be published separately, but they remain the basic building blocks of all higher-level indices.

B.1.2 Weights within elementary aggregates

9.11 In many cases, the explicit revenue weights are not available to calculate the price indices for elementary aggregates. Whenever possible, however, weights should be used that reflect the relative importance of the sampled products, even if the weights are only approximate. Often, the elementary aggregate is simply the lowest level at which reliable weighting information is available. In this case, the elementary index has to be calculated as an unweighted average of the prices of which it consists. However, even in this case it should be noted that when the products are selected with probabilities proportional to the size of some relevant variable such as sales, for example, weights are implicitly introduced by the sample selection procedure. In addition, statistical offices can work with establishment respondents to obtain estimated weight data, as discussed in Chapter 4.

9.12 For certain elementary aggregates, information about output of particular products and market shares from trade and industry sources may be used as explicit weights within an elementary aggregate. Weights within elementary aggregates may be updated independently and possibly more often than the elementary aggregates themselves (which serve as weights for the higher-level indices).

9.13 For example, assume that the number of suppliers of a certain product such as car fuel supplied to garages is limited. The market shares of the suppliers may be known from business survey statistics and can be used as weights in the calculation of an elementary aggregate price index for car fuel. Alternatively, prices for water may be collected from a number of local water supply services where the population in each local region is known. The relative size of the population in each region may then be used as a proxy for the relative revenues to weight the price in each region to obtain the elementary aggregate price index for water.

9.14 A special situation occurs in the case of tariff prices. A tariff is a list of prices for the provision of a particular kind of good or service under different terms and conditions. One example is electricity, for which one price is charged during daytime and a lower price is charged at night. Similarly, a telephone company may charge a lower price for a call on the weekend than a weekday. Another example may be bus tickets sold at one price to regular passengers and at lower prices to children or seniors. In such cases, it is appropriate to assign weights to the different tariffs or prices to calculate the price index for the elementary aggregate.

9.15 The increasing use of electronic recording for transactions in many countries, in which both prices and quantities are maintained as products are sold, means that valuable new sources of information may become increasingly available to statistical offices. This could lead to significant changes in the ways in which price data are collected and processed for PPI purposes. The treatment of electronic data transfer is examined in Chapters 6, 7, and 21.

B.2 Compilation of elementary price indices

9.16 An elementary price index is the price index for an elementary aggregate. Various methods and formulas may be used to calculate elementary price indices. This section provides a summary of pros and cons that statistical offices must evaluate when choosing a formula at the elementary level; Chapter 20 provides a more detailed discussion.

9.17 The methods statistical offices most commonly use are illustrated by means of a numerical example in Table 9.1. In the example, assume that prices are collected for four representative products within an elementary aggregate. The quality of each product remains unchanged over time so that the month-to-month changes compare like with like. No weights can be applied. Assume initially that prices are collected for all four products in every month covered so that there is a complete set of prices. There are no disappearing products, no missing prices, and no replacement products. These are quite strong assumptions because many of the problems encountered in practice are attributable to breaks in the continuity of the price series for the individual transactions for one reason or another. The treatment of disappearing and replacement products is taken up later.

9.18 Three widely used formulas that have been, or still are, in use by statistical offices to calculate elementary price indices are illustrated in Table 9.1. It should be noted, however, that these are not the only possibilities, and some alternative formulas are considered later.

- The first is the *Carli* index for $i = 1,\ldots, n$ products. It is defined as the simple, or unweighted, arithmetic mean of the price relatives, or price ratios, for the two periods, 0 and t, to be compared.

Table 9.1. Calculation of Price Indices for an Elementary Aggregate[1]

	January	February	March	April	May	June	July
				Prices			
Product *A*	6.00	6.00	7.00	6.00	6.00	6.00	6.60
Product *B*	7.00	7.00	6.00	7.00	7.00	7.20	7.70
Product *C*	2.00	3.00	4.00	5.00	2.00	3.00	2.20
Product *D*	5.00	5.00	5.00	4.00	5.00	5.00	5.50
Arithmetic mean prices	5.00	5.25	5.50	5.50	5.00	5.30	5.50
Geometric mean prices	4.53	5.01	5.38	5.38	4.53	5.05	4.98
			Month-to-month price relatives				
Product *A*	1.00	1.00	1.17	0.86	1.00	1.00	1.10
Product *B*	1.00	1.00	0.86	1.17	1.00	1.03	1.07
Product *C*	1.00	1.50	1.33	1.25	0.40	1.50	0.73
Product *D*	1.00	1.00	1.00	0.80	1.25	1.00	1.10
		Current to reference month (January) price relatives					
Product *A*	1.00	1.00	1.17	1.00	1.00	1.00	1.10
Product *B*	1.00	1.00	0.86	1.00	1.00	1.03	1.10
Product *C*	1.00	1.50	2.00	2.50	1.00	1.50	1.10
Product *D*	1.00	1.00	1.00	0.80	1.00	1.00	1.10
Carli index—Arithmetic mean of price relatives							
Month-to-month index	100.00	112.50	108.93	101.85	91.25	113.21	100.07
Chained month-to-month index	100.00	112.50	122.54	124.81	113.89	128.93	129.02
Direct index on January	100.00	112.50	125.60	132.50	100.00	113.21	110.00
Dutot index—Ratio of arithmetic mean prices							
Month-to-month index	100.00	105.00	104.76	100.00	90.91	106.00	103.77
Chained month-to-month index	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Direct index on January	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Jevons index—Geometric mean of price relatives or ratio of geometric mean prices							
Month-to-month index	100.00	110.67	107.46	100.00	84.09	111.45	98.70
Chained month-to-month index	100.00	110.67	118.92	118.92	100.00	111.45	110.00
Direct index on January	100.00	110.67	118.92	118.92	100.00	111.45	110.00

[1]All price indices have been calculated using unrounded figures.

$$(9.1)\quad P_C^{0:t} = \frac{1}{n}\sum\left(\frac{p_i^t}{p_i^0}\right)$$

- The second is the *Dutot* index, which is defined as the ratio of the unweighted arithmetic mean prices.

$$(9.2)\quad P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^t}{\frac{1}{n}\sum p_i^0}$$

- The third is the *Jevons* index, which is defined as the unweighted geometric mean of the price relative or relatives, which is identical to the ratio of the unweighted geometric mean prices.

$$(9.3)\quad P_J^{0:t} = \prod\left(\frac{p_i^t}{p_i^0}\right)^{1/n} = \frac{\prod\left(p_i^t\right)^{1/n}}{\prod\left(p_i^0\right)^{1/n}}$$

The properties of the three indices are examined and explained in some detail in Chapter 20. Here, the purpose is to illustrate how they perform in practice, to compare the results obtained by using the different formulas, and to summarize their strengths and weaknesses.

9.19 Each *month-to-month* index shows the change in the index from one month to the next. The *chained month-to-month* index links together these month-to-month changes by successive multiplication. The *direct* index compares the prices in each successive month directly with those of the reference month, January. By simple inspection of the various indices, it is clear that the choice of formula and method can make a substantial difference in the results obtained. Some results are striking—in particular, the large difference between the chained Carli index for July and each of the direct indices for July, including the direct Carli.

9.20 The properties and behavior of the different indices are summarized in the following paragraphs and explained in more detail in Chapter 20. First, the differences between the results obtained by using the different formulas tend to increase as the variance of the price relatives, or ratios, increases. The greater the dispersion of the price movements, the more critical the choice of index formula and method becomes. If the elementary aggregates are defined so that the price movements within the aggregate are minimized, the results obtained become less sensitive to the choice of formula and method.

9.21 Certain features displayed by the data in Table 9.1 are systematic and predictable and follow from the mathematical properties of the indices. For example, it is well known that an arithmetic mean is always greater than, or equal to, the corresponding geometric mean—the equality holding only in the trivial case in which the numbers being averaged are all the same. The direct Carli indices are therefore all greater than the Jevons indices, except in May and July when the four price relatives based on January are all equal. In general, the Dutot index may be greater or less than the Jevons index but tends to be less than the Carli index.

9.22 One general property of geometric means should be noted when using the Jevons formula. If any one observation out of a set of observations is zero, its geometric mean is zero, whatever the values of the other observations. The Jevons index is sensitive to extreme falls in prices, and it may be necessary to impose upper and lower bounds on the individual price relatives of, say, 10 and 0.1, respectively, when using the Jevons. Of course, extreme observations are often the results of errors of one kind or another, and so extreme price movements should be carefully checked in any case.

9.23 Another important property of the indices illustrated in Table 9.1 is that the Dutot and the Jevons indices are transitive, whereas the Carli index is not. Transitivity means that the chained monthly indices are identical to the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices that link together the month-to-month-indices. The intransitivity of the Carli index is illustrated dramatically in Table 9.1, in which each of the four individual prices in May returns to the same level as it was in January, but the chained Carli index registers an increase of almost 14 percent over January. Similarly, in July, although each individual price is exactly 10 percent higher than in January, the chained Carli index registers an increase of 29 percent. These results would be regarded as perverse and unacceptable in the case of a direct index, but even in the case of the chained index, the results seems so intuitively unreasonable as to undermine the credibility of the chained Carli index. The price changes between March and April illustrate the effects of "price bouncing," in which the same four prices are observed in both periods, but they are switched between the different products. The monthly Carli index from March to April increases, whereas both the Dutot and the Jevons indices are unchanged.

9.24 The message emerging from this brief illustration of the behavior of just three possible formulas is that different index numbers and methods can deliver very different results. Index compilers have to familiarize themselves with the interrelationships between the various formulas at their disposal for the calculation of the elementary price indices so that they are aware of the implications of choosing one formula rather than another. However, knowledge of these interrelationships is not sufficient to determine which formula should be used, even though it makes it possible to make a more informed and reasoned choice. It is necessary to appeal to additional criteria to settle the choice of formula. Two main approaches may be used, the axiomatic and the economic approaches.

B.2.1 Axiomatic approach to elementary price indices

9.25 As explained in Chapters 16 and 20, one way to decide on an appropriate index formula is to require it to satisfy certain specified axioms or tests. The tests throw light on the properties possessed by different kinds of indices, some of which may not be obvious. Four basic tests illustrate the axiomatic approach.

Proportionality Test: If all prices are λ times the prices in the price reference period (January in the example), the index should equal λ. The data for July, when every price is 10 percent higher than in January, show that all three direct indices satisfy this test. A special case of this test is the *identity test,* which requires that if the price of every product is the same as in the reference period, the index should be equal to unity (as in May in the example).

Changes in the Units of Measurement Test (or Commensurability Test): The price index should not change if the quantity units in which the products are measured are changed—for example, if the prices are expressed per liter rather than per pint. The Dutot index fails this test, as explained below, but the Carli and Jevons indices satisfy the test.

Time Reversal Test: If all the data for the two periods are interchanged, then the resulting price index should equal the reciprocal of the original price index. The Carli index fails this test, but the Dutot and the Jevons both satisfy the test. The failure of the Carli index to satisfy the test is not immediately obvious from the example but can easily be verified by interchanging the prices in January and April, for example, in which case the backward Carli index for January based on April is equal to 91.3, whereas the reciprocal of the forward Carli index is 1/132.5, or 75.5.

Transitivity Test: The chained index between two periods should equal the direct index between the same two periods. The example shows that the Jevons and the Dutot indices both satisfy this test, whereas the Carli index does not. For example, although the prices in May have returned to the same levels as in January, the chained Carli index registers 113.9. This illustrates the fact that the Carli index may have a significant built-in upward bias.

9.26 Many other axioms or tests can be devised, as presented in Chapter 16, but the above (summarized in Table 9.2) are sufficient to illustrate the approach and also to throw light on some important features of the elementary indices under consideration here.

9.27 The sets of products covered by elementary aggregates are meant to be as homogeneous as possible. If they are not fairly homogeneous, the failure of the Dutot index to satisfy the units of measurement, or commensurability, test can be a serious disadvantage. Although defined as the ratio of the unweighted arithmetic average prices, the Dutot index may also be interpreted as a weighted arithmetic average of the price relatives in which each ratio is weighted by its price in the base period.[1] However, if the products are not homogeneous, the relative prices of the different products may depend quite arbitrarily on the quantity units in which they are measured.

9.28 Consider, for example, salt and pepper, which are found within the same CPC subclass. Suppose the unit of measurement for pepper is changed from grams to ounces, while leaving the units in which salt is measured (say, kilos) unchanged. Because an ounce of pepper is equal to 28.35 grams, the "price" of pepper increases by more than 28 times, which effectively increases the

[1] This can be seen by rewriting equation (9.1) as

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^0\left(p_i^t/p_i^0\right)}{\frac{1}{n}\sum p_i^0}.$$

Table 9.2. Properties of Main Elementary Aggregate Index Formulas

	Formula		
Formula properties	Carli—Arithmetic mean of price relatives	Dutot—Relative of arithmetic mean prices	Jevons—Geometric mean of price relatives
Proportionality	yes	yes	yes
Change-of-units of measurement	yes	no	yes
Time reversal	no	yes	yes
Transitivity	no	yes	yes
Allows for substitution	no	no	yes

weight given to pepper in the Dutot index by more than 28 times. The price of pepper relative to salt is inherently arbitrary, depending entirely on the choice of units in which to measure the two goods. In general, when there are different kinds of products within the elementary aggregate, the Dutot index is unacceptable conceptually.

9.29 The Dutot index is acceptable only when the set of products covered is homogeneous, or at least nearly homogeneous. For example, the Dutot index may be acceptable for a set of apple prices, even though the apples may be of different varieties, but not for the prices of different kinds of fruits, such as apples, pineapples, and bananas, some of which may be much more expensive per item or per kilo than others. Even when the products are fairly homogeneous and measured in the same units, the Dutot index's implicit weights may still not be satisfactory. More weight is given to the price changes for the more expensive products, but they may well account for only small shares of the total revenue within the aggregate, in practice. Purchasers are unlikely to buy products at high prices if the same products are available at lower prices.

9.30 It may be concluded that from an axiomatic viewpoint, both the Carli and the Dutot indices, although they have been and still are widely used by statistical offices, have serious disadvantages. The Carli index fails the time reversal and transitivity tests. In principle, it should not matter whether we choose to measure price changes forward or backward in time. We would expect the same answer, but this is not the case for the Carli index. Chained Carli indices may be subject to a significant upward bias. The Dutot index is meaningful for a set of homogeneous products but becomes increasingly arbitrary as the set of products becomes more diverse. On the other hand, the Jevons index satisfies all the tests listed above and also emerges as the preferred index when the set of test is enlarged, as shown in Chapter 20. From an axiomatic point of view, the Jevons index is clearly the index with the best properties, even though it may not have been used much until recently. The Jevons index also allows for some substitution effects consistent with a unitary elasticity of substitution. There seems to be an increasing tendency for statistical offices to switch from using Carli or Dutot indices to Jevons.

B.2.2 Economic approach to elementary price indices

9.31 The objective of the economic approach is to estimate for the elementary aggregates an "ideal" (or "true") economic index—that is, one consistent with the economic theory of revenue-maximizing producers explained in Section F of Chapter 20. The products for which respondents provide prices are treated as a basket of goods and services produced by establishments to provide revenue, and producers are assumed to arrive at their decision about the quantities of outputs to produce on the basis of revenue-maximizing behavior. As explained in Chapters 1, 15, and 17, an ideal theoretical economic index measures the ratio of revenues between two periods that an establishment can attain when faced with fixed technologies and inputs. Changes in the index arise only from changes in prices. The technology is assumed to be held fixed, although the revenue-maximizing producer can make substitutions between the products produced in response to changes in their relative prices. In the absence of information about quantities or revenues within an elementary aggregate, an ideal index can be estimated only when certain special conditions are assumed to prevail.

9.32 There are two special cases of some interest. The first case is when producers continue to produce the same *relative* quantities whatever the relative prices. Producers prefer not to make any substitutions in response to changes in relative prices. The cross-elasticities of supply are zero. The technology by which inputs are translated into outputs in economic theory is described by a production function, and a production function with such a restrictive reaction to relative price changes is described in the economics literature as Leontief. With such a production function, a Laspeyres index would provide an exact measure of the ideal economic index. In this case, the Carli index calculated for a random sample of products would provide an estimate of the ideal economic index that the products are selected with probabilities proportional to the population revenue shares.[2]

9.33 The second case occurs when producers are assumed to vary the quantities they produce in inverse proportion to the changes in relative prices. The cross-elasticities of supply between the different products they produce are all unity, the revenue shares remaining the same in both periods. Such an underlying production function is described as Cobb-Douglas. With this production function, the *geometric Laspeyres*[3] index would provide an exact measure of the ideal index. In this case, the Jevons index calculated for a random sample of products would provide an unbiased estimate of the ideal economic index provided that the products are selected with probabilities proportional to the population expenditure shares.

9.34 From the economic approach, the choice between the sample Jevons index and the sample Carli index rests on which is likely to approximate more closely the underlying ideal economic index—in other words, whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the cross-elasticities could take on any value ranging up to $+\infty$ for an elementary aggregate consisting of a set of strictly homogeneous products—that is, perfect substitutes.[4] It may be conjectured that for demand-led industries where producers produce less of a commodity whose relative price has increased to meet the reduced quantity demanded, the average cross-elasticity is likely to be closer to unity. Thus, the Jevons index is likely to provide a closer approximation to the ideal economic index than the Carli index. In this case, the Carli index must be viewed as having an upward bias. However, there are some establishments in industries, including utilities, in which supply is relatively unresponsive to demand changes, and the Carli index would be more appropriate, given that sampling is with probability proportional to base-period revenue shares. And, yet again, there would be establishments in industries in which quantities produced increase as prices increase, and, with probability sampling proportional to base-period revenues, neither the Carli nor the Jevons index would be appropriate from the economic approach.

9.35 The insight provided by the economic approach is that the Jevons and Carli indices can be justified from the economic approach depending on whether a significant amount of substitution is more likely than no substitution, especially as elementary aggregates should be deliberately constructed to group together similar products that are close substitutes for each other.

9.36 The Jevons index does not imply, or assume, that revenue shares remain constant. Obviously, the Jevons can be calculated whether changes do or do not occur in the revenue shares in practice. What the economic approach shows is that *if* the revenue shares remain constant (or roughly constant), *then* the Jevons index can be expected to provide a good estimate of the underlying ideal

[2]It might appear that if the products were selected with probabilities proportional to the population quantity shares, the sample Dutot would provide an estimate of the population Laspeyres. However, if the basket for the Laspeyres index contains different kinds of products whose quantities are not additive, the quantity shares, and hence the probabilities, are undefined.

[3]The geometric Laspeyres is a weighted geometric average of the price relatives, using the revenue shares in the earlier period as weights. (The revenue shares in the second period would be the same in the particular case under consideration.)

[4]It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price "index" is given by the ratio of the unit values in the two periods, as explained below.

economic index. Similarly, *if* the relative quantities remain constant, *then* the Carli index can be expected to provide a good estimate, but the Carli index does not actually imply that quantities remain fixed. Reference should be made to Section F of Chapter 20 for a more rigorous statement of the economic approach.

9.37 It may be concluded that on the economic approach, as well as the axiomatic approach, the Jevons emerges as the preferred index in general, although there may be cases in which little or no substitution takes place within the elementary aggregate, and the Carli might be preferred. The index compiler must make a judgment on the basis of the nature of the products actually included in the elementary aggregate.

9.38 Before leaving this topic, it should be noted that it has thrown light on some of the sampling properties of the elementary indices. If the products in the sample are selected with probabilities proportional to expenditures in the price reference period,

- The sample (unweighted) Carli index provides an unbiased estimate of the population Laspeyres, and
- The sample (unweighted) Jevons index provides an unbiased estimate of the population geometric Laspeyres.

9.39 These results hold, regardless of what the underlying economic index may be.

B.3 Chained versus direct indices for elementary aggregates

9.40 In a direct elementary index, the prices of the current period are compared directly with those of the price reference period—in a chained index, prices in each period are compared with those in the previous period, the resulting short-term indices being chained together to obtain the long-term index, as illustrated in Table 9.1.

9.41 Provided that prices are recorded for the same set of products in every period, as in Table 9.1, any index formula defined as the ratio of the average prices will be transitive—that is, the same result is obtained whether the index is calculated as a direct index or as a chained index. In a chained index, successive numerators and denominators will cancel out, leaving only the average price in the last period divided by the average price in the price reference period, which is the same as the direct index. Both the Dutot and the Jevons indices are therefore transitive. As already noted, however, a chained Carli index is not transitive and should not be used because of its upward bias. Nevertheless, the direct Carli remains an option.

9.42 Although the chained and direct versions of the Dutot and Jevons indices are identical when there are no breaks in the series for the individual products, they offer different ways of dealing with new and disappearing products, missing prices, and quality adjustments. In practice, products continually have to be dropped from the index and new ones included, in which case the direct and the chained indices may differ if the imputations for missing prices are made differently.

9.43 When a replacement product has to be included in a direct index, it often will be necessary to estimate the price of the new product in the price reference period, which may be some time in the past. The same happens if, as a result of an update of the sample, new products have to be linked into the index. Assuming that no information exists on the price of the replacement product in the price reference period, it will be necessary to estimate it using price relatives calculated for the products that remain in the elementary aggregate, a subset of these products, or some other indicator. However, the direct approach should be used only for a limited period. Otherwise, most of the reference prices would end up being imputed, which would be an undesirable outcome. This effectively rules out the use of the Carli index over a long period, because the Carli index can be used only in its direct form anyway, being unacceptable when chained. This implies that, in practice, the direct Carli index may be used only if the overall index is chained annually, or at intervals of two or three years.

9.44 In a chained index, if a product becomes permanently missing, a replacement product can be linked into the index as part of the ongoing index calculation by including the product in the monthly index as soon as prices for two successive months are obtained. Similarly, if the sample is updated and new products have to be linked into the index, this will require successive old and new prices for the present and the preceding month. However, for a chained index, the missing observation will affect the index for two months, since the missing observation is part of two links in the chain. This is not the case for a direct index, where a single, nonestimated missing observation will affect only the index in the current period. For example, when comparing periods 0 and 3, a missing price of a product in period 2 means that the chained index excludes the product for the last link of the index in periods 2 and 3, while the direct index includes it in period 3 (since a direct index will be based on products with prices available in periods 0 and 3). However, in general, the use of a chained index can make the estimation of missing prices and the introduction of replacements easier from a computational point of view, whereas it may be inferred that a direct index will limit the usefulness of overlap methods for dealing with missing observations. This is discussed further in Section B.5.

9.45 The direct and the chained approaches also produce different by-products that may be used for monitoring price data. For each elementary aggregate, a chained index approach gives the latest monthly price change, which can be useful for both editing data and imputing missing prices. By the same token, however, a direct index derives average price levels for each elementary aggregate in each period, and this information may be a useful by-product. However, the availability of cheap computing power and spreadsheets allows such by-products to be calculated whether a direct or a chained approach is applied, so that the choice of formula should not be dictated by considerations regarding by-products.

B.4 Consistency in aggregation

9.46 Consistency in aggregation means that if an index is calculated stepwise by aggregating lower-level indices to obtain indices at progressively higher levels of aggregation, the same overall result should be obtained as if the calculation had been made in one step. For presentational purposes, this is an advantage. If the elementary aggregates are calculated using one formula, and the elementary aggregates are averaged to obtain the higher-level indices using another formula, the resulting PPI is not consistent in aggregation. However, it may be argued that consistency in aggregation is not necessarily an important or even appropriate criterion. Also it may be unachievable, particularly when the amount of information available on quantities and revenues is not the same at the different levels of aggregation. In addition, there may be different degrees of substitution within elementary aggregates compared with the degree of substitution between products in different elementary aggregates.

9.47 As noted in Section B.2.2 above, the Carli index would be consistent in aggregation with the Laspeyres index if the products were to be selected with probabilities proportional to revenues in the price reference period. However, this is typically not the case. The Dutot and the Jevons indices are also not consistent in aggregation with a higher-level Laspeyres. However, as explained below, the PPIs actually calculated by statistical offices are usually not true Laspeyres indices anyway, even though they may be based on fixed baskets of goods and services. As also noted earlier, if the higher-level index were to be defined as a geometric Laspeyres index, consistency in aggregation could be achieved by using the Jevons index for the elementary indices at the lower level, provided that the individual products are sampled with probabilities proportional to revenues. Although unfamiliar, a geometric Laspeyres index has desirable properties from an economic point of view and is considered again later.

B.5 Missing price observations

9.48 The price of a product may not be collected in a particular period, either because the product is missing temporarily or because it has permanently disappeared. The two classes of missing prices require different treatments. Temporary unavailability may occur for seasonal products (particularly for fruit, vegetables, and clothing) because of supply shortages or possibly because of some collection difficulty (for example, an establishment was closed or a respondent was on vacation). The treatment of seasonal products raises a number of particular problems. These are dealt with in Chapter 22 and will not be discussed here.

B.5.1 Treatment of temporarily missing prices

9.49 In the case of temporarily missing observations for products, one of four actions may be taken:

- Omit the product for which the price is missing so that a matched sample is maintained (like is compared with like), even though the sample is depleted.

- Carry forward the last observed price.
- Impute the missing price by the average price change for the prices that are available in the elementary aggregate.
- Impute the missing price by the price change for a particular comparable product from a similar establishment.

Omitting an observation from the calculation of an elementary index is equivalent to assuming that the price would have moved in the same way as the average of the prices of the products that remain included in the index. Omitting an observation changes the implicit weights attached to the other prices in the elementary aggregate.

9.50 Carrying forward the last observed price should be avoided wherever possible and is acceptable only for a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index toward zero change. In addition, there is likely to be a compensating step-change in the index when the price of the missing product is recorded again, which will be wrongly missed by a chained index but will be included in a direct index to return the index to its proper value. The adverse effect on the index will be increasingly severe if the product remains unpriced for some length of time. In general, carryforward is not an acceptable procedure or solution to the problem unless it is certain the price has not changed.

9.51 Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates when the prices can be expected to move in the same direction. The imputation can be made using all the remaining prices in the elementary aggregate. As already noted, this is numerically equivalent to omitting the product for the immediate period, but it is useful to make the imputation so that if the price becomes available again in a later period, the sample size is not reduced in that period. In some cases, depending on the homogeneity of the elementary aggregate, it may be preferable to use only a subset of products from the elementary aggregate to estimate the missing price. In some instances, this may even be a single comparable product from a similar type of establishment whose price change can be expected to be similar to the missing one.

9.52 Table 9.3 illustrates the calculation of the price index for an elementary aggregate consisting of three products, where one of the prices is missing in March. The upper part of Table 9.3 shows the indices where the missing price has been omitted from the calculation. The direct indices are therefore calculated on the basis of *A*, *B*, and *C* for all months except March, where it is calculated on basis of *B* and *C* only. The chained indices are calculated on the basis of all three prices from January to February and from April to May. From February to March and from March to April, the monthly indices are calculated on the basis of *B* and *C* only.

9.53 For both the Dutot and the Jevons, the direct and chain indices now differ from March onward. The first link in the chained index (January to February) is the same as the direct index, so that the two indices are identical numerically. The direct index for March ignores the price decrease of product *A* between January and February, while this is taken into account in the chained index. As a result, the direct index is higher than the chained index for March. On the other hand, in April and May, where all prices again are available, the direct index catches the price development, whereas the chained index fails to track the development in the prices.

9.54 In the lower half of Table 9.3, the missing price for product *A* in March is imputed by the average price change of the remaining products from February to March. While the index may be calculated as a direct index comparing the prices of the present period with the reference period prices, the imputation of missing prices should be made on basis of the average price change from the preceding to the present period, as shown in the table. Imputation on the basis of the average price change from the price reference period to the present period should not be used, since it ignores the information about the price change of the missing product that has already been included in the index. The treatment of imputations is discussed in more detail in Chapter 7.

Table 9.3. Imputation of Temporarily Missing Prices

	January	February	March	April	May
			Prices		
Product *A*	6.00	5.00		7.00	6.60
Product *B*	7.00	8.00	9.00	8.00	7.70
Product *C*	2.00	3.00	4.00	3.00	2.20
Omit missing product from the index calculation					
Carli index—Arithmetic mean of price relatives					
Direct index	100.00	115.87	164.29	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.67	118.18	84.62	91.67
Chained month-to-month index	100.00	106.67	126.06	106.67	97.78
Direct index	100.00	106.67	144.44	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Month-to-month index	100.00	112.62	122.47	81.65	87.31
Chained month-to-month index	100.00	112.62	137.94	112.62	98.33
Direct index	100.00	112.62	160.36	125.99	110.00
Imputation					
Carli index—Arithmetic mean of price relatives					
Impute price for A *in March as 5 (9/8 + 4/3)/2 = 6.15*					
Direct index	100.00	115.87	143.67	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Impute price for A *in March as 5 [(9 + 4)/(8 + 3)] = 5.91*					
Month-to-month index	100.00	106.67	118.18	95.19	91.67
Chained month-to-month index	100.00	106.67	126.06	120.00	110.00
Direct index	100.00	106.67	126.06	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for A *in March as* $5\ (9/8 \times 4/3)^{0.5} = 6.12$					
Month-to-month index	100.00	112.62	122.47	91.34	87.31
Chained month-to-month index	100.00	112.62	137.94	125.99	110.00
Direct index	100.00	112.62	137.94	125.99	110.00

B.5.2 Treatment of products that have permanently disappeared and their replacements

9.55 Products may disappear permanently for various reasons. The product may disappear from the market because new products have been introduced or the establishments from which the price has been collected have stopped selling the product. When products disappear permanently, a replacement product has to be sampled and included in the index. The replacement product should ideally be one that accounts for a significant proportion of sales, is likely to continue to be sold for some time, and is likely to be representative of the sampled price changes of the market that the old product covered.

9.56 The timing of the introduction of replacement products is important. Many new products are initially sold at high prices that then gradually drop over time, especially as the volume of sales increases. Alternatively, some products may be introduced at artificially low prices to stimulate demand. In such cases, delaying the introduction of a new or replacement product until a large volume of sales is achieved may miss some systematic price changes that ought to be captured by PPIs. It may be desirable to try to avoid forced replacements caused when products disappear completely from the market and to try to introduce replacements when sales of the products they replace are decreasing and before they cease altogether.

9.57 Table 9.4 shows an example where product *A* disappears after March and product *D* is included as a replacement from April onward. Products *A* and *D* are not available on the market at the same time, and their price series do not overlap. To in-

Table 9.4. Disappearing Products and Their Replacements with No Overlap

	January	February	March	April	May
			Prices		
Product *A*	6.00	7.00	5.00		
Product *B*	3.00	2.00	4.00	5.00	6.00
Product *C*	7.00	8.00	9.00	10.00	9.00
Product *D*				9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 9 /[(5/3 + 10/7) 0.5] = 5.82*					
Direct index	100.00	99.21	115.08	154.76	155.38
Dutot index—Ratio of arithmetic mean prices					
Impute price for D *in March as 9 /[(5 + 10)/(4 + 9)] = 7.80*					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Impute price for D *in January as 9 /[(5 + 10)/(3 + 7)] = 6.00*					
Direct index	100.00	106.25	112.50	150.00	143.75
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for D *in March as* $9/[(5/4 \times 10/9)^{0.5}] = 7.64$					
Month-to-month index	100.00	96.15	117.13	117.85	98.65
Chained month-to-month index	100.00	96.15	112.62	132.73	130.94
Impute price for D *in January as* $9/[(5/3 \times 10/7)^{0.5}] = 5.83$					
Direct index	100.00	96.15	112.62	154.30	152.22
Omit the price					
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Monthly index	100.00	96.15	117.13	117.85	98.65
Chain month-to-month index	100.00	96.15	112.62	132.73	130.94

clude the new product in the index from April onward, an imputed price needs to be calculated either for the base period (January) if a direct index is being calculated, or for the preceding period (March) if a chained index is calculated. In both cases, the imputation method ensures that the inclusion of the new product does not, in itself, affect the index.

9.58 In the case of a chained index, imputing the missing price by the average change of the available prices gives the same result as if the product is simply omitted from the index calculation until it has been priced in two successive periods. This allows the chained index to be compiled by simply chaining the month-to-month index between periods $t-1$ and t, based on the matched set of prices in those two periods, on to the value of the chained index for period $t-1$. In the example, no further imputation is required after April, and the subsequent movement of the index is unaffected by the imputed price change between March and April.

9.59 In the case of a direct index, however, an imputed price is always required for the reference period to include a new product. In the example, the price of the new product in each month after April still has to be compared with the imputed price for January. As already noted, to prevent a situation in

Table 9.5. Disappearing and Replacement Products with Overlapping Prices

	January	February	March	April	May
			Prices		
Product *A*	6.00	7.00	5.00		
Product *B*	3.00	2.00	4.00	5.00	6.00
Product *C*	7.00	8.00	9.00	10.00	9.00
Product *D*			10.00	9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	99.21	115.08	128.17	131.75
Dutot index—Ratio of arithmetic mean prices					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	106.25	105.88	104.35	95.83
Chained month-to-month index	100.00	106.25	112.50	117.39	112.50
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	106.25	112.50	121.88	118.75
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	106.25	112.50	109.09	104.55
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	96.15	117.13	107.72	98.65
Chained month-to-month index	100.00	96.15	112.62	121.32	119.68
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	96.15	112.62	121.32	119.68
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	96.15	112.62	121.32	119.68

which most of the reference period prices end up being imputed, the direct approach should be used only for a limited period of time.

9.60 The situation is somewhat simpler when there is an overlap month in which prices are collected for both the disappearing and the replacement product. In this case, it is possible to link the price series for the new product to the price series for the old product that it replaces. Linking with overlapping prices involves making an implicit adjustment for the difference in quality between the two products, since it assumes that the relative prices of the new and old product reflect their relative qualities. For perfect or nearly perfect markets, this may be a valid assumption, but for certain markets and products it may not be so reasonable. The question of when to use overlapping prices is dealt with in detail in Chapter 7. The overlap method is illustrated in Table 9.5.

9.61 In the example, overlapping prices are obtained for products *A* and *D* in March. Their relative prices suggest that one unit of product *A* is worth two units of product *D*. If the index is calculated as a direct Carli index, the January base-period price for product *D* can be imputed by dividing the price of product *A* in January by the price ratio of *A* and *D* in March.

9.62 A monthly chained index of arithmetic mean prices will be based on the prices of products *A*, *B*, and *C* until March, and from April onward by *B*, *C*, and *D*. The replacement product is not included until prices for two successive periods are obtained. Thus, the monthly chained index has the advantage that it is not necessary to carry out any explicit imputation of a reference price for the new product.

9.63 If a direct index is calculated as the ratio of the arithmetic mean prices, the price of the new product needs to be adjusted by the price ratio of *A* and *D* in March in every subsequent month, which complicates computation. Alternatively, a reference period price of product *D* for January may be imputed. However, this results in a different index because the price relatives are implicitly weighted by the relative reference period prices in the Dutot index, which is not the case for the Carli or the Jevons index. For the Jevons index, all three methods give the same result, which is an additional advantage of this approach.

B.6 Other formulas for elementary price indices

9.64 A number of other formulas have been suggested for the price indices for elementary aggregates. The most important are presented below and discussed further in Chapter 20.

B.6.1 Laspeyres and geometric Laspeyres indices

9.65 The Carli, Dutot, and Jevons indices are all calculated without the use of explicit weights. However, as already mentioned, in certain cases there may be weighting information that could be exploited or developed in the calculation of the elementary price indices. If the reference period revenues for all the individual products within an elementary aggregate, or estimates thereof, were available, the elementary price index could itself be calculated as a Laspeyres price index, or as a geometric Laspeyres. The Laspeyres price index is defined as

$$(9.4)\quad P_L^{0:t} = \sum w_i^0 \left(\frac{p_i^t}{p_i^0}\right), \quad \sum w_i^0 = 1,$$

where the weights, w_i^0, are the revenue shares for the individual products in the reference period. If all the weights were equal, equation (9.4) would reduce to the Carli index. If the weights were proportional to the prices in the reference period, equation (9.4) would reduce to the Dutot index.

9.66 The geometric Laspeyres index is defined as

$$(9.5)\quad P_{JW}^{0:t} = \prod\left(\frac{p_i^t}{p_i^0}\right)^{W_0^i} = \frac{\prod\left(p_i^t\right)^{W_i^0}}{\prod\left(p_i^0\right)^{W_i^0}}, \quad \sum w_i^0 = 1,$$

where the weights, w_i^0, are again the revenue shares in the reference period. When the weights are all equal, equation (9.5) reduces to the Jevons index. If the revenue shares do not change much between the weight reference period and the current period, then the geometric Laspeyres index approximates a Törnqvist index.

B.6.2 Some alternative index formulas

9.67 Another widely used type of average is the harmonic mean. In the present context, there are two possible versions: either the harmonic mean of price relatives or the ratio of harmonic mean of prices.

9.68 The harmonic mean of price relatives is defined as

$$(9.6)\ P_{HR}^{0:t} = \frac{1}{\frac{1}{n}\sum \frac{p_i^0}{p_i^t}}.$$

The ratio of harmonic mean prices is defined as

$$(9.7)\ P_{RH}^{0:t} = \frac{\sum n/p_i^0}{\sum n/p_i^t}.$$

Equation (9.7), like the Dutot index, fails the commensurability test and would be an acceptable possibility only when the products are all fairly homogeneous. Neither formula appears to be used much in practice, perhaps because the harmonic mean is not a familiar concept and would not be easy to explain to users. However, at an aggregate level, the widely used Paasche index is a weighted harmonic average.

9.69 The ranking of the three common types of mean is always

arithmetic mean ≥ *geometric mean* ≥ *harmonic mean.*

It is shown in Chapter 20 that, in practice, the Carli index, the arithmetic mean of the relatives, is likely to exceed the Jevons index, the geometric mean, by roughly the same amount that the Jevons exceeds the harmonic mean, equation (9.6). The harmonic mean of the price relatives has the same kinds of axiomatic properties as the Carli but with opposite tendencies and biases. It fails the transitivity and time reversal tests discussed earlier. In addition it is very sensitive to "price bouncing," as is the Carli index. As it can be viewed conceptually as the complement, or rough mirror image, of the Carli index, it has been argued that a suitable elementary index would be provided by a geometric mean of the two, in the same way that, at an aggregate level, a geometric mean is taken of the Laspeyres and Paasche indices to obtain the Fisher index. Such an index has been proposed by Carruthers, Sellwood, Ward, and Dalén—namely,

$$(9.8)\ P_{CSWD}^{0:t} = \sqrt{I_C^{0:t} \cdot I_{HR}^{0:t}}.$$

P_{CSWD} is shown in Chapter 20 to have very good axiomatic properties but not quite as good as Jevons index, which is transitive, whereas the P_{CSWD} is not. However, it can be shown to be approximately transitive and, empirically, it has been observed to be very close to the Jevons index.

9.70 More recently, as attention has focused on the economic characteristics of elementary aggregate formulas, consideration has been given to formulas that allow for substitution between products within an elementary aggregate. The increasing use of the geometric mean is an example of this. However, the Jevons index is limited to a functional form that reflects an elasticity of demand equal to one that, while clearly allowing for some substitution, is unlikely to be applicable to all elementary aggregates. A logical step is to consider formulas that allow for different degrees of substitution in different elementary aggregates. One such formula is the unweighted Lloyd-Moulton formula:

$$(9.9)\ P_{LM}^{0:t} = \left[\sum \frac{1}{n}\left(\frac{P_i^t}{P_i^0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the elasticity of substitution. The Carli and the Jevons indices can be viewed as special cases of the P_{LM} in which σ = 0 and σ = 1. The advantage of the P_{LM} formula is that σ is unrestricted. Provided a satisfactory estimate can be made of σ, the resulting elementary price index is likely to approximate the Fisher and other superlative indices. It reduces substitution bias when the objective is to estimate an economic index. The difficulty is in the need to estimate elasticities of substitution, a task that will require substantial development and maintenance work. The formula is described in more detail in Chapter 20.

B.7 Unit-value indices

9.71 The unit-value index is simple in form. The unit value in each period is calculated by dividing total revenue on some product by the related total quantity. It is clear that the quantities must be strictly additive in an economic sense, which implies that they should relate to a single homogeneous product. The unit-value index is then defined as the ratio of unit values in the current period to those in the reference period. It is not a price index as normally understood, since it is essentially a measure of the change in the average price of a *single* product when that product is sold at different prices to different purchasers, perhaps at different times within the same period. Unit values, and unit-value indices, should not be calculated for sets of heterogeneous products.

9.72 However, unit values do play an important part in the process of calculating an elementary price index, because they are the appropriate *average* prices that need to be entered into an elementary price index. Usually, prices are sampled at a particular time or period each month, and each price is assumed to be representative of the average price of that product in that period. In practice, this assumption may not hold. In this case, it is necessary to estimate the unit value for each product, even though this will inevitably be more costly. Thus, having specified the product to be priced in a particular establishment, data should be collected on both the value of the total sales in a particular month and the total quantities sold in order to derive a unit value to be used as the price input into an elementary aggregate formula. It is particularly important to do this if the product is sold at a discount price for part of the period and at the "regular" price in the rest of the period. Under these conditions, neither the discount price nor the regular price is likely to be representative of the average price at which the product has been sold or the price change between periods. The unit value over the whole month should be used. With the possibility of collecting more and more data from electronic records, such procedures may be increasingly used. However, it should be stressed that the product specifications must remain constant through time. Changes in the product specifications could lead to unit-value changes that reflect quantity, or quality, changes and should not be part of price changes.

B.8 Formulas applicable to electronic data

9.73 Respondents may well have computerized management accounting systems that include highly detailed data on sales both in terms of prices and quantities. Their primary advantages are that the number of price observations can be significantly larger and that both price and quantity information are available in real time. Much work has been undertaken on the use of scanner data as an emerging data source for CPI compilation, and there are parallels for the PPI. There are a large number of practical considerations, which are discussed and referenced in the *CPI Manual* (ILO and others, 2004) and also in Chapter 6, Section D, of this *Manual*, but it is relevant to discuss briefly here possible index number formulas that may be applicable if electronic data are collected and used in PPI compilation.

9.74 The existence of quantity and revenue information increases the ability to estimate price changes accurately. It means that traditional index number approaches such as Laspeyres and Paasche can be used, and that superlative formulas such as the Fisher and Törnqvist-Theil indices can also be derived in real time. The main observation made here is that since price and quantity information are available for each period, it may be tempting to produce monthly or quarterly chained indices using one of the ideal formulas mentioned above. However, the compilation of subannual chained indices has been found in some studies to be problematic because it often results in an upward bias referred to as "chain drift."

C. Calculation of Higher-Level Indices

C.1 Target indices

9.75 A statistical office must have some target index at which to aim. Statistical offices have to consider what kind of index they would choose to calculate in the ideal hypothetical situation in which they had complete information about prices and quantities in both time periods compared. If the PPI is meant to be an economic index, then a superlative index such as a Fisher, Walsh, or Törnqvist-Theil would have to serve as the theoretical target, since a superlative index may be expected to approximate the underlying economic index.

9.76 Many countries do not aim to calculate an economic index and prefer the concept of a *basket index*. A basket index is one that measures the change in the total value of a given basket of goods and services between two time periods. This general category of index is described here as a *Lowe* index after the early 19th-century index number pioneer who first proposed this kind of index (see Chapter 15, Section D). The meaning of a Lowe index is clear and can be easily explained to users, important considerations for many statistical offices. It should be noted that, in general, there is no necessity for the basket to be the actual basket in one or other of the two periods compared. If the theoretical target index is to be a basket or Lowe index, the preferred basket might be one that attaches equal importance to the baskets in both periods—for example, the Walsh index.[5] Thus, the same kind of index may emerge as the theoretical target on both the basket and the economic index approaches. In practice, however, a statistical office may prefer to designate the basket index that uses the actual basket in the earlier of the two periods as its target index on grounds of simplicity and practicality. In other words, the Laspeyres index may be a target index.

9.77 The theoretical target index is a matter of choice. In practice, it is likely to be either a Laspeyres or some superlative index. However, even when the target index is the Laspeyres, there may a considerable gap between what is actually calculated and what the statistical office considers to be its target. It is now necessary to consider what statistical offices tend to do in practice.

C.2 PPIs as weighted averages of elementary indices

9.78 Section B discussed alternative formulas for combining individual price observations to calculate the first level of indices, called elementary aggregates. The next steps in compiling the PPI involves taking the elementary indices and combining them, using weights, to calculate successively higher levels of indices as shown in Chapter 4, Figure 4.1.

9.79 A higher-level index is an index for some revenue aggregate above the level of an elementary aggregate, including the overall PPI itself. The inputs into the calculation of the higher-level indices are

- The elementary price indices, and
- Weights derived from the values of elementary aggregates in some earlier year, or years.

The higher-level indices are calculated simply as weighted arithmetic averages of the elementary price indices. This general category of index is described here as a *Young* index after another one of the 19th-century index number pioneers who advocated this type of index (see Chapter 15, Section D).

9.80 The weights typically remain fixed for a sequence of at least 12 months. Some countries revise their weights at the beginning of each year to try to approximate as closely as possible to current production patterns. However, many countries continue to use the same weights for several years. The weights may be changed only every five years or so.

9.81 The use of fixed weights has the considerable practical advantage that the index can make repeated use of the same weights. This saves both time and money. Revising the weights can be both time consuming and costly, especially if it requires new establishment production surveys to be carried out.

9.82 The second stage of calculating the PPI does not involve individual prices or quantities. Instead, a higher-level index is a Young index in which the elementary price indices are averaged using a set of predetermined weights. The formula can be written as follows:

$$(9.10)\quad P_Y^{0:t} = \sum w_i^b I_i^{0:t}\ ,\quad \sum w_i^b = 1,$$

where $P_Y^{0:t}$ denotes the overall PPI, or any higher-level index, from period 0 to t; w_i^b is the weight attached to each of the elementary price indices; and $I_i^{0:t}$ is the corresponding elementary price index. The elementary indices are identified by the subscript i, whereas the higher-level index carries no subscript. As already noted, a higher-level index is any index, including the overall PPI, above the elementary aggregate level. The weights are derived from revenue in period b, which in practice has to precede period 0, the price reference period.

9.83 It is useful to recall that three kinds of reference periods may be distinguished for PPI purposes:

[5]The quantities that make up the basket in the Walsh index are the geometric means of the quantities in the two periods.

- *Weight Reference Period*: The period covered by the revenue statistics used to calculate the weights. Usually, the weight reference period is a year.
- *Price Reference Period*: The period for which prices are used as denominators in the index calculation.
- *Index Reference Period*: The period for which the index is set to 100.

9.84 The three periods are generally different. For example, a PPI might have 1998 as the weight reference *year*, December 2002 as the price reference *month*, and the *year* 2000 as the index reference period. The weights typically refer to a whole year, or even two or three years, whereas the periods whose prices are compared are typically months or quarters. The weights are usually estimated on the basis of an establishment survey that was conducted some time before the price reference period. For these reasons, the weight and the price reference periods are invariably separate periods in practice.

9.85 The index reference period is often a year, but it could be a month or some other period. An index series may also be re-referenced to another period by simply dividing the series by the value of the index in that period, without changing the rate of change of the index. The expression "base period" can mean any of the three reference periods and can sometimes be quite ambiguous. "Base period" should be used only when it is absolutely clear in context exactly which period is referred to.

9.86 Provided the elementary aggregate indices are calculated using a transitive formula such as the Jevons or Dutot, but not the Carli, and provided that there are no new or disappearing products from period 0 to t, equation (9.10) is equivalent to

$$(9.11)\quad P^{0:t} = \sum w_i^b I_i^{0:t-1} I_i^{t-1:t} \ , \quad \sum w_i^b = 1 \ ,$$

where $P^{0:t}$ is a higher-level PPI.

Table 9.6. Aggregation of Elementary Price Indices

Index	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
A	0.20	100.00	102.50	104.88	101.16	101.15	100.00
B	0.25	100.00	100.00	91.67	109.09	101.67	108.20
C	0.15	100.00	104.00	96.15	104.00	101.92	103.77
D	0.10	100.00	92.86	107.69	107.14	100.00	102.67
E	0.30	100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
B	0.25	100.00	100.00	91.67	100.00	101.67	110.00
C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00
Higher-level indices							
G = *A*+*B*+*C*	0.60	100.00	101.83	99.03	103.92	105.53	110.00
H = *D*+*E*	0.40	100.00	99.46	101.25	101.79	104.29	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00

The advantage of this version of the index is that it allows the sampled products within the elementary price index from $t-1$ to t to differ from the sampled products in the periods from 0 to $t-1$. Hence, it allows replacement products and new products to be linked into the index from period $t-1$ without the need to estimate a price for period 0, as explained in Section B.5. For example, if one of the sampled products in periods 0 and $t-1$ is no longer available in period t, and the price of a replacement product is available for $t-1$ at t, the new replacement product can be included in the index using the overlap method.

C.3 A numerical example

9.87 Equation (9.10) applies at each level of aggregation above the elementary index. The index is additive—that is, the overall index is the same whether calculated on the basis of the original elementary price indices or on the basis of the intermediate higher-level indices. This facilitates the presentation of the index.

9.88 Table 9.6 (on the previous page) illustrates the calculation of higher-level indices in the special case where the weight and the price reference period are identical, that is, $b = 0$. The index consists of five elementary aggregate indices (*A–E*), which are calculated using one of the formulas presented in Section 9.B.2, and two intermediate higher-level indices, *G* and *H*. The overall index (Total) and the higher-level indices (*G* and *H*) are all calculated using equation (9.10). Thus, for example, the overall index for April can be calculated from the two intermediate higher-level indices of April as

$$P^{Jan:apr} = 0.6 \times 103.92 + 0.4 \times 101.79 = 103.06$$

or directly from the five elementary indices

$$\begin{aligned} P^{Jan:apr} &= 0.2 \times 108.75 + 0.25 \times 100 + 0.15 \times 104 \\ &\quad + 0.1 \times 107.14 + 0.3 \times 100 \\ &= 103.06 . \end{aligned}$$

Note from equation (9.11) that

$$(9.12)\quad P^{0:t} = \sum w_i^b P_i^{0:t-1} P_i^{t-1:t} \neq P^{0:t-1} \sum w_i^b\ P_i^{t-1:t}$$
$$\Rightarrow \frac{P^{0:t}}{P^{0:t-1}} \neq \sum w_i^b P_i^{t-1:t} .$$

This shows that if the month-to-month indices are averaged using the fixed weights w_i^b, the resulting index is *not* equal to the month-to-month higher-level index. As explained below, to be able to obtain the month-to-month higher-level index, the weights applied to the month-to-month indices need to be updated to reflect the effects of the price changes that have taken place since January.

C.4 Young and Lowe indices

9.89 It is useful to clarify the relationship between the Lowe and Young indices. As already noted, when statistical offices explain their PPIs to users, they often describe them as Lowe indices, which measure the change over time in the value of a fixed basket of goods and services. When they calculate their PPIs, however, the formula they actually use is that of a Young index. The relationship between the two indices is given in equation (9.13), where P_{Lo} is the Lowe index and P_Y is the Young index:

$$(9.13)\ P_{Lo} = \frac{\sum p_j^t q_j^b}{\sum p_j^0 q_j^b} = \frac{\sum p_j^t q_j^b}{\sum p_j^b q_j^b} \Bigg/ \frac{\sum p_j^0 q_j^b}{\sum p_j^b q_j^b}$$
$$= \sum w_j \left(\frac{p_j^t}{p_j^0} \right) = P_Y ,$$

where $w_j = \dfrac{p_j^0 q_j^b}{\sum p_j^0 q_j^b}$.

The individual quantities (q_j^b) in the weight reference period b make up the basket. Assume initially that the weight reference period b has the same duration as that of the two periods 0 and t that are being compared. It can be seen from equation (9.13) that

(i) The Lowe index is equal to a Young index in which the weights are *hybrid* value shares obtained by revaluing the quantities in the weight reference period b (q_j^b), at the prices of the price reference month 0;[6]

(ii) The Lowe index can be expressed as the ratio of the two Laspeyres indices for periods t and 0, respectively, based on month b; and

[6]Since the weights are usually revenues, this is often referred to as price updating the weights to the price reference period and will be discussed further in Section C.6.

(iii) The Lowe index reduces to the Laspeyres index when $b = 0$ and to the Paasche index when $b = t$.

9.90 In practice, the situation is more complicated for actual PPIs because the duration of the reference period b is typically much longer than periods 0 and t. The weights w_j usually refer to the revenues over a year, or longer, while the price reference period is usually a month in some later year. For example, a monthly index may be compiled from January 2003 onward with December 2002 as the price reference month, but the latest available weights during the year 2003 may refer to the year 2000, or even some earlier year.

9.91 Conceptually, a typical PPI may be viewed as a Lowe index that measures the change from month to month in the total revenue of an annual basket of goods and services that may date back several years before the price reference period. Because it uses the fixed basket of an earlier period, it is sometimes loosely described as a "Laspeyres-type" index, but this description is unwarranted. A true Laspeyres index would require the basket to be that purchased in the price reference month, whereas in most PPIs the basket not only refers to a different period from the price reference month but to a period of a year or more. When the weights are annual and the prices are monthly, it is not possible, even retrospectively, to calculate a monthly Laspeyres price index.

9.92 A Lowe index that uses quantities derived from an earlier period than the price reference period is likely to exceed the Laspeyres (see Section D.1 of Chapter 15), and by a progressively larger amount, the further back in time the weight reference period is. The Lowe index is likely to have an even greater upward bias than the Laspeyres index as compared with some target superlative index and the underlying economic index. Inevitably, the quantities in any basket index become increasingly out of date and irrelevant the further back in time the period to which they relate. To minimize the resulting bias, the weights should be updated more frequently, preferably annually.

9.93 A statistical office may not wish to estimate an economic index and may prefer to choose some basket index as its target index. In that case, if the theoretically attractive Walsh index were to be selected as the target index, a Lowe index would have the same bias, as just described, given that the Walsh index is also a superlative index.

C.5 Factoring the Young index

9.94 It is possible to calculate the change in a higher-level Young index between two consecutive periods, such as $t-1$ and t, as a weighted average of the individual price indices between $t-1$ and t, *provided* that the weights are updated to take into account the price changes between the price reference period 0 and the previous period, $t-1$. This makes it possible to factor equation (9.10) into the product of two component indices in the following way:

$$(9.14)\quad P^{0:t} = P^{0:t-1} \sum w_i^{b(t-1)} P_i^{t-1:t} \ ,$$

$$\text{where } w_i^{b(t-1)} = w_i^b P_i^{0:t-1} \Big/ \sum w_i^b P_i^{0:t-1} \ .$$

$I^{0:t-1}$ is the Young index for period $t-1$. The weight $w_i^{b(t-1)}$ is the original weight for elementary aggregate i price updated by multiplying it by the elementary price index for i between 0 and $t-1$, the adjusted weights being rescaled to sum to unity. The price-updated weights are hybrid weights because they implicitly revalue the quantities of b at the prices of $t-1$ instead of at the average prices of b. Such hybrid weights do not measure the actual revenue shares of any period.

9.95 The index for period t can thus be calculated by multiplying the already calculated index for $t-1$ by a separate Young index between $t-1$ and t with hybrid price-updated weights. In effect, the higher-level index is calculated as a chained index in which the index is moved forward period by period. This method gives more flexibility to introduce replacement products and makes it easier to monitor the movements of the recorded prices for errors, since month-to-month movements are smaller and less variable than the total changes since the price reference period.

9.96 Price updating may also occur between the weight reference period to the price reference period, as explained in the next section.

C.6 Updating from weight reference period to price reference period

9.97 When the weight reference period b and the price reference period 0 are different, as is normally the case, the statistical office has to decide whether or not to price update the weights from b to 0. In practice, the price-updated weights can be calculated by multiplying the original weights for period b by elementary indices measuring the price changes between periods b and 0 and rescaling to sum to unity.

9.98 The issues involved are best explained with the help of a numerical example. In Table 9.7, the base period b is assumed to be the year 2000 so that the weights are the revenue shares in 2000. In the upper half of the table, 2000 is also used as the price reference period. However, in practice, weights based on 2000 cannot be introduced until after 2000 because of the time needed to collect and process the revenue data. In the lower half of the table, it is assumed that the 2000 weights are introduced in December 2002, and that this is also chosen as the new price reference base.

9.99 Notice that it would be possible in December 2002 to calculate the indices based on 2000 shown in the upper half of the table, but it is de-

Table 9.7. Price Updating of Weights Between Weight and Price Reference Periods

Index	Weight	2000	Nov 02	Dec 02	Jan 03	Feb 03	Mar 03
Index with 2000 as weight and price reference period							
Elementary price indices							
	w_{00}						
A	0.20	100.00	98.00	99.00	102.00	101.00	104.00
B	0.25	100.00	106.00	108.00	107.00	109.00	110.00
C	0.15	100.00	104.00	106.00	98.00	100.00	97.00
D	0.10	100.00	101.00	104.00	108.00	112.00	114.00
E	0.30	100.00	102.00	103.00	106.00	105.00	106.00
Higher-level indices							
G = A+B+C	0.60	100.00	102.83	104.50	103.08	104.08	104.75
H = D+E	0.40	100.00	101.75	103.25	106.50	106.75	108.00
Total		100.00	102.40	104.00	104.45	105.15	106.05
Index re-referenced to December 2002 and weights price-updated to December 2002							
Elementary price indices							
	$w_{00(Dec02)}$						
A	0.190	101.01	98.99	100.00	103.03	102.02	105.05
B	0.260	92.59	98.15	100.00	99.07	100.93	101.85
C	0.153	94.34	98.11	100.00	92.45	94.34	91.51
D	0.100	96.15	97.12	100.00	103.85	107.69	109.62
E	0.297	97.09	99.03	100.00	102.91	101.94	102.91
Higher-level indices							
G = A+B+C	0.603	95.69	98.41	100.00	98.64	99.60	100.24
H = D+E	0.397	96.85	98.55	100.00	103.15	103.39	104.60
Total		96.15	98.46	100.00	100.43	101.11	101.97
Rescaled to 2000 = 100		100.00	102.40	104.00	104.45	105.15	106.05

cided to make December 2002 the price reference base. This does not prevent the index with the December 2002 price reference period from being calculated backward a few months into 2002, if desired.

9.100 The statistical office compiling the index has two options at the time the new index is introduced. It has to decide whether the weights in the new index should preserve the quantities in 2000 or the revenues in 2000. It cannot do both.

9.101 If it decides to preserve the quantities, the resulting index is a basket, or Lowe, index in which the quantities are those of the year 2000. This implies that the *movements* of the index must be identical with those of the index based on 2000 shown in the upper part of the table. In this case, if the index is to be presented as a weighted average of the elementary price indices with December 2002 as price reference period, the revenue weights for 2000 have to be price updated to December 2002. This is illustrated in the lower half of Table 9.7, where the updated weights are obtained by multiplying the original weights for 2000 in the upper part of the table by the price indices for the elementary aggregates between 2000 and December 2002 and then rescaling the results to sum to unity. These are the weights labeled $w_{00(Dec02)}$ in the table.

9.102 The indices with price-updated weights in the lower part of Table 9.7 are Lowe indices in which b = 2000 and 0 = December 2002. These indices can be expressed as relatives of the indices in the upper part of the table. For example, the overall Lowe index for March 2003 with December 2002 as its price reference base, namely 101.97, is the ratio of the index for March 2003 based on 2000 shown in the upper part of the table, namely 106.05, divided by the index for December 2002 based on 2000, namely 104.00. Thus, the price updating preserves the movements of the indices in the upper part of the table while shifting the price reference period to December 2002.

9.103 On the other hand, it could be decided to calculate a series of Young indices using the revenue weights from 2000 as they stand without updating. If the revenue shares were actually to remain constant, the quantities would have had to move inversely with the prices between 2000 and December 2002. The quantities that make up the basket for the new Young index could not be the same as those of 2000. The movements of this index would have to be slightly different from those of the price-updated Lowe index.

9.104 The issue is whether to use the known quantities of the weight reference period 2000, which are the latest for which firm data have been collected, or to use the known revenue shares of the weight reference period. If the official objective is to measure a Lowe index that uses a fixed basket, the issue is decided and the statistical office is obliged to price update. On the other hand, some statistical offices may have to decide for themselves which option to adopt.

9.105 Updating the prices without updating the quantities does not imply that the resulting revenue weights are necessarily more up to date. When there is a strong inverse relation between movements of price and quantities, price updating on its own could produce perverse results. For example, the price of computers has been declining rapidly in recent years. If the quantities are held fixed while the price is updated, the resulting revenue on computers would also decline rapidly. In practice, however, the share of revenue on computers might actually be rising because of a very rapid increase in quantities of computers purchased.

9.106 When rapid changes take place in relative quantities as well as relative prices, statistical offices are effectively obliged to change their revenue weights more frequently, even if this means conducting more frequent establishment surveys. Price updating on its own cannot cope with this situation. The revenue weights have to be updated with respect to their quantities as well as their prices, which, in effect, implies collecting new revenue data.

C.7 Introduction of new weights and chain linking

9.107 From time to time, the weights for the elementary aggregates have to be revised to ensure that they reflect current revenue shares and business activity. When new weights are introduced, the price reference period for the new index can be the last period of the old index, the old and the new indices being linked together at this point. The old and the new indices make a chained index.

9.108 The introduction of new weights is often a complex operation because it provides the opportunity to introduce new products, new samples, new data sources, new compilation practices, new elementary aggregates, new higher-level indices, or new classifications. These tasks are often undertaken simultaneously at the time of reweighting to minimize overall disruption to the time series and any resulting inconvenience to users of the indices.

9.109 In many countries reweighting and chaining is carried out about every five years, but some countries introduce new weights each year. However, chained indices do not have to be linked annually, and the linking may be done less frequently. The real issue is not whether to chain, but how frequently to chain. Reweighting is inevitable sooner or later, because the same weights cannot continue to be used forever. Whatever the time frame, statistical offices have to address the issue of chain linking sooner or later. It is an inevitable and major task for index compilers.

C.7.1 Frequency of reweighting

9.110 It is reasonable to continue to use the same set of elementary aggregate weights as long as production patterns at the elementary aggregate level remain fairly stable. However, over time purchasers will tend to move away from products whose prices have increased relatively so that, in general, movements in prices and quantities tend to be inversely related. This kind of substitution behavior implies that a Lowe index based on the fixed basket of an earlier period will tend to have an upward bias compared with a basket index using up-to-date weights.

9.111 Another reason why purchasing patterns change is that new products are continually being introduced on the market while others drop out. Over the longer term, purchasing patterns are also influenced by several other factors. These include rising incomes and standards of living, demographic changes in the structure of the population, changes in technology, and changes in tastes and preferences.

9.112 There is wide consensus that regular updating of weights—at least every five years, and more often if there is evidence of rapid changes in production patterns—is a sensible and necessary practice. However, the question of how often to change the weights and chain link the index is not straightforward, because frequent linking can also have some disadvantages. It can be costly to obtain new weights, especially if they require more frequent establishment surveys. On the other hand, annual chaining has the advantage that changes such as the inclusion of new goods can be introduced on a regular basis, although every index needs some ongoing maintenance, whether annually chained or not.

9.113 Purchasers of certain types of products are strongly influenced by short-term fluctuations in the economy. For example, purchases of cars, major durables, expensive luxuries, etc., may change drastically from year to year. In such cases, it may be preferable to base the weight on an average of two or more years' revenue.

C.7.2 Calculation of a chained index

9.114 Assume that a series of fixed-weight Young indices have been calculated with period 0 as the price reference period, and that in a subsequent period, k, a new set of weights has to be introduced in the index. (The new set of weights may, or may not, have been price updated from the new weight reference period to period k.) A chained index is then calculated as

$$(9.15)\quad P^{0:t} = P^{0:k}\,\Sigma w_i^k P_i^{k:t-1} P_i^{t-1:t} = P^{0:k}\,\Sigma w_i^k P_i^{k:t} = P^{0:k} P^{k:t}\,.$$

There are several important features of a chained index.

(i) The chained index formula allows weights to be updated and facilitates the introduction of new products and subindices and removal of obsolete ones.
(ii) To link the old and the new series, an overlapping period (k) is needed in which the index has to be calculated using both the old and the new set of weights.
(iii) A chained index may have two or more links. Between each link period, the index may be calculated as a fixed-weight index using equation (9.10) or any other index formula. The link period may be a month or a year, provided the weights and indices refer to the same period.

Table 9.8. Calculation of a Chained Index

Index	Weight 1998	1998	Nov 02	Dec 02	Weight 2000	Dec 02	Jan 03	Feb 03	Mar 03
		1998 = 100				Dec 2002 = 100			
Elementary price indices									
A	0.20	100.00	120.00	121.00	0.25	100.00	100.00	100.00	102.00
B	0.25	100.00	115.00	117.00	0.20	100.00	102.00	103.00	104.00
C	0.15	100.00	132.00	133.00	0.10	100.00	98.00	98.00	97.00
D	0.10	100.00	142.00	143.00	0.18	100.00	101.00	104.00	104.00
E	0.30	100.00	110.00	124.00	0.27	100.00	103.00	105.00	106.00
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Higher-level indices									
G = A+B+C	0.60	100.00	120.92	122.33	0.55	100.00	100.36	100.73	101.82
H = D+E	0.40	100.00	118.00	128.75	0.45	100.00	102.20	104.60	105.20
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Chaining of higher-level indices to 1998 = 100									
G = A+B+C	0.60	100.00	120.92	122.33	0.55	122.33	122.78	123.22	124.56
H = D+E	0.40	100.00	118.00	128.75	0.45	128.75	131.58	134.67	135.45
Total		100.00	119.75	124.90		124.90	126.39	127.99	129.07

(iv) Chaining is intended to ensure that the individual indices on all levels show the correct development through time.

(v) Chaining leads to nonadditivity. When the new series is chained onto the old as in equation (9.15), the higher-level indices after the link cannot be obtained as weighted arithmetic averages of individual indices using the new weights.[7] Such results need to be carefully explained and presented.

9.115 An example of the calculation of a chained index is presented in Table 9.8. From 1998 to December 2002, the index is calculated with the year 1998 as weight and price reference period. From December 2002 onward, a new set of weights is introduced. The weights may refer to the year 2000, for example, and may or may not have been price updated to December 2002. A new fixed-weight index series is then calculated with December 2002 as price reference month. Finally, the new index series is linked onto the old index with 1998 = 100 by multiplication to get a continuous index from 1998 to March 2003.

9.116 The chained higher-level indices in Table 9.8 are calculated as

$$(9.16)\quad P^{00:t} = P^{98:Dec02} \sum w_i^{00(Dec02)} P_i^{Dec02:t} .$$

Because of the lack of additivity, the overall chained index for March 2003 (129.07), for example, cannot be calculated as the weighted arithmetic mean of the chained higher-level indices *G* and *H* using the weights from December 2002.

[7] If, on the other hand, the index reference period is changed and the index series before the link period are rescaled to the new index reference period, these series cannot be aggregated to higher-level indices by use of the new weights.

Table 9.9. Calculation of a Chained Index Using Linking Coefficients

Index	Weight 1998	1998	Nov 02	Dec 02	Linking coefficient	Jan 03	Feb 03	Mar 03
Elementary price indices (1998 = 100)								
A	0.20	100.00	120.00	121.00	1.2100	121.00	121.00	123.42
B	0.25	100.00	115.00	117.00	1.1700	119.34	120.51	121.68
C	0.15	100.00	132.00	133.00	1.3300	130.34	130.34	129.01
D	0.10	100.00	142.00	143.00	1.4300	144.43	148.72	148.72
E	0.30	100.00	110.00	124.00	1.2400	127.72	130.20	131.44
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07
Higher-level indices (1998 = 100)								
G = A+B+C	0.60	100.00	120.92	122.33	1.2233	122.78	123.22	124.56
H = D+E	0.40	100.00	118.00	128.75	1.2875	131.58	134.67	135.45
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07

Elementary price indices (December 2002 = 100)

Index	Linking coefficient	1998	Nov 02	Dec 02	Weight 2000	Jan 03	Feb 03	Mar 03
A	0.82645	82.65	99.17	100.00	0.25	100.00	100.00	102.00
B	0.85470	85.47	98.29	100.00	0.20	102.00	103.00	104.00
C	0.75188	75.12	99.25	100.00	0.10	98.00	98.00	97.00
D	0.69993	69.99	99.39	100.00	0.18	101.00	104.00	104.00
E	0.80645	80.65	88.71	100.00	0.27	103.00	105.00	106.00
Total	0.80064	80.06	95.88	100.00		101.19	102.47	103.34
Higher-level indices (2000 = 100)								
G = A+B+C	0.81746	81.75	98.85	122.33	0.55	100.36	100.73	101.82
H = D+E	0.77670	77.67	91.65	128.75	0.45	102.20	104.60	105.20
Total	0.80064	80.06	95.88	124.90		101.19	102.47	103.34

C.7.3 Chaining indices using linking coefficients

9.117 Table 9.9 presents an example of chaining indices on new weights to the old reference period (1998 = 100). The linking can be done several ways. As described above, one can take the current index on the new weights and multiply it by the old index level in the overlap month (December 2002). Alternatively, a linking coefficient can be calculated between the old and new series during the overlap period and this coefficient applied to the new index series to bring the index up to the level of the old series. The linking coefficient for keeping the old price reference period is the ratio of the old index in the overlap period to the new index for the same period. For example, the coefficient for the Total index is (124.90 ÷ 100.00) = 1.2490. This coefficient is then applied to the Total index each month to convert it from a December 2002 reference period to the 1998 reference period.[8]

9.118 Another option is to change the index reference period at the time the new weights are introduced. In the current example, the statistical office can shift to a December 2002 reference period and link the old index to the new reference period. This is done by calculating the linking coefficient for each index as the ratio of the new index in the overlap period to the old index. For example, the coefficient for the Total index is (100.00 ÷ 124.90) = 0.80064. This coefficient is applied to the old Total index series to bring it down to the level of the new index. Table 9.9 presents the linking coefficients and the resulting re-reference price indices using the two alternative index reference periods—1998 or December 2002.

C.7.4 Introduction of new elementary aggregates

9.119 First, consider the situation in which new weights are introduced and the index is chain linked in December 2002. The overall coverage of the PPI is assumed to remain the same, but certain products have increased sufficiently in importance to merit recognition as new elementary aggregates. Possible examples are the introduction of new elementary aggregates for mobile telephones or a new multinational company setting up a car factory.

9.120 Consider the calculation of the new index from December 2002 onward, the new price reference period. The calculation of the new index presents no special problems and can be carried out using equation (9.10). However, if the weights are price updated from, say, 2000 to December 2002, difficulties may arise because the elementary aggregate for mobile telephones did not exist before December 2002, so there is no price index with which to price update the weight for mobile telephones. Prices for mobile telephones may have been recorded before December 2002, possibly within another elementary aggregate (communications equipment) so that it may be possible to construct a price series that can be used for price updating. Otherwise, price information from other sources such as business surveys, trade statistics, or industry sources may have to be used. If no information is available, then movements in the price indices for similar elementary aggregates may be used as a proxies for price updating.

9.121 The inclusion of a new elementary aggregate means that the next higher-level index contains a different number of elementary aggregates before and after the linking. Therefore, the rate of change of the higher-level index whose composition has changed may be difficult to interpret. However, failing to introduce new goods or services for this reason would result in an index that does not reflect the actual dynamic changes taking place in the economy. If it is customary to revise the PPI backward, then the prices of the new product and their weights might be introduced retrospectively. If the PPI is not revised backward, however, which is usually the case, little can be done to improve the quality of the chained index. In many cases, the addition of a single elementary aggregate is unlikely to have a significant effect on the next higher-level index into which it enters. If the addition of an elementary aggregate is believed to have a significant impact on the time series of the higher-level index, it may be necessary to discontinue the old series and commence a new higher-level index. These decisions can be made only on a case-by-case basis.

C.7.5 Introduction of new, higher-level indices

9.122 It may be necessary to introduce a new, higher-level index in the overall PPI. This situation may occur if the coverage of the PPI is enlarged or the grouping of elementary aggregates is changed. It then needs to be decided what the initial value the new higher-level index should be when it is included in the calculation of the overall PPI. Take as an example the situation in Table 9.8 and assume that a new higher-level index from January 2003 has to be included in the index. The question is what should be the December 2002 value to which the new higher-level index is linked. There are two options.

- Estimate the value in December 2002 that the new higher-level index would have had with 1998 as price reference period, and link the new series from January 2003 onward onto this value. This procedure will prevent any break in the index series.
- Use 100 in December 2002 as starting point for the new higher-level index. This simplifies the

[8]A linking coefficient is needed for each index series that is being chained.

problem from a calculation perspective, although there remains the problem of explaining the index break to users.

In any case, major changes such as those just described should, so far as possible, be made in connection with the regular reweighting and chaining to minimize disruptions to the index series.

9.123 A final case to consider concerns classification change. For example, a country may decide to change from a national classification to an international one, such as ISIC. The changes in the composition of the aggregates within the PPI may then be so large that it is not meaningful to link them. In such cases, it is recommended that the PPI with the new classification should be calculated backward for at least one year so that consistent annual rates of change can be calculated.

C.7.6 Partial reweighting and introducing new goods

9.124 The weights for the elementary aggregates may be obtained from a variety of sources over a number of different periods. Consequently, it may not be possible to introduce all the new weighting information at the same time. In some cases, it may be preferable to introduce new weights for some elementary aggregates as soon as possible after the information is received. This would be the case for introducing new goods (for example, revolutionary goods, discussed in Chapter 8) into the index when these goods fall within the existing product structure of the index. The introduction of new weights for a subset of the overall index is known as partial reweighting.

9.125 As an example, assume there is a four-digit industry with three major products (*A*, *B*, and *C*) that were selected for the sample in 2000. From the revenue data for 2000, *A* had 50 percent of revenues, *B* had 35 percent, and *C* had 15 percent. From a special industry survey conducted for 2002, the statistical office discovers that *C* now has 60 percent of the revenue and *A* and *B* each have 20 percent. When the new weights are introduced into the index, the procedures discussed in Section C.7.2 for chaining the new index onto the old index can be used. For example, the new product weights for 2002 are used to calculate the index in an overlap month such as April 2003 with a base price reference period of December 2002. For May 2003, the index using the new product weights is again calculated and the price change using the new index is then applied (linked) to the old industry level index for April 2003 (with 2000 = 100) to derive the industry index for May 2003 (2000 = 100). The formula for this calculation is the following:

$$(9.17)\quad P^{00:May03} = P^{00:Apr03}\left[\frac{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:May03}}{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:Apr03}}\right].$$

9.126 Continuing with this example, assume the special survey was conducted because producers are making a new, important product in this industry. The survey finds the new product (*D*) has a significant share of production (perhaps 15 or 20 percent), and it is expected to continue gaining market share. The statistical office would use the same procedure for introducing the new product. In this case, the calculations for the new industry index in April and May would use all four products instead of the original three. The price change in the new sample is linked to the old index as in equation (9.17). The only difference will be that the summations are over m (four products) instead of n (three) products.

9.127 One could also make the same calculations using the linking coefficient approach discussed in Section C.7.3. The linking coefficient is derived by taking the ratio of the old industry index (2000 = 100) to the new industry index (December 2002 = 100) in the overlap period (April 2003):

$$(9.18)\ \text{Linking coefficient} = \frac{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:Apr03}}{\sum_{i=1}^{n} w_i^{00} P_i^{00:Apr03}}.$$

The linking coefficient, computed for the overlap period only, is then applied each month to the new index to adjust it to the level of the old index with an index reference period of 2000.

9.128 Another issue is the weights to use for compiling the index for the product groups represented by *A*, *B*, *C*, and *D*. For example, if indices for products *A* and *B* are combined with products *X* and *Y* to calculate a product group index, the new weights for *A* and *B* present a problem because they represent revenues in a more current period than the weights for *X* and *Y*. Also, the indices have different price reference periods. If we had weights for products X and Y for the same period as A and B, then we could use the same approach as just described for compiling the industry index. Lacking new product weights for X and Y means the statistical office will have to take additional steps. One approach to resolve this problem is to price update the weights for products X and Y from 2000 to 2002 using the change in the respective price indices. Thus, the original weight for product X is multiplied by the change in prices between 2002 and 2000 (that is, the ratio of the average price index of X in 2002 to the average price index of X in 2000). Then use the same base price reference period as for A and B so that the indices for products X and Y are each re-referenced to December 2002. The product group index can then be compiled for April 2003 using the new weights for all four products and their indices with December 2002 = 100. Once the April 2003 index is compiled on the December 2002 price reference period, then the linking coefficient using equation (9.18) can be calculated to adjust the new index level to that of the old index. Alternatively, the price change in the new product group index (December 2002 = 100) can be applied to the old index level each month as shown in equation (9.17).

9.129 As this example demonstrates, partial reweighting has particular implications for the practice of price updating the weights. Weighting information may not be available for some elementary aggregates at the time of reweighting. Thus, it may be necessary to consider price updating the old weights for those elementary aggregates for which no new weights are available. The weights for the latter may have to be price updated over a long period, which, for reasons given earlier, may give rise to some index bias if relative quantities have changed inversely with the relative price changes. Data on both quantity and price changes for the old index weights should be sought before undertaking price updating alone. The disadvantage of partial reweighting is that the implicit quantities belong to different periods than other components of the index, so that the composition of the basket is obscure and not well defined.

9.130 One may conclude that the introduction of new weights and the linking of a new series to the old series is not difficult in principle. The difficulties arise in practice when trying to align weight and price reference periods and when deciding whether higher-level indices comprising different elementary aggregates should be chained over time. It is not possible for this *Manual* to provide specific guidance on decisions such as these, but compilers should consider carefully the economic logic and statistical reliability of the resulting chained series and also the needs of users. To facilitate the decision process, careful thought should be given to these issues in advance during the planning of a reweighting exercise, paying particular attention to which indices are to be published.

C.7.7 Long- and short-term links

9.131 Consider a long-term chained index in which the weights are changed annually. In any given year, the current monthly indices when they are first calculated have to use the latest set of available weights, which cannot be those of the current year. However, when the weights for the year in question become available subsequently, the monthly indices can then be recalculated on basis of the weights for the same year. The resulting series can then be used in the long-term chained index rather than the original indices first published. Thus, the movements of the long-term chained index from, say, any one December to the following December, are based on weights of that same year, the weights being changed each December.[9]

9.132 Assume that each link runs from December to December. The long-term index for month m of year Y with December of year 0 as index reference period is then calculated by the formula[10]

$$(9.19)\quad P^{Dec0:mY} = \left(\prod_{Y=1}^{Y-1} P^{DecY-1:DecY} \right) P^{DecY-1:mY}$$

$$= P^{Dec0:Dec1} \times P^{Dec1:Dec2} \times \ldots \times P^{DecY-2:DecY-1} \times P^{DecY-1:mY}$$

The long-term movement of the index depends on the long-term links only as the short-term links are successively replaced by their long-term counterparts. For example, let the short-term indices for January to December 2001 be calculated as

[9]This method has been developed by the Central Statistical Office of Sweden, where it is applied in the calculation of the CPI. See Statistics Sweden (2001).

[10]In the actual Swedish practice, a factor scaling the index from December year 0 to the average of year 0 is multiplied onto the right-hand side of equation (9.19) to have a full year as reference period.

$$(9.20)\ P^{Dec00:m01} = \sum w_i^{00(Dec00)} P_i^{Dec00:m01},$$

where $W_i^{00(Dec00)}$ are the weights from 2000 price updated to December 2000. At the time when weights for 2001 are available, this is replaced by the long-term link

$$(9.21)\ P^{Dec00:Dec01} = \sum w_i^{01(Dec00)} P_i^{Dec00:Dec01},$$

where $W_i^{01(Dec00)}$ are the weights from 2001 price backdated to December 2000. The same set of weights from 2001 price updated to December 2001 are used in the new short-term link for 2002,

$$(9.22)\ P^{Dec01:m02} = \sum w_i^{01(Dec01)} P_i^{Dec01:m02}.$$

9.133 Using this method, the movement of the long-term index is determined by contemporaneous weights that refer to the same period. The method is conceptually attractive because the weights that are most relevant for most users are those based on production patterns at the time the price changes actually take place. The method takes the process of chaining to its logical conclusion, at least assuming the indices are not chained more frequently than once a year. Since the method uses weights that are continually revised to ensure that they are representative of current production patterns, the resulting index also largely avoids the substitution bias that occurs when the weights are based on the production patterns of some period in the past. The method may therefore appeal to statistical offices whose objective is to estimate an economic index.

9.134 Finally, it may be noted that the method involves some revision of the index first published. In some countries, there is opposition to revising a PPI once it has been first published, but it is standard practice for other economic statistics, including the national accounts, to be revised as more up-to-date information becomes available. This point is considered below.

C.8 Decomposition of index changes

9.135 Users of the index are often interested in how much of the change in the overall index is attributable to the change in the price of some particular product or group of products, such as petroleum or food. Alternatively, there may be interest in what the index would be if food or energy were left out. Questions of this kind can be answered by decomposing the change in the overall index into its constituent parts.

9.136 Assume that the index is calculated as in equation (9.10) or equation (9.11). The relative change of the index from $t-m$ to t can then be written as

$$(9.23)\ \frac{P^{0:t}}{P^{0:t-m}} - 1 = \frac{\sum w_i^b P_i^{0:t-m} P_i^{t-m:t}}{\sum w_i^b P_i^{0:t-m}} - 1.$$

Hence, a subindex from $t-m$ to 0 enters the higher-level index with a weight of

$$(9.24)\ \frac{w_i^b P_i^{0:t-m}}{\sum w_i^b P_i^{0:t-m}} = \frac{w_i^b P_i^{0:t-m}}{P^{0:t-m}}.$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$(9.25)\ \text{Effect} = \frac{w_i^b I_i^{0:t-m}}{I^{0:t-m}}\left(\frac{I_i^{0:t}}{I_i^{0:t-m}} - 1\right)$$
$$= \frac{w_i^b}{P_i^{0:t-m}}\left(P_i^{t:0} - P_i^{0:t-m}\right).$$

With $m = 1$, it gives the effect of a monthly change; with $m = 12$, it gives the effect of the change over the past 12 months.

9.137 If the index is calculated as a chained index, as in equation (9.15), then a subindex from $t-m$ enters the higher-level index with a weight of

$$(9.26)\ \frac{w_i^0 P_i^{k:t-m}}{P^{k:t-m}} = \frac{w_i^0\left(P_i^{0:t-m}/P_i^{0:k}\right)}{\left(P^{0:t-m}/P^{0:k}\right)}.$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$(9.27)\ \text{Effect} = \frac{w_i^0}{P^{k:t-m}}\left(P_i^{k:t} - P_i^{k:t-m}\right)$$
$$= \frac{w_i^0}{\left(I^{0:t-m}/I^{0:k}\right)}\left(\frac{I_i^{0:t} - I_i^{0:t-m}}{I_i^{0:k}}\right).$$

It is assumed that $t - m$ lies in the same link (that is, $t - m$ refers to a period later than k). If the effect of a subindex on a higher-level index is to be calculated across a chain, the calculation needs to be carried out in two steps, one with the old series up to the link period and one from the link period to period t.

9.138 Table 9.10 illustrates an analysis using both the index point effect and contribution of each component index to the overall 12-month change. The next-to-last column in Table 9.10 is calculated using equation (9.25) to derive the effect each component index contributes to the total percentage change. For example, for agriculture the index weight (w_i^b) is 38.73, which is divided by the previous period index ($P_i^{0:t-m}$), or 118.8, and then multiplied by the index point change ($P_i^{t:0} - P_i^{0:t-m}$) between January 2003 and January 2002, 10.5. The result shows that agriculture's effect on the 9.1 percent overall change was 3.4 percent. The change in agriculture contributed 37.3 percent (3.4 ÷ 9.1 × 100) to the total 12-month change.

C.9 Some alternatives to fixed-weight indices

9.139 Monthly PPIs are typically arithmetic weighted averages of the price indices for the elementary aggregates in which the weights are kept fixed over a number of periods, which may range from 12 months to many years. The repeated use of the same weights relating to some past period b simplifies calculation procedures and reduces data collection requirements. It is also cheaper to keep using the results from an old production survey than conduct an expensive new one. Moreover, when the weights are known in advance of the price collection, the index can be calculated immediately after the prices have been collected and processed.

9.140 However, the longer the same weights are used, the less representative they become of current production patterns, especially in periods of rapid technical change when new kinds of goods and services are continually appearing on the market and old ones disappearing. This may undermine the credibility of an index that purports to measure the rate of change in the production value of goods and services typically produced by businesses. Such a basket needs to be representative not only of the producers covered by the index but also of the revenue patterns at the time the price changes occur.

9.141 Similarly, if the objective is to compile an economic index, the continuing use of the same

Table 9.10. Decomposition of Index Change from January 2002 to January 2003

	Industry Sector	2000 weights (w_i^b)	Index (I) 2000	Jan 02	Jan 03	Percent change from Jan 02 to Jan 03	Effect (Contribution) Percentage points of total price change	Percent of total price change
1	Agriculture	38.73	100	118.8	129.3	8.8	3.4	37.3
2	Mining	6.40	100	132.8	145.2	9.3	0.7	7.3
3	Manufacturing	18.64	100	109.6	120.6	10.0	1.7	18.8
4	Transport and Communication	19.89	100	126.3	131.3	4.0	0.8	9.1
5	Services	16.34	100	123.4	141.3	14.5	2.4	26.8
	Total	100.00	100	120.2	131.1	9.1	9.1	100.0

fixed basket is likely to become increasingly unsatisfactory the longer the same basket is used. The longer the same basket is used, the greater the bias in the index is likely to become. It is well known that the Laspeyres index has a substitution bias compared with an economic index. However, a Lowe index between periods 0 and t with weights from an earlier period b will tend to exceed the Laspeyres substitution bias between 0 and t, becoming larger the further back in time period b is (see Chapter 15, Section D).

9.142 There are several possible ways of minimizing, or avoiding, the potential biases from the use of fixed-weight indices. These are outlined below.

9.143 *Annual chaining*. One way to minimize the potential biases from the use of fixed-weight indices is to keep the weights and the base period as up to date as possible by frequent weight updates and chaining. A number of countries have adopted this strategy and revise their weights annually. In any case, as noted earlier, it would be impossible to deal with the changing universe of products without some chaining of the price series within the elementary aggregates, even if the weights attached to the elementary aggregates remain fixed. Annual chaining eliminates the need to choose a base period, because the weight reference period is always the previous year, or possibly the preceding year.

9.144 *Annual chaining with current weights*. When the weights are changed annually, it is possible to replace the original weights based on the previous year, or years, by those of the current year if the index is revised retrospectively as soon as information on current-year revenue becomes available. The long-term movements in the PPI are then based on the revised series. This is the method adopted by the Swedish Statistical Office as explained in Section C.7.7 above. This method could provide unbiased results.

9.145 *Other index formulas*. When the weights are revised less frequently, say, every five years, another possibility would be to use a different index formula for the higher-level indices instead of an arithmetic average of the elementary price indices. One possibility would be a weighted geometric average. This is not subject to the same potential upward bias as the arithmetic average. More generally, a weighted version of the Lloyd-Moulton formula, given in Section B.6 above, might be considered. This formula takes account of the substitutions that purchasers make in response to changes in relative prices and should be less subject to bias for this reason. It reduces to the geometric average when the elasticity of substitution is unity, on average. It is unlikely that such a formula could replace the arithmetic average in the foreseeable future and gain general acceptance, if only because it cannot be interpreted as measuring changes in the value of a fixed basket. However, it could be compiled on an experimental basis and might well provide a useful supplement to the main index. It could at least flag the extent to which the main index is liable to be biased and throw light on its properties.

9.146 *Retrospective superlative indices*. Finally, it is possible to calculate a superlative index retrospectively. Superlative indices such as Fisher and Törnqvist-Theil treat both periods compared symmetrically and require revenue data for both periods. Although the PPI may have to be some kind of Lowe index when it is first published, it may be possible to estimate a superlative index later when much more information becomes available about producers' revenues period by period. At least one office, the U.S. Bureau of Labor Statistics, is publishing such an index for its CPI. The publication of revised or supplementary indices raises matters of statistical policy, but users readily accept revisions in other fields of economic statistics. Moreover, users are already confronted with more than one CPI in the EU where the harmonized index for EU purposes may differ from the national CPI. Thus, the publication of supplementary indices that throw light on the properties of the main index and that may be of considerable interest to some users seems justified and acceptable.

D. Data Editing

9.147 This chapter has been concerned with the methods used by statistical offices to calculate their PPIs. This concluding section considers the data editing carried out by statistical offices, a process closely linked to the calculation of the price indices for the elementary aggregates. Data collection, recording, and coding—the data-capture processes—are dealt with in Chapters 5 through 7. The next step in the production of price indices is the data editing. Data editing is here meant to comprise two steps:

- Detecting possible errors and outliers, and
- Verifying and correcting data.

9.148 Logically, the purpose of detecting errors and outliers is to exclude errors or the effects of outliers from the index calculation. Errors may be falsely reported prices, or they may be caused by recording or coding mistakes. Also, missing prices because of nonresponse may be dealt with as errors. Possible errors and outliers are usually identified as observations that fall outside some prespecified acceptance interval or are judged to be unrealistic by the analyst on some other ground. It may also be the case, however, that even if an observation is not identified as a potential error, it may actually show up to be false. Such observations are sometimes referred to as inliers. On the other hand, the sampling may have captured an exceptional price change, which falls outside the acceptance interval but has been verified as correct. In some discussions of survey data, any extreme value is described as an outlier. The term is reserved here for extreme values that have been verified as being correct.

9.149 When a possible error has been identified, it needs to be verified whether it is in fact an error or not. This can usually be accomplished by asking the respondent to verify the price, or by comparison with the price change of similar products. If it is an error, it needs to be corrected. This can be done easily if the respondent can provide the correct price or, where this is not possible, by imputation or omitting the price from the index calculation. If it proves to be correct, it should be included in the index. If it proves to be an outlier, it can be accepted or corrected according to a predefined practice—for example, omitting or imputation.

9.150 Although the power of computers provides obvious benefits, not all of these activities have to be computerized. However, there should be a complete set of procedures and records that controls the processing of data, even though some or all of it may be undertaken without the use of computers. It is not always necessary for all of one step to be completed before the next is started. If the process uses spreadsheets, for example, with default imputations predefined for any missing data, the index can be estimated and reestimated whenever a new observation is added or modified. The ability to examine the impact of individual price observations on elementary aggregate indices and the impact of elementary indices on various higher-level aggregates is useful in all aspects of the computation and analytical processes.

9.151 It is neither necessary nor desirable to apply the same degree of scrutiny to all reported prices. The price changes recorded by some respondents carry more weight than others, and statistical analysts should be aware of this. For example, one elementary aggregate with a weight of 2 percent, say, may contain 10 prices, while another elementary aggregate of equal weight may contain 100 prices. Obviously, an error in a reported price will have a much smaller effect in the latter, where it may be negligible, while in the former it may cause a significant error in the elementary aggregate index and even influence higher-level indices.

9.152 However, there may be an interest in the individual elementary indices as well as in the aggregates built from them. Since the sample sizes used at the elementary level may often be small, any price collected, and error in it, may have a significant impact on the results for individual products or industries. The verification of reported data usually has to be done on an index-by-index basis, using statistical analysts' experience. Also, for support, analysts will need the cooperation of the survey respondents to help explain unusual price movements.

9.153 Obviously, the design of the survey and questionnaires influences the occurrence of errors. Hence, price reports and questionnaires should be as clear and unambiguous as possible to prevent misunderstandings and errors. Whatever the design of the survey, it is important to verify that the data collected are those that were requested initially. The survey questionnaire should prompt the respondent to indicate if the requested data could not be provided. If, for example, a product is not produced anymore and thus is not priced in the current month, a possible replacement would be requested along with details of the extent of its comparability with the old one. If the respondent cannot supply a replacement, there are a number of procedures for dealing with missing data (see Chapter 7).

D.1 Identifying possible errors and outliers

9.154 One of the ways price surveys are different from other economic surveys is that, although prices are recorded, the measurement concern is with price *changes*. As the index calculations consist of comparing the prices of matching observations from one period to another, editing checks should focus on the price changes calculated from pairs of observations, rather than on the reported prices themselves.

9.155 Identification of unusual price changes can be accomplished by

- Nonstatistical checking of input data,
- Statistical checking of input data, and
- Output checking.

These will be described in turn.

D.1.1 Nonstatistical checking of input data

9.156 Nonstatistical checking can be undertaken by manually checking the input data, by inspecting the data presented in comparable tables, or by setting filters.

9.157 When the price reports or questionnaires are received in the statistical office, the reported prices can be checked manually by comparing these with the previously reported prices of the same products or by comparing them with prices of similar products from other establishments. While this procedure may detect obvious unusual price changes, it is far from sure that all possible errors are detected. It is also extremely time consuming, and it does not identify coding errors.

9.158 After the price data have been coded, the statistical system can be programmed to present the data in a comparable form in tables. For example, a table showing the percentage change for all reported prices from the previous to the current month may be produced and used for detection of possible errors. Such tables may also include the percentage changes of previous periods for comparison and 12-month changes. Most computer programs and spreadsheets can easily sort the observations according to, say, the size of the latest monthly rate of change so that extreme values can easily be identified. It is also possible to group the observations by elementary aggregates.

9.159 The advantage of grouping observations is that it highlights potential errors so that the analyst does not have to look through all observations. A hierarchical strategy whereby all extreme price changes are first identified and then examined in context may save time, although the price changes underlying elementary aggregate indices, which have relatively high weights, should also be examined in context.

9.160 Filtering is a method by which possible errors or outliers are identified according to whether the price changes fall outside some predefined limits, such as ±20 percent or even 50 percent. This test should capture any serious data coding errors, as well as some of the cases where a respondent has erroneously reported on a different product. It is usually possible to identify these errors without reference to any other observations in the survey, so this check can be carried out at the data-capture stage. The advantage of filtering is that the analyst need not look through numerous individual observations.

9.161 These upper and lower limits may be set for the latest monthly change, or change over some other period. Note that the set limits should take account of the context of the price change. They may be specified differently at various levels in the hierarchy of the indices—for example, at the product level, at the elementary aggregate level, or at higher levels. Larger changes for products with prices known to be volatile might be accepted without question. For example, for monthly changes, limits of ±10 percent might be set for petroleum prices, while for professional services the limits might be 0 percent to +5 percent (as any price that falls is suspect), and for computers it might be –5 percent to zero, as any price that rises is suspect. One can also change the limits over time. If it is known that petroleum prices are rising, the limits could be 10 percent to 20 percent, while if they are falling, they might be –10 percent to –20 percent. The count of failures should be monitored regularly to examine the limits. If too many observations are being identified for review, the limits will need to be adjusted, or the customization refined.

9.162 The use of automatic deletion systems is not advised, however. It is a well-recorded phenomenon in pricing that price changes for many products, especially durables, are not undertaken smoothly over time but saved up to avoid what are termed "menu costs" associated with making a price change. These relatively substantial increases may take place at different times for different models of products and have the appearance of extreme, incorrect values. To delete a price change for each model of the product as being "extreme" at the time it occurs is to ignore all price changes for the industry.

D.1.2 Statistical checking of input data

9.163 Statistical checking of input data compares, for some time period, each price change with the change in prices in the same or a similar sample. Two examples of such filtering are given here, the first based on nonparametric summary measures and the second on the log-normal distribution of price changes.

9.164 The first method involves tests based on the median and quartiles of price changes, so they are unaffected by the impact of any single extreme observation. Define the median, first quartile, and third quartile price relatives as R_M, R_{Q1}, and R_{Q3}, respectively. Then, any observation with a price ratio more than a certain multiple C of the distance between the median and the quartile is identified as a potential error. The basic approach assumes price changes are normally distributed. Under this assumption, it is possible to estimate the proportion of price changes that are likely to fall outside given bounds expressed as multiples of C. Under a normal distribution, R_{Q1} and R_{Q3} are equidistant from R_M; thus, if C is measured as $R_M - (R_{Q1} + R_{Q3})/2$, 50 percent of observations would be expected to lie within $\pm C$ from the median. From the tables of the standardized normal distribution, this is equivalent to about 0.7 times the standard deviation (σ). If, for example, C was set to 6, the distance implied is about 4σ of the sample, so about 0.17 percent of observations would be identified this way. With $C = 4$, the corresponding figures are 2.7σ, or about 0.7 percent of observations. If $C = 3$, the distance is 2.02σ, so about 4 percent of observations would be identified.

9.165 In practice, most prices may not change each month, and the share of observations identified as possible errors as a percentage of all changes would be unduly high. Some experimentation with alternative values of C for different industries or sectors may be appropriate. If this test is to be used to identify possible errors for further investigation, a relatively low value of C should be used.

9.166 To use this approach in practice, three modifications should be made. First, to make the calculation of the distance from the center the same for extreme changes on the low side as well as on the high side, a transformation of the relatives should be made. The transformed distance for the ratio of one price observation i, S_i, should be

$$S_i = 1 - R_M/R_i \text{ if } 0 < R_i < R_M \text{ and}$$
$$= R_i/R_M - 1 \text{ if } R_i \geq R_M.$$

Second, if the price changes are grouped closely together, the distances between the median and quartiles may be very small, so that many observations would be identified that had quite small price changes. To avoid this, some minimum distance, say, 5 percent for monthly changes, should be also set. Third, with small samples, the impact of one observation on the distances between the median and quartiles may be too great. Because sample sizes for some elementary indices are small, samples for similar elementary indices may need to be grouped together.[11]

9.167 An alternative method can be used if it is thought that the price changes may be distributed log-normally. To apply this method, the standard deviation of the log of all price changes in the sample (excluding unchanged observations) is calculated and a goodness of fit test (χ^2) is undertaken to identify whether the distribution is log-normal. If the distribution satisfies the test, all price changes outside two times the exponential of the standard deviation are highlighted for further checking. If the test rejects the log-normal hypothesis, all the price changes outside three times the exponential of the standard deviation are highlighted. The same caveats mentioned before about clustered changes and small samples apply.

[11]For a detailed presentation of this method, the reader is referred to Hidiroglou and Berthelot (1986). The method can be expanded also to take into account the level of the prices, so that, for example, a price increase from 100 to 110 is attributed a different weight than a price increase from 10 to 11.

9.168 The second example is based on the Tukey algorithm. The set of price relatives are sorted and the highest and lowest 5 percent flagged for further attention. In addition, having excluded the top and bottom 5 percent, exclude the price relatives that are equal to 1 (no change). The arithmetic (trimmed) mean (AM) of the remaining price relatives is calculated. This mean is used to separate the price relatives into two sets, an upper and a lower one. The upper and lower "mid-means"—that is, the means of each of these sets (AM_L, AM_U)—are then calculated. Upper and lower Tukey limits (T_L, T_U) are then established as the mean ±2.5 times the difference between the mean and the mid-means:

$$T_U = AM + 2.5\ (AM_U - AM),$$
$$T_L = AM - 2.5\ (AM - AM_L).$$

Then, all those observations that fall above T_U and below T_L are flagged for attention.

9.169 This is a similar but simpler method than that based on the normal distribution. Since it excludes all cases of no change from the calculation of the mean, it is unlikely to produce limits that are very close to the mean, so there is no need to set a minimum difference. However, its success will also depend on there being a large number of observations on the set of changes being analyzed. Again, it will often be necessary to group observations from similar elementary indices. For any of these algorithms, the comparisons can be made for any time period, including the latest month's changes, but also longer periods, in particular, 12-month changes.

9.170 The advantage of these two models of filtering compared with the simple method of filtering is that for each period the upper and lower limits are determined by the data itself and hence are allowed to vary over the year, given that the analyst has decided the value of the parameters entering the models. A disadvantage is that, unless one is prepared to use approximations from earlier experience, all the data have to be collected before the filtering can be undertaken. Filters should be set tightly enough so that the percentage of potential errors that turn out to be real errors is high. As with all automatic methods, the flagging of an unusual observation is for further investigation, as opposed to automatic deletion.

D.1.3 Checking by impact, or data output checking

9.171 Filtering by impact, or output editing, is based on calculating the impact an individual price change has on an index to which it contributes. This index can be an elementary aggregate index, the total index, or some other aggregate index. The impact a price change has on an index is its percentage change times its effective weight. In the absence of sample changes, the calculation is straightforward: it is the nominal (reference period) weight, multiplied by the price relative, and divided by the level of the index to which it is contributing. So the impact on the index I of the change of the price of product i from time t to t + 1 is $\pm w_i \left(p_{t+1} / p_t \right) / I_t$, where w_i is the nominal weight in the price reference period. A minimum value for this impact can be set, so that all price changes that cause an impact greater than this change can be flagged for review. If index I is an elementary index, then all elementary indices may be reviewed, but if I is an aggregative index, prices that change by a given percentage will be flagged or not depending on how important the elementary index to which they contribute is in the aggregate.

9.172 However, at the lowest level, births and deaths of products in the sample cause the effective weight of an individual price to change quite substantially. The effective weight is also affected if a price observation is used as an imputation for other missing observations. The evaluation of effective weights in each period is possible, though complicated. However, as an aid to highlighting potential errors, the nominal weights, as a percentage of their sum, will usually provide a reasonable approximation. If the impact of 12-month changes is required to highlight potential errors, approximations are the only feasible filters to use, since the effective weights will vary over the period.

9.173 One advantage of identifying potential errors this way is that it focuses on the results. Another advantage is that this form of filtering also helps the analyst to describe the contributions to change in the price indices. In fact, much of this kind of analysis is done after the indices have been calculated, as the analyst often wishes to highlight those indices that have contributed the most to overall index changes. Sometimes the analysis results in a finding that particular industries have a relatively high contribution to the overall price change, and that is considered unrealistic. The change is traced back to an error, but it may be late in the production cycle and jeopardize the schedule release date. There is thus a case for identifying such unusual contributions as part of the data editing procedures. The disadvantage of this method is that an elementary index's change may be rejected at that stage. It may be necessary to override the calculated index, though this should be a stopgap measure only until the index sample is redesigned.

D.2 Verifying and correcting data

9.174 Some errors, such as data coding errors, can be identified and corrected easily. Ideally, these errors are caught at the first stage of checking, before they need to be viewed in the context of other price changes. Dealing with other potential errors is more difficult. Many results that fail a data check may be judged by the analyst to be quite plausible, especially if the data checking limits are broad. Some editing failures may be resolved only by checking the data with the respondent.

9.175 If a satisfactory explanation can be obtained from the respondent, the data can be verified or corrected. If not, procedures may differ. Rules may be established that if a satisfactory explanation is not obtained, then the reported price is omitted from the index calculation. On the other hand, it may be left to the analyst to make the best judgment as to the price change. However, if an analyst makes a correction to some reported data, without verifying it with the respondent, it may subsequently cause problems with the respondent. If the respondent is not told of the correction, the same error may persist in the future. The correct action depends on a combination of the confidence in the analysts, the revision policy in the survey, and the degree of communication with respondents. Most statistical offices do not want to unduly burden respondents.

9.176 In many organizations, a disproportionate share of activity is devoted to identifying and following up potential errors. If the practice leads to little change in the results, as a result of most reports ending up as being accepted, then the bounds on what are considered to be extreme values should be relaxed. More errors are likely introduced by respondents failing to report changes that occur than from wrongly reporting changes, and the goodwill of respondents should not be unduly undermined.

9.177 Generally, the effort spent on identifying potential errors should not be excessive. Obvious mistakes should be caught at the data-capture stage. The time spent identifying observations to query, unless they are highly weighted and excessive, is often better spent treating those cases in the production cycle where things have changed—quality changes or unavailable prices—and reorganizing activities toward maintaining the relevance of the sample and checking for errors of omission.

9.178 If the price observations are collected in a way that prompts the respondent with the previously reported price, the respondent may report the same price as a matter of convenience. This can happen even though the price may have changed, or even when the particular product being surveyed is no longer available. Because prices for many products do not change frequently, this kind of error is unlikely to be spotted by normal checks. Often the situation comes to light when the contact at the responding outlet changes and the new contact has difficulty in finding something that corresponds to the price previously reported. It is advisable, therefore, to keep a record of the last time a particular respondent reported a price change. When that time has become suspiciously long, the analyst should verify with the respondent that the price observation is still valid. What constitutes too long will vary from product to product and the level of overall price inflation, but, in general, any price that has remained constant for more than a year is suspect.

D.2.1 Treatment of outliers

9.179 Detection and treatment of outliers (extreme values that have been verified as being correct) is an insurance policy. It is based on the fear that a particular data point collected is exceptional by chance, and that if there were a larger survey, or even a different one, the results would be less extreme. The treatment, therefore, is to reduce the impact of the exceptional observation, though not to ignore it, since, after all, it did occur. The methods to test for outliers are the same as those used to identify potential errors by statistical filtering, described above. For example, upper and lower bounds of distances from the median price change are determined. In this case, however, when observations are found outside those bounds, they may be changed to be at the bounds or imputed by the rate of change of a comparable set of prices. This outlier adjustment is sometimes made automatically, on the grounds that the analyst by definition has no additional information on which to base a better estimate. While such automatic adjustment methods are employed, the *Manual* proposes caution in their use. If an elementary aggregate is relatively highly weighted and has a relatively small sample, an adjustment may be made. The general prescription should be to include verified prices and the exception to dampen them.

D.2.2 Treatment of missing price observations

9.180 It is likely that not all the requested data will have been received by the time the index needs to be calculated. It is generally the case that missing data turns out to be delayed. In other cases, the respondent may report that a price cannot be reported because neither the product, nor any similar substitute, is being made anymore. Sometimes, of course, what started as an apparent late report becomes a permanent loss to the sample. Different actions need to be taken depending on whether the situation is temporary or permanent.

9.181 For temporarily missing prices, the most appropriate strategy is to minimize the occurrence of missing observations. Survey reports are likely to come in over a period of time before the indices need to be calculated. In many cases, they follow a steady routine—some respondents will tend to file quickly, others typically will be later in the processing cycle. An analyst should become familiar with these patterns. If there is a good computerized data-capture system, it can flag reports that appear to be later than usual, well before the processing deadline. Also, some data are more important than others. Depending on the weighting system, some respondents may be particularly important, and such products should be flagged as requiring particular scrutiny.

9.182 For those reports for which no estimate can be made, two basic alternatives are considered here (see Chapter 7 for a full range of approaches): imputation, preferably targeted, in which the missing price change is assumed to be the same as some other set of price changes, or an assumption of no change, as the preceding period's price is used (the carryforward method discussed in Chapter 7). However, this latter procedure ignores the fact that some prices will prove to have changed, and if prices are generally moving in one direction, this will mean that the change in the indices will be understated. It is not advised. However, if the index is periodically revised, the carryforward method will lead to less subsequent revisions than making an imputation, since for most products, prices do not generally change in any given period. The standard approach to imputation is to base the estimate of the missing price observation on the change of some similar group of observations.

9.183 There will be situations where the price is permanently missing because the product no longer exists. Since there is no replacement for the missing price, an imputation will have to be made each period until either the sample is redesigned or until a replacement can be found. Imputing prices for permanently missing sample observations is, therefore, more important than in the case of temporarily missing reports and requires closer attention.

9.184 The missing price can be imputed by the change of the remaining price observations in the elementary aggregate, which has the same effect as removing the missing observation from the sample, or by the change of a subset of other price observations for comparable products. The series should be flagged as being based on imputed values.

9.185 Samples are designed on the basis that the products chosen to observe are representative of a wider range of products. Imputations for permanently missing prices are indications of weakness in the sample, and their accumulation is a signal that the sample should be redesigned. For indices where there are known to be a large number of deaths in the sample, the need for replacements should be anticipated.