A common criticism of the first generation of macroeconometric models is that they represented an incomplete macroeconomic paradigm because they failed to integrate a well-articulated “supply side” into the sticky-price Keynesian framework. In particular, because of the failure to impose adequate theoretical restrictions, most of these early models were dynamically unstable or did not have well-defined, long-run equilibrium properties; in some cases, for example, an increase in government spending could lead to implausible, never-ending expansions of output. Consequently, the first-generation econometric models have been discarded in policymaking institutions, and research has focused on investigating more thoroughly the channels through which policies affect macroeconomic aggregates in both the short term and the long term.

The second generation of macroeconomic models that were designed for policy analysis incorporated supply sides of varying complexity. A key element was generally a linear Phillips curve that hypothesized a short-run trade-off between inflation and unemployment while in many cases embodying the restrictions of the long-run natural rate hypothesis (LR-NRH)—that is, the hypothesis that the economy gravitates over the long term toward a natural rate of unemployment that cannot be reduced permanently by increasing either the level or the rate of growth of the money supply.44

The Short-Run Trade-Off Between Inflation and Unemployment

The hypothesis of a short-run trade-off between inflation and unemployment is consistent with considerable evidence that decisions to tighten monetary policy have been followed by reductions in both inflation and output growth (Romer and Romer, 1989). This raises the question of why monetary policy has effects on output in the short run. A plausible and popular answer is that nominal wages and prices are sticky in the short run, reflecting the existence of medium-term and perhaps overlapping wage contracts (Fischer, 1977; Taylor, 1980; Calvo, 1983). More fundamentally, price and wage stickiness may result from costs of changing individual prices or renegotiating wages (see the discussion in Fischer, 1994).

One way to specify a Phillips curve relationship is with the following two equations:

\[ \pi_t = \pi^e_{t+1} + F(y_t) + \varepsilon^p_t \]  

and

\[ \pi^e_{t+1} = \Phi \pi^m_{t+1} + [1 - \Phi] \pi_{t-1}, \]

where \( \pi_t \) is the rate of change of the non-oil GNP deflator during year \( t \); \( \pi^e_{t+1} \) is the expectation during year \( t \) of inflation one year ahead; \( \pi^m_{t+1} \) is the model-consistent solution for inflation one year ahead; \( y_t \) is the output gap; \( \Phi \) is a nonnegative weight on the model-consistent solution; and \( \varepsilon^p \) is a stochastic error term. Equation (7) can be given several different interpretations. One interpretation is that \( (1 - \Phi) \) represents the proportion of myopic agents who always use the previous year’s inflation rate to forecast future inflation, while \( \Phi \) represents the proportion of informed agents. Alternatively, equation (7) can be interpreted as a very simple learning model where \( \Phi \) represents the speed at which agents learn the underlying structure of the economy and exogenous forcing processes. A third interpretation, perhaps more suitable to variants of equation (7) with additional lagged terms, views \( \Phi \) as the proportion of medium-term overlapping wage contracts that are renegotiated in period \( t \).

Many economists have argued that the Phillips curve is not a structural relationship, taking the view that no matter which of the above interpretations is adopted, \( \Phi \) cannot be regarded as a “deep structural parameter” that is insensitive to the behavior of the monetary authorities. If monetary policy becomes highly inflationary, for example, the proportion of myopic agents would decline, the speed of learning...

---

44Thus, the Phillips curve is vertical in the long run, and any attempt to hold unemployment below its natural rate will result in accelerating inflation; see Friedman (1968). The incorporation of a supply side into what was essentially a Keynesian paradigm is sometimes referred to as the “neoclassical synthesis.”
would increase, and the prevalence and average duration of multiperiod nominal contracts would decline.

In practice, however, almost all macro models describe the supply side of the economy with a Phillips curve, given the strength of the empirical correlations in historical data and the lack of more attractive alternatives. This is also the case in MULTIMOD, which treats the Phillips curve as a structural relationship. That being said, we regard it as incumbent on model users to avoid subjecting their models to policy experiments for which the underlying assumptions about inflation expectations would break down.

When specifying a Phillips curve relationship in the form of equations (6) and (7), it is important to place a positive weight on the model-consistent expectation (that is, $\Phi > 0$). Otherwise, the model is inconsistent with the LR-NRH, since with $\Phi = 0$ it would always be possible for the monetary authorities to fool people systematically and raise the level of output in the long run by continuously increasing the rate of growth of the money supply.\(^{45}\) The condition that $\Phi > 0$ implies that the long-run Phillips curve is vertical and that real variables, such as the unemployment rate, are tied down by real factors, such as tastes and technology, and are to a first approximation independent of both the level of the money supply (the condition of monetary neutrality) and the rate of growth of the money supply (monetary superneutrality). These long-run neutrality properties are an important trademark of MULTIMOD and several other prominent modern macroeconomic models.

**Implications of Linear and Convex Phillips Curves**

While the introduction of well-articulated supply sides has succeeded in pinning down the medium-term and long-term properties of macroeconomic models and clarifying the channels through which monetary and fiscal policies affect the economy in both the short run and long run, models with linear Phillips curves have been criticized on two important counts. First, they are inadequate by themselves for explaining certain prominent asymmetries in labor markets, such as the observation that unemployment rates appear to be stuck at high levels in a number of countries while examples of low unemployment equilibria seem rare. Second, they imply that as long as policy authorities succeed on average in achieving a given target (or informal objective) for the rate of inflation, the speed and intensity with which stabilization policies react to unexpected shocks do not have first-order effects on the average levels of output and employment over time.

A number of industrial countries have experienced prolonged upward trends in unemployment that have persisted during much or all of the past three decades. Germany provides a good example. Although the unemployment rate has varied from year to year, each major economic downturn and subsequent recovery, or supply-side shock—such as the oil price hikes of the 1970s and German unification in the early 1990s—has ratcheted unemployment upward. While economists have long recognized that noncyclical unemployment can be the result of job search and market distortions—including regulations (for example, minimum wage rates and restrictive labor laws) and unionization—the observed phenomenon of upward trending unemployment rates has elicited attention over the past decade to theories that link the behavior of unemployment to factors that can plausibly generate a process of asymmetric hysteresis (Summers, 1988; Blanchard and Summers, 1986; Blanchard and Katz, 1997).

Such theories of the history-dependence of unemployment rates—and, in particular, of why an upward-trending proportion of the labor supply has not been absorbed or reabsorbed into the employed labor force—have focused predominantly on insider-outsider effects and human capital depreciation as likely explanatory factors. It has also been emphasized, however, that in the context of a nonlinear Phillips curve, the effectiveness of stabilization policies can interact importantly with other factors in explaining the upward trends in unemployment rates (Isard and Laxton, 1996; and Faruqee, Laxton, and Rose, 1998).

To provide an analytic framework in which stabilization policies can have first-order effects on the average levels of output and employment (at any given average inflation rate), the Mark III version of MULTIMOD continues to work within the confines of the long-run natural rate hypothesis, but features significant convexity in the short-run Phillips curve. This implies that the tradition of decomposing unemployment into structural and cyclical components requires modification, as does the traditional discussion of the nonaccelerating inflation rate of unemployment—the so-called NAIRU. To illustrate, Figure 1 shows a convex (to the origin) short-run Phillips curve, plotted as a relationship between expectations-augmented inflation (vertical axis) and the unemployment rate (horizontal axis), with expectations-augmented inflation corresponding to the difference

\(^{45}\)In early versions of macroeconomic models, it was quite common to set $\Phi$ equal to zero because model builders did not have access to robust solution algorithms for solving models where agents were assumed to have some knowledge of the underlying structure and policy process.
between actual and (ex ante) expected inflation.\footnote{\textsuperscript{46}} The unemployment rate at which expectations-augmented inflation is zero—labeled $u^e$ in Figure 1 and referred to as the DNAIRU or deterministic NAIRU—corresponds to the structural rate of unemployment that would prevail in a deterministic world. It is critical to recognize that the DNAIRU is not a feasible stable-inflation equilibrium in a stochastic economy with convexity. The average rate of unemployment that would be associated with nonaccelerating inflation (and expectations equilibrium) in a stochastic world—labeled $\bar{u}$ in the figure and referred to as the NAIRU—must lie above the DNAIRU. This is because convexity in the short-run Phillips curve means that inflation rises faster when unemployment is below the DNAIRU than it falls when unemployment is commensurately above the DNAIRU.\footnote{\textsuperscript{47}} If $u$ were maintained equal to $u^e$ on average, the asymmetry in the response of inflation to symmetric aggregate demand shocks would make it impossible to maintain a constant average inflation rate.

The convex short-run Phillips curve combined with standard models of inflation expectations implies that stabilization policies that are successful in avoiding boom and bust cycles will reduce the average unemployment rate and raise the average level of output. This can be seen in Figure 1, which has been drawn under the assumption that the unemployment rate is symmetrically distributed around the NAIRU over the range between $u_1$ and $u_2$. The important point is that success in reducing the variability of unemployment will also lower its mean value. One can see this immediately from Figure 1 by imagining a tighter control on the dispersion of unemployment. The line $LL$ would move down and to the left and the gap between $\bar{u}$ and $u^e$ would shrink. The key lesson is that stabilization can matter in the sense that policies that either induce or allow extreme variability in the business cycle will also cause a permanently higher NAIRU.\footnote{\textsuperscript{48}}

With convex models of the Phillips curve, the analysis of unemployment behavior, in addition to identifying the cyclical variation of actual unemployment around its average rate, needs to recognize that the average rate of unemployment exceeds the structural rate of unemployment by an amount that generally reflects both the nature and the magnitude of economic shocks and the effectiveness of stabilization policies. With convexity in the short-run Phillips curve, stabilization policies can have permanent effects on unemployment and output.\footnote{\textsuperscript{49}} When combined with more elaborate models of inflation expectations and imperfect policy credibility, the convex Phillips curve paradigm will hopefully provide a much richer macroeconomic framework for assessing the effectiveness of stabilization policies.\footnote{\textsuperscript{50}}

**Specification of the Mark III Phillips Curve**

The Phillips curve in Mark III follows closely on the heels of some recent empirical work that has extended an estimation methodology proposed in Debelle and Laxton (1997) and applied to a fairly large group of industrial countries.\footnote{\textsuperscript{51}} The empirical strat-
Intrinsic and Expectational Dynamics

The core version of Mark III retains the assumption that the inflation process has important myopic and forward-looking components, as in equation (7). However, an attempt has been made to distinguish between inflation inertia that arises because of overlapping wage and price contracts, which we refer to as intrinsic dynamics, and those dynamics that arise because of expectational rigidities.

The Phillips curve in the core version of Mark III is represented by equation (9).

\[ \pi_t = \delta \pi_{t-1} + (1 - \delta) \pi_{t-1}^{\text{myopic}} + \gamma \frac{(u_t^* - u_t)}{(u_t - \phi_t)} + \epsilon_t. \]  

The nonlinear term in the unemployment rate can be derived by assuming, as implied by various labor market models, that the degree of real wage rigidity increases in the face of higher unemployment and market pressures for a wage decline; recall Box 6. The motivation for inflation inertia in equation (9) reflects an overlapping contracting framework where a fixed proportion of the economy’s population receives nominal wage adjustments linked to lagged inflation, while the remaining proportion negotiates wage increases linked to expected future inflation.53

The Mark III representation of the myopic and forward-looking components model has several desirable properties. First, by incorporating intrinsic dynamics, it is consistent with the view that inflation can be costly to reduce even if agents have model-consistent expectations.54 Second, the Mark III specification can in principle account for some differences in the structure and type of nominal inertia and real wage rigidities across countries, and hence for differences in inflation persistence across countries. Third, by separating the intrinsic and expectational dynamics, it makes it easier to isolate the implications of alternative assumptions about the formation of inflation expectations.

Proxies for Inflation Expectations

There has been some success in utilizing survey measures of inflation expectations to estimate quarterly models of the U.S. unemployment-inflation process that feature convexity.55 Unfortunately, reliable historical survey measures of inflation expectations do not exist for all of the industrial countries. The strategy for Mark III is to use available survey measures of one-year-ahead inflation expectations data for the United States to derive an estimate of the myopic component of inflation expectations \((1 - \Phi)\). We then employ this estimate of \((1 - \Phi)\), combined with data on the yields on long-term government bonds in each country, to extract a proxy for one-
Box 6. Model of a Convex Phillips Curve

A very simple derivation of a nonlinear Phillips curve is presented to provide some further insight into the features of labor and goods markets that may possibly underlie its convexity. To proceed, we borrow from the framework discussed in Layard, Nickell, and Jackman (1991) and Clark and Laxton (1997). Specifically, price-setting and wage-setting behavior are characterized respectively as follows:

\[ p = w + \delta_0 - \delta_1 u, \]  
\[ w = p^e + \phi_0 - \phi_1 u. \]  

Equation (1) specifies that (the logarithm of) the price level \( p \) is set as a (constant) markup over unit labor costs, expressed in terms of the (log) wage rate \( w \) and the rate of unemployment \( u \), which can also be related to output or capacity utilization via Okun’s law. Equation (2) represents a target real wage expression in which \( \phi_0 \) signifies the responsiveness of real wage to the level of unemployment, and \( p^e \) is the expected price level.

In the presence of nominal inertia, equation (1) can be viewed as defining a target price level (denoted with a bar), and the observed price can be assumed to adjust only gradually to the target price according to

\[ \Delta p = \lambda_1 (p - p_{-1}) + \lambda_2 \Delta p_{-1}. \]  

The first term on the right-hand side of equation (3) represents an error-correction mechanism, while the second term introduces (higher-order) inertia in inflation (\( \pi = \Delta p \)) and not just the price level.

Using these three equations with \( \lambda_2 = 1 - \lambda_1 \), one can derive a linear expectations-augmented Phillips curve that summarizes the (reduced-form) relationship between inflation and unemployment:

\[ \pi = \lambda_1 \pi^e + (1 - \lambda_1) \pi_{-1} + \lambda_1 (\phi_1 + \delta_1)(u^*-u), \]  

where expected inflation is defined by \( \pi^e = p^e - p_{-1} \), where in equilibrium, \( \pi_{-1} = \pi^e \) in conjunction with an unemployment rate equal to the DNAIRU, \( u^* \), which coincides with the NAIRU in the linear case.\(^1\)

In equation (4), the responsiveness of \( \pi \) to \( \pi^e \), as measured by \( \lambda_1 \), can be regarded as the degree of nominal flexibility. The coefficient on the unemployment gap reflects both the degree of nominal flexibility and—following the discussion in Layard, Nickell, and Jackman (1991)—the degree of real rigidity \( RR = (\phi_1 + \delta_1)^{-1} \). Note that RR is high when \( \phi_1 \) and \( \delta_1 \) are small or when prices and wages are not very responsive to the level of unemployment. The interaction between the degrees of nominal flexibility and real rigidity determines the slope of the linear Phillips curve, reflecting the constant short-run trade-off between inflation and unemployment. For example, greater nominal flexibility (larger \( \lambda_1 \)), other things being equal, implies a steeper Phillips curve.

Nonlinearity is introduced into the Phillips curve when the parameters that determine the short-run trade-off between inflation and unemployment are variables that change with labor market conditions. For example, if the degree of real wage rigidity is a function of the level of unemployment, such that \( RR = h(u) \) with \( h' > 0 \), the Phillips curve would exhibit convexity. In the simple case where this function has the linear form \( h(u) = Au - \Omega \), we can rewrite the Phillips curve as follows:

\[ \pi = \lambda_1 \pi^e + (1 - \lambda_1) \pi_{-1} + \gamma (\frac{u^*-u}{u-\Omega}); \gamma = \frac{\lambda_1}{\Lambda}. \]  

Note that an increase in nominal flexibility (larger \( \lambda_1 \)) or an increase in real flexibility (smaller \( \Lambda \)) raises the coefficient \( \gamma \) in the above expression. The motivation for assuming that the degree of real wage rigidity increases in the face of higher unemployment can be found in the implications of several labor market models.

For example, in incentive wage models, employers find it desirable to pay an efficiency wage greater than the market-clearing wage in order to induce effort, sustain morale, reduce turnover, avoid adverse selection problems, and so on, which places an effective floor (that is, asymmetry) on adjustment in real wages. To the extent that the wage floor becomes an increasingly binding constraint as unemployment rises, these models have the implication that the degree of real wage rigidity increases with the unemployment rate. Correspondingly, the "wage gap"—between the prevailing (efficiency) wage and the market clearing wage—would increase with the unemployment rate. Asymmetric wage bargaining could also have similar implications. In effect, once market rigidities vary with the level of activity and employment, the wage-price mechanism is no longer linear.

---

1Imposing \( \lambda_1 + \lambda_2 = 1 \) in equation (3) translates it into an error-correction equation for inflation rather than the price level: \( \Delta \pi = \lambda_1 (\pi - \pi_{-1}) \), thus this specification implies inflation persistence, given by \( 1 - \lambda_1 \), and allows for a non-zero (steady-state) equilibrium inflation rate where \( \pi_{-1} = \pi = \bar{\pi} \).

2In expectational equilibrium \(( p = p^e )\) with nonaccelerating prices \(( \Delta \pi = 0 )\), the equilibrium unemployment rate, or NAIRU, is given by

\[ u^* = \frac{\phi_0 + \delta_1}{\phi_1 + \delta_1}. \]
terest rate from a measure of the country’s long-term nominal interest rate.\textsuperscript{56} Second, we regress the Michigan Survey measure of one-year-ahead inflation expectations for the United States\textsuperscript{57} on lagged CPI inflation and the constructed long-term inflation expectation measure $\pi^{LTE}$. This produces an estimate of $(1 – \Phi)$ of 0.47.\textsuperscript{58} Using this estimate of $(1 – \Phi)$, we then construct a set of one-year-ahead inflation expectation proxies for each country using the following equation.

$$\hat{\pi}_{t+1} = \Phi \pi^{LTE} + [1 – \Phi] \pi_{t-1}. \quad (10)$$

This empirical strategy results in highly persistent deviations between the constructed inflation expectation and observed inflation rates. Such "expectation errors" are, in principle, capable of explaining persistence in the business cycle, and, when combined with the convex Phillips curve, this empirical methodology can produce fairly large magnitudes for the cyclical level of unemployment in those countries that have experienced both high unemployment and low policy credibility.\textsuperscript{59}

The fact that these inflation expectation proxies are based on long-term interest rates, which have long-term memory components, does not necessarily imply that agents’ forecasts of inflation are irrational.\textsuperscript{60} Because there is nothing really fundamental in a democratic system to tie down the distribution of future monetary policies—beyond the reputation of today’s policymakers—it may take a considerable amount of time for rational agents to become convinced that governments are committed to low inflation.\textsuperscript{61} Along the transition path, it may still be rational for market participants, when confronted with a new regime, to discount recent inflation performance under the new regime and to place a high weight on long moving averages of past inflation performance until it is evident that policymakers are committed to living with any adverse consequences of low inflation.\textsuperscript{62}

### III The Inflation-Unemployment Nexus

The methodology that we employ to estimate the Mark III Phillips curve is based on estimating model-consistent measures of the labor market tightness term $(u^* - u)$ in equation (11).

$$\pi_t = \delta \hat{\pi}_{t+1} + [1 – \delta] \pi_{t-1} + \gamma \frac{[u^* - u_t]}{[u_t – \Phi]} + \varepsilon_t. \quad (11)$$

Equation (11) can be estimated as a simple time-varying parameter model by assuming that $u^*$, the DNAIRU, follows a random walk. If we define an artificial variable $Z_t$ as follows,

$$Z_t = \frac{\gamma u^*_t}{u_t – \Phi}, \quad (12)$$

then equation (11) can be rewritten as

$$\pi_t = \delta \hat{\pi}_{t+1} + [1 – \delta] \pi_{t-1} + Z_t – \gamma \frac{u_t}{[u_t – \Phi]} + \varepsilon_t. \quad (13)$$

\textsuperscript{56}This first step is based on an assumption that most of the variation in long-term bond yields is a result of variation in long-term inflation expectations. The measure of the equilibrium world real interest rate term is meant to account for the trend increase (low frequency variation) in the equilibrium real interest rate that has been a result of the rise in world government debt; for details, see Debelle and Laxton (1997). Goodfriend (1993) and Barr and Campbell (1996) argue that most of the high-frequency variation in long-term bond yields is driven by inflation scares rather than by historical movements in the ex ante real rate of interest.

\textsuperscript{57}Reliable survey measures of inflation expectations for other countries span substantially shorter time periods than the data for the United States.

\textsuperscript{58}The standard error of this estimate is 0.18. Users of MULTIMOD may vary this parameter in simulation mode.

\textsuperscript{59}The term “policy credibility” refers here to the speed with which inflation expectations adjust in response to announced changes in policy objectives for inflation.

\textsuperscript{60}The estimation strategy employed here is considerably different from what has been employed in the recent U.S. academic literature on Phillips curves. The latter literature imposes a very strong form of the rational expectations hypothesis, where agents do not make serially correlated forecast errors even in small samples. In structural models, this extreme form of rational expectations breaks down quickly with modest amounts of uncertainty, which explains why these models are rejected overwhelmingly by less restricted time-series representations of the data. For evidence of autocorrelated forecast errors and historical regime shifts in the inflation process, see Evans and Wachtel (1993), Laxton, Ricketts, and Rose (1994), and Ricketts and Rose (1995).

\textsuperscript{61}Recent studies focusing on the behavior of long-term interest rates suggest that market participants in some cases revise their expectations of long-term inflation very slowly in response to observed inflation performance. For example, Gagnon (1996) shows that the Fisher equation holds surprisingly well if long moving averages of past inflation are used to measure long-term inflation expectations.

\textsuperscript{62}Using a multiple regime-switching model, Laxton, Ricketts, and Rose (1994) show that, because of historical inflation bias, it may take the monetary authorities a considerable length of time to establish credibility in a low-inflation regime and that, along the transition path, there will be a persistent period of excess supply until credibility has been established. However, in any one particular draw, inflation expectations will converge slowly and then suddenly credibility will improve when the time-series properties of the inflation process become consistent with the underlying policy fundamentals. This view of the importance of policy credibility can account for slow adjustment on the one hand, as well as for cases where long-term interest rates jump because market participants become convinced that the monetary authorities and democratic process are committed to low inflation. This can explain, for example, why it has taken a very long time for long-term inflation expectations in Canada to fall below rates in the United States, but how certain countries that appear to be committed to Economic and Monetary Union have experienced a very rapid decline in their long-term interest rate differentials with Germany.
Under the assumption that $u_t^*$ follows a random walk, equation (13) can be estimated as a time-varying parameter model using standard Kalman filter procedures. It is then straightforward to obtain estimates of $u_t^*$ by solving equation (12) after time-varying estimates of $Z_t$ are derived from the Kalman filtering procedure. Assuming that $\gamma$ and $\phi$ are taken as given, the assumption that $u_t^*$ follows a random walk implies that $Z_t = Z_{t-1} + \varepsilon_t^*$. To implement the Kalman filter routine, it is necessary to specify the signal-to-noise ratio or, more precisely in this instance, the variance of $\varepsilon_t^*$ relative to the variance of $\varepsilon_t$. With annual data, we found that simply using the default option of unity in most econometric packages produced estimates of $u_t^*$ that were not excessively volatile.

However, before equation (13) can be estimated, it is necessary to specify a measurement equation for $\phi$, which can be regarded as the "unemployment wall" parameter. Recall from Figure 1 that $\phi$ represents the lower bound on unemployment, below which the economy cannot operate regardless of the degree of excess demand. In countries where the unemployment rate has drifted upward, it seems quite plausible that $\phi$ has drifted upward as well; recall the discussion of asymmetric hysteresis. The specification adopted for the minimum unemployment wall is the following:

$$\phi = \text{MAX} [0, \hat{u}_t - 4], \quad (14)$$

where $\hat{u}_t$ is a time-varying measure of the natural rate of unemployment. Box 7 provides a discussion of the filtering method that was used to construct the estimates of the $\hat{u}_t$ and shows the estimates of the $\hat{u}_t$. Obviously, there is considerable uncertainty about any estimates of the $\phi$, since econometricians are not likely to have any observations of unemployment rates in its neighborhood. Part of our justification for adopting the specification in equation (14) is that it implies modest convexity, or approximate linearity, of the Phillips curve in the neighborhood of the DNAIRU, $u^*$. In addition, this seems more attractive than setting $\phi = 0$, as was done in the initial application of this functional form (see Debelle and Laxton, 1997). The specification with $\phi = 0$ has some undesirable counterfactual properties because it would suggest that countries with extremely rigid labor markets and high unemployment rates would have lower convexity than countries with low unemployment rates and very flexible labor markets.

Table 1 provides estimates of the key parameters of the short-run Phillips curve model for the seven major industrial countries when equation (13) is estimated over a sample period that starts in 1976 and ends in 1996. Given the small number of observations in our sample and the limited number of business cycles, one should expect significant uncertainty in the parameter estimates. Because of such imprecision in estimated parameters, previous versions of MULTIMOD have reflected decisions to impose the same parameter values across countries in cases where individual country estimates were judged to be implausible.

In Table 1, the estimated parameter on the inflation expectation term ($\lambda_{\pi}$), which reflects the degree of nominal flexibility (recall Box 6), is less than one for France, Germany, Italy, and the United States, indicating that these economies may be characterized by greater nominal rigidities than Canada, Japan, and the United Kingdom, for which the estimated values exceeded one and coefficients of unity were therefore imposed. The parameter on the labor market tightness term ($\gamma_t$) to a large extent reflects the degree of real wage rigidities. The estimates of relatively low

<table>
<thead>
<tr>
<th>Country</th>
<th>$\gamma_t$</th>
<th>$\lambda_{\pi}$</th>
<th>$R^2$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.018*</td>
<td>1.00</td>
<td>0.39</td>
<td>0.016</td>
</tr>
<tr>
<td>France</td>
<td>0.011**</td>
<td>0.75**</td>
<td>0.63</td>
<td>0.009</td>
</tr>
<tr>
<td>Germany</td>
<td>0.008**</td>
<td>0.74**</td>
<td>0.69</td>
<td>0.009</td>
</tr>
<tr>
<td>Italy</td>
<td>0.023*</td>
<td>0.91**</td>
<td>0.61</td>
<td>0.020</td>
</tr>
<tr>
<td>Japan</td>
<td>0.091**</td>
<td>1.00</td>
<td>0.50</td>
<td>0.026</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.024**</td>
<td>1.00</td>
<td>0.25</td>
<td>0.035</td>
</tr>
<tr>
<td>United States</td>
<td>0.013**</td>
<td>0.69**</td>
<td>0.38</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Note: Standard errors reported in parentheses; ** (*) indicates that estimated coefficient is significantly different from zero at the 5 percent (10 percent) significance level.

63 Kutner (1992, 1994) adopts a strategy that is closest to the one we follow, but he assumes that the Phillips curve has a linear specification.

64 The software package that was used to estimate the model was TSP.
values of this parameter for France and Germany suggest that these economies have relatively high degrees of downward real wage resistance. At the other extreme, Japan has a very high value of \( \gamma_{pc} \) suggesting that, of the seven major industrial countries, Japan’s economy has the lowest degree of real wage rigidities. The relative responsiveness in the model of real and nominal variables to monetary shocks will depend intricately on the degrees of real and nominal rigidities in the economy under consideration. For example, output and unemployment will respond more to monetary shocks in countries (such as Germany) that have relatively high degrees of both nominal and real rigidities than in countries (such as Japan) that have very low degrees of both nominal and real rigidities; see Box 8 for a discussion of the simulation properties of the model in response to positive and negative money supply shocks.

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Box 7. A Simple Prior-Consistent Filter for Measuring the Natural Rate

The Mark III Phillips curve embodies model-consistent estimates of the time series of \( u_{pc} \) (the DNAIRU) and \( \phi \) (the unemployment wall parameter) that are generated from estimates of the time series of \( \bar{u} \), the natural rate of unemployment (also called the NAIRU). The latter are generated using a simple prior-consistent filter. The choice of this particular filter was based upon the following considerations. First, we wanted the method to be able to accommodate simple prior assumptions about how the natural rate has evolved over time. For example, for work with data from the industrial countries, we would want to allow for the possibility that the natural rate has drifted up over time. However, at the same time, we would also not want to rule out the possibility that it may have actually fallen in some countries during certain periods. Second, we wanted it to be straightforward to impose prior assumptions about the variance of the natural rate relative to the observed unemployment rate. Third, we wanted an approach that could easily incorporate information from research that has been directed at accounting for trend variation in the natural rate. These considerations ruled out many of the possible techniques and led us to specify a simple prior-consistent (PC) filter.\(^1\)

Estimates of the natural rate from the PC filter are derived by minimizing the squared deviations of the observed unemployment rate, \( u_t \), from the natural rate, \( \bar{u}_t \), subject to a constraint that penalizes squared deviations in the change in the natural rate relative to some prior estimate of the change in the natural rate, which we denote as \( \Delta \bar{u}_{pc} \). Specifically, given a sample of observations that range from 1 to \( T \), the PC filter solves for the sequence \( \{ \bar{u}_t \} \) that minimizes the objective function:

\[
\sum_{t=1}^{T} (u_t - \bar{u}_t)^2 + \lambda_{pc} \sum_{t=1}^{T} \left( (\bar{u}_t - \bar{u}_{t-1}) - \Delta \bar{u}_{pc} \right)^2.
\]

The PC filter is a close cousin to the Hodrick-Prescott (HP) filter (see Hodrick and Prescott, 1981). However, by penalizing deviations from prior estimates of changes in the natural rate instead of just penalizing curvature, the PC filter has the advantage of allowing us to incorporate any priors we may have on the natural rate via the \( \Delta \bar{u}_{pc} \) terms.\(^2\)

Values of the \( \Delta \bar{u}_{pc} \) could perhaps be obtained from structural studies of the natural rate that attempt to identify factors that can explain why the natural rate may have shifted over time.\(^3\) However, without specific information about shifts in the natural rate, we have chosen to set the \( \Delta \bar{u}_{pc} \) terms to zero. This seems to be an appropriate choice for a variable like the unemployment rate that could be subject to permanent shocks but should not be expected to continue to drift upward continuously over time.

The value of \( \lambda_{pc} \) in the PC filter can be determined by forming priors about what would constitute a very large value of the unemployment gap compared to priors about a very large change in the natural rate of unemployment. For example, a very high value of \( \lambda_{pc} \) would imply an approximately constant estimate for the natural rate over the sample, while a very low value for \( \lambda_{pc} \) would allow the natural rate to roughly coincide with the actual unemployment rate. One way of calibrating \( \lambda_{pc} \) in practice is to set it at a level at which a particular pair of extreme values for the unemployment gap and the change in the natural rate would have identical effects on the value of the objective function. For most of the countries in our sample, a 5 percent unemployment gap would be a large unemployment gap,

\(^1\)Univariate filters—such as the one employed here—are clearly a second-best solution to a more sophisticated structural approach that would attempt to develop model-consistent measures of the natural rate that included both deterministic and stochastic elements. However, while a preferred “structural approach” has been implemented with some success in a few industrial countries, it has been problematic for explaining trend variation in the natural rate in several other industrial countries. See Blanchard and Katz (1997).

\(^2\)The objective function for the HP filter can be obtained by replacing \( \Delta \bar{u} \) in the PC objective function with \( \bar{u}_{t+1} - \bar{u}_{t-1} \). One problem with the HP filter is that the estimates of the trend series can be virtually useless at the end of the sample because there are very few meaningful restrictions to tie down the level of the trend series. The PC filter can be potentially more informative in cases where there is a strong prior for steady-state changes in the underlying trend series.

\(^3\)For examples of a more structural approach to estimating the natural rate of unemployment, see Ford and Rose (1989), Adams and Coe (1990), and Layard, Nickell, and Jackman (1991).
The Dynamic Effects of Output Gaps on Unemployment

The Mark III version includes a very simple and parsimonious dynamic Okun’s law equation to translate movements in the output gap \((y - \bar{y})\) into movements in the unemployment gap \((u - \bar{u})\):

\[
[u_t - \bar{u}_t] = \gamma_1(y_t - \bar{y}_t) + \gamma_2[u_{t-1} - \bar{u}_{t-1}] + \epsilon_{ut}. \tag{15}
\]

Equation (15) can be viewed as a derived demand for labor function, taking the demand for output as given.\(^5\) The partial adjustment of unemployment

while a 1 percentage point annual change in the natural rate would also be considered very large. These particular extreme values make equal contributions to the PC penalty function when \(\lambda_{pc}\) equals 25, which is the value we have chosen to use in estimating natural rates for the core Mark III model.\(^4\)

\(^4\)In general, once users have identified their priors for a large value of the unemployment gap and a large change in the natural rate, they can then extract \(\lambda_{pc}\) from the following formula:

\[
\text{(Prior for large gap)}^2 = \lambda_{pc} \times \text{(Prior for large change in natural rate)}.^2
\]

The panels in the figure present estimates of the trend rate of unemployment for the industrial countries in Mark III when \(\lambda_{pc}\) is set at 25. According to these estimates, the natural rate of unemployment has shifted up significantly in all countries except the United States, which seems quite plausible. In addition, unlike attempts to explain the historical behavior of unemployment rates in terms of simple time trends in natural rates (see Blanchard and Quah (1989) for example), the approach taken here allows for the possibility that in some countries the upward drift in unemployment may have come to an end. For the United States, the estimates in the figure suggest that the natural rate has fallen somewhat during the 1990s.

\(^5\)Specifications of the derived demand for labor typically also include a real wage disequilibrium term, as in Bartolini, Razin, and Symansky (1995). However, the estimated parameter on this term is usually found to be fairly small compared with the role of the output gap, and this effect is ignored in Mark III.
Box 8. Asymmetries and Country-Specific Differences in the Real and Nominal Effects of Shocks

The existence of short-run capacity constraints in Mark III implies that the relative magnitudes of the real and nominal effects of shocks to aggregate demand depend on the initial position of the economy. If the economy is initially characterized by a high degree of excess supply, an increase in aggregate demand induced by a change in either monetary or fiscal policy will result in larger changes in real economic activity and unemployment in the short run, and smaller changes in prices, than if the economy is initially characterized by a high degree of excess demand. Indeed, in the limit, if the unemployment rate was at the minimum level feasible under the short-run supply curve in the model, an increase in aggregate demand would result only in inflation without any change in real activity.

The degree of asymmetry in MULTIMOD is illustrated in the table, which contrasts the effects of positive and negative money supply shocks on several countries individually. The focus is on countries that have flexible exchange rates in the base-case version of Mark III—Canada, Germany, Japan, United Kingdom, and United States. The simulations are performed on the individual country models, without taking account of international spillover and feedback effects.

The table reports the shock-minus-control results for real GDP, unemployment, the GNP deflator, and the CPI. The shocks represent positive and negative 10 percent changes in money surplus starting from conditions of full-employment equilibrium. Because the shocks are fairly large, they provide a good indication of the degree of asymmetry in the model when fairly significant movements exist along the short-run aggregate supply function. For all countries, the positive shocks have smaller short-run effects on real variables and larger short-run effects on nominal variables than the negative shocks.

Cross-country comparisons show that the effects of shocks depend on the relative degrees of nominal and real rigidities exist in the different countries. As seen in the table, for countries like Germany, where significant real and nominal rigidities exist, changes in aggregate demand can have persistent effects on real economic activity and unemployment, particularly if the shocks tend to move the economy into a position of excess supply.

The table also provides some estimates of the cumulative changes in real GDP and unemployment over the first five years of the shock. According to these measures, the degree of asymmetry is the largest in Germany and the smallest in Japan.

### The Asymmetric Effects of Positive and Negative Money Supply Shocks
(Responses to 10 percent changes in the money supply)

<table>
<thead>
<tr>
<th>Country</th>
<th>Real GDP</th>
<th>Unemployment rate</th>
<th>GNP deflator</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>United States</strong></td>
<td>+3.6, -4.7</td>
<td>-1.5, 2.2</td>
<td>+2.2, -1.5</td>
<td>+2.8, -2.2</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>+3.4, -4.3</td>
<td>-1.4, 1.8</td>
<td>+3.8, -2.9</td>
<td>+3.0, -2.6</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>+3.8, -4.7</td>
<td>-1.2, 1.6</td>
<td>+1.0, -0.7</td>
<td>+2.4, -2.6</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>+3.4, -3.7</td>
<td>-0.2, 0.2</td>
<td>+2.6, -2.6</td>
<td>+2.8, -2.9</td>
</tr>
<tr>
<td><strong>United Kingdom</strong></td>
<td>+2.6, -3.3</td>
<td>-0.8, 1.1</td>
<td>+4.1, -3.5</td>
<td>+4.4, -4.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>Real GDP</th>
<th>Unemployment rate</th>
<th>GNP deflator</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cumulative Effect</strong></td>
<td>+3.5, -11.0</td>
<td>+3.8, -9.3</td>
<td>+3.8, -9.3</td>
<td>+3.8, -9.3</td>
</tr>
</tbody>
</table>

Note: + denotes positive 10 percent shock, and – denotes negative 10 percent shock.
gaps to output gaps embodied in equation (15) can be motivated by an assumption that it is costly to hire and fire workers, and so firms consequently adjust employment levels slowly in response to changes in the demands for their products.

Table 2 provides the econometric estimates of equation (15) under the assumption that its error term follows a first-order autoregressive process. The measures of the natural rate of unemployment are obtained from the prior-consistent filter described in Box 7, and the measures of potential output are derived from a Hodrick-Prescott filter. Figure 2 provides estimates of the unemployment and output gaps that are based on these filtered estimates of the respective trend components. As can be seen in the figure, there is a striking negative correlation between the output gaps and the unemployment gaps in most countries.

The estimated parameter on the contemporaneous output gap is highly significant in all countries (see Table 2). The average value of this term across all the countries is approximately one-third, indicating that a 1 percentage point change in the excess demand gap in the goods market translates into a contemporaneous change of roughly \( \frac{1}{3} \) of 1 percentage point in the difference between the observed unemployment rate and the natural rate. Because of the dynamics in equation (15), shocks that result in a persistent increase in the output gap would result in a continuing rise in the unemployment gap over time.

There are considerable differences across countries in the responses of unemployment gaps to output gaps. For example, the coefficients on the output gap are considerably smaller for Japan and Italy than for the other countries. For Japan, this may be biased downward, and this can perhaps account for why we would expect that the parameter estimate on the output gap changes in the natural rate to potential output. As a consequence, the estimated potential output fails to capture the direct link from changes in the natural rate to potential output. As a consequence, the model doesn’t fit nearly as well for Italy as it does for the other countries. In future work, we hope to develop system estimates of potential output and the natural rate that will alleviate any problems of this type.

### Table 2. Unemployment Equations

<table>
<thead>
<tr>
<th>Country</th>
<th>( \gamma_u )</th>
<th>( \gamma_y )</th>
<th>( R^2 )</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-0.42** (0.04)</td>
<td>0.32** (0.09)</td>
<td>0.85</td>
<td>0.382</td>
</tr>
<tr>
<td>France</td>
<td>-0.39** (0.06)</td>
<td>0.44** (0.16)</td>
<td>0.68</td>
<td>0.354</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.33** (0.07)</td>
<td>0.18 (0.21)</td>
<td>0.64</td>
<td>0.496</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.09** (0.06)</td>
<td>0.79** (0.25)</td>
<td>0.52</td>
<td>0.507</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.06** (0.02)</td>
<td>0.49** (0.16)</td>
<td>0.71</td>
<td>0.125</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.33** (0.05)</td>
<td>0.69** (0.09)</td>
<td>0.86</td>
<td>0.476</td>
</tr>
<tr>
<td>United States</td>
<td>-0.42** (0.04)</td>
<td>0.21** (0.09)</td>
<td>0.85</td>
<td>0.352</td>
</tr>
</tbody>
</table>

Note: Standard errors reported in parentheses; ** indicates that estimated coefficient is significantly different from zero at the 5 percent significance level.

---

66The model was estimated this way because there was significant residual autocorrelation when the model was estimated with ordinary least squares in a few countries. This autocorrelation is probably a result of inconsistencies in the filtered estimates of the natural rate and potential output. In the future it would be preferable to develop model-consistent estimates of these trend variables in a larger system that embodies equation (15).

67The relatively low parameter estimate on the output gap for Italy presumably is not attributable to extremely low variability in the underlying unemployment rate. One possibility is that Italy has had larger shocks to the natural rate of unemployment than Japan and that the simple univariate filter that is being used to estimate potential output fails to capture the direct link from changes in the natural rate to potential output. As a consequence, we would expect that the parameter estimate on the output gap may be biased downward, and this can perhaps account for why the model doesn’t fit nearly as well for Italy as it does for the other countries. In future work, we hope to develop system estimates of potential output and the natural rate that will alleviate any problems of this type.

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The CPI Equations and Import Price Pass-Through

The Mark II version of MULTIMOD included two broad measures of inflation—the rates of change of the GNP deflator and the absorption deflator—but it did not include measures based on the consumer price index (CPI). Consequently, it was difficult to compare model simulation results with the conventional headline measures of inflation, which typically focus on the CPI. The price index closest to the CPI in the previous version of MULTIMOD was the absorption deflator; and because its behavior is considerably different for many countries than headline CPI-based measures of inflation, it was a source of confusion in some instances. Accordingly, to make simulation results easier to relate to the World Economic Outlook and numbers reported in the popular press, the Mark III version of MULTIMOD includes explicit models for the CPI. Among other things, this makes it possible to include the rate of change of the CPI in monetary policy reaction functions for certain countries that have explicit inflation targets.

In the context of specifying a CPI equation, it is important to pay close attention to the degree of exchange-rate pass-through, recognizing that
Figure 2. Unemployment and Output Gaps

Unemployment gap

Output gap

Canada

France

Germany

Italy

Japan

United Kingdom

United States

Small Industrial Countries
MULTIMOD is regularly subjected to shocks that affect exchange rates. The remainder of this section presents estimated short- and long-term elasticities that are obtained from reduced-form time-series relationships. To help evaluate the plausibility of these econometric estimates, we also present information on import coefficients derived from input-output tables.

The \( CPI \) specification in Mark III includes aggregate import prices \( (PIM) \), domestically produced goods prices \( (PGDP) \), and a lagged dependent variable to allow for partial adjustment of prices at the retail level. The equations were estimated in first differences because the sample period data did not provide evidence that the \( CPI \) was cointegrated with the levels of aggregate measures of import prices and the GDP deflator. The lack of a level condition in the \( CPI \) equation is not likely to cause any problems with the internal macro consistency of MULTIMOD’s structure because the level of the \( CPI \) does not—and is not intended to—feed back on the rest of the model’s structure.

The estimated equations for the \( CPI \) are reported in Table 3. The coefficients on the GNP deflator are highly significant for all countries, and in all cases except Canada the estimated parameters on the import price deflator are significantly greater than zero at the 90 percent confidence level. The equations indicate that there is generally fairly rapid pass-through from both import prices and producer prices to the \( CPI \), but as may be seen by comparing columns 1 and 3, in some cases the long-run elastic-

### Table 3. CPI Equations

\[
\Delta \log (CPI_t) = \gamma_1 \Delta \log (PIM_t) + \gamma_2 \Delta \log (PGDP_t) + (1 - \gamma_1 - \gamma_2) \Delta \log (CPI_{t-1})
\]

Estimation period: 1973–96

<table>
<thead>
<tr>
<th></th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \eta_{PIM}^{LR} )</th>
<th>( R^2 )</th>
<th>SE</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.09</td>
<td>0.59**</td>
<td>0.13</td>
<td>0.88</td>
<td>0.112</td>
<td>2.32</td>
</tr>
<tr>
<td>France</td>
<td>0.13**</td>
<td>0.82**</td>
<td>0.14</td>
<td>0.97</td>
<td>0.007</td>
<td>1.80</td>
</tr>
<tr>
<td>Germany</td>
<td>0.16**</td>
<td>0.56**</td>
<td>0.22</td>
<td>0.87</td>
<td>0.007</td>
<td>1.76</td>
</tr>
<tr>
<td>Italy</td>
<td>0.06</td>
<td>0.85**</td>
<td>0.06</td>
<td>0.98</td>
<td>0.009</td>
<td>2.20</td>
</tr>
<tr>
<td>Japan</td>
<td>0.07**</td>
<td>0.79**</td>
<td>0.08</td>
<td>0.98</td>
<td>0.008</td>
<td>1.65</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.14**</td>
<td>0.74**</td>
<td>0.16</td>
<td>0.97</td>
<td>0.009</td>
<td>2.46</td>
</tr>
<tr>
<td>United States</td>
<td>0.06**</td>
<td>0.98**</td>
<td>0.06</td>
<td>0.90</td>
<td>0.010</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Note: Standard errors reported in parentheses; ** (*) indicates that estimated coefficient is significantly different from zero at the 5 percent (10 percent) significance level. \( \eta_{PIM}^{LR} \) = long-run elasticity of the CPI with respect to import prices.

### Table 4. Estimated Effects on the CPI of a 1 Percent Increase in Import Price

<table>
<thead>
<tr>
<th></th>
<th>Direct effects</th>
<th>Total effects</th>
<th>Short run</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.10</td>
<td>0.21</td>
<td>0.08</td>
<td>0.13</td>
</tr>
<tr>
<td>France</td>
<td>0.09</td>
<td>0.22</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Germany</td>
<td>0.10</td>
<td>0.22</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td>Italy</td>
<td>0.06</td>
<td>0.20</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Japan</td>
<td>0.02</td>
<td>0.11</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.15</td>
<td>0.30</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>United States</td>
<td>0.07</td>
<td>0.10</td>
<td>0.06</td>
<td>0.06</td>
</tr>
</tbody>
</table>

1. The input-output estimates are based on data for 1990 in all countries except Italy, for which they are based on data for 1985.
2. Based on direct import contents of final goods.
3. Reflects direct import contents of both intermediate and final goods; see main text.
4. Estimates reported in Table 3.
ity of the CPI with respect to import prices is quite different from the short-run elasticity.

As can also be seen in Table 3, the parameter estimates on the import deflator are not very precisely estimated in some cases. To judge whether or not the estimated degree of import price pass-through is realistic, one should compare the short- and long-run price elasticities with simple estimates that one can compute from the input-output tables compiled by the Organization for Economic Cooperation and Development.

Table 4 reports some input-output–based estimates of import-price pass-through for the aggregate consumption deflator under two assumptions. The calculations in the first column take account only of the direct import content of final goods, assuming that there is complete pass-through for direct imports but ignoring the effects of higher import costs on the prices of intermediate goods and, hence, the associated indirect effects on the prices of final goods. The calculations in the second column include the latter indirect effects, assuming full pass-through of higher prices at each stage of processing, with no changes in unit profit margins and unit labor costs. The econometric estimates of the short-run and long-run import price elasticities from the equations estimated in Table 3 are also reported in Table 4. The econometric estimates of the long-run import price elasticities (column 4) are generally smaller than the simple measures of full import price pass-through based on the input-output tables (column 2), but the two sets of estimates have reasonably consistent orders of magnitude. Accordingly, it was decided to employ the econometric estimates in Mark III.

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68 These estimates are based on the latest available input-output tables; see the notes to Table 4.