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CALCULATION OF POSITIONS AND INTEREST ON CANADIAN BONDS HELD BY NON-RESIDENTS

Prepared by Réjean Tremblay
Statistics Canada

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CALCULATION OF POSITIONS AND INTEREST ON CANADIAN BONDS HELD BY NON-RESIDENTS

Réjean Tremblay[1]

1. Introduction

In the Canadian balance of payments processing system, four types of value are maintained for bonds: new issue, maturity value, book value and market value. New issue is defined as the value of proceeds received at the time of issue. Par value is defined as the value of proceeds to be redeemed at maturity. Book value is the new issue value plus interest payable. The market value is the value that the instrument has in the market at a certain point in time.

Two broad categories of income are calculated on bonds: interest on coupon and interest from amortization. Interest on coupon is the income generated from the contractual arrangements agreed upon a the time of issue of the instrument (also called the coupon). Interest on amortization applies only when the instrument is issued at a price other than the value at maturity. Interest from amortization will be negative if the instrument is issued at a greater value than its value at maturity: it is then said to have been issued at a premium. If the amortization is positive, the instrument is said to have been issued at a discount.

Most of the examples found in this document are not realistic. The values were chosen in order to provide an effective graphical representation, for example, of the exponential effect of interest from amortization.

2. Book Value

Example #1: The instrument is issued for $25,000. It matures at 24 months, at which time it is worth $100,000. This is what might be called a "deep discount bond" (some might even say "deep deep discount"!). It has a coupon rate of 10% per year, payable every 6 months.

The graph below shows the position book value, interest payable from amortization and interest payable on coupon. The par value is not shown, but it remains $100,000 throughout the life of the instrument.

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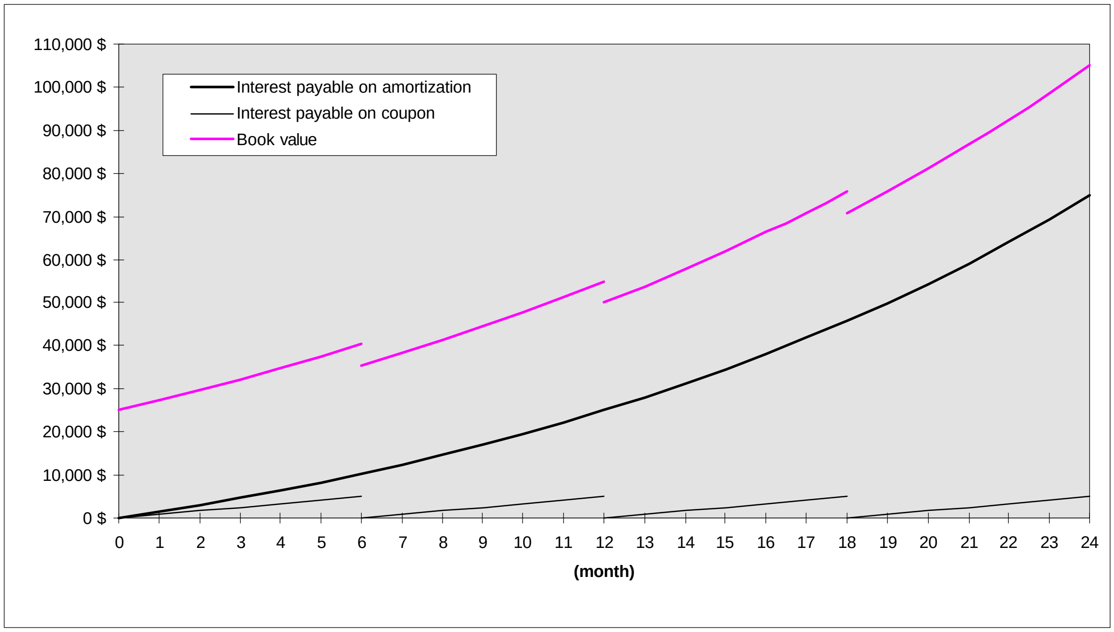
[1] Program analyst at the System Development Division of Statistics Canada. The work was conducted under the direction of Marc Fournier of the same division and Gerard Meagher of the Balance of Payments Division.



3. Maturity value and book value

This section describes what maturity value and book value are. The description of market value will be presented further on.

3.1 Evolution in the value of an instrument over time

If an instrument is issued at the same price as its value at maturity, then there will not be interest from amortization. There may be a difference between the value at time of issue and the value at maturity. An example is a deep discount bond. In such case, the income from amortization is the difference between these two values. The book value changes over time because of amortization and interest payable on coupon. The increase in the book value from the amortization takes place according to a continuing compound interest model. This interest rate is called the "Internal Rate of Return", or IRR. This rate is derived using the following formula:

Equation 1

$$IRR = \frac{\ln\left(\left(\frac{(V2 - V1)}{V1}\right) + 1\right)}{\Delta t}$$

where

V1 is the value at time of issue,
V2 is the value at maturity,
Δt is the number of years between issuance and maturity
ln is the natural logarithm (with base e).

The book value is calculated using the IRR and interest payable on coupon. The formula is as follows:

Equation 2 $$BookValue = V1 + V1(e^{(IRRx\Delta t)} - 1) + IPC$$

where

V1 is the value at time of issue.
e is the napierian number (2.71828...),
Δt is the number of years between issuance and the maturity date
IPC are interest payable on coupon

The second term of the last equation, $V1(e^{(IRR.\Delta t)} - 1)$, represents the interest payable on amortization. The book value is thus made up of the value at time of issue plus interest payable on amortization and on coupon. Equation 2 may be reduced.

Equation 3 $$BookValue = V1e^{(IRRx\Delta t)} + IPC$$

In the examples that follow, we will use Equation 2, simply because it gives a better picture of the makeup of book value: value at time of issue plus interest payable on amortization plus interest payable on coupon.

Example #1a: Say that an instrument is sold at $95,000 with a value at maturity of $100,000 two years later. The instrument does not have a coupon. The rate of increase of the value of the instrument (IRR) will be:

$$IRR = \frac{\ln\left(\left(\frac{(100000 - 95000)}{95000}\right) + 1\right)}{2}$$

IRR= 2.5647%

The book value after 18 months will be:

$$BookValue = 95000 + 95000(e^{(0.025647x18/12)} - 1)$$
$$BookValue = 95000 + 3726$$
$$BookValue = 98726$$

The par value does not change during the life of the instrument unless there are retirements before maturity (see below) or the instrument is issued in tranches (see below). In the above example, the par value is always $100,000.

3.2 Geographical distribution

When an instrument is held in several countries, the calculations of the value for each country will be proportional to the distribution of the holding in these various countries. The basis for determining the various proportions is of course the par value.

Example 1b: Take the same data as in Example 1a. If Canada holds $30,000 of the instrument, Japan holds $50,000 and the United States holds $20,000, the values in each country after 18 months will be as follows:

Country	Par value ($)	Book value ($)
Canada	30,000	98,726*30% = 29,618
Japan	50,000	98,726*50% = 49,363
United States	20,000	98,726*20% = 19,745
Total	100,000	98,726

3.3 Partial Retirement

Retirements are made on the basis of the value at maturity. The value at maturity decreases by an amount equal to the value of the retirement. The book value, which is calculated on the basis of the initial issue value plus interest payable, decreased by the same percentage as the par value. The market value after the retirement will be calculated on the basis of the par value, adjusted by retirements (see the section entitled «Market value» for more detail).

Example #1c: Say that an instrument is sold at $50,000 with a value at maturity of $100,000 two years later. Assume that a retirement of $25,000 (par value) is made at the end of the 18th month. Starting in the 19th month there remains 75% of the initial value of the instrument, that is, $75,000 in par value. For all the values calculated from the 19th month onward, the reference amount will therefore be 50000 x 75%. The IRR for such an instrument is:

$$IRR = \frac{\ln\left(\left(\frac{(100000-50000)}{50000}\right)+1\right)}{2}$$

$$IRR = 34.6574\%$$

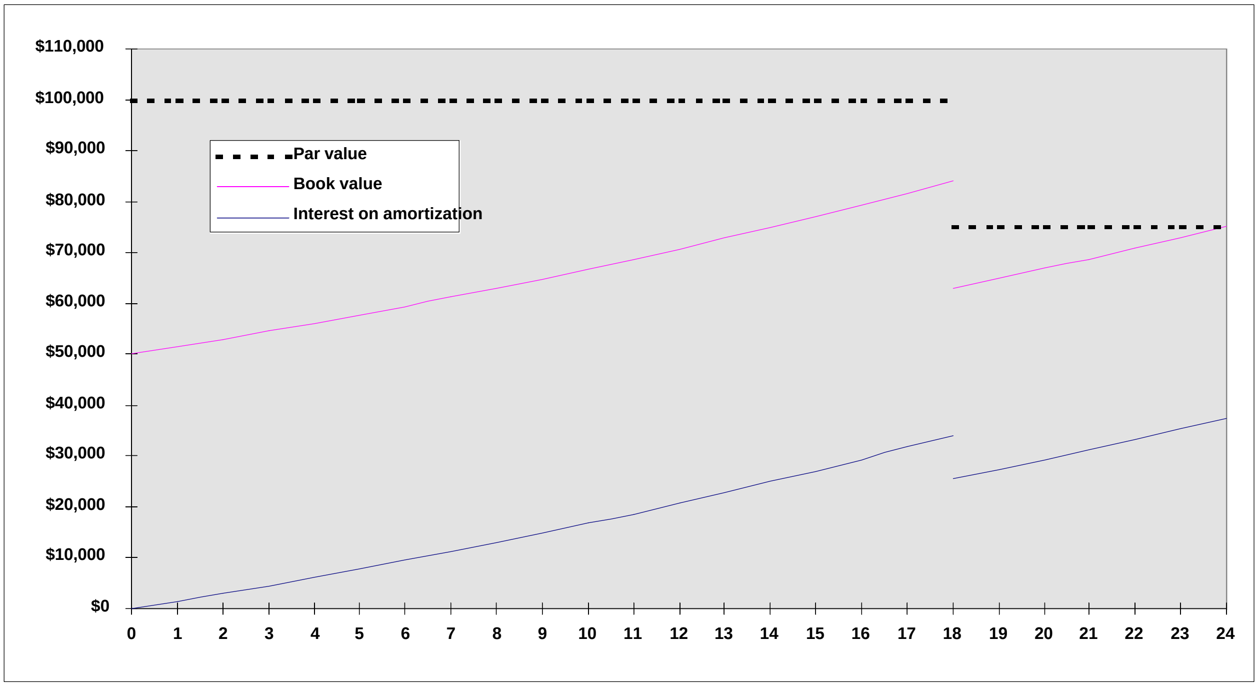
At the end of the 19th month, the book value will be:

$$BookValue = (50000x75\%) + (50000x75\%)(e^{(0.346574x19/12)} - 1)$$

$$BookValue = (37500) + (37500)(e^{(0.346574x19/12)} - 1)$$

$$BookValue = 64915$$

The following graph shows the evolution of the par value and book value as well as interest payable on amortization.



3.4 Issue in tranches

The total value of an instrument issued in tranches is equal to the summed value of the positions of the different tranches. This is the case for the three types of positions: at maturity, at book and at market value. The calculation of maturity value is straightforward. It is calculated as if the instrument was issued in one tranche. For the book value, things are more complicated: each tranche may have a distinct issue price which may differ for each tranche. Each tranche will then have its own rate of increase (IRR). The idea is that various tranches may be sold at different prices. They therefore have a different rate of increase (IRR). For the book value, the formula is as follows:

Equation 4

$$BookValue = (Va1 + Va1(e^{IRR1 \times \Delta t1} - 1) + \ldots + Vax + Vax(e^{IRRx \times \Delta tx} - 1) + IPC$$

$$BookValue = (Va1(e^{IRR1 \times \Delta t1} - 1) + \ldots + Vax(e^{IRRx \times \Delta tx} - 1) + IPC$$

Where

Va1 is the issue value of tranche #1,
Vax is the issue value of the x^e tranche,
IRR1 is the IRR of tranche #1,
IRRx is the IRR of x^e tranche,
Δt1 is the number of years between the issue of tranche #1 and the date at which the book value is calculated,

Δtx is the number of years between the issue of x^e tranche and the date at which the book value is calculated,

IPC are interest payable on coupons.

Example #2a: Say that an instrument with no coupon is sold in two tranches. The first is sold at \$25,000 with a value at maturity of \$50,000. The second is sold one month later at \$35,000 with a value at maturity of \$50,000. The instrument matures 24 months after the issue of the first tranche. The par value during the first month is \$50,000. After the first month, the par value is \$50,000 plus \$50,000 for a total of \$100,000.

The book value at the end of the second month will be the summed book value of the two tranches. The IRR of the first tranche will be:

$$IRR1 = \frac{\ln\left(\left(\frac{(50000-25000)}{25000}\right)+1\right)}{(24/12)}$$
$$IRR1 = 34.6574\%$$

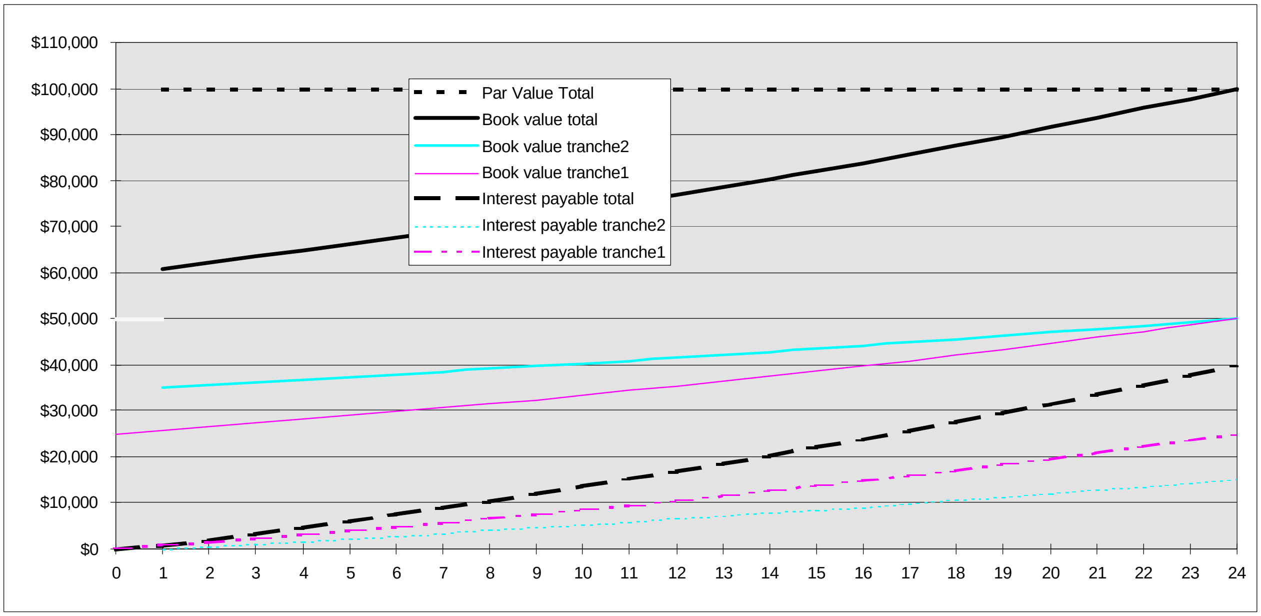
The IRR of the second tranche will be:

$$IRR2 = \frac{\ln\left(\left(\frac{(50000-35000)}{35000}\right)+1\right)}{(23/12)}$$
$$IRR2 = 18.6091\%$$

The book value will be:

$$BookValue = 25000 + 25000(e^{(0.346574x2/12)} - 1) + 35000 + 35000(e^{(0.186091x1/12)} - 1)$$
$$BookValue = 25000 + 1487 + 35000 + 547$$
$$BookValue = 62034$$

The graph below shows the evolution of the par value and the book value as well as interest payable from amortization for each tranche and for the total.



3.5. **Retirements on issue in tranches**

Partial retirements on an instrument issued in tranches will be distributed among the different tranches in proportion to their size, with the proportional weight of each obviously being determined by its par value.

Example #2b: Assume the following data. Two tranches with no coupon are sold, one at $45,000 with a value of $50,000 at maturity and the other at $47,000 one month later with a value of $50,000 at maturity. The maturity is 24 months after the issue of the first tranche. Now assume that a retirement of $25,000 is made at the end of the sixth month. Since the two tranches are equal, 50% of the retirement will be made from the first tranche and 50% from the second tranche.

The par value of the first tranche becomes:

$$ParValue1 = 50000x75\%$$
$$ParValue1 = 37500$$

The par value of the second tranche becomes:

$$ParValue2 = 50000x75\%$$
$$ParValue2 = 37500$$

The total par value will be:

$$ParValue = 37500 + 37500 = 75000$$

The book value of the first tranche at the end of the seventh month becomes:

$$BookValue1 = (45000x75\%) + (45000x75\%)\left(e^{(0.05268x7/12)} - 1\right)$$
$$BookValue1 = 34803$$

The book value of the second tranche becomes:

$$BookValue2 = (47000x75\%) + (47000x75\%)\left(e^{(0.032283x6/12)} - 1\right)$$
$$BookValue2 = 35824$$

The total book value will be:

$$BookValue = 34803 + 35824 = 70627$$

3.6. Geographic distribution of issue by tranche

What has been discussed under "Geographic distribution" above remains valid in the case of a bond issued by tranches. In other words, the calculation of positions for each country is pro-rated according to the total position of the instrument. The calculation of position at maturity is calculated as if the instrument was issued in one single tranche. The calculation of the book value is more complicated. The pro-rating factor is composed of the value at maturity of the country divided by the total value at maturity. By inserting this factor in the calculation of book value, the following is obtained:

Equation 5

$$BookValue = (Va1(e^{IRR1 \times \Delta t1}) + \cdots + Vax(e^{IRRx \times \Delta tx}) \times \frac{vn}{Vnte} + IPC)$$

where

Va1 is the issue value of tranche #1,
Vax is the issue value of x^e tranche,
IRR1 is the IRR of tranche #1,
IRRx is the IRR of x^e tranche,
Δt1 is the number of years between tranche #1 and the date the book value is calculated,
Δtx is the number of years between the issue of x^e and the date the book value is calculated,
Vn is the maturity of value of the specific country,
at the date book value is calculated,
Vnte is the total maturity value (i.e. not taking into account partial retirements) of the instrument at the date book value is calculated,
IPC are interest payable on coupons for the country.

It is important to note that the pro-rate indicator (Vn/Vnte) changes over time reflecting that Vnte changes overtime. After the issue of tranche #1, Vnte is equal to the par value of tranche #1; after the issue of the tranche #2, Vnte is equal to the sum of par values of tranches #1 and #2, etc.

4. Market value

There are two ways to calculate market value; the calculation method to be used depends on whether the instrument was traded during the month. «Traded» as used here means that parts of the instrument changed hands between Canada and other countries.

For instruments traded recently – that is, during the month – see the section entitled «Market value for traded instrument». For other instruments, see the section entitled, «Market value for untraded instrument».

4.1. Market value for traded instruments

If an instrument was traded during the month, when the market value position is taken, the market value is calculated using the following formula with the average of all transactions having taken place in the current month:

$$MarketValue = ParValue \times Average\left(\frac{|TrMarketValue| - ShortTermLoan}{|TrParValue|} \right)$$

where Par Value is the value at maturity
Short TermLoan is equal to interest payable on coupon at the time of the transaction,
TrMarket Value is the price at which the bond traded,
TrparValue is the par value of the bond transacted.

For the purpose of calculating position, we use the aggregate of transactions having occurred during the month. It is likely that these transactions will be done with several countries. For the calculation of position, we will take the average of all the selling countries involved in the aggregate transaction.

4.2. Market value for untraded instruments

For instruments not traded, a yield is determined using reference tables. Two reference tables are used for this purpose: the Benchmark Table contains standard market indexes, while the Differential Table contains an adjustment to be applied to the benchmark values. Shown below are examples of how these two tables look.

4.2.1.1. Benchmark Table

Bench-mark	Sector	Currency	Years to Maturity	Benchmark value per month			
				01/95	02/95	03/95	etc...
BM1	Govt Canada	Cdn$	3-5	8.5	8.5	8.3	
BM2	US Govt	US$	1-3	8.6	8.7	8.4	
BM3	Japan bond	Yen	>5	4.1	4.2	4.2	
BM4	Hydro Que	Euro US	3-5	8.1	8.2	8.1	

4.2.1.2. Differential Table

Sector	Currency	Years to maturity				
		< 1	1-3	3-5	5-7	> 7
Federal Direct	Cdn$	BM1+0.6	BM1+0.5	BM1+0	BM1+0	BM1+2.5
	Euro Cdn	BM1+0.6	BM1+0	BM1+1	BM1+1.5	BM1+3
	US$	etc...				
	etc...					
Federal Enterp	Cdn$	BM1+0.6	BM1+0.5	BM1+0	BM1+0	BM1+2.5
	Euro Cdn	BM2+0.6	BM2+0	BM2+1	BM2+1.5	BM2+3
	US$	etc...				
	etc...					
Prov. direct	Cdn$	etc...				
etc...						

To determine the yield for an untraded instrument, it is first necessary to search through the Differential Table according to the variables Sector, Currency and Years to maturity. The yield is determined by the formula in the corresponding box.

Once the yield is determined according to the above method, the following formula should be used to obtain the market value of the instrument:

$$MarketValue = \frac{CouponRate \times ParValue \times \left(1 - \frac{1}{(1+Yield)^t}\right)}{Yield} + \frac{ParValue}{(1+Yield)^t}$$

where

Coupon rate is the interest rate from coupon
t is the number of years left to maturity
Yield is the factor as calculated above.
Par Value is the value at maturity.

5. Calculation of interest on coupon

5.1. General

Interest on coupon is the income generated by the coupon of an instrument. On the other hand, interest from amortization arises from the difference between the issue and the maturity values. Interest from amortization will be dealt with further on.

The calculation of interest on coupon follows a linear model.

There are two ways to express the interest on coupon generated by an instrument. One is to use an interest rate, while the other is to specify a lump sum for a given period. The following two sections explain how to calculate an amount of interest in general. Specific cases, such as accrued, payable and paid interest, will be dealt with subsequently.

5.1.1. Calculation of interest using an interest rate

When interest is expressed by means of an interest rate, the amount is determined by the following formula:

Equation 6 $$Interest = ParValue \times Tx \times \Delta t$$

where

ParValue is the «ParValue» as calculated above,
Tx is the interest rate,
Δt is the period in years.

Example #3: Say that an instrument has a par value of $100,000. Assume an interest rate of 8.5% payable twice yearly. The value of the accrual at the end of the third month will be:

$$Accrual = 100000 \times 0.085 \times \frac{3}{12}$$
$$Accrual = 2125$$

5.1.2. Calculation of interest on coupon on the basis of a lump sum

If interest is expressed with a lump sum, the following formula determines the amount applicable to a fraction of the payment period.

Equation 7

$$Interest = Payment \times \frac{\Delta t}{PayPer}$$

where

Payment is the amount of the payment,
Δt is the period in years,
PayPer is the total period applicable to the payment.

The amount of payment for a given country is determined on the pro-rata basis using the maturity value. The previous formula becomes:

Equation 8

$$Interest = Payment \times \frac{\Delta t}{PaiePer} \times \frac{Vn}{Vntc}$$

where

Payment is the amount of payment,
Δt is the period expressed in a year,
PaiePer is the total period for which the payment applies,
Vn is the par value for a specific country,
Vntc is the total par value.

Example #4: Say that an instrument has a par value of $100,000. An interest payment of $5,000 is scheduled for the end of the sixth month.

The value of the interest accrued to the end of the third month will be:

$$Accrual = 5000 \times \frac{\frac{3}{12}}{\frac{6}{12}}$$
$$Accrual = 5000 \times \frac{3}{6}$$
$$Accrual = 2500$$

5.1.3. Accruals from coupons

Accruals are the income generated by an instrument for a given period. Ordinarily, accruals are calculated for a given month; in this case the beginning of the period is the first day of the month and the end is the last day. Either equation 6 or equation 7 applies, depending on the case.

In the event that a payment took place during the month, the accruals are equal to the amount of interest accrued between the beginning of the month and the payment and between the payment and the end of the month. In the case of interest on coupon, the fact that a payment takes place during the reference period has no impact on the value of the accruals: the amount is the same as if there were no payment. This is due to the fact that interest on coupon follows a linear model.

Another definition of accruals is the difference between the interest payable at the end of the period (usually one month) and the interest payable at the beginning of the period. This is true only for month with payment.

5.1.4. Payables from coupons

Payables are defined as the interest that has accumulated since the last payment. If the interest is expressed using an interest rate, then Equation 6 applies. If interest is expressed on the basis of a lump sum, then Equation 7 applies. In both cases Δt is the time that has elapsed since the last payment (or issue).

An equivalent definition is that interest is the sum of accruals since the last payment. This is true for the instrument in total but not for each country. In the latter case, interest payable are calculated on a pro-rata basis using the maturity value at the time of calculation.

Payables relate to a date and not a time interval. By contrast, accruals (see below) relate to a time interval.

5.1.5. Interest paid from coupons

The amount of interest paid is equal to the amount of interest payable on the day of payment.

5.2. Change of position value

In the event that there is a change in the value at maturity within an accrual calculation period, the total amount of accruals will be the sum of accruals calculated for each of the stable periods. This is valid for any change, such as a new issue, retirements, geographical change, etc.

6. Calculation of interest on amortization

The concept of interest on amortization was introduced earlier in the document, in the section on the different types of value.

6.1. Accrual on amortization

The following formula is used to accrue interest from amortization:

$$DiscAccrual = V1(e^{IRR\times t2} - e^{IRR\times t1})$$

where

V1 is the issue value of the instrument,
IRR is the rate of change due to amortization,
t1 is the time elapsed between the beginning of the calculation period and the issue period,
t2 is the time elapsed between the end of the calculation period and the issue period.

In a more general way, if one takes into account the tranches and geographical distribution, the previous formula becomes:

$$DiscAccrual = (Va1(e^{IRR1\times t12} - e^{IRR1\times t11}) + \cdots + Vax(e^{IRRx\times t2x} - e^{IRRx\times t1x})\frac{Vn}{Vnte}$$

where

Va1 is the issue price of tranche #1,
Vax is the issue price of x^e tranche,
IRR1 is the IRR of tranche #1,
IRRx is the IRR of x^e tranche,
t11 is the time elapsed between the beginning of the calculation period and the issue of tranche #1,
t12 is the elapsed between the end of the calculation period and the issue of tranche,
t1x is the elapsed between the beginning of the calculation period and the issue of x^e tranche,
t2x is the elapsed between the end of the calculation period and the issue of x^e tranche,
Vn par value held by the country,
Vnte is the total par value issued (i.e. without partial retirement) for the instrument for the date of the position.

6.2. Interest payable on amortization

The same formula used for interest payable on amortization is also used for accruals, except that the variable t1 is equal to zero. Once reduced, the formula becomes:

$$DiscPayable = V1\left(e^{(IRR\times\Delta t)} - 1\right)$$

where

V1 is the issue value of the instrument,
IRR is the rate of increase of amortization,
Δt is the time elapsed between the period of calculation and the issue date.

In more general way, if one takes into account the tranches and geographical distribution, the previous formula becomes:

$$DiscPayable = \left(Va1\left(e^{(IRR1\times\Delta t1)}-1\right)+\ldots+Vax\left(e^{(IRRx\times\Delta tx)}-1\right)\right)\frac{Vn}{Vnte}$$

where

Va1 is the issue price of tranche #1,
Vax is the issue price of x^e tranche,
IRR1 is the IRR of tranche #1,
IRRx is the IRR of x^n tranche,
Δt1 is the time elapsed between the current date and the issue date of tranche #1,
Δtx is the time elapsed between the current date and the issue of x^e tranche,
Vn is the par value held by a country,
Vnte is the total par value issued (i.e. without taking into account partial retirements) of the instrument for the date of position.

6.2 Interest paid on amortization

Most bonds are retired at maturity. The difference between the issue price and the maturity price represents the interest which has been amortized (the interest payable from coupon becomes nil when the coupon is paid out).

When, however, there is a retirement prior to maturity, the interest an amortization will be according to the following formula.

$$DiscPaid = V1(e^{(IRR\times\Delta t)}-1)\frac{Vt}{Vnte}$$

where

Vt is the par value of the retirement
V1 is the book value at the time of withdrawal
IRR is IRR of the tranche
Δt is the time from the new issue to the partial retirement
Vnte is the total par value (i.e. regardless of partial retirement) of the instrument at the time the retirements occurs.

The following is interest paid from amortization when there are several tranches:

$$DiscPaid = \left(Va1\left(e^{(IRR1\times\Delta t1)}-1\right)+\ldots+Vax\left(e^{(IRRx\times\Delta tx)}-1\right)\right)\frac{Vn}{Vnte}$$

where

Va1 is the value of issue of tranche #1,
Vax is the the book value of x^e tranche at the time of withdrawal,
IRR1 is the IRR of tranche #1,
IRRx is the IRR of x^e tranche,
Δt1 is the time is the time from new issue to the partial retirement of tranche #1,
Δtx is the time is the time from new issue to the partial retirement of x^e tranche,
Vnte is the total par value (i.e. regardless of partial retirements) of the instrument at the time the retirement occurs.

Appendix A. Vocabulary

The following definitions of terms apply to this document.

A.1. Untraded instrument

The expression «untraded instrument» is generally used to indicate that the instrument was not sold on the secondary market during a given month. An instrument is said to have been sold on the secondary market if there were one or more transactions on that instrument.

A.2. Accrual

Interest accrued during a given period, such as, say, during March 1995. This is a current account item.

A.3. Payable

Interest accumulated since the last interest payment. This is a position item. The change in interest payable is a financial account item.

A.4. Lump sum

Interest expressed as a fixed amount.

A.5. Book value

Value of an instrument taking into account interest payable. This is a position concept.

A.6. Market value

Value that an instrument trades (or would have if it traded) in the market at a certain point in time. This is a position concept.

A.7. Par value

Value of an instrument at maturity. Also called "maturity value." This is a position concept.

A.8. Interest rate

Percentage of maturity value expressing the amount of interest to be paid at periodic intervals.

Appendix B. Origin of equations to calculate amortization

Equation 1 and Equation 2 are used to accrue income on bonds from coupon and from amortization for a given period. The portion for accruing income on amortization is as follows:

$$Ac = V1\left(\left(e^{IRRxt2} - 1\right)\left(e^{IRRxt1} - 1\right)\right)$$

where

V1 is the initial value of the instrument,
IRR is the interest rate ("Internal Rate of Return»)
t1 is the beginning of the period,
t2 is the end of the period.

The previous equation, once reduced, becomes:

$$Ac = V1(e^{\,IRR\times t2} - e^{\,IRR\times t1})$$

Appendix C. Origin of equation on amortization, formal demonstration

The following demonstration shows in a more rigourous way the origin of the equation on amortization presented in Appendix B. This section shows how to derive equations using the compound interest model on a continuous basis starting with the model of compounded interest on a discrete basis.

The discrete model is according to the following law:

$$V1(1+i)^n = V2$$

where i is the rate used for every capitalization and n is the number of capitalization. Given n is expressed in number of years, j is the annual rate. Hence the rate of capitalization for x period per year is j/x,

The model becomes:

$$V1\left(1+\frac{j}{x}\right)^{xn} = V2$$

The exponential "xn" arise from capitalizing x times a year which in turn means n times x capitalization in n years. Now, what does happen it the capitalization is carried out an infinite time in a one year. The differential calculusindicates that the limit of the expression $\left(1+\frac{j}{x}\right)^{x}$, when x tends towards infinity is e^j. Hence, when x tends toward infinity, the model become

$$V1e^{jn} = V2$$

The latter formula is at the origin of formulas 1 and 2 found in the document. Hence, with a bit of manipulation, we can arrive to the following

$$j = \frac{\ln\left(\frac{V2}{V1}\right)}{n}$$

which is the IRR formula,

$$Ac = V1(e^{jn_2} - e^{jn_1})$$

which is the formula used to accrue interest on amortization.