VIII Seasonal Adjustment and Estimation of Trend-Cycles

A. Introduction

8.1. Seasonal adjustment serves to facilitate an understanding of the development of the economy over time, that is, the direction and magnitude of changes that have taken place. Such understanding can be best pursued through the analyses of *time series*.[1] One major reason for compiling high-frequency statistics such as GDP is to allow timely identification of changes in the business cycle, particularly turning points. If observations of, say, quarterly non-seasonally adjusted GDP at constant prices are put together for consecutive quarters covering several years to form a time series and are graphed, however, it is often difficult to identify turning points and the underlying direction of the data. The most obvious pattern in the data may be a recurrent within-a-year pattern, commonly referred to as the seasonal pattern.

8.2. Seasonal adjustment means using analytical techniques to break down a series into its components. The purpose is to identify the different components of the time series and thus provide a better understanding of the behavior of the time series. In seasonally adjusted data, the impact of the regular within-a-year seasonal pattern, the influences of moving holidays such as Easter and Ramadan, and the number of working/trading days and the weekday composition in each period (the trading-day effect, for short) are removed. By removing the repeated impact of these effects, seasonally adjusted data highlight the underlying trends and short-run movements in the series.

8.3. In trend-cycle estimates, the impact of irregular events in addition to seasonal variations is removed. Adjusting a series for seasonal variations removes the identifiable, regularly repeated influences on the series but not the impact of any irregular events. Consequently, if the impact of irregular events is strong, seasonally adjusted series may not represent a smooth, easily interpretable series. To further highlight the underlying trend-cycle, most standard seasonal adjustment packages provide a smoothed trend line running through the seasonally adjusted data (representing a combined estimate of the underlying long-term trend and the business-cycle movements in the series).

8.4. An apparent solution to get around seasonal patterns would be to look at rates of change from the same quarter of the previous year. This has the disadvantage, however, that turning points are only detected with some delay.[2] Furthermore, these rates of change do not fully exclude all seasonal elements (e.g., Easter may fall in the first or second quarter, and the number of working days of a quarter may differ between succeeding years). Moreover, these year-to-year rates of change will be biased owing to changes in the seasonal pattern caused by institutional or behavioral changes. Finally, these year-to-year rates of change will reflect any irregular events affecting the data for the same period of the previous year in addition to any irregular events affecting the current period. For these reasons, year-to-year rates of change are inadequate for business-cycle analysis.

8.5. Therefore, more sophisticated procedures are needed to remove seasonal patterns from the series. Various well-established techniques are available for this purpose. The most commonly used technique is the Census X-11/X-12 method. Other available seasonal adjustment methods include, among others, TRAMO-SEATS, BV4, SABLE, and STAMP.

[1]Paragraph 1.13 defined time series as a series of data obtained through repeated measurement of the same concept over time that allows different periods to be compared.

[2]The delay can be substantial, on average, two quarters. A numerical example illustrating this point is provided in Annex 1.1.

8.6. A short presentation on the basic concept of seasonal adjustment is given in Section B of this chapter, while the basic principles of the Census X-11/X-12 method are outlined in section C. The final section, Section D, addresses a series of related general seasonal adjustment issues, such as revisions to the seasonally adjusted data and the wagging tail problem, and the minimum length of time series for seasonal adjustment. Section D also addresses a set of critical issues on seasonal adjustment of quarterly national accounts (QNA), such as preservation of accounting identities, seasonal adjustment of balancing items and aggregates, and the relationship between annual data and seasonally adjusted quarterly data. Section D also discusses the presentation and status of seasonally adjusted and trend-cycle data.

B. The Main Principles of Seasonal Adjustment

8.7. For the purpose of seasonal adjustment, a time series is generally considered to be made up of three main components—the trend-cycle component, the seasonal component, and the irregular component—each of which may be made up of several subcomponents:

(a) *The trend-cycle (T_t) component* is the underlying path or general direction reflected in the data, that is, the combined long-term trend and the business-cycle movements in the data.

(b) *The seasonal (S_t^c) component* includes seasonal effects narrowly defined and calendar-related systematic effects that are not stable in annual timing, such as trading-day effects and moving holiday effects.

   (i) The seasonal effect narrowly defined (S_t) is an effect that is reasonably stable[3] in terms of annual timing, direction, and magnitude. Possible causes for the effect are natural factors, administrative or legal measures, social/cultural traditions, and calendar-related effects that are stable in annual timing (e.g., public holidays such as Christmas).

   (ii) Calendar-related systematic effects on the time series that are not stable in annual timing are caused by variations in the calendar from year to year. They include the following:

   - The trading-day effect *(TD_t)*, which is the effect of variations from year to year in the number working, or trading, days and the weekday composition for a particular month or quarter relative to the standard for that particular month or quarter.[4,5]
   - The effects of events that occur at regular intervals but not at exactly the same time each year, such as moving holidays (MH_t), or paydays for large groups of employees, pension payments, and so on.
   - Other calendar effects (OC_t), such as leap-year and length-of-quarter effects.
   - Both the seasonal effects narrowly defined and the other calendar-related effects represent systematic, persistent, predictable, and identifiable effects.

(c) *The irregular component (I_t^c)* captures effects that are unpredictable unless additional information is available, in terms of timing, impact, and duration. The irregular component (I_t^c) includes the following:

   (i) Irregular effects narrowly defined (I_t).

   (ii) Outlier[6] effects (OUT_t).

   (iii) Other irregular effects (OI_t) (such as the effects of unseasonable weather, natural disasters, strikes, and irregular sales campaigns).

The irregular effect narrowly defined is assumed to behave as a stochastic variable that is symmetrically distributed around its expected value (0 for an additive model and 1 for a multiplicative model).

[3]It may be gradually changing over time (moving seasonality).

[4]The period-to-period variation in the standard, or average, number and type of trading days for each particular month or quarter of the year is part of the seasonal effect narrowly defined.

[5]Trading-day effects are less important in quarterly data than in monthly data but can still be a factor that makes a difference.

[6]That is, an unusually large or small observation, caused by either to errors in the data or special events, which may interfere with estimating the seasonal factors.

8.8. The relationship between the original series and its trend-cycle, seasonal, and irregular components can be modeled as additive or multiplicative.[7] That is, the time-series model can be expressed as

Additive Model

$$X_t = S_t^c + T_t + I_t^c \quad (8.1.a)$$

or with some subcomponents specified

$$X_t = (S_t + TD_t + MH_t + OC_t) + T_t + (I_t + OUT_t + OI_t) \quad (8.1.b)$$

where

the seasonal component is
$S_t^c = (S_t + TD_t + MH_t + OC_t)$

the irregular component is
$I_t^c = (I_t + OUT_t + OI_t)$, and

the seasonally adjusted series is
$A_t = T_t + I_t^c = T_t + (I_t + OUT_t + OI_t)$,

or as

Multiplicative Model

$$X_t = S_t^c + T_t + I_t^c \quad (8.2.a)$$

or with some subcomponents specified

$$X_t = (S_t \cdot TD_t \cdot MH_t \cdot OC_t) \cdot T_t \cdot (I_t \cdot OUT_t \cdot OI_t) \quad (8.2.b)$$

where

the seasonal component is $S_t^c = (S_t \cdot TD_t \cdot MH_t \cdot OC_t)$,

the irregular component is $I_t^c = (I_t \cdot OUT_t \cdot OI_t)$, and

the seasonally adjusted series is
$A_t = T_t \cdot I_t^c = T_t \cdot (I_t \cdot OUT_t \cdot OI_t)$.

8.9. The multiplicative model is generally taken as the default. The model assumes that the absolute size of the components of the series are dependent on each other and thus that the seasonal oscillation size increases and decreases with the level of the series, a characteristic of most seasonal macroeconomic series. With the multiplicative model, the seasonal and irregular components will be ratios centered around 1. In contrast, the additive model assumes that the absolute size of the components of the series are independent of each other and, in particular, that the size of the seasonal oscillations is independent of the level of the series.

8.10. Seasonal adjustment means using analytical techniques to break down a series into its components. The purpose is to identify the different components of the time series and thus to provide a better understanding of the behavior of the time series for modeling and forecasting purposes, and to remove the regular within-a-year seasonal pattern to highlight the underlying trends and short-run movements in the series. The purpose is not to smooth the series, which is the objective of trend and trend-cycle estimates. A seasonally adjusted series consists of the trend-cycle plus the irregular component and thus, as noted in the introduction, if the irregular component is strong, may not represent a smooth easily interpretable series.

8.11. Example 8.1 presents the last four years of a time series and provides an illustration of what is meant by seasonal adjustment, the trend-cycle component, the seasonal component, and the irregular component.

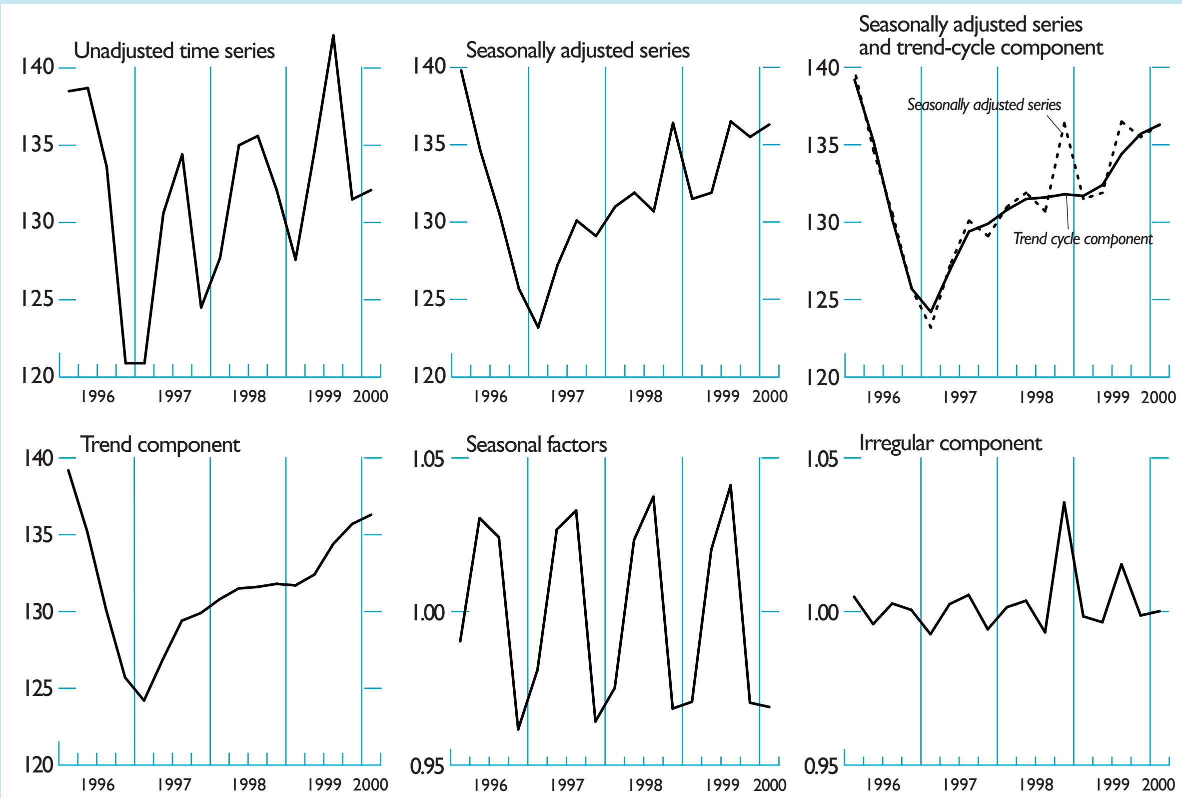
8.12. Seasonal adjustment and trend-cycle estimation represent an analytical massaging of the original data. As such, the *seasonally adjusted* data and the estimated trend-cycle component complement the original data, but, as explained in Section D of Chapter I, *they can never replace the original* data for the following reasons:

- Unadjusted data are useful in their own right. The non-seasonally adjusted data show the actual economic events that have occurred, while the seasonally adjusted data and the trend-cycle estimate represent an analytical elaboration of the data designed to show the underlying movements that may be hidden by the seasonal variations. Compilation of seasonally adjusted data, exclusively, represents a *loss of information*.
- No unique solution exists on how to conduct seasonal adjustment.
- Seasonally adjusted data are subject to revisions as future data become available, even when the original data *are not* revised.
- When compiling QNA, balancing and reconciling the accounts are better done on the original unadjusted QNA estimates.

[7] Other main alternatives exist, in particular, X-12-ARIMA includes a pseudo-additive model $X_t = T_t \cdot (S_t^c + I_t^c - 1)$ tailored to series whose value is zero for some periods. Moreover, within each of the main models, the relationship between some of the subcomponents depends on the exact estimation routine used. For instance, in the multiplicative model, some of the sub-components may be expressed as additive to the irregular effect narrowly defined, e.g., as: $X_t = S_t \cdot T_t \cdot (I_t + OUT_t + OI_t + TR_t + MH_t + OC_t)$.

Example 8.1. Seasonal Adjustment, Trend-Cycle Component, Seasonal Component, and Irregular Component

Multiplicative Seasonal Model



Date	Unadjusted Time Series (X_t) Index 1980 = 100 (1)	Seasonal Factors[1] (S_t) (2)	Irregular Component (I_t) (3)	Seasonally Adjusted Series (X_t/S_t) Index 1980 = 100 (4) = (1)/(2)	Trend-Cycle Component (T_t) Index 1980 = 100 (5) = (4)/(3)
q1 1996	138.5	0.990	1.005	139.8	139.2
q2 1996	138.7	1.030	0.996	134.6	135.2
q3 1996	133.6	1.024	1.003	130.5	130.1
q4 1996	120.9	0.962	1.000	125.7	125.7
q1 1997	120.9	0.981	0.993	123.2	124.2
q2 1998	130.6	1.027	1.002	127.2	126.9
q3 1997	134.4	1.033	1.005	130.1	129.4
q4 1997	124.5	0.964	0.994	129.1	129.9
q1 1998	127.7	0.975	1.001	131.0	130.8
q2 1998	135.0	1.023	1.003	131.9	131.5
q3 1998	135.6	1.037	0.993	130.7	131.6
q4 1998	132.1	0.968	**1.035**	**136.4**	131.8
q1 1999	127.6	0.971	0.998	131.5	131.7
q2 1999	134.6	1.020	0.997	131.9	132.4
q3 1999	142.1	1.041	1.015	136.5	134.4
q4 1999	131.5	0.970	0.999	135.5	135.7
q1 2000	132.1	0.969	1.000	136.3	136.3

With a multiplicative seasonal model, the seasonal factors are ratios centered around 1 and are reasonably stable in terms of annual timing, direction, and magnitude. The irregulars[2] are also centered around 1 but with erratic oscillations.

Observe the particularly strong irregular effect, or outlier, for q4 1998. Examples 8.3 and 8.4 show how an outlier like this causes trouble in early identification of changes in the trend-cycle.

[1]The values of the estimated seasonal component, particularly from the multiplicative model, are often called "seasonal factors."
[2]The irregular component is often referred to as "the irregulars," and the seasonal component is often referred to as "the seasonals."

- While errors in the source data may be more easily detected from seasonally adjusted data, it may be easier to identify the source for the errors and correct the errors working with the unadjusted data.
- Practice has shown that seasonally adjusting the data at the detailed level needed for compiling QNA estimates can leave residual seasonality in the aggregates.

The original unadjusted QNA estimates, the seasonally adjusted estimates, and the trend-cycle component all provide useful information about the economy (see Box 1.1), and, for the major national accounts aggregates, all three sets of data should be presented to the users.

8.13. Seasonal adjustment is normally done using off-the-shelf programs—most commonly worldwide by one of the programs in the X-11 family. Other programs in common use include the TRAMO-SEATS package developed by Bank of Spain and promoted by Eurostat and the German BV4 program. The original X-11 program was developed in the 1960s by the U.S. Bureau of the Census. It has subsequently been updated and improved through the development of X-11-ARIMA[8] by Statistics Canada[9] and X-12-ARIMA by the U.S. Bureau of the Census, which was released in the second half of the 1990s. The core of X-11-ARIMA and X-12-ARIMA is the same basic filtering procedure as in the original X-11.[10]

8.14. For particular series, substantial experience and expertise may be required to determine whether the seasonal adjustment is done properly or to fine-tune the seasonal adjustment. In particularly unstable series with a strong irregular component (e.g., outliers owing to strikes and other special events, breaks, or level shifts), it may be difficult to seasonally adjust properly.

8.15. It is also important to emphasize, however, that many series are well-behaved and easy to seasonally adjust, allowing seasonal adjustment programs to be used without specialized seasonal adjustment expertise. The X-11 seasonal adjustment procedure has in practice proved to be quite robust, and a large number of the seasonally adjusted series published by different agencies around the world are adjusted by running the programs in their default modes, often without special expertise. Thus, lack of experience in seasonal adjustment or lack of staff with particular expertise in seasonal adjustment should not preclude one from starting to compile and publish seasonally adjusted estimates. When compiling seasonally adjusted estimates for the first time, however, keep in mind that the main focus of compilation and presentation should be on the original unadjusted estimates. Over time, staff will gain experience and expertise in seasonal adjustment.

8.16. It is generally recommended that the statisticians who compile the statistics should also be responsible—either solely or together with seasonal adjustment specialists—for seasonally adjusting the statistics. This arrangement should give them greater insight into the data, make their job more interesting, help them understand the nature of the data better, and lead to improved quality of both the original unadjusted data and the seasonally adjusted data. However, it is advisable in addition to set up a small central group of seasonal adjustment experts, because the in-depth seasonal adjustment expertise required to handle ill-behaved series can only be acquired by hands-on experience with seasonal adjustment of many different types of series.

C. Basic Features of the X-11 Family of Seasonal Adjustment Programs

8.17. The three programs in the X-11 family—X-11, X-11-ARIMA, and X-12-ARIMA— follow an iterative estimation procedure, the core of which is based on a series of moving averages.[11] The programs comprise seven main parts in three main blocks of operations. First (part A), the series may optionally be "preadjusted" for outliers, level shifts in the series, the effect of known irregular events, and calendar-related effects using adjustment factors supplied by the user or estimated using built-in estimation procedures. In addition, the series may be extended by backcasts and forecasts so that less asymmetric filters can be used at the beginning and end of the series. Second (parts B, C, and D), the preadjusted series then goes through three rounds of seasonal filtering and extreme value adjustments, the "B, C, and D iterations" in the X-11/X-12 jargon. Third

[8]Autoregressive integrated moving average time-series models. ARIMA modeling represents an optional feature in X-11-ARIMA and X-12-ARIMA to backcast and forecast the series so that less asymmetric filters than in the original X-11 program can be used at the beginning and end of the series (see paragraph. 8.37).

[9]Initially released in 1980, with a major update in 1988, the X-11-ARIMA/88.

[10]The X-12-ARIMA can be obtained by contacting the U.S. Bureau of the Census (as of the time of writing, X-12-ARIMA was available free and could be downloaded with complete documentation and some discussion papers from http://www.census.gov/pub/ts/x12a/). X-11-ARIMA can be obtained by contacting Statistics Canada, and TRAMO-SEATS can be obtained by contacting Eurostat. The original X-11 program is integrated into several commercially available software packages (including among others SAS, AREMOS, and STATSTICA).

[11]Also called "moving average filters" in the seasonal adjustment terminology.

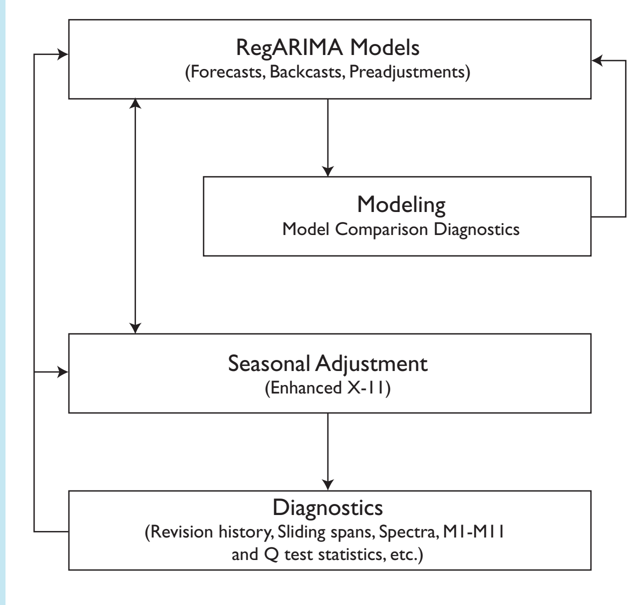
(parts E, F, and G), various diagnostics and quality control statistics are computed, tabulated, and graphed.[12]

8.18. The second block—with the parts B, C, and D seasonal filtering procedure—represents the central (X-11) core of the programs. The filtering procedure is basically the same for all three programs. X-12-ARIMA, however, provides several new adjustment options for the B, C, and D iterations that significantly enhance this part of the program. The main enhancements made in X-12-ARIMA to the central X-11 part of the program include, among others, a pseudo-additive $X_t = T_t \cdot (S_t^c + I_t^c - 1)$ model tailored to series whose value is zero for some periods; new centered seasonal and trend MA filters (see next section); improvements in how trading-day effects and other regression effects—including user-defined effects (a new capability)—are estimated from preliminary estimates of the irregular component (see Subsection 3 below).

8.19. In contrast, the first block, and to some extent the last block (see Subsection 4), differ markedly among the three programs. The original X-11 provided no built-in estimation procedures for preadjustments of the original series besides trading-day adjustments based on regression of tentative irregulars in parts B and C (see Subsection 1), but it provided for user-supplied permanent or temporary adjustment factors. X-11-ARIMA, in addition, provided for built-in procedures for ARIMA-model-based backcasts and forecasts of the series. In contrast, X-12-ARIMA contains an extensive time-series modeling block, the RegARIMA part of the program, that allows the user to preadjust, as well as backcasts and forecasts of the series by modeling the original series. The main components of X-12-ARIMA are shown in Box 8.1.

8.20. The RegARIMA block of the X-12-ARIMA allows the user to conduct regression analysis directly on the original series, taking into account that the non-explained part of the series typically will be autocorrelated, nonstationary, and heteroscedastic. This is done by combining traditional regression techniques with ARIMA modeling into what is labeled RegARIMA modeling.[13] The RegARIMA part of X-12-ARIMA allows the user to provide a set of user-defined

Box 8.1. Main Elements of the X-12-ARIMA Seasonal Adjustment Program



regressor variables. In addition, the program contains a large set of predefined regressor variables to identify, for example, trading-day effects, Easter effects,[14] leap-year effects, length-of-quarter effects, level shifts, point outliers, and ramps in the series. As a simpler alternative to RegARIMA modeling, X-12-ARIMA has retained the traditional X-11 approach of regressing the tentative irregulars on explanatory variables, adding regressors for point outliers and facilities for user-defined regressors to X-11's trading-day and Easter effects (see Subsection 3).

1. Main Aspects of the Core X-11 Moving Average Seasonal Adjustment Filters

8.21. This subsection presents the main elements of the centered moving average filtering procedure in the X-12-ARIMA B, C, and D iterations for estimating the trend-cycle component and the seasonal effects narrowly defined. The moving average filtering procedure implicitly assumes that all effects except the seasonal effects narrowly defined are approximately symmetrically distributed around their expected value (1 for a multiplicative and 0 for an additive model) and thus can be fully eliminated by using the centered moving

[12] Test statistics that users should consult regularly are also included in parts A and D.

[13] The standard seasonal ARIMA model is generalized to include regression parameters with the part not explained by the regression parameters following an ARIMA process, that is, $X_t = \beta' Y_t = Z_t$, where X_t is the series to be modeled, β a parameters vector, Y_t a vector of fixed regressors, and Z_t a pure seasonal ARIMA model.

[14] The user can select from different Easter-effect models.

average filter instead of ending up polluting the estimated trend-cycle component and the seasonal effects narrowly defined. Ideally, all effects that are not approximately symmetrically distributed around the expected value of 1 or 0 should have been removed in the preadjustment part (part A).

8.22. The centered moving average filtering procedure described below only provides estimates of the seasonal effects narrowly defined (S_t), not the other parts of the seasonal component (S_t^c). Subsection 3 briefly discusses the procedures available for estimating the not-captured impact of trading-day effects and other calendar-related systematic effects. It includes, namely, the traditional X-11 approach of regressing the tentative irregulars on explanatory trading-day and other calendar-related variables as part of the B and C iterations, and the X-12-ARIMA option of estimating these effects as part of the RegARIMA-based preadjustment of the series.

8.23. The main steps of the multiplicative version of the filtering procedure for quarterly data in the B, C, and D iterations, assuming preadjusted data, are as follows:[15]

Stage 1. Initial Estimates

(a) Initial trend-cycle. The series is smoothed using a weighted 5-term (2 x 4)[16] centered moving average to produce a first estimate of the trend-cycle. $T_t^1 = {}^1/_8 X_{t-2} + {}^1/_4 X_{t-1} + {}^1/_4 X_t + {}^1/_4 X_{t+1} + {}^1/_8 X_{t+2}$.

(b) *Initial SI ratios*. The "original"[17] series is divided by the smoothed series (T_t^1) to give an initial estimate of the seasonal and irregular component $S_t I_t^1$.

(c) *Initial preliminary seasonal factors*. A time series of initial preliminary seasonal factors is then derived as a weighted 5-term (3 x 3) centered seasonal[18] moving average[19] of the initial SI ratios ($S_t I_t^1$). This method implicitly assumes that I_t behaves as a stochastic variable that is symmetrically distributed around its expected value (1 for a multiplicative model) and therefore can be eliminated by averaging.
$\hat{S}_t^1 = {}^1/_9 SI_{t-8} + {}^2/_9 SI_{t-4} + {}^3/_9 SI_t + {}^2/_9 SI_{t+4} + {}^1/_9 SI_{t+8}$

(d) *Initial seasonal factors*. A time series of initial seasonal factors is then derived by normalizing the initial preliminary seasonal factors.

$$S_t^1 = \frac{\hat{S}_t^1}{{}^1\!/_8 \hat{S}_{t-2}^1 + {}^1\!/_4 \hat{S}_{t-1}^1 + {}^1\!/_4 \hat{S}_t^1 + {}^1\!/_4 \hat{S}_{t+1}^1 + {}^1\!/_8 \hat{S}_{t+2}^1}$$

This step is done to ensure that the annual average of the seasonal factors is close to 1.

(e) *Initial seasonal adjustment*. An initial estimate of the seasonally adjusted series is then derived as $A_t^1 = X_t / S_t^1 = T_t \cdot S_t \cdot I_t / S_t = T_t^1 \cdot I_t$.

Stage 2. Revised Estimates

(a) *Intermediate trend-cycle*. A revised estimate of the trend-cycle (T_t^2) is then derived by applying a Henderson moving average[20] to the initial seasonally adjusted series (A_t^1).

(b) *Revised SI ratios*, are then derived by dividing the "original" series by the intermediate trend-cycle estimate (T_t^2).

(c) *Revised preliminary seasonal factors* are then derived by applying a 3 x 5 centered seasonal moving average[21] to the revised *SI* ratios.

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[15] Adapted from Findley and others (1996), which presents the filters assuming monthly data.

[16] A 2 x 4 moving average

$$\left(\overline{X}_t^{2x4} = {}^1\!/_2 \left(\overline{X}_t^{1x4} + \overline{X}_{t+1}^{1x4}\right)\right)$$

is a 2-term moving average

$$\left(\overline{X}_t^{1x4} + \overline{X}_{t+1}^{1x4}\right)$$

of a 4-term moving average.

$$\left(\overline{X}_t^{1x4} = {}^1\!/_4 \left(X_{t-2} + X_{t-2} + X_t + X_{t+2}\right)\right).$$

[17] The series may be pre-adjusted, and, for the C and D iterations, extreme value adjusted (see below).

[18] A seasonal moving average is a moving average that is applied to each quarter separately, that is, as moving averages of neighboring q1s, q2s, etc.

[19] The 3 x 3 seasonal moving average filter is the default. In addition, users can select a 3 x 5 or 3 x 9 moving average filter (X-12-ARIMA also contains an optional 3 x 15 seasonal moving average filter). The user-selected filter will then be used in both stage 1 and stage 2.

[20] A Henderson moving average is a particular type of weighted moving average in which the weights are determined to produce the smoothest possible trend-cycle estimate. In X-11 and X-11-ARIMA, for quarterly series, Henderson filters of length 5, and 7 quarters could be automatically chosen or user-determined. In X-12-ARIMA, the users can also specify Henderson filters of any odd-number length.

[21] The 3 x 5 seasonal moving average filter is the default. In the D iteration, X-11-ARIMA, and X-12-ARIMA automatically select from among the four seasonal moving average filters (3 x 3, 3 x 5, 3 x 9, and the average of all SI ratios for each calendar quarter (the stable seasonal average)), unless the user has specified that the program should use a particular moving average filter.

(d) *Revised seasonal factors.* A revised time series of initial seasonal factors is then derived by normalizing the initial preliminary seasonal factors as in stage 1.

(e) *Revised seasonal adjustment.* A revised estimate of the seasonally adjusted series is then derived as $A_t^2 = X_t / S_t^2 = T_t^2 \cdot I_t$.

(f) *Tentative irregular.* A tentative estimate of the irregular component is then derived by de-trending the revised seasonally adjusted series: $I_t^2 = A_t^2 / T_t^3$.

Stage 3. Final Estimates (D iteration only)

(a) *Final trend-cycle.* A final estimate of the trend-cycle component (T_t^3) is derived by applying a Henderson moving average to the revised and final seasonally adjusted series (A_t^2).

(b) *Final irregular.* A final estimate of the irregular component is derived by de-trending the revised and final seasonally adjusted series $I_t^3 = A_t^2 / T_t^3$.

8.24. The filtering procedure is made more robust by a series of identifications and adjustments for extreme values. First, for the B and D iterations, when estimating the seasonal factors in steps (b) to (d) (stages 1 and 2) based on analyses of implied irregulars, extreme SI ratios are identified and temporarily replaced. For the B iteration, this is done in both stages 1 and 2, while for the D iteration it is done only at stage 2. Second, after the B and C iterations and before the next round of filtering, based on analyses of the tentative irregular component (I_t^2) derived in step (f) of stage 2, extreme values are identified and temporarily removed from the original (or preadjusted) series (that is, before the C and D iterations, respectively).

2. Preadjustments

8.25. The series may have to be preadjusted before entering the filtering procedure. For the seasonal moving average in step (c) (stages 1 and 2) above to fully isolate the seasonal factors narrowly defined, the series may have to be preadjusted to temporarily remove the following effects:

- outliers;
- level-shifts (including ramps);
- some calendar-related effects, particularly moving holidays, and leap-years;
- unseasonable weather changes and natural disasters; and
- strikes and irregular sale campaigns.

The extreme value adjustments described in paragraph 8.24 will to some extent take care of the distortions caused by point outliers but generally not the other effects. Furthermore, because outliers and the other effects listed cannot be expected to behave as a stochastic variable that is approximately symmetrically distributed around its expected value (1 for a multiplicative model), they will not be fully eliminated by the seasonal moving average filter used in step (c) (stages 1 and 2), and may end up polluting the estimated seasonal factors narrowly defined. For that reason, the impact of these effects cannot be fully identified from the estimated irregular component. Preadjustment can be conducted in a multitude of ways. The user may adjust the data directly based on particular knowledge about the data before feeding them to the program, or, in the case of X-12-ARIMA, use the estimation procedures built into the program.

3. Estimation of Other Parts of the Seasonal Component Remaining Trading-Day and Other Calendar-Related Effects

8.26. The moving average filtering procedure in paragraph 8.23 provides estimates of the seasonal effects narrowly defined (S_t), but not of the other parts of the overall seasonal component (S_t^c). Variations in the number of working/trading days and the weekday composition in each period, as well as the timing of moving holidays and other events that occur at regular calendar-related intervals, can have a significant impact on the series. Parts of these calendar effects will occur on average at the same time each year and affect the series in the same direction and with the same magnitude. Thus, parts of these calendar effects will be included in the (estimated) seasonal effects narrowly defined. Important parts of these systemic calendar effects will not be included in the seasonal effect narrowly defined, however, because (a) moving holidays and other regular calendar-related events may not fall in the same quarter each year and (b) the number of trading days and the weekday composition in each period varies from year to year.

8.27. Seasonally adjusted data should be adjusted for all seasonal variations, not only the seasonal effect narrowly defined. Leaving parts of the overall seasonal component in the adjusted series can be misleading and seriously reduce the usefulness of the seasonally adjusted data. Partly seasonally adjusted series, where the remaining identifiable calendar-related effects have not been removed, can give false signals of what's happening in the economy. For instance, such series may indicate that the economy declined in a particular quarter when it actually increased. Both the seasonal effects narrowly defined and the other calendar-related effects represent systematic, persistent, predictable, and identifiable seasonal effects, and all should be removed when compiling seasonally adjusted data.

8.28. Separate procedures are needed to estimate the remaining impact of the calendar-related systematic effects. X-11 and X-11-ARIMA contain built-in models for estimation of trading-day and Easter effects based on ordinary least-square (OLS) regression analysis of the tentative irregular component (I_t^2). When requested, the program derives preliminary estimates and adjustments for trading days and Easter effects at the end of the B iteration and final estimates and adjustments for trading days and Easter[22] effects at the end of the C iteration. X-12-ARIMA, in addition, provides an option for estimating these effects and others directly from the original data as part of the RegARIMA block of the program.

8.29. X-12-ARIMA's options for supplying user-defined regressors make it possible for users to construct custom-made moving holiday adjustment procedures. This option makes it easier to take into account holidays particular to each country or region, or country-specific effects of common holidays. Typical examples of such regional specific effects are regional moving holidays such as Chinese new year[23] and Ramadan, and the differences in timing and impact of Easter. Regarding the latter, while in some countries Easter is mainly a big shopping weekend creating a peak in retail trade, in other countries most shops are closed for more than a week creating a big drop in retail trade during the holiday combined with a peak in retail trade before the holiday. Also, Easter may fall on different dates in different countries, depending on what calendar they follow.

8.30. Some countries publish as *"non-seasonally adjusted data"* data that have been adjusted for some seasonal effects, particularly the number of working days. It is recommended that this approach not be adopted for two main reasons. First, data presented as non-seasonally adjusted should be fully unadjusted, showing what actually has happened, not partly adjusted for some seasonal effects. Working/trading-day effects are part of the overall seasonal variation in the series, and adjustment for these effects should be treated as an integral part of the seasonal adjustment process, not as a separate process. Partly adjusted data can be misleading and are of limited analytical usefulness. Second, working-day adjustments made outside the seasonal adjustment context are often conducted in a rather primitive manner, using fixed coefficients based on the ratio of the number of working days in the month or quarter to the number of working days in a standard month or quarter. Moreover, it has been shown that the simple proportional method overstates the effect of working days on the series and may render it more difficult to seasonally adjust the series. Parts of these calendar effects will be captured as part of the seasonal effect narrowly defined, and X-11/X-12's trading-days adjustment procedures are able to handle the remaining part of these calendar effects in a much more sophisticated and realistic manner.

4. Seasonal Adjustment Diagnostics

8.31. X-11-ARIMA and, especially, X-12-ARIMA provide a set of diagnostics to assess the outcome, both from the modeling and the seasonal adjustment parts of the programs. These diagnostics range from advanced tests targeted for the expert attempting to fine-tune the treatment of complex series to simple tests that as a minimum should be looked at by all users of the programs. While the programs sometimes are used as a black box without the diagnostics, they should not (and need not be) used that way, because many tests can be readily understood.

8.32. Basic tests that as a minimum should be looked at include F-tests for existence of seasonality and the M- and Q-test statistics introduced with X-11-ARIMA. Other useful tests include tests for residual seasonality (shown in Box 8.2), existence of trading-day effects, other calendar-related effects, extreme values, and tests for fitting an ARIMA model to the series. Box 8.2 shows the parts of the output from X-12-ARIMA for the illustrative series in Example 8.1 regarding the F-tests for existence of seasonality. Similarly, Box 8.3 shows the M- and Q-test statistics for the same illustrative series. Series for which the program cannot find any identifiable seasonality or that fail the M- and Q-test statistics should be left unadjusted. Unfortunately, in these cases, the programs will not abort with a message that the series cannot

[22]Custom-making may be needed to account for country-specific factors (see paragraph 8.29).

[23]The Chinese new year represents a moving holiday effect in monthly data but not in quarterly data, because it always occurs within the same quarter.

Box 8.2. X-11/X-11-ARIMA/X-12-ARIMA Tests for Existence of Seasonality

The following is an edited copy of the relevant parts of X-12-ARIMA's main output file with the basic F-tests for existence of seasonality. The test statistics values are for the full 21 years of the illustrative series, of which the last four years of data were presented in Example 8.1. The D 8.A and D 11 codes refer to the various "output tables" in the main output file from the different programs in the X-11 family, documenting the various steps in the A, B, C, D, E, F, and G parts of the program.

As a minimum, Table D 8.A should be checked to make sure that the program returns an IDENTIFIABLE SEASONALITY PRESENT and not an IDENTIFIABLE SEASONALITY NOT PRESENT statement. The series should generally be left unadjusted if the F-tests indicate that identifiable seasonality is not present.

D8.A F-Tests for Seasonality

Test for the Presence of Seasonality Assuming Stability

	Sum of Squares	Degrees of Freedom	Mean Square	F-Value
Between quarters	809.1996	3	269.73319	43.946**
Residual	497.1645	81	6.13783	
Total	1306.3640	84		

Seasonality present at the 0.1 percent level.

Nonparametric Test for the Presence of Seasonality Assuming Stability

Kruskal-Wallis Statistic	Degrees of Freedom	Probability Level
53.2410	3	0.000%

Seasonality present at the 1 percent level.

Moving Seasonality Test

	Sum of Squares	Degrees of Freedom	Mean Square	F-Value
Between Years	85.8291	20	4.291454	1.857
Error	138.6635	60	2.311058	

Moving seasonality present at the 5 percent level.

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY
IDENTIFIABLE SEASONALITY PRESENT

D 11 Final Seasonally Adjusted Data

Test for the Presence of Residual Seasonality.
No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.03
No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.48
No evidence of residual seasonality in the last 3 years at the 5 percent level.

be properly adjusted. Instead, they will produce "adjusted" data. The only way to detect that these adjusted data should not be used is to look at the diagnostics.

8.33. X-12-ARIMA provides, in addition, a large set of new diagnostic tools to further gauge the quality of the seasonal adjustment and the appropriateness of the seasonal adjustment and modeling options chosen. These new diagnostic tools include features such as sliding span and frequency spectrum estimates, revision history[24] simulations, and options for comparing direct and indirect seasonal adjustments of aggregates.[25] Sliding spans can be used to evaluate the overall quality of the seasonal adjustment in competition with the Q statistics. They can also be used to assess the stability of trading-day estimates, to assess the adequateness of the length of the filters chosen, and to decide between direct and indirect adjustment. Frequency spectrum estimates from the irregular component can help identify residual seasonality narrowly defined and residual trading-day effects in different parts of the series. Revision history simulations can help decide between direct and indirect adjustment, selection of competing RegARIMA models, and identification of optimal length of forecast extension before filtering. The RegARIMA part of X-12-ARIMA also contains a large set of test statistics for model selection and outlier detection.

D. Issues in Seasonality

8.34. This section addresses a series of general and more QNA-specific issues related to seasonal adjustment.

- Subsection 1 explains how changes in the seasonal patterns cause revisions to the seasonally adjusted and trend-cycle estimates—the wagging tail problem. The subsection explains why trend-cycle estimates at the end of the series are particularly prone to revisions and why turning points can be identified only after a lag of several observations, because it is logically impossible to distinguish an outlier from a change in the trend-cycle based on one observation.
- Subsection 2 discusses the minimum length of time-series data required for obtaining seasonally adjusted estimates.

[24]See Section D.1 of this chapter for a discussion of revisions to seasonally adjusted data, and the wagging tail effect.
[25]See Section D.3.a of this chapter for a discussion of direct-versus-indirect seasonal adjustment of balancing items and aggregates.

- Subsection 3 addresses a series of issues related particularly to seasonal adjustment and trend-cycle estimation of QNA data, such as preservation of accounting identities, seasonal adjustment of balancing items and aggregates, and the relationship between annual data and seasonally adjusted quarterly data.
- Finally, Subsection 4 discusses the status and presentation of seasonally adjusted and trend-cycle QNA estimates.

1. Changes in Seasonal Patterns, Revisions, and the Wagging Tail Problem

8.35. Seasonal effects may change over time. The seasonal pattern may gradually evolve as economic behavior, economic structures, and institutional and social arrangements change. The seasonal pattern may also change abruptly because of sudden institutional changes.

8.36. Seasonal filters estimated using centered moving averages allow the seasonal pattern of the series to change over time and allow for a gradual update of the seasonal pattern, as illustrated in Example 8.2. This results in a more correct identification of the seasonal effects influencing different parts of the series.

8.37. Centered moving average seasonal filters also imply, however, that the final seasonally adjusted values depend on both past and future values of the series. Thus, to be able to seasonally adjust the earliest and latest observations of the series, either asymmetric filters have to be used for the earliest and the latest observations of the series or the series has to be extended by use of backcasts and forecasts based on the pattern of the time series. While the original X-11 program used asymmetric filters at the beginning and end of the series, X-12-ARIMA and X-11-ARIMA use ARIMA modeling techniques to extend the series so that less asymmetric filters can be used at the beginning and end.

8.38. Consequently, new observations may result in changes in the estimated seasonal pattern for the latest part of the series and subject seasonally adjusted data to more frequent revisions than the original non-seasonally adjusted series. This is illustrated in Example 8.3 below. Estimates of the underlying trend-cycle component for the most recent parts of the time series in particular may be subject to relatively large revisions at the first updates,[26] however, theoretical and empirical studies indicate that the trend-cycle converges much faster to its final value than the seasonally adjusted series. In contrast, the seasonally adjusted series may be subject to lower revisions at the first updates but not-negligible revisions even after one to two years. There are two main reasons for slower convergence of the seasonal estimates. First, the seasonal moving average filters are significantly longer than the trend-cycle filters.[27] Second, revisions to the estimated regression parameters for calendar-related systematic effects may affect the complete time series. These revisions to the seasonally adjusted and trend-cycle estimates, owing to new observations, are commonly referred to as the "wagging tail problem."

8.39. Estimates of the underlying trend-cycle component for the most recent parts of the series should be interpreted with care, because signals of a change in the trend-cycle at the end of the series may be false. There are two main reasons why these signals may be false. First, outliers may cause significant revisions to the trend-cycle end-point estimates. It is usually not possible from a single observation to distinguish between an outlier and a change in the underlying trend-cycle, unless a particular event from other sources generating an outlier is known to have occurred. In general, several observations verifying the change in the trend-cycle indicated by the first observation are needed. Second, the moving average trend filters used at the end of the series (asymmetric moving average filters with or without ARIMA extension of the series) implicitly assume that the most recent basic trend of the series will persist. Consequently, when a turning point appears at the current end of the series, the estimated trend values at first present a systematically distorted picture, continuing to point in the direction of the former, now invalidated, trend. It is only after a lag of several observations that the change in the trend comes to light. While the trend-cycle component may be subject to large revisions at the first updates, however, it typically converges relatively fast to its final value.[28] An illustration of this can be found by comparing the data presented in Example 8.3 (seasonally adjusted estimates) with that in Example 8.4 (trend-cycle estimates).

[26] Illustrated in Example 8.4.

[27] For instance, the seasonal factors will be final after 2 years with the default 5-term (3 x 3) moving average seasonal filter (as long as any adjustments for calendar effects and outliers are not revised). In contrast, the trend-cycle estimates will be final after 2 quarters with the 5-term Henderson moving average trend-cycle filter (as long as the underlying seasonally adjusted series is not revised).

[32] The trend-cycle estimates will be final after 2 quarters with a 5-term Henderson moving average filter and after 3 quarters with a 7-term filter as long as the underlying seasonally adjusted series is not revised.

Box 8.3. X-11-ARIMA/X-12-ARIMA M- and Q-Test Statistics

The first and third column below are from the F 3 table of X-12-ARIMA's main output file with the M- and Q-test statistics. The test statistic values are for the full 21 years of the illustrative series, of which the last four years of data were presented in Example 8.1. The F 3 and F 2.B codes refer to the various "output tables" in the program's main output file.

The Q-test statistic at the bottom is a weighted average of the M-test statistics.

F 3. Monitoring and Quality Assessment Statistics

All the measures below are in the range from 0 to 3 with an acceptance region from 0 to 1.

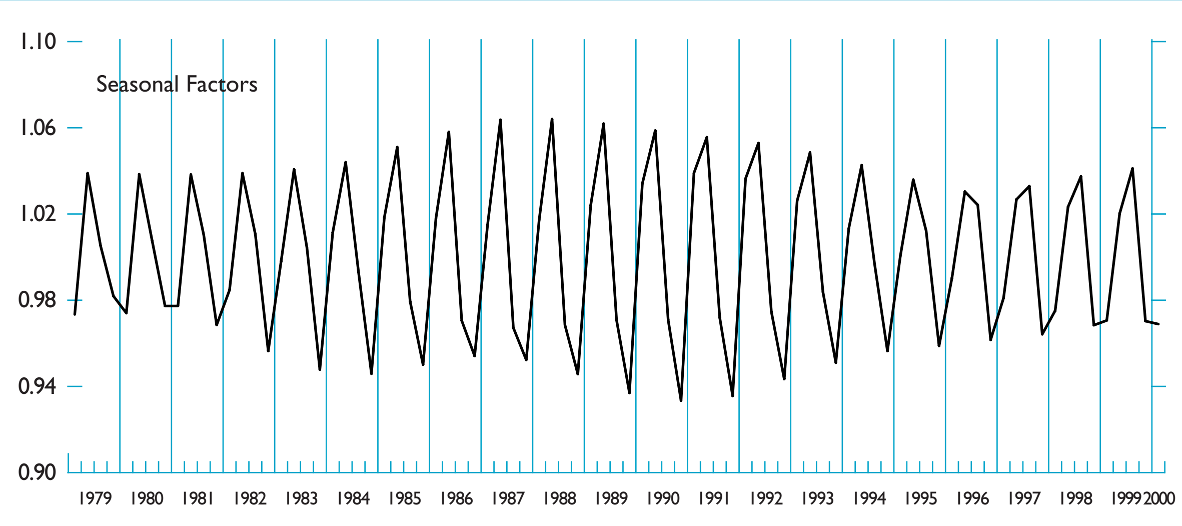
Statistics	Weight in Q	Value
1. The relative contribution of the irregular component over a one-quarter span (from Table F 2.B).	13	M1 = 0.245
2. The relative contribution of the irregular component to the stationary portion of the variance (from Table F 2.F).	13	M2 = 0.037
3. The amount of quarter-to-quarter change in the irregular component compared with the amount of quarter-to-quarter change in the trend-cycle (from Table F2.H).	10	M3 = 0.048
4. The amount of auto-correlation in the irregular as described by the average duration of run (Table F 2.D).	5	M4 = 0.875
5. The number of quarters it takes the change in the trend-cycle to surpass the amount of change in the irregular (from Table F 2.E).	11	M5 = 0.200
6. The amount of year-to-year change in the irregular compared with the amount of year-to-year change in the seasonal (from Table F 2.H).	10	M6 = 0.972
7. The amount of moving seasonality present relative to the amount of stable seasonality (from Table F 2.I).	16	M7 = 0.378
8. The size of the fluctuations in the seasonal component throughout the whole series.	7	M8 = 1.472
9. The average linear movement in the seasonal component throughout the whole series.	7	M9 = 0.240
10. Same as 8, calculated for recent years only.	4	M10= 1.935
11. Same as 9, calculated for recent years only.	4	M11= 1.935

ACCEPTED at the level 0.52 Check the three above measures that failed. Q (without M2) = 0.59 ACCEPTED.

[1]Based on Eurostat (1998).
[2]Based on Statistics Canada's seasonal adjustment course material.

Motivation[1]	Diagnose and Remedy if Fails[2]
The seasonal and irregular components cannot be separated sufficiently if the irregular variation is too high compared with the variation in the seasonal component. M1 and M2 test this property by using two different trend removers.	Series too irregular. Try to preadjust the series.
If the quarter-to-quarter movement in the irregular is too important in the SI component compared with the trend-cycle, the separation of these component can be of low quality.	Irregular too strong compared to trend-cycle. Try to preadjust the series.
Test of randomness of the irregular component. (Be careful, because the estimator of the irregular is not white noise and the statistics can be misleading.)	Irregulars are autocorrelated. Try to change length of the trend filter and (different) preadjustment for trading-day effects. There may be residual trading-day effects in the series.
Similar to M3.	Irregular too strong compared to trend-cycle. Try to preadjust the series.
In one step of the X-11 filtering procedure, the irregular is separated from the seasonal by a 3x5 seasonal moving average. Sometimes, this can be too flexible (I/S ratio is very high) or too restrictive (I/S ratio is very low). If M6 fails, you can try to use the 3x1 or the stable option to adjust for this problem.	Irregular too strong compared with seasonality. Try to change length of seasonal MA filter.
Combined F-test to measure the stable seasonality and the moving seasonality in the final SI ratios. Important test statistics for indicating whether seasonality is identifiable by the program.	Do not seasonally adjust the series. Indicates absence of Seasonality.
Measurement of the random fluctuations in the seasonal factors. A high value can indicate a high distortion in the estimate of the seasonal factors.	Change seasonal moving average filter. Seasonality may be moving too fast.
Because one is normally interested in the recent data, these statistics give insights into the quality of the recent estimates of the seasonal factors. Watch these statistics carefully if you use forecasts of the seasonal factors and not concurrent adjustment.	Look at ARIMA extrapolation. Indicate that the seasonality may be moving too fast at the end of the series

Example 8.2. Moving Seasonality



The chart presents the seasonal factors for the last 21 years of the time series presented in Example 8.1 and illustrates how the seasonal pattern has been changing gradually over time, as estimated by X-12-ARIMA.

8.40. Studies have shown that using ARIMA models to extend the series before filtering generally significantly reduces the size of these revisions compared with using asymmetric filters.[29] These studies have shown that, typically, revisions to the level of the series as well as to the period-to-period rate of change are reduced. Use of RegARIMA models, as offered by X-12-ARIMA, may make the backcasts and forecasts more robust and thus further reduces the size of these revisions compared with using pure ARIMA models. The reason for this is that RegARIMA models allow trading-day effects and other effects captured by the regressors to be taken into account in the forecasts in a consistent way. Availability of longer time series should result in a more precise identification of the regular pattern of the series (the seasonal pattern and the ARIMA model) and, in general, also reduce the size of the revisions.

8.41. Revisions to the seasonally adjusted data can be carried out as soon as new observations become available—concurrent revisions—or at longer intervals. The latter requires use of the one-year-ahead forecasted seasonal factors offered by X-11, X-11-ARIMA, and X-12-ARIMA to compute seasonally adjusted estimates for more recent periods not covered by the last revision. Use of one-year-ahead forecast of seasonal factors was common in the early days of seasonal adjustment with X-11 but is less common today. Besides full concurrent revisions and use of forecasts of seasonal factors, a third alternative is to use period-to-period rates of change from estimates based on concurrent adjustments to update previously released data and only revise data for past periods once a year.

8.42. From a purely theoretical point of view, and excluding the effects of outliers and revisions to the original unadjusted data, concurrent adjustment is always preferable. New data contribute new information about changes in the seasonal pattern that preferably should be incorporated into the estimates as early as possible. Consequently, use of one-year-ahead forecasts of seasonal factors results in loss of information and, as empirical studies[30] have shown and as illustrated in Example 8.5, often in larger, albeit less frequent, revisions to the levels as well as the period-to-period rates of change in the seasonally adjusted data. Theoretical studies[31] support this finding.

8.43. The potential gains from concurrent adjustment can be significant but are not always. In general

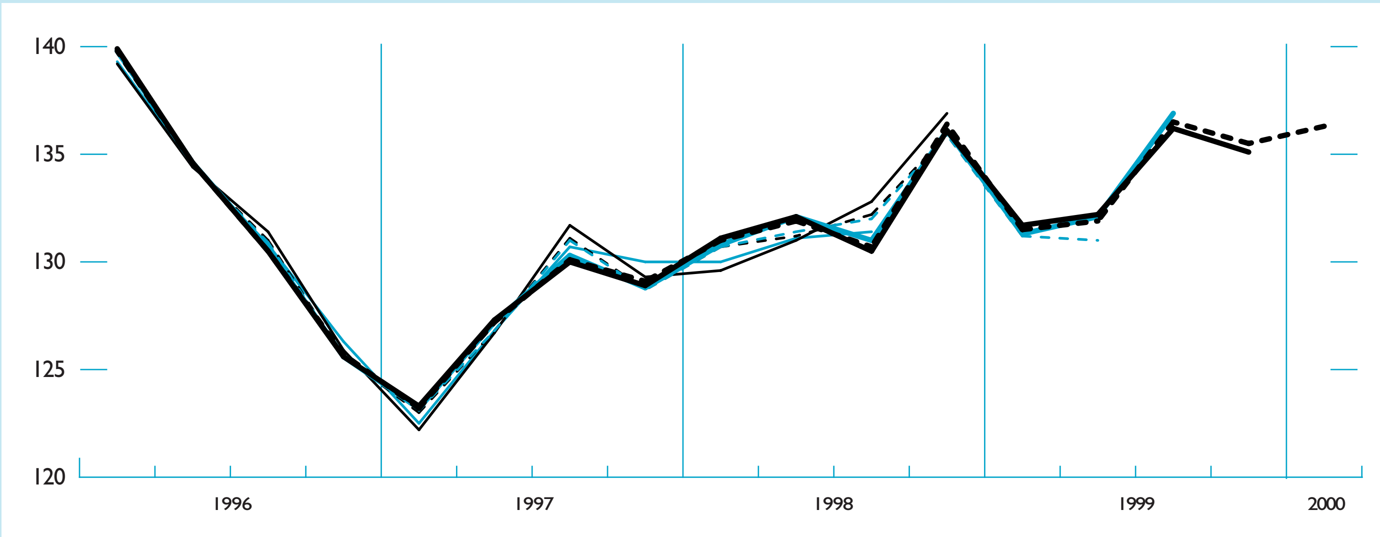
[29]See among others Bobitt and Otto (1990), Dagum (1987), Dagum and Morry (1984), Hout et al. (1986).

[30]See among others Dagum and Morry (1984), Hout and others. (1986), Kenny and Durbin (1982), and McKenzie (1984).
[31]See among others Dagum (1981 and 1982) and Wallis (1982).

Example 8.3. Changes in Seasonal Patterns, Revisions of the Seasonally Adjusted Series, and the Wagging Tail Problem

Revisions to the Seasonally Adjusted Estimates by Adding New Observations

(Original unadjusted data in Example 8.1.)



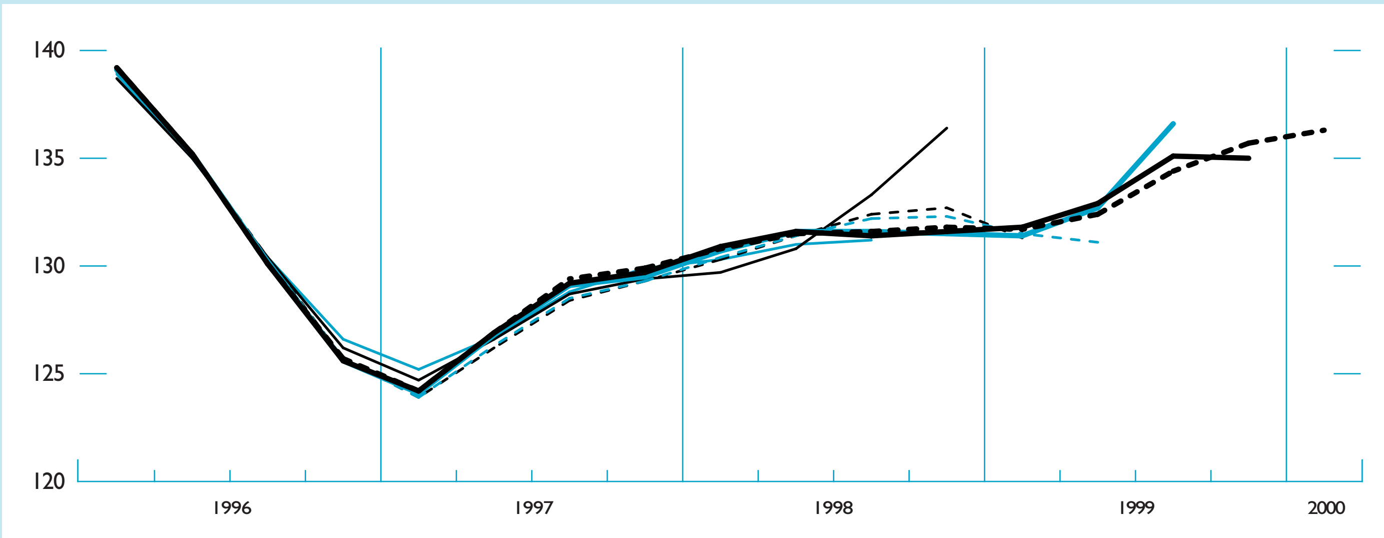
	Data until q1 00		Data until q4 99		Data until q3 99		Data until q2 99		Data until q1 99		Data until q4 98		Data until q3 98	
Date	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change
q1 1996	139.8		139.9		139.8		139.7		139.7		139.2		139.3	
q2 1996	134.6	–3.7%	134.6	–3.7%	134.6	–3.7%	134.5	–3.7%	134.5	–3.7%	134.4	–3.4%	134.5	–3.5%
q3 1996	130.5	–3.1%	130.5	–3.1%	130.6	–3.0%	130.9	–2.7%	131.0	–2.6%	131.4	–2.2%	130.8	–2.7%
q4 1996	125.7	–3.7%	125.6	–3.7%	125.6	–3.8%	125.6	–4.1%	125.6	–4.1%	125.9	–4.2%	126.3	–3.5%
q1 1997	123.2	–2.0%	123.3	–1.9%	123.2	–2.0%	123.1	–2.0%	123.0	–2.0%	122.2	–2.9%	122.5	–3.0%
q2 1997	127.2	3.2%	127.3	3.2%	127.2	3.3%	126.8	3.1%	126.8	3.0%	126.7	3.7%	126.8	3.5%
q3 1997	130.1	**2.3%**	130.0	2.2%	130.3	2.4%	131.0	3.3%	131.1	3.5%	131.7	3.9%	130.7	**3.1%**
q4 1997	129.1	–0.7%	128.9	–0.8%	128.8	–1.1%	128.7	–1.7%	128.7	–1.8%	129.3	–1.8%	130.0	–0.5%
q1 1998	131.0	**1.4%**	131.1	1.7%	130.8	1.6%	130.7	1.6%	130.7	1.5%	129.6	0.2%	130.0	**0.0%**
q2 1998	131.9	0.7%	132.1	0.8%	132.1	0.9%	131.4	0.5%	131.2	0.4%	131.0	1.1%	131.1	0.8%
q3 1998	130.7	–1.0%	130.5	–1.2%	131.0	–0.8%	132.0	0.5%	132.2	0.7%	132.8	1.3%	131.4	0.2%
q4 1998	136.4	4.4%	136.1	4.3%	136.1	3.9%	135.9	3.0%	135.9	2.8%	136.9	3.0%		
q1 1999	131.5	–3.6%	131.7	–3.2%	131.3	–3.5%	131.2	–3.4%	131.2	–3.5%				
q2 1999	131.9	0.3%	132.2	0.4%	132.1	**0.6%**	131.0	**–0.2%**						
q3 1999	136.5	**3.4%**	136.2	**3.0%**	136.9	**3.6%**								
q4 1999	135.5	–0.7%	135.1	–0.8%										
q1 2000	136.3	0.6%												

Note how the seasonally adjusted data (like the trend-cycle data presented in Example 8.4 but less so) for a particular period are revised as later data become available, even when the *unadjusted data for that period were not revised.* In this example, adding q1 2000 results in an upward adjustment of the growth from q2 1999 to q3 1999 in the seasonally adjusted series from an estimate of 3.0 percent to a revised estimate of 3.4 percent. Minor effects on the seasonally adjusted series of adding q1 2000 can be traced all the way back to 1993.

Example 8.4. Changes in Seasonal Patterns, Revisions and the Wagging Tail Problem

Revisions to Trend-Cycle Estimates

(Original unadjusted data in Example 8.1, seasonally adjusted in Example 8.3.)



	Data until q1 00		Data until q4 99		Data until q3 99		Data until q2 99		Data until q1 99		Data until q4 98		Data until q3 98	
Date	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change	Index	Period-to-Period Rate of Change
q1 1996	139.8		139.9		139.8		139.7		139.7		139.2		139.3	
q1 1996	139.2		139.2		139.1		139.0		139.0		138.7		138.9	
q2 1996	135.2	–2.9%	135.2	–2.9%	135.2	–2.8%	135.2	–2.7%	135.2	–2.7%	135.0	–2.7%	135.0	–2.8%
q3 1996	130.1	–3.7%	130.1	–3.8%	130.2	–3.7%	130.3	–3.6%	130.4	–3.6%	130.4	–3.4%	130.4	–3.4%
q4 1996	125.7	–3.4%	125.6	–3.5%	125.6	–3.5%	125.7	–3.6%	125.7	–3.6%	126.2	–3.2%	126.6	–2.9%
q1 1997	124.2	–1.2%	124.2	–1.1%	124.1	–1.2%	123.9	–1.4%	123.9	–1.4%	124.7	–1.2%	125.2	–1.1%
q2 1997	126.9	2.2%	126.9	2.2%	126.8	2.1%	126.3	1.9%	126.2	1.9%	126.6	1.5%	126.7	1.2%
q3 1997	129.4	2.0%	129.2	1.8%	129.1	1.8%	128.5	1.8%	128.4	1.8%	128.7	1.7%	128.8	1.7%
q4 1997	129.9	0.4%	129.7	0.4%	129.5	0.4%	129.3	0.6%	129.3	0.7%	129.4	0.5%	129.9	0.8%
q1 1998	130.8	0.7%	130.9	0.9%	130.7	0.9%	130.4	0.8%	130.3	0.8%	129.7	0.3%	130.3	0.4%
q2 1998	131.5	0.5%	131.6	0.5%	131.6	0.7%	131.4	0.8%	131.4	**0.8%**	130.8	**0.9%**	131.0	0.5%
q3 1998	131.6	0.1%	131.4	–0.2%	131.6	0.0%	132.2	0.5%	132.4	**0.8%**	133.3	**1.9%**	131.2	**0.2%**
q4 1998	131.8	0.2%	131.6	0.2%	131.5	–0.1%	132.3	0.1%	132.7	**0.3%**	136.4	**2.3%**		
q1 1999	131.7	–0.1%	131.8	0.2%	131.4	–0.1%	131.5	–0.6%	131.3	**–1.1%**				
q2 1999	132.4	0.5%	132.9	0.8%	132.7	1.0%	131.1	–0.3%						
q3 1999	134.4	1.5%	135.1	1.6%	136.6	2.9%								
q4 1999	135.7	**1.0%**	135.0	**–0.1%**										
q1 2000	136.3	0.4%												

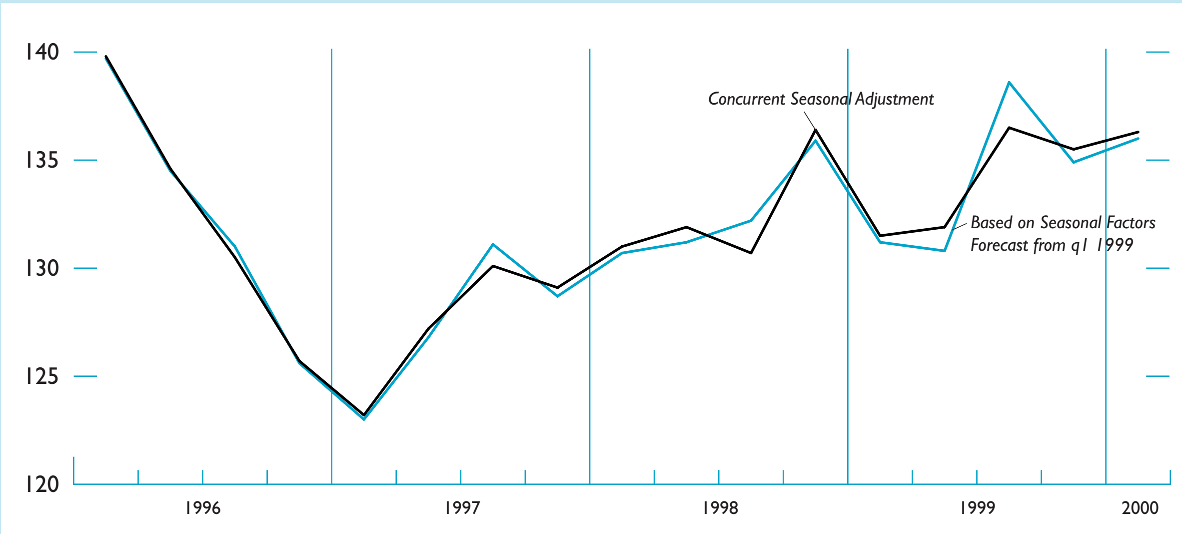
The chart and table demonstrate how the trend-cycle estimates for a particular period may be subject to relatively large revisions as data for new periods become available, even when *the unadjusted data for that period were not revised.* In this example, adding q1 2000 results in an upward adjustment of the change in the estimated trend-cycle component from q3 1999 to q4 1999, from an initial estimate of –0.1 percent to a revised estimate of 1.0 percent.

Also, observe how the strong irregular effect that occurred in q4 1998—an upward turn that disappears in the later trend-cycle estimates—wrongly resulted in an initial estimated strong growth from mid-1998 and onward in the earlier trend-cycle estimates.

Example 8.5. Changes in Seasonal Patterns, Revisions, and the Wagging Tail Problem

Concurrent Adjustment Versus Use of One-Year-Ahead Forecast of Seasonal Factors

(Original unadjusted data in Example 8.1, revisions of last seven quarters with concurrent seasonally adjusted data in Example 8.3.)



Date	Concurrent Seasonal Adjustment	Period-to-Period Rate of Change	Based on Seasonal Factors Forecast from q1 1999	Period-to-Period Rate of Change
q1 1996	139.8		139.7	
q2 1996	134.6	–3.7%	134.5	–3.7%
q3 1996	130.5	–3.1%	131.0	–2.6%
q4 1997	125.7	–3.7%	125.6	–4.1%
q1 1997	123.2	–2.0%	123.0	–2.0%
q2 1997	127.2	3.2%	126.8	3.0%
q3 1997	130.1	2.3%	131.1	3.5%
q4 1997	129.1	–0.7%	128.7	–1.8%
q1 1998	131.0	1.4%	130.7	1.5%
q2 1998	131.9	0.7%	131.2	0.4%
q3 1998	130.7	–1.0%	132.2	0.7%
q4 1998	136.4	4.4%	135.9	2.8%
q1 1999	131.5	–3.6%	131.2	–3.5%
q2 1999	131.9	**0.3%**	130.8	**–0.3%**
q3 1999	136.5	**3.4%**	138.6	**6.0%**
q4 1999	135.5	**–0.7%**	134.9	**–2.7%**
q1 2000	136.3	**0.6%**	136.0	**0.8%**

The chart and table demonstrate the effect of current update (concurrent adjustment) versus use of one-year-ahead forecast seasonal factors. As can be seen by comparing with Example 8.3, use of one-year-ahead forecasts of the seasonal factors results in loss of information and larger, but less frequent, revisions. In particular, in this example, using one-year-ahead forecasts of the seasonal factors gave an initial estimated decline from q3 to q4 1999 in the seasonally adjusted series of –2.7 percent, which is substantially larger compared with the initial estimate of –0.8 percent with current update of the seasonal factors (see Example 8.3).

the potential gains depend on, among other things, the following factors:

- The stability of the seasonal component. A high degree of stability in the seasonal factors implies that the information gain from concurrent adjustment is limited and makes it easier to forecast the seasonal factors. On the contrary, rapidly moving seasonality implies that the information gain can be significant.
- The size of the irregular component. A high irregular component may reduce the gain from concurrent adjustment because there is a higher likelihood for the signals from the new observations about changes in the seasonal pattern to be false, reflecting an irregular effect and not a change in the seasonal pattern.
- The size of revisions to the original unadjusted data. Large revisions to the unadjusted data may

reduce the gain from concurrent adjustment because there is a higher likelihood for the signals from the new observations about changes in the seasonal pattern to be false.

2. Minimum Length of the Time Series for Seasonal Adjustment

8.44. Five years of data and relatively stable seasonality are required in general as a minimum length to obtain properly seasonally adjusted estimates. For series that show particularly strong and stable seasonal movements, it may be possible to obtain seasonally adjusted estimates based on only three years of data.

8.45. A longer time series, however, is required to identify more precisely the seasonal pattern and to adjust the series for calendar variations (i.e., trading days and moving holidays), breaks in the series, outliers, and particular events that may have affected the series and may cause difficulties in properly identifying the seasonal pattern of the series.

8.46. For countries that are setting up a new QNA system, at least five years of retrospective calculations are recommended to conduct seasonal adjustment.

8.47. If a country has gone through severe structural changes resulting in radical changes in the seasonal patterns, it may not be possible to seasonally adjust its data until several years after the break in the series. In such cases, it may be necessary to seasonally adjust the pre-break and post-break part of the series separately.

3. Critical Issues in Seasonal Adjustment of QNA

8.48. When producing seasonally adjusted national account estimates, four critical issues must be decided:

(a) Should balancing items and aggregates be seasonally adjusted directly or derived residually, and should accounting and aggregation relationships be maintained?

(b) Should the relationship among current price value, price indices, and volume estimates be maintained, and, if so, which component should be derived residually?

(c) Should supply and use and other accounting identities be maintained, and, if so, what are the practical implications?

(d) Should the relationship to the annual accounts be strictly preserved?

a. Compilation levels and seasonal adjustment of balancing items and aggregates

8.49. Seasonally adjusted estimates for balancing items and aggregates can be derived directly or indirectly from seasonally adjusted estimates for the different components; generally the results will differ, sometimes significantly. For instance, a seasonally adjusted estimate for value added in manufacturing at current prices can be derived either by seasonally adjusting value added directly or as the difference between seasonally adjusted estimates for output and intermediate consumption at current prices. Similarly, a seasonally adjusted estimate for GDP at current prices can be derived either by seasonally adjusting GDP directly or as the sum of seasonally adjusted estimates for value added by activity (plus taxes on products). Alternatively, a seasonally adjusted estimate for GDP can be derived as the sum of seasonally adjusted estimates for the expenditure components.

8.50. Conceptually, neither the direct approach nor the indirect approach is optimal. There are arguments in favor of both approaches. It is convenient, and for some uses crucial, that accounting and aggregation relationships are preserved.[32] Studies[33] and practice, however, have shown that the quality of the seasonally adjusted series, and especially estimates of the trend-cycle component, may be improved, sometimes significantly, by seasonally adjusting aggregates directly or at least at a more aggregated level. Practice has shown that seasonally adjusting the data at a detailed level can leave residual seasonality in the aggregates, may result in less smooth seasonally adjusted series, and may result in series more subject to revisions. Which compilation level for seasonal adjustment gives the best results varies from case to case and depends on the properties of the particular series.

8.51. For aggregates, the direct approach may give the best results if the component series shows the same seasonal pattern or if the trend-cycles of the series are highly correlated. If the component series shows the same seasonal pattern, aggregation often reduces the impact of the irregular components of the component series, which at the most detailed level (the level of the source data) may be too dominant for

[32]However, for time series of chain-linked price indices and volume data, these accounting relationships are already broken (see Section D.4 of Chapter IX for a discussion of the non-additivity feature of chain-linked measures).

[33]See, among others, Dagum and Morry (1984).

proper seasonal adjustment. This effect may be particularly important for small countries where irregular events have a stronger impact on the data. Similarly, if the component series do not show the same seasonal pattern but their trend-cycles are highly correlated, aggregation reduces the impact of both the seasonal and irregular components of the component series.

8.52. In other cases, the indirect approach may give the best results. For instance, if the component series show very different seasonal patterns and the trend-cycles of the series are uncorrelated, aggregation may increase the appearance of irregular movements in the aggregate. Similarly, aggregation may cause large, highly volatile nonseasonal component series to overshadow seasonal component series, making it difficult or impossible to identify any seasonality that is present in the aggregate series. Moreover, it may be easier to identify breaks, outliers, calendar effects, the seasonal effect narrowly defined, and so on in detailed series with low to moderate irregular components than directly from the aggregates, because at the detailed level these effects may display a simpler pattern.

8.53. For balancing items, there is reason to believe that the indirect approach more often gives better results. Because balancing items are derived as the difference between two groups of component series, in the balancing item, the impact of the irregular components of the component series is more likely to be compounded. In contrast, because aggregates are derived by summation, opposite irregular movements in the component series will cancel each other out.

8.54. Some seasonal adjustment programs, including the X-11-ARIMA and the X-12-ARIMA, offer the possibility of adjusting aggregates using the direct and indirect approach simultaneously and comparing the results. For instance, the X-12-ARIMA, using the Composite series specifications command, adjusts aggregates simultaneously using the direct and indirect approach and provides users with a set of test statistics to compare the results. These test statistics are primarily the M and Q statistics presented in Example 8.4, measures of smoothness, and frequency spectrum estimates from the directly and indirectly estimated irregular component. In addition, sliding span and revision history simulation tests for both the direct and the indirect estimates are available to assess which approach results in estimates less subject to revisions.

8.55. In practice, the choice between direct and indirect seasonal adjustment should be based on the main intended use of the estimates and the relative smoothness and stability of the derived estimates. For some uses, preserved accounting and aggregation relationships in the data may be crucial, and the smoothness and stability of the derived estimates secondary. For other uses, the time-series properties of the derived estimates may be crucial, while accounting and aggregation relationships may be of no importance. If the difference is insignificant, representing a minor annoyance rather than adding any useful information, most compilers will opt for preserving accounting and aggregation relationships between published data.

8.56. Consequently, international practice varies with respect to the choice between direct and indirect seasonal adjustment. Many countries obtain the seasonally adjusted QNA aggregates as the sum of adjusted components, while some also adjust the totals independently, with discrepancies between the seasonally adjusted total and the sum of the component series as a result. Finally, some countries only publish seasonally adjusted estimates for main aggregates and typically seasonally adjust these directly or derive them indirectly by adjusting rather aggregated component series.

b. Seasonal adjustment and the relationship among price, volume, and value

8.57. As for balancing items and aggregates, seasonally adjusted estimates for national accounts price indices, volume measures, and current price data can be derived either by seasonally adjusting the three series independently or by seasonally adjusting two of them and deriving the third as a residual, if all three show seasonal variations.[34] Again, because of nonlinearities in the seasonal adjustment procedures, the alternative methods will give different results; however, the differences may be minor. Preserving the relationship among the price indices, volume measures, and the current price data is convenient for users.[35] Thus, it seems reasonable to seasonally adjust two of them and derive a seasonally adjusted estimate for the third residually. Choosing which series to derive residually must be determined on a case-by-case basis, depending on which alternative seems to produce the most reasonable result.

[34]Experience has shown that the price data may not always show identifiable seasonal variations.
[35]Note that chain-linking preserves this relationship ($V = P \cdot Q$).

c. Seasonal adjustment and supply and use and other accounting identities

8.58. Seasonal adjustment may cause additional statistical discrepancies in seasonally adjusted data between supply and use, GDP estimated from alternative sides, and between the different sides of other accounting identities. These statistical discrepancies are caused by nonlinearities in the seasonal filters, as well as use of different filter length, use of different pre-adjustments, and differences in estimated calendar effects on the various sides of the accounting identity. The statistical discrepancies may be reduced by forcing the programs to choose the same filter length and use the same pre-adjustment factors and calendar-effect factors for all series. This may, however, reduce the smoothness and stability of the individual seasonally adjusted series.

d. Seasonal adjustment and consistency with annual accounts

8.59. Annual totals based on the seasonally adjusted data will not automatically—and often should not conceptually—be equal to the corresponding annual totals based on the original unadjusted data. The number of working days, the impact of moving holidays, and other calendar-related effects vary from year to year. Similarly, moving seasonality implies that the impact of the seasonal effect narrowly defined will vary from year to year. Thus, conceptually, for series with significant calendar-related effects or moving seasonality effects, the annual totals of a seasonally adjusted series *should differ* from the unadjusted series.

8.60. For series without any significant calendar-related or moving seasonality effects, X-11/X-12 will produce seasonally adjusted data that automatically add up to the corresponding unadjusted annual totals if the seasonal components are additive (equation 8.1) but not if the seasonals are multiplicative (equation 8.2). Multiplicative seasonal factors require that a current-period weighted average of the seasonal factors averages to 1 and for the seasonally adjusted data to automatically add up to the corresponding unadjusted annual totals. However, the normalization of the seasonal factors in step (d) of the filtering procedure in stages 1 and 2 described in paragraph 8.23 only ensures that the unweighted, and not the weighted, annual average of the seasonal factors averages 1. It follows that, for series with multiplicative seasonal factors and no significant calendar-related or moving seasonality effects, the difference between the annual totals of the adjusted and unadjusted series will depend on the amplitude of the seasonal variation narrowly defined, the volatility of the seasonally adjusted series, and the pace of the change in the underlying trend-cycle. The difference will be small, and often insignificant, for series with moderate to low seasonal amplitudes and for series with little volatility and trend-cycle change.

8.61. X-11-ARIMA and X-12-ARIMA provide options for forcing the annual totals from the seasonally adjusted data to be equal to the original totals. Seasonal adjustment experts, however, generally recommend not using the forcing option[36] if the series show significant[37] trading-day, other calendar-related, or moving seasonality effects and trading-day or other calendar adjustments are performed. In such cases, consistency with the annual series would be achieved at the expense of the quality of the seasonal adjustment and would be conceptually wrong.

4. Status and Presentation of Seasonally Adjusted and Trend-Cycle QNA Estimates

8.62. The status and presentation of seasonally adjusted and trend-cycle QNA estimates vary. Some countries publish seasonally adjusted estimates for only a few main aggregates and present them as additional (sometimes unofficial) analytical elaborations of the official data. Other countries focus on the seasonally adjusted and trend-cycle estimates and publish an almost complete set of seasonally adjusted and trend-cycle QNA estimates in a reconciled accounting format. They may present the original unadjusted data as supplementary information.

8.63. The mode of presentation also varies substantially. Seasonally adjusted and trend-cycle data can be presented as charts; as tables with the actual data, either in money values or as index series; and as tables with derived measures of quarter-to-quarter rates of change. The last may be presented as the actual rates of change or as annualized rates (see Box 8.4).

[36]The X-12-ARIMA manual explicitly recommends against using the forcing option if trading-day adjustment is performed or if the seasonal pattern is changing rapidly.
[37]Relative to the “adding-up error” introduced by the unweighted, and not the weighted, annual average of the seasonal factors averaging to 1 for multiplicative seasonal adjustment.

8.64. The rates of change are sometimes annualized to make it easier for the layman to interpret the data. Most users have a feel for the size of annual growth rates but not for monthly or quarterly rates. Annualizing growth rates, however, also means that the irregular effects are compounded. Irrespectively of whether the actual or annualized quarterly rates of change are presented, it is important to indicate clearly what the data represent.

8.65. Growth rates representing different measures of change can easily be confused unless it is clearly indicated what the data represent. For instance, terms like "annual percentage change" or "annual rate of growth" can mean (a) the rate of change from one quarter to the next annualized (at annual rate); (b) the change from the same period of the previous year; (c) the change from one year to the next in annual data, or, equivalently, the change from the average of one year to the average of the next year; or (d) the change from the end of one year to the end of the next year.

8.66. Some countries also present the level of quarterly current and constant price data at annualized levels by multiplying the actual data by four. This seems artificial, does not make the data easier to interpret, and may be confusing because annual flow data in monetary terms no longer can be derived as the sum of the quarters. Users not familiar with the practice of annualizing levels of current and constant price data by multiplying the actual data by four may confuse annualized levels with forecast annual data. For these reasons, this practice is not recommended.

8.67. Finally, whether to present seasonally adjusted data or estimates of the trend-cycle component is still the subject of debate between experts in this area. In this manual, it is recommended to present both, preferably in the form of graphs incorporated into the same chart, as illustrated in Example 8.6.

8.68. An integrated graphical presentation highlights the overall development in the two series over time, including the uncertainties represented by the irregular component. In contrast, measures of quarter-to-quarter rates of change (in particular, annualized rates) may result in an overemphasis on the short-term movements in the latest and most uncertain observations at the expense of the general trend in the series. The underlying data and derived measures of quarter-to-quarter rates of change, however, should be provided as supplementary information.

8.69. The presentation should highlight the lower reliability, particularly for the trend-cycle component, of the estimates for the latest observations as discussed in this section. Means of highlighting the lower quality of the end-point estimates include (a) noting past revisions to these estimates; (b) suppressing estimates of the trend-cycle component for the latest observations in graphical presentations, as in Example 8.6; and (c) showing estimates for the latest observations with a trumpet on graphical presentations and with an estimated confidence interval in tabular presentations.

Box 8.4. Annualizing, or Compounding, Growth Rates

Period-to-period rates of change in quarterly data can be annualized using the following compounding formula:

$ar_{q,y} = (1 + r_{q,y})^4 - 1, \qquad r_{q,y} = (X_{q,y} / X_{q-1,y} - 1)$

where:

$ar_{q,y}$ Annualized quarter-to-quarter rate of change for quarter q of year y.

$r_{q,y}$ Original quarter-to-quarter rate of change for quarter q of year y in time series $X_{q,y}$.

The purpose of annualizing the rates of change is to present period-to-period rates of change for different period lengths on the same scale and thus to make it easier for the layman to interpret the data. For instance, annualizing the rates of change may help to clarify that a 0.8 percent growth from one month to the next is equivalent to:

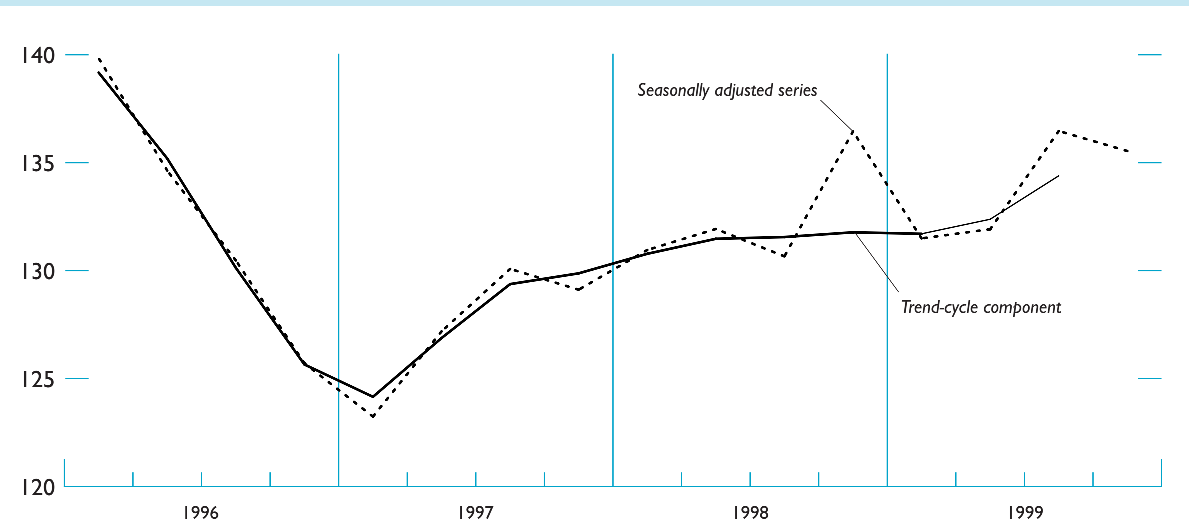
- 2.4 percent growth from one quarter to the next ($2.4\% = [(1 + 0.008)^3 - 1] \cdot 100$), or
- 10.0 percent growth from one year to the next ($10.0\% = [(1 + 0.024)^4 - 1] \cdot 100 = [(1 + 0.008)^{12} - 1] \cdot 100$).

Most users have a feel for annual growth rates and immediately recognize that a 10.0 percent annual growth in, for example, constant price household consumption expenditures is a lot, while 0.8 percent from one month to the next appears meager.

Annualized quarterly growth rates do not indicate what the annual growth will be and are not intended to be simple forecasts of what the annual growth rate would be if this growth continues for four quarters. The quarterly growth rate has to be constant for eight quarters for the annualized quarterly growth rate to be equal to the annual growth rate.

Example 8.6. Presentation of Seasonally Adjusted Series and the Corresponding Trend-Cycle Component

(Based on data from Example 8.1.)



Presenting the seasonally adjusted series and estimates of the trend-cycle components in the same chart highlights the overall development in the two series over time, including the uncertainties represented by the irregular component. Suppressing in the chart the estimates of the trend-cycle component at the end of the series or showing the trend-cycle estimates at the end with a trumpet based on estimated confidence intervals further highlights the added uncertainties at the end of the series.