IX Price and Volume Measures: Specific QNA-ANA Issues

A. Introduction

9.1. This chapter addresses a selected set of issues for constructing time series of price and volume measures that are of specific importance for the quarterly national accounts (QNA). In particular, it discusses the relationship between price and volume measures in the QNA and in the annual national accounts (ANA): namely, (1) how to aggregate price and volume measures over time; (2) how to choose the base period in the QNA; (3) the frequency of chain-linking; and (4) the techniques for annual chain-linking of quarterly data. In addition, the chapter addresses how to deal with nonadditivity and presentation of chain-linked volume measures in the QNA.

9.2. The *1993 SNA* does not contain specific recommendations for price and volume measures for the QNA or the relationship between price and volume measures in the QNA and the ANA. The basic principles for quarterly price and volume measures in the QNA and the ANA are the same, including the *1993 SNA* recommendation of moving away from the traditional constant-price measures[1] to annually chain-linked measures, preferably using superlative index number formulas such as the Fisher and Tornquist formulas. The issues listed above raise new problems, however, many of which have not satisfactorily been dealt with to date in the literature. Conventional intertemporal index number theory has mainly been concerned with price and quantity comparisons between separate pairs of *points in time* and not with price and volume measures in a time-series context. In particular, conventional index number theory has not been concerned with price and quantity comparisons between *periods of time of different duration* (e.g., years and quarters) and the relationship among these price and volume measures for longer time periods, the corresponding measures for the subperiods, and the point-to-point measures.

[1]Constant price measures are fixed-base Laspeyres-type volume measures (fixed-price weights) and the corresponding price deflators are Paasche price indices.

9.3. QNA price and volume measures should be in the form of time series and should be consistent with corresponding ANA estimates. For QNA price and volume measures to constitute a time series, they must meet the following four requirements:

(a) The data should reflect both the *short- and long-term* movements in the series, particularly the timing of any turning points.

(b) The data should allow different *periods* to be compared in a consistent manner. That is, based on the underlying time series, the data should allow measures of change to be derived between any period (i.e., from the previous period, the same period in the previous year, and a particular period several years earlier).

(c) The data should allow *periods of different duration* to be compared in a consistent manner. That is, based on the underlying time series, the data should allow measures of change to be derived between any periods of any length (e.g., between the average of the last two quarters and of the previous two quarters or the same two quarters several years earlier, from the average of the previous year and of a year several years earlier).

(d) The data should allow *subperiods and periods* to be compared in a consistent manner (e.g., quarters with years).

9.4. Consistency between QNA and ANA price and volume measures, in principle, requires either that the ANA measures are derived from quarterly measures or that consistency is forced on the QNA data using benchmarking techniques. This is true even if the basic requirement that the QNA and ANA measures are based on the same methods of compilation and presentation (i.e., same index formula, base year(s), and reference period) is met. Strict consistency between QNA and direct ANA price and volume measures is generally not possible because quarterly indices based on most index formulas, including Paasche and Fisher, do not aggregate exactly to their corresponding direct annual indices. For fixed-base Laspeyres volume indices, or traditional constant price estimates, consistency requires that the estimates are derived by explicitly or implicitly valuing the quantities at the annual quantity-weighted average of the prices charged in different time periods of the base year,[2] effectively implying that the annual volume data are derived from the quarterly data[3] (see Section B) and not directly. Finally, for annually chain-linked Laspeyres volume indices, strict consistency can only be achieved by use of an annual linking technique that can result in a break[4] in the estimates between the fourth quarter of one year and the first quarter of the next year (see Section D).

9.5. Consistency between QNA and ANA price and volume measures also requires that new methods, like chain-linking, are implemented simultaneously in both the QNA and ANA. Although the *1993 SNA* recommends moving to chain-linked volume measures, for countries currently compiling traditional constant price estimates, it would generally be undesirable to complicate the introduction of QNA by also introducing new techniques for constructing and presenting volume measures at the same time. It is recommended for these countries to introduce chaining in a second phase, concurrent with the introduction of chain-linking in the ANA. Thus, for countries currently compiling traditional constant price estimates, only the discussion in Section B of aggregating price and volume measures over time is of immediate importance.

B. Aggregating Price and Volume Measures Over Time

9.6. Aggregation over time means deriving less frequent data (e.g., annual) from more frequent data (e.g., quarterly). Incorrect aggregation of prices, or price indices, over time to derive annual deflators can introduce errors in independently compiled annual estimates and thus can cause inconsistency between QNA and ANA estimates, even when they are derived from the same underlying data. When deriving annual constant price estimates by deflating annual current price data, a common practice is to compute the annual price deflators as a simple unweighted average of monthly or quarterly price indices. This practice may introduce substantial errors in the derived annual constant price estimates, even when inflation is low. This may happen when

- there are seasonal or other within-year variations in prices or quantities, and
- the within-year pattern of variation in either prices or quantities is unstable.

9.7. Volume measures for aggregated periods of time should conceptually be constructed from period-total quantities for each individual homogenous product. The corresponding implicit price measures would be quantity-weighted period-average price measures. For example, annual volume measures for single homogenous products[5] should be constructed as sums of the quantities in each subperiod. The corresponding implicit annual average price, derived as the annual current price value divided by the annual quantity, would therefore be a quantity-weighted average of the prices in each quarter. As shown in Example 9.1, the quantity-weighted average price will generally differ, sometimes significantly, from the unweighted average price. Similarly, for groups of products, conceptually, annual volume measures can be constructed as a weighted aggregate of the annual quantities for each individual product. The corresponding implicit annual price deflator for the group would be a weighted aggregate of the quantity-weighted annual average prices for the individual products. This annual price deflator for the group based on the quantity-weighted annual average prices would generally differ, sometimes significantly, from the annual price deflators derived as a simple unweighted average of monthly or quarterly price indices often used in ANA systems—deflation by the latter may introduce substantial errors in the derived annual constant price estimates.

[2]The corresponding explicit or implicit annual deflators should be derived as current-year quantity-weighted averages of monthly or quarterly fixed-based Paasche price indices.
[3]This is particularly an issue under high inflation and for highly volatile items.
[4]This can occur if there are strong changes in relative quantities and relative prices.

[5]Homogenous products are identical in physical and economic terms to other items in that product group and over time. In contrast, when there are significant variations among items or over time in the physical or economic characteristic of the product group, each version should be treated as a separate product (e.g., out-of-season fruit and vegetables such as old potatoes may be regarded as different products than in-season fruit and vegetables such as new potatoes).

Example 9.1. Weighted and Unweighted Annual Averages of Prices (or Price Indices) When Sales and Price Patterns Through the Year are Uneven

	Quantity (1)	Price (2)	Current Price Value (3)	Unweighted Average Price (4)	Unit Value Weighted Average Price (5) = (3)/(1)	Constant Price Value: At Unweighted Average 1999 Prices (6) = (4)·(1)	Constant Price Value: At Weighted Average 1999 Prices (7) = (5)·(1)
q1	0	80	0			0	0
q2	150	50	7,500			7,500	6,750
q3	50	30	1,500			2,500	2,250
q4	0	40	0			0	0
1999	**200**		**9,000**	**50**	**45**	**10,000**	**9,000**
q1	0	40	0			0	0
q2	180	50	9,000			9,000	8,100
q3	20	30	600			1,000	900
q4	0	40	0			0	0
2000	**200**		**9,600**	**40**	**48**	**10,000**	**9,000**
% Change from 1999 to 2000	0.00%		6.70%	–20.00%	6.70%	0.00%	0.00%

Direct Deflation of Annual Current Price Data

2000 at 1999 prices	9600/(40/50) = 9600/0.8	= 12,000
% change from 1999	(12000/9000-1) · 100	= 33.3%

This example highlights the case of an unweighted annual average of prices (or price indices) being misleading when sales and price patterns through the year are uneven for a single homogenous product. The products sold in the different quarters are assumed to be identical in all economic aspects.

In the example, the annual quantities and the quarterly prices in quarters with nonzero sales are the same in both years, but the pattern of sales shifts toward the second quarter in 1998. As a result, the total annual current price value increases by 6.7 percent.

If the annual deflator is based on a simple average of quarterly prices then the deflator appears to have dropped by 20 percent. As a result, the annual constant price estimates will wrongly show an increase in volume of 33.3 percent.

Consistent with the quantity data, the annual sum of the quarterly constant price estimates for 1999 and 2000, derived by valuing the quantities using their quantity-weighted average 1999 price, shows no increase in volumes (column 7). The change in annual current price value shows up as an increase in the implicit annual deflator, which would be implicitly weighted by each quarter's proportion of annual sales at constant prices.

Price indices typically use unweighted averages as the price base, which corresponds to valuing the quantities using their unweighted average price. As shown in column 6, this results in an annual sum of the quarterly constant price estimates in the base year (1999) that differs from the current price data, which it should not. This difference, however, can easily be removed by a multiplicative adjustment of the complete constant price time series, leaving the period-to-period rate of change unchanged. The adjustment factor is the ratio between the annual current price data and sum of the quarterly constant price data in the base year (9000/10000).

9.8. Consequently, to obtain correct volume measures for aggregated periods of time, deflators should take into account variations in quantities as well as prices within the period. For example, annual deflators could be derived implicitly from annual volume measures derived from the sum of quarterly volume estimates obtained using the following two-step procedure:

(a) Benchmark the quarterly current price data/indicator(s) to the corresponding annual current price data.

(b) Construct quarterly constant price data by deflating the benchmarked quarterly current price data. Equivalently, the annual volume measure could be obtained by deflating using an annual deflator that weights the quarterly price indices by the constant price values of that item for each quarter.

Either way of calculation achieves annual deflators that are quantity-weighted average annual price measures.[6]

9.9. A more difficult case occurs when the annual estimates are based on more detailed price and value information than is available quarterly. In those cases, if seasonal volatility is significant, it would be possible to approximate the correct procedure using weights derived from more aggregated, but closely related, quarterly data.

9.10. The issue of price and quantity variations also apply within quarters. Accordingly, when monthly data are available, quarterly data will better take into

[6] The corresponding formulas are provided in Annex 9.1.

account variations within the period if they are built up from the monthly data.

9.11. In many cases, variation in prices and quantities within years and quarters will be so insignificant that it will not substantially affect the estimates. Primary products and high-inflation countries are cases where the variation can be particularly significant. Of course, there are many cases in which there are no data to measure variations within the period.

9.12. A related problem that can be observed in quarterly data is the annual sum of the quarterly constant price estimates in the base year differing from the annual sum of the current price data, which should not be the case. This difference can be caused by the use of unweighted annual average prices as the price base when constructing monthly and quarterly price indices. As shown in Annex 9.1, deflating quarterly data with deflators constructed with unweighted average prices as the price base corresponds to valuing the quantities using their unweighted annual average price rather than their weighted annual average price. This difference in the base year between the annual sum of the quarterly constant price estimates and the annual sum of the current price data can easily be removed by a multiplicative adjustment of the complete constant price time series, leaving the period-to-period rate of change unchanged. The adjustment factor is the ratio between the annual current price data and the sum of the initial quarterly constant price data based on the unweighted annual average prices in the base year, which, for a single product, is identical to the ratio of the weighted and unweighted average price.

9.13. Two different concepts and measures of annual change in prices are illustrated in Example 9.1, which both are valid measures of economic interest. The first—showing a decline in prices of 20 percent based on unweighted annual average prices—corresponds to a measure of the *average change in prices*. The second—showing an increase in prices of 6.7 percent based on weighted annual average prices—corresponds to *change in average prices*. As shown in Example 9.1, only the latter fits in a value/volume/price measurement framework for *time periods*, as required by the national accounts, in contrast to the measurement framework for *points in time* addressed in conventional index number theory. In Example 9.1, the annual value change is 6.7 percent, and the correct annual volume change is an undisputable 0.0 percent, because the annual sum of the quantities is unchanged and the quantities refer to a single homogenous product.

9.14. An apparent difficulty is that the changes shown by the weighted annual average price measure fail the fundamental index number axiom that the measures should reflect only changes in prices and not changes in quantities. Thus, the weighted annual average price measure appears to be invalid as a measure of price change. The 6.7 increase in *average prices* from 1997 to 1998 results from changes in the quantities transacted at each price and not from increases in the prices, and therefore does not satisfy basic index number tests such as the identity and proportionality tests. For that reason, it can be argued that Example 9.1 shows that, in principle, it is not possible to factor changes in values for time periods into measures of price and quantity changes that are each acceptable as index numbers in their own right. The basic index number tests and conventional index number theory, however, are concerned with price and quantity comparisons between separate pairs of *points in time* rather than with price and quantity comparisons between *time periods* and, consequently, not with measures of the change in average prices from one period to another. To measure the *change in average prices*, for a single homogenous product, each period's average price should be defined as the total value divided by the corresponding quantities within that period; that is, they should be unit values. From Example 9.1, it is clear that annual average prices for national accounting purposes cannot be realistically defined without reference to the corresponding quantities and therefore should be calculated using a weighted average with quarterly/subannual quantities as weights.

C. Choice of Price Weights for QNA Volume Measures

1. Laspeyres-Type Volume Measures

9.15. The time-series requirements and the QNA-ANA consistency requirement imply that the quantity-weighted average prices of a whole year should be used as price weights for ANA and QNA Laspeyres-type volume measures.[7] Use of the prices of one

[7]The term "Laspeyres-type" is used to cover the traditional constant price measures, fixed-base Laspeyres volume indices, and chain-linked Laspeyres volume indices.

particular quarter, the prices of the corresponding quarter of the previous year, the prices of the corresponding quarter of a fixed "base year," or the prices of the previous quarter are not appropriate for time series of Laspeyres-type volume measures in the national accounts for the following reasons:

- Consistency between directly derived ANA and QNA Laspeyres-type volume measures requires that the same price weights are used in the ANA and the QNA, and that the same price weights are used for all quarters of the year.
- The prices of one particular quarter are not suitable as price weights for volume measures in the ANA, and thus in the QNA, because of seasonal fluctuations and other short-term volatilities in relative prices. Use of weighted annual average prices reduces these effects. Therefore, weighted annual average prices are more representative for the other quarters of the year as well as for the year as a whole.
- The prices of the corresponding quarter of the previous year or the corresponding quarter of a fixed "base year" are not suitable as price weights for volume measures in the QNA because the derived volume measures only allow the current quarter to be compared with the same quarter of the previous year or years. Series of year-to-year changes do not constitute time series that allow different periods to be compared and cannot be linked together to form such time series. In particular, because they involve using different prices for each quarter of the year, they do not allow different quarters within the same year to be compared. For the same reason, they do not allow the quarters within the same year to be aggregated and compared with their corresponding direct annual estimates. Furthermore, as shown in Annex 1.1, changes from the same period in the previous year can introduce significant lags in identifying the current trend in economic activity.
- The prices of the previous quarter are not suitable as price weights for Laspeyres-type volume measures for two reasons:
  - (a) The use of different price weights for each quarter of the year does not allow the quarters within the same year to be aggregated and compared with their corresponding direct annual estimates.
  - (b) If the quarter-to-quarter changes are linked together to form a time series, short-term volatility in relative prices may cause the quarterly chain-linked measures to show substantial drift compared to corresponding direct measures. This is illustrated in Example 9.3.

9.16. Quarterly Laspeyres-type volume measures with two different base-period[8] price weights may be used:

(a) The annual average of a fixed-base year, resulting in the traditional constant price measures, which is equivalent to a fixed-based Laspeyres volume index.

(b) The annual average of the previous year, resulting in the annually chain-linked quarterly Laspeyres volume index.

9.17. The traditional volume measures at the constant price of a fixed base year, the fixed-based quarterly Laspeyres volume index, and the short-term link in the annually chain-linked quarterly Laspeyres volume index can be expressed in mathematical terms as the following:

- At the constant "average" prices of a fixed base year:

$$CP_{q,y_{\bar{0}}} = \sum_i \bar{p}_{i,0} \cdot q_{i,q,y} \qquad (9.1.a)$$

- The fixed-based quarterly Laspeyres:

$$LQ_{0\rightarrow(q,y)} = \frac{\sum_i \bar{p}_{i,0} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \qquad (9.1.b)$$

- Short-term link in the annually chain-linked quarterly Laspeyres:

$$LQ_{\overline{(y-1)}\rightarrow(q,y)} = \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}} \qquad (9.1.c)$$

where

$CP_{q,y\bar{0}}$ is the total value in quarter q of year y measured at the annual average prices of year 0.

$LQ_{0\rightarrow(q,y)}$ represents a Laspeyres volume index measuring the volume change from the average of year 0 to quarter q in year y with average of year 0 as base and reference period;[9]

$LQ_{\overline{(y-1)}\rightarrow(q,y)}$ represents a Laspeyres volume index measuring the volume change from the average of year $y-1$ to quarter q in year

[8] The term "base period" is defined in paragraph 9.22 as meaning (1) the base of the price or quantity ratios being weighted together (e.g., period 0 is the base for the quantity ratio), and (2) the pricing year (the base year) for constant price data.

[9] The term "Reference period" is defined in paragraph 9.22 as meaning the period for which the index series is expressed as equal to 100.

y with the average of year $y - 1$ as base and reference period;

$p_{i,q,y}$ is the price of item i in quarter q of year y;

$\bar{p}_{i,y-1}$ is the quantity-weighted arithmetic average of the price of item i in the quarters of year $y - 1$;

$\bar{p}_{i,0}$ is the quantity-weighted arithmetic average of the price of item i in the quarters of year 0

$$\bar{p}_{i,0} = \frac{\sum_q p_{i,q,0} \cdot q_{i,q,0}}{\sum_q q_{i,q,0}};$$

$q_{i,q,y}$ is the quantity of item i in quarter q of year y;

$\bar{q}_{i,y-1}$ is the simple arithmetic average of the quantities of item i in the quarters of $y - 1$; and

$\bar{q}_{i,0}$ is the simple arithmetic average of the quantities of item i in the quarters of year 0.

2. Fisher-Type Volume Indices

9.18. The Fisher volume index, being the geometric average of a Laspeyres and a Paasche volume index, uses price weights from two periods—the base period and the current period. Quarterly Fisher indices with three different base-period weights may be used:

(a) The annual average of a fixed-base year, resulting in the fixed-based quarterly Fisher index.

(b) The annual average of the previous year, resulting in the annually chain-linked quarterly Fisher index.

(c) The average of the previous quarter, resulting in the quarterly chain-linked quarterly Fisher index.

9.19. The fixed-based quarterly Fisher volume index and the short-term links in the annually and quarterly chain-linked quarterly Fisher volume index can be expressed in mathematical terms as the following:

- Fixed-based quarterly Fisher:

$$FQ_{0 \to (q,y)} = \sqrt{LQ_{0 \to (q,y)} \cdot PQ_{0 \to (q,y)}} \qquad (9.2.a)$$

$$\equiv \sqrt{\frac{\sum_i \bar{p}_{i,0} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \cdot \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i p_{i,q,y} \cdot \bar{q}_{i,0}}}$$

- Short-term link in the annually chain-linked quarterly Fisher:

$$FQ_{\overline{(y-1)} \to (q,y)} = \sqrt{LQ_{\overline{(y-1)} \to (q,y)} \cdot PQ_{\overline{(y-1)} \to (q,y)}} \qquad (9.2.b)$$

$$\equiv \sqrt{\frac{\sum_i \bar{P}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{P}_{i,y-1} \cdot \bar{q}_{i,y-1}} \cdot \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i p_{i,q,y} \cdot \bar{q}_{i,y-1}}}$$

- Short-term link in the quarterly chain-linked quarterly Fisher:

$$FQ_{(t-1) \to 1} = \sqrt{LQ_{(t-1) \to (t)} \cdot PQ_{(t-1) \to (t)}} \qquad (9.2.c)$$

$$\equiv \sqrt{\frac{\sum_i p_{i,t-1} \cdot q_{i,t}}{\sum_i p_{i,t-1} \cdot q_{i,t-1}} \cdot \frac{\sum_i p_{i,t} \cdot q_{i,t}}{\sum_i p_{i,t} \cdot q_{i,t-1}}}$$

where

t is a generic symbol for time, which is more convenient to use for period-to period measures than the quarter q in year y notation used for most formulas in this chapter;

$FQ_{A \to (q,y)}$ represents a Fisher volume index measuring the volume change from period A to quarter q in year y with period A as base and reference period;

$LQ_{A \to (q,y)}$ represents a Laspeyres volume index measuring the volume change from period A to quarter q in year y with period A as base and reference period;

$PQ_{A \to (q,y)}$ represents a Paasche volume index measuring the volume change from period A to quarter q in year y with period A as base and reference period; and

$p_{i,A}$ is the price of item i in period A.

Period A is equal to the average of year 0 for the fixed-based Fisher, to the average of the previous year for the annually chain-linked Fisher, and to the previous quarter for the quarterly chain-linked Fisher.

9.20. For the same reasons as for Laspeyres-type volume measures, the following alternative periods are not suitable as base periods for time series of Fisher-type volume indices:

- One particular fixed quarter.
- The corresponding quarter of the previous year.
- The corresponding quarter of a fixed “base year.”

D. Chain-Linking in the QNA

1. General

9.21. The *1993 SNA* recommends moving away from the traditional fixed-base year constant price estimates to chain-linked volume measures. Constant price estimates use the average prices of a particular period,[10] the base period, to weight together the corresponding quantities. Constant price data have the advantage for the users of the component series being additive, unlike alternative volume measures. The pattern of relative prices in the base year, however, is less representative of the economic conditions for periods farther away from the base year. Therefore, from time to time it is necessary to update the base period to adopt weights that better reflect the current conditions (i.e., with respect to production technology and user preferences). Different base periods, and thus different sets of price weights, give different perspectives. When the base period is changed, data for the distant past should not be recalculated (rebased). Instead, to form a consistent time series, data on the old base should be linked to data on the new base.[11] Change of base period and chain-linking can be done with different frequencies; every 10 years, every 5 years, every year, or every quarter/month. The *1993 SNA* recommends changing the base period, and thus conducting the chain-linking, annually.

9.22. The concepts of base, weight, and reference period should be clearly distinguished. Index number terminology is not well established internationally, which can lead to confusion. In particular, the term "base period" is sometimes used for different concepts. Similarly, the terms "base period," "weight period," and "reference period" are sometimes used interchangeably. In this manual, following *1993 SNA* and the current dominant national accounts practice, the following terminology is used:

- *Base period* for (1) the base of the price or quantity ratios being weighted together (e.g., period 0 is the base for the quantity ratio $q_{i,t}/q_{i,0}$), and (2) the pricing year (the base year) for the constant price data.
- *Weight period* for the period(s) from which the weights are taken. The weight period is equal to the base period for a fixed-base Laspeyres index and to the current period for a fixed-base Paasche index. Symmetric fixed-base index formulas like Fisher and Tornquist have two weight periods—the base and the current period.
- *Reference period* for the period for which the index series is expressed as equal to 100. The reference period can be changed by simply dividing the index series with its level in any period chosen as the new reference period.

9.23. Chain-linking means constructing long-run price or volume measures by cumulating movements in short-term indices with different base periods. For example, a period-to-period chain-linked index measuring the changes from period 0 to t (i.e., $CI_{0\to t}$) can be constructed by multiplying a series of short-term indices measuring the change from one period to the next as follows:

$$CI_{0\to t} = I_{0\to 1}\cdot I_{1\to 2}\cdot I_{2\to 3}\cdot I_{3\to 4}\cdot ....I_{(t-1)\to t} \equiv \prod_{\tau=1}^{t} I_{(\tau-1)\to\tau} \qquad (9.3)$$

where $I_{(t-1)\to\tau}$ represents a price or volume index measuring the change from period $t-1$ to t, with period $t-1$ as base and reference period.

9.24. The corresponding run, or time series, of chain-linked index numbers where the links are chained together so as to express the full time series on a fixed reference period is given by

$$\begin{cases} CI_{0\to 0} = 1 \\ CI_{0\to 1} = I_{0\to 1} \\ CI_{0\to 2} = I_{0\to 1}\cdot I_{1\to 2} \\ CI_{0\to 3} = I_{0\to 1}\cdot I_{1\to 2}\cdot I_{2\to 3} \\ \quad\cdots \\ CI_{0\to t} = \prod_{\tau=1}^{t} I_{(\tau-1)\to\tau} \end{cases} \qquad (9.4.a)$$

9.25. Chain-linked indices do not have a *particular base or weight period.* Each link ($I_{(t-1)\to t}$) of the chain-linked index in equation (9.4.a) has a base period and one or two weight periods, and the base and weight period(s) are changing from link to link. By the same token, the full run of index numbers in

[10]The period length should be a year, as recommended in the previous section.

[11]This should be done for each series, aggregates as well as subcomponents of the aggregates, independently of any aggregation or accounting relationship between the series. As a consequence, the chain-linked components will not aggregate to the corresponding aggregates. No attempts should be made to remove this "chain discrepancy," because any such attempt implies distorting the movements in one or several of the series.

equation (9.4.a) derived by chaining each link together does not have a particular base period—it has a fixed reference period.

9.26. The *reference period* can be chosen freely without altering the rates of change in the series. For the chain-linked index time series in equation (9.4.a), period 0 is referred to as the index's reference period and is conventionally expressed as equal to 100. The reference period can be changed simply by dividing the index series with its level in any period chosen as a new reference period. For instance, the reference period for the run of index numbers in equation (9.4.a) can be changed from period 0 to period 2 by dividing all elements of the run by the constant $CI_{0\to2}$ as follows:

$$\begin{cases} CI_{2\to0} = CI_{0\to0}/CI_{0\to2} = 1/I_{0\to1}I_{1\to2} \\ CI_{2\to1} = CI_{0\to1}/CI_{0\to2} = 1/I_{1\to2} \\ CI_{2\to2} = CI_{0\to2}/CI_{0\to2} = 1 \\ CI_{2\to3} = CI_{0\to3}/CI_{0\to2} = I_{1\to2} \\ \quad \ldots \\ CI_{2\to t} = CI_{0\to t}/CI_{0\to2} = \prod_{\tau=1}^{t} I_{(\tau-1)\to\tau} \end{cases} \tag{9.4.b}$$

9.27. The chain-linked index series in equation (9.3) and equations (9.4.a) and (9.4.b) will constitute a period-to-period chain-linked Laspeyres volume index series if, for each link, the short-term indices ($I_{(t-1)\to t}$) are constructed as Laspeyres volume indices with the previous period as base and reference period. That is, if

$$I_{(t-1)\to t} = LQ_{(t-1)\to t} = \sum_i \frac{q_{i,t}}{q_{i,t-1}} \cdot w_{i,t-1} \tag{9.5.}$$

$$\equiv \frac{\sum_i p_{i,t-1} \cdot q_{i,t}}{\sum_i p_{i,t-1} \cdot q_{i,t-1}} \equiv \frac{\sum_i p_{i,t-1} \cdot q_{i,t}}{V_{t-1}}$$

where

$LQ_{(t-1)\to t}$ represents a Laspeyres volume index measuring the volume change from period $t-1$ to t, with period $t-1$ as base and reference period;

$p_{i,t-1}$ is the price of item i in period $t-1$ (the "price weights");

$q_{i,t}$ is the quantity of item i in period t;

$w_{i,t-1}$ is the base period "share weight," that is, the item's share in the total value of period $t-1$; and

V_{t-1} is the total value at current prices in period $t-1$.

9.28. Similarly, the chain-linked index series in equation (9.3) and equations (9.4.a) and (9.4.b) will constitute a period-to-period chain-linked Fisher volume index series if, for each link, the short-term indices ($I_{(t-1)\to t}$) are constructed as Fisher volume indices with the previous period as base and reference period as in equation (9.2.c).

9.29. Any two index series with different base and reference periods can be linked to measure the change from the first to the last year[12] as follows:

$$CI_{0\to t} = I_{0\to(t-h)} \cdot I_{(t-h)\to t} \tag{9.6}$$

That is, each link may cover any number of periods.

9.30. For instance, if in equation (9.6) t = 10 and h = 5, the resulting linked index ($CI_{0\to10}$) constitutes a 5-year chain-linked annual index measuring the change from year 0 to year 10. Example 9.2 provides an illustration of the basic chain-linking technique for annual data with t = 15 and h = 10.

9.31. Growth rates and index numbers computed for series that contain negatives or zeroes—such as changes in inventories and crop harvest data—generally are misleading and meaningless. For instance, consider a series for changes in inventories at constant prices that is –10 in period one and +20 in period two. The corresponding growth rate between these two periods is –300 percent (= ((20/–10) – 1) · 100), which obviously is both misleading and meaningless. Similarly, for a series that is 1 in period one and 10 in period two, the corresponding growth rate from period one to two would be 900 percent. Consequently, for such series, only measures of contribution to percentage change in the aggregates they belong to can be made (see Section D.7. for a discussion of measure of contribution to percentage change in index numbers).

2. Frequency of Chain-Linking in the QNA

9.32. The *1993 SNA* recommends that chain-linking should not be done more frequently than annually. This is mainly because short-term volatility in relative prices (e.g., caused by sampling errors and seasonal effects) can cause volume measures that are chain-linked more frequently than annually to show substantial drift—particularly so

[12]As long as they have one period in common, that is, there is at least one overlapping period. For instance, in equation (9.6) with t = 10 and h = 5, year 5 represents the overlap. Similarly, in Example 9.2, year 10 represents the overlap.

Example 9.2. Basic Chain-Linking of Annual Data

The *1993 SNA* Example

The example is an elaborated version of the illustration provided in the *1993 SNA*.
(*1993 SNA* Table 16.1, pages 386–387.)

Basic Data

	Year 0			Year 10			Year 15		
	p_0	q_0	v_0	p_{10}	q_{10}	v_{10}	p_{15}	q_{15}	v_{15}
Item A	6	5	30	9	12	108	11	15	165
Item B	4	8	32	10	11	110	14	11	154
Total			62			218			319

Constant price Data

	Base Year 0			Base Year 10		
	Year 0	Year 10	Year 15	Year 0	Year 10	Year 15
	$p_0 \cdot q_0$	$p_0 \cdot q_{10}$	$p_0 \cdot q_{15}$	$p_{10} \cdot q_0$	$p_{10} \cdot q_{10}$	$p_{10} \cdot q_{15}$
Item A	30	72	90	45	108	135
Item B	32	44	44	80	110	110
Total	62	116	134	125	218	245

Laspeyres Volume Indices for the Total

Fixed-Based	Year 0	Year 10	Year 15
Year 0 as base and reference	100	187.1	216.1
Period-to-period rate of change		87.1%	15.5%
Year 10 as base and reference	57.3	100	112.4
Period-to-period rate of change		74.4%	12.4%
Re-referenced to year 0 (year 10 as base)	100	174.4	196.0
Chain-Linked Index			
Year 0 =100	100	187.1	210.3= 112.4 • 1.871
Period-to-period rate of change		87.1%	12.4%
Year 10 = 100	100/1.871 = 53.4	100	112.4
Period-to-period rate of change		87.1%	12.4%

The Laspeyres fixed-base volume index for the total with year 0 as base and reference period was derived as

$62/62 \cdot 100 = 100$, $116/62 \cdot 100 = 187.1$, $134/62 \cdot 100 = 216.1$

Similarly, the Laspeyres fixed-base volume index for the total year with 10 as base and reference period was derived as

$125/218 \cdot 100 = 57.3$, $218/218 \cdot 100 = 100$, $245/218 \cdot 100 = 112.4$

And the Laspeyres fixed-base volume index for the total with year 10 as base and year 0 as reference period was derived as

$57.3/57.3 \cdot 100 = 100$, $100/57.3 \cdot 100 = 174.4$, $112.4/57.3 \cdot 100 = 196.0$

for nonsuperlative index formulas like Laspeyres and Paasche—as illustrated in Example 9.3. Similarly, short-term volatility in relative quantities can cause price measures that are chain-linked more frequently than annually to show substantial drift. The purpose of chain-linking is to take into account long-term trends in changes in relative prices, not temporary short-term variations.

9.33. Superlative index formulas, such as the Fisher index formula, are more robust against the drift problem than the other index formulas—as illustrated in Example 9.3. For this reason, a quarterly chain-linked Fisher index may be a feasible alternative to annually chain-linked Fisher or Laspeyres indices for quarterly data that show little or no short-term volatility. The quarterly chain-linked Fisher index does not aggregate exactly to the corresponding direct annual Fisher index.[13] For chain-linked Fisher indices, consistency between QNA and ANA price and volume measures can only be achieved by deriving the ANA measures from the quarterly measures or by forcing consistency on the data with the help of benchmarking techniques. There is no reason to believe that for nonvolatile series the average of an annually chain-linked Fisher will be closer to a direct annual Fisher index than the average of a quarterly chain-linked Fisher.

[13]Neither does the annually-linked, nor the fixed-based, Fisher index.

Example 9.3. Frequency of Chain-Linking and the Problem of "Drift"[1] in the Case of Price and Quantity Oscillation

Observation/Quarter	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Price item A (pA)	2	3	4	2
Price item B (pB)	5	4	2	5
Quantities item A (qA,t)	50	40	60	50
Quantities item B (qB,t)	60	70	30	60
Total value (Vt)	400	400	300	400
Volume Indices	**q1**	**q2**	**q3**	**q4**
Fixed-based Laspeyres (q1-based)	100.0	107.5	67.5	100.0
Fixed-based Paasche (q1-based)	100.0	102.6	93.8	100.0
Fixed-based Fisher (q1-based)	100.0	105.0	79.6	100.0
Quarterly chain-linked Laspeyres	100.0	107.5	80.6	86.0
Quarterly chain-linked Paasche	100.0	102.6	102.6	151.9
Quarterly chain-linked Fisher	100.0	105.0	90.9	114.3

Fixed-Based Laspeyres Index:

$$I_{0\to t} = \frac{\sum_i p_{i,0} \cdot q_{i,t}}{\sum_i p_{i,0} \cdot q_{i,0}} \equiv \frac{\sum_i p_{i,0} \cdot q_{i,t}}{V_0}$$

$$I_{t\to 2} = [2 \cdot 40+5 \cdot 70]/400] \cdot 100 = 107.5$$
$$I_{t\to 2} = [2 \cdot 60+5 \cdot 30]/400] \cdot 100 = 67.5$$
$$I_{t\to 4} = [2 \cdot 50+5 \cdot 60]/400] \cdot 100 = 100.0$$

Fixed-Based Paasche Index:

$$I_{0\to t} = \frac{\sum_i p_{i,t} \cdot q_{i,t}}{\sum_i p_{i,t} \cdot q_{i,0}} \equiv \frac{V_t}{\sum_i p_{i,t} \cdot q_{i,0}}$$

$$I_{t\to 2} = [400/(3 \cdot 50+4 \cdot 60] \cdot 100 = 102.6$$
$$I_{t\to 3} = [300/(4 \cdot 50+2 \cdot 60] \cdot 100 = 93.8$$
$$I_{t\to 4} = [400/(2 \cdot 50+5 \cdot 60] \cdot 100 = 100.0$$

Quarterly Chain-Linked Laspeyres Index:

$$CL_{0,t} = \prod_{\tau=1}^{t} I_{(\tau-1)\to\tau} = \prod_{\tau=1}^{t} \frac{\sum_i p_{i,\tau-1} \cdot q_{i,\tau}}{\sum_i p_{i,\tau-1} \cdot q_{i,\tau-1}}$$

$$I_{t\to 3} = I_{t\to 2} \cdot [3 \cdot 60+4 \cdot 30]/400] = 80.6$$
$$I_{t\to 4} = I_{t\to 3} \cdot [4 \cdot 50+2 \cdot 60]/400] = 86.0$$

Quarterly Chain-Linked Paasche Index:

$$CL_{0,t} = \prod_{\tau=1}^{t} I_{(\tau-1)\to\tau} = \prod_{\tau=1}^{t} \frac{\sum_i p_{i,\tau-1} \cdot q_{i,\tau}}{\sum_i p_{i,\tau-1} \cdot q_{i,\tau-1}}$$

$$I_{t\to 3} = I_{t\to 2} \cdot [300/(4 \cdot 40+2 \cdot 70] = 102.6$$
$$I_{t\to 2} = I_{t\to 3} \cdot [400/(2 \cdot 60+5 \cdot 30] = 151.9$$

In this example, the prices and quantities in quarter 1 are the same as those in quarter 1, that is, the prices and quantities oscillate rather than move as a trend. The fixed-base indices correspondingly show identical values for q1 and q4, but the chain-linked indices show completely different values. This problem can also occur in annual data if prices and quantities oscillate and may make annual chaining inappropriate in some cases. It is more likely to occur in data for shorter periods, however, because seasonal and irregular effects cause those data to be more volatile.

Furthermore, observe that the differences between the q1 and q4 data for the quarterly chain-linked Laspeyres and the quarterly chain-linked Paasche indices are in opposite directions; and, correspondingly, that the quarterly chain-linked Fisher index drifts less. This is a universal result.

[1]The example is based on Szultc (1983).

9.34. For Laspeyres-type volume measures, consistency between QNA and ANA provides an additional reason for not chain-linking more frequently than annually. Consistency between quarterly data and corresponding direct annual indices requires that the same price weights are used in the ANA and the QNA, and consequently that the QNA should follow the same change of base year/chain-linking practice as in the ANA. Under those circumstances, the annual overlap linking technique presented in the next section will ensure that the quarterly data aggregate exactly to the corresponding direct index. Moreover, under the same circumstances, any difference between the average of the quarterly data and the direct annual index caused by the preferred one-quarter overlap technique will be minimized.

9.35. Thus, in the QNA, chain-linked Laspeyres-type volume measures should be derived by compiling quarterly estimates at the average prices of the previous year. These quarterly volume measures for each year should then be linked to form long, consistent time series—the result constitutes an annually chain-linked quarterly Laspeyres index. Alternative linking techniques for such series are discussed in the next section.

3. Choice of Index Number Formulas for Annually Chain-Linked QNA Data

9.36. The *1993 SNA* recommends compiling annually chain-linked price and volume measures, preferably using superlative index number formulas such as the Fisher and Tornquist formulas. The rationale for this recommendation is that index number theory shows that annually chain-linked Fisher and Tornquist indices will most closely approximate the theoretically ideal index. Fisher and Tornquist indices will, in practice, yield almost the same results, and Fisher, being the geometric average of a Laspeyres and a Paasche index, will be within the upper and lower bounds provided by those two index formulas. Most countries[14] that have implemented chain-linking in their national accounts, however, have adopted the annually chain-linked Laspeyres formula for volume measures with the corresponding annually chain-linked Paasche formula for price measures,[15] and the European Union's statistical office (Eurostat) is requiring member states to provide annually chain-linked volume measures using the Laspeyres formula.[16]

9.37. Annual chain-linking of quarterly data implies that each link in the chain is constructed using the chosen index number formula with the average of the previous year ($y-1$) as base and reference period. The resulting short-term quarterly indices must subsequently be linked to form long, consistent time series expressed on a fixed reference period. Alternative annual linking techniques for such series will be discussed in Section D.3. While the discussion in Section D.3 focuses on Laspeyres indices, the techniques illustrated and the issues discussed are applicable to all annually chain-linked index formulas. The Laspeyres, Paasche, and Fisher annually chain-linked quarterly volume index formulas for each short-term link in the chain are given as

- Short-term link in annually chain-linked Laspeyres:

$$LQ_{\overline{(y-1)}\rightarrow(q,y)} = \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}} \equiv \sum_i \frac{q_{i,q,y}}{\bar{q}_{i,y-1}} \cdot w_{i,y-1} \quad (9.7.a)$$

- Short-term link in annually chain-linked Paasche:

$$PQ_{\overline{(y-1)}\rightarrow(q,y)} = \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i p_{i,q,y} \cdot \bar{q}_{i,y-1}} \quad (9.7.b)$$

- Short-term link in annually chain-linked Fisher:

$$FQ_{\overline{(y-1)}\rightarrow(q,y)} = \sqrt{LQ_{\overline{(y-1)}\rightarrow(q,y)} \cdot PQ_{\overline{(y-1)}\rightarrow(q,y)}} = \sqrt{\frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}} \cdot \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i p_{i,q,y} \cdot \bar{q}_{i,y-1}}} \quad (9.7.c)$$

where

$$w_{i,y-1} = \frac{\bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}} \equiv \frac{\sum_q p_{i,q,y-1} \cdot q_{i,q,y-1}}{\sum_i \sum_q p_{i,q,y-1} \cdot q_{i,q,y-1}}$$

is the base period "share weight," that is, the item's share in the total value in year $y-1$; and

$p_{i,q,y-1}$ is the price of item i in quarter q of year $y-1$.

9.38. Countries have opted for the annually chain-linked Laspeyres formula instead of the annually chain-linked Fisher formula for volume measures mainly for the following reasons:

- Experience and theoretical studies indicate that annual chain-linking tends to reduce the index number spread to the degree that the exact choice of index number formula assumes less significance (see, for example, *1993 SNA*, paragraph 16.51).
- The annually chain-linked quarterly Fisher index does not aggregate to the corresponding direct

[14]The use of chain-linked measures for official national accounts data was pioneered by the Netherlands (1985) and Norway (1990). Subsequently, a large number of countries have adopted, or are in the process of adopting, chain-linking for their official measures. Currently, only the United States has opted for a chain-linked Fisher index formula instead of the chain-linked Laspeyres formula. The United States adopted in 1996 an annually chain-linked quarterly "Fisher-like" formula using annual weights in both the Laspeyres and the Paasche part of the index but changed to a standard quarterly chain-linked Fisher index in 1999.

[15]Laspeyres volume measures require that the corresponding price measures are based on the Paasche formula so that the product of the volume and price indices is equal to the corresponding value index.

[16]European Commission Decision of November 30, 1998, clarifying the *European System of Accounts 1995* principles for price and volume measures, and Eurostat (1999) paragraph 3.186.

annual index;[17] the annually chain-linked Laspeyres index linked, using the annually overlap technique presented in Example 9.4.a, does.[18]

- Chain volume measures in monetary terms[19] based on the annually chain-linked Laspeyres formula will be additive in the reference year and the subsequent year,[20] while volume measures based on the Fisher index will not.
- The Laspeyres formula is simpler to work with and to explain to users than the Fisher index. For instance, time series of annually chain-linked Laspeyres indices easily can be converted into series of data valued at the constant average prices of the previous year that are additive if corresponding current price data are made available. This feature makes it easy for users to construct their own aggregates from published data.
- The formulas for computing contribution to percentage change are easier for data based on the chain-linked Laspeyres formula than for data based on the Fisher index.
- The Fisher formula is not consistent in aggregation within each link; it is only approximately consistent in aggregation.
- The Laspeyres formula, in contrast, is additive within each link. This makes it easier to combine chain-linking with compilation analytical tools like supply and use (SU) tables and input-output tables that require additivity of components.[21]

4. Techniques for Annual Chain-Linking of Quarterly Data

9.39. Two alternative techniques for annual chain-linking of quarterly data are usually applied: annual overlaps and one-quarter overlaps. In addition to these two conventional chain-linking techniques, a third technique sometimes is used based on changes from the same period in the previous year (the "over-the-year technique"). While, in many cases, all three techniques give similar results, in situations with strong changes in relative quantities and relative prices, the over-the-year technique can result in distorted seasonal patterns in the linked series. While standard price statistics compilation exclusively uses the one-quarter overlap technique, the annual overlap technique may be more practical for Laspeyres-type volume measures in the national accounts because it results in data that aggregate exactly to the corresponding direct annual index. In contrast, the one-quarter overlap technique and the over-the-year technique do not result in data that aggregate exactly to the corresponding direct annual index. The one-quarter overlap provides the smoothest transition between each link, however, in contrast to the annual overlap technique that may introduce a step between each link. Examples 9.4.a, 9.4.b, 9.4.c, and Chart 9.1 provide an illustration of these three chain-linking techniques. (A formal presentation of the two first methods is given in Annex 9.2.)

9.40. The technique of using annual overlaps implies compiling estimates for each quarter at the weighted annual average prices of the previous year, with subsequent linking using the corresponding annual data to provide linking factors to scale the quarterly data upward or downward. The technique of one-quarter overlaps requires compiling estimates for the overlap quarter at the weighted annual average prices of the current year in addition to estimates at the average prices of the previous year. The ratio between the estimates for the linking quarter at the average prices of the current year and at the average prices of the previous year then provides the linking factor to scale the quarterly data up or down. The over-the-year technique requires compiling estimates for each quarter at the weighted annual average prices of the current year in addition to estimates at the average prices of the previous year. The year-on-year changes in these constant price data are then used to extrapolate the quarterly constant price data of the chosen reference period.

9.41. To conclude, there are no established standards with respect to techniques for annually chain-linking of QNA data, but chain-linking using the one-quarter overlap technique, combined with benchmarking to remove any resulting discrepancies between the quarterly and annual data, gives the best result. In many circumstances, however, the annual overlap technique may give similar results. The over-the-year technique should be avoided.

[17] Neither does the quarterly-chain linked, nor the fixed-based, quarterly Fisher index.

[18] However, this may not be a decisive argument for two reasons. First, simulations indicate that, in practice, the difference between a direct annual Fisher and the average of a quarterly Fisher may often not be significant and may easily be removed using benchmarking techniques. Second, the preferred quarterly overlap technique presented in Section D.3., even when used for Laspeyres indices, also introduces differences between direct annual indices and the average of quarterly indices.

[19] See Section D.7. and particularly paragraph 9.48 for a discussion of chain volume measures in monetary terms.

[20] See Example 9.5.a for an illustration of this and Section D.5. for a discussion of the nonadditivity property of most index number formulas besides the fixed-based Laspeyres formula.

[21] The first two countries to adopt chain-linking for their official national accounts price and volume measures both did it within an SU compilation framework.

Example 9.4.a. Quarterly Data and Annual Chain-Linking
Annual Overlap
Laspeyres Volume Index

Annual sums and averages in bold.

						At Constant Prices of: 1997		1998		1999		Chain-Linked Index 1997=100	
Basic data	Quantities A	Quantities B	Price A	Price B	Total at current prices	Level	Index 1997 =100	Level	Index 1998 = 100	Level	Index 1999 =100	Level	q-q Rate of Change
1997	**251.0**	**236.0**	**7.0**	**6.0**	**3,173.00**	3,173.00	100.00					100.00	
q1	67.4	57.6	6.1	8.0	871.94	817.40	103.04					103.04	3.0%
q2	69.4	57.1	5.7	8.6	885.51	828.40	104.43					104.43	1.3%
q3	71.5	56.5	5.3	9.4	910.05	839.50	105.83					105.83	1.3%
q4	73.7	55.8	5.0	10.0	926.50	850.70	107.24					107.24	1.3%
1998	**282.0**	**227.0**	**5.5**	**9.0**	**3,594.00**	**3,336.00**	**105.14**	**3,594.00**	100.00			**105.14**	
q1	76.0	55.4	4.5	10.7	934.78			916.60	102.01			107.26	0.0%
q2	78.3	54.8	4.3	11.5	963.07			923.85	102.82			108.10	0.8%
q3	80.6	54.2	3.8	11.7	940.42			931.10	103.63			108.95	0.8%
q4	83.1	53.6	3.5	12.1	940.73			939.45	104.56			109.93	0.9%
1999	**318.0**	**218.0**	**4.0**	**11.5**	**3,779.00**			**3,711.00**	**103.26**	**3,779.00**	100.00	**108.56**	
q1	85.5	53.2	3.4	12.5	955.70					953.80	100.96	109.60	–0.3%
q2	88.2	52.7	3.1	13.0	961.70					958.85	101.49	110.18	0.5%
q3	90.8	52.1	2.8	13.8	973.22					962.35	101.86	110.58	0.4%
q4	93.5	52.0	2.7	14.7	1018.36					972.00	102.88	111.69	1.0%
2000	**358.0**	**210.0**	**3.0**	**13.5**	**3,908.97**					**3,847.00**	**101.80**	**110.51**	–1.1%
Independently chain-linked annuals													
1997						3,173.0						**100.00**	
1998						3,336.0	105.1	3,594.0				**105.14**	
1999								3,711.0	103.3	3,779.0		**108.56**	
2000										3,847.0	101.8	**110.51**	

Step 1: Compile estimates for each quarter at the annual average prices of the previous year; the annual data being the sum of the four quarters.

e.g.:
- q1 1998 7.0 · 67.4+6.0 · 57.6 = 817.00
- q4 1998 7.0 · 73.7+6.0 · 55.8 = 850.70
- 1998 817.0 + 828.4 + 839.5 + 850.7 = 3336.00

Step 2: Convert the constant price estimates for each quarter into a volume index with the average of last year = 100.

e.g.:
- q1 1998 [817.0/(3173.0/4)] · 100 = 103.00
- q4 1998 [850.7/(3173.0/4)] · 100 = 107.20
- 1998 3336.0/3173.0 · 100 = 105.10

Step 3: Link the quarterly volume indices with shifting base and reference year using the annual indices as linking factors (using 1997 as the reference period for the chain-linked index).

e.g.:
- q1 1999 102.01 · 1.051 = 107.26
- q4 1999 104.56 · 1.051 = 109.93
- q1 2000 100.9 · 1.0326 · 1.051 = 109.60

Observe that the unweighted annual average of the derived chain-linked quarterly index series is equal to the independently derived chain-linked annual data.

e.g.: 2000 [109.6+110.18+110.58+111.69]/4 = 110.51

Finally, observe that the change from, e.g., q4 1999 to q1 2000, in the chain-linked series based on annual overlap differs from the corresponding change in the chain-linked index based on a one-quarter overlap in the next example.

e.g.:
- q1 2000/q 4 1999 based on annual overlap –0.3%
- ≠ q1 1999/q 4 1998 based on one quarter overlap (and 1999 prices) 0.5%

This is the step in the series introduced by the annual overlap technique.

Example 9.4.b. Quarterly Data and Annual Chain-Linking
One-quarter overlap
Annual sums and averages in bold.

						At Constant Prices of: 1997		1998		1999		Chain-linked index 1997=100	
Basic data	q1	q2	p1	p2	Total at current prices	Level	Index 1997 =100	Level	Index q4 1998 = 100	Level	Index q4 1999 = 100	Level	q-q Rate of Change
1997	**251.0**	**236.0**	**7.0**	**6.0**	**3,173.00**	**3,173.00**	*100.00*					**100.00**	
q1	67.4	57.6				817.40	103.04					103.04	
q2	69.4	57.1				828.40	104.43					104.43	1.3%
q3	71.5	56.5				839.50	105.83					105.83	1.3%
q4	73.7	55.8				850.70	107.24		907.55	100.00		107.24	1.3%
1998	**282.0**	**227.0**	**5.5**	**9.0**	**3,594.00**	**3,336.00**	**105.14**		**3,594.00**			**105.14**	
q1	76.0	55.4							916.60	101.00		108.31	*1.0%*
q2	78.3	54.8							923.85	101.80		109.17	0.8%
q3	80.6	54.2							931.10	102.59		110.03	0.8%
q4	83.1	53.6						939.45	103.51	948.80	100.00	111.01	0.9%
1999	**318.0**	**218.0**	**4.0**	**11.5**	**3,779.00**			**3,711.00**		**3,779.00**		**109.63**	
q1	85.5	53.2								953.80	100.53	111.60	*0.5%*
q2	88.2	52.7								958.85	101.06	112.19	0.5%
q3	90.8	52.1								962.35	101.43	112.60	0.4%
q4	93.5	52.0								972.00	102.45	113.73	1.0%
2000	**358.0**	**210.0**	**3.0**	**13.5**	**3,908.97**					**3,847.00**		**112.53**	

Step 1: Compile estimates for each quarter at the annual average prices of the previous year; the annual data being the sum of the four quarters.
Step 2: Compile estimates for the fourth quarter of each year at the annual average prices of the same year.
e.g.: q4 1998 $5.5 \cdot 73.7 + 9.0 \cdot 55.8 = 907.55$
Step 3: Convert the constant price estimates for the quarters of the first year after the chosen reference year (1997) into a volume index with the average of the reference year = 100
e.g.: q1 1998 $[817.4/(3173.0/4)] \cdot 100 = 103.04$
q4 1998 $[850.7/(3173.0/4)] \cdot 100 = 107.24$
Step 4: Convert the constant price estimates for each of the other quarters into a volume index with the fourth quarter of last year = 100
e.g.: q1 1999 $[916.60/907.55] \cdot 100 = 101.00$
q4 1999 $[936.45/907.55] \cdot 100 = 103.51$
Step 5: Link together the quarterly volume indices with shifting base using the fourth quarter of each year as link.
e.g.: q1 1999 $101.00 \cdot 1.0724 = 108.31$
q4 1999 $103.51 \cdot 1.0724 = 111.01$
q1 2000 $100.53 \cdot 1.1101 = 111.60$
The resulting linked series is referenced to average 1997 = 100.

Finally, observe that the unweighted annual average of the derived chain-linked quarterly index series differs from the independently derived chain-linked annuals in example 9.4.a.
e.g.: 2000 $[111.6+112.19+112.6+113.73]/4 = 112.53 \neq 110.51$

5. Chain-Linked Measures and Nonadditivity

9.42. In contrast to constant price data, chain-linked volume measures are nonadditive. To preserve the correct volume changes, related series should be linked independently of any aggregation or accounting relationships that exist between them; as a result, additivity is lost. Additivity is a specific version of the *consistency in aggregation* property for index numbers. Consistency in aggregation means that an aggregate can be constructed both directly by aggregating the detailed items and indirectly by aggregating subaggregates using the same aggregation formula. Additivity, in particular, implies that at each level of aggregation the volume index for an aggregate takes the form of a weighted arithmetic average of the volume indices for its components with the base-period values as weights (*1993 SNA*, paragraph 6.55). That is the same as requiring that the aggregate be equal to the sum of its components when the current price value of the aggregate and the components in some reference period are multiplied, or extrapolated, with the aggregate index and the component indices, respectively, resulting in *chain volume measures expressed in monetary terms*. It follows that, at the most detailed level, additivity is the same as requiring that the value obtained by extrapolating the aggregate is equal to the sum of the components valued at the reference period's prices. Thus, the additivity requirement effectively defines the fixed-base Laspeyres index and standard constant price data.

Example 9.4.c. Quarterly Data and Annual Chain-Linking
The Over-the-Year Technique
Laspeyres Volume Index

(i) Pair of years at the same prices.

(ii) Chain-linking using changes from the same quarter in the previous year.

Annual sums and averages in bold.

						At Constant Prices of :						Chain-Linked Index	
						1997		1998		1999		1997=100	
Basic Data	Quanti-ties A	Quanti-ties B	Price A	Price B	Total at Current Prices	Level	Index 1997 = 100	Level	q–4 = 1	Level	q–4 = 1	Level	q-q- Rate of Change
1997	**251.0**	**236.0**	**7.0**	**6.0**	**3,173.00**	**3,173.00**	*100.00*					**100.00**	
q1	67.4	57.6				817.40	103.04	889.10				103.04	1.3%
q2	69.4	57.1				828.40	104.43	895.60				104.43	1.3%
q3	71.5	56.5				839.50	105.83	901.75				105.83	1.3%
q4	73.7	55.8				850.70	107.24	907.55		936.50		107.24	1.3%
1998	**282.0**	**227.0**	**5.5**	**9.0**	**3,594.00**	**3,336.00**	**105.14**	**3,594.00**				**105.14**	
									0				
q1	76.0	55.4						916.60	1.0309	941.10		106.23	**–0.9%**
q2	78.3	54.8						923.85	1.0315	943.40		107.73	1.4%
q3	80.6	54.2						931.10	1.0325	945.70		109.28	1.4%
q4	83.1	53.6						939.45	1.0451	948.80		111.01	1.6%
1999	**318.0**	**218.0**	**4.0**	**11.5**	**3,779.00**			**3,711.00**		**3,779.00**		108.56	
									0				
q1	85.5	53.2								953.80	1.0135	107.67	*–3.0%*
q2	88.2	52.7								958.85	1.0164	109.49	1.7%
2													
q3	90.8	52.1								962.35	1.0176	111.20	1.6%
q4	93.5	52.0								972.00	1.0245	113.73	2.3%
2000	**358.0**	**210.0**	**3.0**	**13.5**	**3,908.97**					**3,847.00**		**110.52**	

Step 1: Compile estimates for each quarter at the annual average prices of the previous year.
e.g.: q1 1998 7.0·67.4+6.0·57.6 = 817.00
q4 1998 7.0·73.7+6.0·55.8 = 850.70

Step 2: Compile estimates for each quarter at the annual average prices of the same year.
e.g.: q1 1998 5.5·67.4+9.0·57.6 = 889.10
q4 1998 5.5·73.7+9.0·55.8 = 895.60

Step 3: Convert the constant price estimates for each quarter of the first year after the chosen reference year (1997) into a volume index with the average of the previous year = 100
e.g.: q1 1998 [817.4/(3173.0/4)]·100 = 103.04
q4 1998 [850.7/(3173.0/4)]·100 = 107.24

Step 4: For the other years, based on the constant price estimates derived in steps 1 and 2, calculate the volume change from the same quarter of the proceeding year as the following:
e.g.: q1 1999/q1 1998 916.60 / 889.10 = 1.0309
q4 1999/q4 1998 939.45 / 907.55 = 1.0451

Step 5: Link the quarterly volume indices with shifting base and reference year using the changes from the same period of the previous year as linking factors (extrapolators).
e.g.: q1 1999 1.0309·103.04 = 106.23
q4 1999 1.0451·107.24 = 111.07
q1 2000 1.0135·106.23 = 107.67

Observe that the unweighted annual average of the derived chain-linked quarterly index series is only approximately equal to the independently derived chain-linked annuals.
e.g.: 2000 [107.67+109.49+111.20+113.73]/4 = 110.52 ≠ 110.51

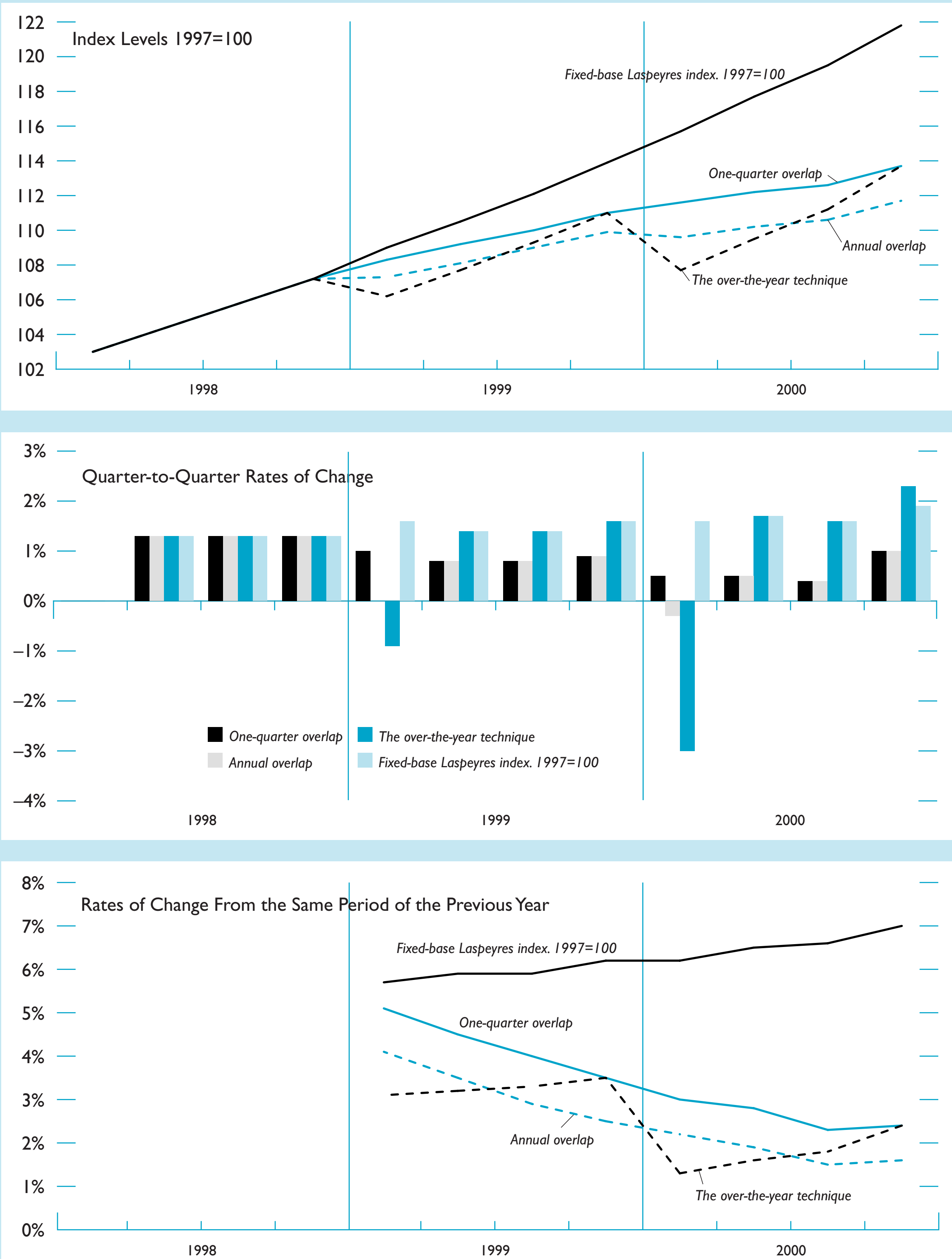
Finally, observe that the rate of change from q4 in one year to q1 in the next year in the chain-linked series based on the over-the-year technique differs substantially from the corresponding changes in chain-linked index based on a one-quarter overlap in the previous example.
e.g.: q1 1999/q 4 1998 based on the over-the-year technique (106.23/107.24–1)·100 = –0.9%
≠ q1 1999/q 4 1998 based on one-quarter overlap (and 1998 prices): (108.31/107.24–1)·100 = 1.0%

q1 2000/q 4 1999 based on the over-the-year technique (107.67/111.01–1)·100 = –3.0%
≠ q1 2000/q 4 1999 based on one-quarter overlap (and 1999 prices): (111.60/111.01–1)·100 = 0.5%

Observe also that the rate of change from q4 in one year to q1 in the next year in the chain-linked series based on the over-the-year technique differs substantially from the corresponding changes in the constant price measures based on the average prices of the current year. That is, q1 1999/q 4 1998 based on average 1999 prices (953.8/936.50–1)·100 = 0.5%

These differences between the q4-to-q1 rate of change in the chain-linked series based on the over-the-year technique and the corresponding rate of change based on direct measurements are the steps in the series introduced by the technique. Notice also that, in this example, the break appears to increase over time, that is, the breaks are cumulative. The breaks will be cumulative if there is a trend-wise change in relative prices and relative quantities, as in this example.

Chart 9.1 Chain-Linking of QNA Data



Example 9.5.a. Chain-Linking and Nonadditivity

This example illustrates the difference between constant price data and chain volume measures presented in monetary terms and shows the loss of additivity stemming from chain-linking.

The basic data are the same as in Examples 9.4.a, b, and c.

	Basic Data				At Constant 1997 Prices				Chain Volume Measures for the Total Referenced to its Average	Chain
	Quanti-ties A (1)	Quanti-ties B (2)	Price A (3)	Price B (4)	Item A (5)=(1)·(3)	Item B (6)=(2)·(4)	Total (7)=(6)+(6)	Chain-Linked Index (8)	Current price Level in 1997 (9)=(8)·3173.0/4	Discre-pancy (10)= (7)–(8)
1997	**251.0**	**236.0**	**7.0**	**6.0**	**1,757.0**	**1,416.0**	**3,173.0**	**100.00**	**3,173.0**	**0.0**
q1 1998	67.4	57.6			471.8	345.6	817.4	103.04	817.4	0.0
q2 1998	69.4	57.1			485.8	342.6	828.4	104.43	828.4	0.0
q3 1998	71.5	56.5			500.5	339.0	839.5	105.83	839.5	0.0
q4 1998	73.7	55.8			515.9	334.8	850.7	107.24	850.7	0.0
q1 1999	76.0	55.4			532.0	332.4	864.4	107.26	850.8	13.6
q2 1999	78.3	54.8			548.1	328.8	876.9	108.10	857.5	19.4
q3 1999	80.6	54.2			564.2	325.2	889.4	108.95	864.3	25.1
q4 1999	83.1	53.6			581.7	321.6	903.3	109.93	872.0	31.3
q1 2000	85.5	53.2			598.5	319.2	917.7	109.60	869.4	48.3
q2 2000	88.2	52.7			617.4	316.2	933.6	110.18	874.0	59.6
q3 2000	90.8	52.1			635.6	312.6	948.2	110.58	877.2	71.0
q4 2000	93.5	52.0			654.5	312.0	966.5	111.69	886.0	80.5

The chain-linked Laspeyres volume index in column 8 was derived in Example 9.4.a.

The chain discrepancies are zero for all quarters in 1998 because the 1998 link in the chain-linked Laspeyres index in column 8 is based on 1997 weights.

Finally, observe that the chain discrepancies for 2000 are substantially larger than for 1999. This is a general result. The chain discrepancies increase the more distant the reference period is if the weight changes are trend-wise and not cyclical.

All other indices in common use are nonadditive.[22] Example 9.5.a. illustrates the difference between constant price data and chain volume measures presented in monetary terms and shows the loss of additivity stemming from chain-linking.

6. Chain-Linking, Benchmarking, Seasonal Adjustment, and Compilation Procedures Requiring Additivity

9.43. Benchmarking and seasonal adjustment require long consistent time series with a fixed reference period at a detailed level, while many standard national accounts compilation methods require additive data. Examples of national accounts compilation methods requiring additive data include estimating value added as the difference between output and intermediate consumption, commodity flow techniques, and use of SU tables as a integrating framework. Both requirements may appear inconsistent with chain-linking, but they may not be.

9.44. In practice, the problem of nonadditivity can in most cases be circumvented by using the following multistep procedure (or its permutations):

Step 1

At the most detailed compilation level, construct long time series of non-seasonally adjusted traditional constant price data with a fixed-base year and corresponding Paasche price deflators using benchmarking, commodity flow, and other standard national accounts compilation techniques. These constant price data may be reconciled within an SU-table framework, if desired.

[22]The reason for non-additivity is that different weights are used for different annual periods, and therefore, will not yield the same results unless there have been no shifts in the weights.

Step 2

Aggregate these detailed constant price data using one of the following two alternative procedures:

A. Annual chain-linked Laspeyres framework

(i) For each year, revalue all detailed constant price data to the constant average prices of the previous year.

(ii) Add together these revalued data measured at the average prices of the previous year to construct the various aggregates and subaggregates at the constant average prices of the previous year.

(iii) Construct long time series with a fixed reference year by chain-linking the aggregates and subaggregates at the constant average prices of the previous year, using the annual overlap technique in Example 9.4.a or the one-quarter overlap technique in Example 9.4.b (preferred).

B. All index formulas

Use the price-quantity version of the relevant index formula,[23] and treat in the formula the detailed constant price data as if they were quantities and the detailed price deflators as if they were prices.

Aggregation procedures A and B in step 2 will give the same results for annually chain-linked Laspeyres indices.

9.45. The multistep procedure outlined above also can be used for indirect seasonal adjustment of aggregates. In that case, to obtain the best seasonally adjusted estimates, aggregation to an intermediate level before seasonally adjusting the various components may be required for the reasons given in Chapter VIII, Section D.3.a, which discusses the pros and cons of direct versus indirect seasonal adjustment of aggregates.

7. Presentation of Chain-Linked Measures

9.46. There are some important aspects to consider in presenting chain-linked measures in publications:

- Whether to present measures of percentage change or time series with a fixed reference period.
- Whether to present time series as index numbers or in monetary terms.
- Terminology to avoid confusing chain-linked measures in monetary terms for constant price data (fixed-based measures).
- Choice of reference year and frequency of reference year change—among others, as a means to reduce the inconvenience of nonadditivity associated with chain-linked measures.
- Whether to present supplementary measures of contribution of components to percentage change in aggregates.

9.47. Chain-linked price and volume measures must, at the minimum, be made available *as time series with a fixed reference period.* The main reason is that data presented with a fixed reference period allow different periods and periods of different duration to be compared and provide measures of long-run changes. Thus, presentation of price and volume measures should not be restricted to presenting only tables with period-to-period or year-on-year percentage change nor tables with each quarter presented as a percentage of a previous quarter. For users, tables with percentage changes derived from the time series may represent a useful supplement to the time series with a fixed reference period and may be best suited for presentation of headline measures. Tables with such data cannot replace the time-series data with a fixed reference period, however, because such tables do not provide the same user flexibility. Tables with each quarter presented as a percentage of a previous quarter (e.g., the previous quarter or the same quarter in the previous year) should be avoided because they are less useful and can result in users confusing the original index with the derived changes. Restricting the presentation of price and volume measures to presenting changes only runs counter to the core idea behind chain-linking, which is to construct long-run measures of change by cumulating a chain of short-term measures.

9.48. Chain-linked volume measures can be presented either as *index numbers* or in *monetary terms.* The difference between the two presentations is in how the reference period is expressed. As explained in paragraph 9.26, the reference period and level can be chosen freely without altering the rates of change in the series. The *index number* presentation shows the series with a fixed reference period that is set to 100, as shown in Examples 9.4.a, b, and c. The presentation is in line with usual index practice. It emphasizes that volume measures fundamentally are measures of relative change and that the choice and form of the reference point, and thus the level of the series, is arbitrary. It also highlights the differences of chain-linked measures from constant price estimates and

[23] For value added, the “double indicator” version of the formulas should be used.

prevents users from treating components as additive. Alternatively, the time series of chain-linked volume measures can be presented in *monetary terms* by multiplying the series by a constant to equal the constant price value in a particular reference period, usually a recent year. While this presentation has the advantage of showing the relative importance of the series, the indication of relative importance can be highly sensitive to the choice of reference year and may thus be misleading.[24] Because relative prices are changing over time, different reference years may give very different measures of relative importance. In addition, volume data expressed in monetary terms may wrongly suggest additivity to users who are not aware of the nature of chain-linked measures. On the other hand, they make it easier for users to gauge the extent of nonadditivity. Both presentations show the same underlying growth rates and both are used in practice.

9.49. Annually chain-linked Laspeyres volume measures in monetary terms are additive in the reference period. The nonadditivity inconvenience of chain volume measures in monetary terms may further be reduced by simultaneously doing the following:

- Using the average of a year and not the level of a particular quarter as reference period.
- Choosing the last complete year as reference year.
- Moving the reference year forward annually.

This procedure may give chain volume measures presented in monetary terms that are approximately additive for the last two years of the series. As illustrated in Example 9.5.a, the chain discrepancy increases (unless the weight changes are cyclical or noise) the more distant the reference year is. Thus, as illustrated in Example 9.5.b, moving the reference year forward can reduce the chain discrepancies significantly for the most recent section of the time series (at the expense of increased nonadditivity at the beginning of the series). For most users, additivity at the end of the series is more important than additivity at the beginning of the series.

9.50. To avoid chain discrepancies completely for the last two years of the series, some countries have adopted a practice of compiling and presenting data for the quarters of the last two years as the weighted annual average prices of the first of these two years. That second-to-last year of the series is also used as reference year for the complete time series. Again the reference year is moved forward annually. This approach has the advantage of providing absolute additivity for the last two years. The disadvantage, however, is that it also involves a series of back-and-forth changes in the price weights for the last two years, with added revisions to the growth rates as a result.

9.51. Chain-linked volume measures presented in monetary terms are *not* constant price measures and should not be labeled as measures at "*Constant xxxx Prices*." Constant prices means estimates based on fixed-price weights, and thus the term should not be used for anything other than true constant price data based on fixed-price weights. Instead, chain-volume measures presented in monetary terms can be referred to as "chain-volume measures referenced to their nominal level in xxxx."

9.52. The inconvenience for users of chain-linked measures being nonadditive can be reduced somewhat by presenting measures of the components' contribution to percentage change in the aggregate. Contributions to percentage change measures are additive and thus allow cross-sectional analysis such as explaining the relative importance of GDP components to overall GDP volume growth. The exact formula for calculating contribution to percentage change depends on the aggregation formula used in constructing the aggregate series considered and the time span the percentage change covers. The following provides a sample of the most common cases:

- Contribution to percentage change from period $t-n$ to t in current and constant price data:

$$\%\Delta_{i,(t-n)\to t} = 100 \cdot \left(X_{i,t} - X_{i,t-n}\right) \Big/ \sum_i X_{i,t-n} \qquad (9.9)$$
$$n \in \{1, 2..\}$$

- Contribution to percentage change from period $t-1$ to t in a period-to-period as well as in an annually chain-linked[25] Laspeyres index series:

$$\%\Delta_{i,(t-1)\to t} = 100 \cdot w_{i,t-1} \cdot \left(I_{i,t} - I_{i,t-1}\right) \Big/ \sum_i w_i \cdot I_{i,t-1} \qquad (9.8)$$

Where $w_{i,t-1}$ is the base period "share weight," that is, the item's share in the total value of period $t-1$.

[24]For the same reason, measuring relative importance from constant price data can be grossly misleading. For most purposes, it is better to make comparisons of relative importance based on data at current prices—these are the prices that are most relevant for the period for which the comparisons are done, and restating the aggregates relative to prices for a different period detracts from the comparison.

[25]The formula assumes that the series is linked using the one-quarter overlap technique.

Example 9.5.b. Choice of Reference Period and Size of the Chain Discrepancy

This example illustrates how moving the reference period forward may reduce the nonadditivity inconvenience of chain volume measures.

The basic data are the same as in Examples 9.4 and 9.5.a.

	Basic Data				At Constant 1999 Prices			Chain Linked-Index	Chain Volume Measures for the Total Referenced to its Average Current price Level in 1999	Chain Discre-pancy
	Quanti-ties A (1)	Quanti-ties B (2)	Price A (3)	Price B (4)	Item A (5)=(1)·(3)	Item B (6)=(2)·(4)	Total (7)=(5)+(6)	1999=100 (8)	(9)=(8)·3394.2/4	(10)=(7)–(8)
q1 1998	67.4	57.6			269.6	662.4	932.0	94.92	896.8	35.2
q2 1998	69.4	57.1			277.6	656.6	934.2	96.20	908.8	25.4
q3 1998	71.5	56.5			286.0	649.7	935.7	97.49	921.0	14.8
q4 1998	73.7	55.8			294.8	641.7	936.5	98.79	933.3	3.2
q1 1999	76.0	55.4			304.0	637.1	941.1	98.80	933.4	7.7
q2 1999	78.3	54.8			313.2	630.2	943.4	99.68	940.8	2.6
q3 1999	80.6	54.2			322.4	623.3	945.7	100.36	948.2	–2.5
q4 1999	83.1	53.6			332.4	616.4	948.8	101.26	956.7	–7.9
1999	**318.0**	**218.0**	**4.0**	**11.5**	**1,272.0**	**2,507.0**	**3,779.0**	**100.00**	**3,779.0**	**0.0**
q1 2000	85.5	53.2			342.0	611.8	953.8	100.96	953.8	0.0
q2 2000	88.2	52.7			352.8	606.0	958.8	101.49	958.8	0.0
q3 2000	90.8	52.1			363.2	599.1	962.3	101.86	962.3	0.0
q4 2000	93.5	52.0			374.0	598.0	972.0	102.88	972.0	0.0

First, the chain-linked index in column 8 is obtained by re-referencing the chain-linked index derived in Example 9.4.a, to average 1999 = 100. The original index series derived in Example 9.4.a was expressed with 1997 = 100. Changing the reference period to 1999 simply means dividing the original index series by its average level in 1999 (102.5).

e.g.:

q1 1998	103.04 / 1.0856 =	94.92
q3 1998	105.83 / 1.0856 =	97.49
q1 1999	107.26 / 1.0856 =	98.80
q4 1999	109.93 / 1.0856 =	101.26
q4 2000	111.69 / 1.0856 =	102.88

The chain discrepancies are zero for all quarters in 2000 because the 2000 link in the original chain-linked Laspeyres index derived in Example 9.4.a is based on 1999 weights.

Finally, observe that the chain discrepancies for 1998 are substantially larger than for 1999. Again, we see that the chain discrepancies increase the more distant the reference period is.

For a period-to-period chain-linked Laspeyres index series, as equation (9.5), the share weights are

$$w_{i,t-1} = p_{i,t-1} \cdot q_{i,t} \Big/ \sum_i p_{i,t-1} \cdot q_{i,t-1}.$$

Correspondingly, for an annually chain-linked Laspeyres index series the share weights are

$$w_{i,y-1} = \overline{p}_{i,y-1} \cdot \overline{q}_{i,y-1} \Big/ \sum_i \overline{p}_{i,y-1} \cdot \overline{q}_{i,y-1},$$

where year y – 1 is the base year for each short-term link in the index as given by equation (9.7.a).

- Contribution to percentage change from period t–1 to t in a period-to-period chain-linked Fisher volume index series:

$$\%\Delta_{i,(t-1)\to 1} = 100 \cdot \frac{\left(p_{i,t}\big/P_t^F + p_{i,t-1}\right) \cdot \left(q_{i,t} - q_{i,t-1}\right)}{\sum_i \left(p_{i,t}\big/P_t^F + p_{i,t-1}\right) \cdot q_{i,t-1}} \quad (9.9)$$

where P_t^F is the Fisher price index for the aggregate in period t with period t – 1 as base and reference period.

9.53. The nonadditivity inconvenience of chain-linking often can be circumvented by simply noting that chain Laspeyres volume measures are additive within each link. For that reason, chain-linked Laspeyres volume measures, for instance, can be combined with analytical tools like constant price SU and IO tables/models that require additivity.[26]

[26]In fact, the first countries to adopt annually chain-linked volume measures as their official national accounts volume measures used SU tables as their integrating GDP compilation framework.

Annex 9.1. Aggregation Over Time and Consistency Between Annual and Quarterly Estimates

A. Introduction

9A1.1. This annex provides a formal presentation of the following conclusions about annual and quarterly Laspeyres-type volume measures with corresponding Paasche deflators reached in Section B of the chapter and illustrated in Example 9.1:

(a) To ensure consistency between quarterly and annual data, annual Paasche deflators should in principle be derived as current period weighted averages of monthly or quarterly price deflators, where the weights represent constant price data.

(b) These annual deflators correspond to quantity-weighted period-average price measures and, equivalently to being derived as in conclusion (a), can be constructed directly from current period quantity-weighted average annual prices.

(c) Quarterly Paasche price indices should be based on the quantity-weighted average of each item's prices in the quarters of the base year and not on unweighted averages as typically used in price index compilations, to ensure that in the base year the annual sum of the quarterly constant price estimates is equal to the annual sum of the current price data.

(d) Deflating quarterly data with deflators constructed with unweighted average prices as the price base corresponds to valuing the quantities using their unweighted annual average price rather than their weighted annual average price.

(e) Valuing the quantities using their unweighted annual average price rather than heir weighted annual average price causes the annual sum of the quarterly constant price estimates in the base year to differ from the annual sum of the current price data.

(f) The error in conclusion (e) can be removed by a multiplicative adjustment of the complete constant price time series. The adjustment factor is the ratio of the annual current price data to the sum of the initial quarterly constant price data based on the unweighted annual average prices in the base year, which, for a single product, is identical to the ratio of the weighted to the unweighted average price.

The two first conclusions are formally shown in Section B and the last four conclusions in Section C of this annex.

B. Relationship Between Quarterly and Annual Deflators

9.A1.2. Quarterly data at current prices, at the "average" prices of the base year (year 0), and the corresponding (implicit) quarterly deflator with the average of year 0 as base and reference period can be expressed mathematically as the following:

- At current prices:

$$V_{q,y} = \sum_i p_{i,q,y} \cdot q_{i,q,y} \qquad (9.A1.1)$$

- At the "average" prices of the base year:

$$CP_{q,y_{\bar{0}}} = \sum_i \bar{p}_{i,0} \cdot q_{i,q,y}, \qquad (9.A1.2)$$

$$\bar{p}_{i,0} = \frac{\sum_q p_{i,q,0} \cdot q_{i,q,0}}{\sum_q q_{i,q,0}} \equiv \sum_q p_{i,q,0} \cdot \left(\frac{q_{i,q,0}}{\sum_q q_{i,q,0}} \right)$$

- Quarterly deflator (quarterly fixed-base Paasche index):[27]

$$PP_{\bar{0} \to (q,y)_{\bar{0}}} = \frac{V_{q,y}}{CP_{q,y_{\bar{0}}}} \equiv \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i \bar{p}_0 \cdot q_{i,q,y}} \qquad (9.A1.3)$$

where

$p_{i,q,y}$ is the price of item i in quarter q of year y;
$q_{i,q,y}$ is the quantity of item i in quarter q of year y;

[27] In the remainder of this annex, index numbers are presented with the following syntax: *Type of index* $_{(Reference\ period) \to (Current\ period)(base\ period)}$. Using the following codes for the elements of the syntax: *LQ* for a Laspeyres volume index, *PP* for a Paasche price index, $\bar{y-1}$ for average year $y-1$, and (q,y) for quarter q of year y.

$V_{q,y}$ is the total value at current prices in quarter q of year y;

$\bar{p}_{i,0}$ is the quantity-weighted annual arithmetic average of the price of item i in each quarter of year 0; and

$CP_{q,y_{\bar{0}}}$ is the total value in quarter q of year y measured at the annual average prices of year 0.

The quarterly deflator can either be implicitly derived as the current price value divided by the constant price value ($V_{q,y}/CP_{q,y_{\bar{0}}}$) or explicitly as a quarterly fixed-base Paasche index with the weighted average prices in year 0 ($\bar{p}_{i,0}$) as the price base.

9.A1.3. Similarly, annual data at current prices, at the "average" prices of the base year (year *0*), and the corresponding (implicit) annual deflator with the average of year 0 as base and reference period can be expressed mathematically as the following:

- At current prices:

$$V_y = \sum_q \sum_i v_{i,q,y} \equiv \sum_q \sum_i p_{i,q,y} \cdot q_{i,q,y} \tag{9.A1.4}$$

- At the "average" prices of the base year:

$$CP_{y_{\bar{0}}} = \sum_q CP_{q,y_{\bar{0}}} = \sum_q \sum_i \bar{p}_{i,0} \cdot q_{i,q,y} \tag{9.A1.5}$$

- Annual deflator (annual fixed-base Paasche index):

$$PP_{\bar{0}\to\bar{y}_{\bar{0}}} = \frac{\sum_q V_{q,y}}{\sum_q CP_{q,y_{\bar{0}}}} = \sum_q PP_{(\bar{0})\to(q,y)_{\bar{0}}} \cdot \left[\frac{CP_{q,y_{\bar{0}}}}{\sum_q CP_{q,y_{\bar{0}}}}\right] \tag{9.A1.6a}$$

where

$v_{i,q,y}$ is the value of item i at current prices in quarter q of year y; and

$CP_{y_{\bar{0}}}$ is the total annual value for year y measured at the annual average prices of year 0.

9.A1.4. Equations (9.A1.1) to (9.A1.6a) show that to ensure consistency between quarterly and annual data, annual Paasche deflators should in principle be current period weighted averages of the quarterly price deflators ($PP_{\bar{0}\to(q,y)_{\bar{0}}}$), where the weights ($CP_{q,y_{\bar{0}}}/\sum_q CP_{q,y_{\bar{0}}}$) are based on current period constant price data, as stated in paragraph 9.A1.1 conclusion (a) above. These current period weighted averages of the quarterly price deflators can either be implicitly derived as the annual sum of the quarterly current price data divided by the annual sum of the quarterly constant price data, or explicitly as a weighted average of monthly or quarterly price indices.

9.A1.5. The implicit annual deflator in equation (9.A1.6a) can, as stated in paragraph 9.A1.1 conclusion (b), equivalently be constructed directly from current period quantity-weighted average annual prices as evident from the following:

$$PP_{\bar{0}\to\bar{y}_{\bar{0}}} = \frac{\sum_q V_{q,y}}{\sum_q CP_{q,y_{\bar{0}}}} = \frac{\sum_q \sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_q \sum_i \bar{p}_{i,0} \cdot q_{i,q,y}} = \frac{\sum_i \bar{p}_{i,y} \cdot \bar{q}_{i,y}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,y}} \tag{9.A1.6b}$$

$$\text{when } \bar{p}_{i,y} = \frac{\sum_q p_{i,q,y} \cdot q_{i,q,y}}{\sum_q q_{i,q,y}},\ \bar{q}_{i,y} = \sum_q q_{i,q,y}$$

where

$\bar{p}_{i,y}$ is the quantity-weighted annual arithmetic average of the price of item i in each quarter of year y; and

$\bar{q}_{i,y}$ is the total annual quantity of item i in year y.

C. Annual Average Prices as Price Base

9.A1.6. In the base year 0, the annual sum of the quarterly constant price data is given by the following:

$$CP_{0_{\bar{0}}} = \sum_q CP_{q,0_{\bar{0}}} = \sum_q \sum_i \bar{p}_{i,0} \cdot q_{i,q,0} \tag{9.A1.7a}$$

9.A1.7. It follows that the annual sum of the quarterly constant price data is equal to the annual sum of the current price data in the base year, if, for each item, the base price is the quantity-weighted average of the item's prices in each quarter of the base year. That is, the base price is derived as $\bar{p}_{i,0} = \sum_q p_{i,q,0} \cdot q_{i,q,0} / \sum_q q_{i,q,0}$. This conclusion is evident from the following:

$$CP_{0_{\bar{0}}} = \sum_i \left(\bar{p}_{i,0} \cdot \sum_q q_{i,q,0} \right) \quad (9.A1.7b)$$

$$= \sum_i \left(\frac{\sum_i p_{i,q,0} \cdot q_{i,q,0}}{\sum_q q_{i,q,0}} \cdot \sum_q q_{i,q,0} \right)$$

$$\equiv \sum_i \sum_q p_{i,q,0} \cdot q_{i,q,0} \equiv V_0$$

9.A1.8. It follows furthermore, as stated in paragraph 9.A1.1 conclusion (c), that the quarterly deflators should be constructed with quantity-weighted average prices as the price base—as in equation (9.A1.3)—to ensure that in the base year the annual sum of the quarterly constant price estimates is equal to the annual sum of the current price data. This conclusion is evident from the following in combination with equation (9.A1.7b):

$$CP_{q,0_{\bar{0}}} = V_{q,0} \Big/ PP_{\bar{0}\to(q,0)_{\bar{0}}} \quad (9.A1.7c)$$

$$= \sum_i p_{i,q,0} \cdot q_{i,q,0} \Big/ \frac{\sum_i p_{i,q,0} \cdot q_{i,q,0}}{\sum_i \bar{p}_0 \cdot q_{i,q,0}}$$

$$\equiv \sum_i \bar{p}_0 \cdot q_{i,q,0}$$

9.A1.9. Deflating quarterly data with deflators constructed with unweighted average prices as the price base, corresponds, as stated in paragraph 9.A1.1 conclusion (d), to valuing the quantities using their unweighted annual average price. This result is evident from the following:

$$CP_{q,y_{\bar{0}}} = V_{q,y} \Big/ \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i \hat{p}_{i,q,0} \cdot q_{i,q,y}} \quad (9.A1.8)$$

$$\equiv \sum_i p_{i,q,y} \cdot q_{i,q,y} \Big/ \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i \hat{p}_{i,q,0} \cdot q_{i,q,y}}$$

$$\equiv \sum_i \hat{p}_{i,0} \cdot q_{i,q,y}$$

where

$p_{i,0} = 1/4 \sum_q p_{i,q,0}$ is the unweighted annual arithmetic average of the price of item *i* in each quarter of year 0; and

$\frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i \hat{p}_{i,q,0} \cdot q_{i,q,y}}$ is a Paasche index (deflator) constructed with unweighted average prices as price base.

9.A1.10. As stated in paragraph 9.A1.1 conclusion (e) above, in the base year, the constant price data derived in equation (9.A1.8), in contrast to the constant price data derived in equation (9.A1.7), do not sum to the annual sum of the current price data. However, as stated in paragraph 9.A1.1 conclusion (f), this error can be removed by a multiplicative adjustment, using the following adjustment factor:

$$\frac{\sum_q CP_{q,0_{\bar{0}}}}{\sum_q CP_{q,0_{\bar{0}}}} = \frac{\sum_q \sum_i \bar{p}_{i,0} \cdot q_{i,q,0}}{\sum_q \sum_i \hat{p}_{i,0} \cdot q_{i,q,0}} \quad (9.A1.9a)$$

$$\equiv \frac{\sum_i \sum_q p_{i,q,0} \cdot q_{i,q,0}}{\sum_i \sum_q \hat{p}_{i,0} \cdot q_{i,q,0}} \equiv \frac{\sum_q V_{q,0}}{\sum_q CP_{q,0_{\bar{0}}}}$$

That is, the ratio of the annual current price data to the sum of the initial quarterly constant price data based on the unweighted annual average prices in the base year. This factor, for a single product, is identical to the ratio of the weighted and unweighted average price:

$$\frac{\sum_q \bar{p}_{i,0} \cdot q_{i,q,0}}{\sum_q \hat{p}_{i,0} \cdot q_{i,q,0}} = \frac{\sum_q \bar{p}_{i,0} \cdot q_{i,q,0} \Big/ \sum_q q_{i,q,0}}{\hat{p}_{i,0}} \quad (9.A1.9b)$$

$$\equiv \frac{\bar{p}_{i,0}}{\hat{p}_{i,0}}$$

Annex 9.2. Annual Chain-Linking of Quarterly Laspeyres Volume Measures: A Formal Presentation of the Annual and One-Quarter Overlap Techniques

A. The Annual Overlap Technique

9.A2.1. Quarterly estimates at the quantity-weighted average prices of the previous year (year $y-1$) are given as

$$CP_{q,y_{\overline{y-1}}} = \sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y}, \tag{9.A2.1}$$

$$\overline{p}_{i,y-1} = \frac{\sum_q p_{i,q,y-1} \cdot q_{i,q,y-1}}{\sum_q q_{i,q,y-1}}$$

where

$p_{i,q,y-1}$ is the price of item i in quarter q of year $y-1$;

$q_{i,q,y-1}$ is the quantity of item i in quarter q in year y;

$\overline{q}_{i,y-1}$ is the simple arithmetic average of the quantities of item i in the quarters of year $y-1$;

$\overline{p}_{i,y-1}$ is the quantity weighted arithmetic average of the price of item i in the quarters of year $y-1$; and

$CP_{q,y_{\overline{y-1}}}$ is the total value in quarter q of year y measured at the average prices of year $y-1$.

9.A2.2. The corresponding short-term quarterly Laspeyres volume index and (implicit) Paasche deflator series with the average of the previous year as base and reference period are given as the following:[28]

- Short-term quarterly Laspeyres volume index:

$$LQ_{\overline{y-1}\rightarrow(q,y)_{\overline{y-1}}} = \frac{CP_{q,y_{\overline{y-1}}}}{\frac{1}{4}\sum_q V_{q,y-1}} \tag{9.A2.2}$$

$$\equiv \frac{\sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y}}{\frac{1}{4}\sum_q \sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y-1}}$$

$$\equiv \frac{\sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \overline{p}_{i,y-1} \cdot \frac{1}{4}\sum_q q_{i,q,y-1}}$$

$$\equiv \frac{\sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \overline{p}_{i,y-1} \cdot \overline{q}_{i,q,y-1}}$$

$$\equiv \sum_i \left(\frac{q_{i,q,y}}{\overline{q}_{i,y-1}}\right) \cdot \frac{\overline{p}_{i,y-1} \cdot \overline{q}_{i,y-1}}{\sum \overline{p}_{i,y-1} \cdot \overline{q}_{i,q,y-1}}$$

$$= \sum_i \frac{q_{i,q,y}}{\overline{q}_{i,y-1}} \cdot w_{i,y-1}$$

- Short-term (implicit) quarterly Paasche deflator:

$$PP_{\overline{y-1}\rightarrow(q,y)_{\overline{y-1}}} = \frac{V_{q,y}}{CP_{q,y_{y-1}}} \tag{9.A2.3}$$

$$= \frac{\sum_i p_{i,q,y} \cdot q_{i,q,y}}{\sum_i \overline{p}_{i,y-1} \cdot q_{i,q,y}}$$

where

$w_{i,y-1}$ is the base-period weight, that is, item i's share of the total value in period $y-1$ at current prices;

$V_{q,y-1}$ is the total value at current prices in quarter q of year y;

$LQ_{(\overline{y-1})\rightarrow(q,y)_{(\overline{y-1})}}$ is a Laspeyres volume index for quarter q of year y with average year y–1 as base and reference period; and

$PP_{(\overline{y-1})\rightarrow(q,y)_{(\overline{y-1})}}$ is a Paasche price index (deflator) for quarter q of year y with average year y–1 as base and reference period.

[28]In the remainder of this annex, index numbers are presented with the following syntax: *Type of index* $_{(Reference\ period)\rightarrow(Current\ period)(base\ period)}$. Using the following codes for the elements of the syntax: *LQ* for a Laspeyres volume index, *CLQ* for a chain-linked Laspeyres volume index, *PP* for a Paasche price index, *CPP* for a chain-linked Paasche Price index, $\overline{y-1}$ for average year $y-1$, and (q,y) for quarter q of year y.

9.A2.3. Similarly, the short-term annual Laspeyres volume index and Paasche deflator series with the average of the previous year as base and reference period are given as the following:

- Short-term annual Laspeyres volume index:

$$LQ_{\overline{y-1}\to\bar{y}_{\overline{y-1}}} = \frac{\sum_q CP_{q,y_{\overline{y-1}}}}{\sum_q V_{q,y-1}} \tag{9.A2.4}$$

$$\equiv \frac{\sum_q \sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_q \sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y-1}}$$

$$\equiv \frac{\sum_i \bar{p}_{i,y-1} \cdot \sum_q q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \sum_q q_{i,q,y-1}}$$

$$\equiv \frac{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,q,y-1}}$$

$$= \sum_i \frac{\sum_q q_{i,q,y}}{\sum_q q_{i,q,y-1}} \cdot w_{i,y-1}$$

$$\equiv \sum_i \frac{\bar{q}_{i,y}}{\bar{q}_{i,y-1}} \cdot w_{i,y-1}$$

- Short-term annual Paasche deflator:

$$PP_{\overline{y-1}\to\bar{y}_{\overline{y-1}}} = \frac{\sum_q \sum_i p_{i,q,y} \cdot p_{i,q,y}}{\sum_q \sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}} \tag{9.A2.5}$$

$$\equiv \frac{\sum_i \bar{p}_{i,y} \cdot \bar{q}_{i,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y}}$$

9.A2.4. Thus, the long-term annually chain-linked quarterly Laspeyres volume index and Paasche deflator can be constructed as the following:

- Long-term annually chain-linked quarterly Laspeyres volume index:

  For measuring the overall change from the average of year 0 (the reference year) to quarter q of year 2:

$$CLQ_{(\bar{0})\to(q,2)} = \frac{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,q,2}}{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}} \tag{9.A2.6a}$$

  For measuring overall change from the average of year 0 (the reference year) to quarter q of year Y:

$$CLQ_{(\bar{0})\to(q,Y)} = \left[\prod_{y=1}^{Y-1} \frac{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y-1}}\right] \cdot \frac{\sum_i \bar{p}_{i,Y-1} \cdot \bar{q}_{i,q,Y}}{\sum_i \bar{p}_{i,Y-1} \cdot \bar{q}_{i,Y-1}} \tag{9.A2.6b}$$

- Long-term annually chain-linked quarterly Paasche deflator:

  For measuring the overall change from the average of year 0 (the reference year) to quarter q of year 2:

$$CPP_{(\bar{0})\to(q,2)} = \frac{V_{q,2}}{V_0} \Big/ CLQ_{(\bar{0})\to(q,2)} \tag{9.A2.7a}$$

$$= \frac{\sum_i p_{i,q,2} \cdot q_{i,q,2}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \Big/ \frac{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,q,2}}{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}}$$

$$= \frac{\sum_i p_{i,q,2} \cdot q_{i,q,2}}{\sum_i \bar{p}_{i,1} \cdot q_{i,q,2}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}$$

  For measuring overall change from the average of year 0 (the reference year) to quarter q of year Y:

$$CPP_{(\bar{0})\to(q,Y)} = \left[\prod_{y=1}^{Y-1} \frac{\sum_i \bar{p}_{i,y} \cdot \bar{q}_{i,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,y}}\right] \cdot s\frac{\sum_i p_{i,q,Y} \cdot q_{i,q,Y}}{\sum_i \bar{p}_{i,Y-1} \cdot q_{i,q,Y}} \tag{9.A2.7b}$$

9.A2.5. The corresponding monetary term chain-volume measure for quarter q of year Y with the average of year 0 as reference base can be constructed as the following:

$$MCQ_{q,Y_{\bar{0}}} = CLQ_{(\bar{0})\to(q,Y)} \cdot \sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0} \tag{9.A2.8}$$

$$= CLQ_{(\bar{0})\to(q,Y)} \cdot {}^1\!/_4 V_0$$

9.A2.6. The monetary term chain-volume measure in equation (9.A2.8) can alternatively be derived by rescaling the constant price levels directly using the corresponding implicit annual Paasche deflator index (annually chain-linked). This follows from the following elaboration of equation (9.A2.8), for simplicity only shown in a three-period context, and Example 9.A2.1 below:

(9.A2.9)

$$MCQ_{q,2_{\bar{0}}} = CLQ_{(\bar{0})\to(q,2)} \cdot \sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}$$

$$\equiv \sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0} \cdot \frac{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,0}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,q,2}}{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}}$$

$$\equiv \frac{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}} \cdot \sum_i \bar{p}_{i,1} \cdot q_{i,q,2}$$

$$\equiv \sum_i \bar{p}_{i,1} \cdot q_{i,q,2} \Bigg/ \frac{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}$$

where

$\dfrac{\sum_i \bar{p}_{i,1} \cdot \bar{q}_{i,1}}{\sum_i \bar{p}_{i,0} \cdot \bar{q}_{i,1}}$ is the corresponding implicit annual Paasche deflator index with period 0 as base and reference period.

B. The One-Quarter Overlap Technique

9.A2.7. The short-term Laspeyres volume index series in (9.A2.2) can be re-referenced to be expressed with the fourth quarter of the previous year as reference period as follows:

(9.A2.10)

$$LQ_{(4,y-1)\to(q,y)_{\overline{(y-1)}}} = \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y-1}} \Bigg/ \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot \bar{q}_{i,q,y-1}}$$

$$\equiv \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,q,y}}{\sum_i \bar{p}_{i,y-1} \cdot q_{i,4,y-1}}$$

where

$LQ_{(4,y-1)\to(q,y)_{\overline{(y-1)}}}$ is a Laspeyres volume index for quarter q of year y with average year $y-1$ as base period and the fourth quarter of the previous year as reference period.

9.A2.8. Thus, the corresponding long-term chain-linked volume index measuring the overall change from the average of year 0 (the reference year) to quarter q of year 2 can be constructed as

$$CLQ_{\overline{(0)}\to(q,2)} = \frac{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,0}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,4,2}}{\sum_i \bar{p}_{i,1} \cdot q_{i,4,1}} \quad (9.A2.11)$$

9.A2.9 And the long-term chain-linked volume index measuring the overall change from the average of year 0 (the reference year) to quarter q of year Y can be constructed as

(9.A2.12)

$$CLQ_{\overline{(0)}\to(q,Y)} = \frac{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,0}} \cdot \left[\prod_{y=2}^{Y-1} \frac{\sum_i \bar{p}_{i,y-1} \cdot q_{i,4,y}}{\sum_i \bar{p}_{i,y-1} \cdot q_{i,4}, y-1}\right] \cdot \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,q,Y}}{\sum_i \bar{p}_{i,1} \cdot q_{i,4,Y-1}}$$

9.A2.10. The corresponding monetary term chain volume measure with the average of year 0 as reference base can be constructed as

$$MCQ_{q,Y_{\bar{0}}} = CLQ_{\overline{(0)}\to(q,Y)} \cdot \sum_i \bar{p}_{i,0} \cdot q_{i,0} \quad (9.A2.13)$$

$$= CLQ_{\overline{(0)}\to(q,Y)} \cdot {}^{1}\!/_{4}\, V_0$$

9.A2.11. The monetary term chain volume measure in (9.A2.13) can alternatively be derived by re-scaling the constant price levels directly using the corresponding implicit fourth quarter weighted annual Paasche deflator. This follows from the following elaboration of equation (9.A2.13), which for simplicity is only shown in a three period context:

$$MCQ_{q,2_{\bar{0}}} = CLQ_{\overline{(0)}\to(q,2)} \cdot \sum_i \bar{p}_{i,0} \cdot q_{i,0} \quad (9.A2.14)$$

$$= \sum_i \bar{p}_{i,0} \cdot q_{i,0} \cdot \frac{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,0}} \cdot \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,q,2}}{\sum_i \bar{p}_{i,1} \cdot q_{i,4,1}}$$

$$\equiv \frac{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}} \cdot \sum_i \bar{p}_{i,1} \cdot q_{i,q,2}$$

$$\equiv \sum_i \bar{p}_{i,1} \cdot q_{i,q,2} \Bigg/ \frac{\sum_i \bar{p}_{i,1} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,q,2}}$$

where

$\dfrac{\sum_i \bar{p}_{i,1} \cdot q_{i,4,1}}{\sum_i \bar{p}_{i,0} \cdot q_{i,4,1}}$ is the corresponding implicit fourth-quarter-weighted annual Paasche deflator index with period 0 as base and reference period.

Example 9.A2.1. Quarterly Data and Annual Chain-Linking
An Alternative "Annual Price Scaling" Version of the Annual Overlap Technique

Annual sums and averages in bold.

The basic data are the same as in Example 9.4.

This example provides an alternative presentation of the annual overlap chain-linking technique presented in Example 9.4. The final results are the same, but the procedure of obtaining the linked time series differs.

	Total at Current Prices	At 1997 Constant Prices 1998 = 100	Implicit Paasche Deflator 1997 = 100	At 1998 Constant Prices	Implicit Paasche Deflator	At 1999 Constant Prices	Chain Volume Measures for the Total in Monetary Terms Referenced to its Average Current Price Level in 1997
1997	**3,173.00**	**3,173.00**	100.00				**3,173.00**
q1 1998	871.94	817.40	106.67				817.40
q2 1998	885.51	828.40	106.89				828.40
q3 1998	910.05	839.60	108.40				839.60
q4 1998	926.50	850.70	108.91				850.70
1998	**3,594.00**	**3,336.00**	**107.73**		3,594.00	**100.00**	**3,336.00**
q1 1999	934.78			916.60	101.98		850.80
q2 1999	963.07			923.85	104.25	857.53	
q3 1999	940.42			931.10	101.00	864.26	
q4 1999	940.73			939.45	100.14	872.01	
1999	**3,779.00**			**3,711.00**	**101.83**	**3,779.00**	**3,444.60**
q1 2000	955.70					953.80	869.40
q2 2000	961.70					958.85	874.00
q3 2000	973.22					962.35	877.19
q4 2000	1018.36					972.00	885.99
2000	**3,908.97**					**3,847.00**	**3,777.78**

Step 1: As in Example 9.4, compile estimates for each quarter at the annual average prices of the previous year; the annual data being the sum of the four quarters.

Step2: Derive the corresponding annual implicit Paasche deflators with the previous year as base and reference period.

1998 [3594.0/3336.0] • 100 = 107.73

1999 [3779.0/3711.0] • 100 = 101.83

Step 3: Scale down the quarterly constant price estimates to measures at previous year's average prices, to the average price level of 1997.

e.g.: q1 1999 916.60/ 1.0773 = 850.80

q4 1999 939.45/ 1.0773 = 872.01

q1 2000 953.80/ (1.0773 • 1.0183) = 869.40

Observe that the resulting monetary termed chain-linked volume measures are identical to the ones derived in Example 9.5.a.