V Consumption and Saving Behavior: A Life-Cycle Perspective

As emphasized in the introduction of this paper, one of MULTIMOD's most important functions is to assist with the IMF's multilateral surveillance over the policies of its members. Among other considerations, MULTIMOD has frequently been used to analyze the implications of changes in fiscal policies. In this context, it is now widely recognized that the analysis of fiscal policy in any macroeconometric model depends critically on the specification of consumption and saving behavior.

Following Keynes (1936), the view that changes in government deficits tended to have first-order effects on the level of economic activity became part of mainstream macroeconomic doctrine for several decades. However, the Keynesian framework was significantly challenged during the 1970s, partly in association with the rational expectations revolution, which generated a revival of the classical Ricardian view of government deficits.[69] From the Ricardian perspective, changes in government debt should have little or no real effects on economic activity, reflecting the view that forward-looking, rational agents would revise intertemporal consumption and saving decisions to offset the implications of changes in the intertemporal pattern of public sector consumption and saving behavior (that is, changes in government deficits).

Consistent with the rational expectations hypothesis that underlies the modern Ricardian view of fiscal policy, MULTIMOD assumes that economic agents behave in a forward-looking manner based on model-consistent (rational) expectations. It also recognizes, however, that the sensitivity of private saving behavior to a change in the public deficit depends importantly on factors other than the nature of expectations. In this context, MULTIMOD Mark III incorporates important changes (compared with Mark II) in the specification of consumption and saving behavior, which now reflect an explicit life-cycle dimension. While retaining the basic framework of forward-looking optimizing behavior, the Mark III specification allows us to explore the implications of the life-cycle paradigm, as well as other relevant considerations such as liquidity constraints (capital market imperfections) for the effectiveness of fiscal policy.

This section describes the theoretical framework and empirical estimates that underlie the specification of life-cycle consumption and saving behavior in MULTIMOD Mark III. It also examines the macroeconomic implications of such behavior for closed and small open economies by comparing a neoclassical paradigm in which agents are characterized as disconnected generations with life-cycle features to a classical (Ricardian) paradigm that views agents as dynastic families. An analytical framework that nests these two alternative views is developed on the basis of an extended version of Blanchard's (1985) overlapping-agents model. From the neoclassical perspective,[70] the analysis shows that changes in public saving can have considerable effects on the level of national saving, with implications for interest rates and asset accumulation. To estimate the quantitative importance of these life-cycle considerations for the effects of deficit finance on real interest rates, the capital stock, and net foreign assets, the model is calibrated to actual age-earnings profiles.

In the presence of life-cycle saving behavior, the rate of saving may vary significantly over time and across individuals, depending on where agents are within their respective life cycles. Younger agents expecting a rising earnings profile may choose to borrow and consume against their permanent income, which may initially exceed current income. At middle age, agents enjoying a relatively higher level of earnings may choose to accumulate assets and save for their eventual retirement. Upon reaching retirement, individuals may tend to run down their assets (dissave) in order to maintain a given level of consumption in the face of declining labor income.

To add an empirical content to this life-cycle dimension of the analysis, the model first incorporates

[69]The revival of the classical view and the Ricardian equivalence proposition was led by Barro (1974). See Barro (1989) for a more recent review.

[70]The seminal paper is Diamond (1965); see Persson (1985) for open economy extensions. See Bernheim (1989) for a more recent review of the neoclassical approach to budget deficits.

a general specification of the relationship between individual earnings profiles and age. In particular, disposable labor income is assumed to generally increase with age as a worker gains experience and seniority before earnings level off and eventually decline with retirement. To calibrate the model, the time profile of labor income is estimated using earnings and employment data across different age groups in the United States.[71] Using the ensuing calibration, the parameters describing the behavior of aggregate consumption are estimated, and the dynamic effects of fiscal policy on interest rates and the accumulation of capital or net foreign assets in a life-cycle context can then be simulated.

In terms of their fiscal implications, life-cycle considerations tend to *augment* the real effects of government debt on the economy (see Faruqee, Laxton, and Symansky, 1997). With no altruistic link between generations, an increase in the fiscal deficit is not fully offset by an increase in private saving, as a share of the debt burden falls on future generations whose marginal propensities to save presently are zero. With life-cycle (eventually declining) earnings and retirement, agents further discount the impact of future tax liabilities from an increase in public debt since the prospective tax base increasingly shifts to future generations with higher taxable incomes.

In a primarily closed economy, the resultant upward pressure on interest rates from an increase in government debt tends to crowd out investment and retard the rate of capital accumulation. In a small open economy, the decline in domestic saving is manifested in a crowding out of net exports and a greater reliance on foreign borrowing. In either case, the increase in current consumption takes place at the expense of lower living standards in the future, whether through a lower level of the capital stock or higher foreign claims on future output.

These results are in sharp contrast to the classical Ricardian view of government deficits. From a Ricardian perspective, the economic consequences of public debt and deficits should be minimal. According to the government's own (intertemporal) budget constraint, deficit financing represents a change only in the timing of taxes, while the present value of taxes remains unaffected, given public expenditure. Thus, the future taxes implied by higher government debt negate the benefits of any current reduction in taxes, leaving consumer demand unchanged. In the case of higher government spending, the increased deficit implies an increase in the present value of tax liabilities, which would act to lower current consumption. In either case, an increase in the budget deficit should be met with an increase in private saving, offsetting or diminishing the effects of lower public saving on aggregate saving and the economy.

The disparate implications of budget deficits within the Ricardian and Neoclassical paradigms stem from their contrasting views of the intergenerational link between individuals. In contrast to the life-cycle view, the Ricardian approach views agents as dynastic families, where current generations are closely linked (through a bequest motive) to their descendants. With stronger intergenerational ties, individuals are more likely to internalize the implications of the government's intertemporal budget constraint and the prospect of higher taxes in the future.[72] With dynastic saving behavior, a form of Say's law would hold for fiscal deficits, wherein an increase in the supply of government bonds (that is, deficit finance) would be met by a corresponding increase in demand (that is, private saving) at an unchanged price (interest rate). In that case, the choice between tax and deficit finance becomes irrelevant (that is, Ricardian equivalence holds) and changes in government debt are neutral in their effects on the economy.

The issue regarding the validity of Ricardian equivalence is part of an ongoing and contentious debate in which no clear consensus has emerged. The empirical evidence on the effects of fiscal deficits and other matters (for example, bequests) related to the Ricardian debate has, thus far, consisted of mixed results rather than definitive conclusions.[73] Many of the empirical tests suffer from low power, unable to reject either Ricardian equivalence *or* its failure in favor of the corresponding alternative. Consequently, the findings of various studies tend to correlate closely with how the empirical tests are structured—that is, which proposition is taken as the null hypothesis. From a policy perspective, however, formulating policy based on the "wrong" hypothesis can lead to serious welfare consequences. In that sense, assuming that Ricardian equivalence fails may be the more appropriate starting point given that the costs of inappropriate policies may significantly outweigh those associated with irrelevant policies. In other words, the welfare costs of inappropriately failing to take fiscal actions when fiscal policies matter (based on a false acceptance of Ricardian equivalence) are likely to exceed the welfare costs of erroneously adjusting fiscal policy when fis-

[71]See Jappelli and Pagano (1989) for an international comparison of age-earnings profiles (and capital market imperfections).

[72]See Bernheim and Bagwell (1988) for a critical review on the dynastic approach.

[73]In a survey of empirical tests for Ricardian equivalence, Seater (1993) claims that the evidence is supportive of the equivalence proposition; at the same time, however, he acknowledges that the results of various studies correlate closely with the political leanings of the investigator (p.184). See Barro (1989) and Bernheim (1989) for opposing interpretations of the empirical evidence.

cal policy is ineffective (based on a false rejection of Ricardian equivalence).

The Basic Model

Following Blanchard (1985), we consider an economy where agents have finite planning horizons (that is, a positive probability of death). In the case of dynasties, the probability of death represents the likelihood that the family line will end; in the case of overlapping agents who are disconnected from each other (that is, no bequest motive), the probability of death is related to an individual's life expectancy.

Specifically, consider an economy populated by finitely lived agents, each facing a constant probability p of dying at each moment in time, and a planning horizon (that is, the expected time until death) given by $1/p$.[74] Also, at each point in time a new generation (or dynasty) is born of relative size normalized to p, so that the size of the population remains constant. Specifically, the number of surviving agents from a cohort born at time s and remaining at time t is equal to $pe^{-p(t-s)}$, leaving the total number of agents—aggregating over all existing cohorts (indexed by s)—constant and normalized to unity.[75]

Consumption

Agents are assumed to maximize expected utility over their lifetimes subject to a budget constraint.[76] Specifically, the evolution of financial wealth $w(s,t)$ for an individual or household is determined by its saving, defined as the difference between income and consumption:

$$\dot{w}(s,t) = [r(t) + p]w(s,t) + y(s,t) - \tau(s,t) - c(s,t), \quad (16)$$

where r is the interest rate, y is labor income, τ is taxes (net of transfers), and c is consumption, all expressed in real terms (units of consumption).[77] In a small open economy, the real interest rate is also assumed to be exogenous and fixed at the world real rate of interest. Ignoring for now capital market imperfections and liquidity constraints, optimal consumption should be based on an agent's permanent income. Explicitly, solving the dynamic optimization problem facing consumers, individual consumption is given by[78]

$$c(s,t) = (\theta + p)[w(s,t) + h(s,t)], \quad (17)$$

where θ is the rate of time preference in utility, and $h(s,t)$ is a measure of an agent's human wealth—equal to the present value of future labor income.[79]

Aggregating over all agents,[80] total consumption as a function of (financial and human) wealth can be expressed as follows:

$$C = (\theta + p)[W + H], \quad (18)$$

where uppercase letters denote economy-wide aggregates. Financial wealth W equals the sum of domestic equity K, bond holdings B, and in an open economy, holdings of net foreign assets F:

$$W \equiv K + B + F. \quad (19)$$

As for aggregate human wealth, its definition depends on the treatment of agents as dynasties or disconnected generations of individuals.

The Behavior of Dynastic Households

When households represent dynasties rather than (disconnected) generations, an individual's planning horizon may far exceed his or her own actual lifetime, as people internalize the welfare and circumstances of their descendants as their own. Correspondingly, human wealth is expressed in terms of the disposable income stream available to the dynastic household. As dynasties themselves do not possess any life-cycle dimension, members from different households (regardless of age) can be treated identically. Consistent with this representative-agent framework, income and taxes are *not* generation-specific (that is, $y(s,t) = Y(t)$, $\tau(s,t) = T(t)$). Consequently, the dynamics for total human wealth under a dynastic interpretation can be written (dropping the time index) as[81]

[74]See Blanchard (1985). If the probability of death goes to zero, agents have infinite horizons.

[75]The case of population (and productivity) growth is addressed later.

[76]Labor supply is taken to be inelastically supplied. Hence, the labor-leisure decision is not part of the consumer's optimization problem. See Ludvigson (1996) for a recent paper on fiscal policy effects with endogenous labor supply.

[77]The term *pw(s,t)* in the dynamic budget constraint reflects the efficient operation of the life insurance or annuities market. See Yaari (1965) or Blanchard (1985).

[78]This result assumes logarithmic utility. The case of constant relative risk aversion (CRRA utility) (see also Blanchard, 1985) and the implications of different intertemporal substitution elasticities are explored in Faruqee, Laxton, and Symansky (1997).

[79]For a given (world) real interest rate, individual human wealth can be written as

$$h(s,t) \equiv \int_t^{\infty} [y(s,v) - \tau(s,v)]e^{-(r+p)(v-t)}dv.$$

[80]To derive aggregate variables, we simply sum over all existing generations (or dynasties). Specifically, aggregate variables, denoted by uppercase letters, are derived by integrating over all existing cohorts or generations (indexed by s):

$$X(t) \equiv \int_{-\infty}^{t} x(s,t)pe^{-p(t-s)}ds.$$

[81]To simplify notation, the time arguments in the equations have been dropped in the text, except where potential ambiguities may arise. The time index is reintroduced in the tables.

$$\dot{H} = (r + p)H - [Y - T]. \tag{20}$$

Under this representative agent assumption, household income (and taxes) is independent of age and can grow indefinitely with long-run productivity growth. With a growing taxable income base over time, agents belonging to dynastic households remain sensitive to changes in future tax burdens falling on themselves *or* their descendants. Indeed, Evans (1991) shows that this variant of the model obeys approximate Ricardian equivalence, and that variations in planning horizons (that is, changes in the birth and death rate p) do not substantially alter this basic property of the model. This result, however, rests crucially on the assumption of dynastic rather than life-cycle behavior, as will become apparent below.

The Life-Cycle Behavior of Disconnected Generations

For the case in which agents represent overlapping generations of individuals (rather than dynasties) disconnected from one another, the planning horizon reflects an individual's expected life span, during which time life-cycle considerations are relevant. To capture these features, the basic framework can be modified to incorporate a time profile of labor income that conforms to empirical observations, rising with age and experience when individuals are young, before eventually declining with retirement when they are old.[82]

To introduce a concave earnings profile over an individual's lifetime, we assume that the income $y(s,t)$ accruing to an individual from generation s at time t, as a proportion of aggregate labor income $Y(t)$, can be expressed using age-dependent weights determined by the sum of two exponential functions (to allow for aggregation) as follows:[83]

$$y(s,t) = [a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)}]Y(t);$$
$$a_1 > 0,\ a_2 < 0,\ \alpha_1,\alpha_2 > 0. \tag{21}$$

The first exponential term, which decays over time, can be interpreted as the effects of the gradually declining endowment of labor (that is, gradual retirement), which is inelastically supplied. The second exponential term, which rises (toward zero) over time, can be interpreted as reflecting the relative productivity and wage gains associated with increased experience (age).

With age-dependent income and lifetime-earnings profiles, the dynamics governing aggregate human wealth are modified accordingly, with equation (20) replaced by

$$H = \beta H_1 + (1 - \beta)H_2, \tag{22}$$

$$\dot{H}_1 = (r + p + \alpha_1)H_1 - [Y - T], \tag{23}$$

and

$$\dot{H}_2 = (r + p + \alpha_2)H_2 - [Y - T]. \tag{24}$$

Human wealth, measuring the present value of future disposable labor income, is now expressed as the sum of two components, reflecting the concave nature of an individual's income over the life cycle.

As will be shown explicitly later, the real effects of government debt on the economy tend to be larger in the case of disconnected generations than in the case of dynastic households. The basic intuition can be seen by comparing equation (20) with equations (23) and (24). In the life-cycle case, by further increasing the wedge (via the α terms) between the public discount rate (r) and private discount rate beyond the wedge resulting from finite horizons ($p > 0$),[84] the model with age-dependent income suggests that the choice between tax financing and deficit financing will have more significant consequences for national consumption and saving.[85] A fall in, say, current taxes has future tax implications in each case, but in the life-cycle case the burden partly falls on future (disconnected) generations, who also will tend to have higher taxable incomes. Hence, in this case, changes in the intertemporal pattern of public saving and consumption behavior will not be negated by fully offsetting changes in private consumption and saving behavior.

The Small Open Economy Case

The small open economy is assumed to be a price taker in the world market for goods and capital; hence, consumption is expressed in terms of a single

[82]Blanchard (1985) examines the case of declining individual income profiles; the more realistic case of nonmonotonic (concave) earnings profiles is mentioned only in passing (footnote 75). Both cases introduce a saving-for-retirement motive and open up the possibility that the economy may be dynamically inefficient (that is, may overaccumulate capital).

[83]As discussed later, the parameters in equation (21) are chosen such that the weighting function is assumed to be nonnegative and initially increasing; by an adding-up constraint, we also require that

$$\frac{a_1 p}{\alpha_1 + p} + \frac{a_2 p}{\alpha_2 + p} = 1.$$

[84]Integrating up equation (23) yields the definition of the human wealth component H_1:

$$H_1(t) \equiv \int_t^\infty [Y(v) - T(v)]e^{-(r+p+\alpha_1)(v-t)}dv,$$

where the following boundary condition is assumed to be satisfied: $\lim_{v\to\infty} H_1(v)e^{-(r+p+\alpha_1)} = 0$; H_2 is derived equivalently from equation (24). Hence, disposable labor income is effectively discounted at a rate that depends on $r + p + \alpha_1$ and $r + p + \alpha_2$.

[85]Another critical parameter affecting the degree of debt nonneutrality is the elasticity of intertemporal substitution, which determines the sensitivity of consumption to changes in interest rates. See Faruqee, Laxton, and Symansky (1997) for a discussion.

internationally traded good (numeraire) whose price is taken as given. In a closed economy (see next subsection), specifying the behavior of consumption and saving given output and technology is sufficient to pin down investment. But in an open economy with international capital mobility, domestic investment is not constrained to equal domestic saving. Thus, to complete our characterization of the national accounts, we need to specify explicitly an investment function and also to describe the public sector and external accounts.

In specifying investment behavior, it is assumed that domestic firms can freely borrow at the (exogenous) world real interest rate. With (convex) installation costs of capital,[86] the investment decision can be derived as having a simple neoclassical specification:

$$I = (q - 1 + \delta)K, \tag{25}$$

where I is gross investment (excluding installation costs), K is the domestic capital stock, δ is the rate of depreciation of capital, and q is the (shadow) value of an additional unit of capital (related to Tobin's q). Total investment expenditure $\tilde{I}$ is given by the sum of gross investment plus adjustment or installation costs A:

$$\tilde{I} = I + A. \tag{26}$$

Domestic investment is independent of domestic saving and consumption behavior. In other words, with the ability to borrow and lend freely at a given world real rate of interest, a small open economy will choose an investment rule that is *separable* from its consumption behavior (Fisherian separability).[87] Net investment—defined as gross investment net of depreciation—determines the incremental change in the domestic capital stock:

$$\dot{K} = I - \delta K, \tag{27}$$

where again δ is the (constant) rate of depreciation or obsolescence for capital.

As for the public sector, it is assumed that government expenditures G are financed either through (lump-sum) taxation T or the issuance of government debt B. Debt accumulation and the government's dynamic budget constraint is given by

$$\dot{B} = rB + G - T, \tag{28}$$

where B is the stock of public debt. In equation (28), the primary deficit plus interest payments on the existing stock determines the government's bond-financing requirements and the corresponding rate of debt issue.

Using national accounting identities, the current account can be expressed in terms of income, saving, and absorption. Domestic production (GDP) is given by $f(K)$, which is a concave, twice-differentiable aggregate production function (labor L normalized to 1),[88] and national income (GNP) is defined by GDP plus net interest income (factor payments) from abroad:

$$GNP = f(K) + rF. \tag{29}$$

In turn, national saving S equals national income less consumption (public and private):

$$S = GNP - C - G. \tag{30}$$

Turning to the external accounts, the difference between domestic production and absorption equals the trade balance NX (that is, net exports):

$$f(K) - C - G - \tilde{I} = NX. \tag{31}$$

The current account CA is the difference between national income and absorption (or between saving and investment):

$$CA = NX + rF = S - \tilde{I}. \tag{32}$$

Since the gap between income and expenditure must be met by international lending or borrowing, the current account also reflects changes in the stock of net foreign assets:

$$\dot{F} = CA. \tag{33}$$

Table 5 summarizes the basic equations and laws of motion for the discrete-time version of the model in a small open economy when individuals exhibit life-cycle behavior. Replacing the discrete-time analogue of equations (22) – (24) in the table with that of equation (20) would characterize the model under the dynastic assumption.

In the life-cycle case, the departure from Ricardian equivalence can be substantial. In a small open economy, these effects tend to be reflected through the net foreign asset position rather than through in-

[86]Following Lucas (1967) and Treadway (1969), installation costs are quadratic in the deviation of the investment-capital ratio from its steady-state value:

$$A = \frac{\chi}{2}\left[\frac{I}{K} - (\delta + g + n)\right]^2 K,$$

where g is the long-run growth rate, n is the long-run rate of population growth, and χ is a scale parameter in the adjustment cost function. For now, we consider the case where $g = n = 0$; population and productivity growth are introduced later. For convenience, we assume $\chi = 1$ throughout this section. Investment behavior is discussed more fully in Section VI.

[87]See also Turnovsky (1996) for a similar small open economy model (with Fisherian separability), but in the context of endogenous growth.

[88]Assuming that $F(K,L)$ is homogeneous-of-degree-one in its arguments, we can write the production function as $LF(K/L,1) = f(K)[\equiv F(K,1)]$ at $L = 1$. Also, the following conditions are assumed to apply to guarantee the existence of an interior steady-state solution: $0 \le \lim_{k\to\infty} f'(K) \le r + \delta \le \lim_{k\to 0} f'(K) \le \infty$. Strict concavity of $f(K)$—an increasing function—guarantees uniqueness.

Table 5. Small Open Economy Model: Behavioral Equations and Laws of Motion

$$C_t = (\theta + p)[W_t + H_t]$$
$$I_t = (q_t - 1 + \delta)K_{t-1}$$
$$\Delta K_t = I_t - \delta K_{t-1}$$
$$\Delta B_t = rB_{t-1} + G_t - T_t$$
$$\Delta F_t = rF_{t-1} + f(K_t) - C_t - G_t - [I_t + A_t]$$
$$H_t = \beta H_{1t} + (1 - \beta)H_{2t}$$
$$\Delta H_{1t} = (r + p + \alpha_1)H_{1t-1} - [Y_t - T_t]$$
$$\Delta H_{2t} = (r + p + \alpha_2)H_{2t-1} - [Y_t - T_t]$$
$$\Delta q_{t+1} = (r + \delta)q_t - \frac{I_t}{K_{t-1}}(q_t - 1) + \frac{A_t}{K_{t-1}} - f'(K_t)$$

Table 6. Closed Economy Model: Behavioral Equations and Laws of Motion

$$C_t = (\theta + p)[W_t + H_t]$$
$$I_t = (q_t - 1 + \delta)K_{t-1}$$
$$\Delta K_t = f(K_t) - C_t - G_t - A_t - \delta K_{t-1}$$
$$\Delta B_t = r_t B_{t-1} + G_t - T_t$$
$$H_t = \beta H_{1t} + (1 - \beta)H_{2t}$$
$$\Delta H_{1t} = (r_t + p + \alpha_1)H_{1t-1} - [Y_t - T_t]$$
$$\Delta H_{2t} = (r_t + p + \alpha_2)H_{2t-1} - [Y_t - T_t]$$
$$\Delta q_{t+1} = \left[(r_t + \delta) - \frac{I_t}{K_{t-1}}\right](q_t - 1) + \frac{1}{2}(q_t - 1)^2$$
$$r_t = f'(K_t) - \delta$$

terest rates and the capital stock. The appendix illustrates the dynamic effects of fiscal policy on a small open economy in the context of life-cycle consumption behavior.

The Closed Economy Case

The discussion thus far has considered the case of a fixed world real interest rate faced by a price-taking small open economy. However, in the context of global shocks (for example, changes in public debt across countries), one might expect the world real interest rate to be affected and to change over time. This phenomenon can be incorporated into the same basic framework by noting that the world as a whole is a closed economy and introducing an endogenous real interest rate to be determined by tastes, technology, and policies.

In a closed economy, domestic saving must equal investment in the absence of international capital flows (that is, zero current account). Hence, the rate of capital accumulation will depend on preferences, or on the willingness of households to forgo current consumption (save), as well as on the return to investment as determined by technology. To ensure that the level of saving equals investment, the domestic real interest rate $r(t)$ must adjust to equate the supply and demand for these funds. Under profit maximization by firms, the real interest rate must also equal the *net* marginal product of capital (that is, net of depreciation):

$$r(t) = f'(K(t)) - \delta. \tag{34}$$

As before, net investment—defined as gross investment net of depreciation—determines the incremental change in the capital stock. But now, in a closed economy, domestic investment is equal to domestic saving. Hence, capital stock dynamics can be written as follows:

$$\dot{K} = f(K) - C - G - A - \delta K. \tag{35}$$

The rate of capital accumulation is determined by that portion of *GDP* (=*GNP*) that is saved or set aside after accounting for private and public consumption, capital depreciation, and installation costs.[89]

The equations characterizing the closed economy are presented in discrete time in Table 6 under the life-cycle interpretation. In comparison with Table 5, for a closed economy, net foreign assets and the current account are identically zero. Also the real interest rate in Table 6 carries a time argument.[90] The modification to incorporate dynastic households into a closed economy follows in exactly the same way as in the small open economy version of the model.

Extensions

Liquidity Constraints

The overlapping-generations framework can be extended to consider the case where capital market

[89]In the case of public investment, the model would need to be revised to include the contribution of the public sector to the domestic capital stock.

[90]With a time-varying rate of interest, the present value of labor income that comprises human wealth is given by

$$H(t) \equiv \int_t^\infty [Y(v) - T(v)]e^{-\int_t^v (r(z)+p)dz}\,dv.$$

Differentiating this expression with respect to time yields the dynamic equation for human wealth shown in Table 6.

imperfections preclude some agents from always borrowing against their future incomes. In particular, it is assumed that younger generations with insufficient collateral in the form of financial wealth are initially denied access to credit markets and, hence, are left to consume out of current resources. Assuming that a generation "graduates" out of this pool just as another is born into it, a fixed proportion λ of liquidity-constrained individuals exists in the economy. For these current-income consumers, consumption is constrained by current disposable income: $c(s,t) = y(s,t) - \tau(s,t)$, where $y(s,t)$, $c(s,t)$, and $\tau(s,t)$ are labor income, consumption, and taxes for a given generation s at time t.

Consequently, overall consumption is characterized by the behavior of *both* permanent- and current-income consumers. Leaving aside life-cycle income momentarily, aggregate consumption can be written compactly as

$$\begin{aligned} C &= (\theta + p)[W + (1 - \lambda)H] + \lambda[Y - T] \\ &= C^p + C^c. \end{aligned} \tag{36}$$

Agents with the ability to borrow choose their consumption (C^p) based on permanent income as before, which consists of financial wealth W and human wealth H.[91] Meanwhile, agents who face borrowing constraints have their consumption (C^c) constrained by current disposable income, where Y is labor income and T is lump-sum taxes. The parameter λ represents the proportion of households in the latter category, and total consumption is simply the sum of consumption by permanent-income and current-income consumers. In equation (36), λ can be interpreted as the degree of *excess sensitivity* of consumption to current disposable income compared with the case where every agent behaves according to the permanent-income hypothesis.

Returning to the life-cycle case, in the presence of age-dependent income (and taxes), as depicted in equation (21), aggregate consumption can be summarized as follows:

$$\begin{aligned} C = {} & (\theta + p)[W + \beta(1 - \lambda_1)H_1 + (1 - \beta)(1 - \lambda_2)H_2] \\ & + (\beta\lambda_1 + (1 - \beta)\lambda_2)[Y - T]. \end{aligned} \tag{37}$$

With generation-specific income, the excess sensitivity of consumption to income now depends on the relative share of aggregate disposable income held by current-income consumers—described by the coefficient $\beta\lambda_1 + (1 - \beta)\lambda_2$—rather than on the proportion of liquidity-constrained consumers in the population.[92]

Population and Productivity Growth

The basic model can also be further extended to the case of population and productivity growth (see also Buiter, 1988, and Weil, 1989). In the case of a growing population, the rate of population growth n is equal to the difference between the birth and death rates: $n = b - p$. The size of the total "population" at each moment in time is given by $N(t) = e^{nt}$, where $N(0)$ is normalized to unity. In the dynastic case, N would represent the number of dynastic families; if the number of members within these family were constant, then the total population would be proportional to the number of dynasties.[93]

Similarly, we can introduce long-run growth in productivity. Assuming (Harrod-neutral) labor-augmenting technical change, labor productivity is assumed to grow at a constant rate g. In other words, labor input L, measured in efficiency units, depends on both the number of workers and the efficiency of each worker: $L(t) = N(t)e^{gt}$. In the case where N represents the number of dynasties (rather than individuals), the labor force would be proportional to the number of these households. As with the population, the level of productivity at $t = 0$ is normalized to unity.

In the case of overlapping generations with life-cycle income, further substantive modifications are needed in the case of population growth. Specifically, to ensure adding up, individual labor income is now expressed as a function of aggregate labor income *per capita*:

$$\begin{gathered} y(s,t) = [a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)}]Y(t)e^{-nt}; \\ \frac{a_1 b}{\alpha_1 + b} + \frac{a_2 b}{\alpha_2 + b} = 1. \end{gathered} \tag{38}$$

The second part of this expression reflects the adding-up restriction on the parameters in terms of the *birth* (rather than death) rate, so that individual incomes sum to aggregate income over all generations; the earlier example in the text showed the simpler case where $b = p$ (that is, stationary population). The dynamic equations for human wealth (in labor

[91]From the equation, permanent-income consumers who comprise $1 - \lambda$ of the population hold all the financial wealth W in the economy. This is because agents are born without wealth and younger agents do not save (that is, accumulate wealth) initially while liquidity constrained.

[92]By adding up, we have $\beta \equiv a_1 p/(p + \alpha_1)$, $\lambda_1 \equiv 1 - e^{-(\alpha_1+p)(t-\tau(t))}$, $\lambda_2 \equiv 1 - e^{-(\alpha_2+p)(t-\iota(t))}$, where $\iota(t)$ is an index of the oldest generation still liquidity constrained at time t. By construction, $\lambda_2 > \lambda_1$, and both parameters can be greater than λ for plausible (hump-shaped) income profiles.

[93]In terms of specific cohorts, the number of individuals (or dynasties) born as part of cohort s is a proportion of the contemporaneous population given by $N(s,s) = bN(s)$, and the number of these individuals surviving at time $t \geq s$ is given by $N(s,t) = bN(s)e^{-p(t-s)}$.

Table 7. Extended Closed Economy Model with Liquidity Constraints and Population and Productivity Growth

$$c_t = (\theta + p)[w_t + \beta(1 - \lambda_1)h_{1t} + (1 - \beta)(1 - \lambda_2)h_{2t}] + (\beta\lambda_1 + (1 - \beta)\lambda_2)[y_t - \tau_t]$$

$$i_t = (q_t - 1 + \delta + n + g)k_{t-1}$$

$$\Delta k_t = f(k_t) - c_t - gv_t - a_t - (\delta + n + g)k_{t-1}$$

$$\Delta b_t = (r_t - n - g)b_{t-1} + gv_t - \tau_t$$

$$h_t = \beta h_{1t} + (1 - \beta)h_{2t}$$

$$\Delta h_{1t} = (r_t + p + \alpha_1 - g)h_1 - [y_t - \tau_t]$$

$$\Delta h_{2t} = (r_t + p + \alpha_2 - g)h_{2t-1} - [y_t - \tau_t]$$

efficiency units) under life-cycle income are derived analogously:[94]

$$h = \beta h_1 + (1 - \beta)h_2, \tag{39}$$

$$\dot{h}_1 = (r + p + \alpha_1 - g)h_1 - [y - \tau], \tag{40}$$

and

$$\dot{h}_2 = (r + p + \alpha_2 - g)h_2 - [y - \tau]. \tag{41}$$

For n and $\mu > 0$, we normalize aggregate variables (denoted by lowercase) in terms of labor measured in efficiency units.[95] Accordingly (normalized), consumption with life-cycle income and liquidity constraints would be modified as follows:

$$c = (\theta + p)[w + \beta(1 - \lambda_1)h_1 + (1 - \beta)(1 - \lambda_2)h_2] + (\beta\lambda_1 + (1 - \beta)\lambda_2)[y - \tau]. \tag{42}$$

Under the assumption of life-cycle behavior, the set of revised equations in discrete time in the closed economy model with population or productivity growth and liquidity constraints is summarized in Table 7.

Income Profiles: Theory and Calibration

To simulate the implications of life-cycle saving in the model, we must calibrate the degree of concavity in age-earnings profiles. This subsection discusses some specification issues and then describes the data set, the estimated earnings profile, and the calibration that we adopt.

Specification Issues

To characterize the time profile of earnings, individual incomes are represented (following the previous discussion) as a time-varying, generation-specific weight $\omega(s,t)$ on income per capita for the economy as whole. Specifically, labor income $y(s,t)$ for a member of generation s at time t ($\geq s$), as a proportion of average income per capita, can be written as

$$y(s,t) = \left[a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)}\right]\frac{Y(t)}{N(t)}, \tag{43}$$

where Y is aggregate labor income, and N is the size of the population. Several characteristics of the earnings profiles and parameter restrictions with respect to equation (43) are worth noting:

- *Nonmonotonicity*. To guarantee that income profiles do not rise or fall monotonically, we require that a_1 and a_2 be of opposite sign. Without loss of generality, we further specify $a_1 > 0$ and $a_2 < 0$. To ensure concavity, two additional restrictions are needed:

  Initially increasing. For incomes to rise initially, the time derivative of $\omega(s,t)$ at $s = t$ must be strictly positive, requiring $\alpha_1 a_1 < -\alpha_2 a_2$.

  Eventually declining. To also ensure that labor earnings eventually fall off with retirement, a sufficient condition has $\alpha_1, \alpha_2 > 0$, which in combination with the previous assumptions will generate a hump-shaped time profile for labor income.

- *Nonnegativity*. For individual incomes to always remain positive given aggregate income (that is, for $\omega(s,t) \geq 0$ for all t), a necessary condition has $a_1 \geq -a_2$, which is also sufficient provided that we also have $\alpha_2 > \alpha_1$.[96]

- *Adding up*. Integrating over all generations, individual labor incomes must add up to aggregate labor income, requiring that

  $$\frac{a_1 b}{\alpha_1 + b} + \frac{a_2 b}{\alpha_2 + b} = 1, \text{ where } b \text{ is the birth rate.}$$

In light of the parameter restrictions on the two exponential terms, the weighting function can be thought of as the sum of two opposing factors:

[94]We now have $\beta \equiv a_1 b/(b + \alpha_1)$, $\lambda_1 \equiv 1 - e^{-(\alpha_1+b)(t-\iota(t))}$, $\lambda_2 \equiv 1 - e^{-(\alpha_2+b)(t-\iota(t))}$.

[95]Lowercase variables with a time and a generation index refer to individual measures, whereas lowercase variables with only a time argument reflect per capita measures (in units of labor efficiency): $x(t) \equiv X(t)/L(t) = X(t)e^{-(n+g)t}$. Government spending in labor efficiency units is denoted gv_t to avoid confusion with the growth rate.

[96]Together, the conditions for nonnegative and initially increasing income profiles imply

$$\alpha_2 a_1 > -\alpha_2 a_2 > \alpha_1 a_1 > -\alpha_1 a_2.$$

(1) declining labor supply (gradual retirement),[97] which by itself reduces labor income over time, and (2) the declining costs of inexperience (wage gains from seniority), which increases labor income over time. In combination, the effects of experience and seniority cause incomes to rise early on, but the effects of (gradual) retirement eventually dominate to lower wage earnings. Together, these "structural" factors can be thought of as underlying the concave earnings profile over an agent's lifetime.

The simple two-exponential specification can also be generalized to allow for a broader range of time profiles for labor income. Specifically, we can expand (43) as follows:

$$y(s,t) = \left[\sum_{i=1}^{k} a_i e^{-\alpha_i(t-s)}\right]\frac{Y(t)}{N(t)}, \qquad (44)$$

for some integer k. This more general specification is used later in the estimation along with the corresponding parameter restrictions on the a_i and α_i terms to ensure adding up and concavity.

Data and Estimation

Using statistics on labor income and employment by age for the United States, a data set was constructed containing the cross-sectional distribution of real labor income at selected ages during each year from 1980 to 1995.[98] The ages used were 20, 30, 40, 50, 60, and 75 years, essentially representing the midpoint (or median) ages for each of six cohort ranges.[99] This provided 16 years of data on the cross-sectional distribution of labor income for each of six age groups, for a total of 96 observations.

To characterize the time profile of labor earnings empirically, we estimate a structural time-series representation of these cross-sectional income distributions. Specifically, we assume that a typical individual's earnings over his or her lifetime follow the time pattern suggested by the average income profile seen in the cross-sectional distribution. The information in the cross-sectional profile is then used to estimate a time-series relationship between labor earnings and age, capturing the life-cycle pattern of an individual's labor income.

In equations (43) and (44), labor income is expressed in absolute terms (that is, in consumption units). However, we can obtain a measure of *relative* income by expressing individual labor income for a particular cohort as a proportion of income per capita for the aggregate economy:

$$ry(s,t) \equiv y(s,t)N(t)/Y(t) = \sum_{i=1}^{k} a_i e^{\alpha_i(t-s)}. \qquad (45)$$

Relative income profiles have the advantage of isolating the parameters of interest and are more likely to reflect the institutional aspects of labor markets (for example, seniority wages, age of retirement, and so on) that interest us. The shapes of the relative income profiles are also likely to be more stable (and thus comparable) over time than absolute income profiles, given time-variation in aggregate labor productivity.[100]

To estimate the shape of the earnings profile, we employ a nonlinear least squares (NLLS) estimation of equation (45) using our data on relative income distributions.[101] However, the specification based on the sum of two (or more) exponential terms has a multiplicity of possible parameterizations (that is, local maxima). Consequently, we make certain identifying restrictions by imposing values for the birth rate b or a given set of coefficients a_1, a_2, and so on, to obtain conditional estimates of the parameters of interest (that is, α's).[102] This narrows the parameter search considerably and provides more robust estimates for alternative starting values for the estimated parameters. Conditional NLLS estimates of equation (45) are shown in Table 8 for one case with $k = 2$ and for two cases with $k = 3$.

The estimates in Table 8 do reasonably well in fitting the cross-sectional income distributions for the United States and are generally sensible. The plots of the fitted income profiles based on each set of estimates are shown in Figure 3. The specifications with an added exponential term ($k = 3$) have somewhat better fits, although the specification with the highest R^2 (that is, in column 2) yields an implausibly high birth rate (6 percent) and eventually turns negative. For these reasons, and others discussed below, the estimates in column 3 of Table 8 are preferred.

Steady-State Calibration

To ascertain whether the model's calibration of birth rates and death rates is sensible, several aspects

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[97]In reality, individuals generally experience a discontinuous fall in labor supply and wage earnings with retirement. However, given that individuals retire at different ages, the representative income profile averaged over many individuals may be approximated with a smooth function. See also Blanchard (1985) and Saint-Paul (1992).

[98]Cross-sectional data on real labor income were readily available only for Canada and the United States. For convenience, only the U.S. data are used, although both data sets appear somewhat similar.

[99]The cohort ranges are 18–24, 25–34, 35–44, 45–54, 55–64, and 65 +.

[100]This will certainly be the case if aggregate labor productivity growth affects absolute labor incomes for all age groups proportionately without affecting the relative (cross-sectional) distribution of income.

[101]NLLS estimates of the vector of parameters (α, a) seek to minimize the sum of squared residuals u from the following regression: $ry_t = f(t,\alpha, \text{a}) + u_t$, where $f(.)$ follows from equation (46).

[102]The imposed parameters are obtained through grid search.

Table 8. Relative Income Profiles: Nonlinear Least Squares Estimates

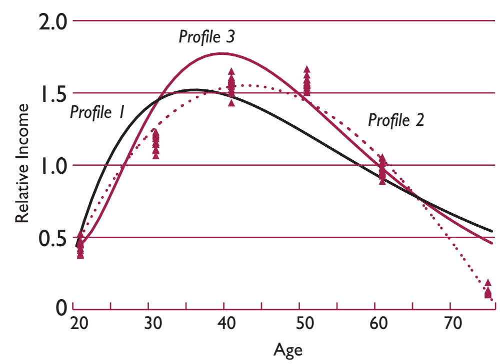
Parameters	(1) $k = 2$	(2) $k = 3$	(3) $k = 3$
α_1	0.051**	0.010**	0.078**
α_2	*0.061*	0.019**	0.121**
α_3	. . .	*–0.004*	*0.091*
a_1	*20.00*	*40.00*	*100.00*
a_2	*–19.56*	*–30.00*	*40.00*
a_3	. . .	*–9.56*	*–139.56*
b	0.031**	0.057**	*0.03*
$\bar{R}^2$	0.78	0.98	0.88
DW	1.95	2.49	1.72

Note: ** indicates significance at the 5 percent level; entries in italics denote parameter values that were imposed (or redundant) in the conditional estimates. The first $k - 1$ exponents (α's) are estimated directly, and the kth term is determined from the adding-up restriction.

Model: $ry_t = \sum_{i=1}^{k} a_i e^{-\alpha_i(t-20)}$

Restrictions: $\sum_{i=1}^{k} \frac{a_i b}{\alpha_i + b} = 1; \ \sum_{i=1}^{k} a_i = \overline{ry}_{20}.$

Figure 3. Estimated Income Profiles



Note: Triangles represent historical data points on the age-earnings distribution, centered at the ages of 20, 30, 40, 50, 60, and 75. The three labor income profiles, which reflect the parameter estimates reported in the corresponding columns of Table 8, depict the per capita incomes for each age cohort relative to average income per capita across all age groups.

need to be considered. First, as the death rate is directly related to the length of an individual's planning horizon in the model, it should broadly reflect measures of life expectancy. Second, the difference between birth and death rates in the model represents the rate of population growth of working-age individuals and retirees, which also guides the choice of these rates. In long-run steady state, to obtain a stationary population, the model further requires *equal* birth and death rates at their long-run (replacement) levels. At those levels, the model's implications for the steady-state age distribution offer yet another guide as to the choice of these magnitudes.

In the presence of a stationary population, the model and its constant death rate assumption can generate steady-state age distributions that broadly mimic those implied by demographic projections. For example, to match an elderly dependency ratio of 44 percent, defined as the population 65 years and older as a share of the working-age population (ages 20–64), the model requires a long-run birth and death rate of about 2½ percent. However, the model's simple assumption that the (conditional) probability of death is identical at all ages implies a steady-state age distribution that tends to overstate the number of individuals in younger age groups and understate the number of middle-age and older individuals,[103] as illustrated in Table 9.[104]

Specifying death rates of 2–2½ percent suggests individual planning horizons of about 40–50 years, or life expectancy (at age 20, when individuals are "born" into the youngest cohort of workers) of about 60–70 years. Away from steady state, the 3 percent birth rates in Table 8 are quite plausible and, given these death rates, imply a growth rate for the (adult) population of about ½ of 1 percent to 1 percent, consistent with the historical figures for the United States during the sample period.[105]

[103]The theoretical distribution also includes an asymptotic tail of (arbitrarily) old people, whereas in reality this distribution is clearly truncated at some finite maximum age. This latter issue is not too severe a problem given that the very old generations form an increasingly small (infinitesimal) proportion of the population.

[104]As a share of the total population, the number of survivors from a generation born at time s remaining at time t is equal to $be^{-p(t-s)}$; with a stationary population in steady state, $b = p$ and $n = 0$. Using this expression, we approximate the model's steady-state age distribution across the (discrete) age groups as shown in the table.

[105]The calculated birth rate—defined as the relative size of new arrivals (that is, youngest cohort) as a share of the existing adult population—varied from 2½ percent to 3 percent for the United States over the sample period. The average growth rate for the adult population was about ½ of 1 percent over the same period, implying a death rate of 2–2½ percent.

Table 9. Theoretical and Projected Steady-State Age Distributions
(In percent of adult population)

	Age Group						Dependency Ratio
	20–24	25–34	35–44	45–54	55–64	65+	
Steady-state model	11.9	19.8	15.4	12.0	9.3	31.6	46
World Bank projections[1]	7.8	15.6	15.5	15.4	15.1	30.6	44

[1]For the United States, based on Bos and others (1994).

Estimates of the Mark III Consumption Function

The previous subsection provides a calibration of the parameters (for example, α_1, α_2) underlying the life-cycle component of the consumption function in Mark III. To complete the calibration of the consumption-saving model, we also require an estimate of the sensitivity parameter λ, representing the share of consumption accounted for by income-constrained individuals, and the intertemporal elasticity of substitution σ^{-1} in consumption, which need not equal unity (that is, log utility case) as assumed (for convenience) in Tables 5 through 7.

Table 10 presents a summary of the system of equations that make up the consumption-saving model in Mark III. The parameters associated with the life-cycle profile of labor income (that is, a_i, α_i, b) are taken from column 3 of Table 8. Estimates of the sensitivity of consumption to disposable income λ and the intertemporal elasticity of substitution σ^{-1} are also shown.

The estimates in Table 10, based on annual data for 1982–96, reflect, for the seven major industrial countries, pooled estimates of the intertemporal elasticity of substitution (σ^{-1}) and country-specific estimates of the share of income-constrained consumption. The model is estimated with instrumental variables, using as instruments $C(t-1)$, $C_{DI}(t-1)$, $WH1(t-1)$, $WH2(t-1)$, and $WH3(t-1)$. The estimated magnitude of the elasticity of substitution is highly significant and broadly consistent with the magnitudes of estimates from other sources.[106] Box

[106]The estimate of 0.41 here relates to the proportion of consumption that is interest sensitive and needs to be multiplied by a factor of $(1-\lambda)$ before comparison with estimates of the elasticity of substitution for aggregate consumption. Patterson and Pesaran (1992) and Attanasio and Weber (1993) place the latter in the range of 0.1 to 0.3; Hall (1988) argues that it may be lower than 0.2.

Table 10. Estimated Parameters of the Mark III Consumption-Saving Model

Aggregate-consumption: $C = C_{DI} + C_{PI}$

Income-constrained consumption:
$$C_{DI} = YD[\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3(1-\beta_1-\beta_2)]$$

Wealth-constrained consumption:
$$C_{PI} = MPC(WK + M/P + B/P + NFA/P + WH)$$

Inverse of marginal propensity to consume:
$$MPC_{t+1}^{-1} = \{1-\sigma^{-1}[(1-\sigma)(rsr_t + p_t) - (\tau_t + p_t)]\}MPC_t^{-1} - 1$$

Human wealth:
$$WH = \beta_1(1-\lambda_1)WH1 + \beta_2(1-\lambda_2)WH2 + (1-\beta_1-\beta_2)(1-\lambda_3)\,WH3$$

i^{th} component of human wealth:
$$WH(i)_{t+1} = WH(i)_t\,(1 + rsr_t + p_t) - YD_t$$

$$\lambda_i = 1 - \exp\{[(\alpha_i + b)/b]\log(1-\lambda)\}; \quad \beta_i = \frac{a_i b}{\alpha_i + b}$$

	σ^{-1}	λ	R^2	SE
Canada	0.41** (0.017)	0.75** (0.09)	0.971	0.022
France	0.41** (0.017)	0.49** (0.01)	0.988	0.010
Germany	0.41** (0.017)	0.46** (0.01)	0.982	0.019
Italy	0.41** (0.017)	0.50** (0.02)	0.874	0.033
Japan	0.41** (0.017)	0.28** (0.01)	0.974	0.026
United Kingdom	0.41** (0.017)	0.46** (0.01)	0.936	0.034
United States	0.41** (0.017)	0.30** (0.09)	0.993	0.015

Note: Standard errors reported in parentheses; ** indicates that estimated coefficient is significantly different from zero at the 5 percent significance level.

Box 9. The Global Crowding-Out Effects of Government Debt

As in most modern macro models, the extent of fiscal crowding out in response to a government debt shock depends critically on (1) the degree to which consumers are assumed to count government bonds as net wealth, (2) the relationship assumed between aggregate consumption and disposable income, and (3) the assumed sensitivity of aggregate consumption to changes in interest rates. If consumers are connected to all future generations by operative intergenerational transfers, increases in government debt will not crowd out private investment because consumers will change their saving rate today to prepare for tax liabilities in the future. This is referred to as the Ricardian equivalence hypothesis because taxes today (that is, tax financing of government spending) are equivalent to taxes in the future (that is, deficit financing of government spending). For reasons discussed in the main text, MULTIMOD does not adopt the Ricardian equivalence hypothesis, but rather reflects the view that households adjust their saving by only part of the higher future tax burden associated with higher levels of government debt. In effect, agents treat a portion of their holdings of government bonds as net wealth because they "excessively" discount the higher tax liabilities in the future required to service the government's interest payments on its debt.

Two explanations why full Ricardian equivalence does not apply in practice are embodied in the properties of Mark III. First, because a significant fraction of consumers cannot borrow against their future labor income, their expenditure is effectively constrained by their current disposable income. Second, consumers who are constrained by wealth rather than by disposable income are assumed not to internalize the tax burden that will be passed on to future generations. Thus, wealth-constrained consumers are assumed to incompletely adjust their saving rates in response to higher future tax liabilities because they realize that future generations will partly share the tax burden associated with higher levels of government debt. Both imperfect capital markets and the disconnectedness of today's generation from future generations imply that higher levels of government debt will be associated with a tendency to overconsume available resources. This tendency to overconsume will result in higher real interest rates and eventually in a lower capital stock and lower sustainable levels of real income and consumption.

The increase in interest rates required to eliminate the tendency to overconsume available resources will depend critically on the interest sensitivity of consumption. If consumption were highly sensitive to changes in real interest rates, then only a small rise in interest rates would be required to induce consumers to adjust their saving rates in response to an increase in government debt. However, the empirical literature, including Mark III estimates, suggests that the interest sensitivity of consumption and saving is low. Thus, this evidence also suggests significant long-run crowding-out effects of government debt.

9, which focuses on the crowding-out effects of government debt, provides some perspective on the sensitivity of these fiscal effects to certain critical assumptions and parameter estimates.

Appendix. Fiscal Policy Effects in a Small Open Economy

Two experiments are considered here to illustrate the dynamic effects of fiscal policy in a small open economy. The simulations use the Canada bloc of MULTIMOD to explore the effects of (1) a permanent increase in government debt that is a result of a temporary tax cut, holding constant the level of real government spending, and (2) a permanent increase in real government spending, holding tax rates constant temporarily before raising them to subsequently stabilize the ratio of government debt to GDP. In both shocks, tax rates are adjusted after the fifth year to raise the debt-to-GDP ratio to a level that is 10 percentage points higher than in the baseline. In each case, the current reduction in public saving (that is, deficit financing) and its consequences for future tax burdens have important macroeconomic effects as private agents are unable, or fail, to fully internalize the implications of the government's intertemporal budget constraint. When consumers "excessively discount" future tax liabilities or are "excessively sensitive" to current disposable income, changes in fiscal policy can have relatively large effects on the real economy, reflecting significant departures from Ricardian equivalence. (See Box 9 for further discussion of the factors underpinning the Ricardian equivalence hypothesis.)

Higher Public Debt Through a Temporary Tax Cut

Figure 4 shows the effects of a temporary five-year tax cut, holding government spending constant, that leads to a permanent increase of 10 percentage points in the ratio of government debt to GDP.[107] Higher public borrowing is accompanied by a pickup in economic activity in the near term, as both

[107]The tax cut takes the form of a lowering of the basic tax rate by 2 percentage points of nominal GDP for five years, holding constant the tax rate on capital income; it thus amounts essentially to a lowering of the tax rate on labor income.

Steady-State Effects of Simultaneous 10 Percentage Point Increases in Ratios of Government Debt to GDP of All Industrial Countries

	Without an Increase in Distortionary Taxes on Capital	With an Increase in Distortionary Taxes on Capital
Output (in percent)	–1.1	–1.6
Consumption (in percent)	–1.0	–1.4
Capital stock (in percent)	–3.4	–4.7
Real interest rate (basis points)	31.0	32.0

The table provides some estimates of the long-run crowding-out effects of simultaneous 10 percentage point increases in ratios of government debt to GDP of all industrial countries. The table includes estimates of the long-run effects on real interest rates, aggregate output, consumption, and the capital stock for all industrial countries. Two cases are shown, representing different assumptions regarding which taxes are raised to meet the higher steady-state interest obligations on government debt. In the first case, the basic tax rate on nominal GDP is raised, holding constant the tax rate on capital, so that the increase in taxes is essentially borne by labor. Unlike capital taxes, labor taxes are nondistortionary in Mark III because the labor supply is assumed to be exogenous in the long run. In the second case, we allow the tax rate on capital income to rise by the same amount as the basic tax rate on GDP.

In the first case, the real interest rate rises by 31 basis points, and the capital stock declines by 3.4 percent. This reduces potential output by 1.1 percent, and the sustainable level of consumption declines by 1.0 percent. When distortionary capital taxes are assumed to partially finance the increase in the debt-service burden, there are slightly larger crowding-out effects on output, consumption, and the capital stock.

private consumption and saving rise with the fall in taxes and the increase in disposable income. An increase in the fiscal deficit thus places upward pressure on interest rates and the exchange rate initially, leading to some crowding out of investment and net exports. Because the increase in public dissaving is not fully offset by a rise in private saving, the current account deficit tends to widen, and the greater reliance on foreign saving leads to an increase in net external debt.

In the long run, private consumption and disposable income are lower, reflecting the higher taxes required to finance higher interest payments on higher public debt. The long-run levels of investment, the capital stock, and output decline very slightly in association with the higher level of debt because the steady-state real interest rate in the individual country models is assumed to be tied down to the (exogenous) world real interest rate.[108] Meanwhile, net exports rise in the steady state—in association with a permanent decline in the real competitiveness index—to finance the higher interest payments to foreigners resulting from higher external debt.

[108]In the full model simulations, the world real interest rate is endogenous and determined by equilibrating world savings and investment. Bayoumi and Laxton (1994) consider the effects of fiscal policy when real interest rate differentials depend on the level of government debt.

Higher Public Debt with Permanently Higher Government Spending

Figure 5 shows the effects of a permanent 2 percentage point increase in the ratio of government spending to GDP, holding tax rates constant for five years and subsequently adjusting the basic tax rate to stabilize the ratio of public debt to GDP at a level that is 10 percentage points higher than in the baseline. This shock also leads to increases in output, interest rates, and the exchange rate in the short term. Higher interest rates and an appreciated currency tend to lower investment and net exports in the near term, and the fall in national (private plus public) saving tends to worsen the current account balance and increase the level of net foreign liabilities.

In the long run, private consumption and disposable income are again lower because of higher taxes. However, the long-run decline in disposable

Figure 4. Higher Public Debt Through a Temporary Tax Cut in Canada
(Deviations from baseline values, in percent)

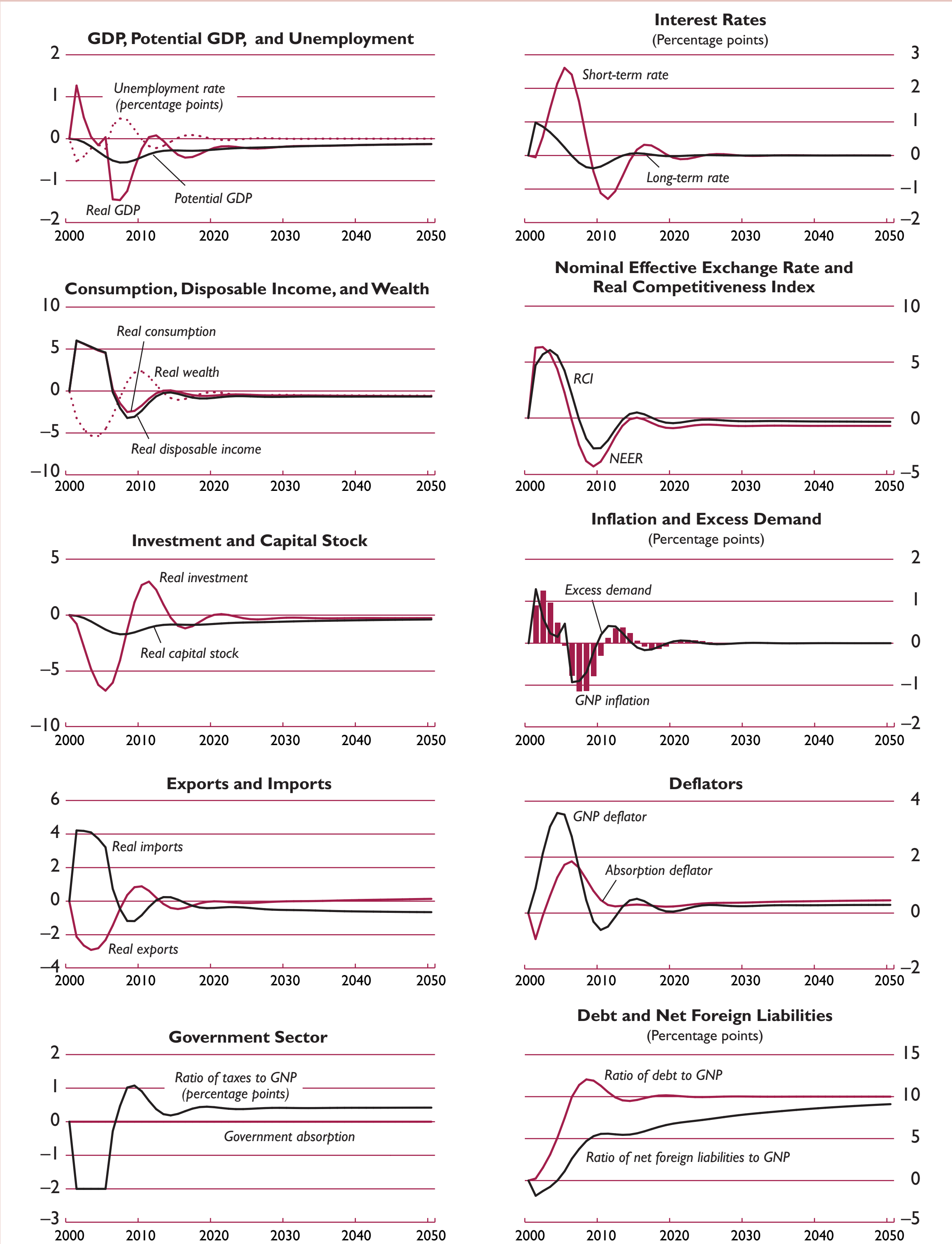
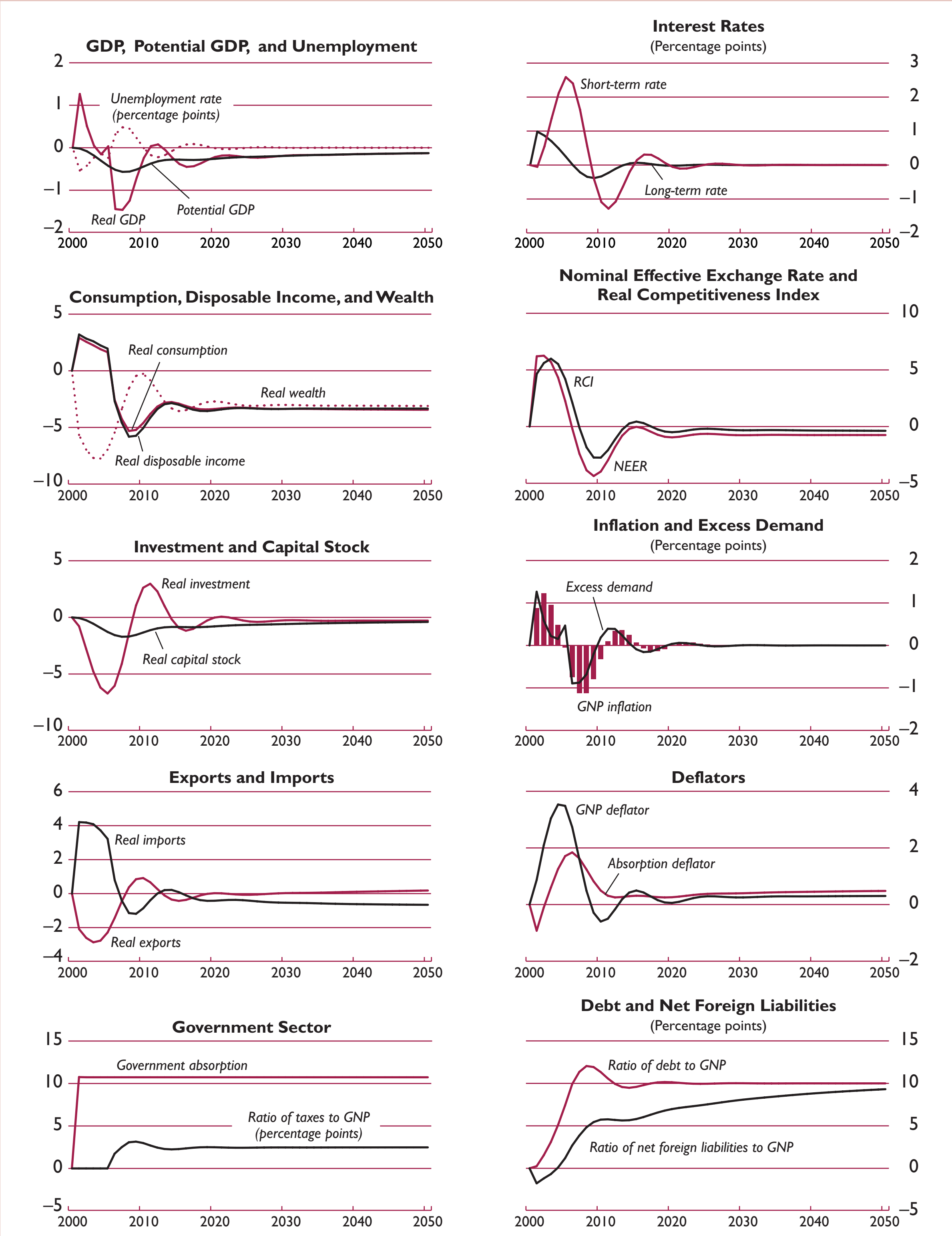


Figure 5. Higher Public Debt with Permanently Higher Fiscal Spending in Canada
(Deviations from baseline values, in percent)



income is somewhat larger for the government expenditure shock, which induces a larger crowding-out effect on private consumption to accommodate higher public consumption. But as was the case for the temporary tax cut, the real interest rate converges back to the world rate of interest as output, investment, and the capital stock return to their baseline levels. Meanwhile, on the external side, a steady-state real depreciation is again required to boost net exports and finance the larger stock of net external debt.

The short-run effects of government expenditure and tax rate shocks are strikingly similar for Canada because the proportion of consumers who base their consumption on disposable income is estimated to be large. For the other major industrial countries, short-run government spending multipliers tend to be significantly greater than the tax multipliers.