SECTION IV. NESTED FORMULAS WITH VULNERABILITY AND STRENGTH VARIABLES

This section presents the methodology and statistical results from applying the nested model to certain vulnerability and strength variables.

Part A presents the Davidson-MacKinnon J test used in estimating the relative weights of strength and vulnerability variables in a two-equation formula system.

Part B presents a summary table and regression results from nested models used to estimate the relative weights of strength and vulnerability variables in the determination of actual quotas.

The regression results for the nested model where the vulnerability model is estimated first (Regression No. 53) indicate that the relative weight for the vulnerability variables (α) is 0.54, while that for the nested model where the strength model is estimated first (Regression No. 54) indicate that the relative weight for the vulnerability variables (α) is 0.43. Since the t statistics for the relative weights in both regressions are statistically significant, neither regression can be rejected. The relative contributions of variables in these regressions, based on the weighted coefficients, indicate that the relative contribution of Y is around 20 percent on average, and that of VC is over 50 percent on average.

The regression equations and resulting quota distributions for members from the nested formulas is presented in Statistical Appendix, Part B, Section III.

A. Davidson-MacKinnon J Test

To estimate the relative weights of strength and vulnerability variables in a two-equation formula system, we used the Davidson-MacKinnon J test. The J test proceeds as follows:

1. Assume the following two models explain actual quotas:

Model A with vulnerability variables: $Y = Z\gamma + v$

where Y = actual quotas, Z = vulnerability variables, γ = coefficients of Z, and v = error term

Model B with strength variables: $Y = X\beta + u$

where X = strength variables, β = coefficients of X, and u = error term.

Models A and B are nonnested if one cannot be derived as a special case of the other. To test whether the models are nonnested, we estimate the artificially nested model C, and then test one or both of the original models against it:

Model C, nesting or encompassing models A and B:

$$Y = (1 - \alpha)\, X\beta + \alpha\, Z\gamma + w$$

where α = relative weight of vulnerability variables, with $0 \leq \alpha \leq 1$, and w = error term.

2. Davidson-MacKinnon J test. Since model C is not estimable, because the parameters α, β, and γ are not separately identifiable, Davidson and MacKinnon[1] suggested that model C be replaced by one in which the unknown parameters of the model that is not being tested are replaced by consistent estimates of those parameters. The idea is that if one model is the correct model, then the fitted values from the other model should not have further explanatory power when estimating that model. Thus, the J testing procedure follows the steps:

(a) Estimate (by OLS) model A and obtain the estimated (fitted) Y values, $\hat{Y}^A$.

(b) Add $\hat{Y}^A$ of step 1 as an additional regressor to model B and estimate (by OLS) the following model D:

Model D: $Y = X\delta + \alpha\, \hat{Y}^A + w_1$

[1] Davidson, R. and J.G. MacKinnon, 1981, "Several Tests for Model Specifications in the Presence of Alternative Hypotheses," Econometrica, Vol. 49. pp. 781–93.

where δ = coefficients of X, with $\delta = \beta(1-\alpha)$, and w_1 = error term.

In this step, we obtain the estimate of α, i.e., the relative weight of the vulnerability variables.

(c) Using the t test, test the hypothesis that $\alpha = 0$.

(d) If the hypothesis that $\alpha = 0$ is not rejected, (i.e., the t statistic on the α is not statistically significant), we can accept (i.e., not reject) model B as the true model because $\hat{Y}^A$ included in model D, which represents the influence of vulnerability variables not included in model B, has no additional explanatory power beyond that contributed by model B. In other words, model B encompasses model A in the sense that the latter model does not contain any additional information that will improve the performance of model B. By the same token, if the null hypothesis is rejected, model B cannot be the true model.

(e) Then, we reverse the order of estimation of models A and B. We now estimate model B first, use the estimated Y values from this model as regressor in model D, repeat step (d), and decide whether to accept model A over model B. More specifically, we estimate the following model E:

Model E: $$Y = Z k + \theta \hat{Y}^B + w_2$$

where $\hat{Y}^B$ = the estimated Y values from model B, θ = coefficients of $\hat{Y}^B$, with $\theta \equiv (1 - \alpha)$, i.e., the relative weight of the strength variables, k = coefficients of Z, with $k = \alpha * \gamma$, and w_2 = error term.

In this step, we obtain the estimate of $(1 - \alpha)$, i.e., the relative weight of the strength variables.

We now test the hypothesis that $\theta = 0$. If this hypothesis is not rejected, we choose model A over B. If the hypothesis that $\theta = 0$ is rejected, choose B over A, as the latter does not improve over the performance of B.

When one does a pair of nonnested tests, there are four possible outcomes, since each of model A and B may or may not be rejected. Furthermore, although it is intuitively appealing, the J test will not be able to provide a clear answer if it leads to the acceptance or rejection of both models. In case both models are rejected, neither model helps to explain the behavior of Y. Similarly, if both models are accepted, as Kmenta notes, the data are apparently not rich enough to discriminate between the two hypotheses [models]."[2] Or, as Davidson and MacKinnon note, "when neither model is rejected, we must conclude that both models apparently fit the data about equally well and that neither

[2] Kmenta, Jan, 1986, Elements of Econometrics, Macmillan, 2d ed., New York, p. 597.

provides evidence that the other is misspecified. Presumably, either the two models are very similar, or the data set is not very informative."[3]

3. Applying the J test to data for the Eleventh Review, we find that neither model A nor B can be rejected,[4] and as noted by Davidson and MacKinnon, a possible interpretation is that the traditional data set does not capture all available information. An important missing element could be the political agreements at the time of the Bretton Woods conference, whose influence has survived through the equiproportional element in subsequent quota increases.

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[3] Davidson, Russell and James G. MacKinnon, 1993, Estimation and Inference in Econometrics, Oxford University Press, New York and Oxford, p. 383.

[4] See Part B of this note.

B. Nested Models With Vulnerability and Strength Variables

The model with the vulnerability variables includes two variables: the variability of current receipts, VC, defined as one standard deviation from a five-year monthly average over a 13-year period (1982–94), and population, POP, in 1994. The model with the strength variables includes four variables: GDP, Y, in 1994; the average monthly reserves with gold valued at market prices, RM, in 1994; the annual average of current receipts over a five-year period (1990–94), C; and the four-year moving average of net private capital flows (1991–94), NNKFL.

The attached table summarizes the regression results of two nested models and a linear equation used to estimate the relative weights of strength and vulnerability variables. The first nested model is estimated using fitted values for the vulnerability model (Regression No. 53), while the second nested model is estimated using fitted values for the strength model (Regression No. 54). The coefficient estimates of the linear equation of actual quotas on both strength and vulnerability variables (Regression No. 55) are OLS estimates. The coefficients shown in this table are weighted coefficients, equal to the product of the estimated coefficients from the respective regressions times their corresponding weights (α) and ($1-\alpha$).

Table IV.1. Comparison of Coefficients from Nested Models and Ordinary Least Squares to Estimate the Relative Weights of Strength and Vulnerability Variables

Dependent Variable: Q	Strength Variables				Vulnerability Variables	
	Y	RM	C	NNKFL	VC	POP
Nested model where a regression of a vulnerability variable is estimated first 1/	0.00253	-0.00510	0.00578	0.03311	0.68187	1.27053
Nested model where a regression of strength variables is estimated first 1/	0.00192	0.00381	0.01271	0.03638	0.54445	2.22456
Ordinary Least Squares	0.00258	-0.01370	0.00905	0.03458	0.61920	2.41307

Q is the actual quota; Y is GDP in a recent year (1994); RM is average monthly reserves with gold valued at market prices in a recent year (1994); C is the annual average current receipts over a recent five-year period (1990-94); NNKFL is the four-year moving average of net private capital flows (1991-94). VC is the variability of current receipts, defined as one standard deviation from a five-year moving average over a recent 13-year period (1982-1994); POP is population in 1994.

1/ Coefficients shown have been multiplied by the estimated relative weights, for the strength and vulnerability variables, i.e., they represent the "net" effect of the variable on the estimated quota.