VI Benchmarking

A. Introduction

6.1. Benchmarking deals with the problem of combining a series of high-frequency data (e.g., quarterly data) with a series of less frequent data (e.g., annual data) for a certain variable into a consistent time series. The problem arises when the two series show inconsistent movements and the less frequent data are considered the more reliable of the two. The purpose of benchmarking is to combine the relative strengths of the low- and high-frequency data. While benchmarking issues also arise in annual data (e.g., when a survey is only conducted every few years), this chapter deals with benchmarking to derive quarterly national accounts (QNA) estimates that are consistent with annual national accounts (ANA) estimates, where the annual data[1] provide the benchmarks.[2] Quarterly data sources often differ from those used in the corresponding annual estimates, and the typical result is that annual and quarterly data sources show inconsistent annual movements. In a few cases, the quarterly data may be superior and so may be used to replace the annual data.[3] More typically, the annual data provide the most reliable information on the overall level and long-term movements in the series, while the quarterly source data provide the only available explicit[4] information about the short-term movements in the series, so that there is a need to combine the information content of both the annual and quarterly sources.

6.2. Benchmarking has two main aspects, which in the QNA context are commonly looked upon as two different topics; these are (a) *quarterization*[5] of annual data to construct time series of historical QNA estimates ("back series") and revise preliminary QNA estimates to align them to new annual data when they become available, and (b) *extrapolation* to update the series from movements in the indicator for the most current period ("forward series"). In this chapter, these two aspects of benchmarking are integrated into one common **benchmark-to-indicator (BI) ratio framework** for converting individual indicator series into estimates of individual QNA variables.

6.3. To understand the relationship between the corresponding annual and quarterly data, it is useful to observe the ratio of the annual benchmark to the sum of the four quarters of the indicator (the annual BI ratio). Movements in the observed annual BI ratio show inconsistencies between the long-term movements in the indicator and in the annual data.[6] As a result, movements in the annual BI ratio can help identify the need for improvements in the annual and quarterly data sources. The technical discussion in this chapter treats the annual benchmarks as binding and, correspondingly, the inconsistencies as caused by errors[7] in the indicator and not by errors in the annual data. Benchmarking techniques that treat the benchmarks as nonbinding are briefly described in Annex 6.1.

[1]That is, the annual source data, or ANA estimates based on a separate ANA compilation system.

[2]A trivial case of benchmarking occurs in the rare case in which annual data are available for only one year. In this case, consistency can be achieved simply by multiplying the indicator series by a single adjustment factor.

[3]One instance is annual deflators that are best built up from quarterly data as the ratio between the annual sums of the quarterly current and constant price data, as discussed in Chapter IX Section B. Another case is that of nonstandard accounting years having a significant effect on the annual data.

[4]The annual data contain implicit information on aspects of the short-term movements in the series.

[5]Quarterization refers to generation of quarterly data for the back series from annual data and quarterly indicators, and encompasses two special cases, namely:

(a) Interpolation—that is, drawing a line between two points—which in the QNA mainly applies to stock data (except in the rare case of periodic quarterly benchmarks).

(b) Temporal distribution, that is, distributing annual flow data over quarters.

[6]See Section B.4 of Chapter II for a further discussion of this issue.

[7]The errors can be systematic ("bias") or irregular ("noise").

6.4. The general objective of benchmarking is

- to preserve as much as possible the short-term movements in the source data under the restrictions provided by the annual data and, at the same time,
- to ensure, for forward series, that the sum of the four quarters of the current year is as close as possible to the unknown future annual data.

It is important to preserve as much as possible the short-term movements in the source data because the short-term movements in the series are the central interest of QNA, about which the indicator provides the only available explicit information.

6.5. In two exceptional cases, the objective should not be to maximally preserve the short-term movements in the source data: (a) if the BI ratio is known to follow a short-term pattern, for example, is subject to seasonal variations; and (b) if a priori knowledge about the underlying error mechanism indicates that the source data for some quarters are weaker than others and thus should be adjusted more than others.

6.6. As a warning of potential pitfalls, this chapter starts off in Section B by explaining the unacceptable discontinuities between years—the "step problem"—caused by distributing annual totals in proportion to the quarterly distribution (pro rata distribution) of the indicator. The same problem arises if preliminary quarterly estimates are aligned to the annual accounts by distributing the differences between the annual sums of the quarterly estimates and independent annual estimates for the same variable evenly, or pro rata, among the four quarters of each year. Techniques that introduce breaks in the time series seriously hamper the usefulness of QNA by distorting the view of developments and possible turning points. They also thwart forecasting and constitute a serious impediment for seasonal adjustment and trend analysis. In addition to explaining the step problem, section B introduces the BI ratio framework that integrates quarterization and extrapolation into one framework.

6.7. Subsequently, the chapter presents a BI ratio-based benchmarking technique that avoids the step problem (the "proportional Denton" technique with extensions).[8] The proportional Denton technique generates a series of quarterly estimates as proportional to the indicator. as possible subject to the restrictions provided by the annual data. The chapter goes on to propose an enhancement to the Denton technique to better deal with the most recent periods. Other enhancements to the Denton are also mentioned and some other practical issues are considered.

6.8. Given the general objective stated above it follows that, for the back series, the proportional Denton is by logical consequence[9] **optimal**, if

- maximal preservation of the short-term movements in the indicator is specified as keeping the quarterly estimates as proportional to the indicator as possible; and
- the benchmarks are binding.

Under the same conditions, it also follows that for the forward series, the enhanced version provides the best way of adjusting for systematic bias and still maximally preserving the short-term movements in the source data. In addition, compared with the alternatives discussed in Annex 6.1, the enhanced proportional Denton technique is relatively simple, robust, and well suited for large-scale applications.

6.9. The technical discussion in this chapter also applies to estimates based on periodically "fixed" ratios in the absence of direct indicators for some variables that also result in a step problem. As mentioned in Chapter III, these cases include cases in which (a) estimates for output are derived from data for intermediate consumption, or, estimates for intermediate consumption are derived from data for output; (b) estimates for output are derived from other related indicators such as inputs of labor or particular raw materials; and (c) ratios are used to gross up for units not covered by a sample survey (e.g., establishments below a certain threshold). In all these cases, the compilation procedure can be expressed in a benchmark-to-(*related*) indicator form, and annual, or less frequent, variations in the ratios result in step problems. The proportional Denton technique can also be used to avoid this step problem and, for the reasons stated above, would generally provide optimal results, except in the case of potential seasonal and cyclical variations in the ratios. This issue is discussed in more detail in Section D.1, which also provides a further enhancement to the proportional Denton that allows for incorporation of a priori known seasonal variations in the BI ratio.[10]

[8]Some of the alternative techniques that have been proposed are discussed in Annex 6.1, which explains the advantages of the proportional Denton technique over these alternatives.

[9]Because the proportional Denton is a mathematical formulation of the stated objective.

[10]Further enhancements, which allow for incorporating a priori knowledge that the source data for some quarters are weaker than others, and thus should be adjusted more than others, are also feasible.

6.10. In the BI ratio benchmarking framework, only the short-term movements—not the format and overall level[11]—of the indicator are important, as long as they constitute continuous time series.[12] The quarterly indicator may be in the form of index numbers (value, volume, or price) with a reference period that may differ from the base period[13] in the QNA; be expressed in physical units; be expressed in monetary terms; or be derived as the product of a price index and a volume indicator expressed in physical units. In the BI framework, the indicator only serves to determine the short-term movements in the estimates, while the annual data determine the overall level and long-term movements. As will be shown, the level and movements in the final QNA estimates will depend on the following:

- The movements, but not the level, in the short-term indicator.
- The level of the annual data—the annual BI ratio—for the current year.
- The level of the annual data—the annual BI ratios—for several preceding and following years.

Thus, it is not of any concern that the BI ratio is not equal to one,[14] and the examples in this chapter are designed to highlight this basic point.

6.11. While the Denton technique and its enhancements are technically complicated, it is important to emphasize that shortcuts generally will not be satisfactory unless the indicator shows almost the same trend as the benchmark. The weaker the indicator is, the more important it is to use proper benchmarking techniques. While there are some difficult conceptual issues that need to be understood before setting up a new system, the practical operation of benchmarking is typically automated[15] and is not problematic or time-consuming. Benchmarking should be an integral part of the compilation process and conducted at the most detailed compilation level. It represents the QNA compilation technique for converting individual indicators into estimates of individual QNA variables.

[11]The overall level of the indicators is crucial for some of the alternative methods discussed in Annex 6.1.
[12]See definition in paragraph 1.13.
[13]For traditional fixed-base constant price data, see Chapter IX.
[14]In the simple case of a constant annual BI ratio, any level difference between the annual sum of the indicator and the annual data can be removed by simply multiplying the indicator series by the BI ratio.
[15]Software for benchmarking using the Denton technique is used in several countries. Countries introducing QNA or improving their benchmarking techniques, may find it worthwhile to obtain existing software for direct use or adaptation to their own processing systems. For example, at the time of writing, Eurostat and Statistics Canada have software that implement the basic version of the Denton technique; however, availability may change.

B. A Basic Technique for Distribution and Extrapolation with an Indicator

6.12. The aim of this section is to illustrate the step problem created by pro rata distribution and relate pro rata distribution to the basic extrapolation with an indicator technique. Viewing the *ratio of the derived benchmarked QNA estimates to the indicator* (the quarterly BI ratio) implied by the pro rata distribution method shows that this method introduces unacceptable discontinuities into the time series. Also, viewing the quarterly BI ratios implied by the pro rata distribution method together with the quarterly BI ratios implied by the basic extrapolation with an indicator technique shows how distribution and extrapolation with indicators can be put into the same BI framework. Because of the step problem, the pro rata distribution technique *is not acceptable.*

1. Pro Rata Distribution and the Step Problem

6.13. In the context of this chapter, distribution refers to the allocation of an annual total of a flow series to its four quarters. A pro rata distribution splits the annual total according to the proportions indicated by the four quarterly observations. A numerical example is shown in Example 6.1 and Chart 6.1.

6.14. In mathematical terms, pro rata distribution can be formalized as follows:

$$X_{q,\beta} = A_\beta \cdot \left(\frac{I_{q,\beta}}{\sum_q I_{q,\beta}} \right) \qquad \text{Distribution presentation (6.1.a)}$$

or

$$X_{q,\beta} = I_{q,\beta} \cdot \left(\frac{A_\beta}{\sum_q I_{q,\beta}} \right) \qquad \text{Benchmark-to-indicator ratio presentation (6.1.b)}$$

where

$X_{q,\beta}$ is the level of the QNA estimate for quarter q of year β;
$I_{q,\beta}$ is the level of the indicator in quarter q of year β; and
A_β is the level of the annual data for year β.

6.15. The two equations are algebraically equivalent, but the presentation differs in that equation (6.1.a) emphasizes the distribution of the annual benchmark (A_β) in proportion to each quarter's proportion of the

Example 6.1. Pro Rata Distribution and Basic Extrapolation

	Indicator				Derived QNA Estimates					
					Distributed Data					
	The Indicator (1)	Period-to-Period Rate of Change	Annual Data (2)	Annual BI ratio (3)	(1)	·	(3)	=	(4)	Period-to-Period Rate of Change
q1 1998	98.2				98.2	·	9.950	=	977.1	
q2 1998	100.8	2.6%			100.8	·	9.950	=	1,003.0	2.6%
q3 1998	102.2	1.4%			102.2	·	9.950	=	1,016.9	1.4%
q4 1998	100.8	−1.4%			100.8	·	9.950	=	1,003.0	−1.4%
Sum	**402.0**		**4000.0**	**9.950**					**4,000.0**	
q1 1999	99.0	***−1.8%***			99.0	·	10.280	=	1,017.7	***1.5%***
q2 1999	101.6	2.6%			101.6	·	10.280	=	1,044.5	2.6%
q3 1999	102.7	1.1%			102.7	·	10.280	=	1,055.8	1.1%
q4 1999	101.5	−1.2%			101.5	·	10.280	=	1,043.4	−1.2%
Sum	**404.8**	**0.7%**	**4161.4**	**10.280**					**4,161.4**	**4.0%**
q1 2000	100.5	−1.0%			100.5	·	10.280	=	1,033.2	−1.0%
q2 2000	103.0	2.5%			103.0	·	10.280	=	1,058.9	2.5%
q3 2000	103.5	0.5%			103.5	·	10.280	=	1,064.0	0.5%
q4 2000	101.5	−1.9%			101.5	·	10.280	=	1,043.4	−1.9%
Sum	**408.5**	**0.9%**	**?**	**?**					**4,199.4**	**0.9%**

Pro Rata Distribution

The annual BI ratio for 1998 of 9.950 is calculated by dividing the annual output value (4000) by the annual sum of the indicator (402.0). This ratio is then used to derive the QNA estimates for the individual quarters of 1998. For example, the QNA estimate for q1 1998 is 977.1, that is, 98.2 times 9.950.

The Step Problem

Observe that quarterly movements are unchanged for all quarters except for q1 1999, where a decline of **1.8%** has been replaced by an increase of **1.5%**. (In this series, the first quarter is always relatively low because of seasonal factors.) This discontinuity is caused by suddenly changing from one BI ratio to another, that is, creating a step problem. The break is highlighted in the charts, with the indicator and adjusted series going in different directions.

Extrapolation

The 2000 indicator data are linked to the benchmarked data for 1999 by carrying forward the BI ratio for the last quarter of 1999. In this case, where the BI ratio was kept constant through 1999, this is the same as carrying forward the annual BI ratio of 10.280. For instance, the preliminary QNA estimate for the second quarter of 2000 (1058.9) is derived as 103.0 times 10.280. Observe that quarterly movements are unchanged for all quarters.

(These results are illustrated in Chart 6.1.)

annual total of the indicator[16] ($I_{q,\beta}/\sum_q I_{4,\beta}$), while equation (6.1.b) emphasizes the raising of each quarterly value of the indicator ($I_{q,\beta}$) by the annual BI ratio ($A_\beta/\sum_q I_{q,\beta}$).

6.16. The **step problem** arises because of discontinuities between years. If an indicator is not growing as fast as the annual data that constitute the benchmark, as in Example 6.1, then the growth rate in the QNA estimates needs to be higher than in the indicator. With pro rata distribution, the entire increase in the quarterly growth rates is put into a single quarter, while other quarterly growth rates are left unchanged. The significance of the step problem depends on the size of variations in the annual BI ratio.

2. Basic Extrapolation with an Indicator

6.17. Extrapolation with an indicator refers to using the movements in the indicator to update the QNA time series with estimates for quarters for which no annual data are yet available (the forward series). A numerical example is shown in Example 6.1 and Chart 6.1 (for 1999).

6.18. In mathematical terms, extrapolation with an indicator can be formalized as follows, when moving from the last quarter of the last benchmark year:

$$X_{4,\beta+1} = X_{4,\beta} \cdot \left(\frac{I_{4,\beta+1}}{I_{4,\beta}}\right) \qquad \text{Moving presentation} \qquad (6.2.a)$$

or

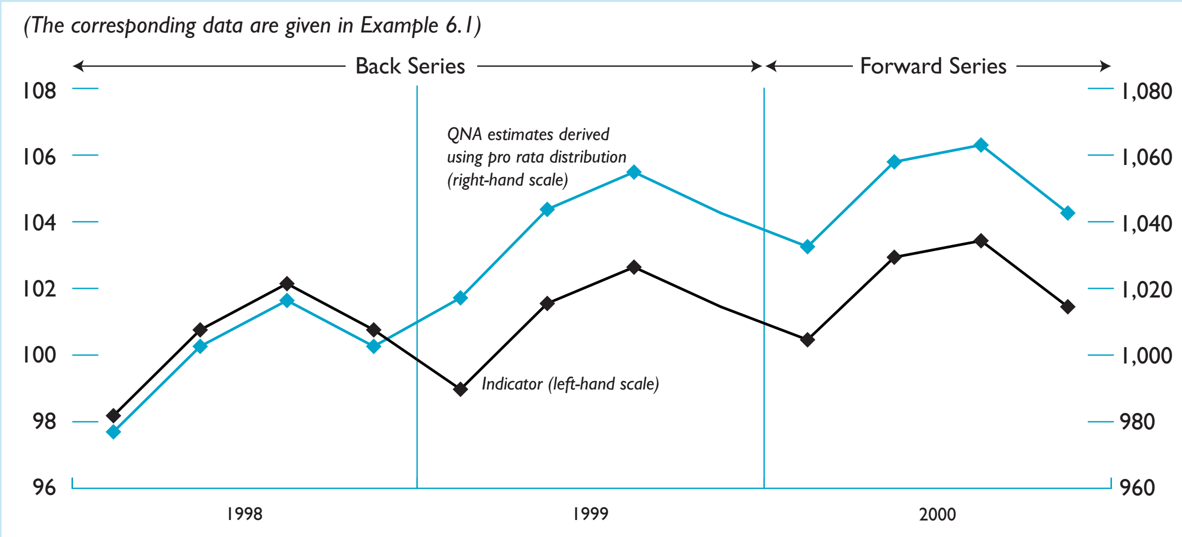
$$X_{4,\beta+1} = I_{4,\beta+1} \cdot \left(\frac{X_{4,\beta}}{I_{4,\beta}}\right) \qquad \text{BI ratio presentation} \qquad (6.2.b)$$

6.19. Again, note that equations (6.2.a) and (6.2.b) are algebraically equivalent, but the presentation differs in that equation (6.2.a) emphasizes that the last quarter of the last benchmark year ($X_{4,\beta}$) is extrapolated by the movements in the indicator from that period to the current quarters ($I_{q,\beta+1}/I_{4,\beta}$), while equation (6.2.b) shows that this is the same as

[16] The formula, as well as all subsequent formulas, applies also to flow series where the indicator is expressed as index numbers.

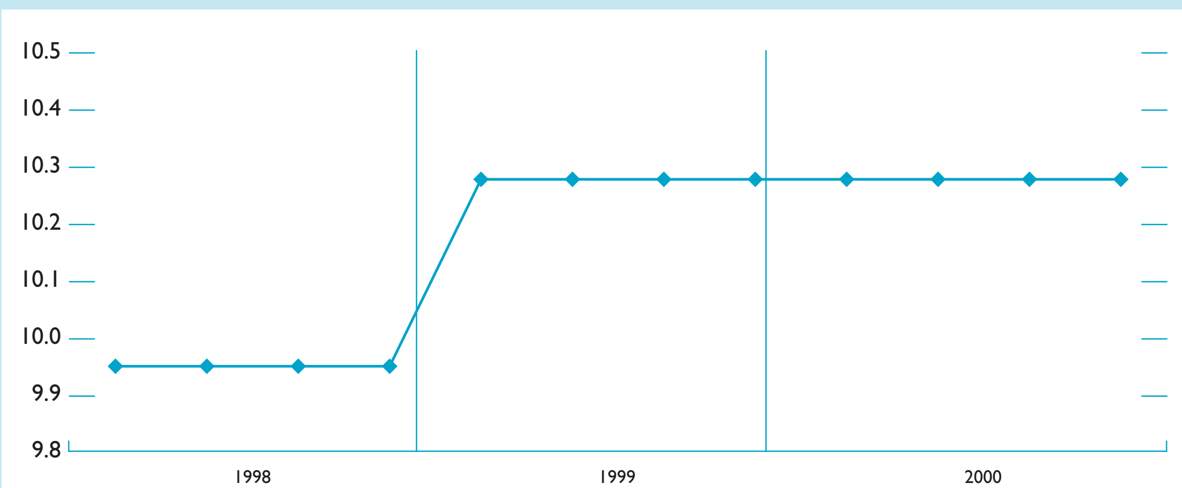
Chart 6.1. Pro Rata Distribution and the Step Problem

The Indicator and the Derived Benchmarked QNA Estimates



In this example, the **step problem** shows up as an increase in the derived series from q4 1998 to q1 1999 that is not matched by the movements in the source data. The quarterized data erroneously show a quarter-to-quarter rate of change for the first quarter of 1999 of **1.5%** while the corresponding rate of change in source data is **–1.8%** (in this series, the first quarter is always relatively low because of seasonal factors).

Benchmark-to-Indicator Ratio



It is easier to recognize the step problem from charts of the BI ratio, where it shows up as abrupt upward or downward steps in the BI ratios between q4 of one year and q1 of the next year. In this example, the step problem shows up as a large upward jump in the BI ratio from q4 1998 to q1 1999.

scaling up or down the indicator ($I_{q,\beta+1}$) by the BI ratio for the last quarter of the last benchmark year ($X_{4,\beta}/I_{4,\beta}$).

6.20. Also, note that if the quarterly estimates for the last benchmark year $X_{4,\beta}$ were derived using the pro rata technique in equation (6.1), for all quarters, the implied quarterly BI ratios are identical and equal to the annual BI ratio. That is, it follows from equation (6.1) that

$$(X_{4,\beta}/I_{4,\beta}) = (X_{q,\beta}/I_{q,\beta}) = (A_{\beta}/\textstyle\sum_q I_{q,\beta}).$$ [17]

6.21. Thus, as shown in equations (6.1) and (6.2), distribution refers to constructing the back series by using the BI ratio for the current year as adjustment factors to scale up or down the QNA source data, while extrapolation refers to constructing the forward series by carrying that BI ratio forward.

C. The Proportional Denton Method

1. Introduction

6.22. The basic distribution technique shown in the previous section introduced a step in the series, and thus distorted quarterly patterns, by making all adjustments to quarterly growth rates to the first quarter. This step was caused by suddenly changing from one BI ratio to another. To avoid this distortion, the (implicit) quarterly BI ratios should change smoothly from one quarter to the next, while averaging to the annual BI ratios.[18] Consequently, all quarterly growth rates will be adjusted by gradually changing, but relatively similar, amounts.

2. The Basic Version of the Proportional Denton Method

6.23. The basic version of the proportional Denton benchmarking technique keeps the benchmarked series as proportional to the indicator as possible by minimizing (in a least-squares sense) the difference in relative adjustment to neighboring quarters subject to the constraints provided by the annual benchmarks. A numerical illustration of its operation is shown in Example 6.2 and Chart 6.2.

6.24. Mathematically, the basic version of the proportional Denton technique can be expressed as[19]

$$\min_{(X_1 \ldots, X_{4\beta}, \ldots \ldots X_T)} \sum_{t=2}^{T} \left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}} \right]^2 \qquad (6.3)$$

$$t \in \{1, \ldots (4\beta), \ldots T\}$$

under the restriction that, for flow series,[20]

$$\sum_{t=2}^{T} X_t = A_y, \; y \in \{1 \ldots\ldots \beta\}.$$

That is, the sum[21] of the quarters should be equal to the annual data for each benchmark year,[22]

where

- t is time (e.g., $t = 4y - 3$ is the first quarter of year y, and $t = 4y$ is the fourth quarter of year y);
- X_t is the derived QNA estimate for quarter t;
- I_t is the level of the indicator for quarter t;
- A_y is the annual data for year y;
- β is the last year for which an annual benchmark is available; and
- T is the last quarter for which quarterly source data are available.

[17] Thus, in this case, it does not matter which period is being moved. Moving from (a) the fourth quarter of the last benchmark year, (b) the average of the last benchmark year, or (c) the same quarter of the last benchmark year in proportion to the movements in the indicator from the corresponding periods gives the same results. Formally, it follows from equation (6.1) that

$$X_{q,\beta+1} = X_{4,\beta} \cdot \left(\frac{I_{q,\beta+1}}{I_{4,\beta}} \right)$$

$$= X_{q,\beta} \cdot \left(\frac{I_{q,\beta+1}}{I_{q,\beta}} \right)$$

$$= A_{\beta} \cdot \left(\frac{I_{q,\beta+1}}{\sum_q I_{q,\beta}} \right)$$

[18] In the standard case of binding annual benchmarks.

[19] This presentation deviates from Denton's original proposal by omitting the requirement that the value for the first period be predetermined. As pointed out by Cholette (1984), requiring that the values for the first period be predetermined implies minimizing the first correction and can in some circumstances cause distortions to the benchmarked series. Also, Denton's original proposal dealt only with estimating the back series.

[20] For the less common case of stock series, the equivalent constraint is that the value of the stock at the end of the final quarter of the year is equal to the stock at the end of the year. For index number series, the constraint can be formulated as requiring the annual average of the quarters to be equal to the annual index or the sum of the quarters to be equal to four times the annual index. The two expressions are equivalent.

[21] Applies also to flow series in which the indicator is expressed as index numbers; the annual total of the indicator should still be expressed as the sum of the quarterly data.

[22] The annual benchmarks may be omitted for some years to allow for cases in which independent annual source data are not available for all years.

Example 6.2. The Proportional Denton Method

Same data as in Example 6.1.

	Indicator: The Indicator	Indicator: Period-to-Period Rate of Change	Annual Data	Annual BI Ratios	Derived QNA Estimates	Estimated Quarterly BI ratios	Period-to-Period Rate of Change
q1 1998	98.2				969.8	9.876	
q2 1998	100.8	2.6%			998.4	9.905	3.0%
q3 1998	102.2	1.4%			1,018.3	9.964	2.0%
q4 1998	100.8	–1.4%			1,013.4	10.054	–0.5%
Sum	**402.0**		**4000.0**	**9.950**	**4,000.0**		
q1 1999	99.0	***–1.8%***			1,007.2	10.174	***–0.6%***
q2 1999	101.6	2.6%			1,042.9	10.264	3.5%
q3 1999	102.7	1.1%			1,060.3	10.325	1.7%
q4 1999	101.5	–1.2%			1,051.0	10.355	–0.9%
Sum	**404.8**	**0.7%**	**4161.4**	**10.280**	**4,161.4**		**4.0%**
q1 2000	100.5	–1.0%			1,040.6	10.355	–1.0%
q2 2000	103.0	2.5%			1,066.5	10.355	2.5%
q3 2000	103.5	0.5%			1,071.7	10.355	0.5%
q4 2000	101.5	–1.9%			1,051.0	10.355	–1.9%
Sum	**408.5**	**0.9%**	**?**	**?**	**4,229.8**		**1.6%**

BI Ratios

- For the back series (1998–1999):
  In contrast to the pro rata distribution method in which the estimated quarterly BI ratio jumped abruptly from 9.950 to 10.280, the proportional Denton method produces a smooth series of quarterly BI ratios in which:
  - The quarterly estimates sum to 4000, that is, the weighted average BI ratio for 1998 is 9.950.
  - The quarterly estimates sum to 4161.4, that is, the weighted average for 1999 is equal to 1.0280.
  - The estimated quarterly BI ratio is increasing through 1998 and 1999 to match the increase in the observed annual BI ratio. The increase is smallest at the beginning of 1998 and at the end of 1999.
- For the forward series (2000), the estimates are obtained by carrying forward the quarterly BI ratio (10.355) for the last quarter of 1999 (the last benchmark year).

Rates of Change

- For the back series, the quarterly percentage changes in 1998 and 1999 are adjusted upwards for all quarters to match the higher rate of change in the annual data.
- For the forward series, the quarterly percentage changes in 1999 are identical to those of the indicator; but note that the rate of change from 1999 to 2000 in the derived QNA series (1.6%) is higher than the annual rate of change in the indicator (0.9%). The next section provides an extension of the method that can be use to ensure that annual rate of change in the derived QNA series equals the annual rate of change in the indicator, if that is desired.

(These results are illustrated in Chart 6.2.)

6.25. The proportional Denton technique *implicitly constructs* from the annual observed BI ratios a time series of *quarterly benchmarked QNA estimates-to-indicator* (quarterly BI) ratios that is as smooth as possible and, in the case of flow series:

- For the back series, ($y \in \{1,...\beta\}$) averages[23] to the annual BI ratios for each year y.
- For the forward series, ($y \in \{\beta + 1.....\}$) are kept constant and equal to the ratio for the last quarter of the last benchmark year.

We will use this interpretation of the proportional Denton method to develop an enhanced version in the next section.

[23]annual weighted average

$$\left(\sum_{q=1}^{4} \frac{X_{q,y}}{I_{q,y}} \cdot w_{q,y} = A_y \Big/ \sum_{q=1}^{4} I_{q,y} \right)$$

where the weights are

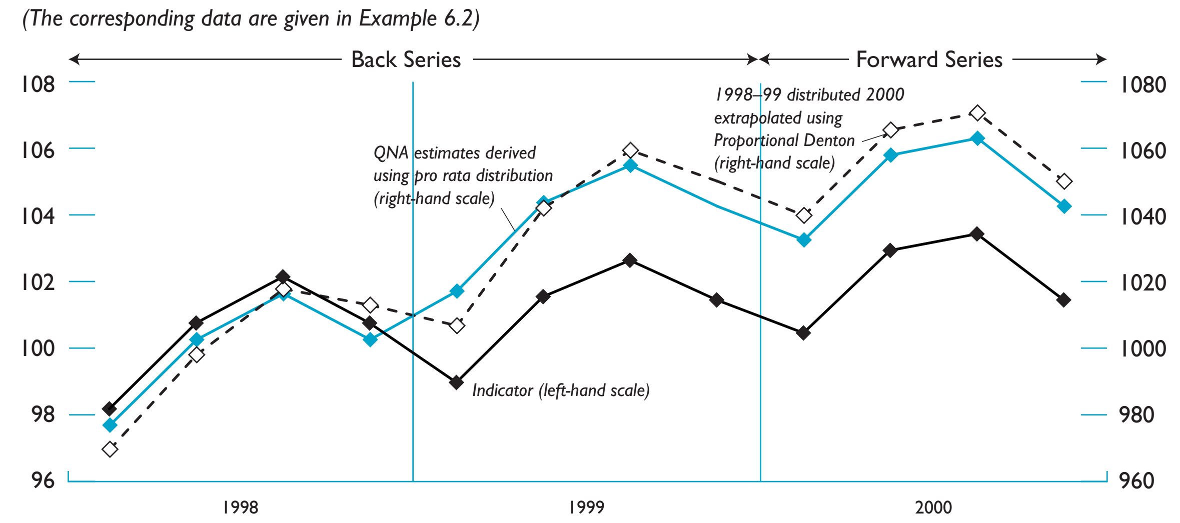
$$w_{q,y} = I_{q,y} \Big/ \sum_{q=1}^{4} I_{q,y}$$

6.26. The proportional Denton technique, as presented in equation (6.3), requires that the indicator contain positive values only. For series that contain zeroes but not negative values, this problem can be circumvented by simply replacing the zeroes with values infinitesimally close to zero. For series that can take both negative and positive values, and are derived as differences between two non-negative series, such as changes in inventories, the problem can be avoided by applying the proportional Denton method to the opening and closing inventory levels rather than to the change. Alternatively, the problem can be circumvented by temporarily turning the indicator into a series containing only positive values by adding a sufficiently large constant to all periods, benchmarking the resulting indicator using equation (6.3), and subsequently deducting the constant from the resulting estimates.

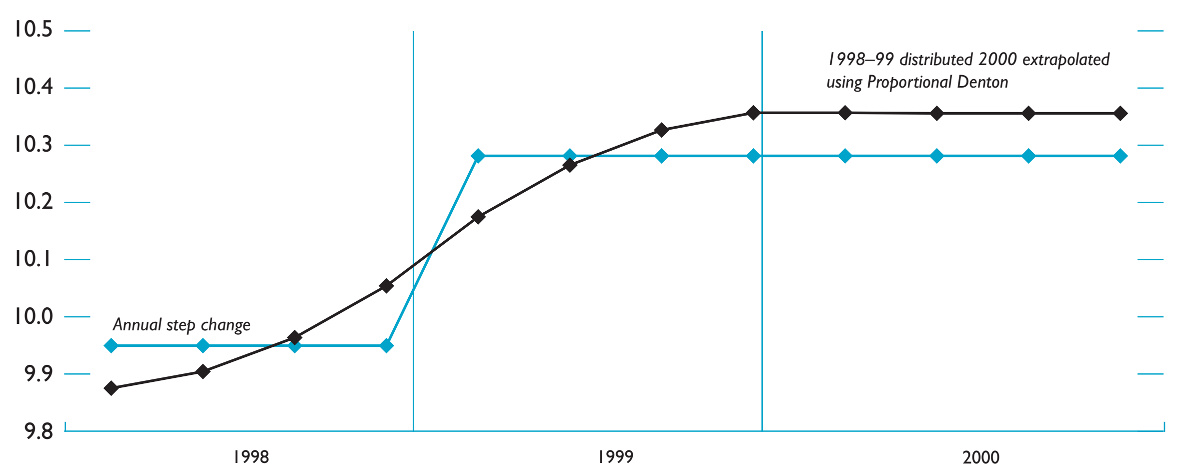
6.27. For the back series, the proportional Denton method results in QNA quarter-to-quarter growth rates that differ from those in the indicator (e.g., see

Chart 6.2. Solution to the Step Problem: The Proportional Denton Method

The Indicator and the Derived Benchmarked QNA Estimates



Benchmark-to-Indicator Ratios



Example 6.2). In extreme cases, the method may even introduce new turning points in the derived series or change the timing of turning points; however, these changes are a necessary and desirable result of incorporating the information contained in the annual data.

6.28. For the forward series, the proportional Denton method results in quarter-to-quarter growth rates that are identical to those in the indicator but also in an annual growth rate for the first year of the forward series that differs from the corresponding growth rate of the source data (see Example 6.2). This difference in the annual growth rate is caused by the way the indicator is linked in. By carrying forward the quarterly BI ratio for the last quarter of the last benchmark year, the proportional Denton method implicitly "forecasts" the next annual BI ratio as different from the last observed annual BI ratio, and equal to the quarterly BI ratio for the last quarter of the last benchmark year. As explained in Annex 6.2, the proportional Denton method will result in the following:

- It will partly adjust for any systematic bias in the indicator's annual rate of change if the bias is sufficiently large relative to any amount of noise, and thus, on average, lead to smaller revisions in the QNA estimates.
- It will create a wagging tail effect with, on average, larger revisions if the amount of noise is sufficiently large relative to any systematic bias in the annual growth rate of the indicator.

The next section presents an enhancement to the basic proportional Denton that better incorporates information on bias versus noise in the indicator's movements.

6.29. For the forward series, the basic proportional Denton method implies moving from the fourth quarter of the last benchmark year (see equation (6.2.a)). As shown in Annex 6.2, other possible starting points may cause a forward step problem, if used together with benchmarking methods for the back series that avoid the step problem associated with pro rata distribution:

- Using growth rates from four quarters earlier. Effectively, the estimated quarterly BI ratio is forecast as the same as four quarters earlier. This method maintains the percentage change in the indicator over the previous four quarters but it does not maintain the quarterly growth rates, disregards the information in past trends in the annual BI ratio, and introduces potential sever steps between the back series and the forward series.
- Using growth rates from the last annual average. Effectively, the estimated quarterly BI ratio is forecast as the same as the last annual BI ratio. This method results in annual growth rates that equal those in the indicator; however, it also disregards the information in past trends in the annual BI ratio and introduces an unintended step between the back series and the forward series.

6.30. When the annual data later become available, the extrapolated QNA data would need to be re-estimated. As a result of the benchmarking process, new data for one year will also lead to changes in the quarterly movements for the preceding year(s). This effect occurs because the adjustment for the errors in the indicator is distributed smoothly over several quarters, not just within the same year. For example, as illustrated in Example 6.3 and Chart 6.3, if the 1999 annual data subsequently showed that the downward error in the indicator for 1998 for Example 6.2 was reversed, then

- the 1999 QNA estimates would be revised down;
- the estimates in the second half of 1998 would be revised down (to smoothly adjust to the 1999 values); and
- the estimates in the first half of 1998 would need to be revised up (to make sure that the sum of the four quarters was still consistent with the 1998 annual total).

While these effects may be complex, it should be emphasized that they are an inevitable and desired implication of incorporating the information provided by the annual data concerning the errors in the long-term movements of the quarterly indicator.

3. Enhancements to the Proportional Denton Method for Extrapolation

6.31. It is possible to improve the estimates for the most recent quarters (the forward series) and reduce the size of later revisions by incorporating information on past systematic movements in the annual BI ratio. It is important to improve the estimates for these quarters, because they are typically of the keenest interest to users. Carrying forward the quarterly BI ratio from the last quarter of the last year is an implicit forecast of the annual BI ratio, but a better forecast can usually be made. Accordingly, the basic Denton technique can be enhanced by adding a forecast of the next annual BI ratio, as follows:

- If the annual growth rate of the indicator is systematically biased compared to the annual data,[24] then, on average, the best forecast of the next year's BI ratio is the previous year's value multiplied by the average relative change in the BI ratio.
- If the annual growth rate of the indicator is unbiased compared to the annual data (i.e., the annual BI follows a random walk process), then, on average, the best forecast of the next year's BI ratio is the previous annual value.

[24]The indicator's annual growth rate is systematically biased if the ratio between (a) the ratio of annual of change in the indicator and (b) the ratio of annual change in the annual data on average is significantly different from one or, equivalently, that the ratio of annual change in the annual BI ratio on average is significantly different from one, as seen from the following expression:

$$\frac{A_y / A_{y-1}}{\sum_{q=1}^{4} I_{q,y} \Big/ \sum_{q=1}^{4} I_{q,y-1}} \Leftrightarrow \frac{A_y \Big/ \sum_{q=1}^{4} I_{q,y}}{A_{y-1} \Big/ \sum_{q=1}^{4} I_{q,y-1}} = \frac{BI_y}{BI_{y-1}}$$

Example 6.3. Revisions to the Benchmarked QNA Estimates Resulting from Annual Benchmarks for a New Year

This example is an extension of Example 6.2 and illustrates the impact on the back series of incorporating annual data for a new year, and subsequent revisions to the annual data for that year.

Assume that preliminary annual data for 2000 become available and the estimate is equal to 4,100.0 (annual data A). Later on, the preliminary estimate for 2000 is revised upwards to 4,210.0 (annual data B). Using the equation presented in (6.3) to distribute the annual data over the quarters in proportion to the indicator will give the following sequence of revised QNA estimates:

	Indicator						Revised QNA Estimates			Quarterized BI Ratios		
Date	The Indicator	Period-to Period rate of Change	Annual Data 2000A	Annual BI Ratio 2000A	Annual Data 2000B	Annual BI Ratio 2000B	Derived in Example 6.2	With 2000A	With 2000B	Derived in Example 6.2	With 2000A	With 2000B
q1 1998	98.2						969.8	968.1	969.5	9.876	9.858	9.873
q2 1998	100.3	2.6%					998.4	997.4	998.3	9.905	9.895	9.903
q3 1998	102.2	1.4%					1,018.3	1,018.7	1,018.4	9.964	9.967	9.965
q4 1998	100.8	–1.4%					1,013.4	1,015.9	1,013.8	10.054	10.078	10.058
Sum	**402.0**		**4,000.0**	**9.950**	**4,000.0**	**9.950**						
q1 1999	99.0	***–1.8%***					1,007.2	1,012.3	1,008.0	10.174	10.225	10.182
q2 1999	101.6	2.6%					1,042.9	1,047.2	1,043.5	10.264	10.307	10.271
q3 1999	102.7	1.1%					1,060.3	1,059.9	1,060.3	10.325	10.321	10.324
q4 1999	101.5	–1.2%					1,051.0	1,042.0	1,049.6	10.355	10.266	10.341
Sum	**404.8**	**0.7%**	**4,161.4**	**10.280**	**4,161.4**	**10.280**						
q1 2000	100.5	–1.0%					1,040.6	1,019.5	1,037.4	10.355	10.144	10.323
q2 2000	103.0	2.5%					1,066.5	1,035.4	1,061.8	10.355	10.052	10.308
q3 2000	103.5	0.5%					1,071.7	1,034.1	1,065.9	10.355	9.991	10.299
q4 2000	101.5	–1.9%					1,051.0	1,011.0	1,044.9	10.355	9.961	10.294
Sum	**408.5**	**0.9%**	**4,100.0**	**10.037**	**4,210.0**	**10.306**	**4,229.8**	**4,100.0**	**4,210.0**			

As can be seen, incorporating the annual data for 2000 results in (a) revisions to both the 1999 and the 1998 QNA estimates, and (b) the estimates for one year depend on the difference in the annual movements of the indicator and the annual data for the previous years, the current year, and the following years.

In **case A**, with an annual estimate for 2000 of 4100.0, the following can be observed:

- The annual BI ratio increases from 9.950 in 1998 to 10.280 in 1999 and then drops to 10.037 in 2000. Correspondingly, the derived quarterly BI ratio increases gradually from q1 1998 through q3 1999 and then decreases through 2000.
- Compared with the estimates obtained in Example 6.2, incorporating the 2000 annual estimate resulted in the following ***revisions to the path of the quarterly BI ratio through 1998 and 1999:***
  - To smooth the transition to the decreasing BI ratios through 2000, which are caused by the drop in the annual BI ratio from 1999 to 2000, the BI ratios for q3 and q4 of 1999 have been revised downwards.
  - The revisions downward of the BI ratios for q3 and q4 of 1999 is matched by an upward revision to the BI ratios for q1 and q2 of 1999 to ensure that the weighted average of the quarterly BI ratios for 1999 is equal to the annual BI ratio for 1999.
  - To smooth the transition to the new BI ratios for 1999, the BI ratios for q3 and q4 of 1998 have been revised upward; consequently, the BI ratios for q1 and q2 of 1998 have been revised downwards.
- As a consequence a turning point in the new time series of quarterly BI ratios has been introduced between the third and the fourth quarter of 1999, in contrast to the old BI ratio time series, which increased during the whole of 1999.

In **case B**, with an annual estimate for 2000 of 4210.0, the following can be observed:

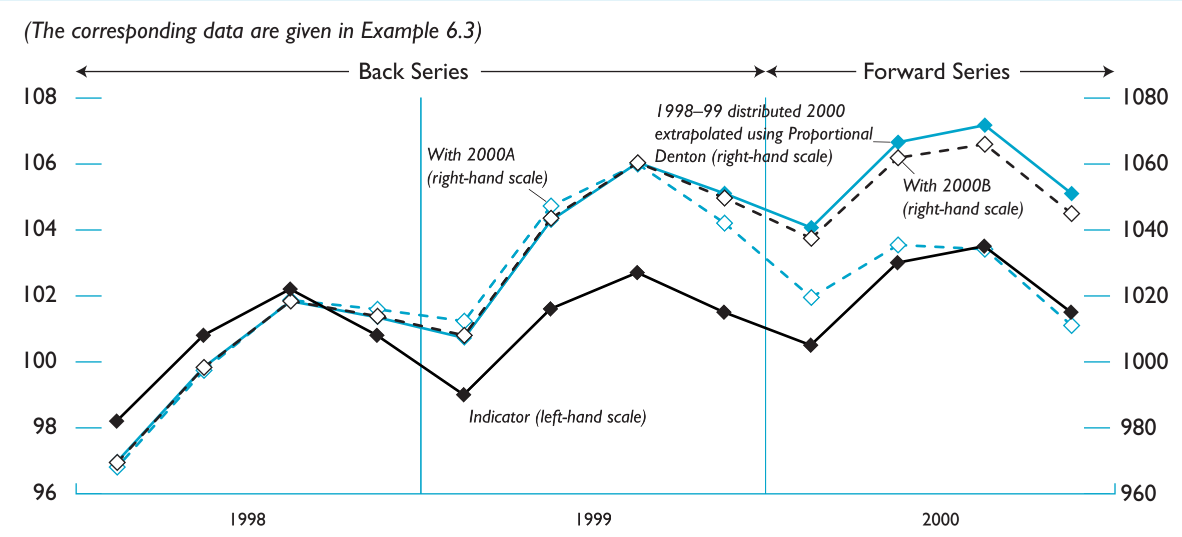
- The annual BI ratio for 1999 of 10.306 is slightly higher than the 1999 ratio of 10.280, but:
  - The ratio is lower than the initial q4 1999 BI ratio of 10.325 that was carried forward in Example 6.2 to obtain the initial quarterly estimates for 2000.
  - Correspondingly, the initial annual estimate for 2000 obtained in Example 6.2 was higher than the new annual estimate for 2000.
- Consequently, compared with the initial estimates from Example 6.2, the BI ratios have been revised downwards from q3 1999 onwards.
- In spite of the fact that the annual BI ratio is increasing, the quarterized BI ratio is decreasing during 2000. This is caused by the steep increase in the quarterly BI ratio during 1999 that was caused by the steep increase in the annual BI ratio from 1998 to 2000.

(These results are illustrated in Chart 6.3.)

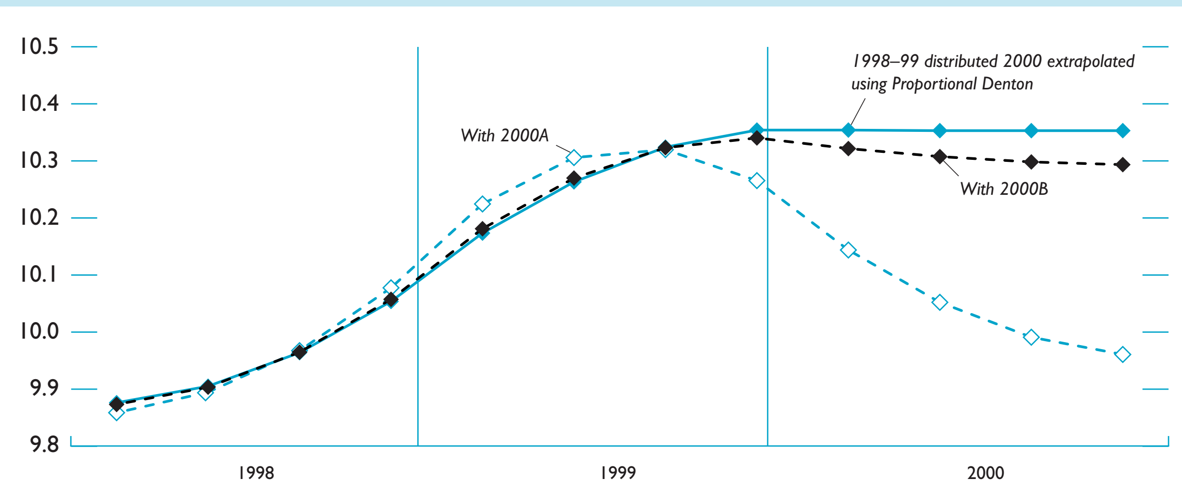
- If the annual BI is fluctuating symmetrically around its mean, on average, the best forecast of the next year's BI ratio is the long-term average BI value.
- If the movements in the annual BI ratio are following a stable, predictable time-series model (i.e., an ARIMA[25] or ARMA[26] model) then, on average, the best forecast of the next year's BI ratio may be obtained from that model.
- If the fluctuations in the annual BI ratio are correlated with the business cycle[27] (e.g., as manifested in the indicator), then, on average, the best forecast of the next year's BI ratio may be obtained by modeling that correlation.

[25]Autoregressive integrated moving average time-series models.
[26]Autoregressive moving average time-series models.
[27]Lags in incorporating deaths and births of businesses in quarterly sample frames may typically generate such correlations.

Chart 6.3. Revisions to the Benchmarked QNA Estimates Resulting from Annual Benchmarks for a New Year



Benchmark-to-Indicator Ratios



Note that only the annual BI ratio and not the annual benchmark value has to be forecast, and the BI ratio is typically easier to forecast than the annual benchmark value itself.

6.32. To produce a series of estimated quarterly BI ratios taking into account the forecast, the same principles of least-square minimization used in the Denton formula can also be used with a series of annual BI ratios that include the forecast. Since the benchmark values are not available, the annual constraint is that the weighted average of estimated quarterly BI ratios is the same as the corresponding observed or forecast annual BI ratios and that period-to-period change in the time series of quarterly BI ratios is minimized.

6.33. In mathematical terms:

$$\min_{(QBI_1,\ldots,QBI_{4\beta},\ldots\ldots,QBI_T)} \sum_{t=2}^{T}\left[QBI_t - QBI_{t-1}\right]^2 \qquad (6.4.a)$$

$$t \in \{1,\ldots(4\beta),\ldots.T\}$$

under the restriction that

(a) $$\sum_{t=4y-3}^{4y} QBI_t \cdot w_t = ABI_y$$

for $t \in \{1,\ldots(4\beta)\}, y \in \{1,\ldots\beta\}$.

and

(b) $$\sum_{t=4y-3}^{4y} QBI_t \cdot w_{t-4} = \hat{A}BI_{y+1}$$

for $t \in \{(4\beta)\ldots\ldots \mathrm{T}\}, y \in \{\beta+1,\ldots.\}$.

Where $w_t = I_t \Big/ \sum_{t=4y-3}^{4y} I_t$ for $t \in \{1,\ldots(4\beta)\}$,

and where

QBI_t is the estimated quarterly BI ratio (X_t/I_t) for period t;

ABI_y is the observed annual BI ratio ($A_t/\sum_q I_{q,y}$) for year $y \in \{1,\ldots\beta\}$; and

$\hat{A}BI_y$ is the forecast annual BI ratio for year $y \in \{\beta + 1\ldots..\}$.

6.34. Once a series of quarterly BI ratios is derived, the QNA estimate can be obtained by multiplying the indicator by the estimated BI ratio.

$$X_t = QBI_t \cdot I_t \qquad (6.4.b)$$

6.35. The following shortcut version of the enhanced Denton extrapolation method gives similar results for less volatile series. In a computerized system, the shortcut is unnecessary, but it is easier to follow in an example (see Example 6.4 and Chart 6.4). This method can be expressed mathematically as

(a) $$\hat{Q}BI_{2,\beta} = QBI_{2,\beta} + 1/4 \cdot \eta \qquad (6.5)$$
$$\hat{Q}BI_{3,\beta} = QBI_{3,\beta} + 1/4 \cdot \eta$$
$$\hat{Q}BI_{4,\beta} = QBI_{4,\beta} - 1/2 \cdot \eta$$

(b) $$\hat{Q}BI_{1,\beta+1} = \hat{Q}BI_{4,\beta} - \eta$$
$$\hat{Q}BI_{q,\beta+1} = \hat{Q}BI_{q-1,\beta+1} - \eta$$

where

$\eta = 1/3(QBI_{4,\beta} - \hat{A}BI_{\beta+1})$ (a fixed parameter for adjustments that ensures that the estimated quarterly BI ratios average to the correct annual BI ratios);

$QBI_{q,\beta}$ is the original BI ratio estimated for quarter q of the last benchmark year;

$\hat{Q}BI_{q,\beta}$ is the adjusted BI ratio estimated for quarter q of the last benchmark year;

$\hat{Q}BI_{q,\beta+1}$ is the forecast BI ratio for quarter q of the following year; and

$\hat{A}BI_{\beta+1}$ is the forecast average annual BI ratio for the following year.

6.36. While national accountants are usually reluctant to make forecasts, all possible methods are based on either explicit or implicit forecasts, and implicit forecasts are more likely to be wrong because they are not scrutinized. Of course, it is often the case that the evidence is inconclusive, so the best forecast is simply to repeat the last observed annual BI ratio.

D. Particular Issues

1. Fixed Coefficient Assumptions

6.37. In national accounts compilation, potential step problems may arise in cases that may not always be thought of as a benchmark-indicator relationship. One important example is the frequent use of assumptions of fixed coefficients relating inputs (total or part of intermediate consumption or inputs of labor and/or capital) to output ("IO ratios"). Fixed IO ratios can be seen as a kind of a benchmark-indicator relationship, where the available series is the indicator for the missing one and the IO ratio (or its inverse) is the BI ratio. If IO ratios are changing from year to year but are kept constant within each year, a step problem is created. Accordingly, the Denton technique can be used to generate smooth time series of quarterly IO ratios based on annual (or less frequent) IO coefficients. Furthermore, systematic trends can be identified to forecast IO ratios for the most recent quarters.

2. Within-Year Cyclical Variations in Coefficients

6.38. Another issue associated with fixed coefficients is that coefficients that are assumed to be fixed may in fact be subject to cyclical variations within the year. IO ratios may vary cyclically owing to inputs that do not

Example 6.4. Extrapolation Using Forecast BI Ratios
Same data as Examples 6.1 and 6.3

				Original estimates from Example 6.2		Extrapolation using forecast BI ratios		Quarter to quarter rates of change		
Date	Indicator	Annual data	Annual BI ratios	BI ratios	QNA estimates for 1997–1998	Forecast BI ratio	Estimate	Original indicator	Original Estimates from Example 6.2	Based on forecast BI ratios
q1 1998	98.2			9.876	969.8					
q2 1998	100.8			9.905	998.4			2.60%	3.00%	3.00%
q3 1998	102.2			9.964	1,018.3			1.40%	2.00%	2.00%
q4 1998	100.8			10.054	1,013.4			–1.40%	–0.50%	–0.50%
Sum	**402.0**	**4,000.0**	**9.950**		**4,000.0**					
q1 1999	99.0			10.174	1,007.2			–1.80%	–0.60%	–0.60%
q2 1999	101.6			10.264	1,042.9	10.253	1,041.7	2.60%	3.50%	3.40%
q3 1999	102.7			10.325	1,060.3	10.314	1,059.2	1.10%	1.70%	1.70%
q4 1999	101.5			10.355	1,051	10.376	1,053.2	–1.20%	–0.90%	–0.20%
Sum	**404.8**	**4,161.4**	**10.280**		**4,161.4**	**10.280**	**4,161.4**	**0.70%**	**4.00%**	**4.00%**
q1 2000	100.5			10.355	1,040.6	10.42	1,047.2	–1.00%	–1.00%	–0.60%
q2 2000	103			10.355	1,066.5	10.464	1,077.8	2.50%	2.50%	2.90%
q3 2000	103.5			10.355	1,071.7	10.508	1,087.5	0.50%	0.50%	0.90%
q4 2000	101.5			10.355	1,051	10.551	1,071	–1.90%	–1.90%	–1.50%
Sum	**408.5**			**10.355**	**4,229.8**	**10.486**	**4,283.5**	**0.90%**	**1.60%**	**2.90%**

This example assumes that, based on a study of movements in the annual BI ratios for a number of years, it is established that the indicator on average understates the annual rate of growth by 2.0%.

The forecast annual and adjusted quarterly BI ratios are derived as follows:

The annual BI ratio for 2000 is forecast to rise to 10.486, (i.e., 10.280 · 1.02).

The adjustment factor (η) is derived as –0.044, (i.e., 1/3 · (10.355 – 10.486).

q2 1999: 10.253 = 10.264 + 1/4 · (–0.044)
q3 1999: 10.314 = 10.325 + 1/4 · (–0.044)
q4 1999: 10.376 = 10.355 – 1/2 · (–0.044)

q1 2000: 10.420 = 10.376 – (–0.044)
q2 2000: 10.464 = 10.420 – (–0.044)
q3 2000: 10.508 = 10.464 – (–0.044)
q4 2000: 10.551 = 10.508 – (–0.044)

Note that for the sum of the quarters, the annual BI ratios are as measured (1999) or forecast (2000), and the estimated quarterly BI ratios move in a smooth way to achieve those annual results, minimizing the proportional changes to the quarterly indicators.

(These results are illustrated in Chart 6.4.)

vary proportionately with output, typically fixed costs such as labor, capital, or overhead such as heating and cooling. Similarly, the ratio between income flows (e.g., dividends) and their related indicators (e.g., profits) may vary cyclically. In some cases, these variations may be according to a seasonal pattern and be known.[28] It should be noted that omitted seasonal variations are only a problem in the original non-seasonally adjusted data, as the variations are removed in seasonal adjustment and do not restrict the ability to pick trends and turning points in the economy. However, misguided attempts to correct the problem in the original data could distort the underlying trends.

[28]Cyclical variations in assumed fixed coefficients may also occur because of variations in the business cycle. These variations cause serious errors because they may distort trends and turning points in the economy. They can only be solved by direct measurement of the target variables.

6.39. To incorporate a seasonal pattern on the target QNA variable, without introducing steps in the series, one of the following two procedures can be used:

(a) **BI ratio-based procedure**

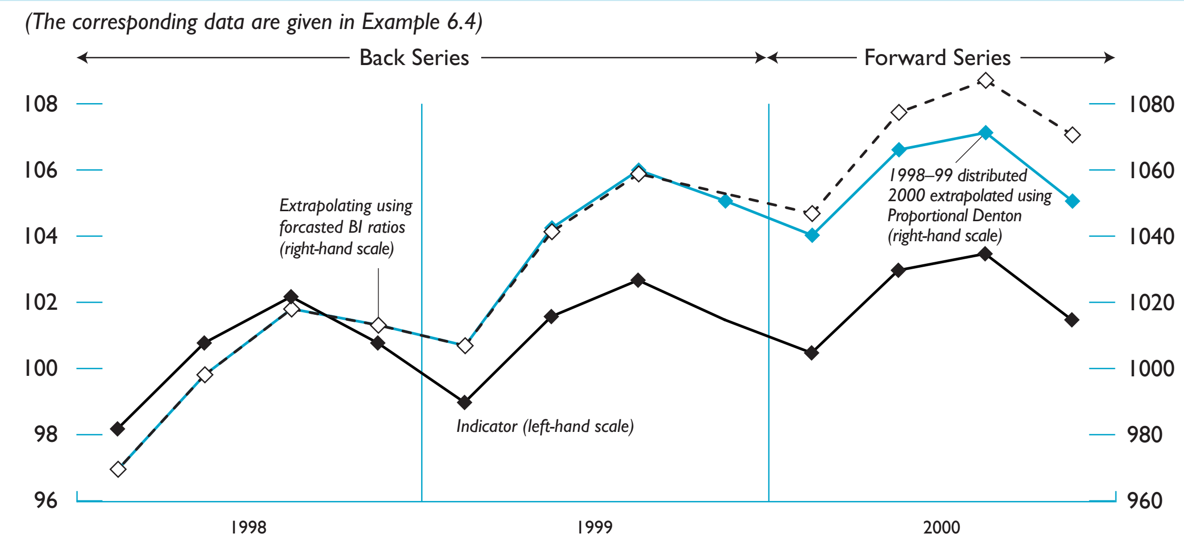
Augment the benchmarking procedure as outlined in equation (6.4) by incorporating the a priori assumed seasonal variations in the estimated quarterly BI ratios as follows:

$$\min_{(QBI_1,\ldots,QBI_{4\beta},\ldots,QBI_T)} \sum_{t=2}^{T} \left[\frac{QBI_t}{SF_t} - \frac{QBI_{t-1}}{SF_{t-1}} \right]^2 \qquad (6.6)$$

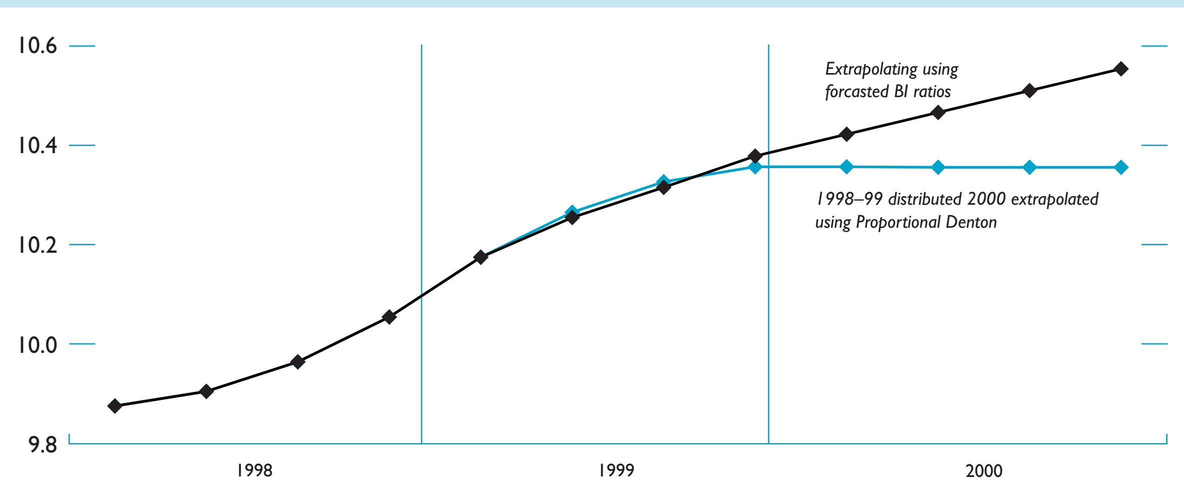
$$t \in \{1,\ldots(4\beta),\ldots.T\}$$

under the same restrictions as in equation (6.4), where SF_t is a time series with a priori assumed seasonal factors.

Chart 6.4. Extrapolation Using Forecast BI Ratios



Benchmark-to-Indicator Ratios



(b) **Seasonal adjustment-based procedure**
- (i) Use a standard seasonal adjustment package to seasonally adjust the indirect indicator.
- (ii) Multiply the seasonally adjusted indicator by the known seasonal coefficients.
- (iii) Benchmark the resulting series to the corresponding annual data.

6.40. The following inappropriate procedure is sometimes used to incorporate a seasonal pattern when the indicator and the target variable have different and known seasonal patterns:

(a) distribute the annual data for one year in proportion to the assumed seasonal pattern of the series, and

(b) use the movements from the same period in the previous year in the indicator to update the series.

6.41. This procedure preserves the superimposed seasonal patterns when used for one year only. When the QNA estimates are benchmarked, however, this procedure will introduce breaks in the series that can remove or distort trends in the series and introduce more severe errors than those that it aims to prevent (see Annex 6.2 for an illustration).

3. Benchmarking and Compilation Procedures

6.42. Benchmarking should be an integral part of the compilation process and should be conducted at the most detailed compilation level. In practice, this may imply benchmarking different series in stages, where data for some series, which have already been benchmarked, are used to estimate other series, followed by a second or third round of benchmarking. The actual arrangements will vary depending on the particularities of each case.

6.43. As an illustration, annual data may be available for all products, but quarterly data are available only for the main products. If it is decided to use the sum of the quarterly data as an indicator for the other products, the ideal procedure would be first to benchmark each of the products for which quarterly data are available to the annual data for that product, and then to benchmark the quarterly sum of the benchmarked estimates for the main products to the total. Of course, if all products were moving in similar ways, this would give similar results to directly benchmarking the quarterly total to the annual total.

6.44. In other cases, a second or third round of benchmarking may be avoided and compilation procedure simplified. For instance, a current price indicator can be constructed as the product of a quantity indicator and a price indicator without first benchmarking the quantity and price indicators to any corresponding annual benchmarks. Similarly, a constant price indicator can be constructed as a current price indicator divided by a price indicator without first benchmarking the current price indicator. Also, if output at constant prices is used as an indicator for intermediate consumption, the (unbenchmarked) constant price output indicator can be benchmarked to the annual intermediate consumption data directly. It can be shown that the result is identical to first benchmarking the output indicator to annual output data, and then benchmarking the resulting benchmarked output estimates to the annual intermediate consumption data.

6.45. To derive quarterly constant price data by deflating current price data, the correct procedure would be first to benchmark the quarterly current price indicator and then to deflate the benchmarked quarterly current price data. If the same price indices are used in the annual and quarterly accounts, the sum of the four quarters of constant price data should be taken as the annual estimate, and a second round of benchmarking is unnecessary. As explained in Chapter IX Section B, annual deflators constructed as unweighted averages of monthly or quarterly price data can introduce an aggregation over time error in the annual deflators and subsequently in the annual constant price data that can be significant if there is quarterly volatility. Moreover, if, in those cases, quarterly constant price data are derived by benchmarking a quarterly constant price indicator derived by deflating the current price indicator to the annual constant price data, the aggregation over time error will be passed on to the implicit quarterly deflator, which will differ from the original price indices. Thus, in those cases, annual constant price data should in principle be derived as the sum of quarterly or even monthly deflated data if possible. If quarterly volatility is insignificant, however, annual constant price estimates can be derived by deflating directly and then benchmarking the quarterly constant price estimates to the annual constant price estimates.

4. Balancing Items and Accounting Identities

6.46. The benchmarking methods discussed in this chapter treat each time series as an independent variable and thus do not take into account any accounting relationship between related time series. Consequently, the benchmarked quarterly time series will not automatically form a consistent set of accounts. For example, quarterly GDP from the production side may differ from quarterly GDP from the expenditure side, even though the annual data are consistent. The annual sum of these discrepancies, however, will cancel out for years where the annual benchmark data are balanced.[29] While multivariate benchmarking methods exist that take the relationship between the time series as an additional constraint, they are too complex and demanding to be used in QNA.

6.47. In practice, the discrepancies in the accounts can be minimized by benchmarking the different parts of the accounts at the most detailed level and building aggregates from the benchmarked components. If the remaining discrepancies between, for

[29]The within-year discrepancies will in most cases be relatively insignificant for the back series.

instance, GDP from the production and expenditure side are sufficiently small,[30] it may be defensible to distribute them proportionally over the corresponding components on one or both sides. In other cases, it may be best to leave them as explicit statistical discrepancies, unless the series causing these discrepancies can be identified. Large remaining discrepancies indicate that there are large inconsistencies between the short-term movements for some of the series.

5. More Benchmarking Options

6.48. The basic version of the proportional Denton technique presented in equation (6.3) can be expanded by allowing for alternative benchmark options, as in the following examples:

- The annual benchmarks may be omitted for some years to allow for cases where independent annual source data are not available for all years.
- Sub-annual benchmarks may be specified by requiring that
  - the values of the derived series are equal to some predetermined values in certain benchmark quarters; or
  - the half-yearly sums of the derived quarterly estimates are equal to half-yearly benchmark data for some periods.
- Benchmarks may be treated as nonbinding.
- Quarters that are known to be systematically more error prone than others may be adjusted relatively more than others.

The formulas for the two latter extensions are provided in Section B.2 of Annex 6.1.

6. Benchmarking and Revisions

6.49. To avoid introducing distortions in the series, incorporation of new annual data for one year will generally require revision of previously published quarterly data for several years. This is a basic feature of all acceptable benchmarking methods. As explained in paragraph 6.30, and as illustrated in Example 6.3, in addition to the QNA estimates for the year for which new annual data are to be incorporated, the quarterly data for one or several preceding and following years, may have to be revised. In principle, previously published QNA estimates for all preceding and following years may have to be adjusted to maximally preserve the short-term movements in the indicator, if the errors in the indicator are large. In practice, however, with most benchmarking methods, the impact of new annual data will gradually be diminishing and zero for sufficiently distant periods.

6.50. One of the advantages of the Denton method compared with several of the alternative methods discussed in Annex 6.1, is that it allows for revisions to as many preceding years as desired. If desired, revisions to some previously published QNA estimates can be avoided by specifying those estimates as "quarterly benchmark restrictions." This option freezes the values for those periods, and thus can be used to reduce the number of years that have to be revised each time new annual data become available. To avoid introducing significant distortions to the benchmarked series, however, at least two to three years preceding (and following) years should be allowed to be revised each time new annual data become available. In general, the impact on more distant years will be negligible.

7. Other Comments

6.51. Sophisticated benchmarking techniques use advanced concepts. In practice, however, they require little time or concern in routine quarterly compilation. In the initial establishment phase, the issues need to be understood and the processes automated as an integral part of the QNA production system. Thereafter, the techniques will improve the data and reduce future revisions without demanding time and attention of the QNA compiler. It is good practice to check the new benchmarks as they arrive each year in order to replace the previous BI ratio forecasts and make new annual BI forecasts. A useful tool for doing so is a table of observed annual BI ratios over the past several years. It will be usual for the BI ratio forecasts to have been wrong to varying degrees, but the important question is whether the error reveals a pattern that would allow better forecasts to be made in the future. In addition, changes in the annual BI ratio point to issues concerning the indicator that will be of relevance to the data suppliers.

[30]That is, so that the impact on the growth rates are negligible.

Annex 6.1. Alternative Benchmarking Methods

A. Introduction

6.A1.1. There are two main approaches to benchmarking of time series: a purely numerical approach and a statistical modeling approach. The numerical differs from the statistical modeling approach by not specifying a statistical time-series model that the series is assumed to follow. The numerical approach encompasses the family of least-squares minimization methods proposed by Denton (1971) and others,[1] the Bassie method,[2] and the method proposed by Ginsburgh (1973). The modeling approach encompasses ARIMA[3] model-based methods proposed by Hillmer and Trabelsi (1987), State Space models proposed by Durbin and Quenneville (1997), and a set of regression models proposed by various Statistics Canada staff.[4] In addition, Chow and Lin (1971) have proposed a multivariable general least-squares regression approach for interpolation, distribution, and extrapolation of time series. While not a benchmarking method in a strict sense, the Chow-Lin method is related to the statistical approach, particularly to Statistics Canada's regression models.

6.A1.2. The aim of this annex is to provide a brief review, in the context of compiling quarterly national accounts (QNA), of the most familiar of these methods and to compare them with the preferred proportional Denton technique with enhancements. The annex is not intended to provide an extensive survey of all alternative benchmarking methods proposed.

6.A1.3. The enhanced proportional Denton technique provides many advantages over the alternatives. It is, as explained in paragraph 6.7, by logical consequence optimal if the general benchmarking objective of maximal preservation of the short-term movements in the indicator is specified as keeping the quarterly estimates as proportional to the indicator as possible and the benchmarks are binding. In addition, compared with the alternatives, the enhanced proportional Denton technique is relatively simple, robust, and well suited for large-scale applications. Moreover, the implied benchmark-indicator (BI) ratio framework provides a general and integrated framework for converting indicator series into QNA estimates through interpolation, distribution, and extrapolation with an indicator that, in contrast to additive methods, is not sensitive to the overall level of the indicators and does not tend to smooth away some of the quarter-to-quarter rates of change in the data. The BI framework also encompasses the basic extrapolation with an indicator technique used in most countries.

6.A1.4. In contrast, the potential advantage of the various statistical modeling methods over the enhanced proportional Denton technique is that they explicitly take into account any supplementary information about the underlying error mechanism and other aspects of the stochastic properties of the series. Usually, however, this supplementary information is not available in the QNA context. Moreover, some of the statistical modeling methods render the danger of over-adjusting the series by interpreting true irregular movements that do not fit the regular patterns of the statistical model as errors, and thus removing them. In addition, the enhancement to the proportional Denton provided in Section D of Chapter VI allows for taking into account supplementary information about seasonal and other short-term variations in the BI ratio. Further enhancements that allow for incorporating any supplementary information that the source data for some quarters are weaker than others, and thus should be adjusted more than others, are provided in Section B.2 of this annex, together with a nonbinding version of the proportional Denton.

6.A1.5. Also, for the forward series, the enhancements to the proportional Denton method developed

[1] Helfand, Monsour, and Trager (1977), and Skjæveland (1985).
[2] Bassie (1958).
[3] Autoregressive integrated moving average.
[4] Laniel, and Fyfe (1990), and Cholette and Dagum (1994).

in Section C.3 of Chapter VI provide more and better options for incorporating various forms of information on past systematic bias in the indicator's movements. The various statistical modeling methods typically are expressed as additive relationships between the levels of the series, not the movements, that substantially limit the possibilities for alternative formulation of the existence of any bias in the indicator. The enhancements to the proportional Denton method developed in Chapter VI express systematic bias in terms of systematic behavior of the relative difference in the annual growth rate of the indicator and the annual series or, equivalently, in the annual BI ratio. This provides for a more flexible framework for adjusting for bias in the indicator.

B. The Denton Family of Benchmarking Methods

1. Standard Versions of the Denton Family

6.A1.6. The Denton family of least-squares-based benchmarking methods is based on the principle of movement preservation. Several least-squares-based methods can be distinguished, depending on how the principle of movement preservation is made operationally. The principle of movement preservation can be expressed as requiring that (1) the quarter-to-quarter growth in the adjusted quarterly series and the original quarterly series should be as similar as possible or (2) the adjustment to neighboring quarters should be as similar as possible. Within each of these two broad groups, further alternatives can be specified. The quarter-to-quarter growth can be specified as absolute growth or as rate of growth, and the absolute or the relative difference of these two expressions of quarter-to-quarter growth can be minimized. Similarly, the difference in absolute or relative adjustment of neighboring quarters can be minimized.

6.A1.7. The proportional Denton method (formula D4 below) is preferred over the other versions of the Denton method for the following three main reasons:

- It is substantially easier to implement.
- It results in most practical circumstances in approximately the same estimates for the back series as formula D2, D3, and D5 below.
- Through the BI ratio formulation used in Chapter VI, it provides a simple and elegant framework for extrapolation using the enhanced proportional Denton method, which fully takes into account the existence of any systematic bias or lack thereof in the year-to-year rate of change in the indicator.
- Through the BI ratio formulation used in Chapter VI, it provides a simple and elegant framework for extrapolation, which supports the understanding of the enhanced proportional Denton method; the Denton method fully takes into account the existence of any systematic bias or lack thereof in the year-to-year rate of change in the indicator.

6.A1.8. In mathematical terms, the following are the main versions[5] of the proposed least-squares benchmarking methods:[6]

MinD1:
$$\min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[(X_t - X_{t-1}) - (I_t - I_{t-1}) \right]^2 \quad (6.A1.1)$$
$$\Leftrightarrow \min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[(X_t - I_t) - (X_{t-1} - I_{t-1}) \right]^2$$

Min D2:
$$\min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[1n\left(\frac{X_t / X_{t-1}}{I_t / I_{t-1}} \right) \right]^2 \quad (6.A1.2)$$
$$\Leftrightarrow \min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[1n\left(\frac{X_t / I_t}{X_{t-1} / I_{t-1}} \right) \right]^2$$
$$\Leftrightarrow \min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[1n(X_t / X_{t-1}) - 1n(I_t / I_{t-1}) \right]^2$$

Min D3:
$$\min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[\frac{X_t}{X_{t-1}} - \frac{I_t}{I_{t-1}} \right]^2 \quad (6.A1.3)$$

Min D4:[7]
$$\min_{(X_1,\dots,X_{4\beta},\dots,X_T)} \sum_{t=2}^{T} \left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}} \right]^2 \quad (6.A1.4)$$

[5]The abbreviations D1, D2, D3, and D4, were introduced by Sjöberg (1982), as part of a classification of the alternative least-squares-based methods proposed by, or inspired by, Denton (1971). D1 and D4 were proposed by Denton; D2 and D3 by Helfand, Monsour, and Trager (1977); and D5 by Skjæveland (1985).

[6]This presentation deviates from the original presentation by the various authors by omitting their additional requirement that the value for the first period is predetermined. Also, Denton's original proposal only dealt with the back series.

[7]This is the basic version of the proportional Denton.

Min D5: $$\min_{(X_1,\ldots,X_{4\beta},\ldots,X_T)} \sum_{t=2}^{T}\left[\frac{X_t/X_{t-1}}{I_t/I_{t-1}}-1\right]^2 \qquad (6.A1.5)$$

$$<=> \min_{(X_1,\ldots,X_{4\beta},\ldots,X_T)} \sum_{t=2}^{T}\left[\frac{X_t/I_t}{X_{t-1}/I_{t-1}}-1\right]^2$$

$$t \in \{1,\ldots(4\beta),\ldots.T\}$$

All versions are minimized under the same restrictions, that for flow series,

$$\sum_{t=4y-3}^{4y} X_1 = A_y, \;\; y \in \{1,\ldots\beta\}.$$

That is, the sum of the quarters should be equal to the annual data for each benchmark year.

6.A1.9. The various versions of the Denton family of least-squares-based benchmarking methods have the following characteristics:

- The D1 formula minimizes the differences in the absolute growth between the benchmarked series X_t and the indicator series I_t. It can also be seen as minimizing the absolute difference of the absolute adjustments of two neighboring quarters.
- The D2 formula minimizes the logarithm of the relative differences in the growth rates of the two series. Formula D2 can also be looked upon as minimizing the logarithm of the relative differences of the relative adjustments of two neighboring quarters and as the logarithm of the absolute differences in the period-to-period growth rates between the two series.
- The D3 formula minimizes the absolute differences in the period-to-period growth rates between the two series.
- The D4 formula minimizes the absolute differences in the relative adjustments of two neighboring quarters.
- The D5 formula minimizes the relative differences in the growth rates of the two series. Formula D5 can also be looked upon as minimizing the relative differences of the relative adjustments of two neighboring quarters.

6.A1.10. While all five formulas can be used for benchmarking, only the D1 formula and the D4 formula have linear first-order conditions for a minimum and thus are the easiest to implement in practice. In practice, the D1 and D4 formulas are the only ones currently in use.

6.A.1.11. The D4 formula—the proportional Denton method—is generally preferred over the D1 formula because it preserves seasonal and other short-term fluctuations in the series better when these fluctuations are multiplicatively distributed around the trend of the series. Multiplicatively distributed short-term fluctuations seem to be characteristic of most seasonal macroeconomic series. By the same token, it seems most reasonable to assume that the errors are generally multiplicatively, and not additively, distributed, unless anything to the contrary is explicitly known. The D1 formula results in a smooth additive distribution of the errors in the indicator, in contrast to the smooth multiplicative distribution produced by the D4 formula. Consequently, as with all additive adjustment formulations, the D1 formula tends to smooth away some of the quarter-to-quarter rates of change in the indicator series. As a consequence, the D1 formula can seriously disturb that aspect of the short-term movements for series that show strong short-term variations. This can occur particularly if there is a substantial difference between the level of the indicator and the target variable. In addition, the D1 formula may in a few instances result in negative benchmarked values for some quarters (even if all original quarterly and annual data are positive) if large negative adjustments are required for data with strong seasonal variations.

6.A1.12. The D2, D3, and D5 formulas are very similar. They are all formulated as an explicit preservation of the period-to-period rate of change in the indicator series, which is the ideal objective formulation, according to several authors (e.g., Helfand, Monsour, and Trager 1977). Although the three formulas in most practical circumstances will give approximately the same estimates for the back series, the D2 formula seems slightly preferable over the other two. In contrast to D2, the D3 formula will adjust small rates of change relatively more than large rates of change, which is not an appealing property. Compared to D5, the D2 formula treats large and small rates of change symmetrically and thus will result in a smoother series of relative adjustments to the growth rates.

2. Further Expansions of the Proportional Denton Method

6.A1.13. The basic version of the proportional Denton technique (D4) presented in the chapter can be further expanded by allowing for alternative or additional benchmark restrictions, such as the following:

- Adjusting relatively more quarters that are known to be systematically more error prone than others.
- Treating benchmarks as nonbinding.

6.A1.14. The following augmented version of the basic formula allows for specifying which quarters should be adjusted more than the others:

$$\min_{(X_1,\ldots,X_{4\beta},\ldots\ldots X_T)} \sum_{t=2}^{T} w_{q_t} \cdot \left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}}\right]^2 \qquad (6.A1.6)$$

$$t \in \{1,\ldots(4\beta),\ldots.T\}$$

under the standard restriction that

$$\sum_{t=4y-1}^{4y} X_t = A_y,\ y \in \{1,\ldots\beta\}.$$

That is, the sum of the quarters should be equal to the annual data for each benchmark year.

Where

w_{q_t} is a set of user-specified quarterly weights that specifies which quarters should be adjusted more than the others.

6.A1.15. In equation (6.A1.6), only the relative value of the user-specified weights (w_{q_t}) matters. The absolute differences in the relative adjustments of a pair of neighboring quarters given a weight that is high relative to the weights for the others will be smaller than for pairs given a low weight.

6.A1.16. Further augmenting the basic formula as follows, allows for treating the benchmarks as non-binding:

$$\min_{(X_1,\ldots,X_{4\beta},\ldots,X_T)} \sum_{t=2}^{T} w_{q_t} \cdot \left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}}\right]^2 - \sum_{y=1}^{\beta} w_{a_y} \cdot \left[\sum_{t=4y-3}^{4y} \frac{X_t}{A_y} - 1\right]^2. \qquad (6.A1.7)$$

Where

w_{a_y} is a set of user-specified annual weights that specifies how binding the annual benchmarks should be treated.

Again, only the relative value of the user-specified weights matters. Relatively high values of the annual weights specify that the benchmarks should be treated as relatively binding.

C. The Bassie Method

6.A1.17. Bassie (1958) was the first to devise a method for constructing monthly and quarterly series whose short-term movements would closely reflect those of a related series while maintaining consistency with annual totals. The Bassie method was the only method described in detail in *Quarterly National Accounts* (OECD, 1979). However, using the Bassie method as presented in OECD (1979) can result in a step problem if data for several years are adjusted simultaneously.

6.A1.18. The Bassie method is significantly less suited for QNA compilation than the proportional Denton technique with enhancements for the following main reasons:

- The proportional Denton method better preserves the short-term movements in the indicator.
- The additive version of the Bassie method, as with most additive adjustment methods, tends to smooth the series and thus can seriously disturb the quarter-to-quarter rates of change in series that show strong short-term variations.
- The multiplicative version of the Bassie method does not yield an exact correction, requiring a small amount of prorating at the end.
- The proportional Denton method allows for the full time series to be adjusted simultaneously, in contrast to the Bassie method, which operates on only two consecutive years.
- The **Bassie method can result in a step problem** if data for several years are adjusted simultaneously and not stepwise.[8]
- The proportional Denton method with enhancements provides a general and integrated framework for converting indicator series into QNA estimates through interpolation, distribution, and extrapolation with an indicator. In contrast, the **Bassie method does not support extrapolation**; it only addresses distribution of annual data.
- The Bassie method results in a more cumbersome compilation process.

6.A1.19. The following is the standard presentation of the Bassie method, as found, among others, in OECD (1979). Two consecutive years are considered. No discrepancies between the quarterly and annual data for the first year are assumed, and the (absolute or relative) difference for the second year is equal to K_2.

6.A1.20. The Bassie method assumes that the correction for any quarter is a function of time, $K_q = f(t)$ and that $f(t) = a + bt + ct^2 + dt^3$. The method then stipulates the following four conditions:

(i) The average correction in year 1 should be equal to zero:

Example 6.A1.1. The Bassie Method and the Step Problem

Date	Original Estimates	Annual Estimates	Rate of Error	Adjustment Coefficients: Adjustment of Year 2	Adjustment Coefficients: Adjustment of Year 3	Adjusted Estimates	Growth Rates	Implied Adjustment Ratio
Year 1								
q1	1,000.0			–0.0981445		990.2		0.990
q2	1,000.0			–0.1440297		985.6	–0.5%	0.986
q3	1,000.0			–0.0083008		999.2	1.4%	0.999
q4	1,000.0			0.25048828		1,025.1	2.6%	1.025
Total year 1	4,000.0	4,000.0	0.00	0.0		4,000.0		
Year 2								
q1	1,000.0			0.57373047	–0.0981445	1,057.4	3.2%	1.057
q2	1,000.0			0.90283203	–0.1440297	1,090.3	3.1%	1.090
q3	1,000.0			1.17911122	–0.0083008	1,117.9	2.5%	1.118
q4	1,000.0			1.34423822	0.25048828	1,134.4	1.5%	1.134
Total year 2	4,000.0	4,400.0	0.10	4.0	0.0	4,400.0		
Year 3								
q1	1,000.0				0.57373047	1,000.0	**–11.9%**	1.000
q2	1,000.0				0.90283203	1,000.0	0.0%	1.000
q3	1,000.0				1.17911122	1,000.0	0.0%	1.000
q4	1,000.0				1.34423822	1,000.0	0.0%	1.000
Total year 3	4,000.0	4,000.0	0.00		4.0			

In the example, revised annual estimates for years 2 and 3 were made available at the same time. As seen, the first–round adjustment of the quarterly series to align the quarterly estimates to the annual estimate for year 2 results in an upward adjustment in the growth through year 1 and year 2 but no adjustments to year 3, leading to a break in the series between q4 of year 2 and q1 of year 3.

The break introduced by the first round of adjustments is not removed in the second round of adjustments to align the series to the annual estimate for year 3. In the example, the error in year 3 is zero, and the Bassie method, applied as described above, results in no further adjustments of the data.

$$\int_0^1 f(t)dt = 0.$$

(ii) The average correction in year 2 should be equal to the annual error in year 2 (K_2):

$$\int_1^2 f(t)dt = K_2.$$

(iii) At the start of year 1, the correction should be zero, so as not to disturb the relationship between the first quarter of year 1 and the fourth quarter of year 0: $f(0) = 0$.

(iv) At the end of year 2, the correction should be neither increasing nor decreasing:

$$\frac{df(2)}{dt} = 0.$$

6.A1.21. These four conditions allow computing the following fixed coefficients to distribute the annual error in year 2 (K_2) over the four quarters of year 2 and to adjust the quarterly pattern within year 1:

To be used for year 1		To be used for year 2	
b_1	-0.098145	c_1	0.573730
b_2	-0.144030	c_2	0.902832
b_3	-0.008301	c_3	1.179111
b_4	0.250488	c_4	1.344238
Sum	0.0		4.0

6.A1.22. The difference between the annual sum of the quarterly estimates and the direct annual estimate in year 2 (K_2) can be expressed either in an additive form or in a multiplicative form. The additive form is as follows:

$$K_2 = A_2 - \sum_{q=1}^{4} X_{q,2} \qquad (6.A1.8)$$

leading to the following additive version of the Bassie adjustment method:

$$Z_{q,1} = X_{q,1} + 0.25 \cdot b_q \cdot K_2 \qquad (6.A1.9)$$
$$Z_{q,2} = X_{q,2} + 0.25 \cdot c_q \cdot K_2$$

[8]This step problem can be reduced, but not removed entirely, by a reformulation of the standard presentation of the method; however, use of the Bassie method is still not advisable.

where

q is used as a generic symbol for quarters;

$Z_{q,y}$ is the level of the adjusted quarterly estimate for quarter q in year 1 (y = 1) and 2 (y = 2);

$X_{q,y}$ is the level of the preliminary quarterly estimate for quarter q in year y; and

A_2 is the level of the direct annual estimate for year 2.

6.A1.23. The multiplicative form is as follows:

$$K_2 = \left(A_2 \Big/ \sum_{q=1}^{4} X_{q,2} \right) - 1 \qquad (6.A1.10)$$

leading to the following multiplicative version of the Bassie adjustment method:

$$Z_{q,1} = X_{q,1} \cdot (1 + b_q \cdot K_2) \qquad (6.A1.11)$$

$$Z_{q,2} = X_{q,2} \cdot (1 + c_q \cdot K_2)$$

The multiplicative version of the Bassie method does not yield an exact correction, and a small amount of prorating is necessary at the end of the computation.

6.A1.24. The Bassie method only works as long as not more than one year is adjusted each time and the quarterly estimates represent a continuous time series. In particular, it should be noted that (contrary to what is stated in *Quarterly National Accounts* (OECD 1979, page 30), when several years are to be adjusted, the process cannot be directly "continued for years 2 and 3, years 3 and 4, etc., applying the correction factors for the 'first year' to year 2 (which has already been corrected once) and the correction factors for 'the second year' for year 3, and 4, etc." That is, the following generalized version of the multiplicative Bassie method *does not work*:

$$Z_{q,y} = X_{q,y} \cdot (1 + c_q \cdot K_y) \cdot (1 + b_q \cdot K_{y+1}) \qquad (6.A1.12)$$

6.A1.25. Example 6.A1.1, using the multiplicative version of the Bassie method, illustrates the working of the Bassie method as described in OECD (1979) and the step problem inherent in this version of the method when used for adjusting several years simultaneously.

6.A1.26. The break introduced by the use of the Bassie method, as applied above, is caused by the fact that the quarterly time series used in aligning the series to year 3 is not continuous. The time series used consists of the original data for year 3 and the data for year 2 aligned or benchmarked to the annual data for year 2. This discontinuity is carried over into the revised series.

D. The Ginsburgh-Nasse Method

6.A1.27. Ginsburgh proposed a three-step method for distribution of annual data using a related quarterly series. He did not address the problem of extrapolation, or estimation of the forward series. By slightly reformulating the original presentation of the method along the lines suggested by Ginsburgh himself, however, the basic version of the QNA "regression-based" compilation system,[9] as originally formulated by Nasse (1973), for estimating both the back and the forward series emerges. In this section the following is shown:

- The Ginsburgh-Nasse method is in essence identical to the additive Denton (D1) method with a prior adjustment of the indicator for any significant average difference between the level of the indicator and the target variable.
- For both the back and forward series, the Ginsburgh-Nasse method and the D1 method with prior level adjustment result in identical estimates.
- The regression component of the Ginsburgh-Nasse method constitutes an unnecessarily complicated and cumbersome way of prior adjusting the indicator for any significant average difference between the level of the indicator and the target variable.
- The same prior level adjustment can be obtained simply by using the ratio between the annual benchmark and the annual sum of the indicator for one year as an adjustment factor.

6.A1.28. Ginsburgh's proposal was to generate the benchmarked quarterly data by using the following three-step procedure:

(a) Estimate the "quarterly trend" of the annual data A_y and the annual sum of the indicator

$$I_y = \sum_q I_{q,y}$$

using a the following least-squares distribution formula:

$$\min_{(Z_1,\ldots,Z_{4\beta})} \sum_{t=2}^{4\beta} \left[Z_t - Z_{t-1} \right]^2$$

under the restriction that

$$\sum_{t=4y-3}^{4\beta} Z_t = A_y \qquad t \in \{1,\ldots(4\beta)\},\ y \in \{1,\ldots\beta\},$$

[9]As presented in for instance Dureau (1995).

where $Z_t = \hat{A}_t$ and $\hat{I}_t$, respectively. Denote the resulting quarterized series $\hat{A}_{q,y}$ and $\hat{I}_{q,y}$.

(b) Use the standard ordinary-least-squares (OLS) technique to estimate the parameters of the following annual linear regression equation:

$$A_y = f(I_y) = a + b \cdot I_y + \varepsilon_y, \qquad (6.A1.13)$$
$$E(\varepsilon_y) = 0, \; y \in \{1,....\beta\}$$

where

ε_y stands for the error term assumed to be random with an expected value equal to zero; and

a and *b* are fixed parameters to be estimated.

(c) Finally, derive the benchmarked data for the back series as follows:

$$X_{q,y} = \hat{A}_{q,y} + \hat{b} \cdot (I_{q,y} - \hat{I}_{q,y}) \qquad (6.A1.14)$$
$$q \in \{1,...4\}, y \in \{1,...\beta\}$$

where $\hat{b}$ is the estimated value of the fixed parameter *b* in equation (6.A1.13).

6.A1.29. As shown by Ginsburgh, the derived benchmarked series in equation (6.A1.14) can equivalently be derived by solving the following least-squares minimization problem:

$$\min_{(X_1...,X_{4\beta})} \sum_{t=2}^{4\beta} \left[(X_t - X_{t-1}) - \hat{b} \cdot (I_t - I_{t-1}) \right]^2 \qquad (6.A1.15)$$

This equation reduces to the additive Denton (D1) formula in equation (6.A1.1) if $\hat{b}$ is close to 1.

6.A1.30. In equation (6.A1.15), the parameter $\hat{b}$ serves to adjust for the average difference between the level of indicator and the target variable and thus helps mitigate one of the major weaknesses of the standard additive Denton formula. The parameter *a*, in the linear regression equation (6.A1.13), serves to adjust for any systematic difference (bias) in the average movements of the indicator and target variable. The parameter *a* does not appear in equations (6.A1.14) or (6.A1.15), however, and thus in the end serves no role in deriving the estimates for the back series.

6.A1.31. The basic set-up of the QNA "regression-based" compilation system proposed by Nasse is the following:

(a) Use an estimated econometric relationship such as in step (b) of the Ginsburgh method above to derive preliminary (nonbenchmarked) QNA time series ($X^p_{q,y}$) as

$$X^p_{q,y} = \hat{a}/4 + \hat{b} \cdot I_{q,y}, \quad y \in \{1,....\beta\} \qquad (6.A1.16)$$

where $\hat{a}$ is the estimated value of the fixed parameter *a* in equation (6.A1.13).

(b) Compute the difference between the annual sums of the quarterly estimates derived by using equation (6.A1.16) and the corresponding independent annual data as follows:

$$\hat{\varepsilon}_y = A_y - \sum_q X^p_{q,y} \neq 0 \qquad (6.A1.17)$$

The OLS estimation technique will ensure that the error term sums to zero over the estimation period ($\sum_y \sum_q \varepsilon_{q,y} = 0$) but will not ensure that the annual sum of the error term is equal to zero.

(c) Generate a smooth continuous time series of error terms for year 1 to β using the following least-squares minimization expression:

$$\min_{(\varepsilon_1...,\varepsilon_{4\beta})} \sum_{t=2}^{4\beta} \left[\varepsilon_t - \varepsilon_{t-1} \right]^2, \qquad (6.A1.18)$$
$$y \in \{1,....\beta\}$$

under the restriction that $\sum_{t=4y-3}^{4y} \varepsilon_t = \varepsilon_y$

(d) Finally, derive the benchmarked data for both the back and the forward series as follows:

For the back series,
$$X_{q,y} = \hat{a}/4 + \hat{b} \cdot I_{q,y} + \hat{\varepsilon}_{q,y} \qquad (6.A1.19)$$
$$y \in \{1,....\beta\}$$

For the forward series,
$$X_{q,y} = \hat{a}/4 + \hat{b} \cdot I_{q,y} + \hat{\varepsilon}_{4,\beta} \qquad (6.A1.20)$$
$$y \in \{\beta + 1,.....\}$$

6.A1.32. By combining equations (6.A1.17), (6.A1.18), (6.A1.19), and (6.A1.20), it can be shown that steps (b) to (d) above reduce to

$$\min_{(X_1...,X_{4\beta},......X_{4y})} \sum_{t=2}^{4y} \left[(X_t - X_{t-1}) - \hat{b} \cdot (I_t - I_{t-1}) \right]^2 \qquad (6.A1.21)$$

and thus become identical to the Ginsburgh method in equation (6.A1.15), expanded slightly to also encompass the forward series. Again, observe that the parameter $\hat{a}$ does not appear in equation (6.A1.21) and thus in the end serves no role in deriving the estimates, even for the forward series.

6.A1.33. Equations (6.A1.15) and (6.A1.21) show that the Ginsburgh-Nasse method does not represent any real difference from the additive Denton (D1) method for the following two reasons. First, and most importantly, the regression approach does not provide any additional adjustment for the existence of any bias in the indicator's movements compared with the basic additive Denton method, neither for the back series nor for the forward series. Second, regression analysis represents an unnecessarily complicated way of adjusting for any significant average difference between the level of the indicator and the target variable. This average-level-difference adjustment can be obtained much more easily by a simple rescaling of the original indicator, using the ratio between the annual benchmark and the annual sum of the indicator for one year as the adjustment factor. Thus, as shown, the Ginsburgh-Nasse method in the end constitutes an *unnecessarily complicated and cumbersome*[10] *way of obtaining for both the back and the forward series the same estimates that can be obtained much easier by using the D1 method.*

6.A1.34. As with most additive adjustment formulations, the Ginsburgh-Nasse and D1 methods tend to smooth away some of the quarter-to-quarter rates of change in the indicator series. As a consequence, they can seriously disturb that aspect of the short-term movements for series that show strong short-term variations.[11] This can particularly occur if there is a substantial difference between the level of the indicator and the target variable.

6.A1.35. The procedure set out in (a) to (d) above has also been criticized (Bournay and Laroque 1979) as being inconsistent in terms of statistical models. OLS regression assumes that the errors are not autocorrelated. This is inconsistent with the smooth distribution of the annual errors in equation (6.A1.18), which implies an assumption that the errors are perfectly autocorrelated with a unit autocorrelation coefficient. This inconsistency may not have any significant impact on the back series but may imply that it is possible to obtain a better estimate for the forward series by incorporating any known information on the errors' autocorrelation structure.

6.A1.36. The procedure can also be criticized for being sensitive to spurious covariance between the series. Formulating the econometric relationship as a relationship between *the level of non-stationary time series* renders the danger of primarily measuring apparent correlations caused by the strong trend usually shown by economic time series.

6.A1.37. Compared with the enhanced version of the proportional Denton method, the Ginsburgh-Nasse and D1 methods have two additional distinct disadvantages, namely:[12]

(a) They will only partly adjust for any systematic bias in the indicator's annual movements if the bias is substantial relative to any amount of noise.

(b) They will, on average, lead to relatively larger revisions (a wagging tail effect) if the amount of noise is substantial relative to any bias in the indicator's annual movements.

6.A1.38. The potential wagging tail effect that the Ginsburgh-Nasse and D1 methods suffer from is associated with the inconsistent use of statistical models mentioned above (paragraph 6.A1.35). In particular, estimating the forward series by carrying forward the estimated error term for the fourth quarter of the last benchmark year $\hat{\varepsilon}_{q,\beta}$ is inconsistent with the assumptions underlying the use of OLS to estimate the parameters of equation (6.A1.13). To see this, assume for the sake of the argument that the statistical model in equation (6.A1.13) is correctly specified and thus that the annual error term ε_y is not autocorrelated and has a zero mean. Then the best forecast for the next annual discrepancy $\hat{\varepsilon}_{\beta+1}$ would be zero and not $4 \cdot \hat{\varepsilon}_{\beta+1}$ as implied by equation (6.A1.20).

[10] In contrast to the D1 method, the regression approach also requires very long time series for all indicators.

[11] Some of the countries using these additive methods partly circumvent the problem by applying them only on seasonally adjusted source data. However, other short-term variations in the data will still be partly smoothed away, and, as explained in Chapter I, loss of the original non-seasonally adjusted estimates is a significant problem in itself.

[12] The basic version of the proportional Denton also suffers from these weaknesses. A detailed discussion of these issues with respect to the D4 formula is provided in Annex 6.2. The discussion in Annex 6.2 is also applicable to the D1 formula, with the only difference being how the annual movements are expressed: as additive changes in the case of the D1 formula and as relative changes (growth rates) in the case of the D4 formula.

E. Arima-Model-Based Methods

6.A1.39. The ARIMA-model-based method proposed by Hillmer and Trabelsi (1987) provides one method for taking into account any known information about the stochastic properties of the series being benchmarked. As for most of the statistical modeling methods, the method was proposed in the context of improving survey estimates, where the survey design may provide identifiable information about parts of the stochastic properties of the series (the sampling part of the underlying error-generating mechanism). Clearly, incorporating any such information, if available, in the estimation procedure may potentially improve the estimates. In the QNA context, however, this information about the stochastic properties of the series is usually non-existent. Furthermore, non-sampling errors in the surveys may often be more important than sampling errors, and incorporating only partial information about the underlying error-generating mechanism may introduce systematic errors.

6.A1.40. The main advantages of the enhanced proportional Denton method over the ARIMA-model-based methods in the QNA compilation context are the following:

- The enhanced proportional Denton method is much simpler, more robust, and better suited for large-scale applications.
- The enhanced proportional Denton method avoids the danger associated with the ARIMA-model-based method of over-adjusting the series by interpreting as errors, and thus removing, true irregular movements that do not fit the regular patterns of the statistical model.
- The enhanced proportional Denton method avoids the danger of substantially disturbed estimates resulting from misspecification of the autocovariance structure of the quarterly and annual error terms in the ARIMA-model-based method.
- The enhanced proportional Denton method allows for extrapolation taking into account fully the existence of any systematic bias or lack thereof in the year-to-year rate of change in the indicator. In contrast, the proposed ARIMA-model-based method does not accommodate any bias in the indicator's movements.

6.A1.41. The core idea behind the Hillmer-Trabelsi ARIMA-model-based method is to assume the following:

(a) That the quarterly time series is observed with an additive error, $I_{q,y} = \theta_{q,y} + \varepsilon_{q,y}$ where $\theta_{q,y}$ represents the true but unknown quarterly values of the series and is assumed to follow an ARIMA model. The error term $\varepsilon_{q,y}$ is assumed to have zero mean and to follow a known ARMA[13] model. Assuming that the error term has zero mean implies that the observed series is assumed to be an unbiased estimate of the true series.

(b) That the annual benchmarks also are observed with an additive error with zero mean and known autocovariance structure. That is, the annual benchmarks follow the model: $A_y = \sum_q \theta_{q,y} + \xi_y$ where ξ_y represents the annual error term, and is assumed independent of $\varepsilon_{q,y}$ and $\eta_{q,y}$.

Based on the assumed time-series models and assumed known autocovariance structures, Hillmer and Trabelsi obtain the quarterly benchmarked series using what the time-series literature refers to as "signal extraction."

F. General Least-Squares Regression Models

6.A1.42. An alternative, and potentially better, method to take into account any known information about the stochastic properties of the underlying error-generating process is represented by the alternative general-least-squares (GLS) regression models proposed by various Statistics Canada staff.

6.A1.43. The advantages of the enhanced proportional Denton method over the GLS regression model methods, in the QNA compilation context, are basically the same as the advantages over the ARIMA-model-based method listed in paragraph 6.A1.40 above.

6.A1.44. The following three models constitute the core of Statistics Canada's benchmarking program "Program Bench":

- The additive model (Cholette and Dagum 1994)

$$I_t = a + \theta_t + \varepsilon_t, \qquad (6.A1.22a)$$

$$E(\varepsilon_t) = 0, \quad E(\varepsilon_t\, \varepsilon_{t-k}) \neq 0$$

$$t \;\&\; k \in \{1,...(4\beta),....T\}, \quad y \in \{1,...\beta\}$$

$$A_y = \sum_{t=4y-3}^{4y} \theta_t + w_y, \qquad (6.A1.22b)$$

$$E\left(w_y\right) = 0, \;\; E\left(w_y w_{t-k}\right) \neq 0$$

[13] Autoregressive moving average.

where

a is an unknown constant bias parameter to be estimated;

θ_t is the true but unknown quarterly values to be estimated;

ε_t is the quarterly error term associated with the observed indicator and is assumed to have zero mean and a known autocovariance structure; and

w_y is the annual error term associated with the observed benchmarks and is assumed to have zero mean and a known autocovariance structure. The benchmarks will be binding if the variance of the annual error term is zero and non-binding if the variance is different from zero.

- The multiplicative model (Cholette 1994)

$$I_t = a \cdot \theta_t \cdot \varepsilon_t, \quad (6.A1.23a)$$

$$E(\varepsilon_t) = 0, \quad E(\varepsilon_t\, \varepsilon_{t-k}) \neq 0$$

$$A_y = \sum_{t=4y-3}^{4y} \theta_t + w_y, \quad (6.A1.23b)$$

$$E(w_y) = 0, \quad E(w_y w_{t-k}) \neq 0$$

- The mixed model (Laniel and Fyfe 1990)

$$I_t = a \cdot \theta_t + \varepsilon_t, \quad (6.A1.24a)$$

$$E(\varepsilon_t) = 0, \quad E(\varepsilon_t\, \varepsilon_{t-k}) \neq 0$$

$$A_y = \sum_{t=4y-3}^{4y} \theta_t + w_y, \quad (6.A1.24b)$$

$$E(w_y) = 0, \quad E(w_y w_{t-k}) \neq 0$$

6.A1.45. Cholette and Dagum (1994) provide the GLS solution to equation (6.A1.22) when the autocovariance structure of the annual and quarterly error terms is known. Similarly, Mian and Laniel (1993) provides the Maximum Likelihood solution to equation (6.A1.24) when the autocovariance structure of the annual and quarterly error terms is known.[14]

6.A1.46. The three GLS models are implemented in Statistics Canada's benchmarking program, assuming that the errors follow the following autocovariance structures:

$$E(\varepsilon_t) = 0, \quad (6.A1.25a)$$

$$E(\varepsilon_t\, \varepsilon_{t-k}) \neq \sigma_{\varepsilon_t}\, \sigma_{\varepsilon_{t-1}}\, \rho_k$$

$$A_y = \sum_{t=4y-3}^{4y} \theta_t + w_y, \quad (6.A1.25b)$$

$$E(w_y) = 0, \quad E(w_y^2) = \sigma_{w_y}^2$$

[14]The solutions are the "best linear unbiased estimates" (BLUE) under the given assumptions.

where

σ_{ε_t} is the standard deviation of the quarterly errors, which may vary with time t, meaning that the errors may be heteroscedastic;

ρ_k is a parameter indicating the degree of autocorrelation in the errors; and

$\sigma_{w_y}^2$ is the variance of the annual errors, which may vary with time y, meaning that the errors may be heteroscedastic.

and where the autocorrelations ρ_k corresponds to those of a stationary and invertible ARMA process whose parameter values are supplied by the users of the program. This is equivalent to assuming that the quarterly errors follow a time-series process given by $\varepsilon_t = \in_t \cdot \sigma_{\varepsilon_t}$ where $\in_t$ follows the selected ARMA process.

6.A1.47. The regression models in equation (6.A1.22) to (6.A1.25) can be used to approximate the D1, D3, and D4 versions of the Denton method above by specifying the autocovariance structure appropriately. The additive regression model in equation (6.A1.22) approximates D1 if

(a) the bias parameter is omitted;
(b) the benchmarks are binding (zero variances);
(c) the variances of the quarterly errors are constant; and
(d) the ARMA model specified approximates a random walk process (that is $\varepsilon_t = \sigma_{\varepsilon_t} \cdot (\varepsilon_{t-1} + v_t)$ where v_t represents "white noise").

Similarly, the additive regression model in equation (6.A1.22) approximates D4 if

(a) the bias parameter is omitted;
(b) the benchmarks are binding;
(c) the coefficients of variation (CVs, $\sigma_{\varepsilon_t}/\overline{\varepsilon}$ (where $\overline{\varepsilon}$ is the average error) of the quarterly errors are constant; and
(d) the ARMA model specified approximates a random walk process (that is $\varepsilon_t = \sigma_{\varepsilon_t} \cdot (\varepsilon_{t-1} + v_t)$ where σ_{ε_t} is given by the constant CVs).

Finally, the multiplicative regression model in equation (6.A1.24) approximates D3 if

(a) the benchmarks are binding;
(b) the coefficients of variation (CVs) of the quarterly errors are constant; and
(c) the ARMA model specified approximates a random walk process (that is, $\varepsilon_t = \sigma_{\varepsilon_t} \cdot (\varepsilon_{t-1} + v_t)$).

G. The Chow-Lin Method

6.A1.48. The Chow-Lin method for distribution and extrapolation of time series is basically a multiple-regression version of the additive GLS model in equation (6.A1.22) above with binding benchmarks. By relating several loosely related indicator series to one annual benchmark series, it does not represent a benchmarking method in a strict sense.

6.A1.49. The main advantages of the enhanced proportional Denton method over the Chow-Lin method are the same as listed above with respect to the GLS regression and ARIMA-model methods. In addition, the Chow-Lin method differs from the above GLS regression methods in the following two fundamental aspects that make it unsuitable for QNA purposes in most circumstances:[15]

- Multiple regression is conceptually fundamentally different from benchmarking. The Chow-Lin method gives the dangerous impression that quarterly estimates of GDP and other national accounts variables can be derived simply by estimating the annual correlation between the national accounts variables and a limited set of some loosely related quarterly source data. In contrast, benchmarking is about combining quarterly and annual source data for the same phenomena. At best, estimating the correlation between, for example, GDP and a set of available quarterly time series is a modeling approach to obtain forecasts or nowcasts of GDP, but it has nothing to do with compiling quarterly national accounts. Furthermore, as a modeling approach for forecasting it is overly simplified and may result in sub-optimal forecasts.
- The multiple-regression approach implicitly assumes that the (net) seasonal pattern of the related series is the same as that of the target aggregate, which is not very likely.

[15]The Chow-Lin multiple-regression method may have an application in filling minor gaps with synthetic data where no direct observations are available.

Annex 6.2. Extrapolation Base and the Forward Step Problem

A. Introduction

6.A2.1. The basic version of the proportional Denton method presented in Chapter VI uses the last quarter of the last benchmark year as the extrapolation base.[16] Arguments have been made for using alternative extrapolation bases. It is sometimes argued that using the last quarter of the last benchmark year as the extrapolation base may make the estimates vulnerable to errors in the source data for that quarter, and thus, it may be better to use the last annual average as the extrapolation base. Similarly, it is sometimes argued that to preserve the seasonal pattern of the series, the same quarter in the previous year should be used as the extrapolation base or, alternatively, that a strong seasonal pattern in the series may cause distortions to the estimates if they are not based on moving from the same quarter of the previous year.

6.A2.2. In this annex we will show that these arguments for using alternative extrapolation bases are not correct and that the alternative extrapolation bases generally should not be used. In particular, we will show that use of different extrapolation bases will result in different estimates only if the implied quarterly benchmark-indicator (BI) ratios for the back series differ from quarter to quarter and from the annual (BI) ratio; which they must do to avoid the back series step problem. In those circumstances:

- *The alternative extrapolation bases introduce a step between the back and forward series* that can seriously distort the seasonal pattern of the series.
- Using the last quarter of the last benchmark year as the extrapolation base will result in the following:[17]
  - It will partly adjust for any systematic bias in the indicator's annual rate of change if the bias is sufficiently large relative to any amount of noise, and thus, on average, lead to smaller revisions in the quarterly national accounts (QNA) estimates.
  - It will create a wagging tail effect with, on average, larger revisions if the amount of noise is sufficiently large relative to any systematic bias in the annual growth rate of the indicator.

The annex also demonstrates that using the last quarter of the last benchmark year as the extrapolation base does not make the estimates more vulnerable to errors in the source data for that quarter. Numerical illustrations of these results are given in Examples 6.A2.1 and 6.A2.2, and Chart 6.A2.1.

B. Alternative Extrapolation Bases

6.A2.3. In mathematical terms the use of the alternative extrapolation bases can be formalized as follows:

(a) Fourth quarter of the last benchmark year as the extrapolation base:

$$X_{q,y} = X_{4,\beta} \cdot \left(\frac{I_{q,y}}{I_{4,\beta}} \right) = I_{q,y} \cdot \left(\frac{X_{4,\beta}}{I_{4,\beta}} \right) \qquad (6.A2.1)$$

$$q \in \{1,\ldots 4\}, \quad y \in \{\beta + 1, \ldots\}$$

(b) Quarterly average of the last benchmark year as the extrapolation base:

$$X_{q,y} = \tfrac{1}{4} \cdot A_\beta \cdot \left(\frac{I_{q,y}}{\tfrac{1}{4} \cdot \sum_q I_{q,\beta}} \right) \qquad (6.A2.2)$$

$$= I_{q,y} \cdot \left(\frac{A_\beta}{\sum_q I_{q,\beta}} \right)$$

$$q \in \{1,\ldots 4\}, \quad y \in \{\beta + 1, \ldots\}.$$

(c) Same quarter of the last benchmark year as the extrapolation base:

$$X_{q,y} = X_{q,\beta} \cdot \left(\frac{I_{q,y}}{I_{q,\beta}} \right) \qquad (6.A2.3)$$

$$= I_{q,y} \cdot \left(\frac{X_{q,\beta}}{I_{q,\beta}} \right)$$

$$q \in \{1,\ldots 4\}, \quad y \in \{\beta + 1, \ldots\}.$$

[16] In contrast, the recommended enhanced version of the proportional Denton presented in section C of Chapter VI does not use any specific extrapolation base.

[2] The enhanced version of the proportional Denton presented in section C of Chapter VI provides means for avoiding the potential wagging tail effect, and for fully adjusting for any systematic bias.

Example 6.A2.1. Extrapolation Bases and the Forward Step Problem

	Indicator	Annual Data	Annual BI Ratios	Quarterized BI Ratios	Estimates for 1998–1999 from 6.2.	Estimates for 2000 (a) Extrapolation of q4 1999: Estimates	Estimates for 2000 (a) Extrapolation of q4 1999: BI Ratio Carried Forward
q1 1998	98.2			9.876	969.8		
q2 1998	100.8			9.905	998.4		
q3 1998	102.2			9.964	1,018.3		
q4 1998	100.8			10.054	1,013.4		
Sum	**402.0**	**4,000.0**	**9.950**	**9.950**	**4,000.0**		
q1 1999	99.0			10.174	1,007.2		
q2 1999	101.6			10.264	1,042.9		
q3 1999	102.7			10.325	1,060.3		
q4 1999	101.5			***10.355***	1,051.0		
Sum	**404.8**	**4,161.4**	**10.280**		**4,161.4**		
q1 2000	100.5					1,040.6	10.355
q2 2000	103.0					1,066.5	10.355
q3 2000	103.5					1,071.7	10.355
q4 2000	101.5					1,051.0	10.355
Sum	**408.5**					**4,229.9**	**10.355**

In this example, the following is worth observing:

First, during 1999 the quarterized BI ratio is increasing gradually (10.174, 10.264, 10.325, and 10.355), and consequently the quarter-to-quarter rate of change in the indicator differs from the quarter-to-quarter rates of change in the derived QNA estimates for 1999.

Second, the three different QNA estimates for 2000 can be derived by carrying forward the 1998 BI ratios as follows:
(a) Extrapolating the fourth quarter of 1999:
q1,00=1040.6 = 100.5 · 10.355 q2,00=1066.5 = 103.0 · 10.355 q4,00=1051.0 = 101.5 · 10.355;
(b) Extrapolating the quarterly average for 1999:
q1,00=1033.2 = 100.5 · 10.280 q2,00=1058.9 = 103.0 · 10.280 q4,00=1043.4 = 101.5 · 10.280; and
(c) Extrapolating the same quarter in 1999:
q1,00=1022.5 = 100.5 · 10.174 q2,00=1057.2 = 103.0 · 10.264 q4,00=1051.0 = 101.5 · 10.355.

Third,
(a) Extrapolating the fourth quarter of 1999:
preserves the quarter-to-quarter rate of changes in the indicator series;
(b) Extrapolating the quarterly average for 1999:
results in **a break** between the fourth quarter of 1999 and the first quarter of 2000 (period-to-period rate of change of –1.7 and not –1.0% as shown in the indicator); and
(c) Extrapolating the same quarter in 1999:
results in an **even more severe break** between the fourth quarter of 1999 and the first quarter of 2000 (period-to-period rate of change of –2.7% and not –1.0% as shown in the indicator).
In addition, the breaks between the fourth quarter of 1999 and the first quarter of 2000 introduced by using extrapolation bases (b) and (c) are introduced by a discontinuity in the time series of quarterized BI ratios. That is, when using extrapolation base (b) the BI ratio changes abruptly from 10.355 in the fourth quarter of 1999 to 10.28 in the first quarter of 2000, and when using extrapolation base (c) the BI ratio changes abruptly from 10.355 in the fourth quarter of 1999 to 10.174 in the first quarter of 2000.

Fourth,
(a) Extrapolating the fourth quarter of 1999:
results in an estimated **annual rate of change** in the QNA series from 1999 to 2000 of 1.6%, which differs from the rate of change from 1999 to 2000 of 0.9% shown in the indicator series;
(b) Extrapolating the quarterly average for 1999:
results in an estimated rate of change from 1999 to 2000, which is identical to the rate of change shown in the indicator series (0.9%); and
(c) Extrapolating the same quarter in 1999:
results in an estimated annual rate of change from 1999 to 2000, which is identical to the rate of change shown in the indicator series (0.9%).

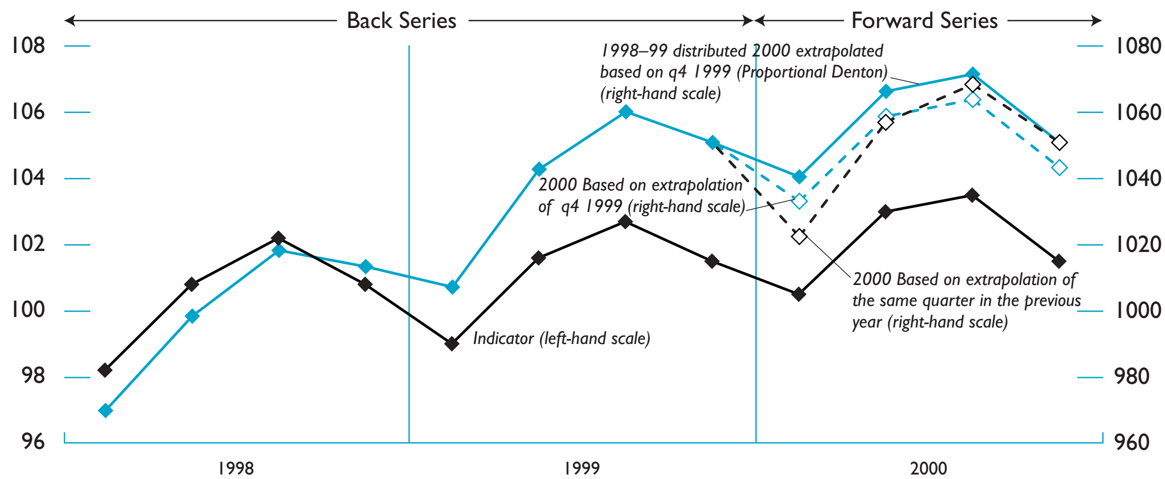
Fifth, if the difference of 3.0 percentage points between the rate of change from 1999 to 2000 in the ANA estimate and in the indicator is due to an average downward bias in the annual movements of the indicator of 3.0 percentage points, then the annual data for 2000 can be expected to show an annual rate of change from 1999 to 1999 of 4.0 percent. Thus, the estimate derived by using extrapolation base (a) will still be downward biased.

(These results are illustrated in Chart 6.A2.1.)

Estimates for 2000				Quarter-to-Quarter Rates of Change			
(b) Extrapolation of the Average Quarter for 1999		(c) Extrapolation of the Same Quarter in the Previous Year					
Estimates	BI Ratio Carried Forward	Estimates	BI ratios Carried Forward	Based on the Indicator	(a) Based q4 1999	(b) Based on average 1999	(c) Extrapolation of the Same Quarter in the Previous Year
				2.6%	3.0%		
				1.4%	2.0%		
				−1.4%	−0.5%		
Identical for All Methods							
				−1.8%	−0.6%		
				2.6%	3.5%		
				1.1%	1.7%		
				−1.2%	−0.9%		
				0.7%	**4.0%**		
1,033.2	10.280	1,022.5	10.174	−1.0%	−1.0%	−1.7%	–2.7%
1,058.9	10.280	1,057.2	10.264	2.5%	2.5%	2.5%	3.4%
1,064.0	10.280	1,068.6	10.325	0.5%	0.5%	0.5%	1.1%
1,043.4	10.280	1,051.0	10.355	−1.9%	−1.9%	−1.9%	−1.6%
4,199.4	**10.280**	**4,199.3**	**10.280**	**0.9%**	**1.6%**	**0.9%**	**0.9%**

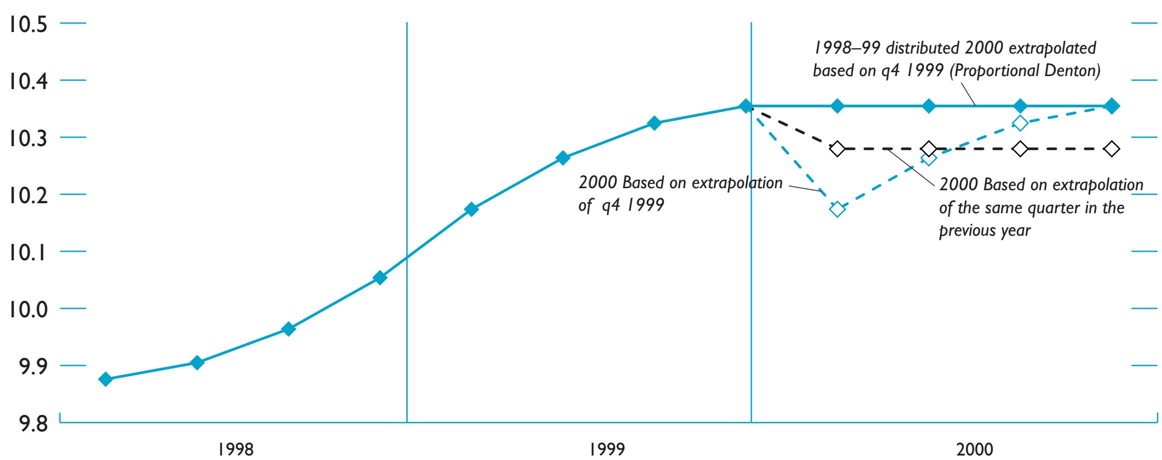
Chart 6.A2.1. Alternative Extrapolation Bases and the Forward Step Problem

(The corresponding data are given in Example 6.A2.1)



In this example, the **step problem** shows up as a decrease in the derived series from q4 1999 to q1 2000 that is not matched by the movements in the source data. The quarter-to-quarter rate of change for the first quarter of 1999 of **–1.0%** in source data is –1.0%. In contrast, the corresponding rate of change in the estimates derived by extrapolating the average of 1999 is **–1.7%**, and the corresponding rate of change in the estimates derived by extrapolating the same quarter of 1999 is **–2.7%**.

Benchmark-to-Indicator Ratio



It is easier to recognize the step problem from charts of the BI ratio, where it shows up as abrupt upward or downward steps in the BI ratios between q4 of one year and q1 of the next year. In this example, the step problem shows up as a large upward jump in the BI ratio between q4 1999 and q1 2000.

6.A2.4. The use of different extrapolation bases will result in different estimates only if the implied quarterly BI ratios for the back series differ from quarter to quarter and from the annual BI ratio. That is, if

$$\left(X_{4,\beta}/I_{4,\beta}\right) \neq \left(X_{q,\beta}/I_{q,\beta}\right) \neq \left(A_{\beta}/\sum_{q} I_{q,\beta}\right).$$

6.A2.5. In Section C of Chapter VI it is explained that to avoid the back series step problem, the implied quarterly BI ratios ($X_{q,y}/I_{q,y}$) must differ from quarter to quarter and from the annual BI ratio. Thus, different extrapolation bases will give different estimates when the back series is derived using benchmarking methods that avoid the (back series) step problem associated with pro rata distribution.

C. The Forward Step Problem

6.A2.6. The forward step problem associated with extrapolation bases (b) and (c) above is caused by a discontinuity in the implied quarterly BI ratios. To keep the benchmarked series as proportional as possible to the original quarterly source data, the proportional Denton method generates quarterly BI ratios that for the last year covered by annual data either increase or decrease gradually. Consequently, the quarterly BI ratio for the last quarter of the last benchmark year may differ significantly from the annual BI ratio and even more from the quarterly BI ratio for the first quarter of the last benchmark year. It follows that:

- Extrapolation base (b) introduces an upward step if

$$\left(A_{\beta}\Big/\sum_{q=1}^{4} I_{q,\beta}\right) > \left(X_{4,\beta}/I_{4,\beta}\right), \text{ or}$$

a downward step if

$$\left(A_{\beta}\Big/\sum_{q=1}^{4} I_{q,\beta}\right) < \left(X_{4,\beta}/I_{4,\beta}\right).$$

- Extrapolation base (c) introduces an upward step if

$$\left(X_{1,b}/I_{1,b}\right) > \left(X_{4,b}/I_{4,b}\right), \text{ or}$$

a downward step if

$$\left(X_{1,b}/I_{1,b}\right) < \left(X_{4,b}/I_{4,b}\right).$$

6.A2.7 It also follows that the step introduced by using the same quarter of the previous year as the extrapolation base (base iii) will always be more severe than the step caused by using the annual average as the extrapolation base (base ii).

D. Annual Rate of Change in the Derived Forward Series

6.A2.8. Using the last quarter of the last benchmark year as the extrapolation base implies adjusting the source data for all subsequent quarters with a factor that systematically differs from the average adjustment in the last benchmark year. This is the cause for the difference between the annual growth rate in the source data and the annual growth rate in the estimates derived by using the basic version of the proportional Denton for the first year of the forward series.[18] It follows that using extrapolation base (a) will result in an annual rate of change for the first year of the forward series that is

- higher than the corresponding change in the source data if

$$\left(A_{\beta}\Big/\sum_{q=1}^{4} I_{q,\beta}\right) < \left(X_{4,\beta}/I_{4,\beta}\right), \text{ or}$$

- lower than the corresponding change in the source data if

$$\left(A_{\beta}\Big/\sum_{q=1}^{4} I_{q,\beta}\right) > \left(X_{4,\beta}/I_{4,\beta}\right).$$

6.A2.9. The relative difference between the annual changes in the derived QNA estimates and the corresponding changes in the indicator is equal to the relative difference between the quarterly BI ratio for the fourth quarter and the annual average BI ratio of the last benchmark year. This can be shown mathematically as follows:

[18] In contrast, it can be shown that the corresponding annual growth rates obtained by using extrapolation base (b) or (c) will for base (b) be identical, and for base (c) approximately identical, to the annual growth rates in the source data. Note that this may not be a desirable property if there is significant bias in the indicator's annual rate of movements.

The ratio of annual change in the derived estimates is equal to

$$\sum_{q=1}^{4} X_{q,y} \Big/ \sum_{q=4}^{4} X_{q,\beta}.$$

The ratio of annual change in the indicator is equal to

$$\sum_{q=1}^{4} I_{q,y} \Big/ \sum_{q=1}^{4} I_{q,\beta} \quad (y=\beta+1).$$

The ratio between these two expressions is equal to the relative difference between the annual changes in the derived estimates and in the indicator, and can be written as

$$\frac{\sum_{q=1}^{4} X_{q,y}}{\sum_{q=1}^{4} X_{q,\beta}} \Bigg/ \frac{\sum_{q=1}^{4} I_{q,y}}{\sum_{q=1}^{4} I_{q,\beta}} = \tag{6A2.4}$$

$$\frac{\sum_{q=1}^{4} \frac{X_{4,\beta}}{I_{4,\beta}} \cdot I_{q,y}}{A_\beta} \Bigg/ \frac{\sum_{q=1}^{4} I_{q,y}}{\sum_{q=1}^{4} I_{q,\beta}} =$$

$$\frac{X_{4,\beta}}{I_{4,\beta}} \Bigg/ \frac{A_\beta}{\sum_{q=1}^{4} I_{q,\beta}}$$

where we have used that

$$X_{q,y} = \frac{X_{4,\beta}}{I_{4,\beta}} \cdot I_{q,y}$$

(from equation (6.A2.1)) and that

$$\sum_{q=1}^{4} X_{q,\beta} = A_\beta.$$

The last expression in equation (6A2.4) is the relative difference between the BI ratio for the fourth quarter and the annual average BI ratio of the last benchmark year.

6.A2.10. Using the last quarter of the last benchmark year as the extrapolation base will result in the following:[19]

- It will partly adjust for any systematic bias in the annual growth rate of the indicator if the bias is sufficiently large relative to any amount of noise and thus, in those circumstances, give on average relatively smaller revisions in the derived QNA estimates.
- It will create a wagging tail effect with, on average, larger revisions in the derived QNA estimates if the amount of noise is sufficiently large relative to any systematic bias in the annual growth rate of the indicator.

6.A2.11. To see this, consider the case in which the annual rate of change in the indicator is consistently downward biased and in which the amount of noise is zero. Then, by definition, the ratio between the annual rate of change in the annual national accounts (ANA) estimates and the annual rate of change in the indicator will be constant and larger than one:

$$(A_y/A_{y-1}) \Big/ \left(\sum_{q=1}^{4} I_{q,y}\right) = \delta,$$

where δ is a fixed bias parameter.

In that case, the annual BI ratio will be increasing with a constant rate from year to year:

$$\left(A_y \Big/ \sum_{q=1}^{4} I_{q,y}\right) = \delta \cdot \left(A_{y-1} \Big/ \sum_{q=1}^{4} I_{q,y-1}\right).$$

6.A2.12. Quarterizing a time series of annual BI ratios that increases with a constant rate will result in a time series of quarterly BI ratios that also increases steadily from quarter to quarter. In particular, the quarterized BI ratio will be increasing through the last benchmark year,[20] and thus, in this case, the BI ratio for the fourth quarter will always be larger than the annual BI ratio for the last benchmark year:

$$(X_{4,\beta}/I_{4,\beta}) > \left(A_\beta \Big/ \sum_{q=1}^{4} I_{q,\beta}\right).$$

6.A2.13. Thus, as explained in paragraph 6.A2.8, in this case, using extrapolation base (a) will result in an annual change in the estimated QNA variable that is higher than the corresponding change in the

[19] Note that the enhanced version presented in Section C of Chapter VI provides means for avoiding the potential wagging tail effect and for fully adjusting for any bias.

[20] The increase will taper off toward the end of the series if the series is based on a first difference least-square expression such as equation (6. 4) in Chapter VI.

indicator, as desired. If the rate of change in the indicator is upward biased, then $\delta < 1$ and the line of arguments in paragraphs 6.A2.11 and 6A2.12 applies in the opposite direction.

6.A2.14 The adjustment for bias in the annual growth rate of the indicator will be partial only because, as can be shown, the BI ratio for the fourth quarter will, at the same time, be smaller than the product of the bias parameter and the last annual BI ratio:

$$\left(X_{4,\beta}/I_{4,\beta}\right) < \delta \cdot \left(A_{\beta}\Big/\sum_{q=1}^{4} I_{q,\beta}\right).$$

To fully correct for the bias in the indicator, the average adjustment of the indicator for the current years should have been equal to the product of the bias parameter and the last annual BI ratio. The enhanced version of the proportional Denton presented in Chapter VI provides means for fully adjusting for any persistent bias.

6.A2.15. The potential wagging tail effect is caused by erratic variations around the fixed bias parameter in the year-to-year increase of the annual BI ratio. As a consequence:

- The BI ratio for the fourth quarter may sometimes be larger than the product of the bias parameter and the last annual BI ratio, resulting in an annual change in the estimated QNA variable that is higher than the expected change in the annual data.
- The quarterized BI ratio may sometimes be decreasing through the last benchmark year, resulting in an annual change in the estimated QNA variable that is lower than in the indicator and lower than the expected change in the annual data.

The enhanced version of the proportional Denton presented in Chapter VI provides means for avoiding this wagging tail effect.

E. Extrapolation Base and Robustness Toward Errors in the Indicator

6.A2.16. Using a single quarter as the extrapolation base does not make the estimates particularly vulnerable to errors in the source data for that quarter. It is sometimes erroneously argued that using extrapolation base (b) gives more robust estimates than using extrapolation base (a). The idea behind this view is that basing the estimates on just one quarter makes them more vulnerable to errors in the indicator. The difference between the estimates derived by using extrapolation base (a) and (b), however, is solely caused by the movements in the quarterized BI ratio during the last benchmark year, which again is mainly a function of the annual BI ratios for that year and the previous years. In particular, as shown in Example 6.A2.2 below, the BI ratio for the fourth quarter of the last benchmark year is almost totally independent of the indicator value for that quarter.

F. Extrapolation Base and Seasonality

6.A2.17. It should be evident from the above that to preserve the seasonal pattern of the series, the same quarter in the previous year generally should not be used as the extrapolation base. As shown, it can introduce an unintended step problem if used together with benchmarking methods that avoid the back series step problem by keeping the derived series as parallel as possible to the source data. In contrast, extrapolation base (a) transmits to the QNA estimate the indicator's seasonal pattern as unchanged as possible, which is what is generally being sought.

6.A2.18. Use of the same quarter in the previous year as the extrapolation base is only acceptable in the following rare circumstance:

- annual benchmarks are not available for more than one year;
- the indicator and the target variable have different seasonal patterns; and
- initial quarterly estimates are available, with a proper seasonal pattern, for a base year.

Example 6.A2.2. Extrapolation Base and Robustness Toward Errors in the Indicator

									Quarter–to–Quarter Rates of Change		
Date	Original Indicator from Example 6.2	Revised Indicator	Annual Data	Annual BI Ratios	Original Estimates from Example 6.2.	Original Quarterized BI Ratios	New Quarterized BI Ratios	Estimates Based on the Revised Indicator	Based on the Original Estimates from Example 6.2	Based on the Revised Indicator	Estimates Based on the Revised Indicator
q1 1998	98.2	98.2			969.8	9.876	9.875	969.7			
q2 1998	100.8	100.8			998.4	9.905	9.904	998.4	3.0%	2.6%	3.0%
q3 1998	102.2	102.2			1,018.3	9.964	9.964	1,018.4	2.0%	1.4%	2.0%
q4 1998	100.8	100.8			1,013.4	10.054	10.055	1,013.6	–0.5%	–1.4%	–0.5%
Sum	**402.0**	**402.0**	**4,000.0**	**9.950**	**4,000.0**			**4,000.0**			
q1 1999	99.0	99.0			1,007.2	10.174	10.176	1,007.5	–0.6%	–1.8%	–0.6%
q2 1999	101.6	101.6			1,042.9	10.264	10.268	1,043.2	3.5%	2.6%	3.5%
q3 1999	102.7	132.7			1,060.3	10.325	10.329	1,370.7	1.7%	30.6%	31.4%
q4 1999	101.5	71.5			1,051.0	10.355	10.350	740.1	–0.9%	–46.1%	–46.0%
Sum	**404.8**	**404.8**	**4,161.4**	**10.280**	**4,161.4**			**4,161.4**			
q1 2000	100.5	100.5			1,040.6	10.355	10.350	1,040.2	–1.0%	40.6%	40.6%
q2 2000	103.0	103.0			1,066.5	10.355	10.350	1,066.1	2.5%	2.5%	2.5%
q3 2000	103.5	103.5			1,071.7	10.355	10.350	1,071.2	0.5%	0.5%	0.5%
q4 2000	101.5	101.5			1,051.0	10.355	10.350	1,050.5	–1.9%	–1.9%	–1.9%
Sum	**408.5**	**408.5**			**4,229.8**	**10.355**	**10.350**	**4,228.0**	**1.6%**	**0.9%**	**1.6%**

In this example the following is worth observing:
First, compared with Example 6.2 the values of the indicator for the third and fourth quarter of 1999 have been substantially changed, but the annual sum of the quarterly values of the indicator, and thus the annual BI ratio, for 1999 is not changed. The data for 2000 are also not changed.

Second, in spite of the big changes in the 1999 data, the quarterized BI ratio for the fourth quarter of 1999 is almost the same as in Example 6.2 (10.350 versus 10.355). This demonstrates that the quarterized BI ratio for the fourth quarter of the last benchmark year is almost totally independent of the value of the indicator for that quarter and that it is mainly a function of the annual BI ratios.

Annex 6.3. First-Order Conditions for the Proportional Denton Benchmarking Formula

6.A3.1. The first-order conditions for a minimum of the proportional Denton adjustment formula can be found with the help of the following Lagrange-function:

$$L\left(X_1 \ldots X_{4y}\right) = \sum_{t=2}^{4y}\left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}}\right]^2 + 2\lambda_y\left[\sum_{t=4y-3}^{4y} X_t - A_y\right], \qquad (6.A3.1)$$

$$t \in \{1, \ldots (4, \beta), \ldots . T\}, \quad y \in \{1, \ldots \beta\}.$$

6.A3.2. Which has the following first order conditions:

$$\frac{\delta L}{\delta X_1} = \frac{1}{I_1^2} \cdot X_1 - \frac{1}{I_1 \cdot I_2} \cdot X_2 + \lambda_1 = 0 \qquad (6.A3.2)$$

$$\frac{\delta L}{\delta X_2} = -\frac{1}{I_1 \cdot I_2} \cdot X_1 + \frac{2}{I_2^2} \cdot X_2 - \frac{1}{I_2 \cdot I_3} \cdot X_3 + \lambda_1 = 0$$

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$$\frac{\delta L}{\delta X_5} = -\frac{1}{I_4 \cdot I_5} \cdot X_4 + \frac{2}{I_5^2} \cdot X_5 - \frac{1}{I_5 \cdot I_6} \cdot X_6 + \lambda_2 = 0$$

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$$\frac{\delta L}{\delta X_t} = -\frac{1}{I_{t-1} \cdot I_t} \cdot X_{t-1} + \frac{2}{I_t^2} \cdot X_t - \frac{1}{I_t \cdot I_{t+1}} \cdot X_{t+1} + \lambda_y = 0, \quad \text{for } t \leq (4\beta\}$$

$$\frac{\delta L}{\delta X_t} = -\frac{1}{I_{t-1} \cdot I_t} \cdot X_{t-1} + \frac{2}{I_t^2} \cdot X_t - \frac{1}{I_t \cdot I_{t+1}} \cdot X_{t+1} = 0, \quad \text{for } t > (4\beta\}$$

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$$\frac{\delta L}{\delta X_T} = -\frac{1}{I_{T-1} \cdot I_T} \cdot X_{T-1} + \frac{1}{I_T^2} \cdot X_T - \frac{1}{I_T \cdot I_{T+1}} \cdot X_{T+1} + \lambda_y = 0, \quad \text{for } T = (4\beta\}$$

$$\frac{\delta L}{\delta X_T} = -\frac{1}{I_{T-1} \cdot I_T} \cdot X_{T-1} + \frac{1}{I_T^2} \cdot X_T = 0, \quad \text{for } t > (4\beta\}$$

6.A3.3. These first-order conditions, together with the benchmark restriction(s)

(in this case, $\sum_{t=4y-3}^{4y} X_t = A_y$),

constitute a system of linear equations. In matrix notation, $I \cdot X = A$, and for a two-year adjustment period with $T=4\beta=8$, matrix I and vector X and A are the following:

$$I = \begin{bmatrix} \frac{1}{I_1^2} & \frac{-1}{I_1 \cdot I_2} & 0 & 0 & 0 & 0 & 0 & 0 & | & 1 & 0 \\ \frac{-1}{I_1 \cdot I_2} & \frac{2}{I_2^2} & \frac{-1}{I_1 \cdot I_2} & 0 & 0 & 0 & 0 & 0 & | & 1 & 0 \\ 0 & \frac{-1}{I_2 \cdot I_3} & \frac{2}{I_3^2} & \frac{-1}{I_3 \cdot I_4} & 0 & 0 & 0 & 0 & | & 1 & 0 \\ 0 & 0 & \frac{-1}{I_3 \cdot I_4} & \frac{2}{I_4^2} & \frac{-1}{I_4 \cdot I_5} & 0 & 0 & 0 & | & 1 & 0 \\ 0 & 0 & 0 & \frac{-1}{I_4 \cdot I_5} & \frac{2}{I_5^2} & \frac{-1}{I_5 \cdot I_6} & 0 & 0 & | & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{-1}{I_5 \cdot I_6} & \frac{2}{I_6^2} & \frac{-1}{I_6 \cdot I_7} & 0 & | & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{I_6 \cdot I_7} & \frac{2}{I_7^2} & \frac{-1}{I_7 \cdot I_8} & | & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{I_7 \cdot I_8} & \frac{1}{I_8^2} & | & 0 & 1 \\ - & - & - & - & - & - & - & - & - & - & - \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & | & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & | & 0 & 0 \end{bmatrix} X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} A = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ A_1 \\ A_2 \end{bmatrix}$$